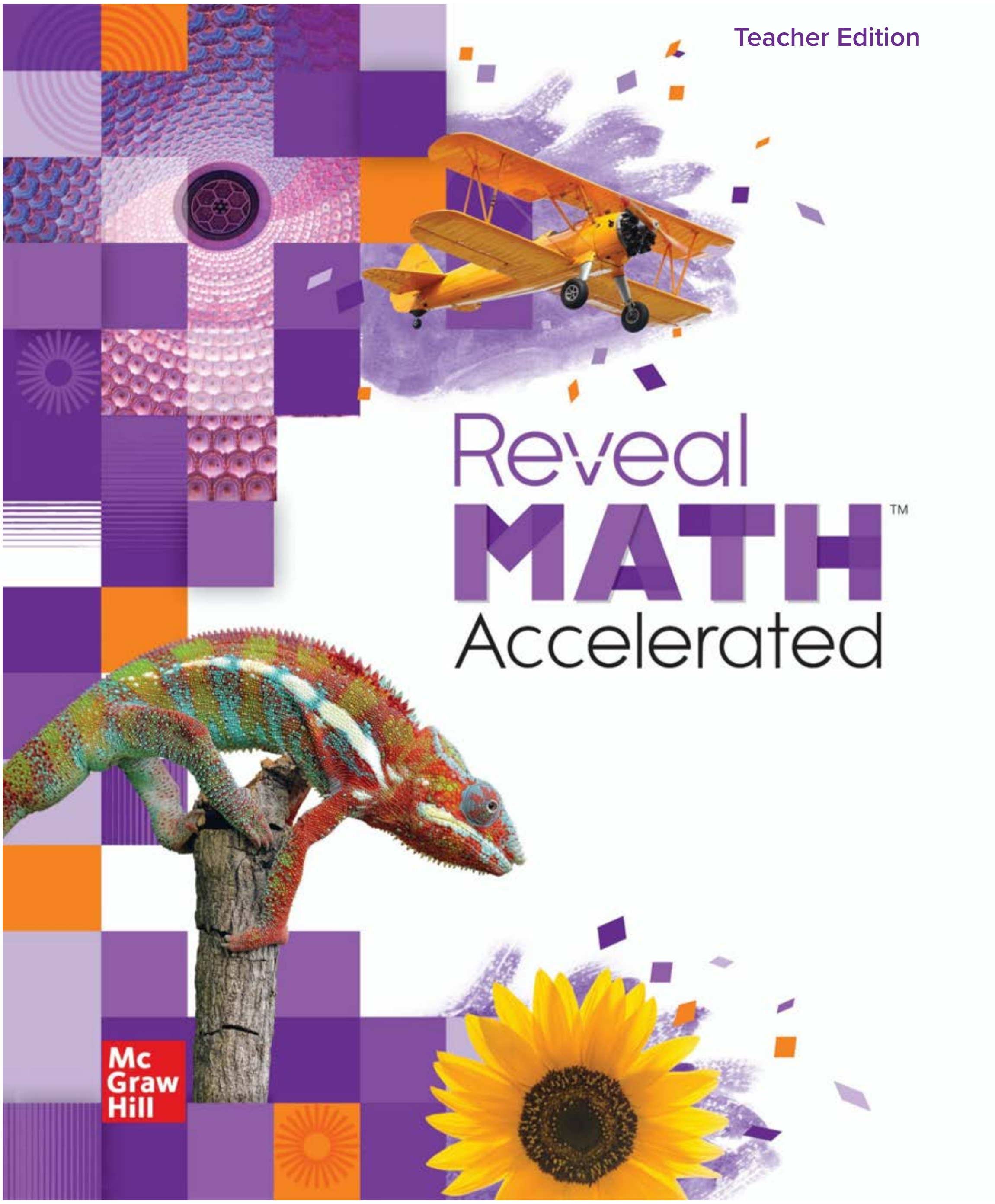
Teacher Edition



Reveal MATH™ Accelerated

McGraw Hill

Teacher Edition

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Contents in Brief

Reveal Math Accelerated,™ Guiding Principles

Academic research and the science of learning provide the foundation for this powerful K–12 math program designed to help reveal the mathematician in every student.

Reveal Math Accelerated is built on a solid foundation of **RESEARCH** that shaped the **PEDAGOGY** of the program.

Reveal Math Accelerated used findings from research on teaching and learning mathematics to develop its instructional model. Based on analyses of research findings, these areas form the foundational structure of the program:

- Rigor
- Productive Struggle
- Formative Assessment
- Rich Tasks
- Mathematical Discourse
- Collaborative Learning

Instructional Model

1 Launch

WARM UP

During the **Warm Up,** students complete exercises to activate prior knowledge and review prerequisite concepts and skills.

LAUNCH THE LESSON

In **Launch the Lesson**, students view a real-world scenario and image to pique their interest in the lesson content. They are introduced to questions that they will be able to answer at the end of the lesson.

EXPLORE

During the **Explore** activity, students work in partners or small groups to explore a rich, real-world or mathematical problem related to the lesson content.

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY

Reveal the full potential
in every student!

2 Explore and Develop

LEARN

In the **Learn** section, students gain the foundational knowledge needed to actively work through upcoming Examples.

EXAMPLES & CHECK

Students work through **Examples** related to the key concepts and engage in mathematical discourse.

Students complete a **Check** after each Example as a quick formative assessment to help teachers adjust instruction as needed.

3 Reflect and Practice

EXIT TICKET

The **Exit Ticket** gives students an opportunity to convey their understanding of the lesson concepts.

PRACTICE

Students complete **Practice** exercises individually or collaboratively to solidify their understanding of lesson concepts or build proficiency with lesson skills.

Reveal Math Accelerated
Key Areas of Focus

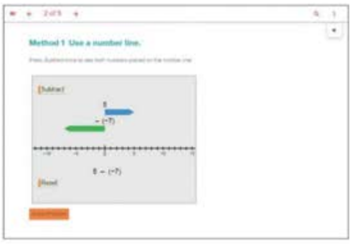
Reveal Math Accelerated has a strong focus on rigor—especially the development of conceptual understanding—an emphasis on student mindset, and ongoing formative assessment feedback loops.

Rigor

Reveal Math Accelerated has been thoughtfully designed to incorporate a balance of the three elements of rigor: conceptual understanding, procedural skills and fluency, and application.

Conceptual Understanding

Explore activities give all students an opportunity to work collaboratively and discuss their thinking as they build conceptual understanding of new topics. In the Explore activity to the left, students use algebra tiles to gain an understanding of operations with positive and negative integers.



Procedural Skills and Fluency

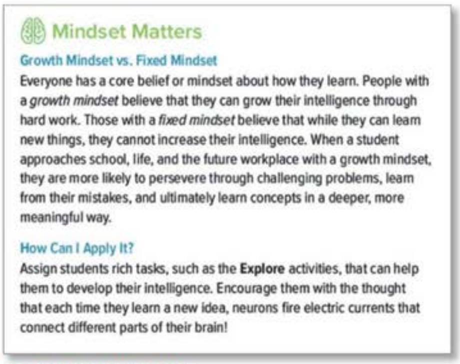
As students move through the lesson, they will use different strategies and tools to build procedural fluency. In the **Example** shown, students use **Web Sketchpad**® to develop proficiency with integer operations.

Application

Real-world examples and practice problems are opportunities for students to apply their learning to new situations. In the real-world example to the left, students apply their understanding of percents to solve a percent error problem.

Student Mindset

Mindset Matters tips located in each module provide specific examples of how *Reveal Math Accelerated* content can be used to promote a growth mindset in all students. Another feature focused on promoting a growth mindset is **Ignite! Activities** developed by Dr. Raj Shah to spark student curiosity about why the math works. An **Ignite!** delivers problem sets that are flexible enough so that students with varying background knowledge can engage with the content and motivates them to ask questions, solve complex problems, and develop a can-do attitude toward math.


Mindset Matters

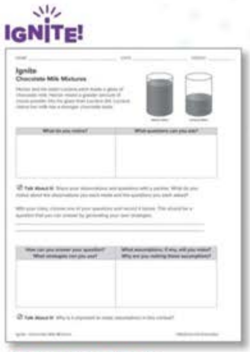
Growth Mindset vs. Fixed Mindset

Everyone has a core belief or mindset about how they learn. People with a *growth mindset* believe that they can grow their intelligence through hard work. Those with a *fixed mindset* believe that while they can learn new things, they cannot increase their intelligence. When a student approaches school, life, and the future workplace with a growth mindset, they are more likely to persevere through challenging problems, learn from their mistakes, and ultimately learn concepts in a deeper, more meaningful way.

How Can I Apply It?

Assign students rich tasks, such as the **Explore** activities, that can help them to develop their intelligence. Encourage them with the thought that each time they learn a new idea, neurons fire electric currents that connect different parts of their brain!

Teacher Edition Mindset Tip



Student Ignite! Activity

Formative Assessment

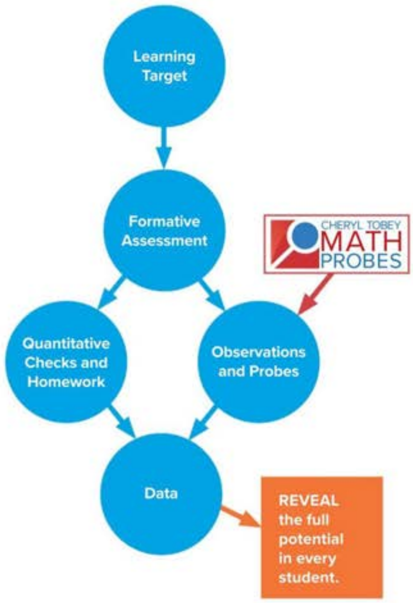
The key to reaching all learners is to adjust instruction based on each student's understanding. *Reveal Math Accelerated* offers powerful formative assessment tools that help teachers to efficiently and effectively differentiate instruction for all students.

Math Probes

Each module includes a **Cheryl Tobey Formative Assessment Math Probe** that is focused on addressing student misconceptions about key math topics. Students can complete these probes at the beginning, middle, or end of a module. The teacher support includes a list of recommended differentiated resources that teachers assign based on students' responses.

Example Checks

Each example is followed by a formative assessment **Check** that students complete on their own that allows teachers to gauge students' understanding of the concept or skill presented. When students complete the Check, the teacher receives resource recommendations, which can be assigned to all students.



A Powerful Blended Learning Experience

The *Reveal Math Accelerated* blended learning experience was designed to include purposeful print and digital components focused on sparking student curiosity and providing teachers with flexible implementation options.

Reveal Math Accelerated has been thoughtfully developed to provide a rich learning experience no matter where a district, school, or classroom falls on the digital spectrum. All of the instructional content can be projected or can be accessed via desktop, laptop, or tablet.

Lesson

1 Launch

WARM UP

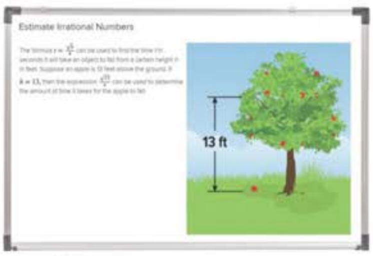
The **Warm Up** exercise can be projected on an interactive whiteboard.

LAUNCH THE LESSON

Launch the Lesson can be projected or assigned to students to access on their own devices.

EXPLORE

The **Explore** Activity can be projected while students record their observations in the Interactive Student Edition or can be assigned for students to complete on individual devices.



Launch the Lesson

Explore

INDIVIDUAL ACTIVITY
GROUP ACTIVITY
CLASS ACTIVITY
INTERACTIVE PRESENTATION
PRINT INTERACTIVE STUDENT EDITION

2 Explore and Develop

LEARN

As students are introduced to the key lesson concepts, they can progress through the **Learn** by recording notes in their Interactive Student Edition or on their own devices.

EXAMPLES & CHECK

In their Interactive Student Edition or on an individual device, students work through one or more **Examples** related to key lesson concepts.

A **Check** follows each Example in either the Interactive Student Edition or on each student device.

Aligned Digital Lesson Presentation to Interactive Student Edition

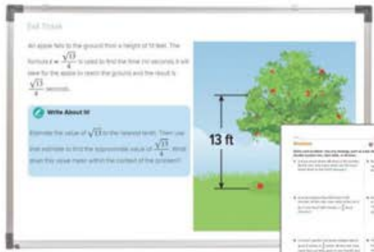
3 Reflect and Practice

EXIT TICKET

The **Exit Ticket** is projected or accessed via student devices to provide students with lesson closure and an opportunity to revisit the lesson concepts.

PRACTICE

Assign students **Practice** problems from their Interactive Student Edition or create a digital assignment for them to work on their device in class or at home to solidify lesson concepts.



Exit Ticket

Practice

Supporting All Learners

The *Reveal Math Accelerated* program was designed so that all students have access to:

- rich tasks that promote productive struggle,
- opportunities to develop proficiency with the habits of mind and thinking strategies of mathematicians, and
- prompts to promote mathematical discourse and build academic language.

Resources for Differentiating Instruction

When needed, resources are available to differentiate math instruction for students who may need to see a concept in a different way, practice prerequisite skills, or are ready to extend their learning.

AL

Approaching Level Resources

- Remediation Activities
- Extra Examples
- *Arrive Math* Take Another Look Mini Lessons

BL

Beyond Level Resources

- Beyond Level Differentiated Activities
- Extension Activities

Resources for English Language Learners

Reveal Math Accelerated also includes student and teacher resources to support students who are simultaneously learning grade-level math and building their English proficiency. Appropriate, research-based language scaffolds are also provided to support students as they engage in rigorous mathematical tasks and discussions.

ELL

English Language Learners

- Spanish Interactive Student Edition
- Spanish Personal Tutors
- Math Language-Building Activities
- Language Scaffolds
- *Think About It!* and *Talk About It!* Prompts
- Multilingual eGlossary
- Audio
- Graphic Organizers
- Web Sketchpad, Desmos, and eTools

Embedded Reteach Support
Arrive Math Booster Mini-Lessons

Reveal Math Accelerated ensures a seamless connection for students who need extra topic support with embedded *Arrive Math Booster* mini-lessons. These mini-lessons, called *Take Another Look*, have been included in *Reveal Math Accelerated* to provide students direct support related to the lesson objective.

- Teacher-assigned option based on Example Check results
- Digital, student-driven lesson
- Gradual release experience in three parts

Part 1: Model

Part 2: Interactive Practice

Part 3: Data Check

Arrive MATH BOOSTER

Complement *Reveal Math Accelerated* with the K-8 *Arrive Math Booster* supplemental intervention to equip teachers with all the resources they need to supplement their instruction and meet the needs of all learners.

Digital mini-lessons
Utilize over 1,160 *Take Another Look* digital mini-lessons for every skill within the K-8 standards.

Hands-On Lesson
Complement the *Take Another Look* lessons with concrete modeling support using hands on, teacher-led activities.

Games
Engage students through exciting math games to become fluent in critical math skills.

Reveal Student Readiness with Individualized Learning Tools

Reveal Math Accelerated incorporates innovative, technology-based tools that are designed to extend the teacher's reach in the classroom to help address a wide range of knowledge gaps, set and align academic goals, and meet student individualized learning needs.

LEARNSMART®

Topic Mastery

With embedded **LearnSmart**,® students have a built-in study partner for topic practice and review to prepare for multi-module, or mid-year tests.

LearnSmart's revolutionary adaptive technology measures students' awareness of their own learning, time on topic, answer accuracy, and suggests alternative resources to support student learning, confidence, and topic mastery.

ALEKS®

Individualized Learning Pathways

Learners of all levels benefit from the use of **ALEKS'** adaptive, online math technology designed to pinpoint what each student knows, does not know, and most importantly, what each student is ready to learn.

When paired with *Reveal Math Accelerated,* **ALEKS** is a powerful tool designed to provide integrated instructionally actionable data enabling teachers to utilize *Reveal Math Accelerated* resources for individual students, groups, or the entire classroom.

Activity Report

Powerful Tools for Modeling Mathematics

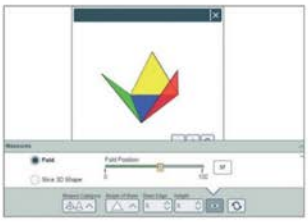
Reveal Math Accelerated has been designed with purposeful, embedded digital tools to increase student engagement and provide unique modeling opportunities.

Web Sketchpad® Activities

The leading dynamic mathematics visualization software has now been integrated with **Web Sketchpad Activities** at point of use within *Reveal Math Accelerated*. Student exploration (and practice) using **Web Sketchpad** encourages problem solving and visualization of abstract math concepts.

desmos

The powerful **Desmos** graphing calculator is available in *Reveal Math Accelerated* for students to explore, model, and apply math to the real-world.



eTools

By using a wide-variety of digital **eTools** embedded within *Reveal Math Accelerated,* students gain additional hands-on experience while they learn and teachers have the option to create problem-based learning opportunities.

Technology-Enhanced Items

Embedded within the digital lesson, technology-enhanced items—such as drag-and-drop, flashcard flips, or diagram completion—are strategically placed to give students the practice with common computer functions needed to master computer-based testing.

TYPE

SWIPE

DRAG & DROP

FLASHCARDS

eTOOLS

MULTI-SELECT

WATCH

EXPAND

Assessment Tools to Reveal Student Progress and Success

Reveal Math Accelerated provides a comprehensive array of assessment tools to measure student understanding and progress. The digital assessment tools include next generation assessment items, such as multiple-response, selected-response, and technology-enhanced items.

Assessment

Reveal Math Accelerated provides embedded, regular formative checkpoints to monitor student learning and provide feedback that can be used to modify instruction and help direct student learning using reports and recommendations based on resulting scores.

Summative assessments built in *Reveal Math Accelerated* evaluate student learning at the module conclusion by comparing it against the state standards covered.

Formative Assessment Resources

- Cheryl Tobey Formative Assessment Math Probes
- Checks
- Exit Tickets
- Put It All Together

Summative Assessment Resources

- Module Tests
- Performance Tasks
- Benchmark Tests
- End-of-Course Tests
- LearnSmart

Or **Build Your Own** assessments focused on standards or objectives. Access to banks of questions, including those with tech-enhanced capabilities, enable a wide range of options to mirror high-stakes assessment formats.

Reporting

Clear, instructionally actionable data will be a click away with the *Reveal Math Accelerated* Reporting Dashboard.

Activity Report Real-time class and student reporting of activities completed by the class. Includes average score, submission rate, and skills covered for the class and each student.

- **Item Analysis Report** Review a detailed analysis of response rates and patterns, answers, and question types in a class snapshot or by student.
- **Standards Report** Performance data by class or individual student is aggregated by standards, skills, or objectives linked to the related activities completed.

Activity Report

Professional Development Support for Continuous Learning

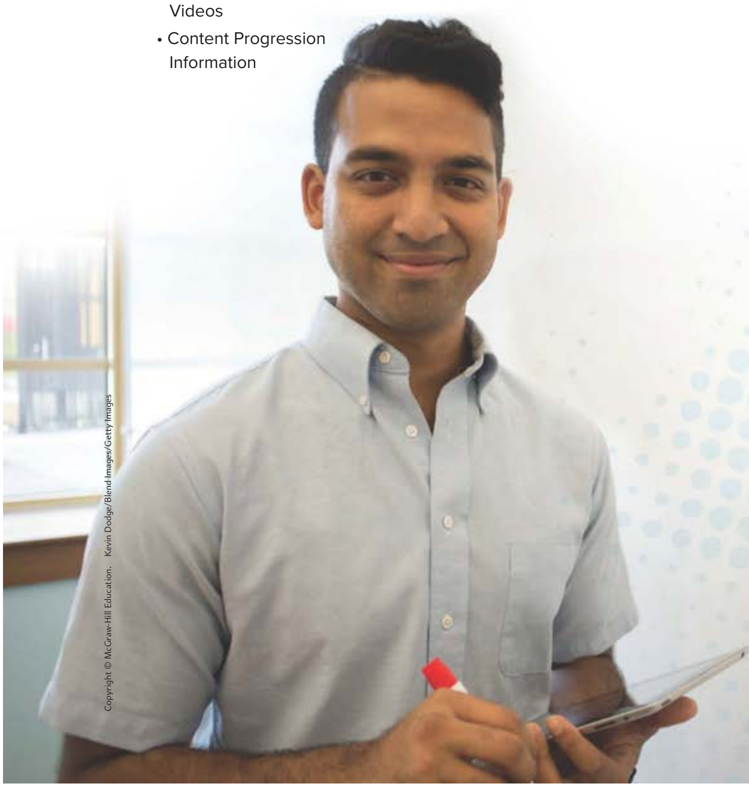
McGraw-Hill Education supports lifelong learning and demonstrates commitment to teachers with a built-in professional learning environment designed for support during planning or extended learning opportunities.

What You Will Find

- Best-practices resources
- Implementation support
- Teaching Strategies
- Classroom Videos
- Math Misconception Videos
- Content and Pedagogy Videos
- Content Progression Information

Why Professional Development is so Important

- Research-based understanding of student learning
- Improved student performance
- Evidence-based instructional best-practices
- Collaborative content strategy planning
- Extended knowledge of program how-to's



Reveal Math Accelerated Expert Advisors

Cathy Seeley, Ed.D.

Austin, Texas

Mathematics educator, speaker, and writer, former Senior Fellow at the Charles A. Dana Center at The University of Texas at Austin, past President of NCTM, former Director of K-12 Mathematics for the State of Texas

Areas of expertise:
Mathematics Teaching, Equity, Assessment, STEM Learning, Informal Learning, Upside-Down Teaching, Productive Struggling, Mathematical Practices, Mathematical Habits of Mind, Family and Community Outreach, Mathematics Education Policy, Advocacy

"We want students to believe deeply that mathematics makes sense–in generating answers to problems, discussing their thinking and other students' thinking, and learning new material."

—Seeley, 2016, Making Sense of Math

Cheryl R. Tobey, M.Ed.

Gardiner, Maine

Senior Mathematics Associate at Education Development Center (EDC)

Areas of expertise:
Formative assessment and professional development for mathematics teachers; tools and strategies to uncovering misconceptions

"Misunderstandings and partial understandings develop as a normal part of learning mathematics. Our job as educators is to minimize the chances of students' harboring misconceptions by knowing the potential difficulties students are likely to encounter, using assessments to elicit misconceptions and implementing instruction designed to build new and accurate mathematical ideas."

— Tobey, et al 2007, 2009, 2010, 2013, 2104, Uncovering Student Thinking Series

Nevels Nevels, Ph.D.

Saint Louis, Missouri

PK-12 Mathematics Curriculum Coordinator for Hazelwood School District

Areas of expertise:
Mathematics Teacher Education; Student Agency & Identity; Socio-Cultural Perspective in Mathematics Learning

"A school building is one setting for learning mathematics. It is understood that all children should be expected to learn meaningful mathematics within its walls. Additionally, teachers should be expected to learn within the walls of this same building. More poignantly, I posit that if teachers are not learning mathematics in their school building, then it is not a school."

—Nevels, 2018

Raj Shah, Ph.D.

Columbus, Ohio

Founder of Math Plus Academy, a STEM enrichment program and founding member of The Global Math Project

Areas of expertise:
Sparking student curiosity, promoting productive struggle, and creating math experiences that kids love

"As teachers, it's imperative that we start every lesson by getting students to ask more questions because curiosity is the fuel that drives engagement, deeper learning and perseverance."

—Shah, 2017

Walter Secada, Ph.D.

Coral Gables, Florida

Professor of Teaching and Learning at the University of Miami

Areas of expertise:
Improving education for English language learners, equity in education, mathematics education, bilingual education, school restructuring, professional development of teachers, student engagement, Hispanic dropout and prevention, and reform

"The best lessons take place when teachers have thought about how their individual English language learners will respond not just to the mathematical content of that lesson, but also to its language demands and mathematical practices."

—Secada, 2018

Ryan Baker, Ph.D.

Philadelphia, Pennsylvania

Associate Professor and Director of Penn Center for Learning Analytics at the University of Pennsylvania

Areas of expertise:
Interactions between students and educational software; data mining and learning analytics to understand student learning

"The ultimate goal of the field of Artificial Intelligence in Education is not to promote artificial intelligence, but to promote education… systems that are designed intelligently, and that leverage teachers' intelligence. Modern online learning systems used at scale are leveraging human intelligence to improve their design, and they're bringing human beings into the decision-making loop and trying to inform them."

—Baker, 2016

Chris Dede, Ph.D.

Cambridge, Massachusetts

Timothy E. Wirth Professor in Learning Technologies at Harvard Graduate School of Education

Areas of expertise:
Provides leadership in educational innovation; educational improvements using technology

"People are very diverse in how they prefer to learn. Good instruction is like an ecosystem that has many niches for alternative types of learning: lectures, games, engaging video-based animations, readings, etc. Learners then can navigate to the niche that best fulfills their current needs."

—Dede, 2017

Dinah Zike, M.Ed.

Comfort, Texas

President of Dinah.com in San Antonio, Texas and Dinah Zike Academy

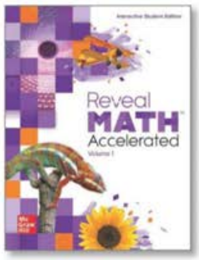
Areas of expertise:
Developing educational materials that include three-dimensional graphic organizers; interactive notebook activities for differentiation; and kinesthetic, cross-curricular manipulatives

"It is education's responsibility to meet the unique needs of students, and not the students' responsibility to meet education's need for uniformity."

—Zike, 2017, InRIGORating Math Notebooks

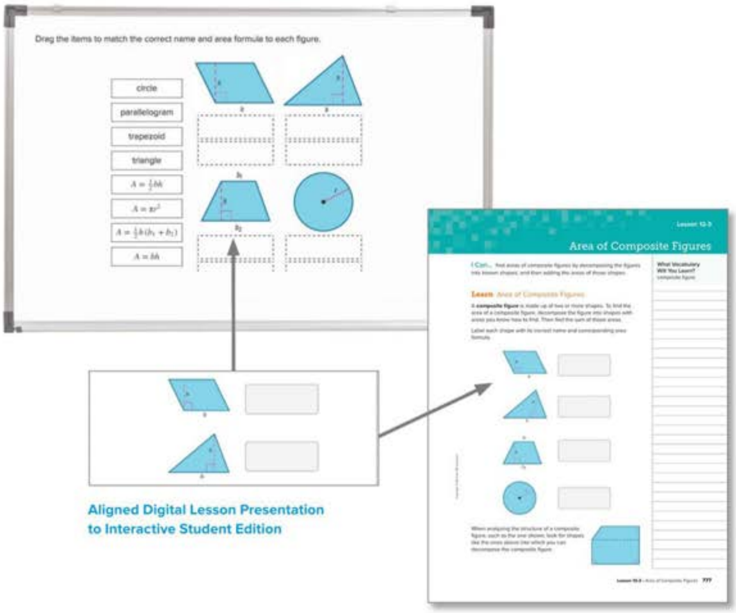
Reveal Everything Needed for Effective Instruction

Reveal Math Accelerated provides both print and innovative, technology-based tools designed to address a wide range of classrooms. No matter whether you're in a 1 : 1 district, or have a classroom projector, *Reveal Math Accelerated* provides you with the resources you need for a rich learning experience.



Blended Classrooms

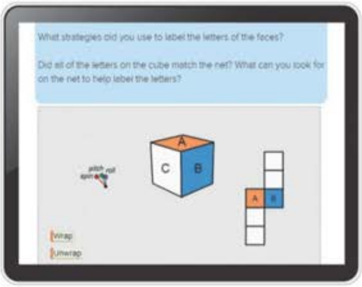
Focused on projection of the **Interactive Presentation**, students follow along taking notes and working through problems in their Interactive Student Edition during class time. Also included in the Interactive Student Edition is a glossary, **Foldables**® at point of use and in the back of the book, selected answers, and a reference sheet.



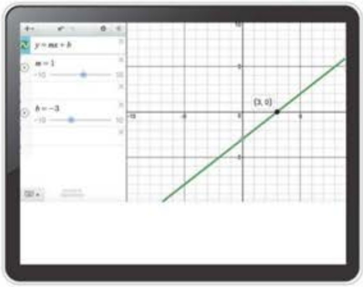
Aligned Digital Lesson Presentation to Interactive Student Edition

Digital Classrooms

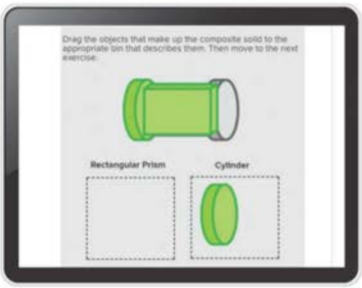
Projection is a focal point for key areas of the course with students interacting with the lesson using their own devices. Each student can access teacher-assigned sections of the lessons for **Explore** activities, **Learn** sections and **Examples**. Point of use videos, animations, as well as interactive content enable students to experience math in interesting and impactful ways.



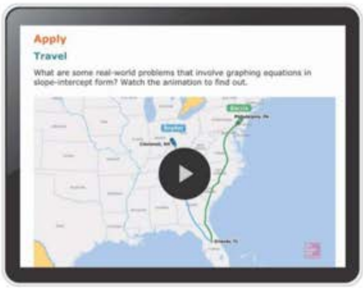
Web Sketchpad



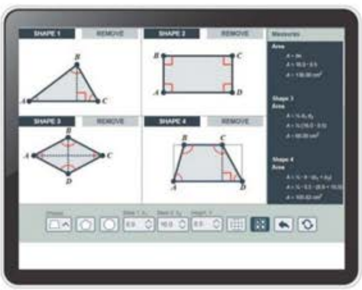
Desmos



Drag-and-Drop



Videos and Animations



eTools

In *Reveal Math Accelerated*, R is for—

- Research
- Rigor
- Relevant Connections

Are you... READY to start?

Module 1

Proportional Relationships

Essential Question

What does it mean for two quantities to be in a proportional relationship?

Module 2

Solve Percent Problems

Essential Question

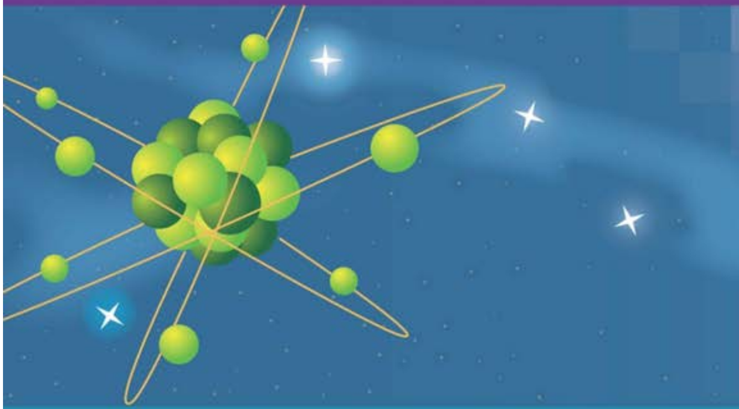
How can percent describe the change of a quantity?

Module 3

Operations with Integers and Rational Numbers

Essential Question

How are operations with rational numbers related to operations with integers?



Module 4

Exponents and Scientific Notation

Essential Question

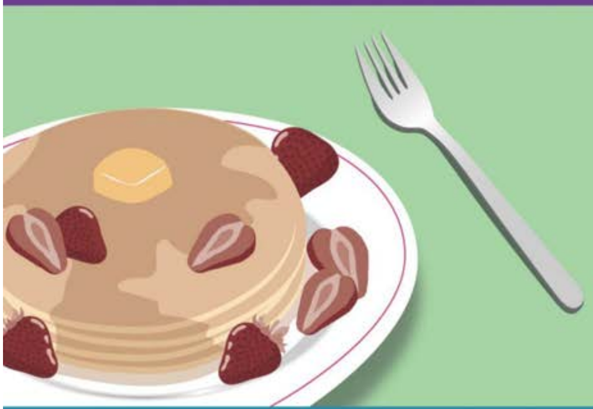
Why are exponents useful when working with very large or very small numbers?

Module 5

Real Numbers

Essential Question

Why do we classify numbers?



Module 6

Algebraic Expressions

Essential Question

Why is it beneficial to rewrite expressions in different forms?

Module 7

Solve Equations

Essential Question

How can equations be used to solve everyday problems?

Module 8

Linear Relationships and Slope

Essential Question

How are linear relationships related to proportional relationships?

Module 9

Probability

Essential Question

How can probability be used to predict future events?

Module 10

Sampling and Statistics

Essential Question

How can you use a sample to gain information about a population?

Module 11

Geometric Figures

Essential Question

How does geometry help to describe objects?

Module 12

Area, Surface Area, and Volume

Essential Question

How can we measure objects to solve problems?

Module 13

Transformations

Essential Question

What does it mean to perform a transformation on a figure?

Standards for Mathematical Content, Accelerated Grade 7

This correlation shows the alignment of *Reveal Math Accelerated* to the Standards for Mathematical Content, Accelerated Grade 7, from the Common Core State Standards for Mathematics. **Primary references are bold.** *Supporting references are italicized.*

Standards for Mathematical Content		Lesson(s)
Unit 1 Rational Numbers and Exponents		
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. (Major Cluster)		
7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	**3-1, 3-2, 3-5, 3-7, 3-9**
	7.NS.A.1.A Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*	**3-1, 3-7**
	7.NS.A.1.B Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	**3-1, 3-7**
	7.NS.A.1.C Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	**3-2, 3-7**
	7.NS.A.1.D Apply properties of operations as strategies to add and subtract rational numbers.	**3-1, 3-2,** *3-4,* **3-5, 3-7,** *3-8,* **3-9**
7.NS.A.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.	**3-3, 3-4, 3-5, 3-6, 3-8, 3-9**
	7.NS.A.2.A Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	**3-3, 3-8**
	7.NS.A.2.B Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.	**3-4, 3-6, 3-8**
	7.NS.A.2.C Apply properties of operations as strategies to multiply and divide rational numbers.	**3-3, 3-4, 3-5, 3-8, 3-9**
	7.NS.A.2.D Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	**3-6**
7.NS.A.3	Solve real-world and mathematical problems involving the four operations with rational numbers. *Computations with rational numbers extend the rules for manipulating fractions to complex fractions.*	*3-1, 3-2, 3-3, 3-4,* **3-5,** *3-6, 3-7,* **3-8, 3-9,** *11-6*
Know that there are numbers that are not rational, and approximate them by rational numbers. (Supporting Cluster)		
8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	**3-6, 5-2, 5-4**

Standards for Mathematical Content		Lesson(s)
8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	**5-3, 5-4**
Work with radicals and integer exponents. (Major Cluster)		
8.EE.A.1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	**4-2, 4-3, 4-4,** *4-6*
8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	**5-1, 5-2,** *5-3*
8.EE.A.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 times 10^8 and the population of the world as 7 times 10^9, and determine that the world population is more than 20 times larger.*	**4-5, 4-6**
8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	**4-5, 4-6**
Unit 2 Proportionality and Linear Relationships		
Analyze proportional relationships and use them to solve real-world and mathematical problems. (Major Cluster)		
7.RP.A.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*	**1-1**
7.RP.A.2	Recognize and represent proportional relationships between quantities.	**1-2, 1-3, 1-4, 1-5, 1-6,** *10-2, 10-3, 11-6*
	7.RP.A.2.A Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	**1-3, 1-4**
	7.RP.A.2.B Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	**1-3, 1-4, 1-5,** *11-6*
	7.RP.A.2.C Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.	**1-5**
	7.RP.A.2.D Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.	**1-4**
7.RP.A.3	Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	**1-6, 2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7,** *10-2, 11-6*

Standards for Mathematical Content		Lesson(s)
Use properties of operations to generate equivalent expressions. (Major Cluster)		
7.EE.A.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	**6-1, 6-2, 6-3, 6-4, 6-5**
7.EE.A.2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*	**2-2, 2-3, 2-4, 2-6,** *3-9*, **6-1**
Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (Major Cluster)		
7.EE.B.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*	*2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7,* **3-1**, *3-2*, **3-3**, *3-4*, **3-5**, *3-6*, **3-7**, *3-8, 3-9, 7-1, 7-2, 7-3, 7-4, 7-6, 7-7, 7-8, 11-1, 11-2, 11-6*
7.EE.B.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	**7-1, 7-2, 7-6, 7-7, 7-8**
	7.EE.B.4.A Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*	**7-1, 7-2,** *11-1, 11-2*
	7.EE.B.4.B Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*	**7-6, 7-7, 7-8**
Understand the connections between proportional relationships, lines, and linear equations. (Major Cluster)		
8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	**8-1,** *8-4*
8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	**8-3, 8-4, 8-5, 8-6**
Analyze and solve linear equations and pairs of simultaneous linear equations. (Major Cluster)		
8.EE.C.7	Solve linear equations in one variable.	**7-1, 7-2, 7-3, 7-4, 7-5**
	8.EE.C.7.A Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).	*7-1, 7-2, 7-3, 7-4,* **7-5**
	8.EE.C.7.B Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	**7-1, 7-2, 7-3, 7-4**

Standards for Mathematical Content		Lesson(s)
Unit 3 Introduction to Sampling and Inference		
Use random sampling to draw inferences about a population. (Supporting Cluster)		
7.SP.A.1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	10-1
7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*	10-1, 10-2, 10-3
Draw informal comparative inferences about two populations. (Additional Cluster)		
7.SP.B.3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*	10-5
7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*	10-4
Investigate chance processes and develop, use, and evaluate probability models. (Supporting Cluster)		
7.SP.C.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	9-1
7.SP.C.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*	9-2, 9-4
7.SP.C.7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	9-2, 9-3, 9-4
	7.SP.C.7.A Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*	9-3, 9-4
	7.SP.C.7.B Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*	9-2, 9-4

Standards for Mathematical Content		Lesson(s)
7.SP.C.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.	9-5, 9-6
	7.SP.C.8.A Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.	9-5
	7.SP.C.8.B Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.	9-5
	7.SP.C.8.C Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*	9-6
Unit 4 Creating, Comparing, and Analyzing Geometric Figures		
Draw, construct, and describe geometrical figures and describe the relationships between them. (Additional Cluster)		
7.G.A.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	11-6
7.G.A.2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	11-4
7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	11-7
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (Additional Cluster)		
7.G.B.4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	12-1, 12-2
7.G.B.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	11-1, 11-2
7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	12-3, 12-4, 12-5, 12-9
Understand congruence and similarity using physical models, transparencies, or geometry software. (Major Cluster)		
8.G.A.1	Verify experimentally the properties of rotations, reflections, and translations:	13-1, 13-2, 13-3, 13-5
	8.G.A.1.A Lines are taken to lines, and line segments to line segments of the same length.	13-1, 13-2, 13-3, 13-5
	8.G.A.1.B Angles are taken to angles of the same measure.	13-5
	8.G.A.1.C Parallel lines are taken to parallel lines.	13-5
8.G.A.2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	13-5

Standards for Mathematical Content		Lesson(s)
8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	**13-1, 13-2, 13-3, 13-4**
8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**13-6, 13-7**
8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**11-3, 11-5, 13-6, 13-7**
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. (Additional Cluster)		
8.G.C.9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	**12-6, 12-7, 12-8, 12-9**

Standards for Mathematical Practice

This correlation shows the alignment of *Reveal Math Accelerated* to the Standards for Mathematical Practice, from the Common Core State Standards.

Standards for Mathematical Practice		Lesson(s)
MP1	**Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	A strong problem-solving strand is present throughout the program with an emphasis on having students explain to themselves and others the meanings of problems and plan their solution strategies. Look for the **Apply** problems and exercises labeled as **Persevere with Problems**. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 2-5, Example 4, Apply • Lesson 2-6, Apply • Lesson 6-2, Apply • Lesson 8-5, Apply • Lesson 12-3, Apply • Lesson 13-1, Practice Exercises 7-8
MP2	**Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	Students are routinely asked to make sense of quantities and their relationships, and attend to the meaning of quantities as opposed to just computing with them. Students are often asked to decontextualize a real-world problem by representing it symbolically as an expression, equation, or inequality. Look for lessons addressing these algebraic topics and the exercises labeled as **Reason Abstractly**. Many ***Talk About It!*** question prompts ask students to reason about relationships between quantities. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-1, Examples 1-2 • Lesson 4-2, Learn *Quotient of Powers*, *Talk About It!* • Lesson 5-3, Example 4, *Talk About It!* • Lesson 7-1, Examples 3-4 • Lesson 7-2, Examples 4-5 • Lesson 7-6, Examples 3-4 • Lesson 8-4, Examples 1-2 • Lesson 12-1, Learn *Circumference of Circles* • Lesson 12-4, Example 5 • Lesson 13-1, Explore activity *Congruence and Transformations*

	Standards for Mathematical Practice	Lesson(s)
MP3	**Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	Students are required to justify their reasoning and to find the errors in another student's reasoning or work. Look for the **Apply** problems (Step 4) and the exercises labeled as **Make a Conjecture, Find the Error, Use a Counterexample, Make an Argument,** or **Justify Conclusions.** Many ***Talk About It!*** question prompts ask students to justify conclusions and/or critique another student's reasoning. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-1, Practice Exercise 14 • Lesson 2-4, Example 1, *Talk About It!* • Lesson 3-7, Practice Exercise 21 • Lesson 4-4, Explore activity *Exponents of Zero* • Lesson 5-3, Practice Exercises 14 and 17 • Lesson 11-4, Learn *Classify Triangles*, *Talk About It!* • Lesson 11-7, Example 3, *Talk About It!* • Lesson 12-6, Example 4 • Lesson 13-2, Practice Exercise 9
MP4	**Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	Students apply the mathematics they know to solve real-world problems by using mathematical modeling. In the **Apply** problems, students determine their own strategy to solve application problems by choosing mathematical models to aid them. Look also for the exercises labeled as **Model with Mathematics**. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 4-5, Apply • Lesson 7-1, Apply • Lesson 7-6, Apply • Lesson 7-7, Apply • Lesson 7-8, Apply • Lesson 8-1, Apply • Lesson 12-4, Apply

Standards for Mathematical Practice		Lesson(s)
MP5	**Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	In addition to traditional tools such as estimation, mental math, or measurement tools, students are encouraged to use digital tools, such as Web Sketchpad, eTools, etc. to help solve problems. Students are routinely asked to compare and contrast methods, tools, and representations and note when one tool might be more advantageous to use than another. Look for selected *Talk About It!* prompts and exercises labeled as **Use Math Tools.** Many **Explore** activities ask students to select and use appropriate tools as they progress through the activities. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 5-2, Practice Exercise 15 • Lesson 6-2, Example 1 • Lesson 7-1, Explore activity *Solve Two-Step Equations Using Algebra Tiles* • Lesson 7-2, Explore activity *Solve Two-Step Equations Using Algebra Tiles* • Lesson 11-4, online Explore activity *Create Triangles* • Lesson 11-4, Learn *Draw Triangles Using Tools*, Examples 2-3 • Lesson 13-3, Example 2, *Talk About It!*
MP6	**Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	Students are routinely required to communicate precisely to partners, the teacher, or the entire class by using precise definitions and mathematical vocabulary. Look for the exercises labeled as **Be Precise.** Many ***Talk About It!*** question prompts ask students to clearly and precisely explain their reasoning. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 1-4, Example 4 • Lesson 3-7, Example 6 • Lesson 4-5, Practice Exercise 14 • Lesson 5-1, Example 4, *Talk About It!* • Lesson 8-1, Learn *Unit Rate and Slope, Talk About It*! • Lesson 11-4, Example 1 • Lesson 12-4, Example 5

	Standards for Mathematical Practice	Lesson(s)
MP7	**Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	Students are routinely encouraged to look for patterns or structure present in problem situations. Look for the exercises labeled as **Identify Structure.** Many ***Talk About It!*** question prompts ask students to study the structure of expressions and figures. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 4-1, Practice Exercise 12 • Lesson 7-5, Learn *Number of Solutions*, *Talk About It!* • Lesson 12-3, Examples 1-2 • Lesson 12-4, Example 5 • Lesson 12-5, Example 2 • Lesson 12-6, Example 3, Learn *Surface Area of Composite Solids*, Example 4
MP8	**Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	Students are encouraged to look for repeated calculations that lead them to sound mathematical conclusions. Look for the exercises labeled as **Identify Repeated Reasoning.** Several ***Talk About It!*** question prompts ask students to look for repeated calculations. In the Teacher Edition, look for the **Teaching the Mathematical Practices** tips labeled as this mathematical practice. *Throughout the program, for example:* *Interactive Student Edition and Teacher Edition:* • Lesson 3-3, Learn *Multiply Integers with the Same Sign* • Lesson 3-6, Examples 1-2 • Lesson 4-2, Practice Exercise 14 • Lesson 5-1, Explore activity *Find Square Roots Using a Square Model* • Lesson 6-3, Learn *Additive Inverses of Expressions* • Lesson 6-3, Example 2 • Lesson 8-2, Explore activity *Slope of Horizontal and vertical Lines*

Module 1

Proportional Relationships

Module Goal

Analyze multiple representations of proportional relationships (tables, graphs, and equations).

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s):

7.RP.A Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standards for Mathematical Content:

7.RP.A.2 Recognize and represent proportional relationships between quantities.

7.RP.A.2.A Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Also addresses 7.RP.A.1, 7.RP.A.2.B, 7.RP.A.2.C, 7.RP.A.2.D, and 7.RP.A.3

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- find unit rates involving whole numbers
- fluently divide fractions and mixed numbers

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students used rate and ratio reasoning to solve real-world and mathematical problems.
6.RP.A.1

Now
Students analyze multiple representations of proportional relationships (tables, graphs, and equations).
7.RP.A.1, 7.RP.A.2, 7.RP.A.3

Next
Students will understand the connection between proportional relationships, lines, and linear equations.
8.EE.B.5

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of ratios and rates to develop *understanding* of proportional relationships. They use this understanding to build *fluency* with proportional relationships by representing them with tables, graphs, and equations, and finding the constant of proportionality.

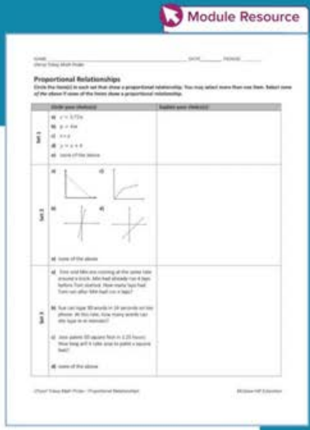
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
1-1	Unit Rates Involving Ratios of Fractions	7.RP.A.1	2	1
1-2	Understand Proportional Relationships	7.RP.A.2	2	1
1-3	Tables of Proportional Relationships	7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B	2	1
1-4	Graphs of Proportional Relationships	7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B, 7.RP.A.2.D	2	1
1-5	Equations of Proportional Relationships	7.RP.A.2, 7.RP.A.2.B, 7.RP.A.2.C	1	0.5
Put It All Together 1: Lessons 1-3 through 1-5			0.5	0.25
1-6	Solve Problems Involving Proportional Relationships	7.RP.A.2, 7.RP.A.3	1	0.5
Put It All Together 2: Lessons 1-3 through 1-6			0.5	0.25
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	14	7

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe
Proportional Relationships



Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which item(s) in each set show a proportional relationship, and explain their choices.

Targeted Concepts Understand proportional relationships in equations, tables, and verbal descriptions in which there is a constant ratio between two quantities.

Targeted Misconceptions

- Students may not recognize a proportional relationship when given a form other than $y = mx + 0$.
- Students may incorrectly assume that any graph that forms a straight line is proportional.

Assign the probe after Lesson 5.

Correct Answers: 1. a, b, c; **2.** b; **3.** b

Collect and Assess Student Answers

If the student selects...	**Then** the student likely...
Set 1: Selects d; Does not select a, b, and/or c	does not understand that, for a relationship to be proportional, y must equal 0 when $x = 0$, or may not recognize a proportional relationship when given a form other than $y = mx + 0$.
Set 2: Selects a and/or d	assumes that all linear relationships are proportional.
Set 3: Selects a and/or c	assumes that a constant rate of change automatically implies a proportional relationship.
Various other incorrect selections	does not recognize proportional relationships represented in symbolic form (Set 1), graphical form (Set 2), or verbal form (Set 3). **Example:** In Set 1, the student does not choose a choice due to the decimal form of the rate of change.

Take Action

After the Probe Design a plan to address any possible misconceptions. Y ou may wish to assign the following resources.

- ALEKS Proportional Relationships
- Lesson 3, Examples 1 and 2
- Lesson 4, Examples 1 and 2
- Lesson 5, Examples 1–3

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

![IGNITE!]

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

What does it mean for two quantities to be in a proportional relationship? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of cooking, fitness center memberships, and exchange rates to introduce the idea of equivalent ratios and proportional relationships. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.

Module 1

Proportional Relationships

Essential Question

What does it mean for two quantities to be in a proportional relationship?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

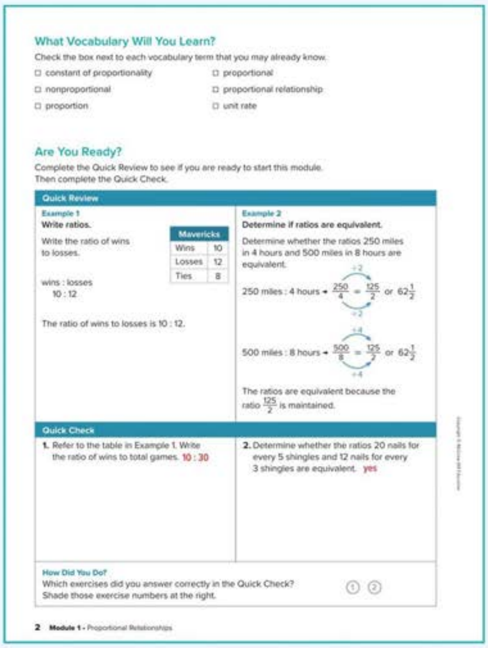
KEY: ○ — I don't know. ○ — I've heard of it. ○ — I know it!

	Before			After		
	○	○	○	○	○	○
computing unit rates involving ratios of fractions						
determining whether a relationship is proportional by looking at a table of values						
determining whether a relationship is proportional by looking at a graph						
finding and interpreting the constant of proportionality						
interpreting the points (0, 0) and (1, r) on the graph of a proportional relationship						
representing proportional relationships with equations						
solving problems involving proportional relationships						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about proportional relationships.

Module 1 • Proportional Relationships 1

Interactive Student Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ constant of proportionality
- ☐ proportional
- ☐ nonproportional
- ☐ proportional relationship
- ☐ proportion
- ☐ unit rate

Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Write ratios.

Write the ratio of wins to losses.

Mavericks	
Wins	10
Losses	12
Ties	8

wins : losses
10 : 12

The ratio of wins to losses is 10 : 12.

Example 2
Determine if ratios are equivalent.

Determine whether the ratios 250 miles in 4 hours and 500 miles in 8 hours are equivalent.

250 miles : 4 hours → $\frac{250}{4} = \frac{125}{2}$ or $62\frac{1}{2}$ (÷2)

500 miles : 8 hours → $\frac{500}{8} = \frac{125}{2}$ or $62\frac{1}{2}$ (÷4)

The ratios are equivalent because the ratio $\frac{125}{2}$ is maintained.

Quick Check

1. Refer to the table in Example 1. Write the ratio of wins to total games. 10 : 30
2. Determine whether the ratios 20 nails for every 5 shingles and 12 nails for every 3 shingles are equivalent. yes

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

2 Module 1 • Proportional Relationships

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine.

Define Two quantities are **proportional** if they have a constant ratio.

Example
The table illustrates the proportional relationship between the number of pizzas ordered and the total cost. For each pair of quantities in the table, the ratio of the cost to number of pizzas is $8 per pizza.

Cost ($)	8	16	24	32	40
Pizzas	1	2	3	4	5

Ask What term do we use to represent a relationship that is not proportional? nonproportional relationship

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- dividing fractions and mixed numbers
- solving unit rate problems with whole numbers
- using ratio reasoning to solve real-world problems
- locating ordered pairs on a coordinate plane
- solving one-step equations
- writing equations to represent real-world problems

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Proportional Relationships** topic—who is ready to learn these concepts and who isn't quite ready to learn them yet—in order to adjust your instruction as appropriate.

Mindset Matters

Growth Mindset vs. Fixed Mindset

Everyone has a core belief or mindset about how they learn. People with a *growth mindset* believe that they can grow their intelligence through hard work. Those with a *fixed mindset* believe that while they can learn new things, they cannot increase their intelligence. When a student approaches school, life, and the future workplace with a growth mindset, they are more likely to persevere through challenging problems, learn from mistakes, and ultimately learn in a deeper, more meaningful way.

How Can I Apply It?

Assign students rich tasks, such as the **Explore** activities, that can help them to develop their intelligence. Encourage them with the thought that each time they learn a new idea, neurons fire electric currents that connect different parts of their brain!

Lesson 1-1

Unit Rates Involving Ratios of Fractions

7.RP.A.1

LESSON GOAL

Students will find unit rates when one or both quantities are fractions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Find Unit Rates with Fractions

Learn: Unit Rates Involving Ratios of Fractions
Example 1: Find Unit Rates
Example 2: Find Unit Rates
Apply: Kayaking

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 1 of the *Language Development Handbook* to help your students build mathematical language related to unit rates associated with ratios of fractions.

ELL You can use the tips and suggestions on page T1 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7 .RP.A** by solving real-world problems involving proportional relationships to find unit rates.
Standards for Mathematical Content: **7. RP.A.1**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found unit rates that involved whole numbers.
6.RP.A.2

Now
Students find unit rates that involve ratios of fractions.
7.RP.A1

Next
Students will use models and ratio reasoning to understand how a proportional relationship can exist between quantities.
7.RP.A.2

Rigor

The Three Pillars of Rigor

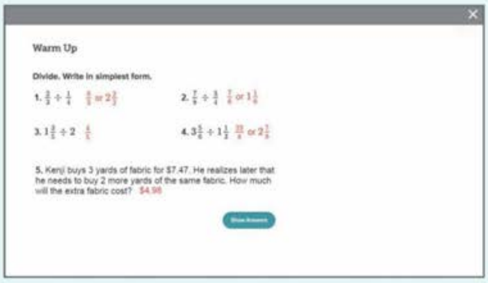
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of fractions and unit rates to build *fluency* with finding unit rates when one or both of the quantities is a fraction. They *apply* their fluency in finding unit rates involving ratios of fractions to solve real-world problems.

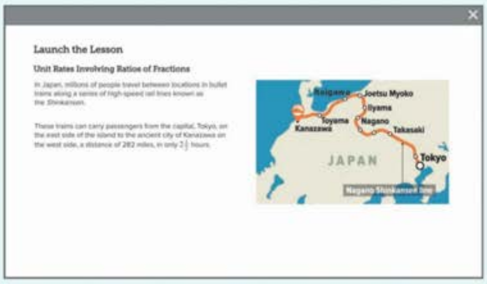
Mathematical Background

To find a unit rate in which one or both quantities are fractions, use a bar diagram, double number line, ratio table, or division. You can use a complex fraction to express the rate. A *complex fraction* is a fraction in which the numerator and/or denominator are also fractions. To find the unit rate, divide the numerator by the denominator.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- dividing fractions and mixed numbers (Exercises 1–4)
- solving unit rate problems (Exercise 5)

Answers

1. $\frac{8}{3}$ or $2\frac{2}{3}$
2. $\frac{7}{6}$ or $1\frac{1}{6}$
3. $\frac{4}{5}$
4. $\frac{23}{8}$ or $2\frac{7}{8}$
5. $4.98

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the rate at which passenger trains can travel in Japan.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What are some synonyms for the term *unit*? Sample answer: element, part, item, piece

Explore Find Unit Rates with Fractions

Objective

Students will use bar diagrams to explore how to find a unit rate when one or both quantities of a given rate are fractions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the rates at which three friends (Sasha, Pedro, and Emily) ran while training for a one-mile race. The distance that each friend ran is written as a fraction of a mile. Throughout this activity, students will use various strategies, including bar diagrams and scaling, to find each friend's unit rate. Students will use their observations to make conjectures about how to find unit rates when one or both quantities are fractions.

Inquiry Question

How can you find a unit rate in which one or both quantities are fractions? Sample answer: I can draw a bar diagram or use scaling to help me find a unit rate that involves ratios of fractions.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* question on Slide 2 are shown.

Talk About It!

SLIDE 2

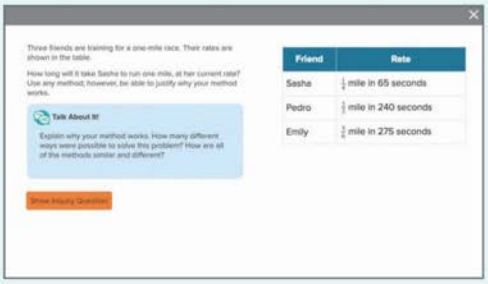
Mathematical Discourse

Explain why your method works. How many different ways were possible to solve this problem? How are all of the methods similar and different? Some students may draw a bar diagram. Others may use mental math and reasoning by adding 65 and 65 to find that Sasha can run $\frac{1}{2}$ mile in 130 seconds, then multiplying that by 2 to find that Sasha can run 1 mile in 260 seconds. Other students may use scaling by multiplying 65 by 4.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 9

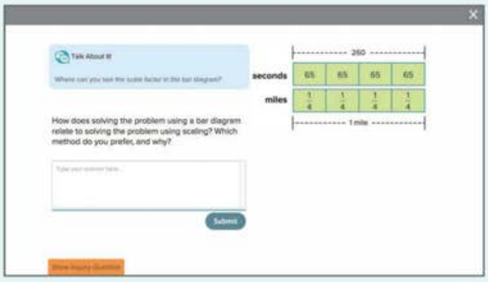


Explore, Slide 2 of 9

TYPE

Throughout the Explore, students complete interactive bar diagrams in order to find unit rates.

Interactive Presentation



Explore, Slide 6 of 9

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Find Unit Rates with Fractions *(continued)*

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
Students will come up with their own strategies for finding each person's unit rate. They may use any strategy they wish, but must be able to justify why their strategy works. They will also use other strategies, such as bar diagrams and scaling, to find each person's unit rate. Students will compare and contrast all of the strategies they used, understand those used by other students, and identify correspondences between the different strategies.

2 Reason Abstractly and Quantitatively Students will apply their knowledge of unit rates and bar diagrams to simplify the complicated situation that arises when one of the quantities in the rate is a fraction. They will identify the important quantities (unit rate and scale factor) and explain how those quantities are illustrated by the bar diagrams.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Where can you see the scale factor in the bar diagram? Sample answer: The scale factor is represented by the number of sections that are unit fractions. There are four $\frac{1}{4}$-mile sections, so the scale factor is 4.

How does solving the problem using a bar diagram compare to solving the problem using scaling? Which method do you prefer, and why? Sample answer: Both methods arrive at the correct unit rate. Drawing a bar diagram helps me to visualize the problem, while it may be faster to use scaling. See student's preferences.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Unit Rates Involving Ratios of Fractions

Objective

Students will learn how to find a unit rate that involves ratios of fractions.

Teaching Notes

SLIDE 1

In a previous grade, students explored and developed the idea of a ratio and what it means to use a ratio to compare two quantities. Students also explored and developed the use of rates to compare two quantities with unlike units. In this course, students will continue these concepts by exploring and developing the use of ratios and rates with fractional quantities. You may wish to review the concepts and definitions of *ratio*, *rate*, and *unit rate* that students have learned in the previous grade. Students should be familiar with finding unit rates when both quantities are whole numbers. In this lesson, students will expand on this concept to find unit rates when one or both quantities is a fraction.

Go Online to find additional teaching notes.

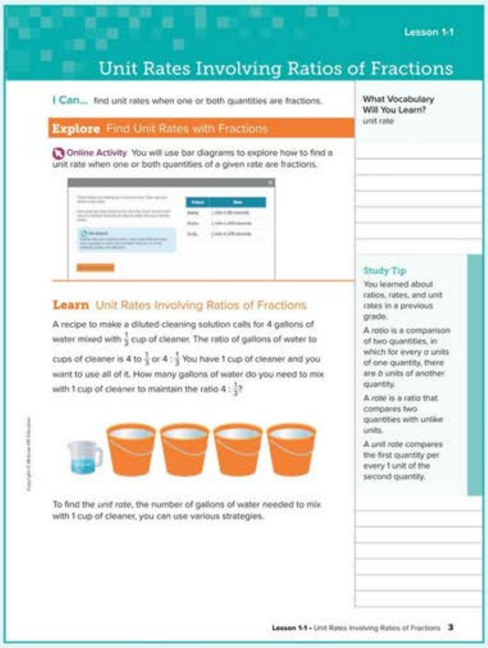
(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

Deepen students' understanding of finding unit rates, when one or both quantities are fractions, by having them respond to the following questions.

- If Jesse can walk $\frac{1}{2}$ mile in $\frac{1}{6}$ hour, what would the complex fraction $\frac{\frac{1}{6}}{\frac{1}{2}}$ represent? Jesse's rate in hours per mile
- How can you mentally simplify $\frac{\frac{1}{6}}{\frac{1}{2}}$? What does this number mean in the context of the problem? Sample answer: Dividing by $\frac{1}{2}$ is the same as multiplying by 2; $\frac{1}{6} \div \frac{1}{2} = \frac{1}{6} \cdot 2 = \frac{2}{6}$, or $\frac{1}{3}$. It will take Jesse $\frac{1}{3}$ hour, or 20 minutes, to walk one mile.



Lesson 1-1

Unit Rates Involving Ratios of Fractions

I Can... find unit rates when one or both quantities are fractions.

Explore Find Unit Rates with Fractions

Online Activity You will use bar diagrams to explore how to find a unit rate when one or both quantities of a given rate are fractions.

Learn Unit Rates Involving Ratios of Fractions

A recipe to make a diluted cleaning solution calls for 4 gallons of water mixed with $\frac{1}{3}$ cup of cleaner. The ratio of gallons of water to cups of cleaner is 4 to $\frac{1}{3}$ or $4 : \frac{1}{3}$. You have 1 cup of cleaner and you want to use all of it. How many gallons of water do you need to mix with 1 cup of cleaner to maintain the ratio $4 : \frac{1}{3}$?

To find the *unit rate*, the number of gallons of water needed to mix with 1 cup of cleaner, you can use various strategies.

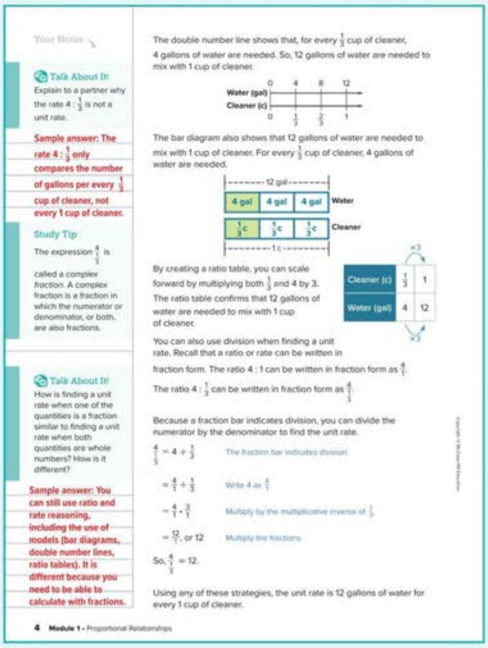
What Vocabulary Will You Learn?
unit rate

Study Tip
You learned about ratios, rates, and unit rates in a previous grade.
A *ratio* is a comparison of two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity.
A *rate* is a ratio that compares two quantities with unlike units.
A *unit rate* compares the first quantity per every 1 unit of the second quantity.

Lesson 1-1 • Unit Rates Involving Ratios of Fractions 3

Interactive Presentation

Learn, Unit Rates Involving Ratios of Fractions, Slide 1 of 6



Your Notes

Talk About It!
Explain to a partner why the rate $4 : \frac{1}{3}$ is not a unit rate.

Sample answer: The rate $4 : \frac{1}{3}$ only compares the number of gallons per every $\frac{1}{3}$ cup of cleaner, not every 1 cup of cleaner.

Study Tip
The expression $\frac{\frac{4}{1}}{\frac{1}{3}}$ is called a complex fraction. A complex fraction is a fraction in which the numerator or denominator, or both, are also fractions.

Talk About It!
How is finding a unit rate when one of the quantities is a fraction similar to finding a unit rate when both quantities are whole numbers? How is it different?

Sample answer: You can still use ratio and rate reasoning, including the use of models (bar diagrams, double number lines, ratio tables). It is different because you need to be able to calculate with fractions.

The double number line shows that, for every $\frac{1}{3}$ cup of cleaner, 4 gallons of water are needed. So, 12 gallons of water are needed to mix with 1 cup of cleaner.

The bar diagram also shows that 12 gallons of water are needed to mix with 1 cup of cleaner. For every $\frac{1}{3}$ cup of cleaner, 4 gallons of water are needed.

By creating a ratio table, you can scale forward by multiplying both $\frac{1}{3}$ and 4 by 3. The ratio table confirms that 12 gallons of water are needed to mix with 1 cup of cleaner.

Cleaner (c)	$\frac{1}{3}$	1
Water (gal)	4	12

You can also use division when finding a unit rate. Recall that a ratio or rate can be written in fraction form. The ratio 4 : 1 can be written in fraction form as $\frac{4}{1}$. The ratio $4 : \frac{1}{3}$ can be written in fraction form as $\frac{\frac{4}{1}}{\frac{1}{3}}$.

Because a fraction bar indicates division, you can divide the numerator by the denominator to find the unit rate.

$\frac{\frac{4}{1}}{\frac{1}{3}} = 4 \div \frac{1}{3}$ The fraction bar indicates division.

$= \frac{4}{1} \div \frac{1}{3}$ Write 4 as $\frac{4}{1}$.

$= \frac{4}{1} \cdot \frac{3}{1}$ Multiply by the multiplicative inverse of $\frac{1}{3}$.

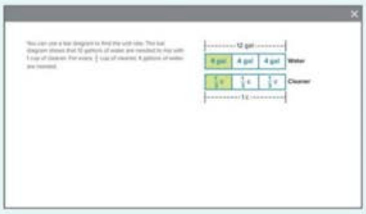
$= \frac{12}{1}$, or 12 Multiply the fractions.

So, $\frac{\frac{4}{1}}{\frac{1}{3}} = 12$.

Using any of these strategies, the unit rate is 12 gallons of water for every 1 cup of cleaner.

4 Module 1 • Proportional Relationships

Interactive Presentation



Learn, Unit Rates Involving Ratios of Fractions, Slide 3 of 6

Learn Unit Rates Involving Ratios of Fractions *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 6, encourage them to make sense of the process of finding a unit rate when one or both quantities are fractions as opposed to when they are whole numbers.

Teaching Notes

SLIDE 5

Encourage students to study each method for finding the unit rate. Each method starts with the quantities given. The double number line, bar diagram, and ratio table require scaling to find the unit rate. Each section of the double number line is also represented in the bar diagram, and could be represented in the ratio table.

Talk About It!

SLIDE 1

Mathematical Discourse

Explain to a partner why the rate $4 : \frac{1}{3}$ is not a unit rate. Sample answer: The rate $4 : \frac{1}{3}$ only compares the number of gallons per every $\frac{1}{3}$ cup of cleaner, not every 1 cup of cleaner.

SLIDE 6

Mathematical Discourse

How is finding a unit rate when one of the quantities is a fraction similar to finding a unit rate when both quantities are whole numbers? How is it different? Sample answer: You can still use ratio and rate reasoning, including the use of models (bar diagrams, double number lines, ratio tables). It is different because you need to be able to calculate with fractions.

Example 1 Find Unit Rates

Objective

Students will find a unit rate in which one of the given quantities is a fraction.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question, encourage them to use reasoning to determine that Tia can paint more than 36 square feet per hour, because she can paint 36 square feet in less than an hour.

7 Look For and Make Use of Structure In Method 4, encourage students to understand that the structure of a complex fraction means that the numerator, the denominator, or both must be fractions.

Questions for Mathematical Discourse

SLIDE 2

AL Which number represents the number of square feet painted? the time, in hours? 36; $\frac{3}{4}$

OL How does the bar diagram represent the ratio? The bar diagram uses two bars to represent the two quantities *36 square feet* and $\frac{3}{4}$ *hour*. The bars are the same length, with the same number of sections, to show that the two quantities are in a ratio.

BL What would the ratio $\frac{\frac{3}{4}}{36}$ represent as a unit rate? the time, in hours, to paint one square foot

SLIDE 3

AL Where on the double number line is the ratio $36 : \frac{3}{4}$ represented? Both number lines begin at 0. The quantities 36 and $\frac{3}{4}$ are located the same distance from their respective 0s.

OL How does this double number line compare to the double bar diagram from Method 1? Both models show the ratio $36 : \frac{3}{4}$ by showing the quantities 36 and $\frac{3}{4}$ as the same location on each bar diagram.

BL How many square feet can Tia paint in three hours? 144 square feet

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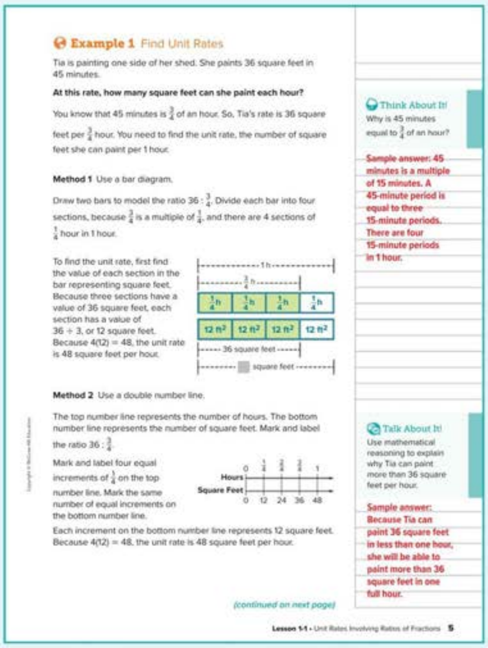


Example 1 Find Unit Rates

Tia is painting one side of her shed. She paints 36 square feet in 45 minutes.

At this rate, how many square feet can she paint each hour?

You know that 45 minutes is $\frac{3}{4}$ of an hour. So, Tia's rate is 36 square feet per $\frac{3}{4}$ hour. You need to find the unit rate, the number of square feet she can paint per 1 hour.

Method 1 Use a bar diagram.

Draw two bars to model the ratio $36 : \frac{3}{4}$. Divide each bar into four sections, because $\frac{3}{4}$ is a multiple of $\frac{1}{4}$, and there are 4 sections of $\frac{1}{4}$ hour in 1 hour.

To find the unit rate, first find the value of each section in the bar representing square feet. Because three sections have a value of 36 square feet, each section has a value of 36 ÷ 3, or 12 square feet. Because 4(12) = 48, the unit rate is 48 square feet per hour.

Method 2 Use a double number line.

The top number line represents the number of hours. The bottom number line represents the number of square feet. Mark and label the ratio $36 : \frac{3}{4}$.

Mark and label four equal increments of $\frac{1}{4}$ on the top number line. Mark the same number of equal increments on the bottom number line.

Each increment on the bottom number line represents 12 square feet. Because 4(12) = 48, the unit rate is 48 square feet per hour.

Think About It! Why is 45 minutes equal to $\frac{3}{4}$ of an hour?

Sample answer: 45 minutes is a multiple of 15 minutes. A 45-minute period is equal to three 15-minute periods. There are four 15-minute periods in 1 hour.

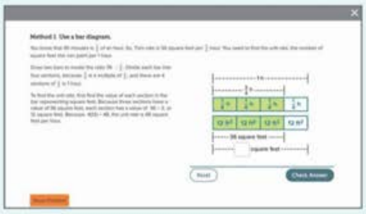
Talk About It! Use mathematical reasoning to explain why Tia can paint more than 36 square feet per hour.

Sample answer: Because Tia can paint 36 square feet in less than one hour, she will be able to paint more than 36 square feet in one full hour.

(continued on next page)

Lesson 1-1 • Unit Rates Involving Ratios of Fractions 5

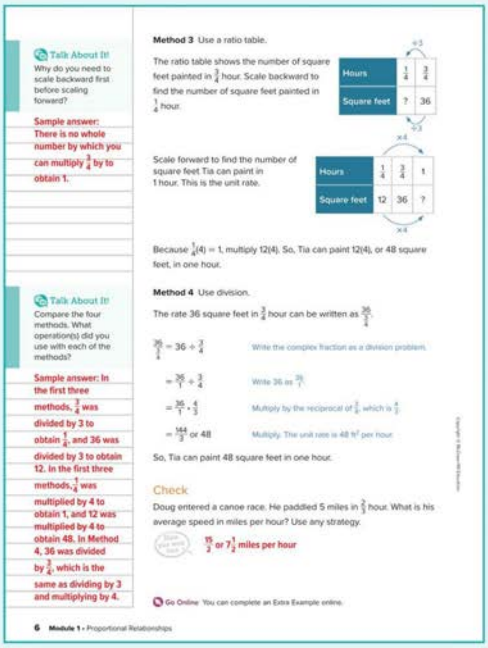
Interactive Presentation



Example 1, Find Unit Rates, Slide 2 of 7

CLICK

On Slide 3, students move through the steps to see how a double number line can be used to solve the problem.



Talk About It! Why do you need to scale backward first before scaling forward?

Sample answer: There is no whole number by which you can multiply $\frac{3}{4}$ by to obtain 1.

Method 3 Use a ratio table.

The ratio table shows the number of square feet painted in $\frac{3}{4}$ hour. Scale backward to find the number of square feet painted in $\frac{1}{4}$ hour.

Hours	$\frac{1}{4}$	$\frac{3}{4}$
Square feet	?	36

Scale forward to find the number of square feet Tia can paint in 1 hour. This is the unit rate.

Hours	$\frac{1}{4}$	$\frac{3}{4}$	1
Square feet	12	36	?

Because $\frac{1}{4}(4) = 1$, multiply 12(4). So, Tia can paint 12(4), or 48 square feet, in one hour.

Talk About It! Compare the four methods. What operation(s) did you use with each of the methods?

Sample answer: In the first three methods, $\frac{3}{4}$ was divided by 3 to obtain $\frac{1}{4}$, and 36 was divided by 3 to obtain 12. In the first three methods, $\frac{1}{4}$ was multiplied by 4 to obtain 1, and 12 was multiplied by 4 to obtain 48. In Method 4, 36 was divided by $\frac{3}{4}$, which is the same as dividing by 3 and multiplying by 4.

Method 4 Use division.

The rate 36 square feet in $\frac{3}{4}$ hour can be written as $\frac{36}{\frac{3}{4}}$.

$\frac{36}{\frac{3}{4}} = 36 \div \frac{3}{4}$ — Write the complex fraction as a division problem.

$= \frac{36}{1} \div \frac{3}{4}$ — Write 36 as $\frac{36}{1}$.

$= \frac{36}{1} \cdot \frac{4}{3}$ — Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$= \frac{144}{3}$ or 48 — Multiply. The unit rate is 48 ft^2 per hour.

So, Tia can paint 48 square feet in one hour.

Check

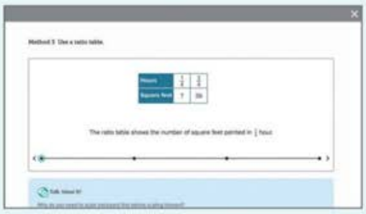
Doug entered a canoe race. He paddled 5 miles in $\frac{2}{3}$ hour. What is his average speed in miles per hour? Use any strategy.

$\frac{15}{2}$ or $7\frac{1}{2}$ miles per hour

Go Online You can complete an Extra Example online.

6 Module 1 • Proportional Relationships

Interactive Presentation



Example 1, Find Unit Rates, Slide 4 of 7

CLICK

On Slide 4, students moves through the steps to see how a ratio table can be used to solve the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 1 Find Unit Rates *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL Why did you not scale back to $\frac{1}{2}$? There is no whole number divisor that you can divide $\frac{3}{4}$ by to obtain $\frac{1}{2}$.

OL Is the ratio $12 : \frac{1}{4}$ equivalent to $36 : \frac{3}{4}$? Explain. yes; Because the same divisor was used to determine the ratio $12 : \frac{1}{4}$, they are equivalent.

BL Would the process be the same if you used $\frac{3}{4}$ hour rather than 45 minutes? Explain. yes; Sample answer: To scale backward, then forward, you would need to find a common factor of 45 minutes and 60 minutes in order to keep the units consistent.

SLIDE 5

AL Why do we write the complex fraction as a division problem? The fraction bar indicates division of the numerator by the denominator.

OL How do you know the answer is reasonable? Three-fourths of an hour is less than 1 hour, but more than half an hour; 36 square feet is less than 48 square feet, but more than 24 square feet, which would represent the number of square feet painted in half an hour.

BL How do you know your answer is square feet per hour, rather than hours per square foot? Sample answer: The ratio $36 : \frac{3}{4}$ was written as the complex fraction $\frac{36}{\frac{3}{4}}$ which shows square feet per hour.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find Unit Rates

Objective

Students will find a unit rate in which both of the given quantities are fractions.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* questions, they will use reasoning and mental math to determine that Josiah's speed will be greater than $\frac{5}{6}$ mile per hour, because he can jog that distance in less than one hour.

Questions for Mathematical Discourse

SLIDE 2

AL How are the two quantities different? Sample answer: One of the quantities is a whole number, while the other quantity is a fraction.

OL Why did you multiply $\frac{5}{6}$ mile by 4? Because the bottom bar diagram was divided into four sections, I need to find the total number of miles represented by all four sections.

BL How could you find the unit rate mentally? The rate is $\frac{5}{6}$ mile per $\frac{1}{4}$ hour. There are 4 quarter hours in one hour. So, to calculate the unit rate per hour, multiply $\frac{5}{6}$ mile by 4, which yields a result of $3\frac{1}{3}$ miles per hour.

SLIDE 3

AL Why was the top number line extended to 1 hour? I need to find the average speed in miles per 1 hour.

OL How is the double number line similar to the double bar diagram from Method 1? Sample answer: Both models show the ratio $\frac{5}{6} : \frac{1}{4}$ because each quantity is located the same distance from its respective 0. Both diagrams also show the corresponding number of miles after 1 hour.

BL How can you use the double number line to find the number of miles Josiah can jog in $1\frac{1}{2}$ hours? Sample answer: I can extend the double number line from 1 hour to $1\frac{1}{2}$ hours. Josiah would jog $\frac{30}{6}$ or 5 miles.

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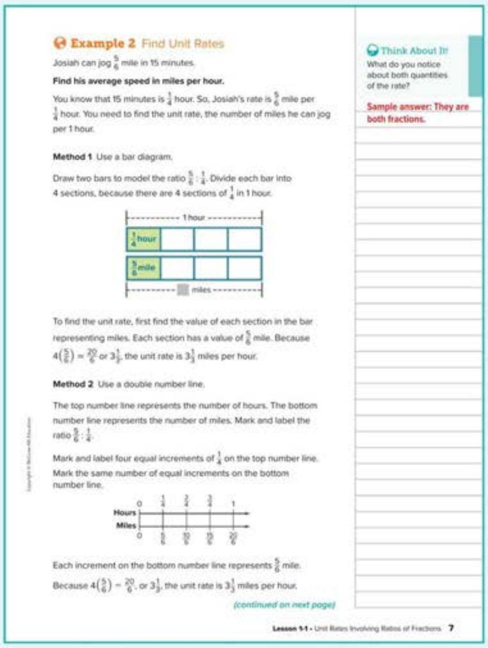


Example 2 Find Unit Rates

Josiah can jog $\frac{5}{6}$ mile in 15 minutes.

Find his average speed in miles per hour.

You know that 15 minutes is $\frac{1}{4}$ hour. So, Josiah's rate is $\frac{5}{6}$ mile per $\frac{1}{4}$ hour. You need to find the unit rate, the number of miles he can jog per 1 hour.

Method 1 Use a bar diagram.

Draw two bars to model the ratio $\frac{5}{6} : \frac{1}{4}$. Divide each bar into 4 sections, because there are 4 sections of $\frac{1}{4}$ in 1 hour.

To find the unit rate, first find the value of each section in the bar representing miles. Each section has a value of $\frac{5}{6}$ mile. Because $4\left(\frac{5}{6}\right) = \frac{20}{6}$ or $3\frac{1}{3}$, the unit rate is $3\frac{1}{3}$ miles per hour.

Method 2 Use a double number line.

The top number line represents the number of hours. The bottom number line represents the number of miles. Mark and label the ratio $\frac{5}{6} : \frac{1}{4}$.

Mark and label four equal increments of $\frac{1}{4}$ on the top number line. Mark the same number of equal increments on the bottom number line.

Each increment on the bottom number line represents $\frac{5}{6}$ mile. Because $4\left(\frac{5}{6}\right) = \frac{20}{6}$, or $3\frac{1}{3}$, the unit rate is $3\frac{1}{3}$ miles per hour.

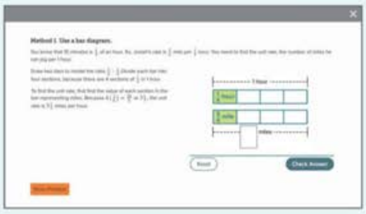
(continued on next page)

Think About It! What do you notice about both quantities of the rate?

Sample answer: They are both fractions.

Lesson 1-1 • Unit Rates Involving Ratios of Fractions 7

Interactive Presentation



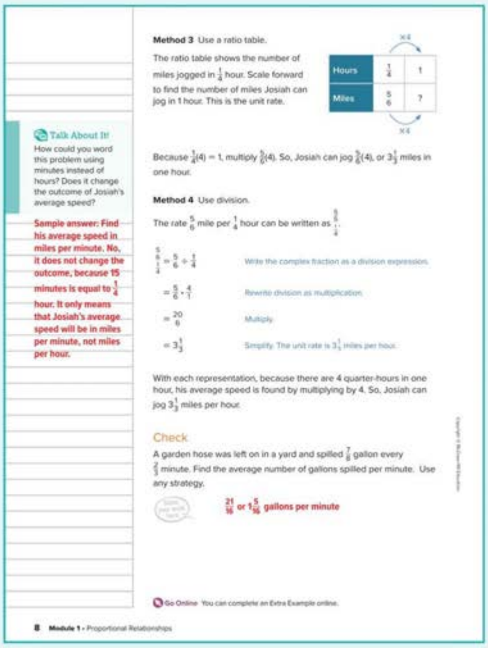
Example 2, Find Unit Rates, Slide 2 of 7

TYPE

On Slide 2, students label the bar diagram.

CLICK

On Slide 3, students follow the steps to see how a double number line can be used to find the unit rate.



Method 3 Use a ratio table.

The ratio table shows the number of miles jogged in $\frac{1}{4}$ hour. Scale forward to find the number of miles Josiah can jog in 1 hour. This is the unit rate.

Hours	$\frac{1}{4}$	1
Miles	$\frac{5}{6}$	?

×4

Because $\frac{1}{4}(4) = 1$, multiply $\frac{5}{6}(4)$. So, Josiah can jog $\frac{5}{6}(4)$, or $3\frac{1}{3}$ miles in one hour.

Method 4 Use division.

The rate $\frac{5}{6}$ mile per $\frac{1}{4}$ hour can be written as $\frac{\frac{5}{6}}{\frac{1}{4}}$.

$\frac{\frac{5}{6}}{\frac{1}{4}} = \frac{5}{6} \div \frac{1}{4}$ Write the complex fraction as a division expression.

$= \frac{5}{6} \cdot \frac{4}{1}$ Rewrite division as multiplication.

$= \frac{20}{6}$ Multiply.

$= 3\frac{1}{3}$ Simplify. The unit rate is $3\frac{1}{3}$ miles per hour.

With each representation, because there are 4 quarter-hours in one hour, his average speed is found by multiplying by 4. So, Josiah can jog $3\frac{1}{3}$ miles per hour.

Talk About It! How could you word this problem using minutes instead of hours? Does it change the outcome of Josiah's average speed?

Sample answer: Find his average speed in miles per minute. No, it does not change the outcome, because 15 minutes is equal to $\frac{1}{4}$ hour. It only means that Josiah's average speed will be in miles per minute, not miles per hour.

Check

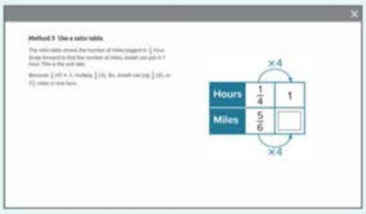
A garden hose was left on in a yard and spilled $\frac{7}{8}$ gallon every $\frac{2}{3}$ minute. Find the average number of gallons spilled per minute. Use any strategy.

$\frac{21}{16}$ or $1\frac{5}{16}$ gallons per minute

Go Online You can complete an Extra Example online.

8 Module 1 • Proportional Relationships

Interactive Presentation



Example 2, Find Unit Rates, Slide 4 of 7

TYPE

On Slide 4, students complete the table.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find Unit Rates *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL Why is multiplying by 4 considered scaling forward? Y ou are going from a lesser number, $\frac{1}{4}$, to a greater number, 1.

OL Why is the number of miles Josiah jogs in 1 hour considered the unit rate? The unit rate is always a quantity compared to 1.

BL How could you use the ratio table to find the number of hours it would take him to jog 10 miles? Sample answer: I could find the unit rate of $3\frac{1}{3}$ miles and then scale forward to 10 miles. It would take him 3 hours.

SLIDE 5

AL Why is the complex fraction not written as $\frac{\frac{1}{4}}{\frac{5}{6}}$? By writing the complex fraction in that way, it is representing the number of hours per mile.

OL How is using division similar to using a ratio table? Sample answer: Both methods scale up by multiplying by 4.

BL How could you use division to find the unit rate if he jogs $3\frac{1}{2}$ miles in $1\frac{1}{4}$ hours? Sample answer: I can divide $3\frac{1}{2} \div 1\frac{1}{4}$. His unit rate is $2\frac{4}{5}$ miles per hour.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Kayaking

Objective

Students will come up with their own strategy to solve an application problem involving kayaking.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

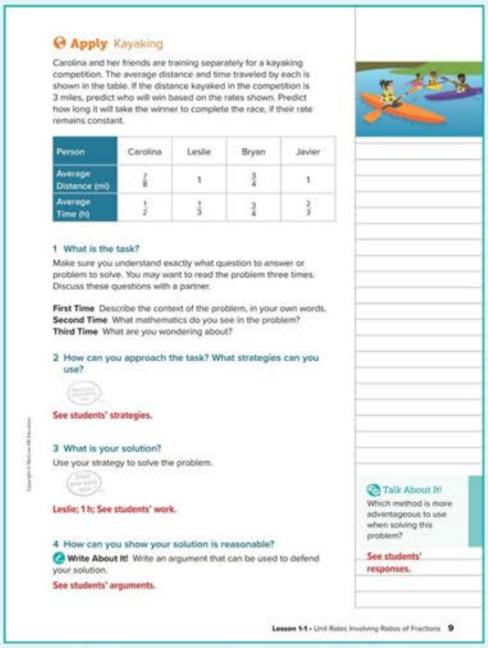
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you use reasoning to find how long it will take Leslie to complete the race? Javier?
- How do you find the unit rate, or speed, for each person?
- How can you use a double bar diagram, double number line, ratio table, or division to find each unit rate?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Kayaking

Carolina and her friends are training separately for a kayaking competition. The average distance and time traveled by each is shown in the table. If the distance kayaked in the competition is 3 miles, predict who will win based on the rates shown. Predict how long it will take the winner to complete the race, if their rate remains constant.

Person	Carolina	Leslie	Bryan	Javier
Average Distance (mi)	$\frac{7}{8}$	1	$\frac{3}{4}$	1
Average Time (h)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{2}{3}$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Leslie; 1 h; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Which method is more advantageous to use when solving this problem?

See students' responses.

Lesson 1-1 • Unit Rates Involving Ratios of Fractions 9

Interactive Presentation

Apply, Kayaking

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A walk-a-thon was held at a local middle school. The table gives the average distances and times for four walkers for certain periods of the walk-a-thon. If the total distance of the route was 2 miles, who completed the route first? How long did it take her to complete the route if she walked at a constant rate?

Person	Distance (mi)	Time (min)
Lakeisha	$\frac{7}{8}$	18
Baydan	$\frac{9}{10}$	22
Madison	$\frac{4}{5}$	17

Lakeisha; 41.14 min

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare the process for finding unit rates involving fractions with what you know about dividing fractions. How are they similar? How are they different?

See students' observations.

10 Module 1 • Proportional Relationships

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. What is the train's average unit rate in miles per hour? Write a mathematical argument that can be used to defend your solution. $112\frac{4}{5}$ miles per hour; Sample answer: Simplify the complex fraction $\frac{282}{2\frac{1}{2}}$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11, odd, 12–15
- ALEKS Ratios and Unit Rates

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 4–7, 11, 12, 14
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Multiplication and Division with Fractions

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Multiplication and Division with Fractions

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

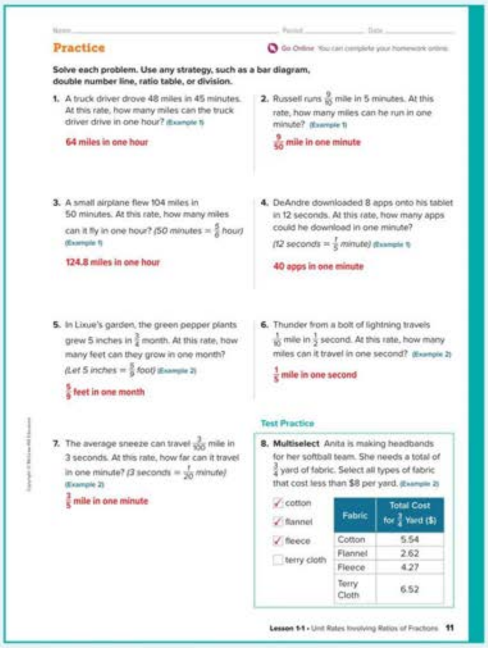
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find unit rates that involve ratios of fractions (one quantity is a fraction)	1–5
2	find unit rates that involve ratios of fractions (both quantities are fractions)	6, 7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving unit rates	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may incorrectly set up the complex fraction when finding the unit rate. Remind students that the order in which the units are represented must correspond to the order in which the numerical values are represented. In Exercise 6, finding the unit rate for $\frac{\frac{1}{10}}{\frac{1}{2}}$ would provide the number of miles in 1 second. Using $\frac{\frac{1}{2}}{\frac{1}{10}}$ would provide the number of seconds in 1 mile. Encourage students to pay attention to which value should be in the numerator and which value should be in the denominator, as students may transpose these.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each problem. Use any strategy, such as a bar diagram, double number line, ratio table, or division.

1. A truck driver drove 48 miles in 45 minutes. At this rate, how many miles can the truck driver drive in one hour? (Example 1)
64 miles in one hour

2. Russell runs $\frac{9}{10}$ mile in 5 minutes. At this rate, how many miles can he run in one minute? (Example 1)
$\frac{9}{50}$ mile in one minute

3. A small airplane flew 104 miles in 50 minutes. At this rate, how many miles can it fly in one hour? *(50 minutes = $\frac{5}{6}$ hour)* (Example 1)
124.8 miles in one hour

4. DeAndre downloaded 8 apps onto his tablet in 12 seconds. At this rate, how many apps could he download in one minute? *(12 seconds = $\frac{1}{5}$ minute)* (Example 1)
40 apps in one minute

5. In Lixue's garden, the green pepper plants grew 5 inches in $\frac{3}{4}$ month. At this rate, how many feet can they grow in one month? *(Let 5 inches = $\frac{5}{9}$ foot)* (Example 2)
$\frac{5}{9}$ feet in one month

6. Thunder from a bolt of lightning travels $\frac{1}{10}$ mile in $\frac{1}{2}$ second. At this rate, how many miles can it travel in one second? (Example 2)
$\frac{1}{5}$ mile in one second

7. The average sneeze can travel $\frac{3}{100}$ mile in 3 seconds. At this rate, how far can it travel in one minute? *(3 seconds = $\frac{1}{20}$ minute)* (Example 2)
$\frac{3}{5}$ mile in one minute

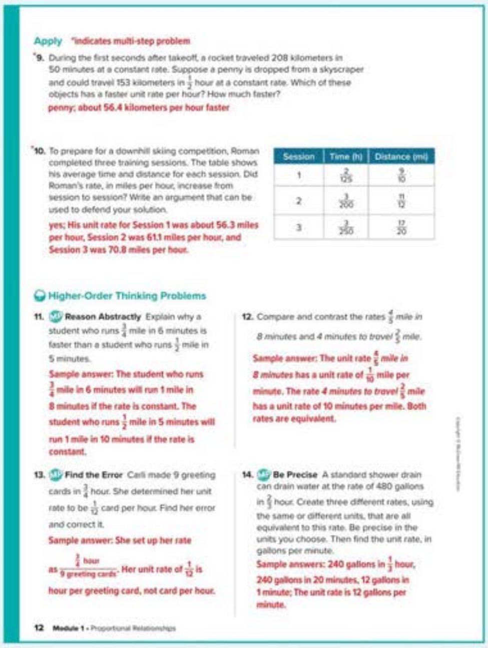
Test Practice

8. **Multiselect** Anita is making headbands for her softball team. She needs a total of $\frac{3}{4}$ yard of fabric. Select all types of fabric that cost less than $8 per yard. (Example 2)

✓ cotton
✓ flannel
✓ fleece
☐ terry cloth

Fabric	Total Cost for $\frac{3}{4}$ Yard ($)
Cotton	5.54
Flannel	2.62
Fleece	4.27
Terry Cloth	6.52

Lesson 1-1 • Unit Rates Involving Ratios of Fractions 11



Apply *indicates multi-step problem

*9. During the first seconds after takeoff, a rocket traveled 208 kilometers in 50 minutes at a constant rate. Suppose a penny is dropped from a skyscraper and could travel 153 kilometers in $\frac{1}{2}$ hour at a constant rate. Which of these objects has a faster unit rate per hour? How much faster?

penny; about 56.4 kilometers per hour faster

*10. To prepare for a downhill skiing competition, Roman completed three training sessions. The table shows his average time and distance for each session. Did Roman's rate, in miles per hour, increase from session to session? Write an argument that can be used to defend your solution.

Session	Time (h)	Distance (mi)
1	$\frac{2}{125}$	$\frac{9}{10}$
2	$\frac{3}{200}$	$\frac{11}{12}$
3	$\frac{3}{250}$	$\frac{17}{20}$

yes; His unit rate for Session 1 was about 56.3 miles per hour, Session 2 was 61.1 miles per hour, and Session 3 was 70.8 miles per hour.

Higher-Order Thinking Problems

11. **Reason Abstractly** Explain why a student who runs $\frac{3}{4}$ mile in 6 minutes is faster than a student who runs $\frac{1}{2}$ mile in 5 minutes.

Sample answer: The student who runs $\frac{3}{4}$ mile in 6 minutes will run 1 mile in 8 minutes if the rate is constant. The student who runs $\frac{1}{2}$ mile in 5 minutes will run 1 mile in 10 minutes if the rate is constant.

12. Compare and contrast the rates $\frac{4}{5}$ *mile in 8 minutes* and *4 minutes to travel* $\frac{2}{5}$ *mile*.

Sample answer: The unit rate $\frac{4}{5}$ *mile in 8 minutes* has a unit rate of $\frac{1}{10}$ mile per minute. The rate *4 minutes to travel* $\frac{2}{5}$ mile has a unit rate of 10 minutes per mile. Both rates are equivalent.

13. **Find the Error** Carli made 9 greeting cards in $\frac{3}{4}$ hour. She determined her unit rate to be $\frac{1}{12}$ card per hour. Find her error and correct it.

Sample answer: She set up her rate as $\frac{\frac{3}{4}\text{ hour}}{9\text{ greeting cards}}$. Her unit rate of $\frac{1}{12}$ is hour per greeting card, not card per hour.

14. **Be Precise** A standard shower drain can drain water at the rate of 480 gallons in $\frac{2}{3}$ hour. Create three different rates, using the same or different units, that are all equivalent to this rate. Be precise in the units you choose. Then find the unit rate, in gallons per minute.

Sample answers: 240 gallons in $\frac{1}{3}$ hour, 240 gallons in 20 minutes, 12 gallons in 1 minute; The unit rate is 12 gallons per minute.

12 Module 1 • Proportional Relationships

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 11, students will reason about two different rates. They will analyze the rates given and explain how one rate is faster than another despite the time being longer. They will understand that, in order to compare rates, the units must be the same.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students will analyze the work of another student to both diagnose and correct the error.

6 Attend to Precision In Exercise 14, students will create three rates that are equivalent to 480 gallons in $\frac{2}{3}$ hour. Encourage them to be careful about specifying the units of measure.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own problem.

Use with Exercises 9–10 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Clearly and precisely explain.

Use with Exercise 11 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of methods, such as using unit rates or bar diagrams, in their responses.

Lesson 1-2

Understand Proportional Relationships

7.RP.A.2

LESSON GOAL

Students will use models and ratio reasoning to understand how a proportional relationship can exist between quantities.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Proportional Relationships
Example 1: Identify Proportional Relationships
Example 2: Identify Proportional Relationships
Apply: Construction

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 2 of the *Language Development Handbook* to help your students build mathematical language related to understanding proportional relationships.

ELL You can use the tips and suggestions on page T2 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7. RP.A** by recognizing and representing proportional relationships between quantities.
Standards for Mathematical Content: 7. RP.A.2
Standards for Mathematical Practice: MP 1, MP2, MP3, MP4, MP6, MP7

Coherence

Vertical Alignment

Previous
Students found unit rates that involved ratios of fractions.
7.RP.A.1

Now
Students use models and ratio reasoning to understand how a proportional relationship can exist between quantities.
7.RP.A.2

Next
Students will analyze the relationship between two quantities represented in tables to determine proportionality.
7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B

Rigor

The Three Pillars of Rigor

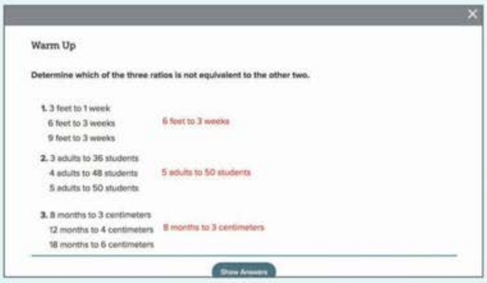
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of proportional relationships and what it means for a relationship between two quantities to be proportional. They build *fluency* with identifying proportional and nonproportional relationships given real-world contexts.

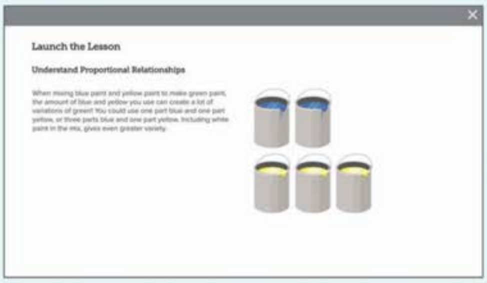
Mathematical Background

Two quantities are in a *proportional relationship* if the two quantities vary and yet have a constant ratio between them. If a recipe calls for 3 cups of flour for every 2 cups of sugar, the relationship between cups of flour and sugar is constant, no matter how many batches are made. Some relationships are not proportional, because a constant ratio is not maintained.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding equivalent ratios (Exercises 1–3)

Answers

1. 6 feet to 3 weeks
2. 5 adults to 50 students
3. 8 months to 3 centimeters

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about proportional relationships between the ratio of blue paint to yellow paint to create a specific color of green.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What are some synonyms of the word *relationship*? Sample answers: association, link, correlation

Learn Proportional Relationships

Objective

Students will understand what makes a relationship between two quantities a proportional relationship.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure A s students discuss the *Talk About It!* question on Slide 2, encourage them to look for the structure of the ratio and what it means to have a ratio be maintained.

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to use number sense to think about whether Pedro will ever be twice as old as his brother again.

Teaching Notes

SLIDE 1

You may wish to ask students if they have used a recipe to double or triple the ingredients to make 2 or 3 batches. Some students may have some experience with this and other students may not be familiar. Present the recipe for pizza dough shown in the Learn. Ask students what the relationship is between cups of flour and cups of water to make one batch of dough. Be sure students can see this ratio in the bar diagram. Ask students why the ratio is maintained when making 2 and 3 batches of the dough.

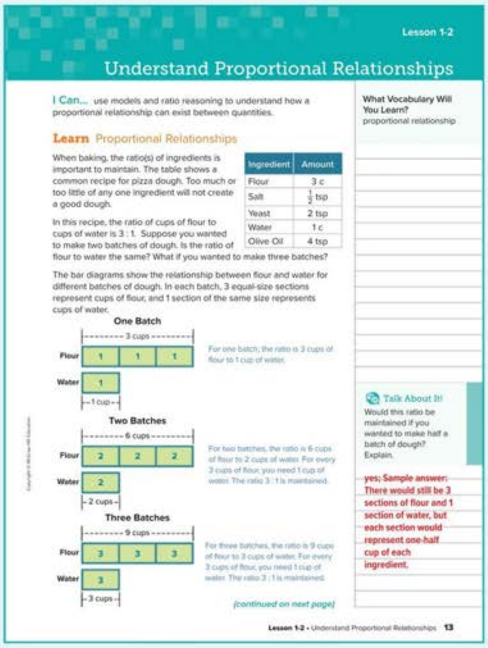
Talk About It!

SLIDE 2

Mathematical Discourse

Would this ratio be maintained if you wanted to make half a batch of dough? Explain. yes; Sample answer: There would still be 3 sections of flour and 1 section of water, but each section would represent one-half cup of each ingredient.

(continued on next page)



Lesson 1-2

Understand Proportional Relationships

I Can... use models and ratio reasoning to understand how a proportional relationship can exist between quantities.

Learn Proportional Relationships

When baking, the ratio(s) of ingredients is important to maintain. The table shows a common recipe for pizza dough. Too much or too little of any one ingredient will not create a good dough.

Ingredient	Amount
Flour	3 c
Salt	$\frac{1}{2}$ tsp
Yeast	2 tsp
Water	1 c
Olive Oil	4 tsp

In this recipe, the ratio of cups of flour to cups of water is 3 : 1. Suppose you wanted to make two batches of dough. Is the ratio of flour to water the same? What if you wanted to make three batches?

The bar diagrams show the relationship between flour and water for different batches of dough. In each batch, 3 equal-size sections represent cups of flour, and 1 section of the same size represents cups of water.

(continued on next page)

What Vocabulary Will You Learn?
proportional relationship

Talk About It!
Would this ratio be maintained if you wanted to make half a batch of dough? Explain.
yes; Sample answer: There would still be 3 sections of flour and 1 section of water, but each section would represent one-half cup of each ingredient.

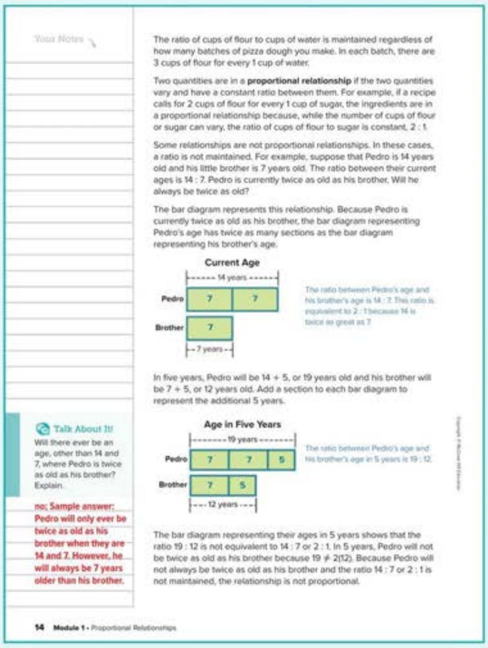
Lesson 1-2 • Understand Proportional Relationships 13

Interactive Presentation

Learn, Proportional Relationships, Slide 1 of 4

CLICK

On Slide 3, students learn how a bar diagram can be used to solve the problem.



Your Notes

The ratio of cups of flour to cups of water is maintained regardless of how many batches of pizza dough you make. In each batch, there are 3 cups of flour for every 1 cup of water.

Two quantities are in a **proportional relationship** if the two quantities vary and have a constant ratio between them. For example, if a recipe calls for 2 cups of flour for every 1 cup of sugar, the ingredients are in a proportional relationship because, while the number of cups of flour or sugar can vary, the ratio of cups of flour to sugar is constant, 2 : 1.

Some relationships are not proportional relationships. In these cases, a ratio is not maintained. For example, suppose that Pedro is 14 years old and his little brother is 7 years old. The ratio between their current ages is 14 : 7. Pedro is currently twice as old as his brother. Will he always be twice as old?

The bar diagram represents this relationship. Because Pedro is currently twice as old as his brother, the bar diagram representing Pedro's age has twice as many sections as the bar diagram representing his brother's age.

In five years, Pedro will be 14 + 5, or 19 years old and his brother will be 7 + 5, or 12 years old. Add a section to each bar diagram to represent the additional 5 years.

The bar diagram representing their ages in 5 years shows that the ratio 19 : 12 is not equivalent to 14 : 7 or 2 : 1. In 5 years, Pedro will not be twice as old as his brother because 19 ≠ 2(12). Because Pedro will not always be twice as old as his brother and the ratio 14 : 7 or 2 : 1 is not maintained, the relationship is not proportional.

Talk About It! Will there ever be an age, other than 14 and 7, where Pedro is twice as old as his brother? Explain.

no; Sample answer: Pedro will only ever be twice as old as his brother when they are 14 and 7. However, he will always be 7 years older than his brother.

14 Module 1 • Proportional Relationships

Interactive Presentation

Learn, Proportional Relationships, Slide 3 of 4

7.RP.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Proportional Relationships *(continued)*

Teaching Notes

SLIDE 3

Point out that not all relationships that exist between two quantities are proportional. After presenting the relationship of Pedro's age to his brother's age, ask students if they can think of other real-world relationships that are not proportional. For example, suppose you currently have twice as many pens as pencils in your desk drawer, because you have 4 pens and 2 pencils. If you add 3 pens and 3 pencils to your collection, you will now have 7 pens and 5 pencils, and you no longer have twice as many pens as pencils. This relationship is not proportional because the ratio 2 : 1 is not maintained.

Talk About It!

SLIDE 4

Mathematical Discourse

Will there ever be an age, other than 14 and 7, where Pedro is twice as old as his brother? Explain. no; Sample answer: Pedro will only ever be twice as old as his brother when they are 14 and 7. However, he will always be 7 years older than his brother.

DIFFERENTIATE

Language Development Activity ELL

To further students understanding of the term proportional, write the Spanish terms *proporciónal* and *proporción* the board. You may wish to have your students who are native Spanish speakers or who are studying Spanish explain what these terms mean in English. The Spanish term *proporciónal* means *proportional* and the Spanish term *proporción* means *ratio*. Even for students who are not native Spanish speakers or who are not studying Spanish, understanding what these Spanish terms mean will allow them to have a greater understanding that there is a connection between ratios and proportional relationships. In order for a relationship to be proportional, there must be a constant ratio.

Example 1 Identify Proportional Relationships

Objective

Students will identify whether a relationship is proportional by determining if the ratio between the two quantities is maintained.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the quantities given in the problem in order to determine if the ratio would be maintained.

Questions for Mathematical Discourse

SLIDE 2

AL What is the ratio of vinegar to water in the recipe for homemade glass cleaner? 1 : 4

AL What must be maintained in order for a relationship to be considered proportional? Explain. the ratio between the quantities; Sample answer: The ratio between parts of vinegar to parts of water must be constant in order for the relationship to be considered proportional.

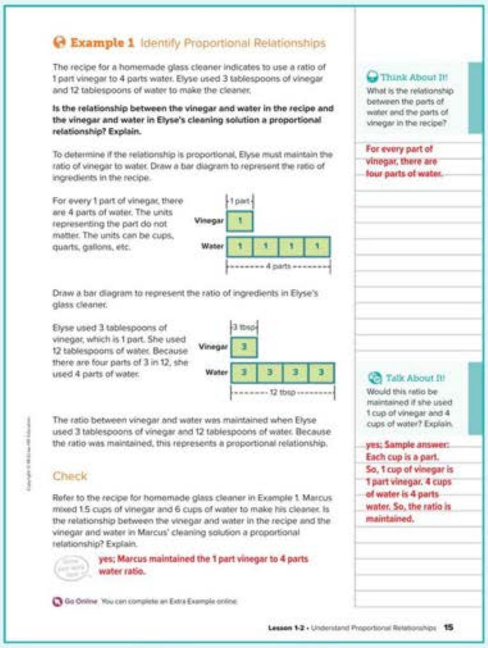
OL Why is each section in the bar diagram representing the recipe labeled with the number 1? Each section represents a part, and the recipe indicates to use 1 part vinegar to 4 parts water, so the bar diagram represents 1 section of vinegar to 4 sections of water.

OL Why is each section in the bar diagram representing Elyse's glass cleaner labeled with the number 3? Elyse used 3 tablespoons of vinegar and 3 × 4, or 12 tablespoons of water. So, each section represents 3 tablespoons.

BL Why does it not matter what unit of measure the parts represent? Sample answer: The ratio between vinegar and water is the ratio of part to part, which doesn't specify the unit of measure. The ratio describes how much of one quantity in relation to how much of another quantity.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Identify Proportional Relationships

The recipe for a homemade glass cleaner indicates to use a ratio of 1 part vinegar to 4 parts water. Elyse used 3 tablespoons of vinegar and 12 tablespoons of water to make the cleaner.

Is the relationship between the vinegar and water in the recipe and the vinegar and water in Elyse's cleaning solution a proportional relationship? Explain.

To determine if the relationship is proportional, Elyse must maintain the ratio of vinegar to water. Draw a bar diagram to represent the ratio of ingredients in the recipe.

For every 1 part of vinegar, there are 4 parts of water. The units representing the part do not matter. The units can be cups, quarts, gallons, etc.

Draw a bar diagram to represent the ratio of ingredients in Elyse's glass cleaner.

Elyse used 3 tablespoons of vinegar, which is 1 part. She used 12 tablespoons of water. Because there are four parts of 3 in 12, she used 4 parts of water.

The ratio between vinegar and water was maintained when Elyse used 3 tablespoons of vinegar and 12 tablespoons of water. Because the ratio was maintained, this represents a proportional relationship.

Think About It! What is the relationship between the parts of water and the parts of vinegar in the recipe?

For every part of vinegar, there are four parts of water.

Talk About It! Would this ratio be maintained if she used 1 cup of vinegar and 4 cups of water? Explain.

yes; Sample answer: Each cup is a part. So, 1 cup of vinegar is 1 part vinegar. 4 cups of water is 4 parts water. So, the ratio is maintained.

Check

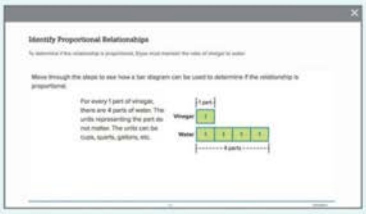
Refer to the recipe for homemade glass cleaner in Example 1. Marcus mixed 1.5 cups of vinegar and 6 cups of water to make his cleaner. Is the relationship between the vinegar and water in the recipe and the vinegar and water in Marcus' cleaning solution a proportional relationship? Explain.

yes; Marcus maintained the 1 part vinegar to 4 parts water ratio.

Go Online You can complete an Extra Example online.

Lesson 1-2 • Understand Proportional Relationships 15

Interactive Presentation



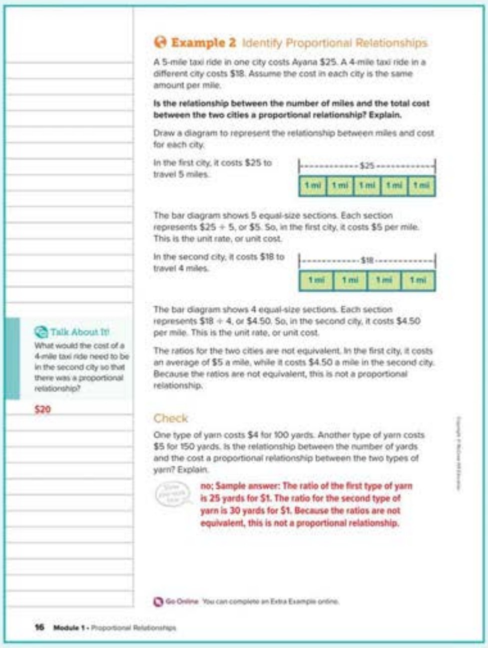
Example 1, Identify Proportional Relationships, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to see how a bar diagram can be used to represent the ratio of ingredients for each cleaner.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 2 Identify Proportional Relationships

A 5-mile taxi ride in one city costs Ayana \$25. A 4-mile taxi ride in a different city costs \$18. Assume the cost in each city is the same amount per mile.

Is the relationship between the number of miles and the total cost between the two cities a proportional relationship? Explain.

Draw a diagram to represent the relationship between miles and cost for each city.

In the first city, it costs \$25 to travel 5 miles.

The bar diagram shows 5 equal-size sections. Each section represents \$25 ÷ 5, or \$5. So, in the first city, it costs \$5 per mile. This is the unit rate, or unit cost.

In the second city, it costs \$18 to travel 4 miles.

The bar diagram shows 4 equal-size sections. Each section represents \$18 ÷ 4, or \$4.50. So, in the second city, it costs \$4.50 per mile. This is the unit rate, or unit cost.

The ratios for the two cities are not equivalent. In the first city, it costs an average of \$5 a mile, while it costs \$4.50 a mile in the second city. Because the ratios are not equivalent, this is not a proportional relationship.

Talk About It!
What would the cost of a 4-mile taxi ride need to be in the second city so that there was a proportional relationship?

\$20

Check

One type of yarn costs \$4 for 100 yards. Another type of yarn costs \$5 for 150 yards. Is the relationship between the number of yards and the cost a proportional relationship between the two types of yarn? Explain.

no; Sample answer: The ratio of the first type of yarn is 25 yards for \$1. The ratio for the second type of yarn is 30 yards for \$1. Because the ratios are not equivalent, this is not a proportional relationship.

Go Online You can complete an Extra Example online.

16 Module 1 • Proportional Relationships

Interactive Presentation

Example 2, Identify Proportional Relationships, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to see how a bar diagram can be used to represent the ratio of cost to miles.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 2 Identify Proportional Relationships

Objective

Students will identify whether a relationship is proportional by determining if the ratio between the two quantities is maintained.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 3, encourage them to consider equivalent ratios when determining the cost of a 4-mile taxi ride.

Questions for Mathematical Discourse

SLIDE 2

AL Why does the first bar diagram show five equal sections? The cost of the taxi ride is for five miles, and each mile is the same distance, so the diagram shows five equal sections.

OL How can you use a ratio table to check your answer? Sample answer: The first column of the ratio table would show the ratio \$25 to 5 miles. I can scale backward to \$5 to 1 mile. Repeating the same step for the second city, I can see that \$18 to 4 miles scales backward to \$4.50 to 1 mile. The ratios are not equivalent.

BL Suppose a taxi ride in a third city cost \$28 for 7 miles. How can you use mental math to determine if the relationship between the number of miles and the total cost is a proportional relationship with either the first city or the second city? Sample answer: The ratio \$28 to 7 miles is equivalent to \$4 to 1 mile. Neither city has this same ratio, so a proportional relationship does not exist with either city.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Construction

Objective

Students will come up with their own strategy to solve an application problem involving constructing a deck.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

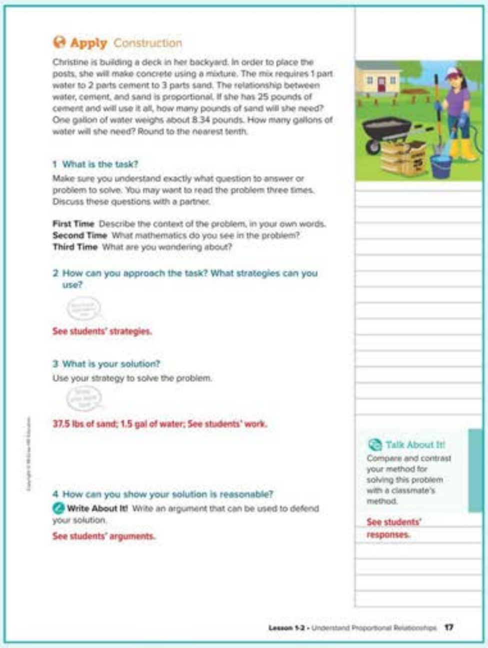
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the ratio of cement to sand?
- What is the ratio of water to cement?
- How can you use a ratio table to help you solve this problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Construction

Christine is building a deck in her backyard. In order to place the posts, she will make concrete using a mixture. The mix requires 1 part water to 2 parts cement to 3 parts sand. The relationship between water, cement, and sand is proportional. If she has 25 pounds of cement and will use it all, how many pounds of sand will she need? One gallon of water weighs about 8.34 pounds. How many gallons of water will she need? Round to the nearest tenth.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

37.5 lbs of sand; 1.5 gal of water; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Compare and contrast your method for solving this problem with a classmate's method.

See students' responses.

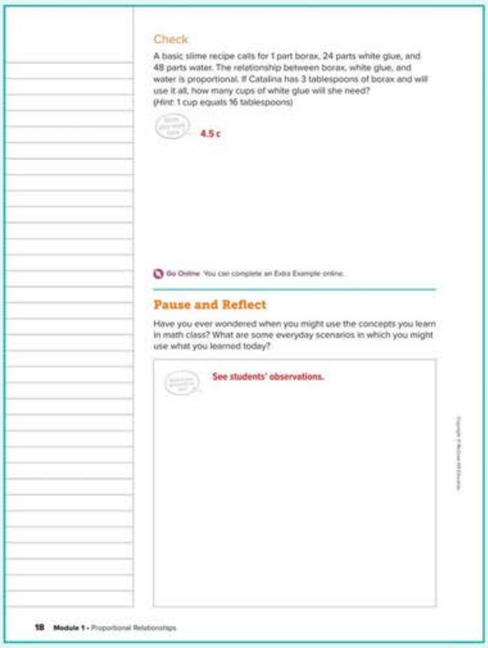
Lesson 1-2 • Understand Proportional Relationships 17

Interactive Presentation

Apply, Construction

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

A basic slime recipe calls for 1 part borax, 24 parts white glue, and 48 parts water. The relationship between borax, white glue, and water is proportional. If Catalina has 3 tablespoons of borax and will use it all, how many cups of white glue will she need? (Hint: 1 cup equals 16 tablespoons)

4.5 c

Go Online You can complete an Extra Example online.

Pause and Reflect

Have you ever wondered when you might use the concepts you learn in math class? What are some everyday scenarios in which you might use what you learned today?

See students' observations.

18 Module 1 • Proportional Relationships

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

What does it mean for two quantities to be in a proportional relationship?

In this lesson, students learned how to determine if situations represented proportional relationships. Encourage them to discuss with a partner why they must check for proportionality by checking for equivalent ratios.

Exit Ticket

Refer to the Exit Ticket slide. Suppose one person used 2 parts blue paint to 3 parts yellow paint. Another person used 4 parts blue paint to 6 parts yellow paint. Do the ratios of blue paint to yellow paint form a proportional relationship? Explain. yes; The ratio of blue paint to yellow paint was maintained at 2 parts blue paint to 3 parts yellow paint.

ASSESS AND DIFFERENTIATE

Use the data from the Checks to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7, 9–13
- ALEKS® Proportions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 9, 10, 12
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–2
- ALEKS® Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS® Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

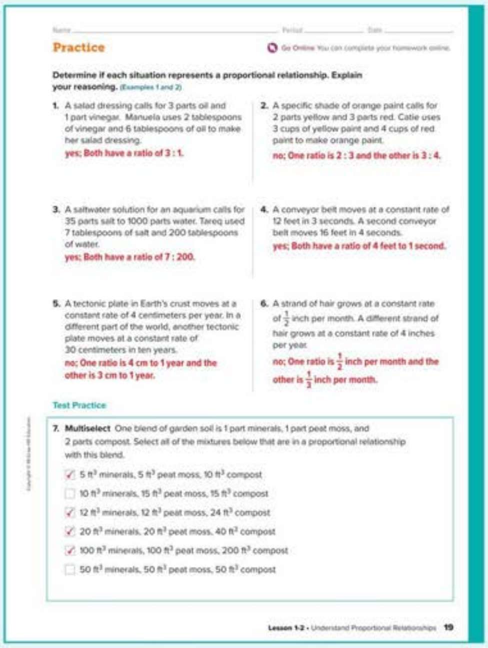
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	determine if each situation represents a proportional relationship	1–6
2	extend concepts learned in class to apply them to new contexts	7
3	solve application problems involving proportional relationships	8, 9
3	higher-order and critical thinking skills	10–13

Common Misconception

Remind students that, when comparing unit rates to determine if a proportional relationship exists, the second quantity must be the same unit as the first quantity. For example, in Exercise 6, rates of "per month" and "per year" cannot be compared. One quantity must be rewritten in the same units as the other quantity, either both *months* or *years*.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Determine if each situation represents a proportional relationship. Explain your reasoning. (Examples 1 and 2)

1. A salad dressing calls for 3 parts oil and 1 part vinegar. Manuela uses 2 tablespoons of vinegar and 6 tablespoons of oil to make her salad dressing.
yes; Both have a ratio of 3 : 1.

2. A specific shade of orange paint calls for 2 parts yellow and 3 parts red. Catie uses 3 cups of yellow paint and 4 cups of red paint to make orange paint.
no; One ratio is 2 : 3 and the other is 3 : 4.

3. A saltwater solution for an aquarium calls for 35 parts salt to 1000 parts water. Tareq used 7 tablespoons of salt and 200 tablespoons of water.
yes; Both have a ratio of 7 : 200.

4. A conveyor belt moves at a constant rate of 12 feet in 3 seconds. A second conveyor belt moves 16 feet in 4 seconds.
yes; Both have a ratio of 4 feet to 1 second.

5. A tectonic plate in Earth's crust moves at a constant rate of 4 centimeters per year. In a different part of the world, another tectonic plate moves at a constant rate of 30 centimeters in ten years.
no; One ratio is 4 cm to 1 year and the other is 3 cm to 1 year.

6. A strand of hair grows at a constant rate of $\frac{1}{2}$ inch per month. A different strand of hair grows at a constant rate of 4 inches per year.
no; One ratio is $\frac{1}{2}$ inch per month and the other is $\frac{1}{3}$ inch per month.

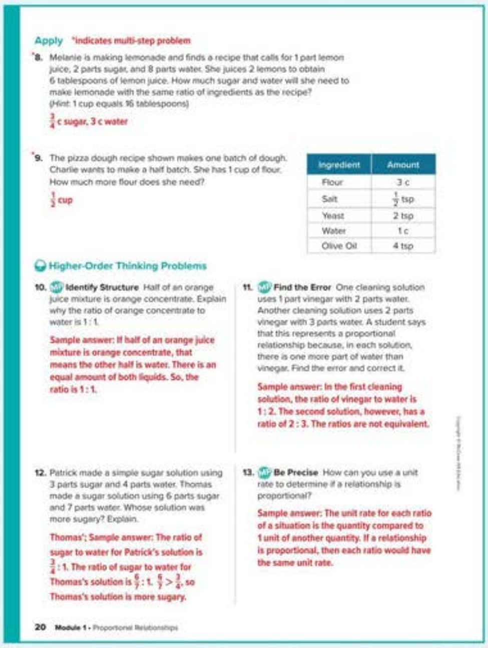
Test Practice

7. **Multiselect** One blend of garden soil is 1 part minerals, 1 part peat moss, and 2 parts compost. Select all of the mixtures below that are in a proportional relationship with this blend.

- [x] 5 ft^3 minerals, 5 ft^3 peat moss, 10 ft^3 compost
- [ ] 10 ft^3 minerals, 15 ft^3 peat moss, 15 ft^3 compost
- [x] 12 ft^3 minerals, 12 ft^3 peat moss, 24 ft^3 compost
- [x] 20 ft^3 minerals, 20 ft^3 peat moss, 40 ft^3 compost
- [x] 100 ft^3 minerals, 100 ft^3 peat moss, 200 ft^3 compost
- [ ] 50 ft^3 minerals, 50 ft^3 peat moss, 50 ft^3 compost

Lesson 1-2 • Understand Proportional Relationships 19

1 CONCEPTUAL UNDERSTANDING	**2 FLUENCY**	**3 APPLICATION**



Apply *indicates multi-step problem

*8. Melanie is making lemonade and finds a recipe that calls for 1 part lemon juice, 2 parts sugar, and 8 parts water. She juices 2 lemons to obtain 6 tablespoons of lemon juice. How much sugar and water will she need to make lemonade with the same ratio of ingredients as the recipe? (Hint: 1 cup equals 16 tablespoons)

$\frac{3}{4}$ c sugar, 3 c water

*9. The pizza dough recipe shown makes one batch of dough. Charlie wants to make a half batch. She has 1 cup of flour. How much more flour does she need?

$\frac{1}{2}$ cup

Ingredient	Amount
Flour	3 c
Salt	$\frac{1}{2}$ tsp
Yeast	2 tsp
Water	1 c
Olive Oil	4 tsp

Higher-Order Thinking Problems

10. MP Identify Structure Half of an orange juice mixture is orange concentrate. Explain why the ratio of orange concentrate to water is 1 : 1.

Sample answer: If half of an orange juice mixture is orange concentrate, that means the other half is water. There is an equal amount of both liquids. So, the ratio is 1 : 1.

11. MP Find the Error One cleaning solution uses 1 part vinegar with 2 parts water. Another cleaning solution uses 2 parts vinegar with 3 parts water. A student says that this represents a proportional relationship because, in each solution, there is one more part of water than vinegar. Find the error and correct it.

Sample answer: In the first cleaning solution, the ratio of vinegar to water is 1 : 2. The second solution, however, has a ratio of 2 : 3. The ratios are not equivalent.

12. Patrick made a simple sugar solution using 3 parts sugar and 4 parts water. Thomas made a sugar solution using 6 parts sugar and 7 parts water. Whose solution was more sugary? Explain.

Thomas'; Sample answer: The ratio of sugar to water for Patrick's solution is $\frac{3}{4}$: 1. The ratio of sugar to water for Thomas's solution is $\frac{6}{7}$: 1. $\frac{6}{7} > \frac{3}{4}$, so Thomas's solution is more sugary.

13. MP Be Precise How can you use a unit rate to determine if a relationship is proportional?

Sample answer: The unit rate for each ratio of a situation is the quantity compared to 1 unit of another quantity. If a relationship is proportional, then each ratio would have the same unit rate.

20 Module 1 • Proportional Relationships

Teaching the Mathematical Practices

7 Look For and Make Use of Structure In Exercise 10, students use their knowledge of the structure of ratios to explain how the ratio 1 : 1 describes the mixture.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students determine the error in a student's reasoning and explain how to correct it.

6 Attend to Precision In Exercise 13, students use precise mathematical language, including the definition of *unit rate*, to explain how to use unit rate when determining if a relationship is proportional.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercise 8 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Solve the problem another way.
Use with Exercise 12 Have students work in groups of 3–4. After completing Exercise 12, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is viable. If the methods were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class and explain why each method is a correct method.

Lesson 1-3

Tables of Proportional Relationships

7.RP.A.2

LESSON GOAL

Students will analyze the relationship between two quantities represented in tables to determine proportionality.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Ratios in Tables

Learn: Proportional Relationships and Tables
Example 1: Proportional Relationships and Tables
Example 2: Proportional Relationships and Tables
Learn: Identify the Constant of Proportionality
Example 3: Identify the Constant of Proportionality
Example 4: Identify the Constant of Proportionality
Apply: Sales Tax

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Constant Rate of Change-Tables		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 3 of the *Language Development Handbook* to help your students build mathematical language related to tables of proportional relationships.

ELL You can use the tips and suggestions on page T3 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7. RP.A** by analyzing the relationship between two quantities shown in tables to determine proportionality.
Standards for Mathematical Content: **7 .RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students used models and ratio reasoning to understand how a proportional relationships can exist between quantities.
7.RP.A.2

Now
Students analyze the relationship between two quantities represented in tables to determine proportionality.
7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B

Next
Students will analyze the relationship between two quantities graphed on a coordinate plane to determine proportionality.
7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B, 7.RP.A.2.D

Rigor

The Three Pillars of Rigor

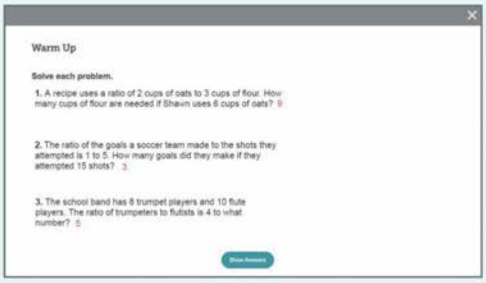
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of proportional relationships and how to identify the constant of proportionality from a table. They use the tables to build *fluency* with identifying the constant of proportionality, and relate it to unit rate.

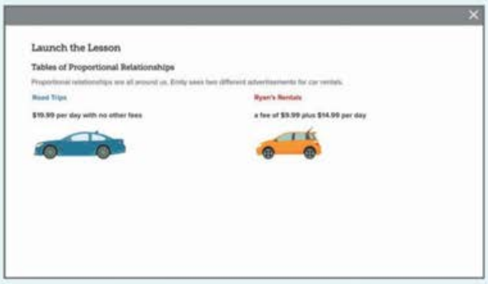
Mathematical Background

Two quantities are in a *proportional relationship* if the two quantities vary and yet have a constant ratio between them. In relationships where these ratios are not equivalent, the two quantities are *nonproportional*. You can check for equivalent ratios by using a table. If all of the ratios have the same unit rate, then the relationship is proportional.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- using ratio reasoning to solve real-world problems (Exercises 1–3)

Answers

1. 9
2. 3
3. 5

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about proportional relationships between two car rental companies.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *proportional* is made up of the suffix *–al* and what other term? proportion
- What does the prefix *non-* mean ? Sample answer: *Non-* means not.
- What is the everyday meaning of the word *constant*? Sample answer: Constant means not changing or varying.

Explore Ratios in Tables

Objective

Students will use a table to explore how to determine if the ratios between two quantities are equivalent.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with real-world scenarios involving the cost of different numbers of pizzas. Throughout this activity, students will find the relationship between the total cost and the number of pizzas. They will compare the ratios in two scenarios.

Inquiry Question

How can organizing information in a table help you determine if the ratios between two quantities are equivalent? Sample answer: I can write the ratios showing the relationships in the tables. I can then compare the ratios and see if they are equivalent.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

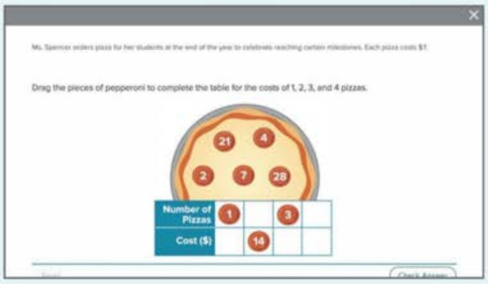
Mathematical Discourse

What method did you use to compare the two values? What do you notice about the comparisons? Sample answer: The ratios are equivalent. See students' methods.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

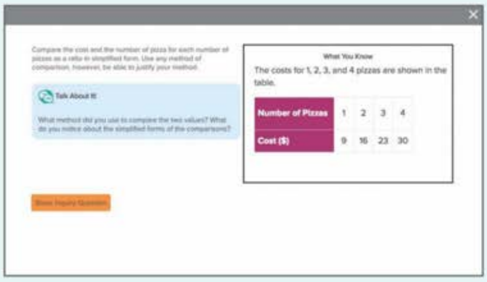


Explore, Slide 2 of 7

DRAG & DROP

On Slide 2, students drag the pieces of pepperoni to complete the table for the costs of 1, 2, 3, and 4 pizzas.

Interactive Presentation



Explore, Slide 5 of 7

DRAG & DROP

On Slide 4, students drag the pieces of pepperoni to complete the table for the costs of 1, 2, 3, and 4 pizzas including the delivery charge.

TYPE

On Slide 6, students respond to a question about why the ratios are equivalent in the first scenario but not in the second.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Ratios in T ables *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to explore how using a table to represent the ratios can help them determine if the ratios are equivalent. Students should notice and understand what happens to the ratios in the table when a delivery charge is added.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What method did you use to compare the two values? What do you notice about the comparisons? Sample answer: The ratios are not equivalent. See students' methods.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Proportional Relationships and Tables

Objective

Students will learn how to identify a proportional relationship from a table.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to clearly explain that all ratios shown in a table must simplify to the same unit rate for the relationship to be proportional.

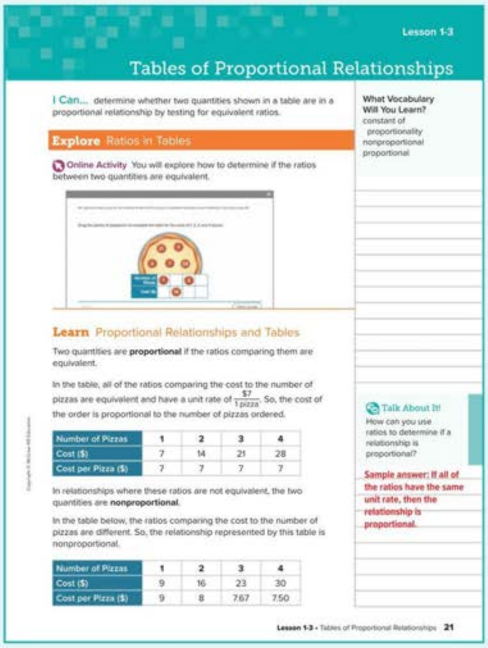
Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

How can you use ratios to determine if a relationship is proportional? Sample answer: If all of the ratios have the same unit rate, then the relationship is proportional.



Lesson 1-3

Tables of Proportional Relationships

I Can... determine whether two quantities shown in a table are in a proportional relationship by testing for equivalent ratios.

What Vocabulary Will You Learn?
constant of proportionality
nonproportional
proportional

Explore Ratios in Tables

Online Activity You will explore how to determine if the ratios between two quantities are equivalent.

Learn Proportional Relationships and Tables

Two quantities are **proportional** if the ratios comparing them are equivalent.

In the table, all of the ratios comparing the cost to the number of pizzas are equivalent and have a unit rate of $\frac{\$7}{1 \text{ pizza}}$. So, the cost of the order is proportional to the number of pizzas ordered.

Number of Pizzas	1	2	3	4
Cost ($)	7	14	21	28
Cost per Pizza ($)	7	7	7	7

In relationships where these ratios are not equivalent, the two quantities are **nonproportional**.

In the table below, the ratios comparing the cost to the number of pizzas are different. So, the relationship represented by this table is nonproportional.

Number of Pizzas	1	2	3	4
Cost ($)	9	16	23	30
Cost per Pizza ($)	9	8	7.67	7.50

Talk About It!
How can you use ratios to determine if a relationship is proportional?
Sample answer: If all of the ratios have the same unit rate, then the relationship is proportional.

Lesson 1-3 • Tables of Proportional Relationships 21

DIFFERENTIATE

Reteaching Activity AL

If students are struggling to determine if relationships are proportional or nonproportional, have them first identify pairs from the table. Then calculate the unit ratios. Have students complete the following simplified exercise.

Number of Chapters	1	2	3	4
Pages	8	16	24	32

What is the unit ratio of pages to chapters based on the first chapter-page pair?

$\frac{8 \text{ pages}}{1 \text{ chapter}}$

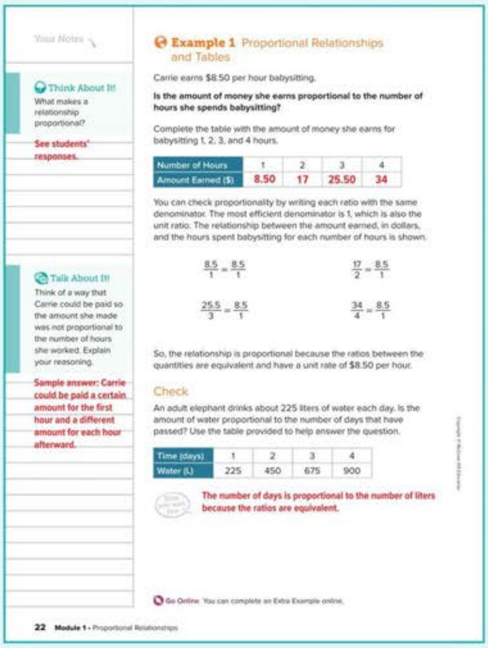
What is the unit ratio of pages to chapters based on the second chapter-page pair?

$\frac{16 \text{ pages}}{2 \text{ chapters}} = \frac{8 \text{ pages}}{1 \text{ chapter}}$

The number of chapters and number of pages are proportional if all of the unit ratios are equal. Check the other unit ratios. Are the two quantities proportional? yes

Interactive Presentation

Learn, Proportional Relationships and Tables, Slide 1 of 3



Your Notes

Think About It! What makes a relationship proportional?

See students' responses.

Talk About It! Think of a way that Carrie could be paid so the amount she made was not proportional to the number of hours she worked. Explain your reasoning.

Sample answer: Carrie could be paid a certain amount for the first hour and a different amount for each hour afterward.

Example 1 Proportional Relationships and Tables

Carrie earns $8.50 per hour babysitting.

Is the amount of money she earns proportional to the number of hours she spends babysitting?

Complete the table with the amount of money she earns for babysitting 1, 2, 3, and 4 hours.

Number of Hours	1	2	3	4
Amount Earned ($)	8.50	17	25.50	34

You can check proportionality by writing each ratio with the same denominator. The most efficient denominator is 1, which is also the unit ratio. The relationship between the amount earned, in dollars, and the hours spent babysitting for each number of hours is shown.

$\frac{8.5}{1} = \frac{8.5}{1}$ $\frac{17}{2} = \frac{8.5}{1}$

$\frac{25.5}{3} = \frac{8.5}{1}$ $\frac{34}{4} = \frac{8.5}{1}$

So, the relationship is proportional because the ratios between the quantities are equivalent and have a unit rate of $8.50 per hour.

Check

An adult elephant drinks about 225 liters of water each day. Is the amount of water proportional to the number of days that have passed? Use the table provided to help answer the question.

Time (days)	1	2	3	4
Water (L)	225	450	675	900

The number of days is proportional to the number of liters because the ratios are equivalent.

Go Online You can complete an Extra Example online.

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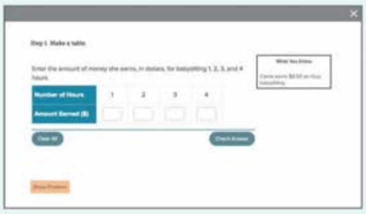
Interactive Presentation



Example 1, Proportional Relationships and Tables, Slide 2 of 5

TYPE

On Slide 2, students complete a table to show the amount of money earned for babysitting different amounts of time.

FLASHCARDS

On Slide 3, students use Flashcards to find the ratios of amount earned and time spent babysitting.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Proportional Relationships and Tables

Objective

Students will identify a proportional relationship from a table.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the quantities given in the problem, and understand the connection between the ratios and what it means for a relationship to be proportional.

6 Attend to Precision Encourage students to understand that all, not one or two, of the ratios must have the same unit rate in order for the relationship to be proportional. Students should carefully examine each of the ratios accurately.

Questions for Mathematical Discourse

SLIDE 2

AL What are you trying to determine? if the money earned is proportional to the number of hours spent babysitting

AL What two quantities are you comparing? money earned to hours spent babysitting

OL How will you determine if the ratios are proportional? Determine if all ratios are equivalent to the unit rate.

BL Would the amount earned change if she was paid $10 for coming over, and then $5.25 an hour? Would the money earned be proportional to the hours spent babysitting? Explain. Sample answer: The amount earned would change. She would earn $15.25 for 1 hour, $20.50 for 2 hours, $25.75 for 3 hours, and $31.00 for 4 hours. The relationship would not be proportional.

SLIDE 3

AL Look at each ratio. How do the denominators compare to one another? How do the numerators compare to one another? Sample answer: The denominators are 1, 2, 3, and 4. The numerators are multiples of 8.5 (8.5, 17, 25.5, and 34).

OL Is the relationship proportional? Explain. yes; All of the ratios are equivalent to the unit rate, $8.50 per hour.

BL Can you determine Carrie's earnings for 5, 8, and 10 hours spent babysitting? Explain. yes; Sample answer: Because the relationship is proportional, I can multiply any number of hours she works by the unit rate, $8.50 per hour, to determine her earnings; $42.50, $68, $85.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Proportional Relationships and Tables

Objective

Students will identify a nonproportional relationship from a table.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the quantities given in the problem in order to set up each of the ratios, and understand the connection between the ratios and what it means for a relationship to be proportional.

6 Attend to Precision Encourage students to understand that all, not some, of the ratios must have the same unit rate in order for the relationship to be proportional. Students should carefully examine each of the ratios.

Questions for Mathematical Discourse

SLIDE 2

AL What are you trying to determine? if the cost of an order is proportional to the number of tickets ordered

OL How will you use the table to determine if the relationship is proportional? Determine whether each ratio is equivalent to the unit rate.

BL If seven tickets are purchased, what is the total cost of the order? $181

SLIDE 3

AL What is the cost of 1 ticket? $31

OL Is the relationship proportional? Explain. No; the ratios are not equivalent.

BL Write an expression that represents the cost, in dollars, of buying x tickets. $25x + 6$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Proportional Relationships and Tables

A ticket agency charges a $6 service fee on any order. Each ticket for a concert costs $25.

Is the cost of an order proportional to the number of tickets ordered?

The table shows the relationship between the total cost of an order and the number of tickets ordered.

Number of Tickets	1	2	3	4
Total Cost of Order ($)	31	56	81	106

The cost of each additional ticket is an additional $25 because the service fee is only charged once per order.

For each number of tickets, write the relationship of the total cost to the number of tickets as a ratio with a denominator of 1. The first two are done for you.

$\frac{31}{1} = \frac{31}{1}$ $\frac{56}{2} = \frac{28}{1}$

$\frac{81}{3} = \frac{27}{1}$ $\frac{106}{4} = \frac{26.50}{1}$

Because the ratios between the quantities are not the same, the cost of an order is not proportional to the number of tickets ordered.

Think About It! How would you begin solving the problem? See students' responses.

Talk About It! What part of the scenario made the situation nonproportional? Explain your reasoning. the service fee; Sample answer: The $6 service fee results in each ratio having a different numerator when the denominator is 1.

Check

The table shows how long it took Maria to run laps around the school track. Is the number of laps she ran proportional to the time it took her? Explain.

Laps	2	4	6
Time (s)	150	320	580

no; Sample answer: The ratios of number of laps to the time are not equivalent. $\frac{2}{150} \neq \frac{4}{320} \neq \frac{6}{580}$

Go Online You can complete an Extra Example online.

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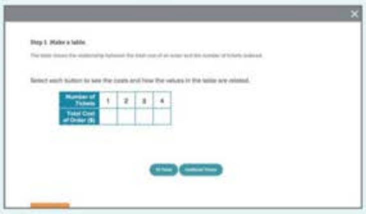
Interactive Presentation

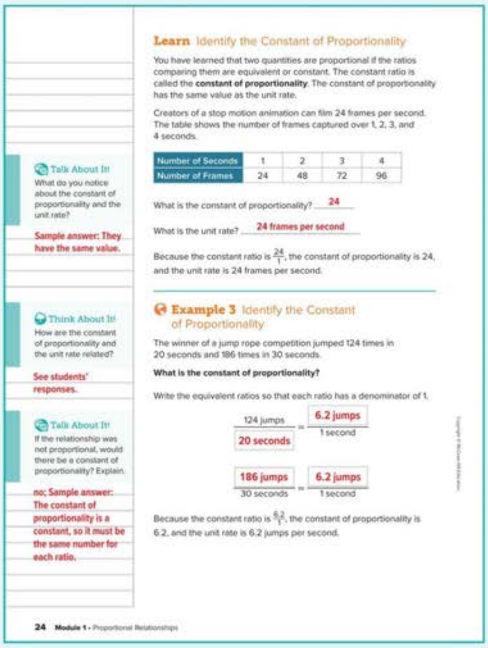


Example 2, Proportional Relationships and Tables, Slide 2 of 5

CLICK On Slide 2, students select buttons to see how the quantities in the table are related.

TYPE On Slide 3, students write ratios as unit ratios.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Learn Identify the Constant of Proportionality

You have learned that two quantities are proportional if the ratios comparing them are equivalent or constant. The constant ratio is called the **constant of proportionality**. The constant of proportionality has the same value as the unit rate.

Creators of a stop motion animation can film 24 frames per second. The table shows the number of frames captured over 1, 2, 3, and 4 seconds.

Number of Seconds	1	2	3	4
Number of Frames	24	48	72	96

What is the constant of proportionality? 24

What is the unit rate? 24 frames per second

Because the constant ratio is $\frac{24}{1}$, the constant of proportionality is 24, and the unit rate is 24 frames per second.

Talk About It! What do you notice about the constant of proportionality and the unit rate? Sample answer: They have the same value.

Example 3 Identify the Constant of Proportionality

The winner of a jump rope competition jumped 124 times in 20 seconds and 186 times in 30 seconds.

What is the constant of proportionality?

Write the equivalent ratios so that each ratio has a denominator of 1.

$\frac{124 \text{ jumps}}{20 \text{ seconds}} = \frac{6.2 \text{ jumps}}{1 \text{ second}}$

$\frac{186 \text{ jumps}}{30 \text{ seconds}} = \frac{6.2 \text{ jumps}}{1 \text{ second}}$

Because the constant ratio is $\frac{6.2}{1}$, the constant of proportionality is 6.2, and the unit rate is 6.2 jumps per second.

Think About It! How are the constant of proportionality and the unit rate related? See students' responses.

Talk About It! If the relationship was not proportional, would there be a constant of proportionality? Explain. no; Sample answer: The constant of proportionality is a constant, so it must be the same number for each ratio.

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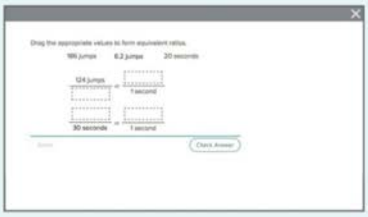
Interactive Presentation



Example 3, Identify the Constant of Proportionality, Slide 2 of 4

EXPAND

On Slide 1 of the Learn, students expand to compare their responses to the correct answers.

DRAG & DROP

On Slide 2 of Example 3, students drag appropriate values to form equivalent ratios.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Identify the Constant of Proportionality

Objective

Students will learn how to find the constant of proportionality from a table or verbal description.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Find a sample answer for the *Talk About It!* question.

Example 3 Identify the Constant of Proportionality

Objective

Students will find the constant of proportionality from a verbal description.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the constant of proportionality and what it means in order to understand and be able to explain why a nonproportional relationship does not have a constant of proportionality.

6 Attend to Precision Encourage students to understand that the constant of proportionality and unit rate have the same value. They should be able to identify the constant of proportionality accurately and efficiently.

Questions for Mathematical Discourse

SLIDE 2

AL What are you trying to find? the constant of proportionality

AL What does the term *constant of proportionality* mean? It is the constant ratio or unit rate between two variable quantities in a proportional relationship.

OL How can you determine the constant of proportionality? Determine if the ratios are equivalent. The constant ratio is the constant of proportionality.

BL Generate a jump rate that has the same unit rate as those in this example. Sample answer: 62 jumps in 10 seconds

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Identify the Constant of Proportionality

Objective

Students will find the constant of proportionality from a table.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to be able to accurately and efficiently identify the constant of proportionality by writing all of the ratios accurately. Students should understand that the unit rate and the constant of proportionality have the same value.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the denominators in each ratio? They are all multiples of 5.

AL What do you notice about the numerators in each ratio? They are all multiples of 9.

OL Are all of the ratios equivalent? What is the constant of proportionality? Yes, the ratios are equivalent. The constant of proportionality is 1.8.

BL How far will the lava flow in 25 seconds? 45 meters

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Students may wonder why the terms *constant of proportionality* and *unit rate* refer to the same value, yet are two different vocabulary terms. Encourage them to consider the context in which each term applies. When talking about the equation of a proportional relationship being in the form $y = kx$, the term *constant of proportionality* is most often used to represent k, although it is not incorrect to use the term *unit rate*. In this context, the units are not typically used, as the value of k is a numerical constant. The term *unit rate* is most often used when given a rate and asked to find how many of one quantity there are per 1 unit of the second quantity. In this context, the unit rate usually includes the units of each quantity.

Have students work with a partner to create a graphic organizer comparing the terms *constant of proportionality* and *unit rate*, including real-world examples of each. Ask them to share their graphic organizers with another pair of students.

Check

The cost of a birthday party at a skating rink is proportional to the number of guests. The skating rink charges $82.50 for 10 guests.

The choices show the number of guests and the total cost for four different parties that were hosted at different locations. Select each party that was hosted with the same constant of proportionality, or unit rate, as the skating rink's unit rate.

- [x] 12 guests for $99.00
- [ ] 8 guests for $52.00
- [ ] 15 guests for $97.50
- [x] 6 guests for $49.50

Go Online You can complete an Extra Example online.

Example 4 Identify the Constant of Proportionality

After a volcanic eruption, lava flows down the slopes of the volcano. The distance the lava flows is proportional to the time.

Time (min)	Distance (m)
5	9
10	18
15	27
20	36

What is the constant of proportionality of the flow of lava?

For each time, find the ratio $\frac{\text{number of meters}}{\text{number of seconds}}$ and rewrite it with a denominator of 1.

$$\frac{9}{5} = \frac{1.8}{1} \qquad \frac{18}{10} = \frac{1.8}{1}$$

$$\frac{27}{15} = \frac{1.8}{1} \qquad \frac{36}{20} = \frac{1.8}{1}$$

Because the constant ratios are $\frac{1.8}{1}$, the constant of proportionality is 1.8, and the unit rate is 1.8 meters per second.

Think About It! Why is a table a good way to organize the information?

See students' responses.

Talk About It! What does the constant of proportionality, 1.8, mean in the context of the problem?

Sample answer: The lava flows 1.8 meters per second.

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Interactive Presentation

Example 4, Identify the Constant of Proportionality, Slide 2 of 4

FLASHCARDS

On Slide 2, students use Flashcards to check their answers after finding the unit ratios.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

2 EXPLORE AND DEVELOP

7.RP.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Math History Minute

Erika Tatiana Camacho (1974-) is a Mexican-American mathematical biologist and Associate Professor at Arizona State University. In 2014, she won the Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring. Her high school teacher and mentor was Jaime Escalante, the subject of the 1988 movie *Stand and Deliver.*

Check

The tables show the amount that three friends earn during a bake sale. Write the correct unit rate that each friend earned in the spaces provided.

Allie

Earnings ($)	17.00	34.00	51.00
Time (h)	1	2	3

Ben

Earnings ($)	9.00	18.00	27.00
Time (h)	0.75	1.5	2.25

Sri

Earnings ($)	40.00	80.00	120.00
Time (h)	2.5	5	7.5

Allie	Ben	Sri
$17 per hour	$12 per hour	$16 per hour

Go Online You can complete an Extra Example online.

Apply Sales Tax

Objective

Students will come up with their own strategy to solve an application problem involving sales tax.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the Write About It! prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does it mean for quantities to be in a proportional relationship?
- How can you find the sales tax for an item that costs $1?
- How can you set up equivalent ratios to find the amount of sales tax for an $84 purchase?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Sales Tax

Jalen went shopping for school clothes. The table shows the sales tax for various purchase amounts. Is the sales tax proportional to the purchase amount? What is the total cost, in dollars, for a purchase amount of $84?

Purchase Amount ($)	12	24	36	48
Sales Tax ($)	0.60	1.20	1.80	2.40

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

yes; $88.20; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! If the relationship was not proportional, could you solve this problem? Explain.

no; Sample answer: If there was not a constant ratio of purchase amount to sales tax, then I could not find the sales tax for a purchase amount of $84.

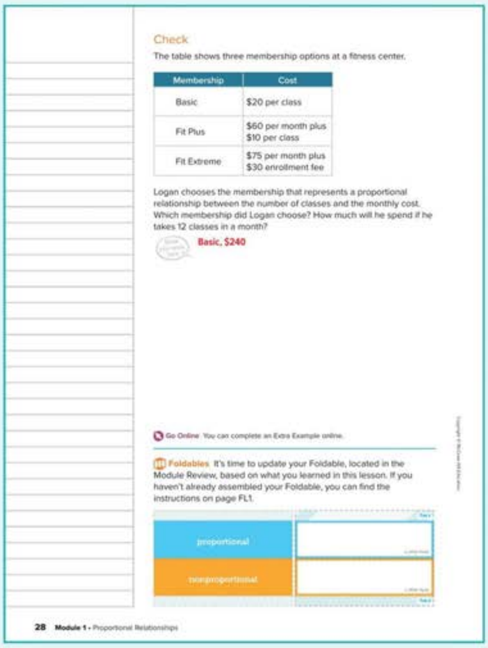
Lesson 1-3 • Tables of Proportional Relationships 27

Interactive Presentation

Apply, Sales Tax

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows three membership options at a fitness center.

Membership	Cost
Basic	$20 per class
Fit Plus	$60 per month plus $10 per class
Fit Extreme	$75 per month plus $30 enrollment fee

Logan chooses the membership that represents a proportional relationship between the number of classes and the monthly cost. Which membership did Logan choose? How much will he spend if he takes 12 classes in a month?

Basic, $240

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

proportional

nonproportional

28 Module 1 • Proportional Relationships

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add examples of proportional and nonproportional relationships expressed as tables. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What does it mean for two quantities to be in a proportional relationship?

In this lesson, students learned how to determine whether two quantities are in a proportional relationship by testing for equivalent ratios in tables. Encourage them to discuss with a partner why they must test *every pair of values* in order to determine if the relationship is proportional.

Exit Ticket

Refer to the Exit Ticket slide. Explain how to determine which relationship is proportional. Sample answer: A relationship is proportional if the ratio between the two variable quantities is constant. Ryan's Rentals has an additional fee on top of the daily rental fee, so the relationship is not proportional. The relationship for Road Trips is proportional because the ratio between the number of days and cost is always $19.99.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 11–14
- Extension: Constant Rate of Change-Tables
- ALEKS Proportions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 9, 12, 13
- Extension: Constant Rate of Change-Tables
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

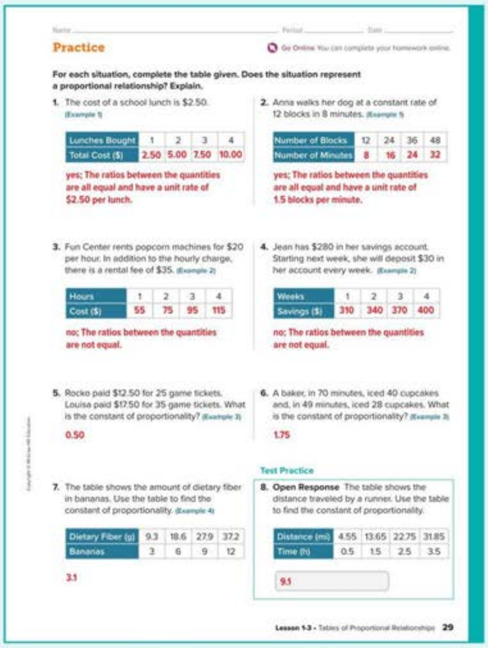
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	identify a proportional relationship from a table	1, 2
2	identify a nonproportional relationship from a table	3, 4
2	find the constant of proportionality from a verbal description	5, 6
2	find the constant of proportionality from a table	7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving proportional relationships and tables	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Students may calculate the reciprocal of the constant of proportionality rather than the correct value. In Exercise 5, students may find the constant of proportionality using the ratio of tickets to cost rather than cost to tickets. In this case, the constant of proportionality may be calculated as 2 rather than 0.5. Before calculating the constant of proportionality, have students identify the independent and dependent variables.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

For each situation, complete the table given. Does the situation represent a proportional relationship? Explain.

1. The cost of a school lunch is $2.50. (Example 1)

Lunches Bought	1	2	3	4
Total Cost ($)	2.50	5.00	7.50	10.00

yes; The ratios between the quantities are all equal and have a unit rate of $2.50 per lunch.

2. Anna walks her dog at a constant rate of 12 blocks in 8 minutes. (Example 1)

Number of Blocks	12	24	36	48
Number of Minutes	8	16	24	32

yes; The ratios between the quantities are all equal and have a unit rate of 1.5 blocks per minute.

3. Fun Center rents popcorn machines for $20 per hour. In addition to the hourly charge, there is a rental fee of $35. (Example 2)

Hours	1	2	3	4
Cost ($)	55	75	95	115

no; The ratios between the quantities are not equal.

4. Jean has $280 in her savings account. Starting next week, she will deposit $30 in her account every week. (Example 2)

Weeks	1	2	3	4
Savings ($)	310	340	370	400

no; The ratios between the quantities are not equal.

5. Rocko paid $12.50 for 25 game tickets. Louisa paid $17.50 for 35 game tickets. What is the constant of proportionality? (Example 3)

0.50

6. A baker, in 70 minutes, iced 40 cupcakes and, in 49 minutes, iced 28 cupcakes. What is the constant of proportionality? (Example 3)

1.75

7. The table shows the amount of dietary fiber in bananas. Use the table to find the constant of proportionality. (Example 4)

Dietary Fiber (g)	9.3	18.6	27.9	37.2
Bananas	3	6	9	12

3.1

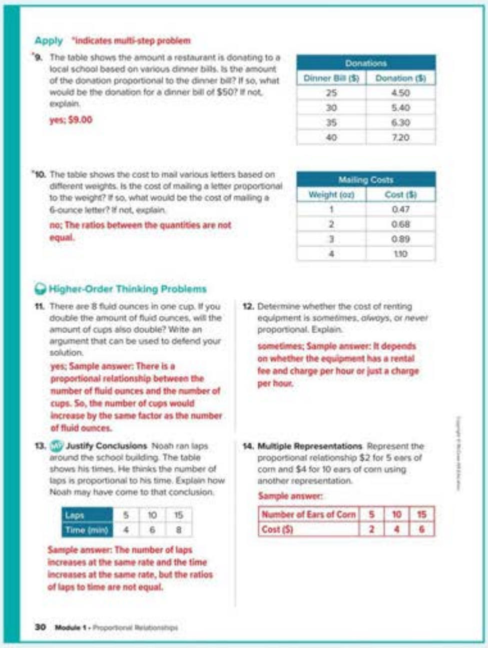
Test Practice

8. **Open Response** The table shows the distance traveled by a runner. Use the table to find the constant of proportionality.

Distance (mi)	4.55	13.65	22.75	31.85
Time (h)	0.5	1.5	2.5	3.5

9.1

Lesson 1-3 • Tables of Proportional Relationships 29



Apply *indicates multi-step problem

*9. The table shows the amount a restaurant is donating to a local school based on various dinner bills. Is the amount of the donation proportional to the dinner bill? If so, what would be the donation for a dinner bill of $50? If not, explain.

yes; $9.00

Donations	
Dinner Bill ($)	Donation ($)
25	4.50
30	5.40
35	6.30
40	7.20

*10. The table shows the cost to mail various letters based on different weights. Is the cost of mailing a letter proportional to the weight? If so, what would be the cost of mailing a 6-ounce letter? If not, explain.

no; The ratios between the quantities are not equal.

Mailing Costs	
Weight (oz)	Cost ($)
1	0.47
2	0.68
3	0.89
4	1.10

Higher-Order Thinking Problems

11. There are 8 fluid ounces in one cup. If you double the amount of fluid ounces, will the amount of cups also double? Write an argument that can be used to defend your solution.

yes; Sample answer: There is a proportional relationship between the number of fluid ounces and the number of cups. So, the number of cups would increase by the same factor as the number of fluid ounces.

12. Determine whether the cost of renting equipment is *sometimes*, *always*, or *never* proportional. Explain.

sometimes; Sample answer: It depends on whether the equipment has a rental fee and charge per hour or just a charge per hour.

13. **Justify Conclusions** Noah ran laps around the school building. The table shows his times. He thinks the number of laps is proportional to his time. Explain how Noah may have come to that conclusion.

Laps	5	10	15
Time (min)	4	6	8

Sample answer: The number of laps increases at the same rate and the time increases at the same rate, but the ratios of laps to time are not equal.

14. **Multiple Representations** Represent the proportional relationship $2 for 5 ears of corn and $4 for 10 ears of corn using another representation.

Sample answer:

Number of Ears of Corn	5	10	15
Cost ($)	2	4	6

30 Module 1 • Proportional Relationships

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students examine a conclusion and hypothesize how the conclusion might have been drawn based on the given table.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.

Use with Exercises 9–10 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 9 might be "How much money does the restaurant donate for every dollar on a dinner bill?"

Create your own higher-order thinking problem.

Use with Exercises 11–14 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Lesson 1-4

Graphs of Proportional Relationships

7.RP.A.2

LESSON GOAL

Students will analyze the relationship between two quantities graphed on a coordinate plane to determine proportionality.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Proportional Relationships, Tables, and Graphs

Learn: Proportional Relationships and Graphs
Examples 1-2: Proportional Relationships and Graphs
Learn: Find the Constant of Proportionality from Graphs
Example 3: Find the Constant of Proportionality from Graphs

Explore: Analyze Points

Learn: Analyze Points on a Graph
Example 4: Analyze Points on a Graph
Apply: Fundraising

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Constant Rate of Change-Graphs		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 4 of the *Language Development Handbook* to help your students build mathematical language related to graphs of proportional relationships.

ELL You can use the tips and suggestions on page T4 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7. RP.A** by analyzing the relationship between two quantities graphed on a coordinate plane to determine proportionality.
Standards for Mathematical Content: **7 .RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B, 7.RP.A.2.D**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP8**

Coherence

Vertical Alignment

Previous
Students analyzed the relationship between two quantities represented in tables to determine proportionality.
7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B

Now
Students analyze the relationship between two quantities graphed on a coordinate plane to determine proportionality.
7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B, 7.RP.A.2.D

Next
Students will write equations to represent proportional relationships.
7.RP.A.2, 7.RP.A.2.B, 7.RP.A.2.C

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of proportional relationships and how to identify the constant of proportionality from a graph. They come to understand that proportional relationships can be represented on a graph by a straight line that passes through the origin, and that in the point (1, *r*), *r* represents the constant of proportionality. They use the graph to build *fluency* with identifying the constant of proportionality.

Mathematical Background

One way to determine whether two quantities are proportional is to graph the quantities on the coordinate plane. The relationship between two quantities is *proportional* if the graph of the data points lies on a straight line that passes through the origin. In a proportional relationship, the point (1, *r*) represents the unit rate *r*.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- locating ordered pairs on a coordinate plane (Exercises 1–5)

Answers

1. (6, 2) **4.** (−1, −12)
2. (−3, 5) **5.** (4, −3)
3. (9 , −8)

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a proportional relationship between job pay and hours worked.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *proportional* consists of the term *proportion* plus the suffix *-al*. The suffix *-al* indicates that this term is what part of speech? Sample answer: The suffix *-al* indicates that the term *proportional* is an adjective.
- How does the meaning of the prefix *non-* help you understand how the meaning of *nonproportional* relates to the meaning of proportional? Sample answer: *Non* means not. So, nonproportional means not proportional.
- What are some synonyms for the term *constant*? Sample answer: Constant means not changing, or something that remains the same.

Explore Proportional Relationships, Tables, and Graphs

Objective

Students will use Web Sketchpad to explore the graphs of proportional and nonproportional linear relationships.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a real-world problem involving four students' online comments and the number of replies they receive. Throughout this activity, students will use the information to identify and compare ratios, determine which data sets are proportional, and compare the graphs of proportional and nonproportional relationships. Students will use Web Sketchpad to explore the Inquiry Question.

Inquiry Question

How are the graphs of proportional and nonproportional linear relationships alike? How are they different? Sample answer: The graphs of proportional and nonproportional linear relationships are straight lines. Only the graphs of proportional linear relationships pass through the origin.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

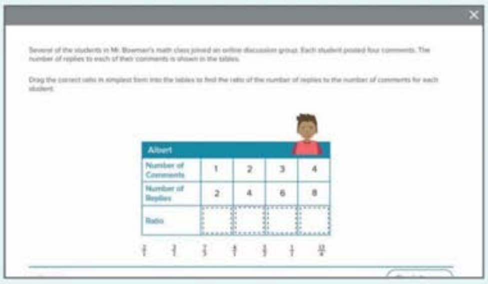
Mathematical Discourse

What do you notice about the ratios for each of the students? Sample answer: The ratios of the number of replies to the number of comments for each of Albert's and David's posts have the same unit ratio. The ratios for Bianca and Connie have different unit ratios.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 5



Explore, Slide 2 of 5

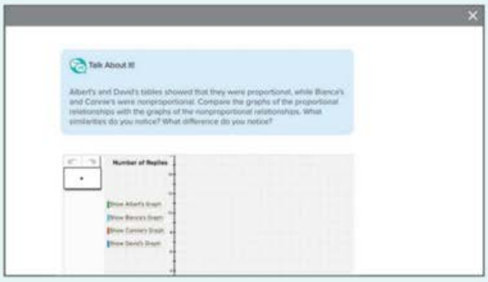
DRAG & DROP

On Slide 2, students drag ratios to corresponding locations in the tables.

DRAG & DROP

On Slide 3, students drag to sort relationships as proportional or nonproportional.

Interactive Presentation



Explore, Slide 4 of 5

WEB SKETCHPAD

On Slide 4, students use Web Sketchpad to explore how the graphs of proportional and nonproportional relationships are alike and different.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Proportional Relationships, Tables, and Graphs *(continued)*

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others Students should be able to make a conjecture as to what the graphs of proportional relationships and nonproportional relationships have in common, and how they are different.

5 Use Appropriate Tools Strategically Encourage students to use the tables and graphs to examine the similarities among the graphs of proportional and nonproportional linear relationships.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Albert's and David's tables showed that they were proportional, while Bianca's and Connie's were nonproportional. Compare the graphs of the proportional relationships with the graphs of the nonproportional relationships. What similarities do you notice? What difference do you notice? Sample answer: The graphs of the proportional relationships, from Albert's and David's tables, pass through the origin, while the graphs of the nonproportional relationships, from Bianca's and Connie's tables, do not pass through the origin.

Learn Proportional Relationships and Graphs

Objective

Students will learn how to identify a proportional relationship from a graph.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to accurately and efficiently use clear and precise mathematical language in their explanation for why the graph of a proportional relationship needs to be a straight line.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

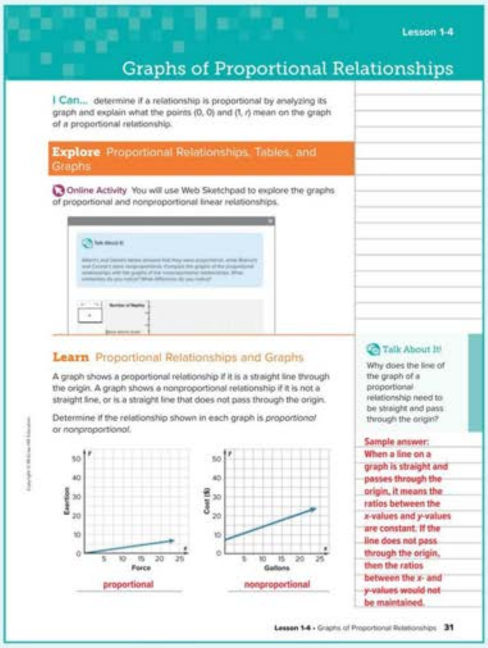
Why does the line of the graph of a proportional relationship need to be straight and pass through the origin? Sample answer: When a line on a graph is straight and passes through the origin, it means the ratios between the *x*-values and *y*-values are constant. If the line does not pass through the origin, then the ratios between the *x*- and *y*-values would not be maintained.

DIFFERENTIATE

Language Development Activity ELL

If students are struggling to determine proportional relationships from graphs, have them consider the four situations shown in the table below and determine whether or not each would represent a proportional relationship. Some students may benefit from drawing an example of a graph for each category.

	Through Origin	Not Through Origin
Line	proportional	nonproportional
Curve	nonproportional	nonproportional



Lesson 1-4

Graphs of Proportional Relationships

I Can... determine if a relationship is proportional by analyzing its graph and explain what the points (0, 0) and (1, *r*) mean on the graph of a proportional relationship.

Explore Proportional Relationships, Tables, and Graphs

Online Activity You will use Web Sketchpad to explore the graphs of proportional and nonproportional linear relationships.

Learn Proportional Relationships and Graphs

A graph shows a proportional relationship if it is a straight line through the origin. A graph shows a nonproportional relationship if it is not a straight line, or is a straight line that does not pass through the origin.

Determine if the relationship shown in each graph is *proportional* or *nonproportional*.

proportional

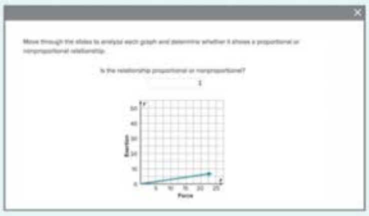
nonproportional

Talk About It! Why does the line of the graph of a proportional relationship need to be straight and pass through the origin?

Sample answer: When a line on a graph is straight and passes through the origin, it means the ratios between the *x*-values and *y*-values are constant. If the line does not pass through the origin, then the ratios between the *x*- and *y*-values would not be maintained.

Lesson 1-4 • Graphs of Proportional Relationships 31

Interactive Presentation



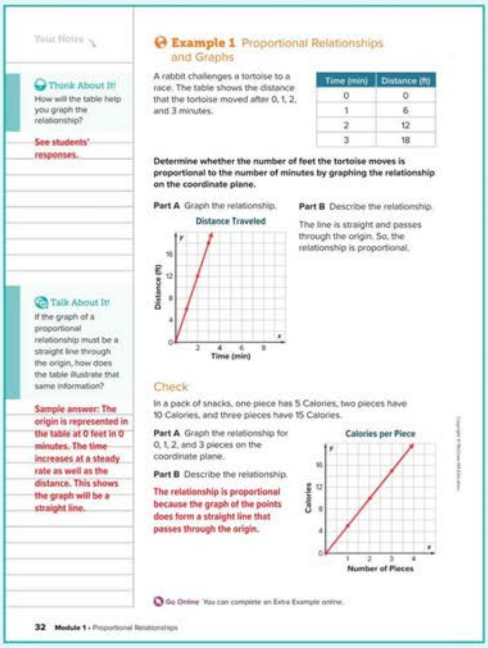
Learn, Proportional Relationships and Graphs, Slide 2 of 3

FLASHCARDS On Slide 1, students use Flashcards to learn how to determine if a relationship is proportional from a graph.

CLICK On Slide 2, students move through the slides to analyze a selection of graphs.

7.RP.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Your Notes

Think About It!
How will the table help you graph the relationship?

See students' responses.

Talk About It!
If the graph of a proportional relationship must be a straight line through the origin, how does the table illustrate that same information?

Sample answer: The origin is represented in the table at 0 feet in 0 minutes. The time increases at a steady rate as well as the distance. This shows the graph will be a straight line.

Example 1 Proportional Relationships and Graphs

A rabbit challenges a tortoise to a race. The table shows the distance that the tortoise moved after 0, 1, 2, and 3 minutes.

Time (min)	Distance (ft)
0	0
1	6
2	12
3	18

Determine whether the number of feet the tortoise moves is proportional to the number of minutes by graphing the relationship on the coordinate plane.

Part A Graph the relationship.

Part B Describe the relationship.

The line is straight and passes through the origin. So, the relationship is proportional.

Check

In a pack of snacks, one piece has 5 Calories, two pieces have 10 Calories, and three pieces have 15 Calories.

Part A Graph the relationship for 0, 1, 2, and 3 pieces on the coordinate plane.

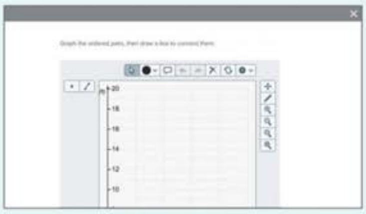
Part B Describe the relationship.

The relationship is proportional because the graph of the points does form a straight line that passes through the origin.

Go Online You can complete an Extra Example online.

32 Module 1 • Proportional Relationships

Interactive Presentation



Example 1, Proportional Relationships and Graphs, Slide 2 of 5

eTOOLS

On Slide 2, students use the Coordinate Graphing eTool to graph a relationship on a coordinate plane.

CLICK

On Slide 3, students select from drop-down menus to describe why a relationship is proportional.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Proportional Relationships and Graphs

Objective

Students will graph and identify a proportional relationship on the coordinate plane.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to reason that, because a proportional relationship has a constant ratio or unit rate, time and distance in the table increase at a steady rate.

5 Use Appropriate Tools Strategically Encourage students to use the Coordinate Graphing eTool embedded within this example in order to graph the relationship on the coordinate plane.

6 Attend to Precision Students should use clear and precise mathematical language to describe why the relationship shown on the graph is proportional.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to determine? if the time and distance of the tortoise's movement are proportional

AL What are the ordered pairs listed in the table? (0, 0), (1, 6), (2, 12), (3, 18)

OL How will the graph show whether or not the relationship is proportional? If the graph forms a straight line that passes through the origin, then the relationship is proportional.

OL Is the relationship proportional? Explain. yes; The graph is a straight line that passes through the origin.

BL How can you tell by studying the table that the relationship is proportional? Sample answer: The relationship will pass through the origin because the point (0, 0) is listed in the table and the quantities are in a constant ratio.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Proportional Relationships and Graphs

Objective

Students will graph and identify a nonproportional relationship on the coordinate plane.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the quantities given in the real-world scenario in order to identify what part of the scenario makes the relationship nonproportional.

5 Use Appropriate Tools Strategically Encourage students to use the Coordinate Graphing eTool embedded within this example in order to graph the relationship on the coordinate plane.

6 Attend to Precision Students should use clear and precise mathematical language to describe why the relationship shown on the graph is nonproportional.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to determine? if the time and distance of the rabbit's movement are proportional

AL What are the ordered pairs listed in the table? (0, 0), (1, 8), (2, 8), (3, 15)

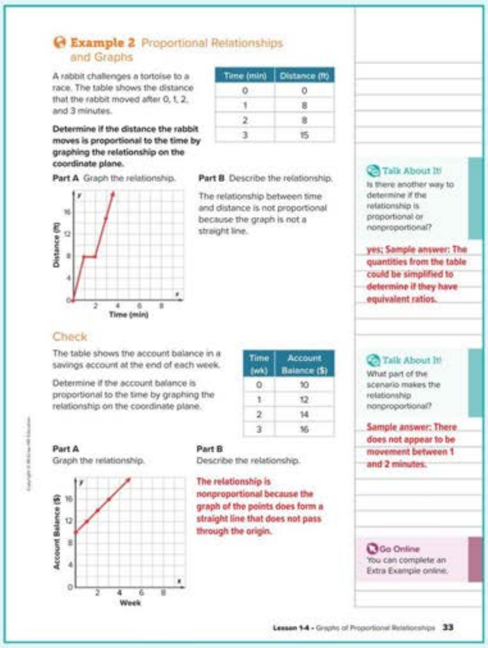
OL How will the graph show whether or not the relationship is proportional? If the graph forms a straight line that passes through the origin, then the relationship is proportional.

OL Is the relationship proportional? no; The graph is not a straight line.

BL How can you tell by studying the table that the relationship is not proportional? Sample answer: The table contains the same distance of 8 feet for both 1 and 2 minutes. The quantities 8 to 1 and 8 to 2 will not be in the same ratio.

Go Online

- Find additional teaching notes and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Proportional Relationships and Graphs

A rabbit challenges a tortoise to a race. The table shows the distance that the rabbit moved after 0, 1, 2, and 3 minutes.

Time (min)	Distance (ft)
0	0
1	8
2	8
3	15

Determine if the distance the rabbit moves is proportional to the time by graphing the relationship on the coordinate plane.

Part A Graph the relationship.

Part B Describe the relationship.

The relationship between time and distance is not proportional because the graph is not a straight line.

Talk About It!
Is there another way to determine if the relationship is proportional or nonproportional?

yes; Sample answer: The quantities from the table could be simplified to determine if they have equivalent ratios.

Check

The table shows the account balance in a savings account at the end of each week.

Determine if the account balance is proportional to the time by graphing the relationship on the coordinate plane.

Time (wk)	Account Balance ($)
0	10
1	12
2	14
3	16

Part A
Graph the relationship.

Part B
Describe the relationship.

The relationship is nonproportional because the graph of the points does form a straight line that does not pass through the origin.

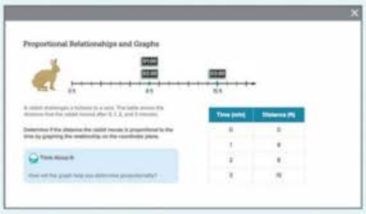
Talk About It!
What part of the scenario makes the relationship nonproportional?

Sample answer: There does not appear to be movement between 1 and 2 minutes.

Go Online
You can complete an Extra Example online.

Lesson 1-4 • Graphs of Proportional Relationships 33

Interactive Presentation



Example 2, Proportional Relationships and Graphs, Slide 1 of 5

eTOOLS

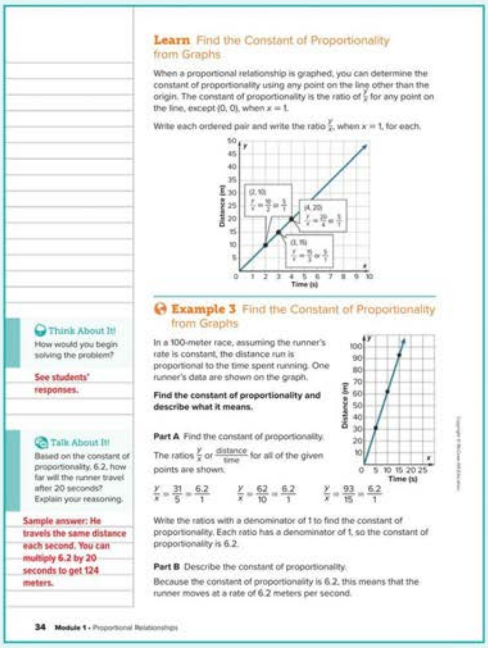
On Slide 2, students use the Coordinate Graphing eTool to graph a nonproportional relationship on a coordinate plane.

CLICK

On Slide 3, students select to explain why the relationship is nonproportional.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! How would you begin solving the problem?

See students' responses.

Talk About It! Based on the constant of proportionality, 6.2, how far will the runner travel after 20 seconds? Explain your reasoning.

Sample answer: He travels the same distance each second. You can multiply 6.2 by 20 seconds to get 124 meters.

Learn Find the Constant of Proportionality from Graphs

When a proportional relationship is graphed, you can determine the constant of proportionality using any point on the line other than the origin. The constant of proportionality is the ratio of $\frac{y}{x}$ for any point on the line, except (0, 0), when $x = 1$.

Write each ordered pair and write the ratio $\frac{y}{x}$, when $x = 1$, for each.

Example 3 Find the Constant of Proportionality from Graphs

In a 100-meter race, assuming the runner's rate is constant, the distance run is proportional to the time spent running. One runner's data are shown on the graph.

Find the constant of proportionality and describe what it means.

Part A Find the constant of proportionality.

The ratios $\frac{y}{x}$ or $\frac{\text{distance}}{\text{time}}$ for all of the given points are shown.

$\frac{y}{x} = \frac{31}{5} = \frac{6.2}{1}$ $\frac{y}{x} = \frac{62}{10} = \frac{6.2}{1}$ $\frac{y}{x} = \frac{93}{15} = \frac{6.2}{1}$

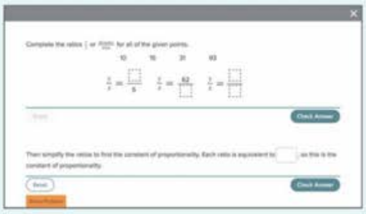
Write the ratios with a denominator of 1 to find the constant of proportionality. Each ratio has a denominator of 1, so the constant of proportionality is 6.2.

Part B Describe the constant of proportionality.

Because the constant of proportionality is 6.2, this means that the runner moves at a rate of 6.2 meters per second.

34 Module 1 • Proportional Relationships

Interactive Presentation



Example 3, Find the Constant of Proportionality from Graphs, Slide 2 of 5

CLICK

On Slide 1 of the Learn, students select points to see the constant of proportionality simplified from each ordered pair.

DRAG & DROP

On Slide 2 of Example 3, students drag values to complete ratios of distance to time.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Find the Constant of Proportionality from Graphs

Objective

Students will learn how to identify the constant of proportionality from a graph.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

What do you notice about the constant of proportionality and the coordinates of points *A*(2, 10) and *B*(3, 15)? Sample answer: The difference between each *y*-coordinate from one point to the next on a graph is equal to the constant of proportionality.

Example 3 Find the Constant of Proportionality from Graphs

Objective

Students will find the constant of proportionality from a graph.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the constant of proportionality

AL Does the graph show a proportional relationship? Explain. Y es, the graph is a straight line through the origin.

OL Because you know the relationship is proportional, how can you use the point (5, 31) to find the constant of proportionality? Sample answer: Find the ratio of 31 to 5, which is 6.2. Check the other ordered pairs to verify this ratio is constant.

BL The problem asks you to find the constant of proportionality of a runner in a 100-meter race based on the first 15 seconds of the race. In real life, what problem(s) could you have encountered when solving a problem similar to this one? Sample answer: The speed might not be constant.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Analyze Points

Objective

Students will use Web Sketchpad to explore and analyze the points (0, 0) and (1, *r*) on a graph of a proportional relationship.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a sketch that models the time and distance that a toy car travels. Throughout this activity, students will observe graphs showing the relationship between the time and the distance traveled. They will determine what various points on the graph of a proportional relationship represent and compare the constant of proportionality, the unit rate, and the *y*-value of the point at (1, *r*). They will use Web Sketchpad to explore the Inquiry Question.

Inquiry Question

What are special points on a graph of a proportional relationship and what do they represent? Sample answer: The two special points in a proportional relationship are (0, 0), which represents the origin, and (1, *r*), which represents the unit rate.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What do you notice about the constant of proportionality, the unit rate, and the coordinates for point *B*? Sample answer: The value for the constant of proportionality is the same as the value for the unit rate. Point *B* has a *y*-value that is equal to the unit rate and an *x*-value equal to 1.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6

Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore special points on the graphs of proportional relationships.

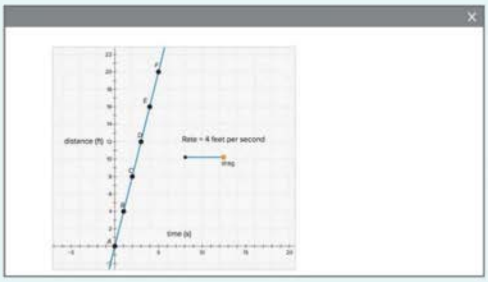
TYPE

On Slide 2, students indicate the distance the front of the car moved in 5 seconds.

CLICK

On Slide 3, students select the point on a graph that represents where the car began.

Interactive Presentation



Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Analyze Points *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore and examine the meaning of the ordered pairs (0, 0) and (1, r) on the graph of a proportional relationship.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Use the point labeled *drag* to change the rate. Where is point B located now? How does this compare to the unit rate and constant of proportionality? Sample answer: Point B is now located at (1, 2). The y-coordinate represents the unit rate in feet per second and has the value of the constant of proportionality.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Analyze Points on a Graph

Objective

Students will understand the significance of the points (0, 0) and (1, *r*) on a graph of a proportional relationship.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to use clear and precise mathematical language when explaining the significance of the point (0, 0) on the graph of a proportional relationship.

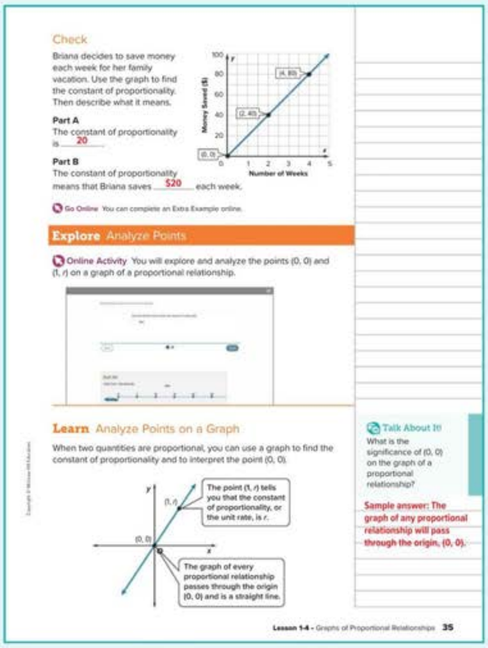
Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

What is the significance of (0, 0) on the graph of a proportional relationship? Sample answer: The graph of any proportional relationship will pass through the origin, (0, 0).


Check

Briana decides to save money each week for her family vacation. Use the graph to find the constant of proportionality. Then describe what it means.

Part A
The constant of proportionality is 20.

Part B
The constant of proportionality means that Briana saves $20 each week.

Go Online You can complete an Extra Example online.

Explore Analyze Points

Online Activity You will explore and analyze the points (0, 0) and (1, *r*) on a graph of a proportional relationship.

Learn Analyze Points on a Graph

When two quantities are proportional, you can use a graph to find the constant of proportionality and to interpret the point (0, 0).

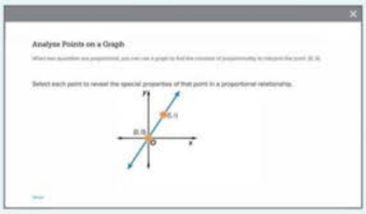
The point (1, *r*) tells you that the constant of proportionality, or the unit rate, is *r*.

The graph of every proportional relationship passes through the origin (0, 0) and is a straight line.

Talk About It!
What is the significance of (0, 0) on the graph of a proportional relationship?
Sample answer: The graph of any proportional relationship will pass through the origin, (0, 0).

Lesson 1-4 • Graphs of Proportional Relationships 35

Interactive Presentation



Learn, Analyze Points on a Graph, Slide 1 of 2

CLICK

On Slide 1, students select points to reveal the special properties of those points in a proportional relationship.

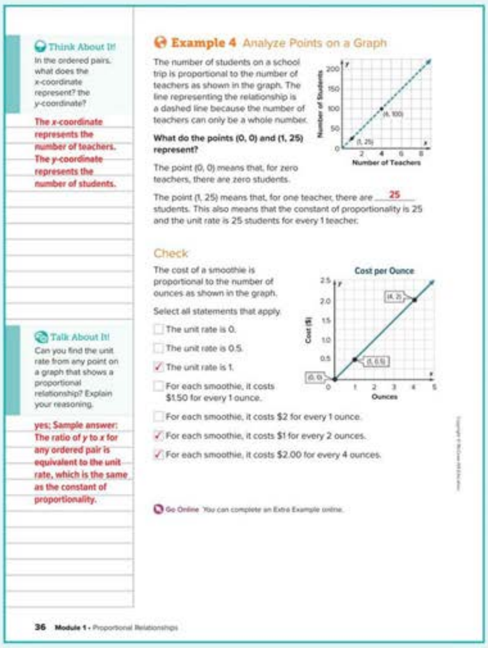
DIFFERENTIATE

Enrichment Activity BL

To help students further their understanding of special points on the graphs of proportional relationships, have them answer the following questions.

The unit rate of a proportional relationship is 3 miles per minute. What two important points are on the graph of the relationship? What do those points mean? (0, 0), (1, 3); The graph will pass through the origin, (0, 0). The unit rate, 3 miles per minute, is represented by the point (1, 3).

Suppose the graph of a proportional relationship passes through the point $\left(1, \frac{5}{6}\right)$. What is the constant of proportionality? $\frac{5}{6}$



Think About It! In the ordered pairs, what does the x-coordinate represent? the y-coordinate?

The x-coordinate represents the number of teachers. The y-coordinate represents the number of students.

Example 4 Analyze Points on a Graph

The number of students on a school trip is proportional to the number of teachers as shown in the graph. The line representing the relationship is a dashed line because the number of teachers can only be a whole number.

What do the points (0, 0) and (1, 25) represent?

The point (0, 0) means that, for zero teachers, there are zero students.

The point (1, 25) means that, for one teacher, there are 25 students. This also means that the constant of proportionality is 25 and the unit rate is 25 students for every 1 teacher.

Check

The cost of a smoothie is proportional to the number of ounces as shown in the graph.

Select all statements that apply.

- [ ] The unit rate is 0.
- [ ] The unit rate is 0.5.
- [x] The unit rate is 1.
- [ ] For each smoothie, it costs $1.50 for every 1 ounce.
- [ ] For each smoothie, it costs $2 for every 1 ounce.
- [x] For each smoothie, it costs $1 for every 2 ounces.
- [x] For each smoothie, it costs $2.00 for every 4 ounces.

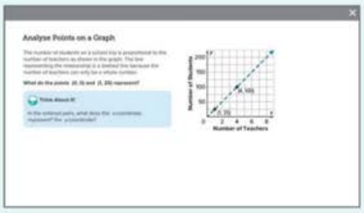
Talk About It! Can you find the unit rate from any point on a graph that shows a proportional relationship? Explain your reasoning.

yes; Sample answer: The ratio of y to x for any ordered pair is equivalent to the unit rate, which is the same as the constant of proportionality.

Go Online You can complete an Extra Example online.

36 Module 1 • Proportional Relationships

Interactive Presentation



Example 4, Analyze Points on a Graph, Slide 1 of 4

CLICK On Slide 2, students select points to see visual representations of the values.

TYPE On Slide 2, students enter the missing value to complete the interpretation of an ordered pair.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Analyze Points on a Graph

Objective

Students will identify and describe the significance of the points (0, 0) and (1, *r*) on the graph of a proportional relationship.

Teaching Notes

Data that can be graphed as any real number are continuous. Data that can only be graphed as whole numbers are discrete. In this Example, a dashed line is used to indicate the graph of discrete data.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make connections between the points (0, 0) and (1, *r*) on the graph of a proportional relationship, what those points mean within the context of the real-world scenario, and how the point (1, *r*) is related to the unit rate or constant of proportionality.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use precise mathematical vocabulary, such as *ratio of y to x*, *ordered pair*, *unit rate*, and/or *constant of proportionality*, as they explain how they can find the unit rate from any point on the graph of a proportional relationship.

Questions for Mathematical Discourse

SLIDE 2

AL In the ordered pair (1, 25), what does the *x*-value represent? the *y*-value? 1 teacher; 25 students

OL Could you use the ordered pair (4, 100) to find the constant of proportionality? Explain. yes; Sample answer: The ratio of *y* to *x* for any ordered pair is equivalent to the unit rate, which is the same as the constant of proportionality, as long as the relationship is proportional.

BL Use the graph to determine how many teachers are needed if there are 200 students. 8 teachers

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Fundraising

Objective

Students will come up with their own strategy to solve an application problem involving fundraising.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

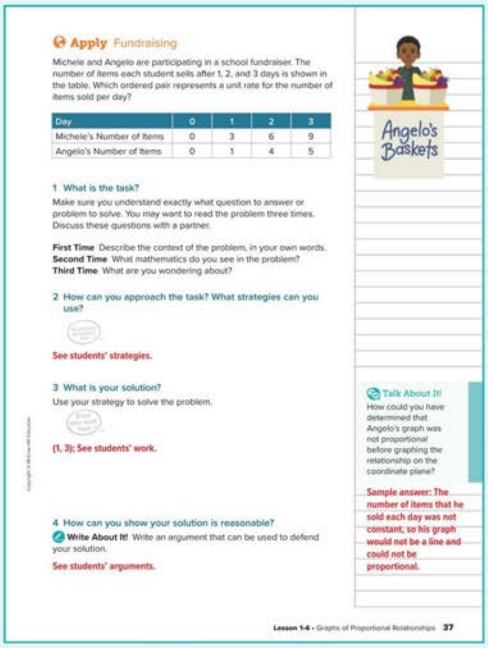
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you graph each set of data on a coordinate plane?
- Which relationship shows a proportional relationship?
- What is the unit rate of the proportional relationship?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Fundraising

Michele and Angelo are participating in a school fundraiser. The number of items each student sells after 1, 2, and 3 days is shown in the table. Which ordered pair represents a unit rate for the number of items sold per day?

Day	0	1	2	3
Michele's Number of Items	0	3	6	9
Angelo's Number of Items	0	1	4	5

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

(1, 3); See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
How could you have determined that Angelo's graph was not proportional before graphing the relationship on the coordinate plane?

Sample answer: The number of items that he sold each day was not constant, so his graph would not be a line and could not be proportional.

Lesson 1-4 • Graphs of Proportional Relationships 37

Interactive Presentation

Apply, Fundraising

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The heights of two plants are recorded after 1, 2, and 3 weeks. The data are shown in the table. Which ordered pair represents a unit rate for the number of inches grown per week?

Time (weeks)	Plant 1 Height (in.)	Time (weeks)	Plant 2 Height (in.)
0	0	0	0
1	5	1	3
2	6	2	6
3	7	3	9

(1, 3)

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

proportional

nonproportional

38 Module 1 • Proportional Relationships

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add examples of proportional and nonproportional relationships expressed as graphs. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What does it mean for two quantities to be in a proportional relationship?

In this lesson, students learned how to determine whether two quantities are in a proportional relationship by observing whether the graph is a straight line through the origin. Encourage them to discuss with a partner why a graph has to possess *both* qualities (straight line, passes through the origin) in order to be considered proportional.

Exit Ticket

Refer to the Exit Ticket slide. Explain how to determine the constant of proportionality from a graph. Sample answer: The constant of proportionality of a graph is the y-value of the ordered pair with a corresponding x-value of 1 if the graph passes through (0, 0).

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1, 3, 5–9
- Extension: Constant Rate of Change-Graphs
- ALEKS® Proportions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 7, 8
- Extension: Constant Rate of Change-Graphs
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

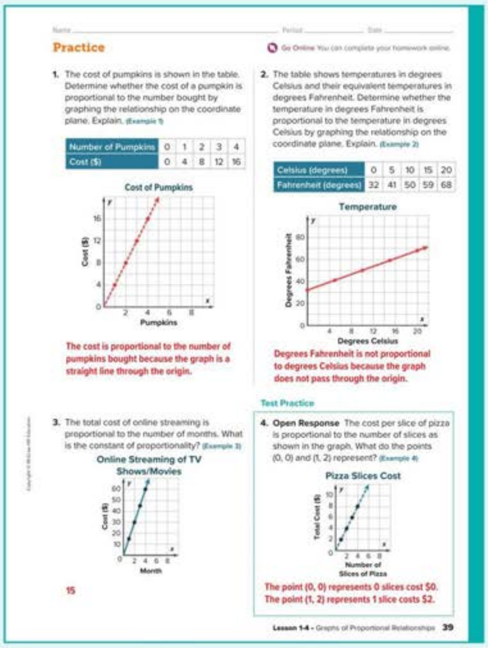
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	graph and identify a proportional relationship on the coordinate plane	1
2	graph and identify a nonproportional relationship on the coordinate plane	2
2	find the constant of proportionality from a graph	3
2	identify and describe the significance of the points (0, 0) and (1, *r*) on the graph of a proportional relationship	4
3	solve application problems involving proportional relationships and graphs	5
3	higher-order and critical thinking skills	6–9

Common Misconception

When determining the constant of proportionality from a graph, students may not take all points into consideration when determining (1, *r*). For example, in Exercise 3, the *y*-value when the *x*-value is 1 could be interpreted as 14, 15, or 16 as the scale does not show those specific values. Encourage students to use the other points graphed, such as (2, 30), to help determine (1, *r*).


Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

1. The cost of pumpkins is shown in the table. Determine whether the cost of a pumpkin is proportional to the number bought by graphing the relationship on the coordinate plane. Explain. (Example 1)

Number of Pumpkins	0	1	2	3	4
Cost ($)	0	4	8	12	16

The cost is proportional to the number of pumpkins bought because the graph is a straight line through the origin.

2. The table shows temperatures in degrees Celsius and their equivalent temperatures in degrees Fahrenheit. Determine whether the temperature in degrees Fahrenheit is proportional to the temperature in degrees Celsius by graphing the relationship on the coordinate plane. Explain. (Example 2)

Celsius (degrees)	0	5	10	15	20
Fahrenheit (degrees)	32	41	50	59	68

Degrees Fahrenheit is not proportional to degrees Celsius because the graph does not pass through the origin.

3. The total cost of online streaming is proportional to the number of months. What is the constant of proportionality? (Example 3)

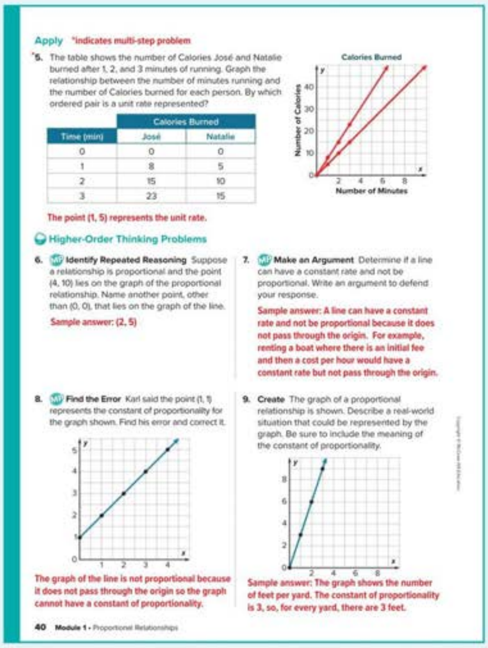
15

Test Practice

4. **Open Response** The cost per slice of pizza is proportional to the number of slices as shown in the graph. What do the points (0, 0) and (1, 2) represent? (Example 4)

The point (0, 0) represents 0 slices cost $0. The point (1, 2) represents 1 slice costs $2.

Lesson 1-4 • Graphs of Proportional Relationships 39



Apply *indicates multi-step problem

*5. The table shows the number of Calories José and Natalie burned after 1, 2, and 3 minutes of running. Graph the relationship between the number of minutes running and the number of Calories burned for each person. By which ordered pair is a unit rate represented?

	Calories Burned	
Time (min)	José	Natalie
0	0	0
1	8	5
2	15	10
3	23	15

The point (1, 5) represents the unit rate.

Higher-Order Thinking Problems

6. **MP Identify Repeated Reasoning** Suppose a relationship is proportional and the point (4, 10) lies on the graph of the proportional relationship. Name another point, other than (0, 0), that lies on the graph of the line.

Sample answer: (2, 5)

7. **MP Make an Argument** Determine if a line can have a constant rate and not be proportional. Write an argument to defend your response.

Sample answer: A line can have a constant rate and not be proportional because it does not pass through the origin. For example, renting a boat where there is an initial fee and then a cost per hour would have a constant rate but not pass through the origin.

8. **MP Find the Error** Karl said the point (1, 1) represents the constant of proportionality for the graph shown. Find his error and correct it.

The graph of the line is not proportional because it does not pass through the origin so the graph cannot have a constant of proportionality.

9. **Create** The graph of a proportional relationship is shown. Describe a real-world situation that could be represented by the graph. Be sure to include the meaning of the constant of proportionality.

Sample answer: The graph shows the number of feet per yard. The constant of proportionality is 3, so, for every yard, there are 3 feet.

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Teaching the Mathematical Practices

8 Look For and Express Regularity in Repeated Reasoning
In Exercise 6, students identify a point on a graph by recognizing and using patterns and relationships in the ordered pairs of graphs of proportional relationships.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 7, students construct an argument for whether or not a line can have a constant rate and not be proportional.
In Exercise 8, students analyze the work of another to diagnose and correct the error.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercise 5 Have students work in pairs. After completing the application problem, have students write two true statements and one false statement about the situation. An example of a true statement might be "Natalie ran at a constant rate for each of the three minutes." An example of a false statement might be "José ran at a constant rate for each of the three minutes." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 8 Have students work together to prepare a brief explanation that illustrates Karl's flawed reasoning. For example, Karl might think that the graph represents a proportional relationship since it is a straight line. Have each pair or group of students present their explanations to the class.

Lesson 1-5

Equations of Proportional Relationships

7.RP.A.2

LESSON GOAL

Students will write equations to represent proportional relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Proportional Relationships and Equations

Learn: Identify the Constant of Proportionality in Equations
Example 1: Identify the Constant of Proportionality in Equations
Learn: Proportional Relationships and Equations
Example 2: Proportional Relationships and Equations
Example 3: Proportional Relationships and Equations
Apply: Running

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Hooke's Law		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 5 of the *Language Development Handbook* to help your students build mathematical language related to equations of proportional relationships.

ELL You can use the tips and suggestions on page T5 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7. RP.A** by writing equations of proportional relationships.
Standards for Mathematical Content: **7.RP.A.2, 7.RP.A.2.B, 7.RP.A.2.C**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students analyzed the relationship between two quantities graphed on a coordinate plane to determine proportionality.
7.RP.A.2, 7.RP.A.2.A, 7.RP.A.2.B, 7.RP.A.2.D

Now
Students write equations to represent proportional relationships.
7.RP.A.2, 7.RP.A.2.B, 7.RP.A.2.C

Next
Students will use proportional relationships to solve problems.
7.RP.A.2, 7.RP.A.3

Rigor

The Three Pillars of Rigor

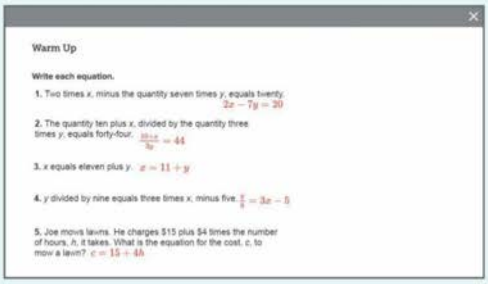
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of proportional relationships and how to identify the constant of proportionality from an equation. They come to understand that proportional relationships can be represented by an equation in the form $y = kx$, where k represents the constant of proportionality or unit rate. They use equations to build *fluency* with identifying the constant of proportionality, or unit rate, and writing equations to find missing values in a proportional relationship.

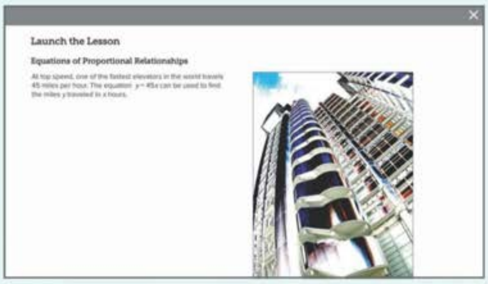
Mathematical Background

Proportional relationships can be represented using equations in the form $y = kx$, where k is the constant of proportionality (unit rate).

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing equations from verbal descriptions (Exercises 1–4)
- writing equations to represent real-world problems (Exercise 5)

Answers

1. $2x - 7y = 20$
2. $\frac{10 + x}{3y} = 44$
3. $x = 11 + y$
4. $\frac{y}{9} = 3x - 5$
5. $c = 15 + 4h$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the proportional relationship of one of the fastest elevators in the world.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does it mean when there is a relationship between two quantities? Sample answer: It means that the two quantities are related or connected in some way.
- Define *portion* in your own words. Sample answer: A portion is a part of a whole.

Explore Proportional Relationships and Equations

Objective

Students will use Web Sketchpad to explore the equations of proportional relationships.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch to graph the time and distance of balloons that have been launched. Throughout this activity, students will observe the graphs and make conjectures about the equations that represent proportional and nonproportional relationships.

Inquiry Question

How are equations of proportional relationships different from those of nonproportional relationships? Sample answer: The equation for the nonproportional relationship has a number added to it as a constant, such as $y = 0.58x + 10$. In equations of proportional relationships there is only a value multiplied by x, such as $y = 0.345x$.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

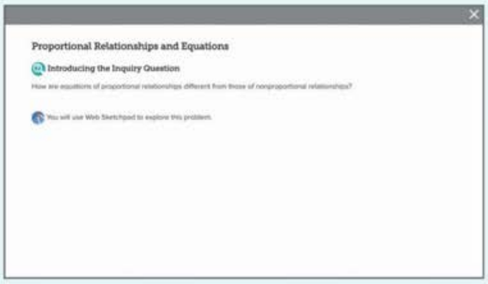
SLIDE 2

Mathematical Discourse

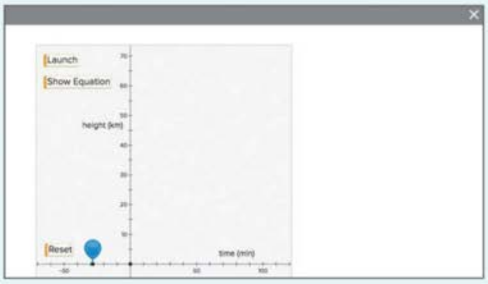
Press the *Show Equation* button. What do you notice about the equation? What do you notice about the graph? Sample answer: The equation is a one-step multiplication equation that has a coefficient of 0.345. The graph passes through the origin and is a straight line.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6

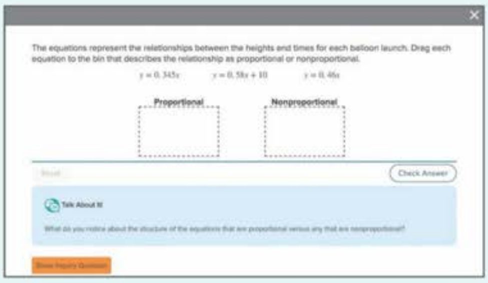


Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how equations of proportional relationships are different from those of nonproportional relationships.

Interactive Presentation



Explore, Slide 5 of 6

DRAG & DROP

On Slide 5, students drag to sort equations as proportional or nonproportional.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Proportional Relationships and Equations *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use the sketch to explore the difference between the equations of proportional relationships and nonproportional relationships.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What do you notice about the structure of the equations that are proportional? Sample answer: The equations that are proportional have a single value multiplied by a variable without an added constant.

Learn Identify the Constant of Proportionality in Equations

Objective

Students will learn how to identify the constant of proportionality in equations.

MP Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to clearly explain that the equation in the form $y = kx$ only represents proportional relationships.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Can you write an equation in the form $y = kx$ for a line that does not pass through the origin? Explain. no; Sample answer: Any value of k when multiplied by the x-value of zero will result in a y-value of zero. This ordered pair passes through the origin, (0, 0), on the graph of any line.

DIFFERENTIATE

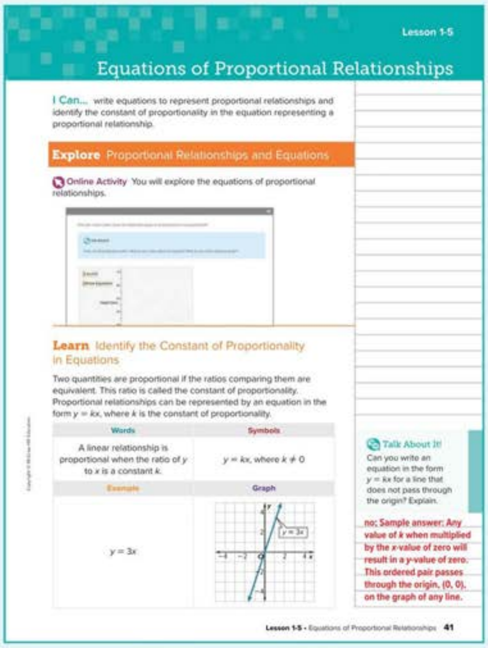
Reteaching Activity AL

If students are struggling to relate the equation $y = kx$ to proportionality, have them think about how they calculate the constant of proportionality from an ordered pair (x, y). For the following ordered pairs, have students calculate the constant of proportionality and write an equation of the form $y = kx$.

(1, 5) $k = \frac{y}{x} = \frac{5}{1} = 5;\ 5 = 5 \cdot 1;\ y = 5x$

(6, 3) $k = \frac{y}{x} = \frac{3}{6} = \frac{1}{2};\ 3 = \frac{1}{2} \cdot 6;\ y = \frac{1}{2}x$

(9, 4) $k = \frac{y}{x} = \frac{4}{9} = \frac{4}{9};\ 4 = \frac{4}{9} \cdot 9;\ y = \frac{4}{9}x$



Lesson 1-5

Equations of Proportional Relationships

I Can... write equations to represent proportional relationships and identify the constant of proportionality in the equation representing a proportional relationship.

Explore Proportional Relationships and Equations

Online Activity You will explore the equations of proportional relationships.

Learn Identify the Constant of Proportionality in Equations

Two quantities are proportional if the ratios comparing them are equivalent. This ratio is called the constant of proportionality. Proportional relationships can be represented by an equation in the form $y = kx$, where k is the constant of proportionality.

Words	Symbols
A linear relationship is proportional when the ratio of y to x is a constant k.	$y = kx$, where $k \neq 0$
Example	**Graph**
$y = 3x$	[graph of $y = 3x$]

Talk About It! Can you write an equation in the form $y = kx$ for a line that does not pass through the origin? Explain.

no; Sample answer: Any value of k when multiplied by the x-value of zero will result in a y-value of zero. This ordered pair passes through the origin, (0, 0), on the graph of any line.

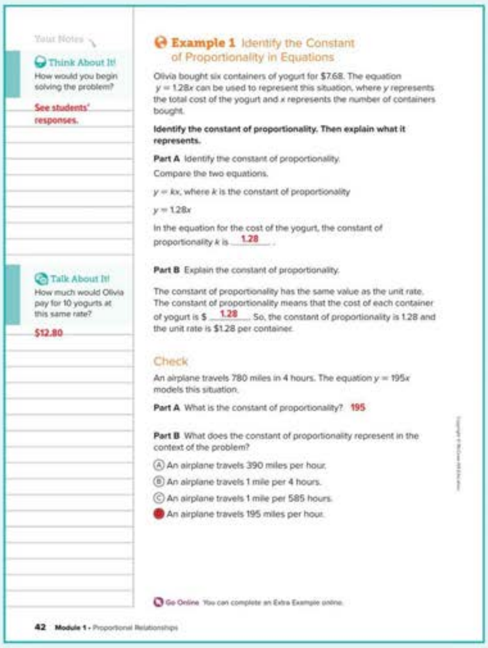
Lesson 1-5 • Equations of Proportional Relationships 41

Interactive Presentation

Learn, Identify the Constant of Proportionality in Equations, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to learn about the multiple representations of proportional relationships.



Your Notes

Think About It!
How would you begin solving the problem?
See students' responses.

Talk About It!
How much would Olivia pay for 10 yogurts at this same rate?
$12.80

Example 1 Identify the Constant of Proportionality in Equations

Olivia bought six containers of yogurt for $7.68. The equation $y = 1.28x$ can be used to represent this situation, where y represents the total cost of the yogurt and x represents the number of containers bought.

Identify the constant of proportionality. Then explain what it represents.

Part A Identify the constant of proportionality.

Compare the two equations.

$y = kx$, where k is the constant of proportionality

$y = 1.28x$

In the equation for the cost of the yogurt, the constant of proportionality k is 1.28.

Part B Explain the constant of proportionality.

The constant of proportionality has the same value as the unit rate. The constant of proportionality means that the cost of each container of yogurt is $ 1.28. So, the constant of proportionality is 1.28 and the unit rate is $1.28 per container.

Check

An airplane travels 780 miles in 4 hours. The equation $y = 195x$ models this situation.

Part A What is the constant of proportionality? 195

Part B What does the constant of proportionality represent in the context of the problem?

(A) An airplane travels 390 miles per hour.
(B) An airplane travels 1 mile per 4 hours.
(C) An airplane travels 1 mile per 585 hours.
(D) An airplane travels 195 miles per hour.

Go Online You can complete an Extra Example online.

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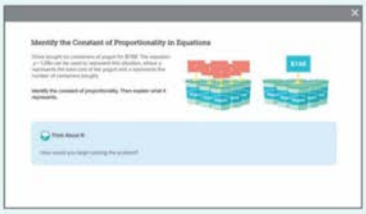
Interactive Presentation



Example 1, Identify the Constant of Proportionality in Equations, Slide 1 of 5

TYPE

On Slide 2, students enter missing values to identify and interpret the constant of proportionality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Identify the Constant of Proportionality in Equations

Objective

Students will identify the constant of proportionality in equations.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make the connection between the constant of proportionality and how it is represented in an equation. As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the quantities given in the question, relate them to the equation, and be able to evaluate the equation for an x-value of 10.

Questions for Mathematical Discourse

SLIDE 2

AL Is the equation $y = 1.28x$ in the form $y = kx$? yes

OL What does k represent? What is the value of k in this scenario? k is the constant of proportionality; in this scenario, its value is 1.28

BL What is another name for the constant of proportionality? Sample answer: the unit rate

SLIDE 3

AL What is the cost of each container of yogurt? $1.28

OL What does k represent, in the context of the problem? the cost of each container

BL How can you use the constant of proportionality to find the cost of 12 containers of yogurt? 15 containers? n containers? Sample answer: Since the cost of one container is $1.28, multiply that by 12, 15, and n to obtain the cost of those containers; $15.36; $19.20; $1.28n$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Proportional Relationships and Equations

Objective

Students will learn how to write equations to represent proportional relationships.

Go Online to find teaching notes, Teaching the Mathematical Practices, and a sample answer for the *Talk About It!* question.

Example 2 Proportional Relationships and Equations

Objective

Students will write equations to represent proportional relationships.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities given in the real-world problem, decontextualize them, and represent them symbolically with the correct equation.

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to explain clearly how they can use the equation to determine the cost for 15 gallons of gas.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that this relationship is proportional? Sample answer: Each gallon of gasoline costs the same amount.

OL How will you find the constant of proportionality? Sample answer: I can write a ratio that compares the cost of the gasoline to the number of gallons.

OL What is the constant of proportionality? 3.89

BL Use the constant of proportionality to find the number of gallons of gasoline used, if the total cost was $54.46. 14 gallons

SLIDE 3

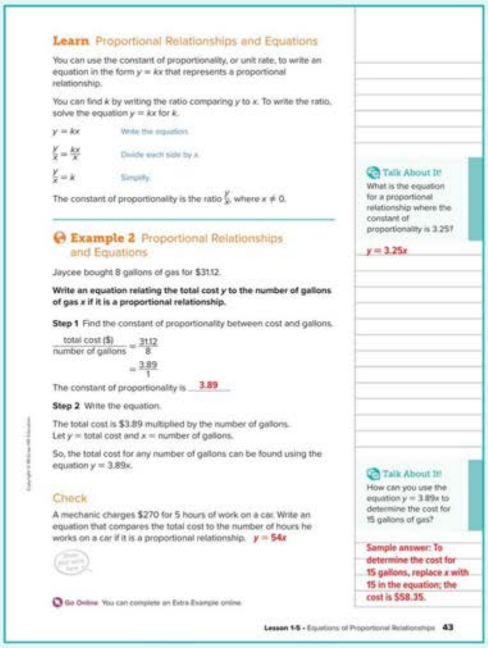
AL What is the form of an equation that represents a proportional relationship? $y = kx$

OL What is the value of k? 3.89

BL Suppose you were to graph this equation. Describe the graph. Name two points that the graph would pass through. Sample answer: The graph would be a straight line that passes through the origin (0, 0), and the unit rate (1, 3.89).

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Proportional Relationships and Equations

You can use the constant of proportionality, or unit rate, to write an equation in the form $y = kx$ that represents a proportional relationship.

You can find k by writing the ratio comparing y to x. To write the ratio, solve the equation $y = kx$ for k.

$y = kx$ Write the equation.

$\frac{y}{x} = \frac{kx}{x}$ Divide each side by x.

$\frac{y}{x} = k$ Simplify.

The constant of proportionality is the ratio $\frac{y}{x}$, where $x \neq 0$.

Talk About It! What is the equation for a proportional relationship where the constant of proportionality is 3.25? $y = 3.25x$

Example 2 Proportional Relationships and Equations

Jaycee bought 8 gallons of gas for $31.12.

Write an equation relating the total cost y to the number of gallons of gas x if it is a proportional relationship.

Step 1 Find the constant of proportionality between cost and gallons.

$\frac{\text{total cost (\$)}}{\text{number of gallons}} = \frac{31.12}{8} = \frac{3.89}{1}$

The constant of proportionality is 3.89.

Step 2 Write the equation.

The total cost is $3.89 multiplied by the number of gallons. Let y = total cost and x = number of gallons.

So, the total cost for any number of gallons can be found using the equation $y = 3.89x$.

Talk About It! How can you use the equation $y = 3.89x$ to determine the cost for 15 gallons of gas? Sample answer: To determine the cost for 15 gallons, replace x with 15 in the equation; the cost is $58.35.

Check

A mechanic charges $270 for 5 hours of work on a car. Write an equation that compares the total cost to the number of hours he works on a car if it is a proportional relationship. $y = 54x$

Go Online You can complete an Extra Example online.

Lesson 1-5 • Equations of Proportional Relationships 43

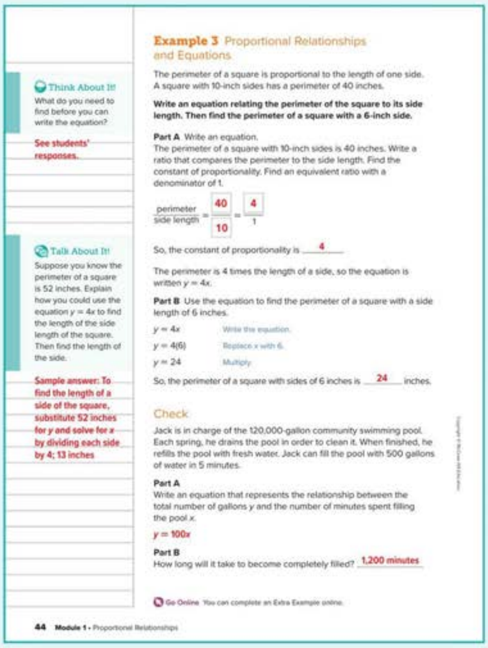
Interactive Presentation

Example 2, Proportional Relationships and Equations, Slide 3 of 5

TYPE On Slide 2, students enter the missing value to identify the constant of proportionality.

FLASHCARDS On Slide 3, students use Flashcards to see the steps for writing an equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.


Example 3 Proportional Relationships and Equations

The perimeter of a square is proportional to the length of one side. A square with 10-inch sides has a perimeter of 40 inches.

Write an equation relating the perimeter of the square to its side length. Then find the perimeter of a square with a 6-inch side.

Part A Write an equation.
The perimeter of a square with 10-inch sides is 40 inches. Write a ratio that compares the perimeter to the side length. Find the constant of proportionality. Find an equivalent ratio with a denominator of 1.

$\frac{\text{perimeter}}{\text{side length}} = \frac{40}{10} = \frac{4}{1}$

So, the constant of proportionality is 4.

The perimeter is 4 times the length of a side, so the equation is written $y = 4x$.

Part B Use the equation to find the perimeter of a square with a side length of 6 inches.

$y = 4x$ Write the equation.
$y = 4(6)$ Replace x with 6.
$y = 24$ Multiply.

So, the perimeter of a square with sides of 6 inches is 24 inches.

Think About It! What do you need to find before you can write the equation? See students' responses.

Talk About It! Suppose you know the perimeter of a square is 52 inches. Explain how you could use the equation $y = 4x$ to find the length of the side length of the square. Then find the length of the side. Sample answer: To find the length of a side of the square, substitute 52 inches for y and solve for x by dividing each side by 4; 13 inches

Check

Jack is in charge of the 120,000-gallon community swimming pool. Each spring, he drains the pool in order to clean it. When finished, he refills the pool with fresh water. Jack can fill the pool with 500 gallons of water in 5 minutes.

Part A
Write an equation that represents the relationship between the total number of gallons y and the number of minutes spent filling the pool x.
$y = 100x$

Part B
How long will it take to become completely filled? 1,200 minutes

Go Online You can complete an Extra Example online.

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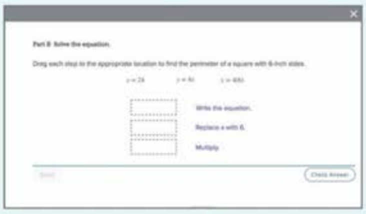
Interactive Presentation



Example 3, Proportional Relationships and Equations, Slide 3 of 5

TYPE

On Slide 2, students enter the missing value to identify the constant of proportionality.

DRAG & DROP

On Slide 3, students drag to match equations with steps in solving a problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Proportional Relationships and Equations

Objective

Students will write equations for proportional relationships to find a missing value.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the quantities given in the problem, decontextualize them, and represent them symbolically with the correct equation.

6 Attend to Precision Students should be able to clearly explain how they can use the equation to find the length of the side of the square with the given perimeter, as they respond to the *Talk about It!* question.

Questions for Mathematical Discourse

SLIDE 2

AL How many sides does a square have? What do you know about the lengths of the sides of a square? 4 sides; They are all of equal length.

OL What is the ratio of perimeter to side length? What does this number represent? 4 : 1; This number is the constant of proportionality.

BL Would the constant of proportionality change if the length of one side of the square was 20 inches? Explain. no; Sample answer: The perimeter would now be 80 inches, and the ratio of 80 to 20 would still remain 4.

SLIDE 3

AL What is the form of the equation for a proportional relationship? $y = kx$

OL What does x represent in the context of the problem? the length of the side, in inches

OL What does y represent in the context of the problem? the perimeter, in inches

BL What is the perimeter of square with a side length of 4.5 inches? 18 inches

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Running

Objective

Students will come up with their own strategy to solve an application problem involving running.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

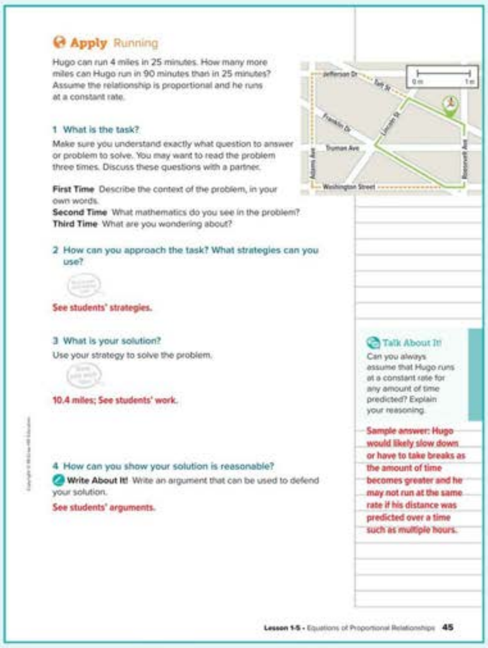
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the unit rate?
- How many miles can Hugo run in 90 minutes?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Running

Hugo can run 4 miles in 25 minutes. How many more miles can Hugo run in 90 minutes than in 25 minutes? Assume the relationship is proportional and he runs at a constant rate.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

10.4 miles; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Can you always assume that Hugo runs at a constant rate for any amount of time predicted? Explain your reasoning.

Sample answer: Hugo would likely slow down or have to take breaks as the amount of time becomes greater and he may not run at the same rate if his distance was predicted over a time such as multiple hours.

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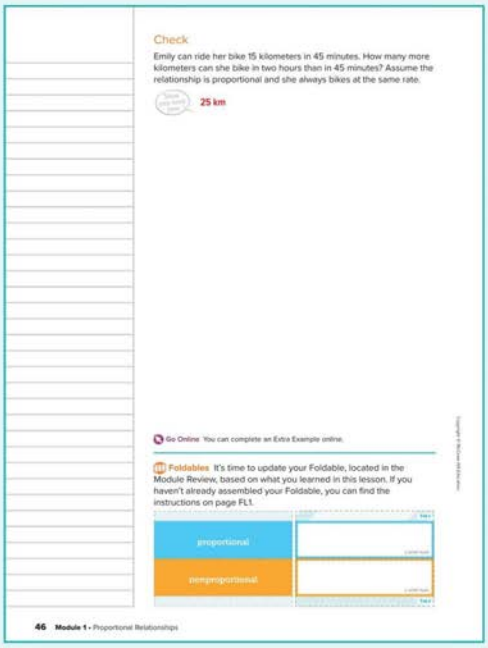
Interactive Presentation

Apply, Running

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Emily can ride her bike 15 kilometers in 45 minutes. How many more kilometers can she bike in two hours than in 45 minutes? Assume the relationship is proportional and she always bikes at the same rate.

25 km

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

proportional

nonproportional

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could add examples of proportional and nonproportional relationships expressed as equations. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What does it mean for two quantities to be in a proportional relationship?

In this lesson, students learned how to represent proportional relationships by equations. Encourage them to discuss with a partner why an equation such as $y = 2x$ indicates a proportional relationship, and why an equation such as $y = 2x + 3$ indicates a nonproportional relationship.

Exit Ticket

Refer to the Exit Ticket slide. The distance traveled by another elevator is given by the equation $y = 5x$ where x is the number of hours. What is the constant of proportionality? Explain the meaning of the constant of proportionality. 5; Sample answer: The constant of proportionality is the speed the elevator travels in miles per hour. So, the elevator travels at a speed of 5 miles per hour.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 10–13
- Extension: Hooke's Law
- ALEKS Proportions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 9
- Extension: Hooke's Law
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

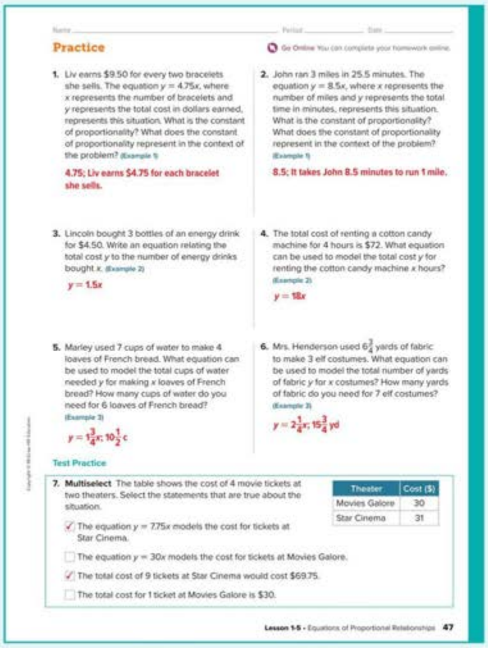
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	identify the constant of proportionality in equations	1, 2
2	write equations to represent proportional relationships	3, 4
2	write equations for proportional relationships to find a missing value	5, 6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems involving proportional relationships and equations	8, 9
3	higher-order and critical thinking skills	10–13

Common Misconception

Students may incorrectly write an equation in the form $x = ky$ or $y = \frac{1}{k}x$ rather than $y = kx$. After students calculate the constant of proportionality and write the equation, encourage them to check their work by substituting the known values into the equation. In Exercise 3, students may write the incorrect equation $y = \frac{2}{3}x$. Students can test this equation by substituting 3 for x and 4.5 for y based on the information given in the question. This results in the equation $4.5 \neq \frac{2}{3}(3)$ or $4.5 \neq 2$. Because the simplified equation is not true, the original equation must be incorrect.



Name ________ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Liv earns $9.50 for every two bracelets she sells. The equation $y = 4.75x$, where x represents the number of bracelets and y represents the total cost in dollars earned, represents this situation. What is the constant of proportionality? What does the constant of proportionality represent in the context of the problem? (Example 1)

   4.75; Liv earns $4.75 for each bracelet she sells.

2. John ran 3 miles in 25.5 minutes. The equation $y = 8.5x$, where x represents the number of miles and y represents the total time in minutes, represents this situation. What is the constant of proportionality? What does the constant of proportionality represent in the context of the problem? (Example 1)

   8.5; It takes John 8.5 minutes to run 1 mile.

3. Lincoln bought 3 bottles of an energy drink for $4.50. Write an equation relating the total cost y to the number of energy drinks bought x. (Example 2)

   $y = 1.5x$

4. The total cost of renting a cotton candy machine for 4 hours is $72. What equation can be used to model the total cost y for renting the cotton candy machine x hours? (Example 2)

   $y = 18x$

5. Marley used 7 cups of water to make 4 loaves of French bread. What equation can be used to model the total cups of water needed y for making x loaves of French bread? How many cups of water do you need for 6 loaves of French bread? (Example 3)

   $y = 1\frac{3}{4}x$; $10\frac{1}{2}$ c

6. Mrs. Henderson used $6\frac{3}{4}$ yards of fabric to make 3 elf costumes. What equation can be used to model the total number of yards of fabric y for x costumes? How many yards of fabric do you need for 7 elf costumes? (Example 3)

   $y = 2\frac{1}{4}x$; $15\frac{3}{4}$ yd

Test Practice

7. **Multiselect** The table shows the cost of 4 movie tickets at two theaters. Select the statements that are true about the situation.

Theater	Cost ($)
Movies Galore	30
Star Cinema	31

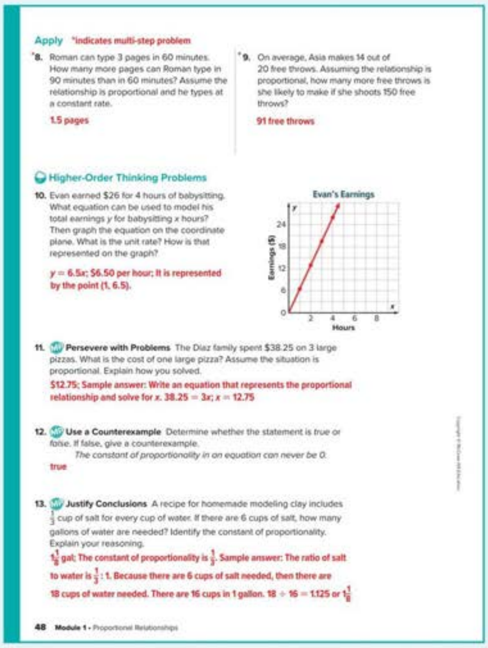
- [x] The equation $y = 7.75x$ models the cost for tickets at Star Cinema.
- [ ] The equation $y = 30x$ models the cost for tickets at Movies Galore.
- [x] The total cost of 9 tickets at Star Cinema would cost $69.75.
- [ ] The total cost for 1 ticket at Movies Galore is $30.

Lesson 1-5 • Equations of Proportional Relationships 47

3 REFLECT AND PRACTICE

7.RP.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*8. Roman can type 3 pages in 60 minutes. How many more pages can Roman type in 90 minutes than in 60 minutes? Assume the relationship is proportional and he types at a constant rate.

1.5 pages

*9. On average, Asia makes 14 out of 20 free throws. Assuming the relationship is proportional, how many more free throws is she likely to make if she shoots 150 free throws?

91 free throws

Higher-Order Thinking Problems

10. Evan earned $26 for 4 hours of babysitting. What equation can be used to model his total earnings y for babysitting x hours? Then graph the equation on the coordinate plane. What is the unit rate? How is that represented on the graph?

$y = 6.5x$; $6.50 per hour; It is represented by the point (1, 6.5).

11. **Persevere with Problems** The Diaz family spent $38.25 on 3 large pizzas. What is the cost of one large pizza? Assume the situation is proportional. Explain how you solved.

$12.75; Sample answer: Write an equation that represents the proportional relationship and solve for x. $38.25 = 3x$; $x = 12.75$

12. **Use a Counterexample** Determine whether the statement is true or false. If false, give a counterexample.

The constant of proportionality in an equation can never be 0.

true

13. **Justify Conclusions** A recipe for homemade modeling clay includes $\frac{1}{3}$ cup of salt for every cup of water. If there are 6 cups of salt, how many gallons of water are needed? Identify the constant of proportionality. Explain your reasoning.

$1\frac{1}{8}$ gal; The constant of proportionality is $\frac{1}{3}$. Sample answer: The ratio of salt to water is $\frac{1}{3}$: 1. Because there are 6 cups of salt needed, then there are 18 cups of water needed. There are 16 cups in 1 gallon. $18 \div 16 = 1.125$ or $1\frac{1}{8}$

48 Module 1 • Proportional Relationships

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 11, students need to make sense of the problem by understanding what it means to be proportional.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students need to determine the truth of a statement, and use a counterexample to justify their reasoning if the statement is false.
In Exercise 13, students must justify their reasoning as they calculate a constant of proportionality and find the number of gallons of water needed.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercise 8–9 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 8 might be “At this rate, Roman can type 6 pages in 2 hours.” An example of a false statement might be “At this rate, Roman can type 9 pages in 2.5 hours.” Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Clearly explain your strategy.
Use with Exercise 13 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain to their partner their strategy for how they would find the number of gallons of water needed, without actually solving the problem. Have each student use their partner’s strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Lesson 1-6

7.RP.A.2, 7.RP.A.3

Solve Problems Involving Proportional Relationships

LESSON GOAL

Students will solve problems involving proportional relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Proportions
Example 1: Solve Problems Involving Proportional Relationships
Example 2: Solve Problems Involving Proportional Relationships
Apply: Blood Drives

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Solve Proportions Involving Two-Step Equations		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 6 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving proportional relationships.

ELL You can use the tips and suggestions on page T6 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7. RP.A** by solving problems involving proportional relationships.
Standards for Mathematical Content: **7. RP.A.2, 7.RP.A.3**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students wrote equations to represent proportional relationships.
7.RP.A.2, 7.RP.A.2.B, 7.RP.A.2.C

Now
Students solve problems involving proportional relationships.
7.RP.A.2, 7.RP.A.3

Next
Students will use proportional relationships to solve percent problems.
7.RP.A.3

Rigor

The Three Pillars of Rigor

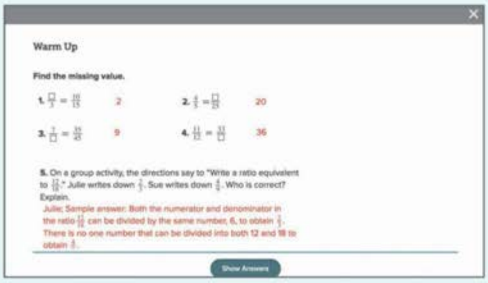
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their *understanding* of proportional relationships by writing and solving problems involving proportional relationships. They build *fluency* by representing a relationship using a proportion, and then solving the problem. They *apply* their understanding of proportions to solve real-world problems.

Mathematical Background

A *proportion* is an equation stating that two ratios are equivalent. If two quantities are in a proportional relationship, you can write and use a proportion to find missing values. Suppose a recipe calls for 3 cups of flour for every 2 cups of sugar. You have 4 cups of sugar. You can use the proportion $\frac{3}{2} = \frac{f}{4}$ to find the number of cups of flour f needed for 4 cups of sugar. You can solve the proportion for f by using ratio reasoning. Because $2 \times 2 = 4$, multiply 3 by 2 to find f, which is 6. So, 6 cups of flour are needed.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 1

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding equivalent ratios (Exercises 1–5)

Answers

1. 2

2. 20

3. 9

4. 36

5. Julie; Sample answer: Both the numerator and denominator in the ratio $\frac{12}{18}$ can be divided by the same number, 6, to obtain $\frac{2}{3}$. There is no one number that can be divided into both 12 and 18 to obtain $\frac{4}{9}$.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using information from an infographic on the Golden Ratio to find forearm length.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What other term have you learned in this module that might relate to the term *proportion*? What does that term mean? Sample answer: proportional; Two quantities are proportional if they vary and have a constant ratio between them.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Proportions

Objective

Students will understand how to make a table, use a graph, or write an equation to solve problems involving proportional relationships.

MP Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language in order to accurately explain how each representation shows the unit rate.

Teaching Notes

SLIDES 1–2

Students also learned how to write an equation, in the form $y = kx$, to represent a proportional relationship where k is the constant of proportionality (unit rate). In this lesson, students will learn about another equation they can write to represent proportional relationships. This type of equation is called a *proportion* and it states that two ratios are equivalent.

Ask students to study the proportion $\frac{3}{4} = \frac{6}{8}$. Ask them to explain how they know that $\frac{6}{8}$ is equal to $\frac{3}{4}$. They should notice that by multiplying both the numerator 3 and the denominator 4 of the ratio $\frac{3}{4}$ by 2, they obtain the ratio $\frac{6}{8}$. Because they multiplied both quantities by the same number, the ratio $\frac{3}{4}$ was maintained.

Have students study the multiple representations table to look for correspondences between each representation (words, ratio table, graph, and example) using the equation $y = \frac{3}{4}x$. Have them explain how each representation is a valid method for finding the number of cups of milk needed if 3 cups of flour are used in the recipe.

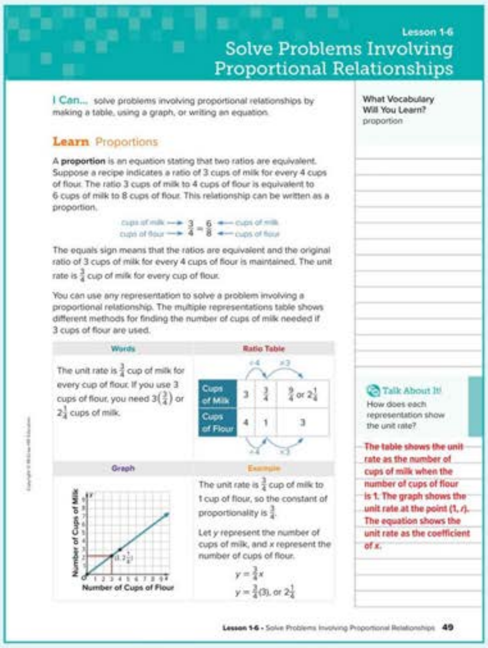
Go Online to find the *Talk About It!* question to promote mathematical discourse.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of proportions, ask them to work with a partner to determine how they can write a proportion and use it to find the number of cups of milk needed if 3 cups of flour are used. Then have them share their proportions and methods with another pair of students or the entire class. A sample method is shown.

$\frac{3}{4} = \frac{m}{3}$	Write a proportion.
$3\left(\frac{3}{4}\right) = \frac{m}{3}$	Multiply each side by 3 to find *m*.
$\frac{9}{4} = m$	Simplify.
$2\frac{1}{4} = m$	Simplify. $2\frac{1}{4}$ cups of milk are needed.



Lesson 1-6

Solve Problems Involving Proportional Relationships

I Can... solve problems involving proportional relationships by making a table, using a graph, or writing an equation.

What Vocabulary Will You Learn?
proportion

Learn Proportions

A **proportion** is an equation stating that two ratios are equivalent. Suppose a recipe indicates a ratio of 3 cups of milk for every 4 cups of flour. The ratio 3 cups of milk to 4 cups of flour is equivalent to 6 cups of milk to 8 cups of flour. This relationship can be written as a proportion.

cups of milk → $\frac{3}{4} = \frac{6}{8}$ ← cups of milk
cups of flour → ← cups of flour

The equals sign means that the ratios are equivalent and the original ratio of 3 cups of milk for every 4 cups of flour is maintained. The unit rate is $\frac{3}{4}$ cup of milk for every cup of flour.

You can use any representation to solve a problem involving a proportional relationship. The multiple representations table shows different methods for finding the number of cups of milk needed if 3 cups of flour are used.

Words: The unit rate is $\frac{3}{4}$ cup of milk for every cup of flour. If you use 3 cups of flour, you need $3\left(\frac{3}{4}\right)$ or $2\frac{1}{4}$ cups of milk.

Ratio Table

Cups of Milk	3	$\frac{3}{4}$	$\frac{9}{4}$ or $2\frac{1}{4}$
Cups of Flour	4	1	3

Graph

Example: The unit rate is $\frac{3}{4}$ cup of milk to 1 cup of flour, so the constant of proportionality is $\frac{3}{4}$.

Let y represent the number of cups of milk, and x represent the number of cups of flour.

$y = \frac{3}{4}x$
$y = \frac{3}{4}(3)$, or $2\frac{1}{4}$

Talk About It! How does each representation show the unit rate?

The table shows the unit rate as the number of cups of milk when the number of cups of flour is 1. The graph shows the unit rate at the point (1, *r*). The equation shows the unit rate as the coefficient of *x*.

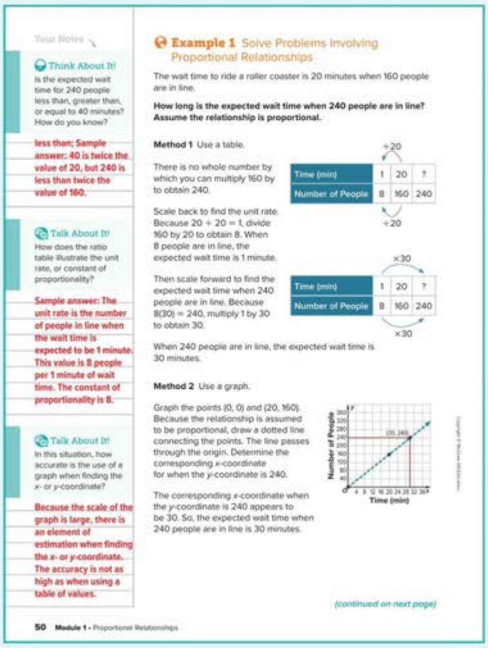
Lesson 1-6 • Solve Problems Involving Proportional Relationships 49

Interactive Presentation

Learn, Proportions, Slide 2 of 3

FLASHCARDS

On Slide 2, students use Flashcards to learn how to solve a problem using multiple representations.



Your Notes

Think About It!
Is the expected wait time for 240 people less than, greater than, or equal to 40 minutes? How do you know?

less than; Sample answer: 40 is twice the value of 20, but 240 is less than twice the value of 160.

Talk About It!
How does the ratio table illustrate the unit rate, or constant of proportionality?

Sample answer: The unit rate is the number of people in line when the wait time is expected to be 1 minute. This value is 8 people per 1 minute of wait time. The constant of proportionality is 8.

Talk About It!
In this situation, how accurate is the use of a graph when finding the x- or y-coordinate?

Because the scale of the graph is large, there is an element of estimation when finding the x- or y-coordinate. The accuracy is not as high as when using a table of values.

Example 1 Solve Problems Involving Proportional Relationships

The wait time to ride a roller coaster is 20 minutes when 160 people are in line.

How long is the expected wait time when 240 people are in line? Assume the relationship is proportional.

Method 1 Use a table.

There is no whole number by which you can multiply 160 by to obtain 240.

Time (min)	1	20	?
Number of People	8	160	240

Scale back to find the unit rate. Because $20 \div 20 = 1$, divide 160 by 20 to obtain 8. When 8 people are in line, the expected wait time is 1 minute.

Then scale forward to find the expected wait time when 240 people are in line. Because $8(30) = 240$, multiply 1 by 30 to obtain 30.

Time (min)	1	20	?
Number of People	8	160	240

When 240 people are in line, the expected wait time is 30 minutes.

Method 2 Use a graph.

Graph the points (0, 0) and (20, 160). Because the relationship is assumed to be proportional, draw a dotted line connecting the points. The line passes through the origin. Determine the corresponding x-coordinate for when the y-coordinate is 240.

The corresponding x-coordinate when the y-coordinate is 240 appears to be 30. So, the expected wait time when 240 people are in line is 30 minutes.

(continued on next page)

50 Module 1 • Proportional Relationships

Interactive Presentation

Example 1, Solve Problems Involving Proportional Relationships, Slide 2 of 6

CLICK

On Slide 2, students move through slides to see how a table can be used.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Solve Problems Involving Proportional Relationships

Objective

Students will solve problems involving proportional relationships using a table, a graph, or an equation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively, 5 Use Appropriate Tools Strategically Encourage students to analyze each representation of the proportional relationship and reason how each illustrates the unit rate. Have them explain how accurate using the graph is when finding the desired coordinates.

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to pay careful attention to the scale.

Questions for Mathematical Discourse

SLIDE 2

AL What is the greatest factor 20 and 160 have in common? 20

OL How is the unit rate represented in the table? What does the unit rate mean? By scaling back to 8 people per minute, the unit rate is found to be 8 people per minute. This means that when 8 people are in line, the expected wait time is 1 minute.

OL Is there a way to solve the problem without scaling backward first? What is the advantage to scaling backward first? yes; Sample answer: Scale from 160 to 240 by multiplying by 1.5. Scaling backward helps to find the unit rate. Knowing the unit rate means you can solve other problems, other than this one.

BL Why is 30 minutes the expected wait time and not the actual wait time? Sample answer: The unit rate of 8 people per minute is assuming the ratio of people to wait time is constant. In real life, the wait times may vary.

SLIDE 3

AL What do the points (0, 0) and (20, 160) represent? The point (0,0) represents a wait time of 0 minutes when 0 people are in line. The point (20, 160) represents a wait time of 20 minutes when 160 people are in line.

OL How do you know the point (30, 240) will lie on the same line as (0, 0) and (20, 160)? Sample answer: Because the relationship is proportional, the graph that represents the relationship is a straight line. The relationship between the x-values and y-values will always be constant.

BL How can you use the graph to estimate the number of people in line when the wait time is 15 minutes? Sample answer: I can find the corresponding y-value when the x-value is 15. 120 people

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Solve Problems Involving Proportional Relationships *(continued)*

Teaching Notes

Another way to solve problems involving proportional relationships is to use cross products. You may or may not choose to show this method to students. While using cross products may be more efficient, it does not convey the meaningful quantities in relation to the context of the problem.

In the proportion, the ratios $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent. The process of setting them equal to each other creates a proportion. The cross products of 3×8 and 4×6 are both 24, so the cross products are equal. This is one way to show a proportion. This method, however, does not fully explain why the ratios are equivalent or that the equivalent ratios were maintained.

Questions for Mathematical Discourse

SLIDE 4

AL What does the variable k represent? Why is it a variable and not a known number? Sample answer: k represents the constant of proportionality, or unit rate, of people per minute. It is a variable because we do not know its value.

OL After solving for k, how can you check your work? Sample answer: I can create a table of values to determine if the number of people in line is 8 times the number of minutes waited.

OL Describe an advantage of using an equation. Sample answer: By writing an equation, I can use it to find other values, not just the one asked for in this Example.

BL Suppose a classmate wrote the equation $20 = k(160)$. Is this a correct equation? Explain. no; Sample answer: The unit rate k is the number of people per minute. If k is the coefficient of 160, then it is saying that the wait time is 160 minutes.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Method 3 Use an equation.

The equation that represents a proportional relationship is of the form $y = kx$, where k is the constant of proportionality, or unit rate.

Find the unit rate, the number of people in line when the expected wait time is 1 minute. Let y represent the number of people in line and x represent the number of minutes.

$y = kx$ — Write the equation.

$160 = k(20)$ — Replace y with 160 and x with 20.

$\frac{160}{20} = \frac{k(20)}{20}$ — Divide each side by 20 to find the value of k.

$8 = k$ — Simplify. The unit rate, or constant of proportionality, is 8.

When 8 people are in line, the expected wait time is 1 minute. Use the equation $y = 8x$ to find the expected wait time when 240 people are in line.

$y = 8x$ — Write the equation. The constant of proportionality is 8.

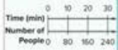
$240 = 8x$ — Replace y with 240.

$\frac{240}{8} = \frac{8x}{8}$ — Divide each side by 8 to find the value of x.

$30 = x$ — Simplify.

So, the expected wait time when 240 people are in line is 30 minutes.

Talk About It! Suppose a classmate drew the double number line shown to represent and solve this problem. Is this a valid method? Explain. Does this method show the unit rate? Is the unit rate necessary to solve the problem? Explain.



Sample answer: The double number line is a valid method. Each number line is divided into equal-size increments of 10 minutes and 80 people. The number line shows that when 240 people are in line, the expected wait time is 30 minutes. The unit rate, 8 people per 1 minute, is not shown on the number line, but because increments are of equal size, you do not need it to solve the problem.

Check

Matthew paid $49.45 for five used video games of equal cost. The relationship between the number of video games and the total cost is proportional. What is the total cost for 11 used video games? Use any strategy. $108.79

Go Online You can complete an Extra Example online.

Lesson 1-6 • Solve Problems Involving Proportional Relationships 51

Interactive Presentation

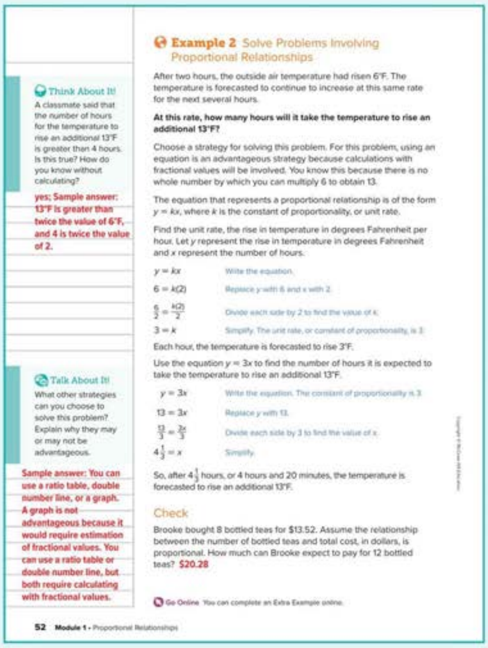
Example 1, Solve Problems Involving Proportional Relationships, Slide 4 of 6

CLICK

On Slide 4, students move through the slides to see how to use an equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 2 Solve Problems Involving Proportional Relationships

After two hours, the outside air temperature had risen 6°F. The temperature is forecasted to continue to increase at this same rate for the next several hours.

At this rate, how many hours will it take the temperature to rise an additional 13°F?

Choose a strategy for solving this problem. For this problem, using an equation is an advantageous strategy because calculations with fractional values will be involved. You know this because there is no whole number by which you can multiply 6 to obtain 13.

The equation that represents a proportional relationship is of the form $y = kx$, where k is the constant of proportionality, or unit rate.

Find the unit rate, the rise in temperature in degrees Fahrenheit per hour. Let y represent the rise in temperature in degrees Fahrenheit and x represent the number of hours.

$y = kx$ Write the equation.

$6 = k(2)$ Replace y with 6 and x with 2.

$\frac{6}{2} = \frac{k(2)}{2}$ Divide each side by 2 to find the value of k.

$3 = k$ Simplify. The unit rate, or constant of proportionality, is 3.

Each hour, the temperature is forecasted to rise 3°F.

Use the equation $y = 3x$ to find the number of hours it is expected to take the temperature to rise an additional 13°F.

$y = 3x$ Write the equation. The constant of proportionality is 3.

$13 = 3x$ Replace y with 13.

$\frac{13}{3} = \frac{3x}{3}$ Divide each side by 3 to find the value of x.

$4\frac{1}{3} = x$ Simplify.

So, after $4\frac{1}{3}$ hours, or 4 hours and 20 minutes, the temperature is forecasted to rise an additional 13°F.

Think About It! A classmate said that the number of hours for the temperature to rise an additional 13°F is greater than 4 hours. Is this true? How do you know without calculating? **yes; Sample answer: 13°F is greater than twice the value of 6°F, and 4 is twice the value of 2.**

Talk About It! What other strategies can you choose to solve this problem? Explain why they may or may not be advantageous. **Sample answer: You can use a ratio table, double number line, or a graph. A graph is not advantageous because it would require estimation of fractional values. You can use a ratio table or double number line, but both require calculating with fractional values.**

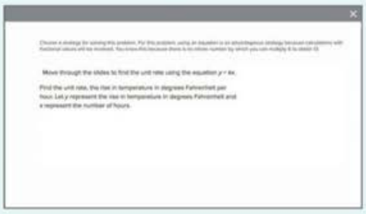
Check

Brooke bought 8 bottled teas for $13.52. Assume the relationship between the number of bottled teas and total cost, in dollars, is proportional. How much can Brooke expect to pay for 12 bottled teas? $20.28

Go Online You can complete an Extra Example online.

52 Module 1 • Proportional Relationships

Interactive Presentation



Example 2, Solve Problems Involving Proportional Relationships, Slide 2 of 5

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 2 Solve Problems Involving Proportional Relationships

Objective

Students will choose a strategy for solving a problem involving a proportional relationship, such as writing an equation.

Questions for Mathematical Discourse

SLIDE 2

AL In the equation $y = kx$, what does the variable y represent? x? y represents the rise in temperature; x represents the number of hours

OL What is the constant of proportionality, or unit rate? 3 degrees per hour

OL How can you use a different representation to check your answer? Sample answer: I can create a table of equivalent ratios and scale backward to determine the number of degrees per hour.

BL A classmate found the constant of proportionality to be $\frac{1}{3}$. What was the likely error? Sample answer: The classmate confused the dependent and independent variables. Time is the independent variable.

SLIDE 3

AL In the equation $y = kx$, what does the variable k represent? the constant of proportionality, 3

OL How can you check your answer? Sample answer: I can multiply the unit rate, 3, by the number of hours, $4\frac{1}{3}$, to determine the number of degrees, 13.

OL Why is $4\frac{1}{3}$ equal to 4 hours and 20 minutes? $\frac{1}{3}$ of an hour, or 60 minutes, is 20 minutes

BL Choose a strategy to find the number of degrees the expected outside temperature will have risen after 5 hours. Explain. Sample answer: I chose the equation because I already know the value of k, which is 3; $y = 3x$; when $x = 5$, $y = 3(5)$, or 15. The temperature is expected to rise an additional 15°F after 5 hours.

BL Will the temperature always continue to rise at this rate? Explain. no; Sample answer: Temperatures fluctuate throughout the day due to changes in weather patterns and time of day or night.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the checks.
- Assign or present an Extra Example.

Apply Blood Drives

Objective

Students will come up with their own strategy to solve an application problem involving blood drives.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

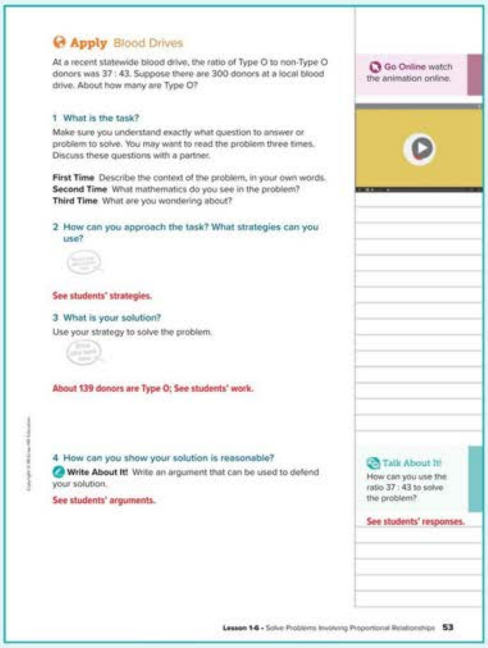
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the ratio of Type O donors to the total donors?
- What is the unit rate of total donors per Type O donor?
- How can you set up a table or equation to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Blood Drives

At a recent statewide blood drive, the ratio of Type O to non-Type O donors was 37 : 43. Suppose there are 300 donors at a local blood drive. About how many are Type O?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

About 139 donors are Type O; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

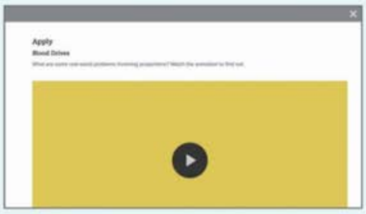
Go Online watch the animation online.

Talk About It! How can you use the ratio 37 : 43 to solve the problem?

See students' responses.

Lesson 1-6 • Solve Problems Involving Proportional Relationships 53

Interactive Presentation



Apply, Blood Drives

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The ratio of girls to boys in a school is 2 : 3. The relationship between the number of girls and the number of boys is proportional. How many boys are there if there are 345 students in the school?

207 boys

Go Online You can complete an Extra Example online.

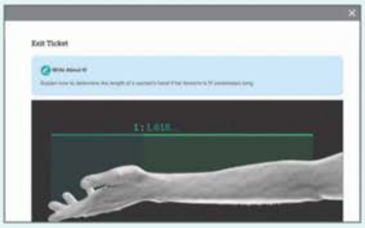
Pause and Reflect

Compare and contrast the different methods presented in this lesson for solving problems involving proportional relationships.

See students' observations.

54 Module 1 • Proportional Relationships

Interactive Presentation



Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Explain how to determine the length of a woman's hand if her forearm is 17 centimeters long.

Sample answer: Find the unit rate using the Golden Ratio. The Golden Ratio is $\frac{\text{human forearm length}}{\text{human hand length}} \approx \frac{1.618}{1}$. So, the unit rate is 1.618. Solve the equation $17 = 1.618x$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–11 odd, 12–15
- Extension: Solve Proportions Involving Two-Step Equations
- ALEKS® Proportions

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 4–7, 11, 12, 14
- Extension: Solve Proportions Involving Two-Step Equations
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Ratios and Unit Rates

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ALEKS® Ratios and Unit Rates

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

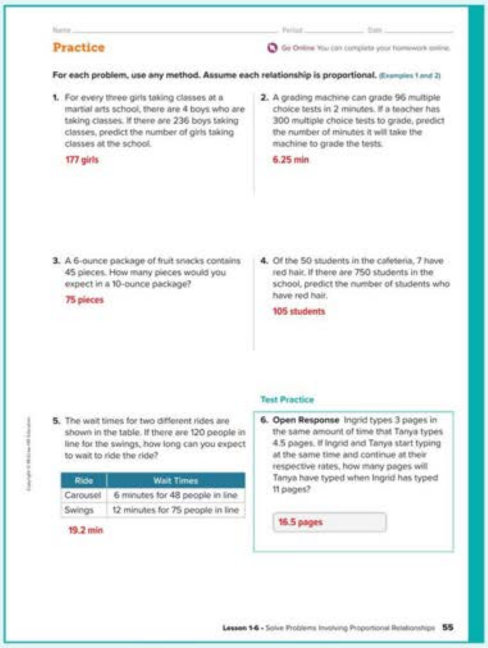
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	solve problems involving proportional relationships using any strategy	1–4
2	extend concepts learned in class to apply them in new contexts	5, 6
3	solve application problems involving proportional relationships	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

Students may incorrectly think of unit rate as an additive relationship. For example, in Exercise 6, students may assume that Tanya has typed 1.5 more pages than Ingrid at all stages. Students may think, then, that Tanya will have typed 12.5 pages in the same amount of time that Ingrid typed 11 pages. Encourage students to use a method learned earlier in the module, such as double number lines, bar diagrams, or ratio tables, to check their work.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

For each problem, use any method. Assume each relationship is proportional. (Examples 1 and 2)

1. For every three girls taking classes at a martial arts school, there are 4 boys who are taking classes. If there are 236 boys taking classes, predict the number of girls taking classes at the school.
177 girls

2. A grading machine can grade 96 multiple choice tests in 2 minutes. If a teacher has 300 multiple choice tests to grade, predict the number of minutes it will take the machine to grade the tests.
6.25 min

3. A 6-ounce package of fruit snacks contains 45 pieces. How many pieces would you expect in a 10-ounce package?
75 pieces

4. Of the 50 students in the cafeteria, 7 have red hair. If there are 750 students in the school, predict the number of students who have red hair.
105 students

5. The wait times for two different rides are shown in the table. If there are 120 people in line for the swings, how long can you expect to wait to ride the ride?

Ride	Wait Times
Carousel	6 minutes for 48 people in line
Swings	12 minutes for 75 people in line

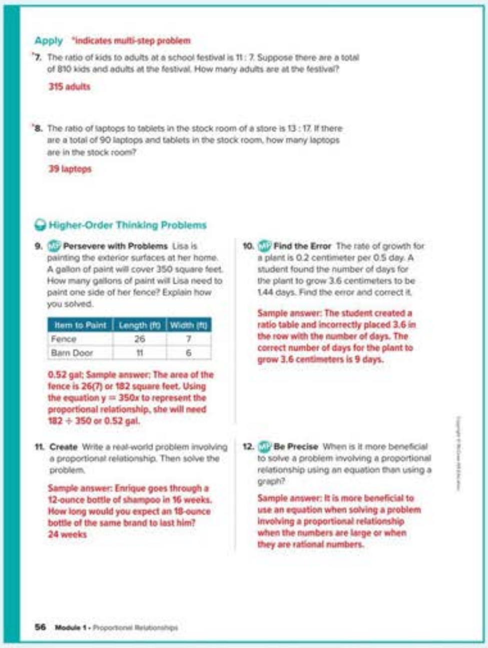
19.2 min

Test Practice

6. **Open Response** Ingrid types 3 pages in the same amount of time that Tanya types 4.5 pages. If Ingrid and Tanya start typing at the same time and continue at their respective rates, how many pages will Tanya have typed when Ingrid has typed 11 pages?
16.5 pages

Lesson 1-6 • Solve Problems Involving Proportional Relationships 55

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*7. The ratio of kids to adults at a school festival is 11 : 7. Suppose there are a total of 810 kids and adults at the festival. How many adults are at the festival?

315 adults

*8. The ratio of laptops to tablets in the stock room of a store is 13 : 17. If there are a total of 90 laptops and tablets in the stock room, how many laptops are in the stock room?

39 laptops

Higher-Order Thinking Problems

9. **MP Persevere with Problems** Lisa is painting the exterior surfaces at her home. A gallon of paint will cover 350 square feet. How many gallons of paint will Lisa need to paint one side of her fence? Explain how you solved.

Item to Paint	Length (ft)	Width (ft)
Fence	26	7
Barn Door	11	6

0.52 gal; Sample answer: The area of the fence is 26(7) or 182 square feet. Using the equation $y = 350x$ to represent the proportional relationship, she will need $182 \div 350$ or 0.52 gal.

10. **MP Find the Error** The rate of growth for a plant is 0.2 centimeter per 0.5 day. A student found the number of days for the plant to grow 3.6 centimeters to be 1.44 days. Find the error and correct it.

Sample answer: The student created a ratio table and incorrectly placed 3.6 in the row with the number of days. The correct number of days for the plant to grow 3.6 centimeters is 9 days.

11. **Create** Write a real-world problem involving a proportional relationship. Then solve the problem.

Sample answer: Enrique goes through a 12-ounce bottle of shampoo in 16 weeks. How long would you expect an 18-ounce bottle of the same brand to last him? 24 weeks

12. **MP Be Precise** When is it more beneficial to solve a problem involving a proportional relationship using an equation than using a graph?

Sample answer: It is more beneficial to use an equation when solving a problem involving a proportional relationship when the numbers are large or when they are rational numbers.

56 Module 1 • Proportional Relationships

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 9, students will need to identify important information, find the area of the fence, and determine how many gallons of paint are needed.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students analyze a student's solution in order to both diagnose and correct the error.

6 Attend to Precision In Exercise 12, students use clear and precise mathematical language to explain when it is more beneficial to use one method over another.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 7–8 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 7 might be "What is the ratio of adults to total number of people?"

Solve the problem another way.
Use with Exercise 9 Have students work in groups of 3–4. After completing Exercise 9, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used. Have students compare and contrast the different methods, and determine if each method is viable. If the methods were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class and explain why each method is a correct method.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of proportional relationships written as tables, graphs, and equations. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

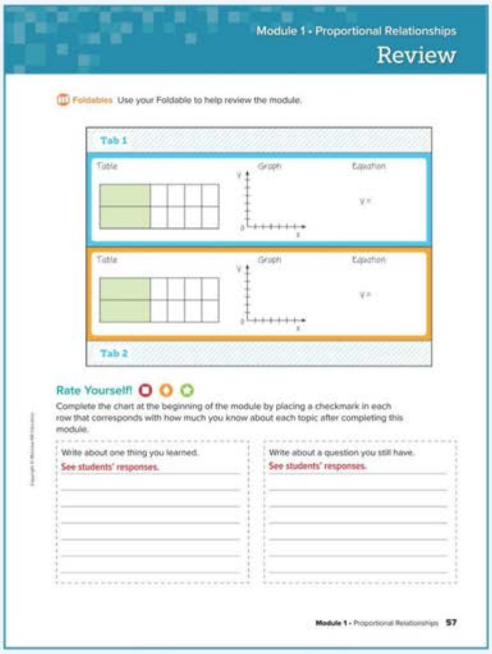
Assessment Resources

Put It All Together 1: Lessons 1-3 through 1-5
Put It All Together 2: Lesson 1-6
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Ratios and Proportional Relationships**.

- Unit Rate
- Proportional Relationships
- Equations of Proportional Relationships
- Applications of Proportional Relationships


Module 1 • Proportional Relationships

Review

Foldables Use your Foldable to help review the module.

Tab 1

Table Graph Equation

y =

Table Graph Equation

y =

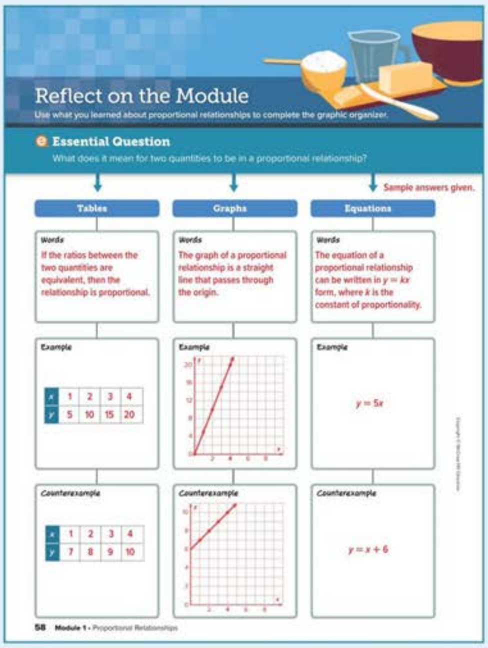
Tab 2

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.	Write about a question you still have.
See students' responses.	See students' responses.

Module 1 • Proportional Relationships 57



Reflect on the Module

Use what you learned about proportional relationships to complete the graphic organizer.

Essential Question

What does it mean for two quantities to be in a proportional relationship?

Sample answers given.

Tables

Words: If the ratios between the two quantities are equivalent, then the relationship is proportional.

Example:

x	1	2	3	4
y	5	10	15	20

Counterexample:

x	1	2	3	4
y	7	8	9	10

Graphs

Words: The graph of a proportional relationship is a straight line that passes through the origin.

Example:

Counterexample:

Equations

Words: The equation of a proportional relationship can be written in $y = kx$ form, where k is the constant of proportionality.

Example: $y = 5x$

Counterexample: $y = x + 6$

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

What does it mean for two quantities to be in a proportional relationship? See students' graphic organizers.

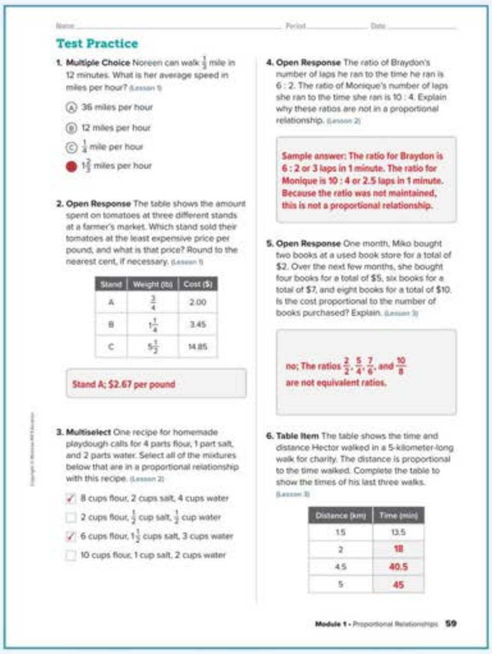
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–10 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1
Multiselect	Multiple answers may be correct. Students must select all correct answers.	3, 7
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	9, 10
Table Item	Students complete a table by entering in the correct values.	6
Grid	Students create a graph on an online coordinate plane.	8
Open Response	Students construct their own response in the area provided.	2, 4, 5

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.RP.A.1	1-1	1, 2
7.RP.A.2	1-2, 1-3, 1-4, 1-5, 1-6	5–10
7.RP.A.2.A	1-3, 1-4	5–8
7.RP.A.2.B	1-3, 1-4, 1-5	5–9
7.RP.A.2.C	1-5	9
7.RP.A.2.D	1-4	7, 8
7.RP.A.3	1-6	10



Name ________ Period ________ Date ________

Test Practice

1. **Multiple Choice** Noreen can walk $\frac{1}{3}$ mile in 12 minutes. What is her average speed in miles per hour? (Lesson 1)
   - (A) 36 miles per hour
   - (B) 12 miles per hour
   - (C) $\frac{1}{4}$ mile per hour
   - (D) $1\frac{2}{3}$ miles per hour

2. **Open Response** The table shows the amount spent on tomatoes at three different stands at a farmer's market. Which stand sold their tomatoes at the least expensive price per pound, and what is that price? Round to the nearest cent, if necessary. (Lesson 1)

Stand	Weight (lb)	Cost ($)
A	$\frac{3}{4}$	2.00
B	$1\frac{1}{4}$	3.45
C	$5\frac{1}{2}$	14.85

Stand A; $2.67 per pound

3. **Multiselect** One recipe for homemade playdough calls for 4 parts flour, 1 part salt, and 2 parts water. Select all of the mixtures below that are in a proportional relationship with this recipe. (Lesson 2)
   - ☑ 8 cups flour, 2 cups salt, 4 cups water
   - ☐ 2 cups flour, $\frac{1}{2}$ cup salt, $\frac{1}{2}$ cup water
   - ☑ 6 cups flour, $1\frac{1}{2}$ cups salt, 3 cups water
   - ☐ 10 cups flour, 1 cup salt, 2 cups water

4. **Open Response** The ratio of Braydon's number of laps he ran to the time he ran is 6 : 2. The ratio of Monique's number of laps she ran to the time she ran is 10 : 4. Explain why these ratios are not in a proportional relationship. (Lesson 2)

Sample answer: The ratio for Braydon is 6 : 2 or 3 laps in 1 minute. The ratio for Monique is 10 : 4 or 2.5 laps in 1 minute. Because the ratio was not maintained, this is not a proportional relationship.

5. **Open Response** One month, Miko bought two books at a used book store for a total of $2. Over the next few months, she bought four books for a total of $5, six books for a total of $7, and eight books for a total of $10. Is the cost proportional to the number of books purchased? Explain. (Lesson 3)

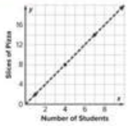
no; The ratios $\frac{2}{2}$, $\frac{5}{4}$, $\frac{7}{6}$, and $\frac{10}{8}$ are not equivalent ratios.

6. **Table Item** The table shows the time and distance Hector walked in a 5-kilometer-long walk for charity. The distance is proportional to the time walked. Complete the table to show the times of his last three walks. (Lesson 3)

Distance (km)	Time (min)
1.5	13.5
2	18
4.5	40.5
5	45

Module 1 • Proportional Relationships **59**

7. **Multiselect** The relationship between the number of slices of pizza purchased and the number of students served is shown in the graph. Select all of the statements that are true.



- [x] The relationship is proportional.
- [x] The point (9, 18) satisfies this relationship.
- [ ] The constant of proportionality is $\frac{1}{2}$.
- [x] The constant of proportionality is 2.

8. **Grid** The cost of dance lessons is \$12 for 1 lesson, \$22 for 2 lessons, and \$32 for 3 lessons.

A. Graph the ordered pairs on the coordinate plane.

B. Determine whether the cost is proportional to the number of lessons. Explain your reasoning.

The ratios of lesson to cost are not equivalent, so the cost is not proportional to the number of lessons.

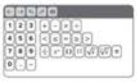
9. **Equation Editor** Mrs. Jameson paid \$202.50 for a group of 9 students to visit an amusement park.

A. Write an equation relating the total cost y and the number of students x attending the park.

$y = 22.5x$

B. What would be the total cost if four more students wanted to join the group?

\$292.50



10. **Equation Editor** A homeowner whose house was assessed at \$120,000 pays \$1,800 in taxes. At the same rate, what is the tax on a house assessed at \$135,000?

2,025

60 **Module 1** • Proportional Relationships

Module 2
Solve Percent Problems

Module Goal
Solve multi-step percent problems.

Focus
Domain: Ratios and Proportional Relationships
Major Cluster(s): **7.RP.A** Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.EE.A Use properties of operations to generate equivalent expressions.
Standards for Mathematical Content:
7.RP.A.3 Use p roportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *Also addresses 7.EE.B.3.*
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover
Students need to have a thorough understanding of the prerequisite skills required for this module.

- operations with decimals and percents
- finding the percent of a number
- finding the whole, given the part and the percent

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence
Vertical Alignment

Previous
Students used ratio and rate reasoning to find the percent of a number, and to find the whole, given the part and the percent.
6.RP.A.3

Now
Students solve multi-step ratio and percent problems.
7.RP.A.3, 7.EE.A.2

Next
Students will use ratios to find the probability of an event occurring.
7.SP.C.7

Rigor
The Three Pillars of Rigor
In this module, students draw on their *understanding* of proportional relationships to build *fluency* with using ratio reasoning and properties of operations to solve algebraic equations involving percents. They *apply* their fluency to solve multi-step ratio and percent problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

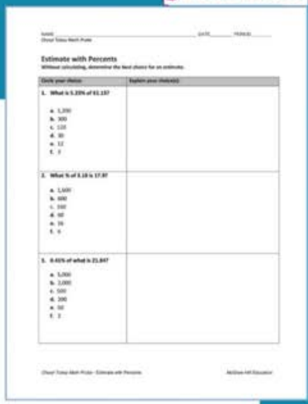
Suggested Pacing

Lesson		Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video			**1**	**0.5**
2-1	Percent of Change	7.RP.A.3, *Also addresses 7.EE.B.3*	1	0.5
2-2	Tax	7.RP.A.3, 7.EE.A.2, *Also addresses 7.EE.B.3*	1	0.5
2-3	Tips and Markups	7.RP.A.3, 7.EE.A.2, *Also addresses 7.EE.B.3*	1	0.5
2-4	Discounts	7.RP.A.3, 7.EE.A.2, *Also addresses 7.EE.B.3*	1	0.5
Put It All Together 1: Lessons 2-1 through 2-4			**0.5**	**0.25**
2-5	Interest	7.RP.A.3, *Also addresses 7.EE.B.3*	1	0.5
2-6	Commission and Fees	7.RP.A.3, 7.EE.A.2, *Also addresses 7.EE.B.3*	1	0.5
2-7	Percent Error	7.RP.A.3, *Also addresses 7.EE.B.3*	1	0.5
Put It All Together 2: Lessons 2-5 through 2-7			**0.5**	**0.25**
Module Review			**1**	**0.5**
Module Assessment			**1**	**0.5**
		Total Days	**11**	**5.5**

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe
Estimate with Percents

Module Resource



Correct Answers: **1.** f; **2.** b; **3** a.

Analyze the Probe

Review the probe prior to assigning it to your students.
In this probe, students will determine the best choice for an estimate for each item (without calculating), and explain their choices.

Targeted Concept Determining an estimate involves reasoning about the size of numbers and the relationship among the percent, the whole, and the part.

Targeted Misconceptions

- Students may apply incorrect reasoning, such as 5.23% of a quantity is about 0.5 or half of the quantity.
- Students may convert a percent to a decimal without understanding that they cannot simply just drop the percent symbol.
- Students may revert to using an algorithm (correctly or incorrectly) to solve the problem.

Assign the probe before Lesson 1.

Collect and Assess Student Work

If the student selects...	Then the student likely...
1. e **2.** f **3.** f	Is dividing the larger number by the smaller number without reasoning about the size of the percent or the relationship among the percent, the part, and the whole.
1. d	thinks of 5.23% of some quantity as 0.5 or half of the quantity.
2. d, e, f	does not understand that 17.9 is almost 6 times as great as 3.18, therefore the percent must be greater than 100.
3. e	incorrectly converts 0.41% to 0.41 instead of 0.0041. 0.41% is less than half of 1%. The correct estimate would have to be very large.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Percents
- Lesson 1, Examples 1–3

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can percent describe the change of a quantity? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition.* They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of training to run a race and sales tax to introduce the idea of percents. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.

Module 2

Solve Percent Problems

Essential Question

How can percent describe the change of a quantity?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

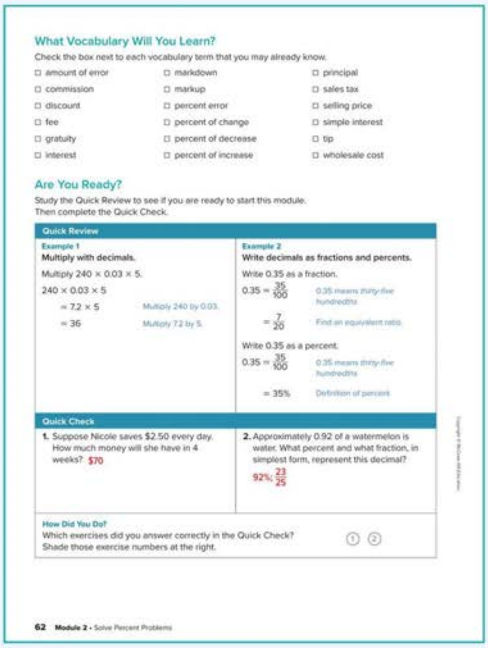
KEY: ○ — I don't know. ○ — I've heard of it. ○ — I know it!

	Before			After		
	○	○	○	○	○	○
finding percent of change						
solving problems involving taxes						
solving problems involving tips and markups						
solving problems involving discounts						
solving problems involving interest						
solving problems involving commissions and fees						
finding percent error						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about solving problems involving percent.

Module 2 • Solve Percent Problems 61

Interactive Student Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ amount of error	☐ markdown	☐ principal
☐ commission	☐ markup	☐ sales tax
☐ discount	☐ percent error	☐ selling price
☐ fee	☐ percent of change	☐ simple interest
☐ gratuity	☐ percent of decrease	☐ tip
☐ interest	☐ percent of increase	☐ wholesale cost

Are You Ready?

Study the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1
Multiply with decimals.

Multiply $240 \times 0.03 \times 5$.

$240 \times 0.03 \times 5$

$= 7.2 \times 5$ Multiply 240 by 0.03.

$= 36$ Multiply 7.2 by 5.

Example 2
Write decimals as fractions and percents.

Write 0.35 as a fraction.

$0.35 = \frac{35}{100}$ 0.35 means thirty-five hundredths

$= \frac{7}{20}$ Find an equivalent ratio

Write 0.35 as a percent.

$0.35 = \frac{35}{100}$ 0.35 means thirty-five hundredths

$= 35\%$ Definition of percent

Quick Check

1. Suppose Nicole saves $2.50 every day. How much money will she have in 4 weeks? $70

2. Approximately 0.92 of a watermelon is water. What percent and what fraction, in simplest form, represent this decimal? 92%; $\frac{23}{25}$

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

62 **Module 2** • Solve Percent Problems

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define The **percent of change** is a ratio, written as a percent, which compares the change in quantity to the original amount.

Example The cost of an item goes from $20 to $25. The amount of change is $5, which is 25% of the original amount, $20. So, the percent of change is 25%.

Ask If the cost of an item goes from $30 to $24, what is the percent of change? 20%

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- operations with decimals
- converting percents to decimals
- converting decimals to percents
- finding the percent of a number
- finding the percent, when given the part and the whole
- finding the whole, when given the part and the percent

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Percents** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

View Challenges as Opportunities

Part of cultivating a growth mindset in math involves viewing challenging problems or tasks as opportunities to learn and make new connections in your brain.

How Can I Apply It?

Encourage students to embrace challenges by trying problems that are thought provoking, such as the **Apply Problems** and **Higher-Order Thinking Problems** in the **Practice** section of each lesson. Remember to regularly remind students that each new challenge is an opportunity to grow!

Lesson 2-1

Percent of Change

7.RP.A.3

LESSON GOAL

Students will solve problems involving percent of increase and percent of decrease.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Percent of Change

Learn: Percent of Increase
Example 1: Percent of Increase
Example 2: Percent of Increase
Learn: Percent of Decrease
Example 3: Percent of Decrease
Apply: Movies

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 7 of the *Language Development Handbook* to help your students build mathematical language related to percent of change.

ELL You can use the tips and suggestions on page T7 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min 0.5 day
45 min 1 day

Focus

Domain: Ratios and Proportional Relationships

Major Cluster(s): In this lesson, students address major clusters **7. RP.A** and **7.EE.B** by solving problems involving percent of increase and percent of decrease.

Standards for Mathematical Content: **7. RP.A.3,** *Also addresses 7.EE.B.3*

Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4**

Coherence

Vertical Alignment

Previous
Students solved problems involving proportional relationships.
7.RP.A.2

Now
Students solve problems involving percent of increase and percent of decrease. **7.RP .A.3**

Next
Students will solve multi-step ratio and percent problems involving tax.
7.RP.A.3, 7.EE.A.2

Rigor

The Three Pillars of Rigor

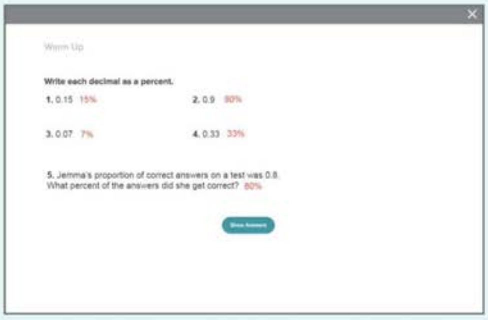
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their *understanding* of proportional relationships and percents to build *fluency* with determining the percent of change when a quantity either increases or decreases. They *apply* their understanding of percents of change to solve real-world problems.

Mathematical Background

A *percent of change* is a ratio, written as a percent, that compares the amount of change in a quantity to the original amount. If the original quantity is less than the new quantity, it is a *percent of increase*. If the original quantity is greater than the new quantity, it is a *percent of decrease*.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- converting decimals to percents (Exercises 1–5)

Answers

1. 15% **4.** 33%
2. 90% **5.** 80%
3. 7%

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about inflation of ticket prices as a percent of change.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- How does knowing the meaning of the word *change* help you determine the meaning of *percent of change*? Sample answer: Change means to become different. So, percent of change means the new amount is different than the original amount by a certain percent.
- What does *increase* mean ? Sample answer: Increase means to become greater in amount.
- What do you think *percent of decrease* means ? Sample answer: Decrease means to become lesser in amount. So, percent of decrease means the new amount is less that the original amount by a certain percent.

Explore Percent of Change

Objective

Students will use bar diagrams to explore percent of change.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with two real-world percent of change problems. Throughout this activity, students will use bar diagrams and sliders to represent and solve the problems.

Inquiry Question

How can you use a percent to describe a change when a quantity increases or decreases? Sample answer: I can use a percent to compare the increase or decrease to the value of the original quantity.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

You know the price increased by $16. How can you use the bar diagram to find the percent that corresponds to $16? Sample answer: I can split the bar diagram into 5 equal parts, with each part representing $16.

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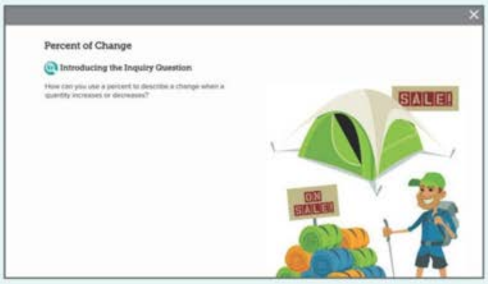
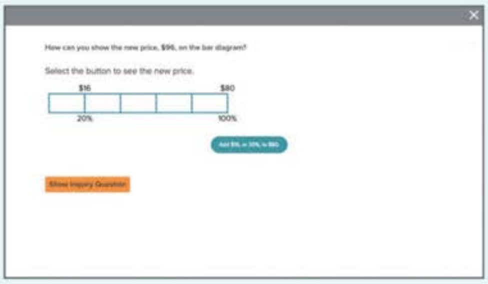
Interactive Presentation



Explore, Slide 1 of 10



Explore, Slide 5 of 10

TYPE

On Slide 2, students type to specify by how much a price increases.

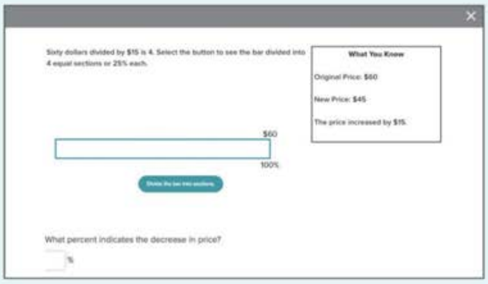
TYPE

On Slide 4, students type to indicate a percent increase.

CLICK

On Slide 5, students select values to interpret the bar diagram.

Interactive Presentation



Explore, Slide 8 of 10

TYPE

On Slide 6, students type to indicate by how much a price decreases.

CLICK

On Slide 9, students select to divide a bar diagram into sections and show a decrease in price.

TYPE

On Slide 10, students respond to the Inquiry Question and view a sample answer.

Explore Percent of Change *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the quantities given in each situation, and how a bar diagram can represent them and be used to find the missing quantity. Students should understand why the bar diagrams are divided into the number of sections indicated in the Explore.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

You know the price decreased by $15. How can you use the bar diagram to find the percent that corresponds to $15? Sample answer: I can split the bar diagram into 4 equal parts, with each part representing $15.

Learn Percent of Increase

Objective

Students will understand how percent of change (increase) compares the change in quantity to the original amount.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage them to make sense of the two quantities. If the quantities have different units, for example dollars and cents, you cannot subtract them to find the amount of change.

Teaching Notes

SLIDE 1

Be sure students understand that the percent of increase compares the amount of increase to the original amount. Some students may incorrectly compare the amount of increase to the new amount. Point out that the percent of change ratio compares the amount of increase, $9, to the *original amount*, $36.

Talk About It!

SLIDE 2

Mathematical Discourse

When finding percent of change, why is it important that the two quantities have the same unit of measure? Sample answer: When determining how much the quantity increased by, you need to use subtraction. You cannot subtract values that have different units of measure.

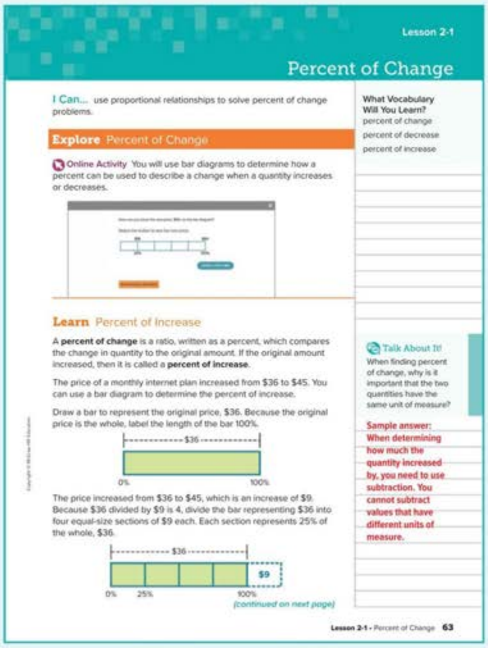
(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

If students are struggling to understand how to find the percent of change, have them first focus on finding the amount of change. In each of the following exercises, have students identify the amount of change and then identify if the change is an increase or decrease.

1. The price of a keyboard changed from $15 to $18. $3; increase
2. Eliza ran 45 minutes yesterday and 53 minutes today. 8 minutes; increase
3. It rained 0.4 inch last week and 0.1 inch this week. 0.3 inch; decrease
4. Student enrollment this year is 12,500. Student enrollment next year is expected to be 13,200. 700; increase



Lesson 2-1

Percent of Change

I Can... use proportional relationships to solve percent of change problems.

What Vocabulary Will You Learn?
percent of change
percent of decrease
percent of increase

Explore Percent of Change

Online Activity You will use bar diagrams to determine how a percent can be used to describe a change when a quantity increases or decreases.

Learn Percent of Increase

A **percent of change** is a ratio, written as a percent, which compares the change in quantity to the original amount. If the original amount increased, then it is called a **percent of increase**.

The price of a monthly internet plan increased from $36 to $45. You can use a bar diagram to determine the percent of increase.

Draw a bar to represent the original price, $36. Because the original price is the whole, label the length of the bar 100%.

The price increased from $36 to $45, which is an increase of $9. Because $36 divided by $9 is 4, divide the bar representing $36 into four equal-size sections of $9 each. Each section represents 25% of the whole, $36.

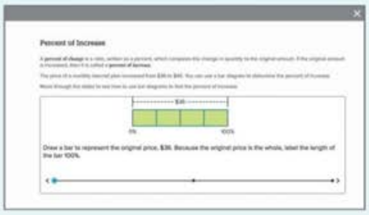
(continued on next page)

Talk About It! When finding percent of change, why is it important that the two quantities have the same unit of measure?

Sample answer: When determining how much the quantity increased by, you need to use subtraction. You cannot subtract values that have different units of measure.

Lesson 2-1 • Percent of Change 63

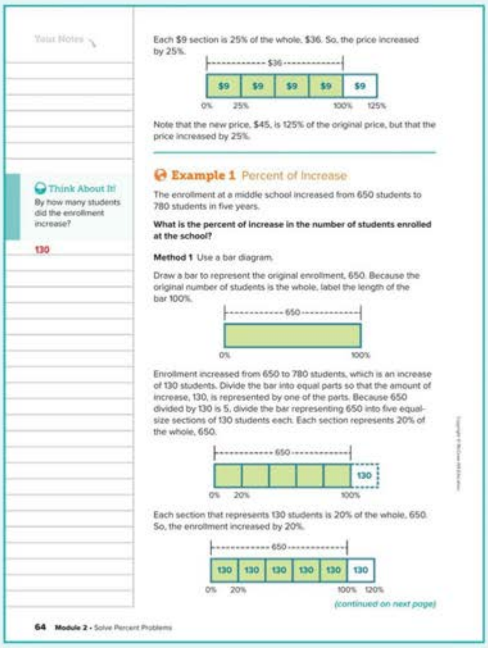
Interactive Presentation



Learn, Percent of Increase, Slide 1 of 2

CLICK

On Slide 1, students move through the slides to see how to use bar diagrams to find the percent of increase.


Your Notes

Think About It! By how many students did the enrollment increase?

130

Each $9 section is 25% of the whole, $36. So, the price increased by 25%.

Note that the new price, $45, is 125% of the original price, but that the price increased by 25%.

Example 1 Percent of Increase

The enrollment at a middle school increased from 650 students to 780 students in five years.

What is the percent of increase in the number of students enrolled at the school?

Method 1 Use a bar diagram.

Draw a bar to represent the original enrollment, 650. Because the original number of students is the whole, label the length of the bar 100%.

Enrollment increased from 650 to 780 students, which is an increase of 130 students. Divide the bar into equal parts so that the amount of increase, 130, is represented by one of the parts. Because 650 divided by 130 is 5, divide the bar representing 650 into five equal-size sections of 130 students each. Each section represents 20% of the whole, 650.

Each section that represents 130 students is 20% of the whole, 650. So, the enrollment increased by 20%.

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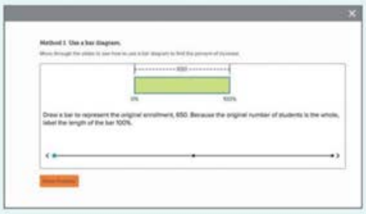
Interactive Presentation



Example 1, Percent of Increase, Slide 2 of 5

CLICK

On Slide 2, students move through the slides to use a bar diagram to find the percent of increase.

Example 1 Percent of Increase

Objective

Students will find the percent of increase in a real-world context.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities given in the Example, and to understand how a bar diagram or equivalent ratios can be used to represent the amount of increase and the percent of increase. Students should understand that the new amount, 780, is 120% of the original amount, but that the number of students increased by 20%.

As students discuss the *Talk About It!* question on Slide 3, encourage them to reason about the quantities given, so they can determine which value represents the whole.

Questions for Mathematical Discourse

SLIDE 2

AL Does 100% represent the original number of students, 650, or the new number of students, 780? the original number of students, 650

AL Why does this situation represent a percent increase? The number of students increased.

OL How do you know how many sections into which to divide the bar diagram? Sample answer: The amount of increase is 130, which is $\frac{1}{5}$ of the original amount 650. So, you need to divide the bar diagram into 5 sections.

BL In order for the percent of increase to be 10%, by what amount would the number of students have to increase? 65

BL Suppose the number of students increased at a constant rate over the five years. Explain why the percent of increase for each year is not 20 ÷ 5, or 4%. If the rate is constant, then the number of students increased by 130 ÷ 5, or 26 students per year. The original amount, then, would change for each year.

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Percent of Increase *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Which value represents the part? the amount of increase, 130

OL How is the amount of increase used to find the percent of increase? Sample answer: The amount of increase is the part and the original amount is the whole. Divide the amount of increase by the original amount to find the percent of increase.

BL What is the percent of increase if the original amount is 650, but the new amount is 845? 30%

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

In Example 1, some students may confuse the following two concepts.

- The new enrollment, 780, is *120% of the original enrollment*.
- The enrollment *increased by 20%*.

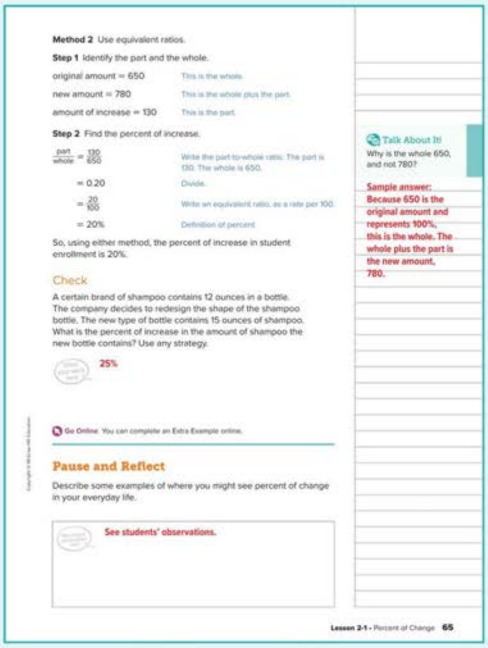
They may incorrectly say that the new enrollment is 20% of the original enrollment, or that the enrollment increased by 120%.

Have students work with a partner to study the vocabulary used in each sentence and use reasoning to determine when to use 120% and when to use 20%. If the percent of increase is 20%, then the enrollment *increased by* that same percent, 20%. However, the new enrollment is not 20% of the original enrollment. If it was, the enrollment would not have increased. The nature of the enrollment increasing means that the new enrollment is greater than 100% of the original enrollment.

You may wish to provide students with sentence frames that they can use when describing percent of increase, such as the ones below.

The new [amount] is _____% of the original [amount].

The [amount] increased by _____%.



Method 2 Use equivalent ratios.

Step 1 Identify the part and the whole.

original amount = 650 — This is the whole.

new amount = 780 — This is the whole plus the part.

amount of increase = 130 — This is the part.

Step 2 Find the percent of increase.

$\frac{\text{part}}{\text{whole}} = \frac{130}{650}$ — Write the part-to-whole ratio. The part is 130. The whole is 650.

$= 0.20$ — Divide.

$= \frac{20}{100}$ — Write an equivalent ratio, as a rate per 100.

$= 20\%$ — Definition of percent

So, using either method, the percent of increase in student enrollment is 20%.

Talk About It! Why is the whole 650, and not 780?

Sample answer: Because 650 is the original amount and represents 100%, this is the whole. The whole plus the part is the new amount, 780.

Check

A certain brand of shampoo contains 12 ounces in a bottle. The company decides to redesign the shape of the shampoo bottle. The new type of bottle contains 15 ounces of shampoo. What is the percent of increase in the amount of shampoo the new bottle contains? Use any strategy.

25%

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe some examples of where you might see percent of change in your everyday life.

See students' observations.

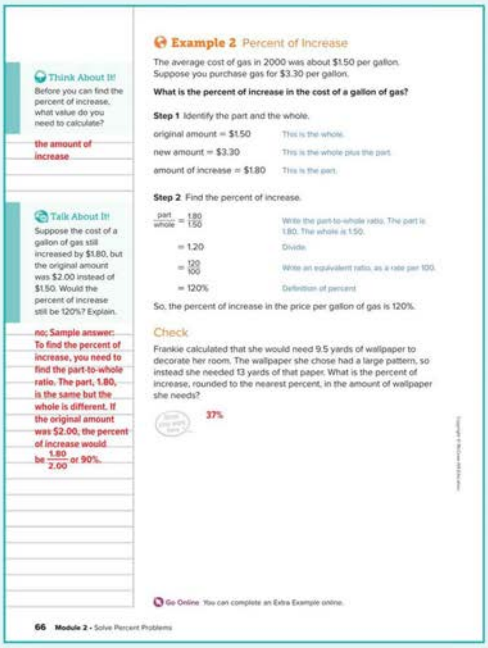
Lesson 2-1 • Percent of Change 65

Interactive Presentation

Example 1, Percent of Increase, Slide 3 of 5

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! Before you can find the percent of increase, what value do you need to calculate?

the amount of increase

Talk About It! Suppose the cost of a gallon of gas still increased by \$1.80, but the original amount was \$2.00 instead of \$1.50. Would the percent of increase still be 120%? Explain.

no; Sample answer: To find the percent of increase, you need to find the part-to-whole ratio. The part, 1.80, is the same but the whole is different. If the original amount was \$2.00, the percent of increase would be $\frac{1.80}{2.00}$ or 90%.

Example 2 Percent of Increase

The average cost of gas in 2000 was about \$1.50 per gallon. Suppose you purchase gas for \$3.30 per gallon.

What is the percent of increase in the cost of a gallon of gas?

Step 1 Identify the part and the whole.

original amount = \$1.50 — This is the whole.

new amount = \$3.30 — This is the whole plus the part.

amount of increase = \$1.80 — This is the part.

Step 2 Find the percent of increase.

$\frac{\text{part}}{\text{whole}} = \frac{1.80}{1.50}$ — Write the part-to-whole ratio. The part is 1.80. The whole is 1.50.

$= 1.20$ — Divide.

$= \frac{120}{100}$ — Write an equivalent ratio, as a rate per 100.

$= 120\%$ — Definition of percent

So, the percent of increase in the price per gallon of gas is 120%.

Check

Frankie calculated that she would need 9.5 yards of wallpaper to decorate her room. The wallpaper she chose had a large pattern, so instead she needed 13 yards of that paper. What is the percent of increase, rounded to the nearest percent, in the amount of wallpaper she needs?

37%

Go Online You can complete an Extra Example online.

66 **Module 2** • Solve Percent Problems

Interactive Presentation

Example 2, Percent of Increase, Slide 2 of 4

CLICK

On slide 2, students move through the slides to use ratio reasoning to find the percent of increase.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Percent of Increase

Objective

Students will find the percent of increase in a real-world context.

Questions for Mathematical Discourse

SLIDE 2

AL How do you find the amount of increase? Subtract the original amount from the new amount.

OL Why is the percent of increase greater than 100? Sample answer: Because the price of a gallon of gas increases by an amount that is greater than the original amount, the part is greater than the whole and the percent of increase will be greater than 100%.

BL Suppose the new price for a gallon of gas is \$2.90. Would the percent of increase still be greater than 100%? Explain. no; Sample answer: If the new price is \$2.90 and the original price is \$1.50, the amount of increase is \$1.40. Because \$1.40 is less than \$1.50, the percent of increase would be less than 100%.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Some students may confuse the percent of increase (120%) in Example 2 with the percent of increase in Example 1 (20%).

- **Example 1:** The new enrollment *is 120% of the original enrollment*. The enrollment *increased by* 20%.
- **Example 2:** The price per gallon of gas *increased by* 120%.

Without taking into context the language, a student may incorrectly think the percents of increase are the same because they see 120% in each Example. Have students complete a table like the one shown. While the percent in the last cell, 220%, is not presented in Example 2, have students use reasoning to generate it and provide an explanation defending the percent they chose. A sample explanation is shown.

	Example 1	Example 2
Percent of Increase "increased by ___%"	20%	120%
The new amount is ___% of the original amount.	120%	220%

In Example 2, the percent of increase is 120%. The new amount, \$3.30, is more than twice the original amount, \$1.50. For an amount to be twice its original amount, the percent of increase is 200%. So, the percent of increase must be greater than 200%.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Percent of Decrease

Objective

Students will understand how percent of change (decrease) compares the change in quantity to the original amount.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage them to attend to the meaning of the quantities given in the problem. Students should understand that Jordan's new time is 80% of his original time, but that his time decreased by 20%.

Talk About It!

SLIDE 2

Mathematical Discourse

The percent of decrease is 20%. Explain why it is not 80%. Sample answer: Twenty-eight is 80% of 35, but this does not represent the percent of change. The time changed by 7 minutes, which is 20% of 35 minutes.

Go Online to find additional teaching notes.

Example 3 Percent of Decrease

Objective

Students will find the percent of decrease in a real-world context.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities given in the example. Even though the starting amount is greater than the new amount, when finding the percent of change, the *amount* of change is a positive number. The fact that the starting amount is greater than the new amount indicates that the percent of change is a percent of decrease.

As students discuss the *Talk About It!* question, encourage them to make sense of the quantities given, in order to be able to estimate the percent of change and check their answer for reasonableness.

Questions for Mathematical Discourse

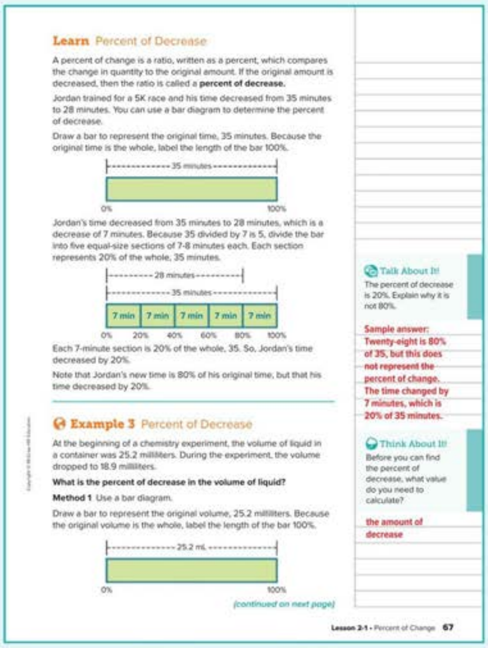
SLIDE 2

AL Does 25.2 milliliters represent the part or the whole? whole

OL Does 6.3 milliliters represent the part or the whole? part

BL How could you use a ratio table to check your work? Sample answer: I can scale backward from 25.2 to 6.3 by dividing by 4. Because 25.2 is 100%, I can divide 100% by 4 to obtain a percent decrease of 25%.

(continued on next page)



Learn Percent of Decrease

A percent of change is a ratio, written as a percent, which compares the change in quantity to the original amount. If the original amount is decreased, then the ratio is called a **percent of decrease**.

Jordan trained for a 5K race and his time decreased from 35 minutes to 28 minutes. You can use a bar diagram to determine the percent of decrease.

Draw a bar to represent the original time, 35 minutes. Because the original time is the whole, label the length of the bar 100%.

Jordan's time decreased from 35 minutes to 28 minutes, which is a decrease of 7 minutes. Because 35 divided by 7 is 5, divide the bar into five equal-size sections of 7-8 minutes each. Each section represents 20% of the whole, 35 minutes.

Each 7-minute section is 20% of the whole, 35. So, Jordan's time decreased by 20%.

Note that Jordan's new time is 80% of his original time, but that his time decreased by 20%.

Talk About It! The percent of decrease is 20%. Explain why it is not 80%. Sample answer: Twenty-eight is 80% of 35, but this does not represent the percent of change. The time changed by 7 minutes, which is 20% of 35 minutes.

Example 3 Percent of Decrease

At the beginning of a chemistry experiment, the volume of liquid in a container was 25.2 milliliters. During the experiment, the volume dropped to 18.9 milliliters.

What is the percent of decrease in the volume of liquid?

Method 1 Use a bar diagram.

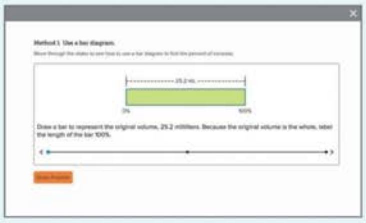
Draw a bar to represent the original volume, 25.2 milliliters. Because the original volume is the whole, label the length of the bar 100%.

Think About It! Before you can find the percent of decrease, what value do you need to calculate? the amount of decrease

(continued on next page)

Lesson 2-1 • Percent of Change 67

Interactive Presentation

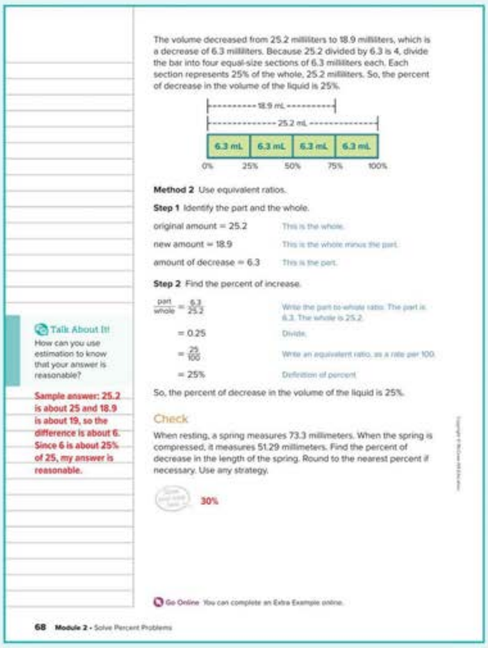


Example 3, Percent of Decrease, Slide 2 of 5

CLICK

Students will move through the slides to use a bar diagram to find the percent of decrease.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



The volume decreased from 25.2 milliliters to 18.9 milliliters, which is a decrease of 6.3 milliliters. Because 25.2 divided by 6.3 is 4, divide the bar into four equal-size sections of 6.3 milliliters each. Each section represents 25% of the whole, 25.2 milliliters. So, the percent of decrease in the volume of the liquid is 25%.

Method 2 Use equivalent ratios.

Step 1 Identify the part and the whole.

original amount = 25.2 — This is the whole.

new amount = 18.9 — This is the whole minus the part.

amount of decrease = 6.3 — This is the part.

Step 2 Find the percent of increase.

$\frac{\text{part}}{\text{whole}} = \frac{6.3}{25.2}$ — Write the part-to-whole ratio. The part is 6.3. The whole is 25.2.

$= 0.25$ — Divide.

$= \frac{25}{100}$ — Write an equivalent ratio, as a rate per 100.

$= 25\%$ — Definition of percent

So, the percent of decrease in the volume of the liquid is 25%.

Talk About It! How can you use estimation to know that your answer is reasonable?

Sample answer: 25.2 is about 25 and 18.9 is about 19, so the difference is about 6. Since 6 is about 25% of 25, my answer is reasonable.

Check

When resting, a spring measures 73.3 millimeters. When the spring is compressed, it measures 51.29 millimeters. Find the percent of decrease in the length of the spring. Round to the nearest percent if necessary. Use any strategy.

30%

Go Online You can complete an Extra Example online.

68 Module 2 • Solve Percent Problems

Interactive Presentation

Example 3, Percent of Decrease, Slide 3 of 5

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Percent of Decrease *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL What two quantities are we comparing in the ratio? amount of decrease and original amount

AL What is the amount of decrease? What is the original amount? The amount of decrease is 6.3 mL. The original amount is 25.2 mL.

OL Suppose a classmate stated that the percent of decrease was 0.25. Describe the error they made. Sample answer: They did not write the part-to-whole ratio as a rate per 100, or a percent. After dividing the part-to-whole ratio, the decimal they found needs to be written as a percent.

BL In order for the percent of decrease to be 50%, by what amount would the volume of liquid have to decrease? 12.6 mL

Go Online

- Find the additional teaching notes and *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To better students' understanding of percent of change, have them complete the exercise below to ensure that they are able to correctly identify the amount of change and the original amount.

Anita invested $400 in a certain company's stock. After one year, her investment had grown to $460. After one more year, her investment had decreased from the previous amount to $414.

What was the percent of change from the initial investment to the end of the first year? 15% increase

What was the percent of change from the end of the first year to the end of the second year? 10% decrease

Apply Movies

Objective

Students will come up with their own strategy to solve an application problem involving the change in the length of movies over time.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What do you notice about the units of time used in the problem?
- Can you find the amount of increase when the units are different?
- Do you think the percent of change will be less than, greater than, or equal to 100%? Why?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Movies

The first known motion picture was filmed in 1888 and lasted for only 2.11 seconds. Today, we watch movies that last an average of about two hours. What is the percent of change in the times from 1888 to today? Round your answer to the nearest whole percent if necessary.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

341,132%; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Why is the percent of change such a large number?

Sample answer: 2 hours, or 7,200 seconds, is a much greater amount of time than 2.11 seconds. Therefore, the percent of change is also going to be large.

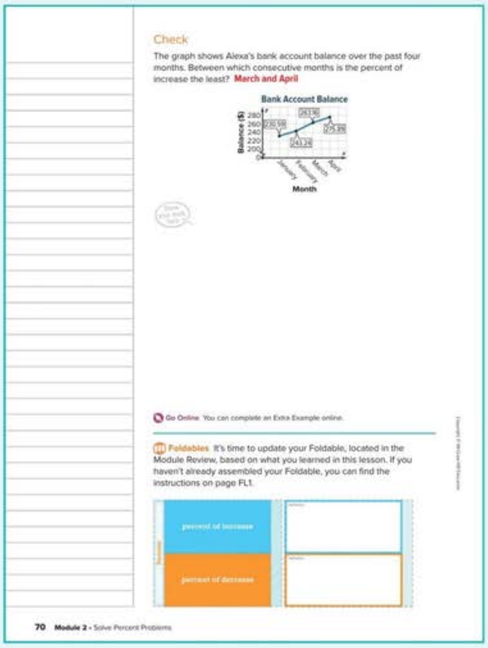
Lesson 2-1 • Percent of Change 69

Interactive Presentation

Apply, Movies

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can give the definition of percent of increase and percent of decrease. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can percent describe the change of a quantity?

In this lesson, students learned how to find the percent of change between two quantities, and how to identify a percent of increase or a percent of decrease. Encourage them to discuss with a partner why it is important to compare the amount of increase or decrease to the original quantity, and how the same amount of increase may not be the same for different situations.

Exit Ticket

Refer to the Exit Ticket slide. Will the percent of change in movie ticket prices between 1985 and 2000 be the same as the percent of change in movie ticket prices between 2000 and 2015? Write a mathematical argument that can be used to defend your solution. no; Sample answer: The amount of change in movie ticket prices between 1985 and 2000 is different than the amount of change in movie ticket prices between 2000 and 2015. Also, the original amount is different in 1985 and 2000.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11 odd, 13–16
- ALEKS® Percent Increase and Decrease

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 15
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Understanding Percents

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

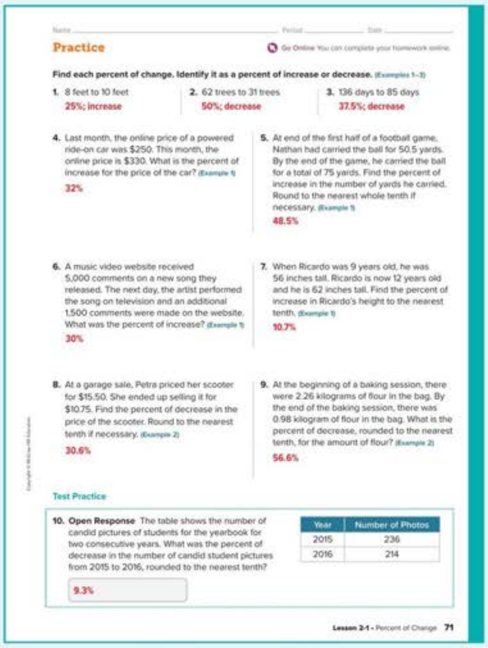
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the percent of change	1–3
2	find the percent of increase	4–7
2	find the percent of decrease	8, 9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving the percent of change	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Students may incorrectly determine the part in the part-to-whole ratio. For example, in Exercise 6, the phrase *an additional 1,500 comments* suggests the amount of change rather than the end result. In this case, students are given the amount of change.



Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

Find each percent of change. Identify it as a percent of increase or decrease. (Examples 1–3)

1. 8 feet to 10 feet
   25%; increase
2. 62 trees to 31 trees
   50%; decrease
3. 136 days to 85 days
   37.5%; decrease
4. Last month, the online price of a powered ride-on car was $250. This month, the online price is $330. What is the percent of increase for the price of the car? (Example 1)
   32%
5. At end of the first half of a football game, Nathan had carried the ball for 50.5 yards. By the end of the game, he carried the ball for a total of 75 yards. Find the percent of increase in the number of yards he carried. Round to the nearest whole tenth if necessary. (Example 1)
   48.5%
6. A music video website received 5,000 comments on a new song they released. The next day, the artist performed the song on television and an additional 1,500 comments were made on the website. What was the percent of increase? (Example 1)
   30%
7. When Ricardo was 9 years old, he was 56 inches tall. Ricardo is now 12 years old and he is 62 inches tall. Find the percent of increase in Ricardo's height to the nearest tenth. (Example 1)
   10.7%
8. At a garage sale, Petra priced her scooter for $15.50. She ended up selling it for $10.75. Find the percent of decrease in the price of the scooter. Round to the nearest tenth if necessary. (Example 2)
   30.6%
9. At the beginning of a baking session, there were 2.26 kilograms of flour in the bag. By the end of the baking session, there was 0.98 kilogram of flour in the bag. What is the percent of decrease, rounded to the nearest tenth, for the amount of flour? (Example 2)
   56.6%

Test Practice

10. **Open Response** The table shows the number of candid pictures of students for the yearbook for two consecutive years. What was the percent of decrease in the number of candid student pictures from 2015 to 2016, rounded to the nearest tenth?

Year	Number of Photos
2015	236
2016	214

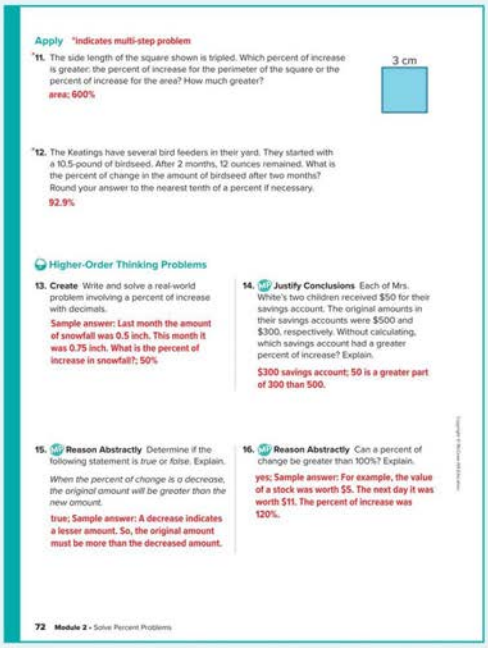
9.3%

Lesson 2-1 • Percent of Change 71

3 REFLECT AND PRACTICE

7.RP.A.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*11. The side length of the square shown is tripled. Which percent of increase is greater: the percent of increase for the perimeter of the square or the percent of increase for the area? How much greater?

3 cm

area; 600%

*12. The Keatings have several bird feeders in their yard. They started with a 10.5-pound of birdseed. After 2 months, 12 ounces remained. What is the percent of change in the amount of birdseed after two months? Round your answer to the nearest tenth of a percent if necessary.

92.9%

Higher-Order Thinking Problems

13. **Create** Write and solve a real-world problem involving a percent of increase with decimals.

Sample answer: Last month the amount of snowfall was 0.5 inch. This month it was 0.75 inch. What is the percent of increase in snowfall?; 50%

14. **Justify Conclusions** Each of Mrs. White's two children received $50 for their savings account. The original amounts in their savings accounts were $500 and $300, respectively. Without calculating, which savings account had a greater percent of increase? Explain.

$300 savings account; 50 is a greater part of 300 than 500.

15. **Reason Abstractly** Determine if the following statement is *true* or *false*. Explain.

When the percent of change is a decrease, the original amount will be greater than the new amount.

true; Sample answer: A decrease indicates a lesser amount. So, the original amount must be more than the decreased amount.

16. **Reason Abstractly** Can a percent of change be greater than 100%? Explain.

yes; Sample answer: For example, the value of a stock was worth $5. The next day it was worth $11. The percent of increase was 120%.

72 **Module 2** • Solve Percent Problems

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Ex ercise 14, students use ratio reasoning and mental math to explain why one savings account had a greater percent of increase.

2 Reason Abstractly and Quantitatively In Exercise 15, students use reasoning to analyze a generalized statement to determine if it is true or false.

2 Reason Abstractly and Quantitatively In Exercise 16, students use an example to explain their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.

Use with Exercises 11–12 Have pairs of students interview each other as they complete these apply problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 12 might be "How many ounces are in a pound?"

Clearly and precisely explain.

Use with Exercise 14 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as using part, whole, and percent in their responses.

Lesson 2-2

Tax

7.RP.A.3, 7.EE.A.2

LESSON GOAL

Students will solve multi-step ratio and percent problems involving taxes.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Sales Tax

Learn: Sales Tax
Example 1: Sales Tax
Example 2: Hotel Tax
Example 3: Sales Tax
Apply: Shopping

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 8 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving tax.

ELL You can use the tips and suggestions on page T8 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min 0.5 day
45 min 1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major clusters **7. RP.A** and **7.EE.A** by solving problems involving taxes.
Standards for Mathematical Content: **7. RP.A.3, 7.EE.A.2,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP7**

Coherence

Vertical Alignment

Previous
Students solved problems involving percent of increase and percent of decrease.
7.RP.A.3

Now
Students solve multi-step ratio and percent problems involving tax.
7.RP.A.3, 7.EE.A.2

Next
Students will solve multi-step ratio and percent problems involving tips and markups.
7.RP.A.3, 7.EE.A.2

Rigor

The Three Pillars of Rigor

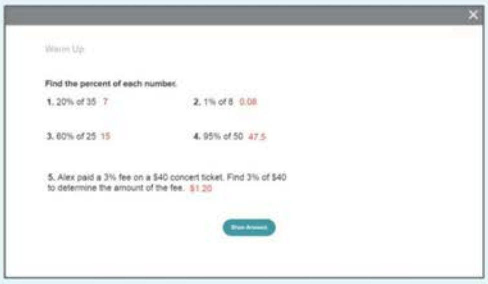
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students *apply* their *understanding* of ratios and percent to solve problems involving tax, such as sales tax and hotel tax rates. Students develop *understanding* of different methods that can be used to find the total cost of an item, including tax. They build *fluency* in using ratio reasoning and/or properties of operations to find the amount of tax and the total cost.

Mathematical Background

Sales tax is a state or local tax that is added to the price of an item or service. *Sales tax* is calculated as a percent of the price. It can be represented as a percent of increase because the total cost increases by that percent. Total cost can be calculated by adding the sales tax to the price of an item. Total cost can also be found by adding the percent of tax to 100% and then using ratio reasoning and/or properties of operations.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding the percent of a number (Exercises 1–5)

Answers

1. 7	**4.** 47 .5
2. 0.08	**5.** $1.20
3. 15	

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about taxes and how they are related to percents.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- *Sales tax* is an additional amount of money charged on purchased items. When have you had to pay *sales tax*? Sample answer: I paid sales tax when I purchased a new book.

Explore Sales T ax

Objective

Students will use Web Sketchpad to explore how sales tax affects the total cost of an item.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch that calculates the sales tax and the total cost of an item. Throughout this activity, students will use sketches to find the sales tax and total cost of items for different sales tax rates.

Inquiry Question

How does sales tax change the total cost to purchase an item? Sample answer: Sales tax is added to the price of an item. Sales tax can be a small amount added, like when buying a shirt, or it can be a large amount, like when buying a car.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Research the sales tax in your location. If you live in a state with no sales tax, research a city in a nearby state that does have sales tax. What is the sales tax? Sample answer: 6.75%

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 9

Explore, Slide 3 of 9

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how sales tax changes the total cost to purchase an item.

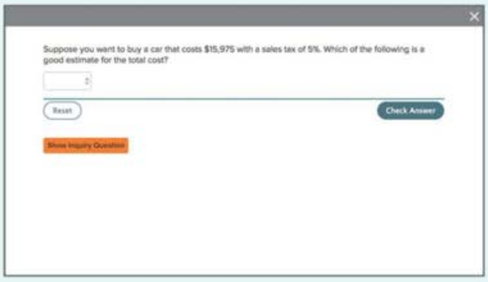
TYPE

On Slide 3, students type to indicate the total cost, the sales tax, and the sales tax rate.

CLICK

On Slide 4, students select from a drop-down menu a good estimate for the total cost.

Interactive Presentation



Explore, Slide 7 of 9

CLICK

On Slide 7, students select from a drop-down menu a good estimate for the total cost.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Sales T ax *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore how sales tax changes the total cost of an item.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 8 is shown.

Talk About It!

SLIDE 8

Mathematical Discourse

Change the tax rate to match the rate you researched at the beginning of the Explore, and drag the car into the cart. How did that change the amount of sales tax and the total cost? Sample answer: Increasing the sales tax to 6.75% increased the total sales tax and the total cost.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Sales Tax

Objective

Students will learn how to find sales tax.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage them to attend to the quantities used in the problem. Students should understand that finding the amount of tax does not give the total cost. The sales tax needs to be added to the cost of the item or items you are purchasing to find the total cost.

Teaching Notes

SLIDE 1

Ask students to generate examples of sales tax they may have encountered in their everyday lives. You may wish to ask them what their city or state sales tax rate is, and then calculate the amount of sales tax on a $10 T-shirt using that rate.

Encourage them to make sense of the proportion presented in the Learn, $\frac{t}{10} = \frac{7.5}{100}$, and to use ratio reasoning to find the amount of tax t. Because the ratios are equivalent, the same number must be multiplied or divided by the numerator and denominator to maintain the equivalence. Have students reason about the quantities to verify the amount of tax.

Another way to solve proportions is to use *cross products*. You may or may not choose to show this method to students. While using cross products is mathematically correct, they may not convey meaningful quantities in relation to the context of the problem. In the proportion $\frac{t}{10} = \frac{7.5}{100}$, the product of t and 100 and the product of 10 and 7.5 are called *cross products*. The cross products of a proportion are equivalent. To solve a proportion by using cross products, write the corresponding equation stating that the cross products are equivalent. Then solve the equation.

$\frac{t}{10} = \frac{7.5}{100}$	Write the proportion.
$100t = 10(7.5)$	Find the cross products.
$\frac{100t}{100} = \frac{75}{100}$	Simplify. Divide each side by 100.
$t = 0.75$	Simplify. The tax is $0.75.

Have students compare and contrast using ratio reasoning and cross products to solve proportions. Ask them to decide which method is more meaningful and allows them to reason about the quantities.

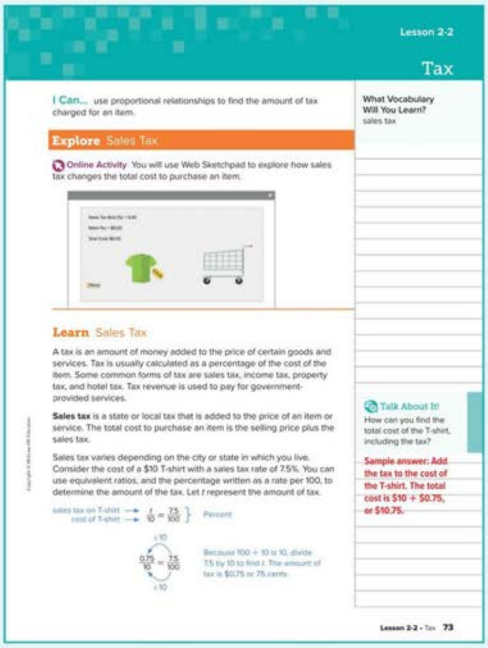
Talk About It!

SLIDE 2

Mathematical Discourse

How can you find the total cost of the T-shirt, including the tax?

Sample answer: Add the tax to the cost of the T-shirt. The total cost is $10 + $0.75, or $10.75.


Lesson 2-2

Tax

I Can... use proportional relationships to find the amount of tax charged for an item.

What Vocabulary Will You Learn?
sales tax

Explore Sales Tax

Online Activity You will use Web Sketchpad to explore how sales tax changes the total cost to purchase an item.

Learn Sales Tax

A tax is an amount of money added to the price of certain goods and services. Tax is usually calculated as a percentage of the cost of the item. Some common forms of tax are sales tax, income tax, property tax, and hotel tax. Tax revenue is used to pay for government-provided services.

Sales tax is a state or local tax that is added to the price of an item or service. The total cost to purchase an item is the selling price plus the sales tax.

Sales tax varies depending on the city or state in which you live. Consider the cost of a $10 T-shirt with a sales tax rate of 7.5%. You can use equivalent ratios, and the percentage written as a rate per 100, to determine the amount of the tax. Let t represent the amount of tax.

sales tax on T-shirt → / cost of T-shirt → $\frac{t}{10} = \frac{7.5}{100}$ } Percent

$\frac{0.75}{10} = \frac{7.5}{100}$ (÷10)

Because 100 ÷ 10 is 10, divide 7.5 by 10 to find t. The amount of tax is $0.75 or 75 cents.

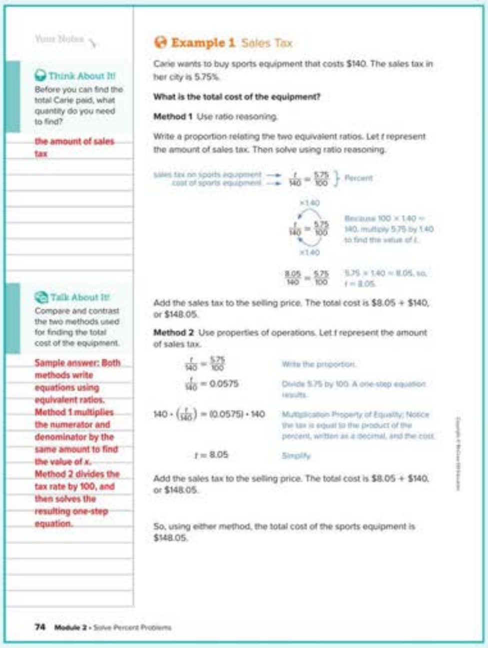
Talk About It! How can you find the total cost of the T-shirt, including the tax?

Sample answer: Add the tax to the cost of the T-shirt. The total cost is $10 + $0.75, or $10.75.

Lesson 2-2 • Tax 73

Interactive Presentation

Learn, Sales Tax, Slide 1 of 2



Your Notes

Think About It! Before you can find the total Carie paid, what quantity do you need to find?

the amount of sales tax

Example 1 Sales Tax

Carie wants to buy sports equipment that costs $140. The sales tax in her city is 5.75%.

What is the total cost of the equipment?

Method 1 Use ratio reasoning.

Write a proportion relating the two equivalent ratios. Let t represent the amount of sales tax. Then solve using ratio reasoning.

sales tax on sports equipment → cost of sports equipment → $\frac{t}{140} = \frac{5.75}{100}$ } Percent

×1.40

$\frac{t}{140} = \frac{5.75}{100}$ Because 100 × 1.40 = 140, multiply 5.75 by 1.40 to find the value of t.

×1.40

$\frac{8.05}{140} = \frac{5.75}{100}$ 5.75 × 1.40 = 8.05, so, $t = 8.05$.

Add the sales tax to the selling price. The total cost is $8.05 + $140, or $148.05.

Method 2 Use properties of operations. Let t represent the amount of sales tax.

$\frac{t}{140} = \frac{5.75}{100}$ Write the proportion.

$\frac{t}{140} = 0.0575$ Divide 5.75 by 100. A one-step equation results.

$140 \cdot \left(\frac{t}{140}\right) = (0.0575) \cdot 140$ Multiplication Property of Equality; Notice the tax is equal to the product of the percent, written as a decimal, and the cost.

$t = 8.05$ Simplify.

Add the sales tax to the selling price. The total cost is $8.05 + $140, or $148.05.

So, using either method, the total cost of the sports equipment is $148.05.

Talk About It! Compare and contrast the two methods used for finding the total cost of the equipment.

Sample answer: Both methods write equations using equivalent ratios. Method 1 multiplies the numerator and denominator by the same amount to find the value of x. Method 2 divides the tax rate by 100, and then solves the resulting one-step equation.

74 Module 2 • Solve Percent Problems

Interactive Presentation

Example 1, Sales Tax, Slide 2 of 5

TYPE

On Slide 3, students determine the total cost.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.RP.A.3, 7.EE.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Sales T ax

Objective

Students will find the total cost for an item given the item's cost and the percent of sales tax.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively St udents should understand the mathematical reasoning behind why either method is acceptable to solving the problem and why they both yield the same solution.

In both methods, encourage students to decontextualize the information given in the problem by representing it symbolically with a proportion.

Questions for Mathematical Discourse

SLIDE 2

AL When setting up a proportion, what values do you know? What value is unknown? Sample answer: You know the percent, which can be written as a rate per 100, and you know the whole, which is the cost of the sports equipment. The unknown is the amount of tax.

OL Estimate the amount of sales tax. Explain. Sample answer: 5.75% is close to 5%, and 5% is half of 10%. Because 10% of $140 is $14, then 5% would be $7. The amount of sales tax is close to $7.

BL Is the amount of sales tax less than or greater than $7? Explain. greater than; Sample answer: 5% of $140 is $7, but the sales tax is 5.75%, which is greater than 5%.

SLIDE 3

AL Why do you need to multiply each side of the equation by 140? Sample answer: Because $\frac{t}{140} = 0.0575$ is a one-step equation, you can isolate the variable t by undoing the division with multiplication.

BL How is Method 2 different than Method 1? Sample answer: In Method 1, you use scaling to determine the unknown value in the proportion. In Method 2, the proportion becomes a one-step equation that you can solve using the properties of operations.

OL How could you find the sales tax without first setting up a proportion? Sample answer: You can find the product of the percent written as a decimal, and the cost of the sports equipment. So, multiply 140 by 0.0575.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Hotel T ax

Objective

Students will find the total cost of a hotel room given the cost of the room and the percent of tax.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
Encourage students to plan a solution pathway, prior to jumping into a solution attempt. Students should understand that finding the total cost of a hotel room with the hotel tax is the similar to finding the total cost of an item with sales tax. Students should be able to explain why Method 1 and Method 2 both arrive at the same solution, even though the steps are different.

When discussing the *Talk About It!* question, encourage students to analyze the steps in Method 2 to determine an efficient way of solving any tax problem.

Questions for Mathematical Discourse

SLIDE 2

AL When setting up a proportion, what values do you know? What value is unknown? Sample answer: You know the percent, which can be written as a rate per 100, and you know the whole, which is the cost of the hotel room. The unknown is the amount of tax.

OL A classmate wrote the proportion $\frac{280}{t} = \frac{12}{100}$. Is this a correct proportion? Why or why not? no; Sample answer: The ratios $\frac{280}{t}$ and $\frac{12}{100}$ are not equivalent. The variable t is the amount of tax which represents the part and not the whole, 280. In the ratio $\frac{12}{100}$, 12 is the part and 100 is the whole.

OL How can you determine what value you should multiply 12 by to find the value of t? Sample answer: You can work backwards to determine what value you multiply 100 by to obtain 280. So, divide 280 by 100, which is 2.80.

BL If the tax on the bill was $42, what was the tax rate? 15%

(continued on next page)

Check

Ashley is buying a laptop that sells for $749. The sales tax rate in her city is $8\frac{1}{4}$%. What is the total cost for the laptop? Round your answer to the nearest cent. Use any strategy.

$810.79

Go Online You can complete an Extra Example online.

Example 2 Hotel Tax

The cost of a hotel room rented for 2 nights is $280. There is also a 12% hotel room tax.

What is the total cost of the hotel room?

Method 1 Use ratio reasoning.

Write a proportion. Let t represent the amount of tax. Then solve using ratio reasoning.

tax on hotel room → / cost of hotel room → $\frac{t}{280} = \frac{12}{100}$ } Percent

$\frac{t}{280} = \frac{12}{100}$ (×2.80) Because $100 \times 2.80 = 280$, multiply 12 by 2.80 to find the value of t.

$\frac{33.60}{280} = \frac{12}{100}$ $12 \times 2.80 = 33.60$, so $t = 33.60$.

Add the hotel tax to the cost of the hotel room. The total cost is $280 + $33.60, or $313.60.

Think About It! Is the tax less than, greater than, or equal to $28? How do you know?

greater than; Sample answer: 10% of $280 is $28 and 12% > 10%.

(continued on next page)

Interactive Presentation

Example 2, Hotel Tax, Slide 1 of 5

CLICK

On Slide 2, students move through the slides to use ratio reasoning to find hotel tax.

Talk About It!

In Method 2, how could you use the steps in solving the equation to find the tax rate, or percentage, of any value?

Sample answer: You can multiply the tax rate percentage, written as a decimal, by the cost of the item.

Method 2 Use properties of operations. Let t represent the amount of tax.

$\frac{t}{280} = \frac{12}{100}$	Write the proportion.
$\frac{t}{280} = 0.12$	Divide 12 by 100. A one-step equation results.
$280 \cdot \left(\frac{t}{280}\right) = (0.12) \cdot 280$	Multiplication Property of Equality: Notice the tax is equal to the product of the percent, written as a decimal, and the cost.
$t = 33.60$	Simplify.

Add the hotel tax to the cost of the hotel room. The total cost is \$280 + \$33.60, or \$313.60.

So, using either method, the total cost of the hotel room is \$313.60.

Check

The cost of a hotel room for 5 nights is \$610. There is a 9.5% hotel tax. What is the total cost of the hotel room? Use any strategy.

\$667.95

Go Online You can complete an Extra Example online.

Pause and Reflect

Use the Internet, or another source, to research the sales tax rate for your city or state. Describe an item you may want to purchase and research the cost of the item. Trade with a partner and use the sales tax rate to find the total cost of the item, including sales tax.

See students' observations.

76 Module 2 • Solve Percent Problems

Interactive Presentation

Example 2, Hotel Tax, Slide 3 of 5

CLICK

On Slide 2, students move through the slides to use ratio reasoning to find the total cost.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Hotel T ax *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL In Method 2, why is it advantageous to divide 12 by 100? Sample answer: Dividing 12 by 100 is advantageous because it results in a one-step equation.

OL After finding t, how can you check your work? Sample answer: I can substitute 33.60 into the proportion, then use division to determine if 33.60 ÷ 280 is equivalent to 12 ÷ 100, which it is.

OL How could you use estimation to determine if your answer is reasonable? Sample answer: I know that 12% is close to 10% and 10% of \$280 is \$28. Because \$28 is close to \$33.60, I know my answer is reasonable.

BL A classmate stated that you do not need to write $\frac{12}{100}$ as 0.12 before multiplying both sides of the equation by 280. Is this correct? Explain. What advantage is there in writing the fraction as a decimal? Yes, this is correct; Sample answer: You can multiply both sides of the original proportion by 280 to isolate the variable t. The only advantage to writing the fraction as a decimal is to perform the operations with decimals, instead of fractions.

Talk About It!

SLIDE 4

Mathematical Discourse

In Method 2, how could you use the steps in solving the equation to find the tax rate, or percentage, of any value? Sample answer: Y ou can multiply the tax rate percentage, written as a decimal, by the cost of the item.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Reteaching Activity AL

To help students who may be struggling to use properties of operations to solve proportions, remind them that solving a proportion is similar to solving a one-step division equation. Have them solve the following division equations, that increase in difficulty, yet are each solved the same way.

$\frac{x}{5} = 3$ $x = 15$

$\frac{x}{5} = 0.3$ $x = 1.5$

$\frac{x}{500} = 0.3$ $x = 150$

Example 3 Sales Tax

Objective

Students will find the total cost of an item given the item's cost and the percent of sales tax.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should understand the mathematical reasoning behind why any of the three methods is acceptable to solving the problem and why they yield the same solution. Encourage students to see the connection between the methods, which includes understanding that 100% of a quantity is the quantity itself.

When discussing the *Talk About It!* question, encourage students to analyze the three methods for solving the problem to see which one would be most efficient.

Questions for Mathematical Discourse

SLIDE 2

AL What does t represent? Is it the part or the whole? t represents the amount of sales tax, which is the part

OL How can you check the amount of sales tax for reasonableness? Sample answer: 7.5% is a little less than 10%. 10% of $56.00 is $5.60. So, the amount of tax should be a little less than $5.60.

BL If the amount of sales tax of a $56 purchase is $3.92, what is the sales tax rate? 7%

SLIDE 3

AL Explain the difference between Method 1 and Method 2. Sample answer: In Method 1, I find the sales tax first, then add it to the selling price. In Method 2, I add the sales tax percent to 100%, then calculate the total cost.

OL When using Method 2, what does 107.5% of $56 represent? Sample answer: 100% of $56 represents the total cost and 7.5% of $56 represents the sales tax. So, 107.5% of $56 represents the cost of the clothing plus the amount of sales tax.

BL Explain how to mentally estimate 107.5% of $56. Sample answer: 100% of $56 is $56. Then estimate 7.5% of $56 by finding 10% of $56. Because 10% of $56 is $5.60, an estimate for 107.5% of $56 is $56 + $5.60 or $61.60.

(continued on next page)

Example 3 Sales Tax

Henry purchases $56.00 worth of clothing at the store. The sales tax in his city is 7.5%.

What is the total cost of the clothing?

Method 1 Find the sales tax.

Use a proportion to find the amount of tax. Let t represent the amount of tax. Then add the tax to the cost of the clothing.

$\frac{t}{56} = \frac{7.5}{100}$ Write the proportion.

$\frac{t}{56} = 0.075$ Divide 7.5 by 100.

$56 \cdot \left(\frac{t}{56}\right) = (0.075) \cdot 56$ Multiplication Property of Equality; Notice the tax is equal to the product of the percent, written as a decimal, and the cost.

$t = 4.20$ Simplify.

Add the sales tax to the selling price.

$4.20 + $56.00 = $60.20 The total cost is $60.20.

Method 2 Add the sales tax percent to 100%.

Because 100% represents the selling price, add the sales tax percent to 100%. The total percent is 100% + 7.5% or 107.5%.

$\frac{c}{56} = \frac{107.5}{100}$ Write the proportion. Let c represent the total cost.

$\frac{c}{56} = 1.075$ Divide 107.5 by 100.

$56 \cdot \left(\frac{c}{56}\right) = (1.075) \cdot 56$ Multiplication Property of Equality; Notice the total cost is equal to the product of the total percent, written as a decimal, and the cost.

$c = 60.20$ Simplify.

So, the total cost of the clothing is $60.20.

Think About It! How would you begin solving the problem?

See students' responses.

(continued on next page)

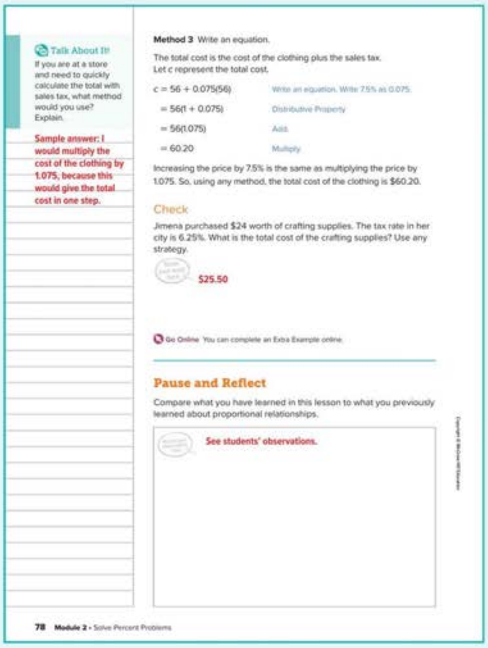
Lesson 2-2 • Tax 77

Interactive Presentation

Example 3, Sales Tax, Slide 1 of 6

CLICK

On Slide 2, students move through the slides to use the properties of operations to find the amount of sales tax.



Talk About It! If you are at a store and need to quickly calculate the total with sales tax, what method would you use? Explain.

Sample answer: I would multiply the cost of the clothing by 1.075, because this would give the total cost in one step.

Method 3 Write an equation.

The total cost is the cost of the clothing plus the sales tax. Let c represent the total cost.

$c = 56 + 0.075(56)$	Write an equation. Write 7.5% as 0.075.
$= 56(1 + 0.075)$	Distributive Property
$= 56(1.075)$	Add.
$= 60.20$	Multiply.

Increasing the price by 7.5% is the same as multiplying the price by 1.075. So, using any method, the total cost of the clothing is $60.20.

Check

Jimena purchased $24 worth of crafting supplies. The tax rate in her city is 6.25%. What is the total cost of the crafting supplies? Use any strategy.

$25.50

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you have learned in this lesson to what you previously learned about proportional relationships.

See students' observations.

78 Module 2 • Solve Percent Problems

Interactive Presentation

Example 3, Sales Tax, Slide 3 of 6

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 3 Sales T ax *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL What does each part of the equation represent? Sample answer: c represents the total cost including sales tax; 56 represents the cost of the clothing, or 100%; 0.075(56) represents the amount of sales tax, or 7.5% of the cost of the clothing.

OL When using Method 3, why are you able to use the Distributive Property in the second step? Sample answer: Because each term on the right side of the equation is multiplied by 56, you can rewrite the expression as a product of 56 and its remaining factors.

BL How could you use the steps in Method 2 or 3 to find the total cost including sales tax for any purchase you make? Sample answer: In both Methods 2 and 3 the cost of the clothing $56, is multiplied by 1.075, which is the sales tax percent added to 100% andthen written as a decimal. For any purchase you make, you can multiply the cost by the percent written as a decimal to find the total, where the percent is the sales tax rate plus 100%.

Talk About It!

SLIDE 5

Mathematical Discourse

If you are at a store and need to quickly calculate sales tax, what method would you use? Explain. Sample answer: I would multiply the cost of the clothing by 1.075, because this would give the total cost in one step.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Shopping

Objective

Students will come up with their own strategy to solve an application problem involving the total cost of a purchase at a grocery store.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

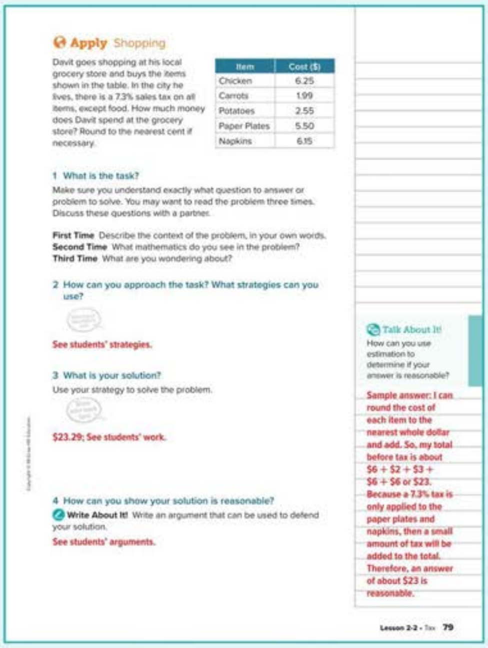
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Which items have sales tax, and which do not?
- How do you calculate sales tax on the non-food items?
- What do you need to do with the cost of the food items?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Shopping

Davit goes shopping at his local grocery store and buys the items shown in the table. In the city he lives, there is a 7.3% sales tax on all items, except food. How much money does Davit spend at the grocery store? Round to the nearest cent if necessary.

Item	Cost ($)
Chicken	6.25
Carrots	1.99
Potatoes	2.55
Paper Plates	5.50
Napkins	6.15

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

$23.29; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
How can you use estimation to determine if your answer is reasonable?

Sample answer: I can round the cost of each item to the nearest whole dollar and add. So, my total before tax is about $6 + $2 + $3 + $6 + $6 or $23. Because a 7.3% tax is only applied to the paper plates and napkins, then a small amount of tax will be added to the total. Therefore, an answer of about $23 is reasonable.

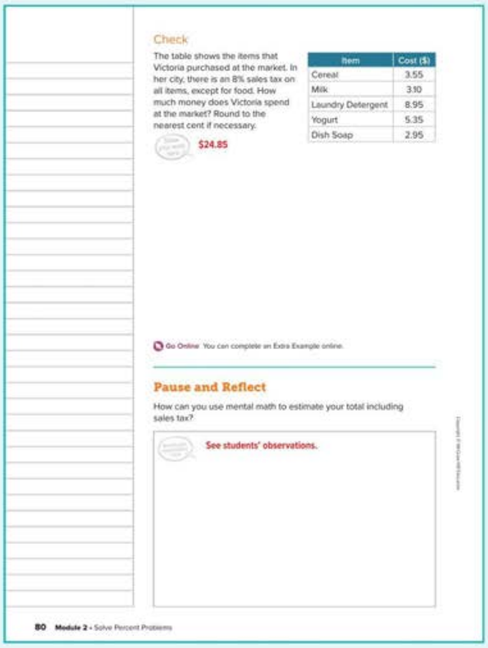
Lesson 2-2 • Tax 79

Interactive Presentation

Apply, Shopping

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows the items that Victoria purchased at the market. In her city, there is an 8% sales tax on all items, except for food. How much money does Victoria spend at the market? Round to the nearest cent if necessary.

Item	Cost ($)
Cereal	3.55
Milk	3.10
Laundry Detergent	8.95
Yogurt	5.35
Dish Soap	2.95

$24.85

Go Online You can complete an Extra Example online.

Pause and Reflect

How can you use mental math to estimate your total including sales tax?

See students' observations.

80 Module 2 • Solve Percent Problems

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Suppose you purchase school supplies for $20 and lunch meat for $5 at a store. Sales tax of 6% is added to all non-food items in your state. What is the total cost of all the items? Write a mathematical argument that can be used to defend your solution.
$26.20; Sample answer: Multiply the price of the school supplies, $20, by the sales tax 6%, or 0.06: $1.20. Then add the tax to the price of the school supplies: $21.20. Then add the price of school supplies including tax to the price of the lunch meat: $26.20.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11 odd, 12–15
- ALEKS Percent Increase and Decrease

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–10, 12, 15
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Understanding Percents

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

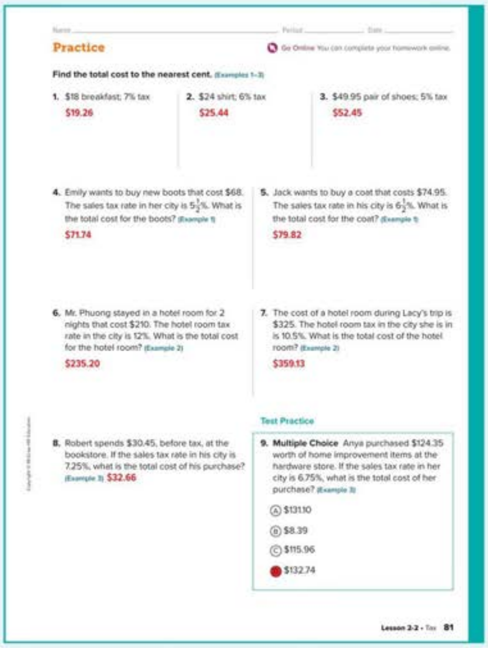
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the total cost for an item given the item's cost and the percent of sales tax	1–3
2	find the total cost for an item given the item's cost and the percent of sales tax	4, 5, 8, 9
2	find the total cost of a hotel room with the hotel room tax	6, 7
3	solve application problems involving tax	10, 11
3	higher-order and critical thinking skills	12–15

Common Misconception

Students may have trouble rewriting percents that include fractions as decimals. For example, in Exercises 4 and 5, students may not correctly rewrite $5\frac{1}{2}\%$ and $6\frac{1}{2}\%$ as 0.055 and 0.065, respectively. Remind them that $\frac{1}{2}\%$ is equivalent to 0.5% which is equivalent to 0.005.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find the total cost to the nearest cent. (Examples 1–3)

1. $18 breakfast; 7% tax
$19.26

2. $24 shirt; 6% tax
$25.44

3. $49.95 pair of shoes; 5% tax
$52.45

4. Emily wants to buy new boots that cost $68. The sales tax rate in her city is $5\frac{1}{2}\%$. What is the total cost for the boots? (Example 1)
$71.74

5. Jack wants to buy a coat that costs $74.95. The sales tax rate in his city is $6\frac{1}{2}\%$. What is the total cost for the coat? (Example 1)
$79.82

6. Mr. Phuong stayed in a hotel room for 2 nights that cost $210. The hotel room tax rate in the city is 12%. What is the total cost for the hotel room? (Example 2)
$235.20

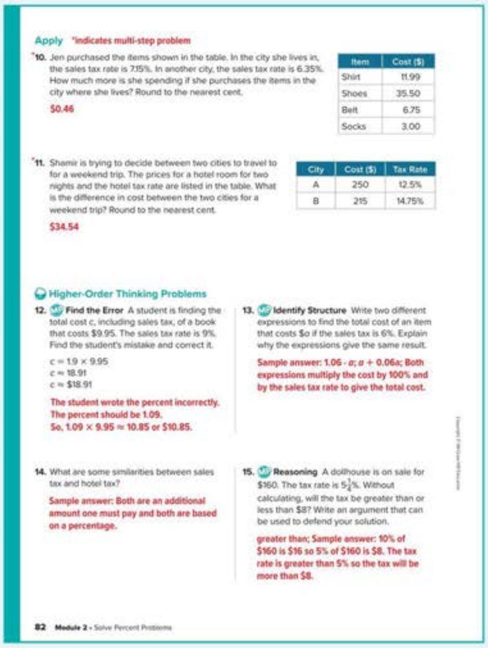
7. The cost of a hotel room during Lacy's trip is $325. The hotel room tax in the city she is in is 10.5%. What is the total cost of the hotel room? (Example 2)
$359.13

8. Robert spends $30.45, before tax, at the bookstore. If the sales tax rate in his city is 7.25%, what is the total cost of his purchase? (Example 3) $32.66

Test Practice

9. **Multiple Choice** Anya purchased $124.35 worth of home improvement items at the hardware store. If the sales tax rate in her city is 6.75%, what is the total cost of her purchase? (Example 3)
Ⓐ $131.10
Ⓑ $8.39
Ⓒ $115.96
● $132.74

Lesson 2-2 • Tax 81



Apply *indicates multi-step problem

*10. Jen purchased the items shown in the table. In the city she lives in, the sales tax rate is 7.15%. In another city, the sales tax rate is 6.35%. How much more is she spending if she purchases the items in the city where she lives? Round to the nearest cent.

$0.46

Item	Cost ($)
Shirt	11.99
Shoes	35.50
Belt	6.75
Socks	3.00

*11. Shamir is trying to decide between two cities to travel to for a weekend trip. The prices for a hotel room for two nights and the hotel tax rate are listed in the table. What is the difference in cost between the two cities for a weekend trip? Round to the nearest cent.

$34.54

City	Cost ($)	Tax Rate
A	250	12.5%
B	215	14.75%

Higher-Order Thinking Problems

12. **Find the Error** A student is finding the total cost c, including sales tax, of a book that costs $9.95. The sales tax rate is 9%. Find the student's mistake and correct it.

$c = 1.9 \times 9.95$
$c \approx 18.91$
$c \approx \$18.91$

The student wrote the percent incorrectly. The percent should be 1.09. So, $1.09 \times 9.95 \approx 10.85$ or $10.85.

13. **Identify Structure** Write two different expressions to find the total cost of an item that costs $a if the sales tax is 6%. Explain why the expressions give the same result.

Sample answer: $1.06 \cdot a$; $a + 0.06a$; Both expressions multiply the cost by 100% and by the sales tax rate to give the total cost.

14. What are some similarities between sales tax and hotel tax?

Sample answer: Both are an additional amount one must pay and both are based on a percentage.

15. **Reasoning** A dollhouse is on sale for $160. The tax rate is $5\frac{1}{4}$%. Without calculating, will the tax be greater than or less than $8? Write an argument that can be used to defend your solution.

greater than; Sample answer: 10% of $160 is $16 so 5% of $160 is $8. The tax rate is greater than 5% so the tax will be more than $8.

82 Module 2 • Solve Percent Problems

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Ex ercise 12, students find a student's mistake and correct it.

7 Look For and Make Use of Structure In Exercise 13, students must identify the operational structure used to find total cost including sales tax as well as algebraic structure.

2 Reason Abstractly and Quantitatively In Exercise 15, students use quantitative reasoning to estimate sales tax.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercise 10 Have students work in groups of 3–4. After completing Exercise 10, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 11.

Listen and ask clarifying questions.

Use with Exercise 12 Have students work in pairs. Have students individually read Exercise 12 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 13.

Lesson 2-3

Tips and Markups

7.RP.A.3, 7.EE.A.2

LESSON GOAL

Students will solve multi-step ratio and percent problems involving tips and markups.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Tips
Example 1: Tips
Learn: Markup
Example 2: Markup
Example 3: Markup
Apply: Dining Out

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 9 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving tips and markups.

ELL You can use the tips and suggestions on page T9 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major clusters **7. RP.A** and **7.EE.A** by solving problems involving tips and markups.
Standards for Mathematical Content: **7. RP.A.3, 7.EE.A.2,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP7**

Coherence

Vertical Alignment

Previous
Students solved multi-step ratio and percent problems involving tax.
7.RP.A.3, 7.EE.A.2

Now
Students solve multi-step ratio and percent problems involving tips and markups.
7.RP.A.3, 7.EE.A.2

Next
Students will solve multi-step ratio and percent problems involving discounts.
7.RP.A.3, 7.EE.A.2

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

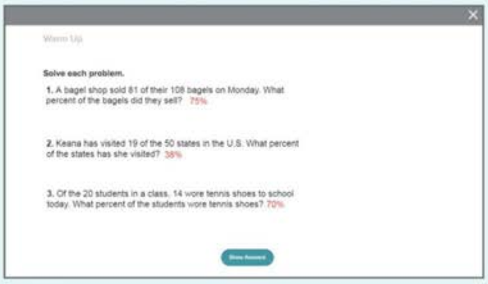
Conceptual Bridge In this lesson, students *apply* their *understanding* of ratios and percent to solve problems involving tips and markups. They build *fluency* in using ratio reasoning and/or properties of operations to find the amount of tip or markup and the total amount paid. They *apply* their *understanding* of markup to find the percent of markup given the selling and wholesale prices of an item.

Mathematical Background

A *tip*, or *gratuity*, is an additional amount of money paid in return for a service. Tips are calculated as a percent of the service. A tip can be represented as a percent of increase because the final amount paid increases by that percent.

Stores typically sell items for more than they pay for them. The amount a store pays for an item is called the *wholesale cost*. The amount of increase is called the *markup*. The *selling price* is the amount the customer pays for an item. A markup can also be represented as a percent of increase.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding the percent when given the part and the whole (Exercises 1–3)

Answers

1. 75%
2. 38%
3. 70%

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about tips for services in everyday life.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- The term *gratuity* comes from the Latin root *gratus*, which means pleasing or thankful. How have you heard the term gratuity used in everyday life? How do you think it might relate to its Latin root? Sample answer: A gratuity is a tip we leave for our servers at a restaurant. We leave gratuities because we are thankful for their service.
- How could you use the word *tip* in a sentence ? Sample answer: I paid the server a tip for serving me at a restaurant.
- How does the word *up* help you understand markup ? Sample answer: Up means to go higher. So, markup means to raise something higher.
- The amount a store pays for an item is the wholesale cost. Who do you think pays the *selling price*? Sample answer: The customer pays the selling price of an item.

Learn Tips

Objective

Students will understand that tips are usually based on a percent of the service provided.

Go Online to find additional teaching notes.

Example 1 Tips

Objective

Students will find the total cost of a service including a percent tip.

Questions for Mathematical Discourse

SLIDE 2

AL What does t represent? t represents the part, which is the amount of the tip

OL Explain how you can check the amount of the tip for reasonableness. Sample answer: 18% is a little less than 20%. 20% of $125 would be twice 10% of $125, or 2($12.50), which is $25. So, the amount of the tip should be a little less than $25.

BL If the tip on a $125 bill was $20, what was the percent of the tip? 16%

SLIDE 3

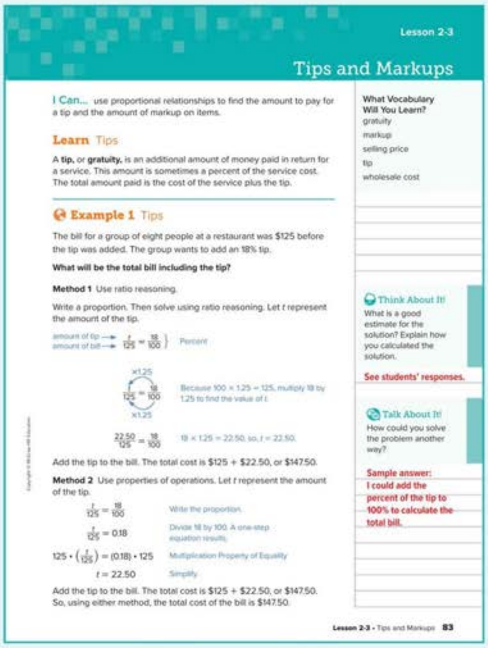
AL Explain the main difference between Method 1 and Method 2. Sample answer: In Method 1, scaling is used to find the value of t in the proportion. In Method 2, properties of operations are used to find the value of t.

OL When using Method 2, why are you able to use the Multiplication Property of Equality? Sample answer: Because a one-step equation results after dividing 18 by 100, you can isolate the variable t by undoing the division with multiplication.

BL Suppose you want to leave a tip of 20%. How could you mentally compute the amount of tip? Sample answer: 10% of $125 is $12.50, so 20% would be twice that or $25.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 2-3

Tips and Markups

I Can... use proportional relationships to find the amount to pay for a tip and the amount of markup on items.

Learn Tips

A **tip**, or **gratuity**, is an additional amount of money paid in return for a service. This amount is sometimes a percent of the service cost. The total amount paid is the cost of the service plus the tip.

Example 1 Tips

The bill for a group of eight people at a restaurant was $125 before the tip was added. The group wants to add an 18% tip.

What will be the total bill including the tip?

Method 1 Use ratio reasoning.

Write a proportion. Then solve using ratio reasoning. Let t represent the amount of the tip.

amount of tip → / amount of bill → $\frac{t}{125} = \frac{18}{100}$ } Percent

$\frac{t}{125} = \frac{18}{100}$ (×1.25) Because 100 × 1.25 = 125, multiply 18 by 1.25 to find the value of t.

$\frac{22.50}{125} = \frac{18}{100}$ 18 × 1.25 = 22.50, so, t = 22.50.

Add the tip to the bill. The total cost is $125 + $22.50, or $147.50.

Method 2 Use properties of operations. Let t represent the amount of the tip.

$\frac{t}{125} = \frac{18}{100}$ Write the proportion.

$\frac{t}{125} = 0.18$ Divide 18 by 100. A one-step equation results.

$125 \cdot \left(\frac{t}{125}\right) = (0.18) \cdot 125$ Multiplication Property of Equality

$t = 22.50$ Simplify.

Add the tip to the bill. The total cost is $125 + $22.50, or $147.50. So, using either method, the total cost of the bill is $147.50.

What Vocabulary Will You Learn? gratuity, markup, selling price, tip, wholesale cost

Think About It! What is a good estimate for the solution? Explain how you calculated the solution. See students' responses.

Talk About It! How could you solve the problem another way? Sample answer: I could add the percent of the tip to 100% to calculate the total bill.

Lesson 2-3 • Tips and Markups 83

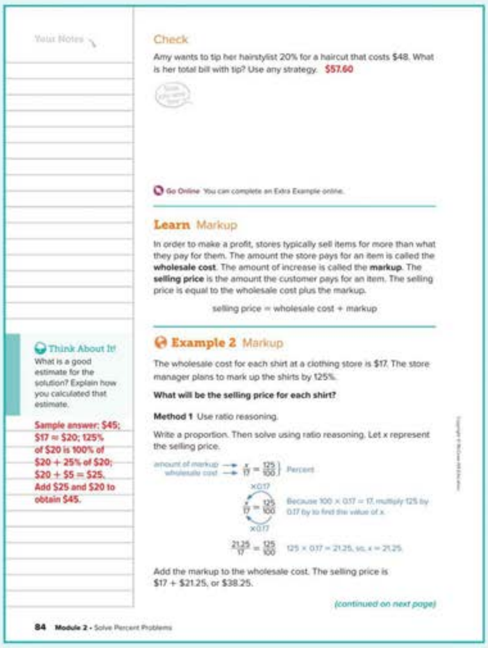
Interactive Presentation

Example 1, Tips, Slide 2 of 5

CLICK On Slide 2 of Example 1, students move through the slides to solve the proportion.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

7.RP.A.3, 7.EE.A.2



Your Notes

Check

Amy wants to tip her hairstylist 20% for a haircut that costs $48. What is her total bill with tip? Use any strategy. $57.60

Go Online You can complete an Extra Example online.

Learn Markup

In order to make a profit, stores typically sell items for more than what they pay for them. The amount the store pays for an item is called the **wholesale cost**. The amount of increase is called the **markup**. The **selling price** is the amount the customer pays for an item. The selling price is equal to the wholesale cost plus the markup.

selling price = wholesale cost + markup

Think About It! What is a good estimate for the solution? Explain how you calculated that estimate.

Sample answer: $45; $17 ≈ $20; 125% of $20 is 100% of $20 + 25% of $20; $20 + $5 = $25. Add $25 and $20 to obtain $45.

Example 2 Markup

The wholesale cost for each shirt at a clothing store is $17. The store manager plans to mark up the shirts by 125%.

What will be the selling price for each shirt?

Method 1 Use ratio reasoning.

Write a proportion. Then solve using ratio reasoning. Let x represent the selling price.

amount of markup → / wholesale cost → $\frac{x}{17} = \frac{125}{100}$ } Percent

$\frac{x}{17} = \frac{125}{100}$ (×0.17) Because 100 × 0.17 = 17, multiply 125 by 0.17 by to find the value of x.

$\frac{21.25}{17} = \frac{125}{100}$ 125 × 0.17 = 21.25, so, x = 21.25

Add the markup to the wholesale cost. The selling price is $17 + $21.25, or $38.25.

(continued on next page)

84 **Module 2** • Solve Percent Problems

Interactive Presentation

Learn, Markup

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Markup

Objective

Students will understand that the selling price of an item is equal to the item's wholesale cost plus the percent markup.

Teaching Notes

SLIDE 1

Students will learn the terms *wholesale cost, markup,* and *selling price*. You may wish to point out to students that markups are a form of percent of increase. Ask students why markups are necessary in the marketplace. Students should understand that a store usually cannot make a profit unless they sell the item for more than what they paid for it.

Example 2 Markup

Objective

Students will find the selling price of an item given the wholesale cost of an item and a percent markup.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to consider alternative methods to solving the problem, and to identify correspondences between the methods. Students should understand that 100% of a quantity is the quantity itself, and be able to explain why Method 1 and Method 2 both arrive at the same solution, even though the steps are different.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question, encourage them to make a case for when Method 1 might be the more helpful method to use. Students should listen to the explanations of others and decide whether or not their reasoning makes sense and ask clarifying questions, if needed.

Questions for Mathematical Discourse

SLIDE 2

AL What does *x* represent? *x* represents the part, which is the amount of the markup

OL Explain how you can check the amount of the markup for reasonableness. Sample answer: 125% is 100% + 25%. 100% of $17 is $17. 25% of $17 will be about $4, because $17 is close to $16, and one fourth of $16 is $4. So, the amount of the markup should be close to $17 + $4, or $21.

BL What does it tell you about the selling price of the shirt if the markup is 125%? Sample answer: The price of the shirt will more than double. It will be greater than $34.

(continued on next page)

Example 2 Markup *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL When using Method 2, why are you able to use the Multiplication Property of Equality? Sample answer: Because a one-step equation results after dividing 125 by 100, you can isolate the variable *t* by undoing the division with multiplication.

OL How can you solve the problem another way? Sample answer: I can add the percent of markup to 100% then set up a proportion and solve. Because 125% + 100% is 225%, then one of the ratios in the proportion would be $\frac{225}{100}$.

BL What will be the store's profit for selling 10 shirts? Explain. $212.50; Sample answer: The profit for one shirt is $21.25, so the profit for 10 shirts would be 10 • $21.25, or $212.50.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Markup

Objective

Students will find the percent of markup of an item given the selling price and the wholesale cost.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should be able to reason that finding the percent of markup is the same as finding the percent of increase. Encourage students to use their prior knowledge to determine the part and the whole in order to write the part-to-whole ratio.

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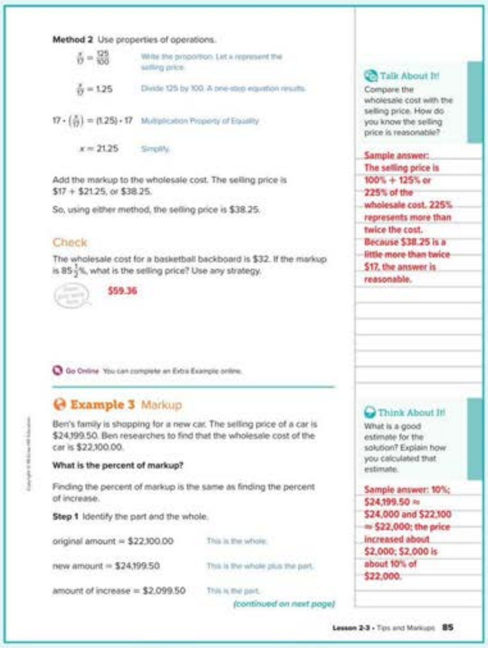


Method 2 Use properties of operations.

$\frac{x}{17} = \frac{125}{100}$ Write the proportion. Let x represent the selling price.

$\frac{x}{17} = 1.25$ Divide 125 by 100. A one-step equation results.

$17 \cdot \left(\frac{x}{17}\right) = (1.25) \cdot 17$ Multiplication Property of Equality

$x = 21.25$ Simplify.

Add the markup to the wholesale cost. The selling price is $17 + $21.25, or $38.25.

So, using either method, the selling price is $38.25.

Check

The wholesale cost for a basketball backboard is $32. If the markup is $85\frac{1}{2}$%, what is the selling price? Use any strategy.

$59.36

Talk About It! Compare the wholesale cost with the selling price. How do you know the selling price is reasonable?

Sample answer: The selling price is 100% + 125% or 225% of the wholesale cost. 225% represents more than twice the cost. Because $38.25 is a little more than twice $17, the answer is reasonable.

Go Online You can complete an Extra Example online.

Example 3 Markup

Ben's family is shopping for a new car. The selling price of a car is $24,199.50. Ben researches to find that the wholesale cost of the car is $22,100.00.

What is the percent of markup?

Finding the percent of markup is the same as finding the percent of increase.

Step 1 Identify the part and the whole.

original amount = $22,100.00 This is the whole.

new amount = $24,199.50 This is the whole plus the part.

amount of increase = $2,099.50 This is the part.

(continued on next page)

Think About It! What is a good estimate for the solution? Explain how you calculated that estimate.

Sample answer: 10%; $24,199.50 ≈ $24,000 and $22,100 ≈ $22,000; the price increased about $2,000; $2,000 is about 10% of $22,000.

Lesson 2-3 • Tips and Markups 85

Interactive Presentation

Example 2, Markup, Slide 2 of 5

CLICK

On Slide 2 of Example 2, students move through the steps to solve the proportion.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Talk About It!
How is finding the percent of markup different than finding the selling price of an item?

Sample answer: When finding the percent of markup, you use the part and the whole. When finding the selling price, you know the percent and the whole.

Step 2 Find the percent of increase.

$\frac{\text{part}}{\text{whole}} = \frac{2{,}099.50}{22{,}100.00}$ Write the part-to-whole ratio. The part is 2,099.50. The whole is 22,100.00.

$= 0.095$ Divide.

$= \frac{9.5}{100}$ Write an equivalent ratio, as a rate per 100.

$= 9.5\%$ Definition of percent

So, the percent of markup for the wholesale price of the car is 9.5%.

Check

Mikka is making jewelry for a craft show. The wholesale cost of a bracelet is $12.50. If she sells them for $20, what is the percent of markup?

60%

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare and contrast tips and markups. Where have you seen or used tips and markups in your everday life?

See students' observations.

86 Module 2 • Solve Percent Problems

Interactive Presentation

Example 3, Markup, Slide 1 of 4

TYPE

On Slide 2, students determine the wholesale cost.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Markup *(continued)*

Questions for Mathematical Discourse

SLIDE 2

AL What does *wholesale cost* mean? Does the wholesale cost represent the original amount or the new amount? the amount a store pays for an item; it represents the original amount

OL How do you find the amount of increase? Does this represent the part or the whole? subtract the wholesale cost from the selling price; it represents the part

BL Suppose a classmate stated that the percent of increase is 95%. Describe the error they made. Sample answer: They incorrectly wrote the rate per 100 for 0.095 as $\frac{95}{100}$ instead of $\frac{9.5}{100}$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Dining Out

Objective
Students will come up with their own strategy to solve an application problem involving tips.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use
Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.
- Does it matter if you find the tip or the tax first? Why or why not?
- Is the sales tax applied to just the cost of the food, or the cost of the food plus the tip?

Write About It!
Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Dining Out

Kerry and three friends went out for dinner. They split a large pizza, and each person had a salad and a soda. They want to leave a 15% tip on the cost of the food, and the sales tax is $8\frac{1}{4}$%. How much will each person pay if they split the bill evenly?

Pizza	\$18.60
Salad	\$2.50
Soda	\$2.25

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?
See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.
\$11.59; See students' work.

Talk About It! What steps should you take before splitting the bill?
Sample answer: You need to find the total cost of the bill including the tax and the tip.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.
See students' arguments.

Lesson 2-3 • Tips and Markups 87

Interactive Presentation

Apply, Dining Out

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

Brian has $24 worth of pizza delivered to his house. He pays the bill plus a 15% tip and 7% sales tax. He also pays a $3 delivery fee that is charged after the tax and tip. How much change does he receive, if he pays with two $20 bills?

$7.72

Go Online You can complete an Extra Example online.

Pause and Reflect

Explain how tips and markups are percents of increase.

See students' observations.

88 Module 2 • Solve Percent Problems

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can percent describe the change of a quantity?

In this lesson, students learned about tips, wholesale cost, markups, and selling price. Encourage them to discuss with a partner how a markup changes the price of an item, and why markups can best be represented as a percent. For example, have students explain how a store can use percents to represent markups and why using percents makes it easier to compare different types of markups.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you had lunch with a friend at a restaurant. The lunch cost $32.00, and sales tax was $4.45. If you plan to tip 20%, find the total amount you need to pay for lunch. Write a mathematical argument that can be used to defend your solution.
$42.85; Sample answer: Find 20% of $32.00, which is $6.40. Add $6.40 to $32.00, which is $38.40. Then add the sales tax of $4.45. $38.40 + $4.45 = $42.85

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11 odd, 13–16
- ALEKS® Percent Increase and Decrease

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 14, 15
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Understanding Percents

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

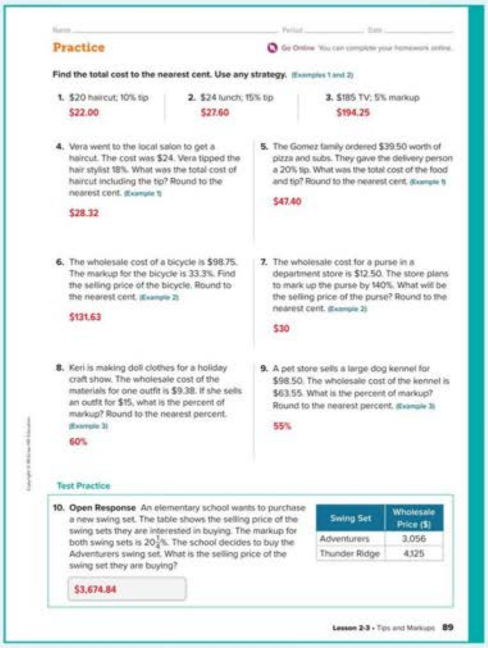
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the total cost including a tip or markup based on a percent	1–3
2	find the total cost of a service including a tip based on a percent	4, 5
2	find the selling price of an item given the wholesale cost of an item and a markup based on a percent	6, 7
2	find the percent of markup of an item given the selling price of an item and the wholesale cost	8, 9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving tips or markups	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

When finding the percent of markup, students may not use the correct value for the whole. For example, in Exercise 9, the selling price of the dog kennel is $98.50 while the wholesale cost is $63.55. After finding the amount of increase, $34.95, students may use $98.50 as the whole. In this case, the whole, or original cost of the dog kennel, is $63.55.



Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Find the total cost to the nearest cent. Use any strategy. (Examples 1 and 2)

1. $20 haircut; 10% tip
$22.00

2. $24 lunch; 15% tip
$27.60

3. $185 TV; 5% markup
$194.25

4. Vera went to the local salon to get a haircut. The cost was $24. Vera tipped the hair stylist 18%. What was the total cost of haircut including the tip? Round to the nearest cent. (Example 1)
$28.32

5. The Gomez family ordered $39.50 worth of pizza and subs. They gave the delivery person a 20% tip. What was the total cost of the food and tip? Round to the nearest cent. (Example 1)
$47.40

6. The wholesale cost of a bicycle is $98.75. The markup for the bicycle is 33.3%. Find the selling price of the bicycle. Round to the nearest cent. (Example 2)
$131.63

7. The wholesale cost for a purse in a department store is $12.50. The store plans to mark up the purse by 140%. What will be the selling price of the purse? Round to the nearest cent. (Example 2)
$30

8. Keri is making doll clothes for a holiday craft show. The wholesale cost of the materials for one outfit is $9.38. If she sells an outfit for $15, what is the percent of markup? Round to the nearest percent. (Example 3)
60%

9. A pet store sells a large dog kennel for $98.50. The wholesale cost of the kennel is $63.55. What is the percent of markup? Round to the nearest percent. (Example 3)
55%

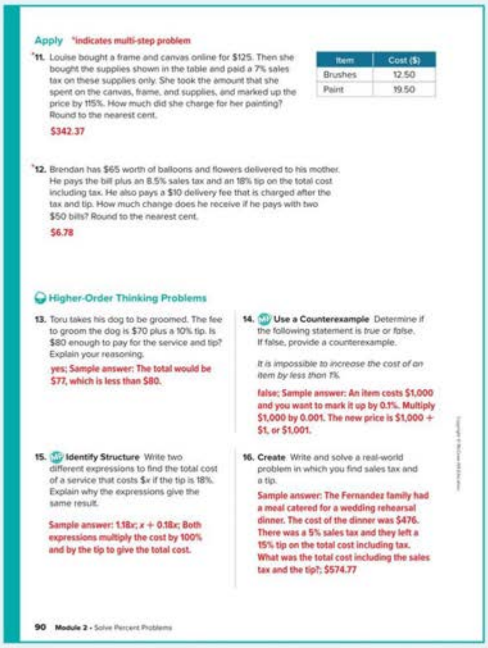
Test Practice

10. **Open Response** An elementary school wants to purchase a new swing set. The table shows the selling price of the swing sets they are interested in buying. The markup for both swing sets is $20\frac{1}{4}$%. The school decides to buy the Adventurers swing set. What is the selling price of the swing set they are buying?

Swing Set	Wholesale Price ($)
Adventurers	3,056
Thunder Ridge	4,125

$3,674.84

Lesson 2-3 • Tips and Markups 89



Apply *indicates multi-step problem

*11. Louise bought a frame and canvas online for $125. Then she bought the supplies shown in the table and paid a 7% sales tax on these supplies only. She took the amount that she spent on the canvas, frame, and supplies, and marked up the price by 115%. How much did she charge for her painting? Round to the nearest cent.

Item	Cost ($)
Brushes	12.50
Paint	19.50

$342.37

*12. Brendan has $65 worth of balloons and flowers delivered to his mother. He pays the bill plus an 8.5% sales tax and an 18% tip on the total cost including tax. He also pays a $10 delivery fee that is charged after the tax and tip. How much change does he receive if he pays with two $50 bills? Round to the nearest cent.

$6.78

Higher-Order Thinking Problems

13. Toru takes his dog to be groomed. The fee to groom the dog is $70 plus a 10% tip. Is $80 enough to pay for the service and tip? Explain your reasoning.

yes; Sample answer: The total would be $77, which is less than $80.

14. **Use a Counterexample** Determine if the following statement is true or false. If false, provide a counterexample.

It is impossible to increase the cost of an item by less than 1%.

false; Sample answer: An item costs $1,000 and you want to mark it up by 0.1%. Multiply $1,000 by 0.001. The new price is $1,000 + $1, or $1,001.

15. **Identify Structure** Write two different expressions to find the total cost of a service that costs $$x$ if the tip is 18%. Explain why the expressions give the same result.

Sample answer: $1.18x$; $x + 0.18x$; Both expressions multiply the cost by 100% and by the tip to give the total cost.

16. **Create** Write and solve a real-world problem in which you find sales tax and a tip.

Sample answer: The Fernandez family had a meal catered for a wedding rehearsal dinner. The cost of the dinner was $476. There was a 5% sales tax and they left a 15% tip on the total cost including tax. What was the total cost including the sales tax and the tip?; $574.77

90 Module 2 • Solve Percent Problems

Teaching the Mathematical Practices

3 Construct a Viable Argument and Critique the Reasoning of Others In Ex ercise 14, students determine if a statement is true or false, providing a counterexample if it is false.

7 Look For and Make Use of Structure In Exercise 15, students write two different expressions to calculate the total cost of a service with a tip, and explain why they are equivalent.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 11–12 Have pairs of students interview each other as they complete these apply problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 11 might be "How much did Louise spend on supplies only, before tax?"

Listen and ask clarifying questions.
Use with Exercise 14 Have students work in pairs. Have students individually read Exercise 14 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 15.

Lesson 2-4
Discounts

7.RP.A.3, 7.EE.A.2

LESSON GOAL

Students will solve multi-step ratio and percent problems involving discounts.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Discounts
Example 1: Discounts
Example 2: Combined Discounts
Example 3: Find the Original Price
Apply: Shopping

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Arrive**MATH** Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 10 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving discounts.

ELL You can use the tips and suggestions on page T10 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min — 0.5 day
45 min — 1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major clusters **7. RP.A** and **7.EE.A** by solving problems involving discounts.
Standards for Mathematical Content: **7. RP.A.3, 7.EE.A.2,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MPI, MP2, MP3, MP4, MP6,MP7**

Coherence

Vertical Alignment

Previous
Students solved multi-step ratio and percent problems involving tips and markups.
7.RP.A.3, 7.EE.A.2

Now
Students solve multi-step ratio and percent problems involving discounts.
7.RP.A.3, 7.EE.A.2

Next
Students will solve problems involving simple interest.
7.RP.A.3

Rigor

The Three Pillars of Rigor

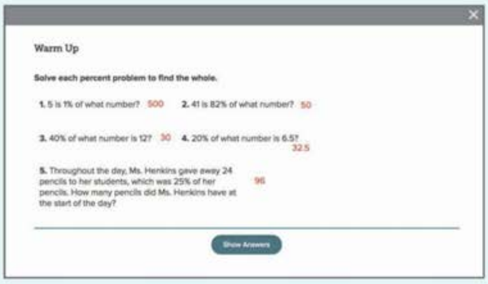
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students *apply* their *understanding* of ratios and percent to solve problems involving discounts. Students come to understand that multiple discounts on an item are not equivalent to a discount of the combined percents. They build *fluency* in using ratio reasoning to find the amount of the discount and the final selling price, using different methods, including subtracting the percent of discount from 100%.

Mathematical Background

Discount or *markdown* is the amount by which the regular price of an item is reduced. The sale price is the original price minus the discount. The percent of discount is a percent of decrease, because the final cost of the item decreases by that percent.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding the whole when given the part and the percent (Exercises 1–5)

Answers

1. 500
2. 50
3. 30
4. 32.5
5. 96

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about discounts in everyday life.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does the prefix *dis-* mean? Sample answer: opposite of, remove, or undo
- How does the key word *down* help you understand the term *markdown*? Sample answer: Down means to decrease or to go lower. So, markdown means to make something decrease.

Learn Discounts

Objective

Students will understand that a discount is an amount by which the price of an item is decreased and is often represented as a percent of the original price.

Teaching Notes

SLIDE 1

Students will learn the terms *discount* and *markdown*. You may wish to ask students what the term *discount* means in their own words. They should note that discount is the amount that is taken off the regular price.

Students previously learned about markups. Have them explain how the terms markup and markdown are constructed and how that can help them remember whether each one is a percent of increase or decrease.

Example 1 Discounts

Objective

Students will find the sale price of an item given the original cost and a percent discount.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them Encourage students to consider alternative methods to solving the problem, and to identify correspondences between the methods.

2 Reason Abstractly and Quantitatively Students should understand that 100% of the original price is the original price itself, and use reasoning to explain why the sale price is 35% of the original price if there is a 65% discount.

Questions for Mathematical Discourse

SLIDE 2

AL What does d represent? *d* represents the amount of the discount

OL Will the amount of the discount be less than or greater than $30? Explain. greater than; Sample answer: $59 is close to $60; If the discount was $30, then the percent of discount would be about 50%. Because 65% > 50%, the discount will be greater than $30.

BL If the amount of the discount was $14.75, what would be the percent of discount? 25%

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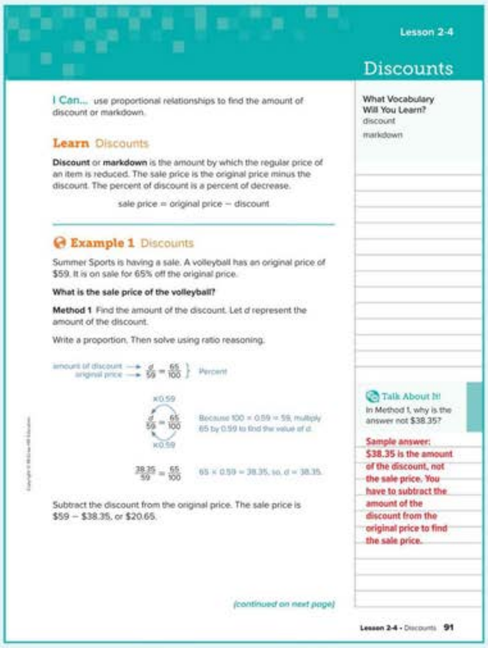


Lesson 2-4

Discounts

I Can... use proportional relationships to find the amount of discount or markdown.

What Vocabulary Will You Learn?
discount
markdown

Learn Discounts

Discount or **markdown** is the amount by which the regular price of an item is reduced. The sale price is the original price minus the discount. The percent of discount is a percent of decrease.

sale price = original price − discount

Example 1 Discounts

Summer Sports is having a sale. A volleyball has an original price of $59. It is on sale for 65% off the original price.

What is the sale price of the volleyball?

Method 1 Find the amount of the discount. Let d represent the amount of the discount.

Write a proportion. Then solve using ratio reasoning.

amount of discount → / original price → $\frac{d}{59} = \frac{65}{100}$ } Percent

$\frac{d}{59} = \frac{65}{100}$ (×0.59) Because 100 × 0.59 = 59, multiply 65 by 0.59 to find the value of d.

$\frac{38.35}{59} = \frac{65}{100}$ 65 × 0.59 = 38.35, so d = 38.35.

Subtract the discount from the original price. The sale price is $59 − $38.35, or $20.65.

Talk About It! In Method 1, why is the answer not $38.35?

Sample answer: $38.35 is the amount of the discount, not the sale price. You have to subtract the amount of the discount from the original price to find the sale price.

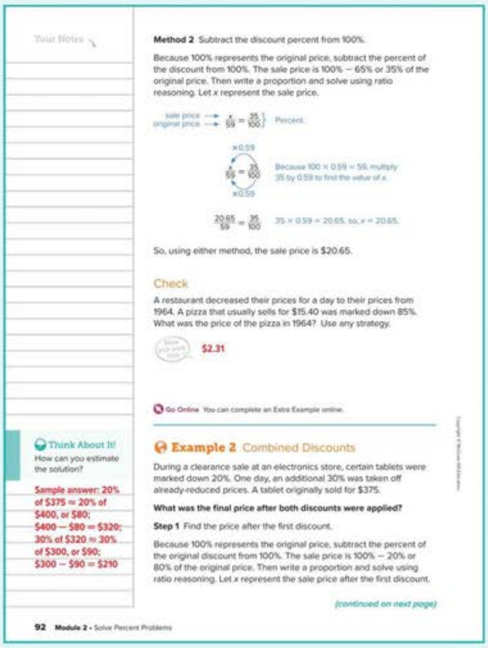
(continued on next page)

Lesson 2-4 • Discounts 91

Interactive Presentation

Example 1, Discounts, Slide 2 of 5

CLICK On Slide 2, students move through the slides to solve the proportion.



Your Notes

Method 2 Subtract the discount percent from 100%.

Because 100% represents the original price, subtract the percent of the discount from 100%. The sale price is 100% − 65% or 35% of the original price. Then write a proportion and solve using ratio reasoning. Let x represent the sale price.

sale price → / original price → $\frac{x}{59} = \frac{35}{100}$ } Percent

×0.59

$\frac{x}{59} = \frac{35}{100}$ Because 100 × 0.59 = 59, multiply 35 by 0.59 to find the value of x.

×0.59

$\frac{20.65}{59} = \frac{35}{100}$ 35 × 0.59 = 20.65, so, x = 20.65.

So, using either method, the sale price is $20.65.

Check

A restaurant decreased their prices for a day to their prices from 1964. A pizza that usually sells for $15.40 was marked down 85%. What was the price of the pizza in 1964? Use any strategy.

$2.31

Go Online You can complete an Extra Example online.

Think About It! How can you estimate the solution?

Sample answer: 20% of $375 ≈ 20% of $400, or $80; $400 − $80 = $320; 30% of $320 ≈ 30% of $300, or $90; $300 − $90 = $210

Example 2 Combined Discounts

During a clearance sale at an electronics store, certain tablets were marked down 20%. One day, an additional 30% was taken off already-reduced prices. A tablet originally sold for $375.

What was the final price after both discounts were applied?

Step 1 Find the price after the first discount.

Because 100% represents the original price, subtract the percent of the original discount from 100%. The sale price is 100% − 20% or 80% of the original price. Then write a proportion and solve using ratio reasoning. Let x represent the sale price after the first discount.

(continued on next page)

92 Module 2 • Solve Percent Problems

Interactive Presentation

Example 1, Discounts, Slide 3 of 5

CLICK

On Slide 3 of Example 1, students move through the slides to find the sale price.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.RP.A.3, 7.EE.A.2

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Example 1 Discounts *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL If the discount is 65%, what percent of the original price will actually be paid? 35%

OL What is the main difference between Method 1 and Method 2? Sample answer: In Method 1, first find the amount of the discount, then subtract that from the original price. In Method 2, first subtract the percent of discount from 100% to find the percent that represents the sale price, then multiply that by the original price to find the sale price.

BL When would Method 1 be more useful? Method 2? Sample answer: Method 1 would be more useful when you need to find the amount of the discount, but not the sale price; Method 2 would be more useful when you only need to find the sale price.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Combined Discounts

Objective

Students will find the sale price of an item given the original cost and more than one discount applied.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the fact that the two discounts must be applied separately. Students should understand that the second discount is taken after the first discount has been applied which results in a different whole, or starting price prior to the second discount.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question, encourage them to construct a plausible argument for why the two discounts must be applied separately. They should use examples and/or counterexamples to demonstrate their reasoning.

(continued on next page)

Example 2 Combined Discounts *(continued)*

Questions for Mathematical Discourse

SLIDE 2

AL How many discounts are given in this problem? 2

AL How do we know which discount to apply first? The first discount is that the tablets were marked down 20%.

OL Why do we find 80% of the original price? If the discount is 20%, then the sale price will be 80% of the original price.

OL Is there another way to find the clearance price after the first discount? Explain. yes; Sample answer: I can find 20% of $375 and then subtract that from $375.

OL Why is there a new starting amount after the first discount was applied? The additional 30% discount was taken off of already-reduced prices, so the original price is now $300 when completing Step 2.

BL Suppose a classmate stated that the clearance price was 120% of the original price. Explain their error. Sample answer: They added 20% to 100% instead of subtracting. It does not make sense that the clearance price would be greater than the original price.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Reteaching Activity AL

To help students that may be struggling with discounts, have them first identify the amount of discount. Finding the amount of discount might be calculated by taking a percent of the original price. Have students find the amount of discount for each of the following.

1. A pair of jeans costs $40 and is discounted $10. $10
2. A clock regularly costs $12 and is on sale for $3 off the regular price. $3
3. A sofa regularly costs $800 is discounted 10%. $80
4. A grill costs $150 and is on sale for 20% off the regular price. $30

$\frac{\text{sale price}}{\text{original price}} \rightarrow \frac{x}{375} = \frac{80}{100}$ } Percent

$\frac{x}{375} = \frac{80}{100}$ (×3.75) Because 100 × 3.75 = 375, multiply 80 by 3.75 to find the value of *x*.

$\frac{300}{375} = \frac{80}{100}$ 80 × 3.75 = 300, so, *x* = 300. The price after the first discount is $300.

Step 2 Find the final price after the additional discount.

Because 100% now represents the price after the first discount, subtract the percent of the additional discount from 100%. The sale price is 100% − 30% or 70% of the price after the first discount. Then write a proportion and solve using ratio reasoning. Let *x* represent the sale price after the additional discount.

$\frac{\text{sale price}}{\text{original price}} \rightarrow \frac{x}{300} = \frac{70}{100}$ } Percent

$\frac{x}{300} = \frac{70}{100}$ (×3) Because 100 × 3 = 300, multiply 70 by 3 to find the value of *x*.

$\frac{210}{300} = \frac{70}{100}$ 70 × 3 = 210, so, *x* = 210.

So, the final price of the tablet after both discounts were applied was $210.

Talk About It! The final price had a discount of 20% followed by a discount of 30%. Is this the same as finding 20% + 30% or 50% of the original price? Use the values in the Example to justify your reasoning.

Sample answer: No; the answer $210 is not 50% of the original price of $375. The second discount is taken on 80% of the original price, not 100% of the original price. The final price is actually 56% of the original price.

Check

A clothing store marked all of their summer clothes down 50%. A sign in the store indicates that an additional 20% is to be taken off clearance prices. What is the final price of a top that originally sold for $58?

$23.20

Go Online You can complete an Extra Example online.

Lesson 2-4 • Discounts 93

Interactive Presentation

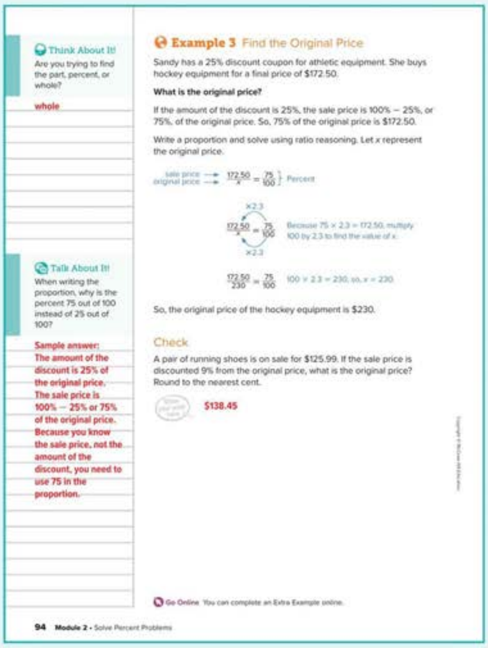
Example 2, Combined Discounts, Slide 3 of 5

CLICK

On Slide 2, students move through the slides to find the first discount.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It!
Are you trying to find the part, percent, or whole?

whole

Talk About It!
When writing the proportion, why is the percent 75 out of 100 instead of 25 out of 100?

Sample answer: The amount of the discount is 25% of the original price. The sale price is 100% − 25% or 75% of the original price. Because you know the sale price, not the amount of the discount, you need to use 75 in the proportion.

Example 3 Find the Original Price

Sandy has a 25% discount coupon for athletic equipment. She buys hockey equipment for a final price of $172.50.

What is the original price?

If the amount of the discount is 25%, the sale price is 100% − 25%, or 75%, of the original price. So, 75% of the original price is $172.50.

Write a proportion and solve using ratio reasoning. Let x represent the original price.

sale price → original price → $\frac{172.50}{x} = \frac{75}{100}$ } Percent

$\frac{172.50}{x} = \frac{75}{100}$ ($\times 2.3$) Because $75 \times 2.3 = 172.50$, multiply 100 by 2.3 to find the value of x.

$\frac{172.50}{230} = \frac{75}{100}$ $100 \times 2.3 = 230$, so, $x = 230$

So, the original price of the hockey equipment is $230.

Check

A pair of running shoes is on sale for $125.99. If the sale price is discounted 9% from the original price, what is the original price? Round to the nearest cent.

$138.45

Go Online You can complete an Extra Example online.

94 Module 2 • Solve Percent Problems

Interactive Presentation

Example 3, Find the Original Price, Slide 2 of 4

TYPE

On Slide 2, students determine the original price of the hockey equipment.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Find the Original Price

Objective

Students will find the original price given the percent of discount and the sale price.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to use clear and precise mathematical language, such as *original price*, *discount*, and *sale price*.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we subtract 25% from 100%? We are only paying 75% of the original price.

OL Will the original price be less than or greater than $172.50? Explain. greater than; Sample answer: $172.50 is the price after receiving a discount, so the original price will be greater than the discounted price.

OL Will the original price be less than or greater than $345? Explain. less than; Sample answer: If the original price was $345, then the discount would be 50%. The discount was only 25%, so the original price must be less than $345.

BL A classmate divided $172.50 by 3 to find that 25% of the original price is $57.50. Why is this helpful? What might the classmate do next? Sample answer: If you know that 25% of the original price is $57.50, you can multiply $57.50 by 4 to find 100% of the original price, which is the original price; $57.50 × 4 = $230.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Shopping

Objective

Students will come up with their own strategy to solve an application problem that involves comparing prices after a discount.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

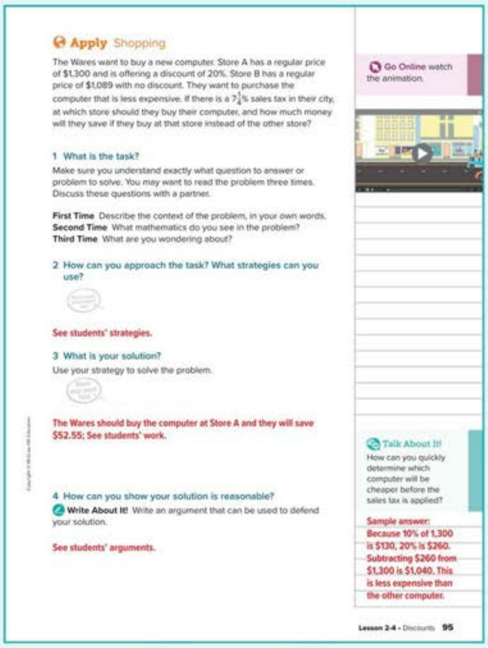
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you break up this problem into smaller problems to solve or questions to answer?
- Why do you think the sales tax will be important?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Shopping

The Wares want to buy a new computer. Store A has a regular price of $1,300 and is offering a discount of 20%. Store B has a regular price of $1,089 with no discount. They want to purchase the computer that is less expensive. If there is a $7\frac{1}{4}$% sales tax in their city, at which store should they buy their computer, and how much money will they save if they buy at that store instead of the other store?

Go Online watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

The Wares should buy the computer at Store A and they will save $52.55; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How can you quickly determine which computer will be cheaper before the sales tax is applied?

Sample answer: Because 10% of 1,300 is $130, 20% is $260. Subtracting $260 from $1,300 is $1,040. This is less expensive than the other computer.

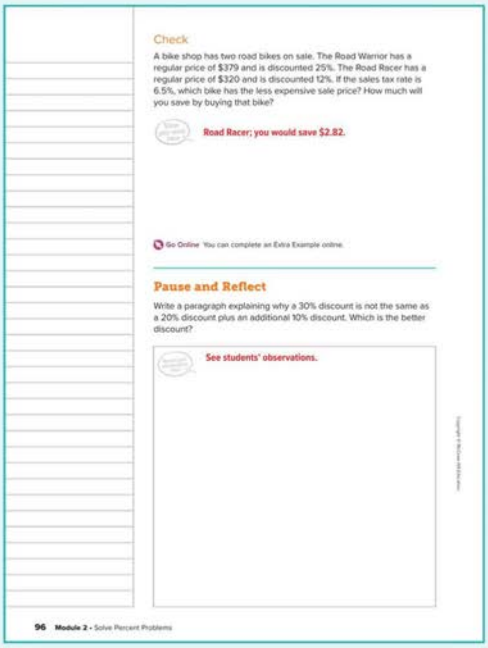
Lesson 2-4 • Discounts 95

Interactive Presentation

Apply, Shopping

WATCH Students watch an animation that illustrates the problem they are about to solve.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can percent describe the change of a quantity?

In this lesson, students learned about discounts. Encourage them to discuss with a partner how a discount changes the cost of an item, and why discounts can best be represented as a percent. For example, have students explain how a store uses percents to represent discounts and how those affect the selling price of an item.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you see a T-shirt you would like to buy that was originally $19.99 and is on sale for 25% off. You have $15 with you. Will you have enough money to buy the T-shirt, before tax? Write a mathematical argument that can be used to defend your solution. yes; Sample answer: To find the amount of discount, multiply the original price of the T-shirt, $19.99, by the percent of discount, 25% or 0.25. To find the discounted price of the T-shirt before tax, subtract the amount of discount from the original price of the T-shirt. The discounted price of the T-shirt is $14.99, which is less than $15, so you have enough to purchase the T-shirt, before tax.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11 odd, 13–16
- ALEKS® Percent Increase or Decrease

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 12–14
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Understanding Percents

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

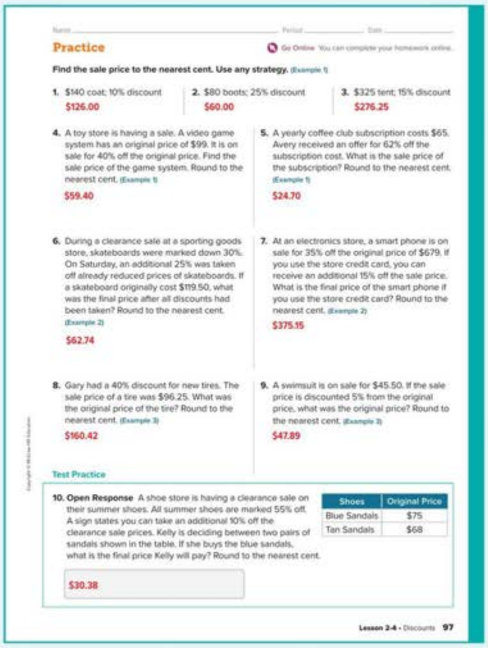
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the sale price of an item given the original cost and a discount based on a percent	1–3
2	find the sale price of an item given the original cost and a discount based on a percent	4, 5
2	find the sale price of an item given the original cost and more than one discount based on a percent	6, 7
2	find the original price given the percent of discount and the sale price	8, 9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving discounts	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

When finding the final price after multiple discounts are applied, students may not use the new price after the first discount is applied when finding the final price after a second discount. In Exercise 6, students may incorrectly use $119.50 as the whole, or original amount, in both steps of finding the price after a discount is applied. Remind students to find the price after one discount is applied, then use that new price as the whole when finding the price after another discount is applied.



Practice

Find the sale price to the nearest cent. Use any strategy. (Example 1)

1. $140 coat; 10% discount
   $126.00
2. $80 boots; 25% discount
   $60.00
3. $325 tent; 15% discount
   $276.25
4. A toy store is having a sale. A video game system has an original price of $99. It is on sale for 40% off the original price. Find the sale price of the game system. Round to the nearest cent. (Example 1)
   $59.40
5. A yearly coffee club subscription costs $65. Avery received an offer for 62% off the subscription cost. What is the sale price of the subscription? Round to the nearest cent. (Example 1)
   $24.70
6. During a clearance sale at a sporting goods store, skateboards were marked down 30%. On Saturday, an additional 25% was taken off already reduced prices of skateboards. If a skateboard originally cost $119.50, what was the final price after all discounts had been taken? Round to the nearest cent. (Example 2)
   $62.74
7. At an electronics store, a smart phone is on sale for 35% off the original price of $679. If you use the store credit card, you can receive an additional 15% off the sale price. What is the final price of the smart phone if you use the store credit card? Round to the nearest cent. (Example 2)
   $375.15
8. Gary had a 40% discount for new tires. The sale price of a tire was $96.25. What was the original price of the tire? Round to the nearest cent. (Example 3)
   $160.42
9. A swimsuit is on sale for $45.50. If the sale price is discounted 5% from the original price, what was the original price? Round to the nearest cent. (Example 3)
   $47.89

Test Practice

10. **Open Response** A shoe store is having a clearance sale on their summer shoes. All summer shoes are marked 55% off. A sign states you can take an additional 10% off the clearance sale prices. Kelly is deciding between two pairs of sandals shown in the table. If she buys the blue sandals, what is the final price Kelly will pay? Round to the nearest cent.

Shoes	Original Price
Blue Sandals	$75
Tan Sandals	$68

$30.38

Lesson 2-4 • Discounts 97

Apply *indicates multi-step problem

*11. A recreational outlet has two trampolines on sale. The table shows the original prices. The Skye Bouncer is discounted 15% and the Ultimate is discounted 13%. If the sales tax rate is 7.5%, which trampoline has the lower sale price? How much will you save by buying that trampoline? Round to the nearest cent.

Trampoline Model	Original Price ($)
Skye Bouncer	1,480
Ultimate	1,450

Skye Bouncer; $3.76

*12. Pets Plus and Pet Planet are having a sale on the same aquarium. At Pets Plus the aquarium is on sale for 30% off the original price and at Pet Planet it is discounted by 25%. If the sales tax rate is 8%, which store has the lower sale price? How much will you save by buying the aquarium there? Round to the nearest cent.

Store	Original Price of Aquarium ($)
Pets Plus	118
Pet Planet	110

Pet Planet; $0.11

Higher-Order Thinking Problems

13. **Persevere with Problems** Suppose an online store has an item on sale for 10% off the original price. By what percent does the store have to increase the price of the item in order to sell it for the original amount? Explain.

$11\frac{1}{9}$%; Sample answer: Suppose the cost of the item is $50. So, 10% of $50 is $5 and $50 − $5 = $45. Then set up a proportion to find what percent of $45 is $5. Solve for r: $\frac{5}{45} = \frac{r}{100}$, $r = 11\frac{1}{9}$%

14. **Identify Structure** A shoe store buys packs of socks wholesale for $5 each and marks them up by 40%. The store decides to discount the packs of socks by 40%. Is the discounted price $5? Explain your reasoning.

no; The packs of socks are marked up to $7 and then discounted to $4.20.

15. A model car is for sale online. The owner discounts the price by 5% each day until it sells. On the third day the car sells. If the original price of the model car was $40, how much did the car sell for? What percent discount does this represent from the original price?

$34.30; about 14%

16. Describe a real-world problem where an item is discounted by 75%. Then find the original price.

Sample answer: A book bag is on sale for $9. The original price was $36.

98 Module 2 • Solve Percent Problems

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 13, students use multiple steps to find the percent of increase needed to raise the price of a 10%-off item to its original price.

7 Look For and Make Use of Structure In Exercise 14, students use the patterns of percent of change to explain why a percent of increase and a percent of decrease are not the same.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 11 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.
Use with Exercises 13–14 Have students work in groups of 3–4 to solve the problem in Exercise 13. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 14.

Lesson 2-5

Interest

7.RP.A.3

LESSON GOAL

Students will solve problems involving simple interest.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Interest

Learn: Simple Interest
Example 1: Find Simple Interest
Example 2: Find Simple Interest
Example 3: Find Simple Interest
Apply: Car Shopping

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Compound Interest		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 11 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving interest.

ELL You can use the tips and suggestions on page T11 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7. RP.A** by solving problems involving simple interest.
Standards for Mathematical Content: **7. RP.A.3,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6**

Coherence

Vertical Alignment

Previous
Students solved multi-step ratio and percent problems involving discounts.
7.RP.A.3, 7.EE.A.2

Now
Students solve problems involving simple interest.
7.RP.A.3

Next
Students will solve problems involving commission and fees.
7.RP.A.3, 7.EE.A.2

Rigor

The Three Pillars of Rigor

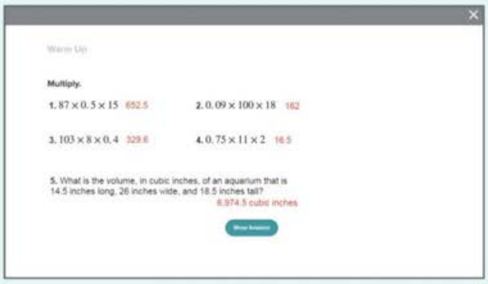
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their *understanding* of percents to build *fluency* with using the simple interest formula, and *apply* that fluency to solve real-world problems involving simple interest.

Mathematical Background

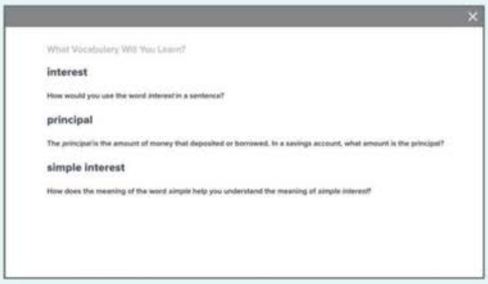
Simple interest is an amount paid or earned for the use of money. Itcan be found by using the formula $I = prt$ where I is the interest, p is the principal, r is the annual interest rate (written as a decimal), and t is the time (in years).

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- multiplying decimals and whole numbers (Exercises 1–5)

Answers

1. 652.5
2. 162
3. 329 .6
4. 16.5
5. 6,974.5 cubic inches

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about everyday life situations involving interest.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- How would you use the word *interest* in a sentence? Sample answer: I earned $7.55 in interest on my savings this year.
- The *principal* is the amount of money that deposited or borrowed. In a savings account, what amount is the principal? The amount that was originally deposited.
- How does the meaning of the word *simple* help you understand the meaning of *simple interest*? Sample answer: Simple means something that is easily understood or done. So, simple interest is interest that is easily calculated.

D. Hurst/Alamy Stock Photo

Explore Interest

Objective
Students will use Web Sketchpad to explore the simple interest formula.

Ideas for Use
Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity
Students will be presented with a real-world problem involving a deposit into an account that earns simple interest. Throughout this activity, students will use the information to graph and describe points, investigate the relationship between total interest and time, and write equations that represent the relationship.

Inquiry Question
How is the amount of interest earned on a deposit related to the length of time and the interest rate? Sample answer: The amount of interest is the product of the interest rate, the length of time, and the amount deposited in an account.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

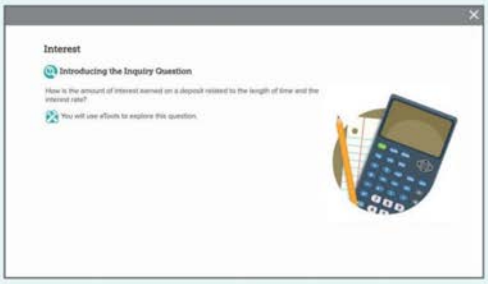
Talk About It!
SLIDE 2

Mathematical Discourse

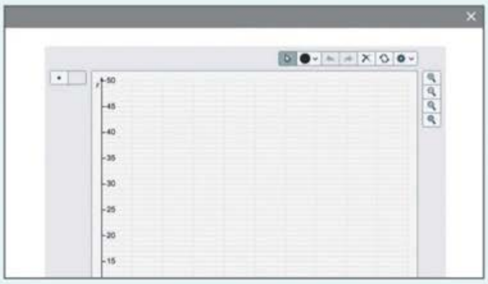
Is the relationship proportional? Explain. yes; Sample answer: The total amount of interest increases by the same dollar amount each year.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8



Explore, Slide 2 of 8

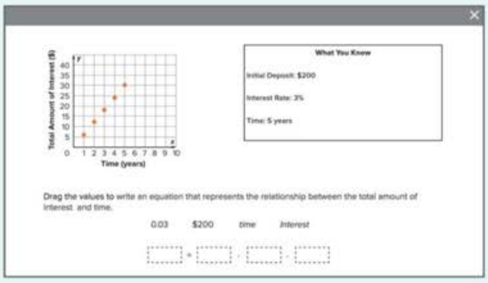
TYPE

On Slide 4, students identify the constant of proportionality and write the simple interest equation.

eTOOLS

On slide 2, students use the Coordinate Graphing eTool to graph the data.

Interactive Presentation



Explore, Slide 6 of 8

DRAG & DROP

On Slide 6, students drag values to write an equation.

TYPE

On Slide 7, students test the equation they wrote and explain whether or not it works for other values.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Interest *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to use the data values on the graph to search for a relationship between interest amount, time, and interest rate. Students should be able to explain the correspondences between the graph, table, and equation.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

How is the constant of proportionality, 6, related to the values in the problem? Sample answer: 6 is 3% of 200.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Simple Interest

Objective

Students will understand what simple interest is, and learn how to use the simple interest formula.

Teaching Notes

SLIDE 1

Students will learn the terms *principal, interest,* and *simple interest*. You may wish to ask students what factors affect the amount of interest that is paid or earned. They should note the amount of money borrowed, the time that the money is borrowed, and the interest rate.

Go Online

Have students watch the animation on Slide 2. The animation illustrates the simple interest formula.

SLIDE 2

Be sure to point out that the simple interest formula expresses the rate of interest as a decimal, and the time in terms of years. Students should use precision when working with the simple interest formula to ensure they adhere to these specifications.

SLIDE 3

Have students select the *Words* and *Symbols* flashcards to view how simple interest is expressed using these multiple representations. Prior to having students select the flashcards, ask them to describe simple interest in their own words.

DIFFERENTIATE

Reteaching Activity L

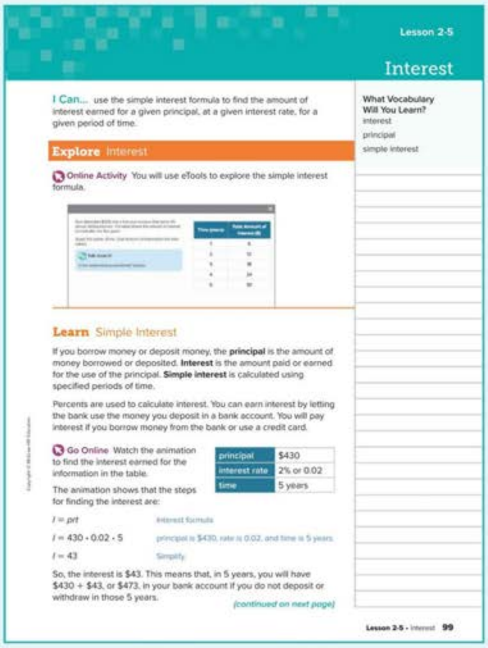
To help students learn about simple interest, encourage them to first consider the interest earned in only one period. Then they can find the total amount of interest by multiplying by the number of time periods. Have students find the interest for the first year for each of the following.

1. \$500 at 8% \$40

2. \$1,000 at 2% \$20

3. \$100 at 10% \$10

4. \$2,000 at 5% \$100


Lesson 2-5

Interest

I Can... use the simple interest formula to find the amount of interest earned for a given principal, at a given interest rate, for a given period of time.

What Vocabulary Will You Learn?
interest
principal
simple interest

Explore Interest

Online Activity You will use eTools to explore the simple interest formula.

Learn Simple Interest

If you borrow money or deposit money, the **principal** is the amount of money borrowed or deposited. **Interest** is the amount paid or earned for the use of the principal. **Simple interest** is calculated using specified periods of time.

Percents are used to calculate interest. You can earn interest by letting the bank use the money you deposit in a bank account. You will pay interest if you borrow money from the bank or use a credit card.

Go Online Watch the animation to find the interest earned for the information in the table.

principal	\$430
interest rate	2% or 0.02
time	5 years

The animation shows that the steps for finding the interest are:

$I = prt$ Interest formula

$I = 430 \cdot 0.02 \cdot 5$ principal is \$430, rate is 0.02, and time is 5 years.

$I = 43$ Simplify.

So, the interest is \$43. This means that, in 5 years, you will have \$430 + \$43, or \$473, in your bank account if you do not deposit or withdraw in those 5 years.

(continued on next page)

Lesson 2-5 • Interest 99

Interactive Presentation

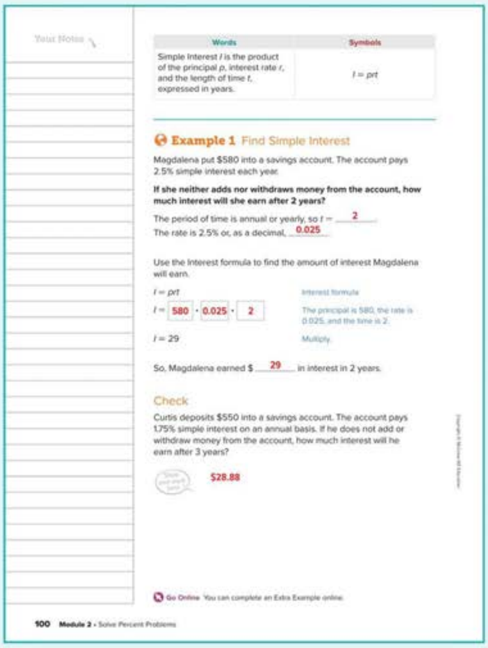
Learn, Simple Interest, Slide 2 of 3

WATCH

On Slide 2, students watch an animation that explains how interest, principal, rate, and time are related.

FLASHCARDS

On Slide 3, students use Flashcards to view multiple representations of simple interest.



Your Notes

Words	Symbols
Simple Interest I is the product of the principal p, interest rate r, and the length of time t, expressed in years.	$I = prt$

Example 1 Find Simple Interest

Magdalena put $580 into a savings account. The account pays 2.5% simple interest each year.

If she neither adds nor withdraws money from the account, how much interest will she earn after 2 years?

The period of time is annual or yearly, so t = 2

The rate is 2.5% or, as a decimal, 0.025

Use the Interest formula to find the amount of interest Magdalena will earn.

$I = prt$ — Interest formula

$I = 580 \cdot 0.025 \cdot 2$ — The principal is 580, the rate is 0.025, and the time is 2.

$I = 29$ — Multiply.

So, Magdalena earned $ 29 in interest in 2 years.

Check

Curtis deposits $550 into a savings account. The account pays 1.75% simple interest on an annual basis. If he does not add or withdraw money from the account, how much interest will he earn after 3 years?

$28.88

Go Online You can complete an Extra Example online.

100 Module 2 • Solve Percent Problems

Interactive Presentation

Example 1, Simple Interest, Slide 1 of 2

CLICK

On Slide 1, students select boxes to use the Interest formula to find the amount of interest earned.

TYPE

On Slide 1, students determine the amount of interest earned.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Find Simple Interest

Objective

Students will use an annual simple interest rate to find the amount of simple interest owed when the time is written as a whole number.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to pay careful attention to writing the percent as a decimal prior to performing the calculations.

Questions for Mathematical Discourse

SLIDE 1

AL What is the principal amount? $580

AL What is the interest rate? 2.5%

OL Why do we write the interest rate as a decimal in the simple interest formula? Sample answer: A percent is a ratio per 100. To calculate with percents, the ratio needs to be written as a single number without the % symbol, such as a decimal or a fraction.

OL How do you know that your answer is reasonable? Sample answer: Round $580 to $600; 2.5% of $100 is $2.50, so 2.5% of $600 is $2.50 × 6, or $15 and $15 × 2 = $30.

BL At this rate, how much interest will she make in 5 years? Assume she neither adds nor withdraws money. $72.50

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find Simple Interest

Objective

Students will use an annual simple interest rate to find the amount of simple interest owed when the time is written as a fraction.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the given quantities, noting that 6 months is equivalent to half of a year.

6 Attend to Precision Students should pay careful attention to units given for time in order to ensure they are calculating the simple interest correctly.

Questions for Mathematical Discourse

SLIDE 2

AL For how long will Magdalena put her money into the savings account? 6 months

AL What unit of time is used in the simple interest formula? What does this tell you that you need to do? years; convert 6 months to years

OL If we had left the time as 6, and not 0.5, what would this represent in the context of the problem? We would have found the interest earned after 6 years, not 6 months.

BL If Magdalena left the money in the account for 9 months, what would you use for the time in the formula? Explain. $\frac{3}{4}$ or 0.75; 9 months is $\frac{9}{12}$, or $\frac{3}{4}$ year. This is equivalent to 0.75 year.

SLIDE 3

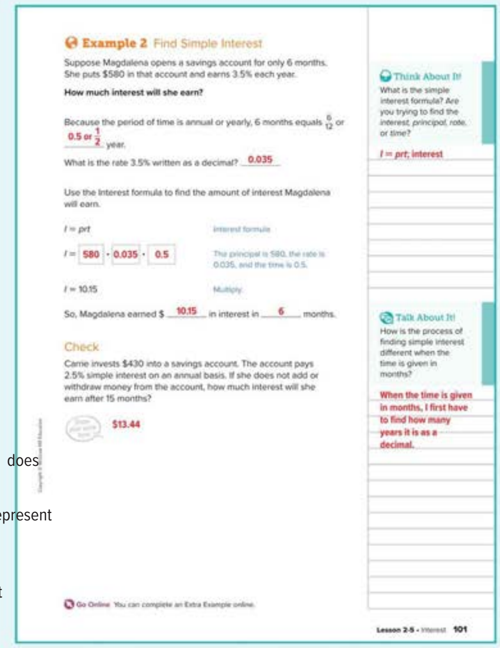
AL Identify the values we should substitute for p, r, and t. $p = 580$, $r = 0.035$, $t = 0.5$

OL If we had represented r as 3.5, what would that mean in the context of the problem? That would mean a 350% interest rate, which is an unreasonable rate.

BL Would doubling the principal double the interest earned? Explain. yes; Sample answer: Doubling the principal would double the interest earned because (580 • 2) • 0.035 • 0.5 = (580 • 0.035 • 0.5) • 2.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Find Simple Interest

Suppose Magdalena opens a savings account for only 6 months. She puts $580 in that account and earns 3.5% each year.

How much interest will she earn?

Because the period of time is annual or yearly, 6 months equals $\frac{6}{12}$ or 0.5 or $\frac{1}{2}$ year.

What is the rate 3.5% written as a decimal? 0.035

Use the Interest formula to find the amount of interest Magdalena will earn.

$I = prt$ Interest formula

$I = 580 \cdot 0.035 \cdot 0.5$ The principal is 580, the rate is 0.035, and the time is 0.5.

$I = 10.15$ Multiply.

So, Magdalena earned $ 10.15 in interest in 6 months.

Check

Carrie invests $430 into a savings account. The account pays 2.5% simple interest on an annual basis. If she does not add or withdraw money from the account, how much interest will she earn after 15 months?

$13.44

Think About It! What is the simple interest formula? Are you trying to find the interest, principal, rate, or time?

$I = prt$; interest

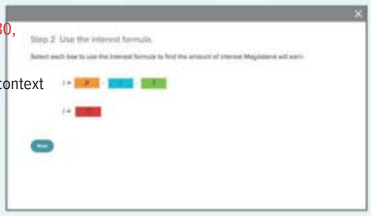
Talk About It! How is the process of finding simple interest different when the time is given in months?

When the time is given in months, I first have to find how many years it is as a decimal.

Go Online You can complete an Extra Example online.

Lesson 2-5 • Interest 101

Interactive Presentation



Example 2, Simple Interest, Slide 3 of 5

CLICK

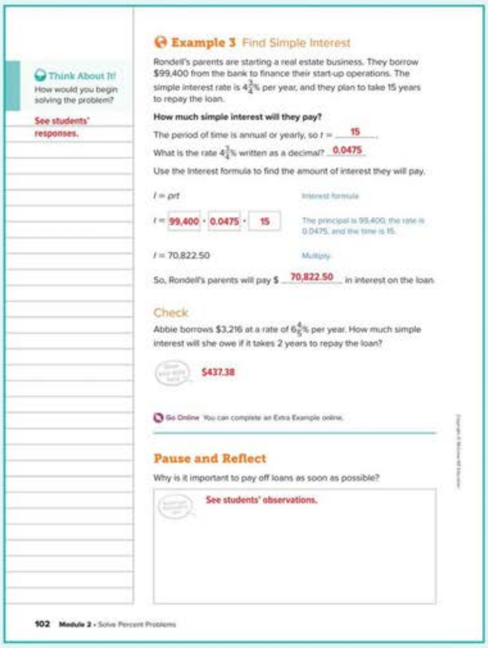
On Slide 3, students select boxes to use the Interest formula to find the amount of interest earned.

TYPE

On Slide 3, students determine the amount of interest earned.

CHECK

Students completethe Check exercise online to determine if they are ready to move on.



Example 3 Find Simple Interest

Think About It! How would you begin solving the problem? See students' responses.

Rondell's parents are starting a real estate business. They borrow \$99,400 from the bank to finance their start-up operations. The simple interest rate is $4\frac{3}{4}$% per year, and they plan to take 15 years to repay the loan.

How much simple interest will they pay?

The period of time is annual or yearly, so t = 15.

What is the rate $4\frac{3}{4}$% written as a decimal? 0.0475

Use the Interest formula to find the amount of interest they will pay.

$I = prt$ Interest formula

$I = 99{,}400 \cdot 0.0475 \cdot 15$ The principal is 99,400, the rate is 0.0475, and the time is 15.

$I = 70{,}822.50$ Multiply.

So, Rondell's parents will pay \$ 70,822.50 in interest on the loan.

Check

Abbie borrows \$3,216 at a rate of $6\frac{4}{5}$% per year. How much simple interest will she owe if it takes 2 years to repay the loan?

\$437.38

Go Online You can complete an Extra Example online.

Pause and Reflect

Why is it important to pay off loans as soon as possible?

See students' observations.

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Interactive Presentation

Example 3, Simple Interest, Slide 2 of 4

TYPE

On Slide 2, students type to indicate variable values and a rate written as a decimal.

CLICK

On Slide 3, students select boxes to use the Interest formula to find the amount of interest earned.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Find Simple Interest

Objective

Students will use an annual simple interest rate to find the amount of simple interest owed when the rate is written as a fraction.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to pay careful attention to the given quantities and express the interest rate as a decimal in the simple interest formula. Students should note that because the time was given as 15 years, they do not need to convert the time before proceeding with calculations. Encourage students to make sense of their final solutions within the context of the problem.

Questions for Mathematical Discourse

SLIDE 2

AL What is the interest rate written as a decimal? 0.0475

OL If we had represented r as 4.75, what would that mean in the context of the problem? That would mean a 475% interest rate, which is an unreasonable rate.

BL How would the value of p change if Rondell's parents decided to use \$15,000 of their own money, and only take out the loan on the remaining amount? p would become 84,400

SLIDE 3

AL Identify the values we should substitute for p, r, and t. $p = 99{,}400$, $r = 0.0475$, $t = 15$

OL Compare the amount of interest they have to pay to the amount of the loan. What do you notice? Sample answer: The amount of interest is a lot of money (about \$70,000), compared to the amount of the loan (about \$100,000).

BL How much interest would they not have to pay if Rondell's parents decided to pay off the loan in 7 years? They would save \$37,772 in interest.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Car Shopping

Objective

Students will come up with their own strategy to solve an application problem that involves purchasing a car.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

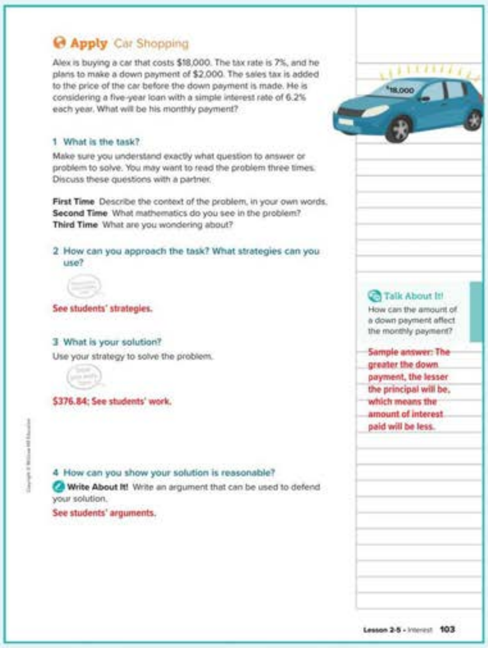
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What values should you use to calculate the sales tax?
- How do you determine the amount of principal in order to calculate the interest?
- How many monthly payments will Alex need to make? How do you know?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Car Shopping

Alex is buying a car that costs $18,000. The tax rate is 7%, and he plans to make a down payment of $2,000. The sales tax is added to the price of the car before the down payment is made. He is considering a five-year loan with a simple interest rate of 6.2% each year. What will be his monthly payment?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$376.84; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How can the amount of a down payment affect the monthly payment?

Sample answer: The greater the down payment, the lesser the principal will be, which means the amount of interest paid will be less.

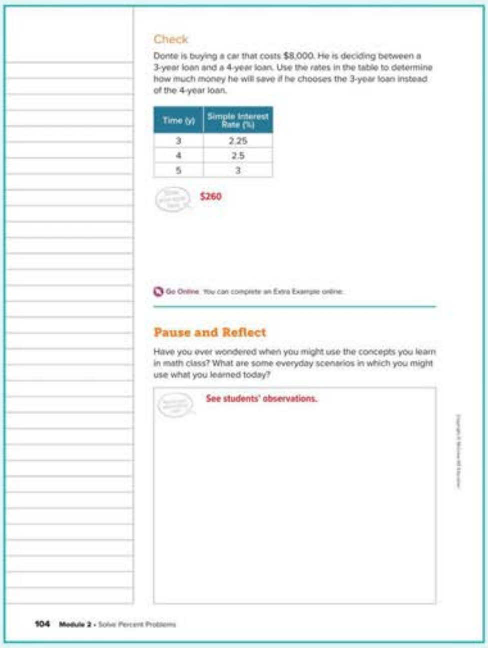
Lesson 2-5 • Interest 103

Interactive Presentation

Apply, Car Shopping

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Donte is buying a car that costs $8,000. He is deciding between a 3-year loan and a 4-year loan. Use the rates in the table to determine how much money he will save if he chooses the 3-year loan instead of the 4-year loan.

Time (y)	Simple Interest Rate (%)
3	2.25
4	2.5
5	3

$260

Go Online You can complete an Extra Example online.

Pause and Reflect

Have you ever wondered when you might use the concepts you learn in math class? What are some everyday scenarios in which you might use what you learned today?

See students' observations.

104 Module 2 • Solve Percent Problems

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can percent describe the change of a quantity?

In this lesson, students learned about how to find simple interest. Encourage them to discuss with a partner how different simple interest rates or lengths of time can affect the total amount they may pay for an item. For example, have students explain the differences in the total amount paid if they purchase an item for $500 and pay 12% simple interest for a year versus if they purchase the same item at a simple interest rate of 18% for a year.

Exit Ticket

Refer to the Exit Ticket slide. Simar is deciding between two accounts to deposit money. Account A pays 2.25% annual interest if the deposit is $600 for one year. Account B pays 2.5% annual interest if the deposit is $800 for one year. Which account will pay more to deposit Simar's money over the course of a year? Write a mathematical argument that can be used to defend your solution. Account B; Sample answer: Use the simple interest formula. For Account A, the interest earned is 600 multiplied by 0.0225, multiplied by 1. The interest earned is $13.50. For Account B, the interest earned is 800 multiplied by 0.025, multiplied by 1. The interest earned is $20. So, Account B earns more interest over the course of one year.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11 odd, 13–16
- Extension: Compound Interest
- ALEKS Interest

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 13, 15
- Extension: Compound Interest
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Understanding Percents

D. Hurst/Alamy Stock Photo

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

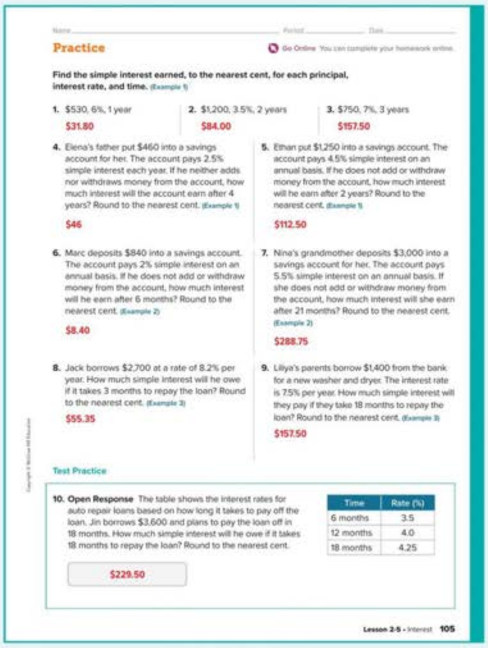
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use an annual simple interest rate to find the amount of simple interest owed when time is written as a whole number	1–3
2	use an annual simple interest rate to find the amount of simple interest owed when time is written as a whole number	4, 5
2	use an annual simple interest rate to find the amount of simple interest owed when time is written as a fraction	6, 7
2	use an annual simple interest rate to find the amount of simple interest owed when the rate is written as a fraction	8, 9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving simple interest	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Remind students that, when finding simple interest, time is represented in years. Students may need to rewrite time that is given in months as a part of a year. For example, in Exercise 6, students should use 0.5 for t because 6 months is $\frac{1}{2}$, or 0.5, of a year.



Name ____ Period ____ Date ____

Practice Go Online You can complete your homework online.

Find the simple interest earned, to the nearest cent, for each principal, interest rate, and time. (Example 1)

1. $530, 6%, 1 year — $31.80
2. $1,200, 3.5%, 2 years — $84.00
3. $750, 7%, 3 years — $157.50

4. Elena's father put $460 into a savings account for her. The account pays 2.5% simple interest each year. If he neither adds nor withdraws money from the account, how much interest will the account earn after 4 years? Round to the nearest cent. (Example 1) — $46

5. Ethan put $1,250 into a savings account. The account pays 4.5% simple interest on an annual basis. If he does not add or withdraw money from the account, how much interest will he earn after 2 years? Round to the nearest cent. (Example 1) — $112.50

6. Marc deposits $840 into a savings account. The account pays 2% simple interest on an annual basis. If he does not add or withdraw money from the account, how much interest will he earn after 6 months? Round to the nearest cent. (Example 2) — $8.40

7. Nina's grandmother deposits $3,000 into a savings account for her. The account pays 5.5% simple interest on an annual basis. If she does not add or withdraw money from the account, how much interest will she earn after 21 months? Round to the nearest cent. (Example 2) — $288.75

8. Jack borrows $2,700 at a rate of 8.2% per year. How much simple interest will he owe if it takes 3 months to repay the loan? Round to the nearest cent. (Example 3) — $55.35

9. Liliya's parents borrow $1,400 from the bank for a new washer and dryer. The interest rate is 7.5% per year. How much simple interest will they pay if they take 18 months to repay the loan? Round to the nearest cent. (Example 3) — $157.50

Test Practice

10. **Open Response** The table shows the interest rates for auto repair loans based on how long it takes to pay off the loan. Jin borrows $3,600 and plans to pay the loan off in 18 months. How much simple interest will he owe if it takes 18 months to repay the loan? Round to the nearest cent. — $229.50

Time	Rate (%)
6 months	3.5
12 months	4.0
18 months	4.25

Lesson 2-5 • Interest 105

3 REFLECT AND PRACTICE

Apply *indicates multi-step problem

*11. Jarvis is buying a boat that costs $6,000. He has $500 for a down payment. He is deciding between a 2-year loan and a 3-year loan. Use the rates in the table to determine how much money he will save if he chooses the 2-year loan instead of the 3-year loan. Round to the nearest cent.

Time (years)	Simple Interest Rate (%)
1	1.5
2	2.25
3	3.0

$247.50

*12. Evelyn is buying a motorcycle that costs $14,000. The tax rate is 6.75%, and she plans to make a down payment of $1,500. The sales tax is added before the down payment is applied. She is considering a three-year loan. What will be her monthly payments? Round to the nearest cent.

Time (years)	Simple Interest Rate (%)
1	3.25
3	6.75
5	8.5

$449.10

Higher-Order Thinking Problems

13. **Persevere with Problems** Suppose Serena invests $2,500 for 3 years and 6 months and earns $328.13. What was the rate of interest? Explain how you solved.

3.75%; Use the simple interest formula and solve for r. $328.13 = 2,500 \cdot 3.5 \cdot r$; $r = 3.75\%$

14. A student stated that an interest rate could not be less than 1%. Do you agree? Why or why not?

no; Sample answer: A percent such as an interest rate can be less than 1%. For example, credit card companies offer low interest rates of 0.5% for the first year.

15. **Justify Conclusions** Suppose you earn 2% on $1,000 for 2 years. Explain how the simple interest is affected if the rate is doubled. What happens if the time is doubled?

Sample answer: If the rate is doubled, then the interest is doubled to $80. If the time is doubled, then the interest is also doubled to $80.

16. Name a principal and interest rate where the amount of simple interest earned in 10 years would be $200. Justify your answer.

Sample answer: $1,000 at 2%. Using the simple interest formula $I = \$1,000 \cdot 0.02 \cdot 10$ or $200.

106 Module 2 • Solve Percent Problems

7.RP.A.3

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 13, students determine a strategy they can use to find the interest rate.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 15, students explain what happens if different variables are doubled when finding simple interest.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercise 12 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Clearly and precisely explain.
Use with Exercise 15 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as using percent models or bar diagrams.

Lesson 2-6

Commission and Fees

7.RP.A.3, 7.EE.A.2

LESSON GOAL

Students will solve problems involving commission and fees.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Commission and Fees
Example 1: Find Commission
Example 2: Find the Amount of Sales
Example 3: Fees
Apply: Personal Finance

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

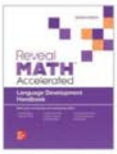
View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Finance Charges		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 12 of the *Language Development Handbook* to help your students build mathematical language related to solving problems involving commission and fees.

ELL You can use the tips and suggestions on page T12 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major clusters **7. RP.A** and **7.EE.A** by solving problems involving commission and fees.
Standards for Mathematical Content: **7. RP.A.3, 7.EE.A.2** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6**

Coherence

Vertical Alignment

Previous
Students solved problems involving simple interest.
7.RP.A.3

Now
Students solve problems involving commission and fees.
7.RP.A.3, 7.EE.A.2

Next
Students will solve problems involving percent error.
7.RP.A.3

Rigor

The Three Pillars of Rigor

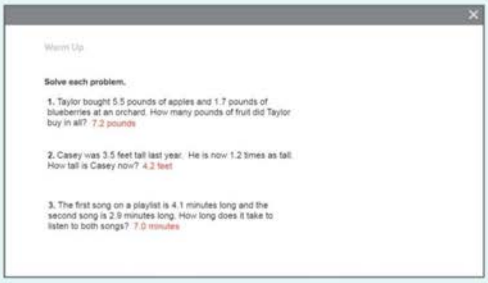
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students *apply* their *understanding* of percents to solve real-world problems involving commissions and fees. They build *fluency* in using ratio reasoning and/or the properties of operations to find the amount of commission, the amount of a fee, or the amount of sales needed to earn a given commission.

Mathematical Background

A *commission* is an amount earned based on a percent of the cost of goods or services sold. A *fee* is payment for a service. It can be a fixed amount, a percent of the charge, or both.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- adding and multiplying decimals (Exercises 1–3)

Answers

1. 7 .2 pounds
2. 4.2 feet
3. 7.0 minutes

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about wages paid as commission.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Use the word *commission* in a sentence. Sample answer: A car salesman earned a 5% commission on the amount of the car sale.
- If you pay money to a person or business, you are paying a *fee*. What is an example of a time when you might pay a *fee*? Sample answer: You may have to pay a fee when you don't return a library book on time.

Learn Commission and Fees

Objective

Students will understand what commission and fees are, and how they are often calculated using percents.

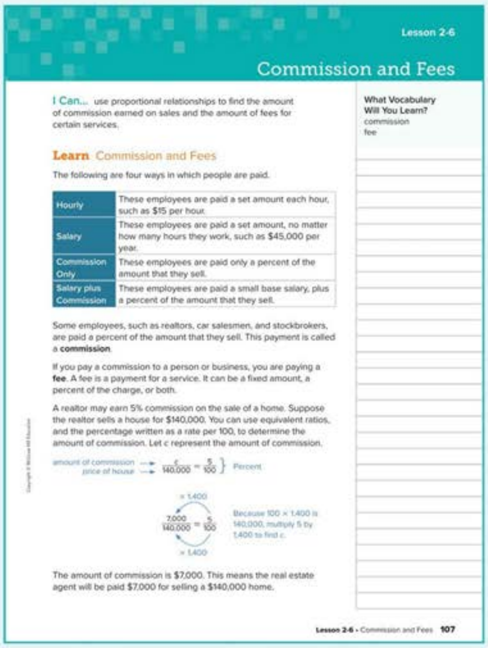
Teaching Notes

SLIDE 1

Explain to your students that employees can be paid in different ways. Some types of careers pay their employees, in part or in whole, using commission. Have students expand to reveal the different ways in which employees can be paid. Have students list several types of jobs or careers that might fall into each category.

SLIDE 2

One type of career in which employees are paid by commission is in real estate. Realtors often earn commission on the sale of a home. Have students use equivalent ratios to determine what a 5% commission on the sale of a home that sells for $140,000 would be.



Lesson 2-6

Commission and Fees

I Can... use proportional relationships to find the amount of commission earned on sales and the amount of fees for certain services.

What Vocabulary Will You Learn?
commission
fee

Learn Commission and Fees

The following are four ways in which people are paid.

Hourly	These employees are paid a set amount each hour, such as $15 per hour.
Salary	These employees are paid a set amount, no matter how many hours they work, such as $45,000 per year.
Commission Only	These employees are paid only a percent of the amount that they sell.
Salary plus Commission	These employees are paid a small base salary, plus a percent of the amount that they sell.

Some employees, such as realtors, car salesmen, and stockbrokers, are paid a percent of the amount that they sell. This payment is called a **commission**.

If you pay a commission to a person or business, you are paying a **fee**. A fee is a payment for a service. It can be a fixed amount, a percent of the charge, or both.

A realtor may earn 5% commission on the sale of a home. Suppose the realtor sells a house for $140,000. You can use equivalent ratios, and the percentage written as a rate per 100, to determine the amount of commission. Let c represent the amount of commission.

amount of commission → / price of house → $\frac{c}{140{,}000} = \frac{5}{100}$ } Percent

$\frac{7{,}000}{140{,}000} = \frac{5}{100}$ (× 1,400)

Because 100 × 1,400 is 140,000, multiply 5 by 1,400 to find c.

The amount of commission is $7,000. This means the real estate agent will be paid $7,000 for selling a $140,000 home.

Lesson 2-6 • Commission and Fees 107

Interactive Presentation

Learn, Commission and Fees, Slide 1 of 2

EXPAND

On Slide 1, students expand to reveal different ways in which people are paid.

CLICK

On Slide 2, students click to use equivalent ratios to determine the amount of commission made.

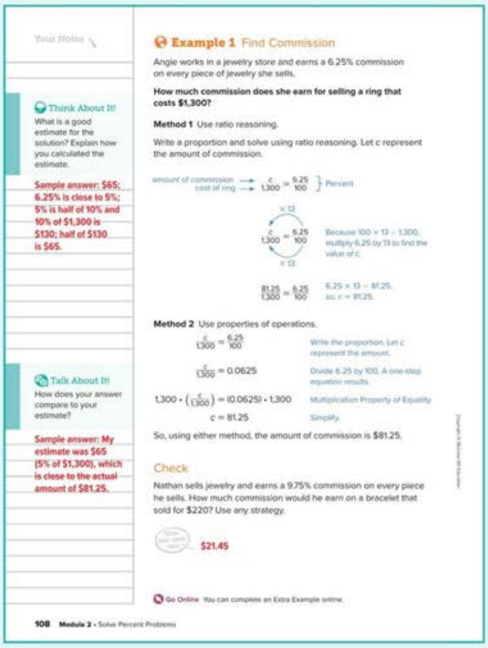
DIFFERENTIATE

Language Development Activity ELL

To further students' understanding of commission, have them work with a partner to research careers in which employees are paid either entirely by commission, or by a salary plus commission. Have them create a presentation that highlights the main responsibilities of those careers, as well as a brief explanation how they earn their pay. Students' presentations should include examples from at least three different types of careers. Have students share their presentations to the class. Some students may be uncomfortable speaking in front of others. Encourage them to make appropriate eye contact and articulate loudly enough for the class to hear.

7.RP.A.3, 7.EE.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION


Your Notes

Think About It!
What is a good estimate for the solution? Explain how you calculated the estimate.

Sample answer: $65; 6.25% is close to 5%; 5% is half of 10% and 10% of $1,300 is $130; half of $130 is $65.

Talk About It!
How does your answer compare to your estimate?

Sample answer: My estimate was $65 (5% of $1,300), which is close to the actual amount of $81.25.

Example 1 Find Commission

Angie works in a jewelry store and earns a 6.25% commission on every piece of jewelry she sells.

How much commission does she earn for selling a ring that costs $1,300?

Method 1 Use ratio reasoning.

Write a proportion and solve using ratio reasoning. Let c represent the amount of commission.

amount of commission → / cost of ring → $\frac{c}{1{,}300} = \frac{6.25}{100}$ } Percent

$\frac{c}{1{,}300} = \frac{6.25}{100}$ (× 13) Because 100 × 13 = 1,300, multiply 6.25 by 13 to find the value of c.

$\frac{81.25}{1{,}300} = \frac{6.25}{100}$ 6.25 × 13 = 81.25, so, c = 81.25

Method 2 Use properties of operations.

$\frac{c}{1{,}300} = \frac{6.25}{100}$ Write the proportion. Let c represent the amount.

$\frac{c}{1{,}300} = 0.0625$ Divide 6.25 by 100. A one-step equation results.

$1{,}300 \cdot \left(\frac{c}{1{,}300}\right) = (0.0625) \cdot 1{,}300$ Multiplication Property of Equality

$c = 81.25$ Simplify.

So, using either method, the amount of commission is $81.25.

Check

Nathan sells jewelry and earns a 9.75% commission on every piece he sells. How much commission would he earn on a bracelet that sold for $220? Use any strategy.

$21.45

Go Online You can complete an Extra Example online.

108 Module 2 • Solve Percent Problems

Interactive Presentation

Example 1, Find Commission, Slide 2 of 5

CLICK

On Slide 3, students move through the steps to solve the proportion.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find Commission

Objective

Students will find the amount of commission, given the total sales and the percent of commission.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to use estimation to make sense of the quantities given in the example, and to verify their answer for reasonableness. Students should be able to reason that 10% of $1,300 is $130 and that 5% of $1,300 is half that amount, $65. Because the percent of commission 6.25% is between 5% and 10%, Angie's commission will be between $65 and $130.

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to use the estimate they made at the beginning of the example as they construct their explanation. Their explanations should use clear and precise mathematical language.

Questions for Mathematical Discourse

SLIDE 2

AL What is the percent of commission Angie will earn for selling the ring? 6.25%

AL What is the cost of the ring? $1,300

OL Without calculating, will her commission be less than or greater than $130? Explain. less than; Sample answer: $130 is 10% of $1,300 and her commission is less than 10%, so she will earn less than $130 in commission.

BL Will Angie earn more than $65 in commission? Explain. Yes, because 10% of $1,300 is $130, then 5% of $1,300 is $65. Because Angie earns more than 5%, she will earn more than $65 in commission.

SLIDE 3

AL What is 6.25 divided by 100? Why is the division helpful? 0.0625; Sample answer: Dividing results in a one-step equation.

OL How do you know that your answer is reasonable? Sample answer: $81.25 is less than $130, and $130 is 10% of $1,300

BL How much does Angie need to sell to earn $1,200 a week? Sample answer: $19,200 worth of jewelry

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 2 Find the Amount of Sales

Objective

Students will find the total amount of sales, given the amount of commission and the percent of commission.

Questions for Mathematical Discourse

SLIDE 2

AL What is the part, the whole, and the percent in this problem? 1,200 is the part, *a* is the whole, and 8 is the percent.

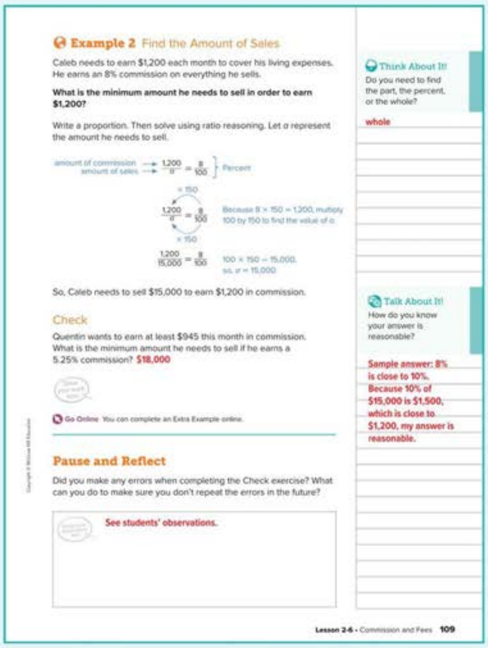
OL Without calculating, will Caleb need to sell more than $12,000? Explain. yes; Sample answer: 10% of $12,000 is $1,200. Because 8% < 10%, Caleb will earn less than $1,200 if the sales are $12,000. So, he needs to sell more than $12,000.

OL A classmate wrote the equation $1{,}200 = 0.08x$. Is this equation equivalent to the proportion $\frac{1{,}200}{x} = \frac{8}{100}$? Explain. yes; Sample answer: By writing $\frac{8}{100}$ as 0.08 and multiplying both sides by *x*, the one-step equation $1{,}200 = 0.08x$ results and is equivalent to the proportion.

BL How much would Caleb need to sell to earn $2,000 each month? $25,000

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Find the Amount of Sales

Caleb needs to earn $1,200 each month to cover his living expenses. He earns an 8% commission on everything he sells.

What is the minimum amount he needs to sell in order to earn $1,200?

Write a proportion. Then solve using ratio reasoning. Let *a* represent the amount he needs to sell.

amount of commission → $\frac{1{,}200}{a} = \frac{8}{100}$ } Percent
amount of sales →

$\frac{1{,}200}{a} = \frac{8}{100}$ (× 150) Because 8 × 150 = 1,200, multiply 100 by 150 to find the value of *a*.

$\frac{1{,}200}{15{,}000} = \frac{8}{100}$ 100 × 150 = 15,000, so, *a* = 15,000

So, Caleb needs to sell $15,000 to earn $1,200 in commission.

Check

Quentin wants to earn at least $945 this month in commission. What is the minimum amount he needs to sell if he earns a 5.25% commission? $18,000

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you make any errors when completing the Check exercise? What can you do to make sure you don't repeat the errors in the future?

See students' observations.

Think About It! Do you need to find the part, the percent, or the whole? whole

Talk About It! How do you know your answer is reasonable? Sample answer: 8% is close to 10%. Because 10% of $15,000 is $1,500, which is close to $1,200, my answer is reasonable.

Lesson 2-6 • Commission and Fees 109

Interactive Presentation

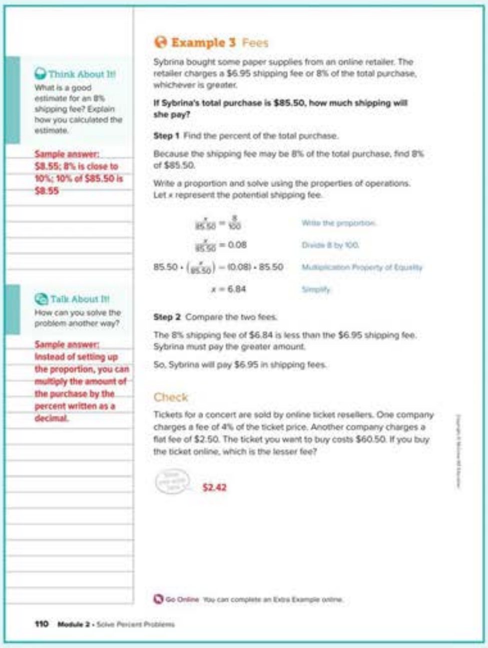
Example 2, Find the Amount of Sales, Slide 2 of 4

CLICK

On Slide 2, students move through steps to construct a proportion.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Fees

Sybrina bought some paper supplies from an online retailer. The retailer charges a $6.95 shipping fee or 8% of the total purchase, whichever is greater.

If Sybrina's total purchase is $85.50, how much shipping will she pay?

Step 1 Find the percent of the total purchase.

Because the shipping fee may be 8% of the total purchase, find 8% of $85.50.

Write a proportion and solve using the properties of operations. Let x represent the potential shipping fee.

$\frac{x}{85.50} = \frac{8}{100}$ Write the proportion.

$\frac{x}{85.50} = 0.08$ Divide 8 by 100.

$85.50 \cdot \left(\frac{x}{85.50}\right) = (0.08) \cdot 85.50$ Multiplication Property of Equality

$x = 6.84$ Simplify.

Step 2 Compare the two fees.

The 8% shipping fee of $6.84 is less than the $6.95 shipping fee. Sybrina must pay the greater amount.

So, Sybrina will pay $6.95 in shipping fees.

Think About It! What is a good estimate for an 8% shipping fee? Explain how you calculated the estimate.

Sample answer: $8.55; 8% is close to 10%; 10% of $85.50 is $8.55

Talk About It! How can you solve the problem another way?

Sample answer: Instead of setting up the proportion, you can multiply the amount of the purchase by the percent written as a decimal.

Check

Tickets for a concert are sold by online ticket resellers. One company charges a fee of 4% of the ticket price. Another company charges a flat fee of $2.50. The ticket you want to buy costs $60.50. If you buy the ticket online, which is the lesser fee?

$2.42

Go Online You can complete an Extra Example online.

110 Module 2 • Solve Percent Problems

Interactive Presentation

Example 3, Fees, Slide 2 of 5

CLICK — On Slide 2, students determine 8% of $85.50.

TYPE — On Slide 3, students determine how much will be paid in shipping fees.

CHECK — Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Fees

Objective

Students will find the amount of a fee, given the conditions on which the fee is based and the percent of the fee.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them Encourage students to explain to themselves and others the meaning of the problem, and what they are asked to find, prior to jumping into a solution attempt. Students should understand that there are two different ways to calculate the shipping fee, and that the greater amount is the one that will be applied.

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to defend their response with clear and accurate mathematical calculations and explanations.

Questions for Mathematical Discourse

SLIDE 2

AL Explain this problem, in your own words. Sample answer: There are two options for calculating the shipping. It could either be a flat fee, $6.95, or it could be 8% of the purchase amount. The greater amount will be the shipping charge.

OL How do you know that 8% of $85.50 will be less than $8.55? 10% of $85.50 is $8.55, and 8% is less than 10%.

BL If the shipping fee was 10% of $85.50, would you need to perform any actual calculations in order to solve the problem? Sample answer: No; I can mentally find 10% of $85.50, which is $8.55. I know that is greater than $6.95, so she will pay the greater amount.

SLIDE 3

AL Why do we compare the two fees? Sybrina must pay the greater amount of shipping fees.

OL If Sybrina's total purchases were $90, which shipping fee would she pay? Explain. 8% of $90, or $7.20; Sample answer: She must pay the greater amount, and an 8% fee, or $7.20, is greater than $6.95.

BL If Sybrina's total purchases were x dollars, she will pay $6.95 in shipping. Find two possible values of x. Explain. Sample answers: $80 and $85; 8% of $80 and 8% of $85 are both less than $6.95 and Sybrina must pay the grater amount.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Personal Finance

Objective

Students will come up with their own strategy to solve an application problem that involves commission rates.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you break up this problem into smaller problems to solve or questions to answer?
- What are the two types of pay that David could choose?
- How much sales over $7,500 does David make on average?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Personal Finance

David is starting a new job in sales. He can choose to earn only a 10% commission on sales each month, or earn a monthly base salary of $1,500 with a 3% commission on sales over $7,500. Which pay method would earn him more money if he has an average sales of $16,000 each month?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

base salary with commission; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How do you determine a 3% commission on sales over $7,500?

Sample answer: You first need to subtract $7,500 from the amount of actual sales, and then find 3% of the difference.

Lesson 2-6 • Commission and Fees 111

Interactive Presentation

Apply, Personal Finance

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Kelly has a job in sales that pays only a commission of 24% of her total monthly sales. She is offered a new job with a monthly salary of $3,750 with a commission of 9.5% on her total monthly sales over $12,500. If she estimates her total monthly sales will average $20,000, which statement is correct?

Ⓐ The new job will pay more because she would earn $4,462.50, and her current job only pays $4,220.

Ⓑ The new job will pay more because she would earn $4,742.75, and her current job only pays $4,220.

● Her current job will pay more because she will earn $4,800, and the new job would only pay $4,462.50.

Ⓓ Her current job will pay more because she will earn $4,800, and the new job would only pay $4,742.75.

Go Online You can complete an Extra Example online.

Pause and Reflect

How are *parts* and *wholes* represented in fees and commissions? Give examples to support your answer.

See students' observations.

112 Module 2 • Solve Percent Problems

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Suppose you are offered a job that pays a base salary of $300 plus 5% commission on sales per week. Explain how to determine your weekly pay. Sample answer: Multiply the amount in sales by 5%, or 0.05. Then add the base salary of $300.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9, 11–14
- Extension: Finance Charges
- ALEKS Percent of a Number

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 10, 11, 14
- Extension: Finance Charges
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Understanding Percents

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

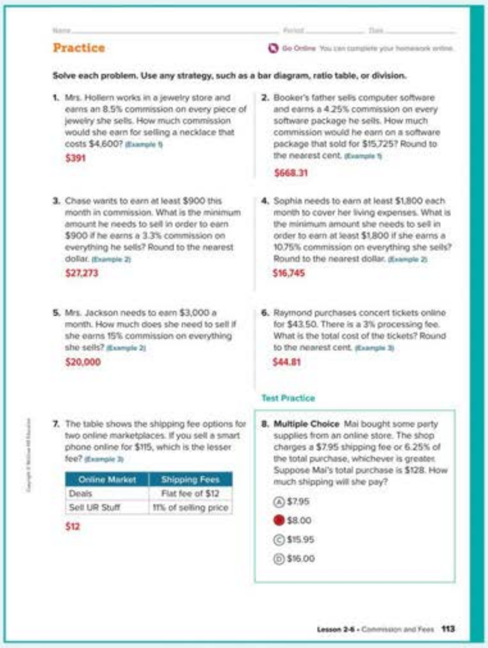
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find the amount of commission given total sales and the percent commission	1, 2
2	find the total amount of sales given the amount of commission and the percent commission	3, 4, 5
2	find the amount of a fee given the amount on which the fee is based and the percent of the fee	6, 7, 8
3	solve application problems involving commission or fees	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Students may use an incorrect part or whole when solving problems involving commission. In Exercise 3, the amount he needs to sell is the whole, while the amount of commission is the part. So, $900 is the part.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each problem. Use any strategy, such as a bar diagram, ratio table, or division.

1. Mrs. Hollern works in a jewelry store and earns an 8.5% commission on every piece of jewelry she sells. How much commission would she earn for selling a necklace that costs $4,600? (Example 1)
   $391
2. Booker's father sells computer software and earns a 4.25% commission on every software package he sells. How much commission would he earn on a software package that sold for $15,725? Round to the nearest cent. (Example 1)
   $668.31
3. Chase wants to earn at least $900 this month in commission. What is the minimum amount he needs to sell in order to earn $900 if he earns a 3.3% commission on everything he sells? Round to the nearest dollar. (Example 2)
   $27,273
4. Sophia needs to earn at least $1,800 each month to cover her living expenses. What is the minimum amount she needs to sell in order to earn at least $1,800 if she earns a 10.75% commission on everything she sells? Round to the nearest dollar. (Example 2)
   $16,745
5. Mrs. Jackson needs to earn $3,000 a month. How much does she need to sell if she earns 15% commission on everything she sells? (Example 2)
   $20,000
6. Raymond purchases concert tickets online for $43.50. There is a 3% processing fee. What is the total cost of the tickets? Round to the nearest cent. (Example 3)
   $44.81
7. The table shows the shipping fee options for two online marketplaces. If you sell a smart phone online for $115, which is the lesser fee? (Example 3)

Online Market	Shipping Fees
Deals	Flat fee of $12
Sell UR Stuff	11% of selling price

   $12

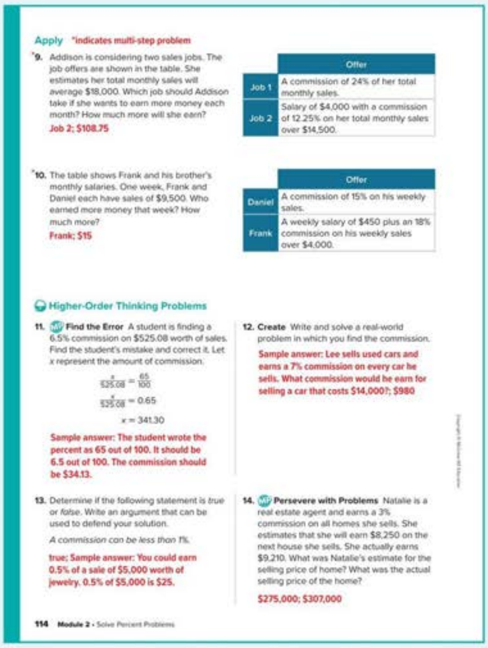
Test Practice

8. **Multiple Choice** Mai bought some party supplies from an online store. The shop charges a $7.95 shipping fee or 6.25% of the total purchase, whichever is greater. Suppose Mai's total purchase is $128. How much shipping will she pay?
   Ⓐ $7.95
   ● $8.00
   Ⓒ $15.95
   Ⓓ $16.00

Lesson 2-6 • Commission and Fees 113

7.RP.A.3, 7.EE.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Apply *indicates multi-step problem

*9. Addison is considering two sales jobs. The job offers are shown in the table. She estimates her total monthly sales will average $18,000. Which job should Addison take if she wants to earn more money each month? How much more will she earn?

Job 2; $108.75

	Offer
Job 1	A commission of 24% of her total monthly sales.
Job 2	Salary of $4,000 with a commission of 12.25% on her total monthly sales over $14,500.

*10. The table shows Frank and his brother's monthly salaries. One week, Frank and Daniel each have sales of $9,500. Who earned more money that week? How much more?

Frank; $15

	Offer
Daniel	A commission of 15% on his weekly sales.
Frank	A weekly salary of $450 plus an 18% commission on his weekly sales over $4,000.

Higher-Order Thinking Problems

11. MP **Find the Error** A student is finding a 6.5% commission on $525.08 worth of sales. Find the student's mistake and correct it. Let x represent the amount of commission.

$$\frac{x}{525.08} = \frac{65}{100}$$
$$\frac{x}{525.08} = 0.65$$
$$x = 341.30$$

Sample answer: The student wrote the percent as 65 out of 100. It should be 6.5 out of 100. The commission should be $34.13.

12. **Create** Write and solve a real-world problem in which you find the commission.

Sample answer: Lee sells used cars and earns a 7% commission on every car he sells. What commission would he earn for selling a car that costs $14,000?; $980

13. Determine if the following statement is *true* or *false*. Write an argument that can be used to defend your solution.

A commission can be less than 1%.

true; Sample answer: You could earn 0.5% of a sale of $5,000 worth of jewelry. 0.5% of $5,000 is $25.

14. MP **Persevere with Problems** Natalie is a real estate agent and earns a 3% commission on all homes she sells. She estimates that she will earn $8,250 on the next house she sells. She actually earns $9,210. What was Natalie's estimate for the selling price of home? What was the actual selling price of the home?

$275,000; $307,000

114 Module 2 • Solve Percent Problems

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Ex ercise 11, students find a student's mistake and correct it.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 14, students plan a solution pathway to solve a problem involving multiple steps.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercises 9–10 Have students work in pairs. After completing the apply problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 9 might be "The commission percent can be written as 0.24." An example of a false statement might be "The commission percent applies to $4,000." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Clearly explain your strategy.
Use with Exercise 11 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would find a 6.5% commission on $525.08, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Lesson 2-7

Percent Error

7.RP.A.3

LESSON GOAL

Students will solve problems involving percent error.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Percent Error

Learn: Percent Error
Example 1: Percent Error
Apply: Sports

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 13 of the *Language Development Handbook* to help your students build mathematical language related to percent error.

ELL You can use the tips and suggestions on page T13 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Ratios and Proportional Relationships
Major Cluster(s): In this lesson, students address major cluster **7. RP.A** by solving problems involving percent error.
Standards for Mathematical Content: **7. RP.A.3,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5**

Coherence

Vertical Alignment

Previous
Students solved problems involving commission and fees.
7.RP.A.3, 7.EE.A.2

Now
Students solve problems involving percent error.
7.RP.A.3

Next
Students will use ratios to find the probability of an event occurring.
7.SP.C.7

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of proportional relationships and percents to build *fluency* with determining the percent error when given an estimated value for a quantity and the actual value. They *apply* their understanding of percent error to solve real-world problems.

Mathematical Background

Sometimes, it is not necessary to find an exact value. Instead, you can use an estimate. To determine if the estimate is reasonable, find the percent error. The *percent error* is a ratio, written as a percent, which compares the inaccuracy of an estimate, or amount of error, to the actual amount. The *amount of error* is the positive difference between the estimate and the actual amount. Subtract the lesser amount from the greater amount, because the amount of error is a positive value.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- subtracting and dividing decimals (Exercises 1–5)

Answers

1. 0.58 **4.** 14
2. 0.13 **5.** 0.15 mile
3. 190

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using a percent to compare an estimate to an actual value in everyday life situations.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What are three words that have the same meaning as the word *error*? Sample answer: mistake, miscalculation, inaccuracy.
- Make a conjecture as to what you think the *amount of error* of an estimate might mean. Use your everyday knowledge of the words *amount* and *error*. Sample answer: The amount of error of an estimate might mean how far off the estimate is from the actual amount.

Explore Percent Error

Objective

Students will use Web Sketchpad to explore percent error.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch that simulates shooting arrows at an archery target. Throughout this activity, students will make estimates and use the archery target sketch to determine the amount of error and the percent error for their estimates.

Inquiry Question

How can the amount of error and percent error help you know if your estimates are reasonable? Sample answer: The amount of error and the percent error can tell you how close to the actual amount the estimate is. The lower the percent error, the closer the estimate.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

Based on the amount of error and the percent error, explain whether or not your estimate was reasonable. Sample answer: For 100 arrows shot, I estimated 50 hits. My percent error was high – above 100%. For 200 hits, I kept my solution at 50 hits. My percent error was much lower.

(continued on next page)

Interactive Presentation

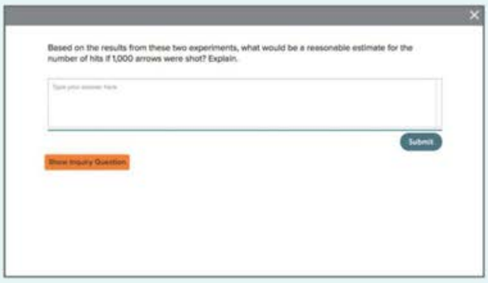
Explore, Slide 1 of 5

Explore, Slide 3 of 5

WEB SKETCHPAD

On Slides 2 and 3, students use Web Sketchpad to explore percent error.

Interactive Presentation



Explore, Slide 4 of 5

TYPE

On Slide 4, students respond to a paragraph entry question about a reasonable estimate based on experiments.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Percent Error *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to investigate how close their estimates are by comparing the estimated number of archery target hits to the actual number of hits. Encourage them to form an understanding of how the amount of error and percent error can help them know if their estimates are reasonable.

Go Online to find additional teaching notes.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Percent Error

Objective

Students will understand that percent error can help them compare the inaccuracy of an estimate, or amount of error, to the actual amount.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively S tudents should understand that percent error is found by writing the ratio between the amount of error and the actual amount. Encourage students to make sense of the quantities given in the problem, so that they understand that the amount of error, 200 is 20% of the actual amount, 1,000. As students discuss the *Talk About It!* question, encourage them to use reasoning about why the amount of error is a positive value.

Teaching Notes

SLIDE 1

You may wish to ask students if they have ever made an estimate that turned out not to be equivalent to the actual amount. For example, suppose a restaurant manager estimated that there would be 300 customers at her restaurant on Friday night, but only 286 customers actually came. Have students explain how to find the amount of error in the restaurant manager's estimate (300 – 286 = 14). The percent error is found by writing the ratio of the error, 14, to the actual number of customers, 300, and then expressing that ratio as a percent.

Talk About It!

SLIDE 2

Mathematical Discourse

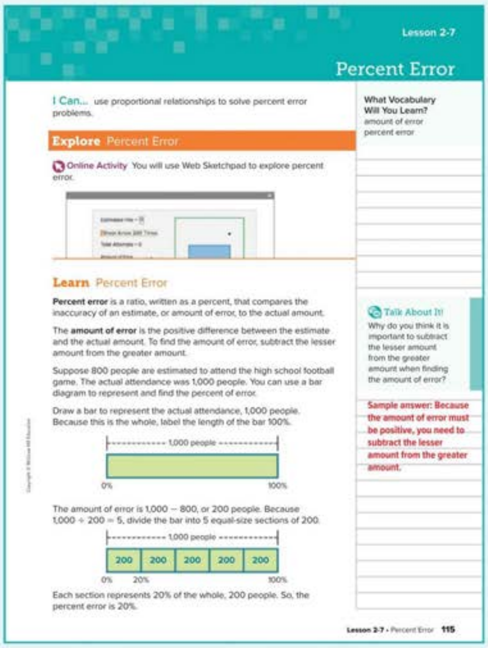
Why do you think it is important to subtract the lesser amount from the greater amount when finding the amount of error? Sample answer: Because the amount of error must be positive, you need to subtract the lesser amount from the greater amount.

DIFFERENTIATE

Reteaching Activity AL

If students are struggling to understand how to find the percent error, have them first practice identifying the amount of error and the actual amount using the following exercises.

1. Keara estimates that there are 24 students in the classroom. There are actually 27 students in the classroom. amount of error: 3; actual: 27
2. A meteorologist forecasted a high temperature of 56 degrees, but the actual high temperature was 54 degrees. amount of error: 2; actual 54
3. A basketball player predicted his team would score 87 points, but they actually scored 101 points. amount of error: 14; actual 101



Lesson 2-7

Percent Error

I Can... use proportional relationships to solve percent error problems.

What Vocabulary Will You Learn?
amount of error
percent error

Explore Percent Error

Online Activity You will use Web Sketchpad to explore percent error.

Learn Percent Error

Percent error is a ratio, written as a percent, that compares the inaccuracy of an estimate, or amount of error, to the actual amount.

The **amount of error** is the positive difference between the estimate and the actual amount. To find the amount of error, subtract the lesser amount from the greater amount.

Suppose 800 people are estimated to attend the high school football game. The actual attendance was 1,000 people. You can use a bar diagram to represent and find the percent of error.

Draw a bar to represent the actual attendance, 1,000 people. Because this is the whole, label the length of the bar 100%.

1,000 people
0% 100%

The amount of error is 1,000 − 800, or 200 people. Because 1,000 ÷ 200 = 5, divide the bar into 5 equal-size sections of 200.

1,000 people
200 200 200 200 200
0% 20% 100%

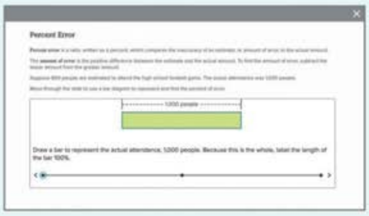
Each section represents 20% of the whole, 200 people. So, the percent error is 20%.

Talk About It!
Why do you think it is important to subtract the lesser amount from the greater amount when finding the amount of error?

Sample answer: Because the amount of error must be positive, you need to subtract the lesser amount from the greater amount.

Lesson 2-7 • Percent Error 115

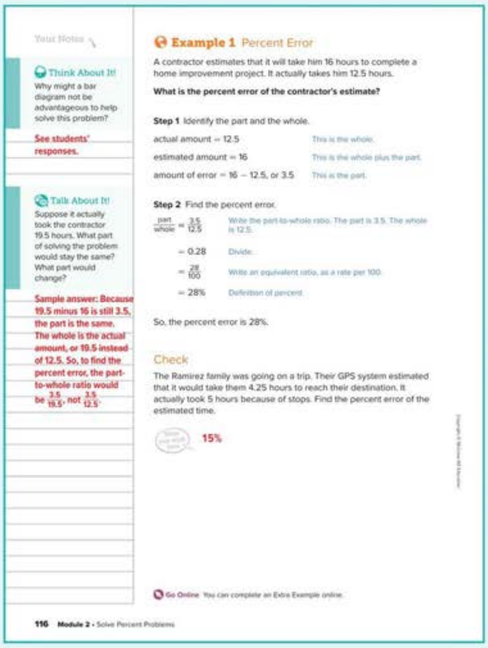
Interactive Presentation



Learn, Percent Error, Slide 1 of 2

CLICK

On Slide 1, students click to see how a bar diagram can be used to find percent error.


Your Notes

Think About It! Why might a bar diagram not be advantageous to help solve this problem?

See students' responses.

Talk About It! Suppose it actually took the contractor 19.5 hours. What part of solving the problem would stay the same? What part would change?

Sample answer: Because 19.5 minus 16 is still 3.5, the part is the same. The whole is the actual amount, or 19.5 instead of 12.5. So, to find the percent error, the part-to-whole ratio would be $\frac{3.5}{19.5}$, not $\frac{3.5}{12.5}$.

Example 1 Percent Error

A contractor estimates that it will take him 16 hours to complete a home improvement project. It actually takes him 12.5 hours.

What is the percent error of the contractor's estimate?

Step 1 Identify the part and the whole.

actual amount = 12.5 — This is the whole.

estimated amount = 16 — This is the whole plus the part.

amount of error = 16 − 12.5, or 3.5 — This is the part.

Step 2 Find the percent error.

$\frac{\text{part}}{\text{whole}} = \frac{3.5}{12.5}$ — Write the part-to-whole ratio. The part is 3.5. The whole is 12.5.

$= 0.28$ — Divide.

$= \frac{28}{100}$ — Write an equivalent ratio, as a rate per 100.

$= 28\%$ — Definition of percent.

So, the percent error is 28%.

Check

The Ramirez family was going on a trip. Their GPS system estimated that it would take them 4.25 hours to reach their destination. It actually took 5 hours because of stops. Find the percent error of the estimated time.

15%

Go Online You can complete an Extra Example online.

116 Module 2 • Solve Percent Problems

Interactive Presentation

Example 1, Percent Error, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the percent error.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Percent Error

Objective

Students will find the percent error given the estimated value and the actual amount.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the quantities given in the problem (the estimate and the actual value), so they can determine what values represent the part and the whole.

As students discuss the *Talk About It!* question, encourage them to understand that even if the part is the same, the whole has changed, so the percent error will change as well.

Questions for Mathematical Discourse

SLIDE 2

AL How do you find the amount of error? subtract the lesser amount, or the actual amount, from the greater amount, or the estimate.

OL Do you think that the contractor's estimate was close to the actual time it took him to complete the project? Explain. Sample answer: The contractor's estimate was not very close to the actual value. The difference, about 4 hours, compared to the actual time, about 12 hours, is close to 30%.

BL Provide an estimate for the amount of time to complete the project that would have a percent error less than 10%. Sample Answer: 13.5 hours

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY **3 APPLICATION**

Apply Sports

Objective
Students will come up with their own strategy to solve an application problem involving percent error of estimating the number of wins a sports team will have in a season.

Teaching the Mathematical Practices
1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use
Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.
- Why is it important to know that the team has currently won 8 of their 10 games so far?
- How many games are they expected to win out of 20 games? 5 games?
- What is the amount of error in the newspaper's estimate?

Write About It!
Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Sports

A school newspaper estimates that their basketball team will win 23 out of 25 games for the season. After 10 games, they have won 8. If the team continues winning at this rate, what will be the percent error of the newspaper's estimate once the season is over?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

15%; See students' work.

4 How can you show that your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
How can you predict the actual number of wins the team has at the end of the season?

I can use the rate, 8 wins out of 10 games, to calculate the number of wins at the end of the season.

Lesson 2-7 • Percent Error 117

Interactive Presentation

Apply, Sports

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

It is predicted that a softball team will win 35 out of their 50 games for their summer season. After 20 games, they have won 16. If the team continues to win at this rate, what will be the percent error of the prediction once the season is over? 12.5%

Go Online You can complete an Extra Example online.

Math History Minute

Marjorie Lee Browne (1914–1979) was one of the first African-American women to receive a doctorate degree in mathematics. While she was the head of the mathematics department at North Carolina College, she wrote a grant for $60,000 to IBM (International Business Machines) so that the university could have its first computer.

Pause and Reflect

Explain how ratio and proportional reasoning can be used to solve problems involving percent.

See students' observations.

118 Module 2 • Solve Percent Problems

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. A jar has 654 jelly beans. Emily says the jar has 710 jelly beans. Find the percent error of Emily's estimate. Round to the nearest tenth if necessary. Explain how you found your answer and state whether or not you think Emily's estimate is close to the actual value. 8.6%; Sample answer: Subtract Emily's estimate from the actual amount to find the amount of error: $710 - 654 = 56$. Then divide the amount of error by the actual amount: $\frac{56}{654} \approx 0.086$. Multiply by 100 to write the decimal as a percent; 8.6%. Emily's estimate is close to the actual value because her percent error is less than 10%.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–9 odd, 11–14
- ALEKS Percent of a Number

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 10, 11, 14
- Remediation: Review Resources
- Personal Tutor
- Extra Example 1
- ALEKS Understanding Percents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Understanding Percents

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

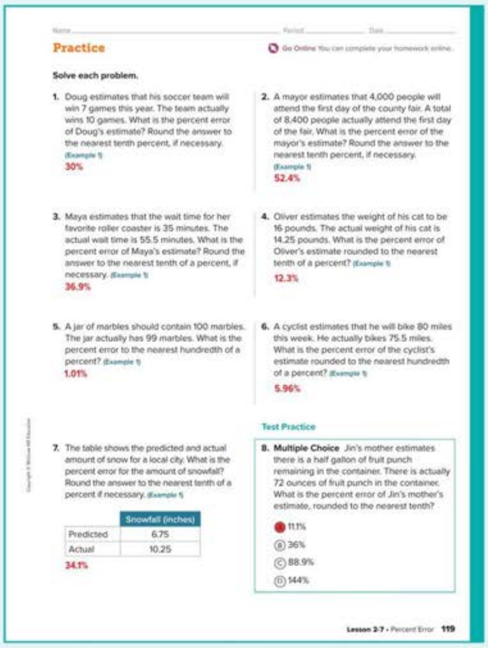
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find the percent error given the estimated value and the actual amount	1–7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving percent error	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Students may attempt to find the percent error by dividing the actual value by the estimated value. In Exercise 1, students might divide 10 by 7 and claim that the percent error is 42.9% because 10 divided by 7 is approximately 1.429. Remind students that they must calculate the amount of error before finding the percent error. Have them use reasoning to explain why a 42.9% error does not seem reasonable if the estimate was off by 3 games out of 10 games.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each problem.

1. Doug estimates that his soccer team will win 7 games this year. The team actually wins 10 games. What is the percent error of Doug's estimate? Round the answer to the nearest tenth percent, if necessary. (Example 1)
30%

2. A mayor estimates that 4,000 people will attend the first day of the county fair. A total of 8,400 people actually attend the first day of the fair. What is the percent error of the mayor's estimate? Round the answer to the nearest tenth percent, if necessary. (Example 1)
52.4%

3. Maya estimates that the wait time for her favorite roller coaster is 35 minutes. The actual wait time is 55.5 minutes. What is the percent error of Maya's estimate? Round the answer to the nearest tenth of a percent, if necessary. (Example 1)
36.9%

4. Oliver estimates the weight of his cat to be 16 pounds. The actual weight of his cat is 14.25 pounds. What is the percent error of Oliver's estimate rounded to the nearest tenth of a percent? (Example 1)
12.3%

5. A jar of marbles should contain 100 marbles. The jar actually has 99 marbles. What is the percent error to the nearest hundredth of a percent? (Example 1)
1.01%

6. A cyclist estimates that he will bike 80 miles this week. He actually bikes 75.5 miles. What is the percent error of the cyclist's estimate rounded to the nearest hundredth of a percent? (Example 1)
5.96%

7. The table shows the predicted and actual amount of snow for a local city. What is the percent error for the amount of snowfall? Round the answer to the nearest tenth of a percent if necessary. (Example 1)

	Snowfall (inches)
Predicted	6.75
Actual	10.25

34.1%

Test Practice

8. **Multiple Choice** Jin's mother estimates there is a half gallon of fruit punch remaining in the container. There is actually 72 ounces of fruit punch in the container. What is the percent error of Jin's mother's estimate, rounded to the nearest tenth?
(A) 11.1%
(B) 36%
(C) 88.9%
(D) 144%

Lesson 2-7 • Percent Error 119

Apply *indicates multi-step problem

*9. A school newspaper estimates that their academic team will win 25 out of 30 matches for the season. After 15 matches, they have won 12. If the team continues winning at this rate, what will be the percent error of the newspaper's estimate once the season is over? Round to the nearest percent.

4%

*10. A toy company that makes bubbles fills its 8-ounce bottles using a machine. To check that the machine fills the bottles with the proper amount, the company randomly checks bottles off the assembly line. A bottle passes inspection if the percent error of the amount is 2% or less. What is the range of values that a bottle could contain to pass inspection? Round to the nearest hundredth.

7.84 oz and 8.16 oz

Higher-Order Thinking Problems

11. **Find the Error** A student is finding the percent error for an estimated length of 22 inches with an actual length of 25 inches. Find the student's mistake and correct it.

$25 \text{ in.} - 22 \text{ in.} = 3 \text{ in.}$

$\frac{3}{25} = 0.12$

$= 0.12\%$

The student forgot to write the amount of error as a percent. 0.12 = 12%

12. A student population of 1,200 was estimated to increase by 15% in the next five years. The population actually increased by 20%. Find the estimated and actual student populations and describe the percent error.

1,380 students; 1,440 students; The percent of error is about 4.2%.

13. **Make an Argument** Make an argument for why you cannot find the percent error when the actual value is 0. Explain.

The denominator would be 0 in the calculation. A denominator of 0 is undefined so the percent of error would be undefined.

14. **Use a Counterexample** Determine if the following statement is true or false. If false, provide a counterexample.

Percent error can never be greater than 100%.

false; Sample answer: For example, the estimated wait time is 12 minutes and the actual wait time is 5 minutes. The percent of error is 12 − 5 or 7, 7 ÷ 5 = 1.4 or 140%.

120 **Module 2** • Solve Percent Problems

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Ex ercise 11, students find and correct a student's mistake.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students make an argument for why you cannot find the percent error when the actual value is 0.

In Exercise 14, students determine if a statement is true or false, and provide a counterexample if it is false.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.

Use with Exercise 9 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Make sense of the problem.

Use with Exercise 11 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise forgot to write the amount of error as a percent. Have each pair or group of students present their explanations to the class.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of percent of increase, percent of decrease, and real world problems involving percents. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

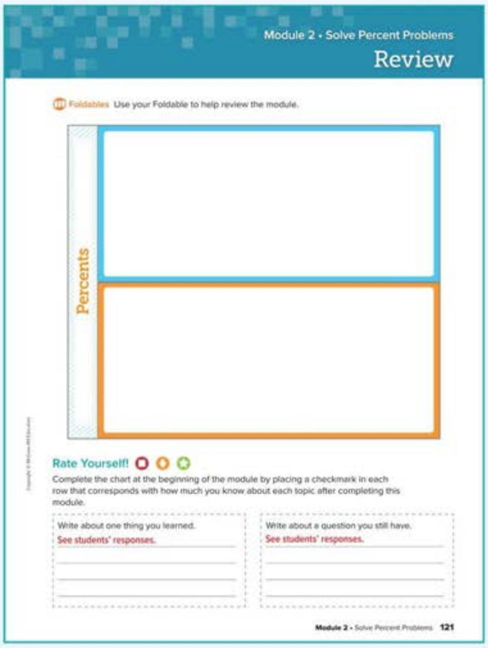
Assessment Resources

Put It All Together 1: Lessons 2-1 through 2-4
Put It All Together 2: Lessons 2-5 through 2-7
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

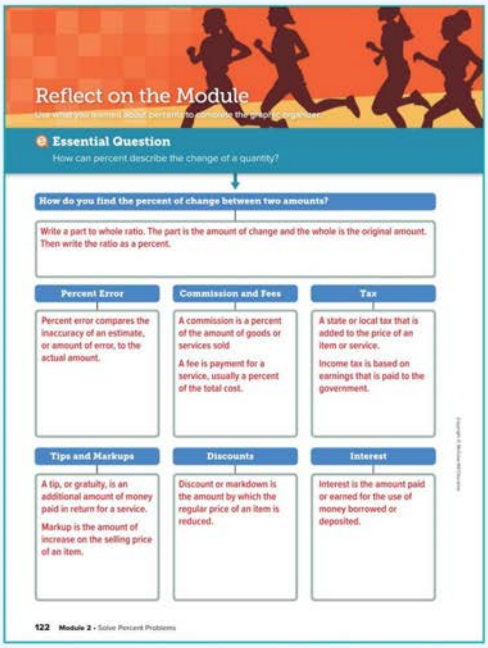
*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with this topic for **Ratios and Proportional Relationships**.

- Applications of Proportional Relationships


Module 2 • Solve Percent Problems
Review
Foldables Use your Foldable to help review the module.

Rate Yourself!
Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.
Write about one thing you learned.
See students' responses.
Write about a question you still have.
See students' responses.
Module 2 • Solve Percent Problems 121



Reflect on the Module

Essential Question

How can percent describe the change of a quantity?

How do you find the percent of change between two amounts?

Write a part to whole ratio. The part is the amount of change and the whole is the original amount. Then write the ratio as a percent.

Percent Error

Percent error compares the inaccuracy of an estimate, or amount of error, to the actual amount.

Commission and Fees

A commission is a percent of the amount of goods or services sold

A fee is payment for a service, usually a percent of the total cost.

Tax

A state or local tax that is added to the price of an item or service.

Income tax is based on earnings that is paid to the government.

Tips and Markups

A tip, or gratuity, is an additional amount of money paid in return for a service.

Markup is the amount of increase on the selling price of an item.

Discounts

Discount or markdown is the amount by which the regular price of an item is reduced.

Interest

Interest is the amount paid or earned for the use of money borrowed or deposited.

122 Module 2 • Solve Percent Problems

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can a percent describe the change of a quantity? See students' graphic organizers.

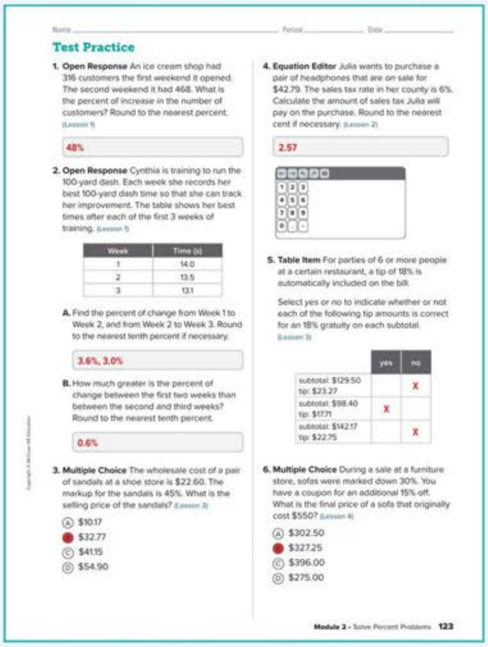
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–12 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	3, 6, 12
Multiselect	Multiple answers may be correct. Students must select all correct answers.	9
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	4, 7, 10
Table Item	Students complete a table.	5
Open Response	Students construct their own response in the area provided.	1, 2, 8, 11

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.RP.A.3	2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 2-7	1–12
7.EE.A.2	2-2, 2-3, 2-4, 2-6	3–7, 10–11


Name _____ Period _____ Date _____

Test Practice

1. **Open Response** An ice cream shop had 316 customers the first weekend it opened. The second weekend it had 468. What is the percent of increase in the number of customers? Round to the nearest percent. (Lesson 1)

   48%

2. **Open Response** Cynthia is training to run the 100-yard dash. Each week she records her best 100-yard dash time so that she can track her improvement. The table shows her best times after each of the first 3 weeks of training. (Lesson 1)

Week	Time (s)
1	14.0
2	13.5
3	13.1

   A. Find the percent of change from Week 1 to Week 2, and from Week 2 to Week 3. Round to the nearest tenth percent if necessary.

   3.6%, 3.0%

   B. How much greater is the percent of change between the first two weeks than between the second and third weeks? Round to the nearest tenth percent.

   0.6%

3. **Multiple Choice** The wholesale cost of a pair of sandals at a shoe store is $22.60. The markup for the sandals is 45%. What is the selling price of the sandals? (Lesson 3)

   Ⓐ $10.17
   ● $32.77
   Ⓒ $41.15
   Ⓓ $54.90

4. **Equation Editor** Julia wants to purchase a pair of headphones that are on sale for $42.79. The sales tax rate in her county is 6%. Calculate the amount of sales tax Julia will pay on the purchase. Round to the nearest cent if necessary. (Lesson 2)

   2.57

5. **Table Item** For parties of 6 or more people at a certain restaurant, a tip of 18% is automatically included on the bill.

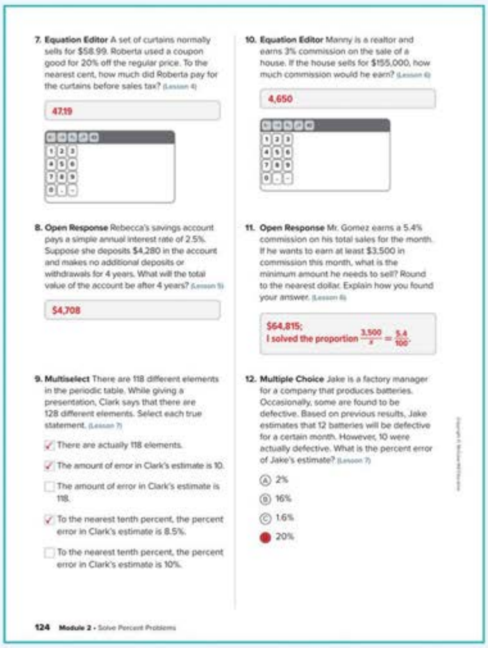
   Select yes or no to indicate whether or not each of the following tip amounts is correct for an 18% gratuity on each subtotal. (Lesson 3)

	yes	no
subtotal: $129.50 tip: $23.27		X
subtotal: $98.40 tip: $17.71	X	
subtotal: $142.17 tip: $22.75		X

6. **Multiple Choice** During a sale at a furniture store, sofas were marked down 30%. You have a coupon for an additional 15% off. What is the final price of a sofa that originally cost $550? (Lesson 4)

   Ⓐ $302.50
   ● $327.25
   Ⓒ $396.00
   Ⓓ $275.00

Module 2 • Solve Percent Problems 123



7. Equation Editor A set of curtains normally sells for \$58.99. Roberta used a coupon good for 20% off the regular price. To the nearest cent, how much did Roberta pay for the curtains before sales tax? (Lesson 4)

47.19

8. Open Response Rebecca's savings account pays a simple annual interest rate of 2.5%. Suppose she deposits \$4,280 in the account and makes no additional deposits or withdrawals for 4 years. What will the total value of the account be after 4 years? (Lesson 5)

\$4,708

9. Multiselect There are 118 different elements in the periodic table. While giving a presentation, Clark says that there are 128 different elements. Select each true statement. (Lesson 7)

- [x] There are actually 118 elements.
- [x] The amount of error in Clark's estimate is 10.
- [ ] The amount of error in Clark's estimate is 118.
- [x] To the nearest tenth percent, the percent error in Clark's estimate is 8.5%.
- [ ] To the nearest tenth percent, the percent error in Clark's estimate is 10%.

10. Equation Editor Manny is a realtor and earns 3% commission on the sale of a house. If the house sells for \$155,000, how much commission would he earn? (Lesson 6)

4,650

11. Open Response Mr. Gomez earns a 5.4% commission on his total sales for the month. If he wants to earn at least \$3,500 in commission this month, what is the minimum amount he needs to sell? Round to the nearest dollar. Explain how you found your answer. (Lesson 6)

\$64,815;
I solved the proportion $\frac{3{,}500}{x} = \frac{5.4}{100}$.

12. Multiple Choice Jake is a factory manager for a company that produces batteries. Occasionally, some are found to be defective. Based on previous results, Jake estimates that 12 batteries will be defective for a certain month. However, 10 were actually defective. What is the percent error of Jake's estimate? (Lesson 7)

Ⓐ 2%
Ⓑ 16%
Ⓒ 1.6%
● 20%

124 **Module 2** • Solve Percent Problems

Module 3

Operations with Integers and Rational Numbers

Module Goal

Add, subtract, multiply, and divide integers and rational numbers.

Focus

Domain: The Number System

Major Cluster(s):

7.NS.A A pply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Standards for Mathematical Content:

7.NS.A.1 A pply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

Also addresses 7.NS.A.3, 7.EE.A.2, 7.EE.B.3, and 8.NS.A.1.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- find the absolute value of integers
- graph integers on a number line
- simplifing numerical expressions involving whole numbers

Use the Module Pretest to diagnose students' readiness for this module.

Coherence

Vertical Alignment

Previous

Students applied and extended previous understandings of numbers to the system of rational numbers. **6.NS.C**

Now

Students add, subtract, multiply, and divide integers and rational numbers.
7.NS.A.1, 7.NS.A.2, 7.NS.A.3, 8.NS.A.1

Next

Students develop and use the Laws of Exponents to evaluate, simplify, and perform computations with expressions with powers..
8.EE.A.1, 8.EE.A.3, 8.EE.A.4

Rigor

The Three Pillars of Rigor

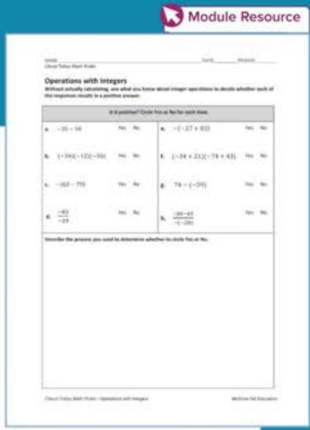
In this module, students draw on their knowledge of rational numbers (gained in Grade 6) to develop *understanding* of operations with integers and rational numbers. They use this understanding to build *fluency* with rational number operations and the order of operations. They will *apply* their fluency to solve multi-step problems involving integers and rational numbers.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
3–1	Add Integers	7.NS.A.1, 7.NS.A.1.A, 7.NS.A.1.B, 7.NS.A.1.D	1	0.5
3–2	Subtract Integers	7.NS.A.1, 7.NS.A.1.C	2	1
3–3	Multiply Integers	7.NS.A.2, 7.NS.A.2.A, 7.NS.A.2.C	1	0.5
3–4	Divide Integers	7.NS.A.2, 7.NS.A.2.B, 7.NS.A.2.C	1	0.5
3–5	Apply Integer Operations	7.NS.A.1, 7.NS.A.1.D, 7.NS.A.2, 7.NS.A.2.C, 7.NS.A.3, 7.EE.B.3	1	0.5
Put It All Together 1: Lessons 3-1 through 3-5			0.5	0.25
3-6	Rational Numbers	7.NS.A.2, 7.NS.A.2.B, 7.NS.A.2.D, 8.NS.A.1	1	0.5
3-7	Add and Subtract Rational Numbers	7.NS.A.1, 7.NS.A.1.A, 7.NS.A.1.B, 7.NS.A.1.C, 7.NS.A.1.D, 7.NS.A.2, 7.NS.A.2.B, 7.EE.B.3	1	0.5
3-8	Multiply and Divide Rational Numbers	7.NS.A.2, 7.NS.A.2.A, 7.NS.A.1.B, 7.NS.A.2.C, 7.NS.A.3	1	0.5
Put It All Together 2: Lessons 3-6 through 3-8			0.5	0.25
3-9	Apply Rational Number Operations	7.NS.A.1, 7.NS.A.1.D, 7.NS.A.2, 7.NS.A.2.C, 7.NS.A.3	1	0.5
Module Review and Assessment			2	1
		Total Days	14	7

Formative Assessment Math Probe
Operations with Integers



Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine whether each simplified expression is positive or not, without actually calculating.

Targeted Concept Determining whether the result is positive or negative involves understanding the effects of integer addition, subtraction, multiplication and division.

Targeted Misconceptions

- Students may overgeneralize or mix up the rules governing rational number operations.
- Students may think that when opposites are multiplied or divided, the result is zero.
- Explanations are ambiguous. For example, students may indicate “two negatives is always a positive” without specifying for which operations.

Assign the probe after Lesson 4.

Correct Answers: a. No; **b.** No; **c.** Yes; **d.** Yes; **e.** No; **f.** Yes; **g.** Yes; **h.** No

Collect and Assess Student Work

If the student selects...	**Then** the student likely...
Yes for a, c, e, f, g, or h	misapplied the rule of “two negatives make a positive” to all operations.
e. Yes	distributed the negative to the first integer inside the parentheses, but not to the second.
h. Yes	thinks a negative divided by a negative results in a positive answer, without simplifying each part of the problem first.
“Two negatives cancel each other out.”	overgeneralized in their explanations.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Whole Numbers, Integers
- Lesson 1, Examples 1–7
- Lesson 2, Examples 1–5
- Lesson 3, Examples 1–6
- Lesson 4, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How are operations with rational numbers related to operations with integers? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

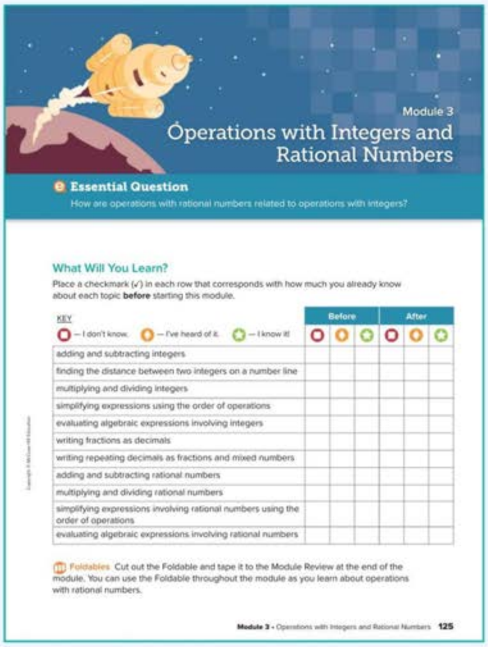
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of measurement, altitude, and the stock market to introduce the idea of operations with rational numbers. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.


Module 3

Operations with Integers and Rational Numbers

Essential Question

How are operations with rational numbers related to operations with integers?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

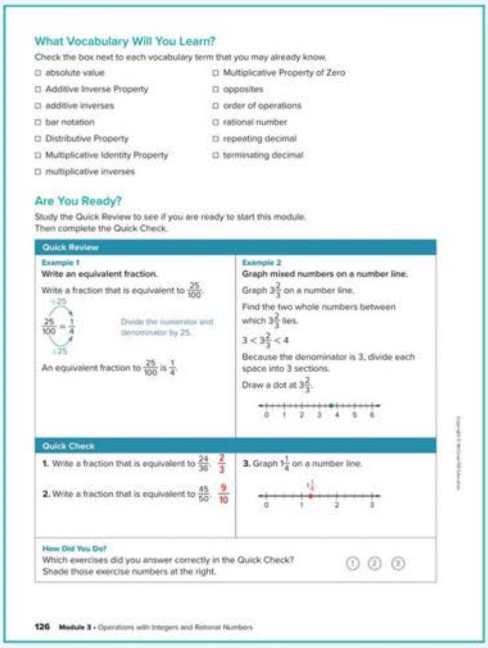
KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know.	I've heard of it.	I know it!	I don't know.	I've heard of it.	I know it!
adding and subtracting integers						
finding the distance between two integers on a number line						
multiplying and dividing integers						
simplifying expressions using the order of operations						
evaluating algebraic expressions involving integers						
writing fractions as decimals						
writing repeating decimals as fractions and mixed numbers						
adding and subtracting rational numbers						
multiplying and dividing rational numbers						
simplifying expressions involving rational numbers using the order of operations						
evaluating algebraic expressions involving rational numbers						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about operations with rational numbers.

Module 3 • Operations with Integers and Rational Numbers 125

Interactive Student Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ absolute value
- ☐ Additive Inverse Property
- ☐ additive inverses
- ☐ bar notation
- ☐ Distributive Property
- ☐ Multiplicative Identity Property
- ☐ multiplicative inverses
- ☐ Multiplicative Property of Zero
- ☐ opposites
- ☐ order of operations
- ☐ rational number
- ☐ repeating decimal
- ☐ terminating decimal

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Write an equivalent fraction.
Write a fraction that is equivalent to $\frac{25}{100}$.
$\frac{25}{100} = \frac{1}{4}$ (÷25)
Divide the numerator and denominator by 25.
An equivalent fraction to $\frac{25}{100}$ is $\frac{1}{4}$.

Example 2
Graph mixed numbers on a number line.
Graph $3\frac{2}{3}$ on a number line.
Find the two whole numbers between which $3\frac{2}{3}$ lies.
$3 < 3\frac{2}{3} < 4$
Because the denominator is 3, divide each space into 3 sections.
Draw a dot at $3\frac{2}{3}$.

Quick Check

1. Write a fraction that is equivalent to $\frac{24}{36}$. $\frac{2}{3}$
2. Write a fraction that is equivalent to $\frac{45}{50}$. $\frac{9}{10}$
3. Graph $1\frac{1}{4}$ on a number line.

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ② ③

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine.

Define A **rational number** is any number that can be written in the form $\frac{a}{b}$ where a and b are integers, and $b \neq 0$.

Example
The numbers -8, $\frac{5}{9}$, $-3\frac{3}{4}$, 1.75, 32%, and $-5.\overline{6}$ are all examples of rational numbers.

Ask Explain why 32% is considered a rational number.
Sample answer: 32% can be written as the ratio of two integers, $\frac{32}{100}$, and the denominator is not 0.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- operations with whole numbers
- operations with fractions
- operations with decimals
- using the order of operations to evaluate expressions

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Whole Numbers**, **Integers**, and **Fractions** topics – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Regular Reflection

When students are asked to regularly explain their thinking about a strategy they used to solve a problem, they are engaging in thought organization, concise consolidation of knowledge, and deductive and inductive reasoning.

How Can I Apply It?

Use the **Think About It!** and **Talk About It!** questions throughout each lesson to encourage students to reflect about what they just learned, or what they might do next.

Throughout the lesson, **Pause and Reflect** questions are included at point-of-use in the *Interactive Student Edition*. Encourage students to not skip over these questions, but to actually *pause* and *reflect* on the concept(s) they just learned and what questions they still might have.

Have students complete the **Exit Tickets** to reflect on their learning about the topics covered in each lesson. Have students share their reflections with a partner or in small groups.

Lesson 3-1

Add Integers

7.NS.A.1

LESSON GOAL

Students will solve problems adding integers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Algebra Tiles to Add Integers

Learn: Add Integers with the Same Sign
Example 1: Add Integers with the Same Sign
Example 2: Add Integers with the Same Sign
Learn: Find Additive Inverses
Example 3: Find Additive Inverses
Learn: Add Integers with Different Signs
Example 4: Add Integers with Different Signs
Example 5: Add Integers with Different Signs
Example 6: Add Three or More Integers
Example 7: Add Three or More Integers

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Balancing a Checkbook		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 14 of the *Language Development Handbook* to help your students build mathematical language related to addition of integers.

ELL You can use the tips and suggestions on page T14 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7. NS.A** by adding integers.
Standards for Mathematical Content: 7 .NS.A.1, 7.NS.A.1.A, 7.NS.A.1.B, 7.NS.A.1.D, 7.EE.B.3, *Also addresses 7.NS.A.3*
Standards for Mathematical Practice: MP 2, MP3, MP5, MP6

Coherence

Vertical Alignment

Previous
Students used integers to describe real-world situations.
6.NS.C.5

Now
Students solve problems involving adding integers.
7.NS.A1.A, 7.NS.A1.B, 7.NS.A.1.D

Next
Students will solve problems involving subtracting integers.
7.NS.A1, 7.NS.A.1.C

Rigor

The Three Pillars of Rigor

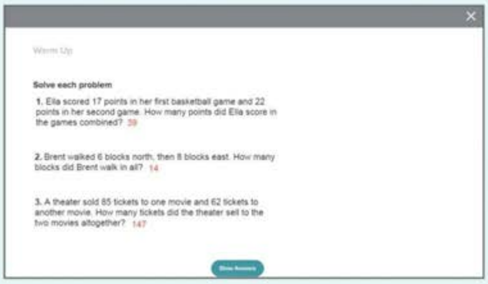
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of rational numbers to develop *understanding* of addition of integers and finding additive inverses. They use this understanding to gain *fluency* in adding multiple signed numbers. They *apply* their knowledge of adding integers in real-world applications.

Mathematical Background

The set of integers consists of whole numbers and their opposites. {..., −3, −2, −1, 0, 1, 2, 3, ...} Numbers that are the same distance from zero on a number line, but on opposite sides of zero have the same *absolute value*. *Opposites* have the same absolute value but different signs. Two integers that are opposites are called *additive inverses* and their sum is always zero. To add two integers with the same sign, add their absolute values. The sum has the same sign as the addends. To add two integers with different signs, subtract their absolute values. The sum has the same sign as the addend with the greater absolute value.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- solving word problems involving adding whole numbers (Exercises 1–3)

Answers

1. 39

2. 14

3. 147

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about integers as they relate to football.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Give an example of *opposites* in everyday language. Sample answer: Walking forward and walking backward are opposites.
- What does the root word *add* in the word additive mean ? Sample answers: *Add* means to join, unite, or bring together.
- What do you think *property* means in mathematics ? Sample answer: A property is an attribute common to all members of a group, as in a set of numbers.

Explore Use Algebra Tiles to Add Integers

Objective

Students will use algebra tiles to explore how to add integers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with algebra tiles representing 1 and −1. Throughout this activity, students will use the algebra tiles to add integers.

Inquiry Question

How can algebra tiles be used to model integer addition? Sample answer: Use algebra tiles to represent each integer. Remove any zero pairs. The value that remains represents the sum of the integers.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

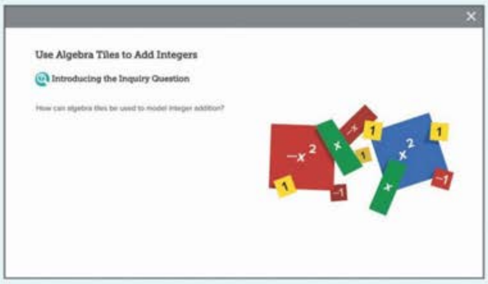
SLIDE 2

Mathematical Discourse

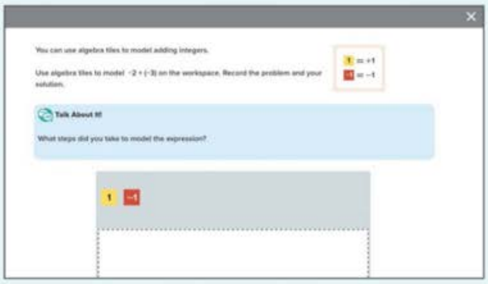
What steps did you take to model the expression? Sample answer: Begin by placing two −1 tiles on the workspace. Then add three −1 tiles. The sum of the tiles on the workspace is five −1 tiles.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 7



Explore, Slide 2 of 7

DRAG & DROP

On Slides 2 and 3, students drag algebra tiles to model the addition of integers.

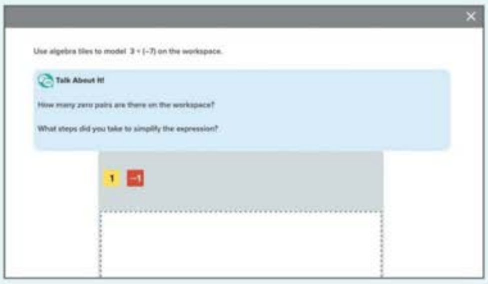
WATCH

On Slide 2, students watch a video that explains how to add integers with algebra tiles.

TYPE

On Slide 3, students type to make a conjecture about the sum of two negative addends.

Interactive Presentation



Explore, Slide 5 of 7

DRAG & DROP

On Slides 5 and 6, students drag algebra tiles to model the addition of integers.

WATCH

On Slide 4, students watch a video that demonstrates zero pairs.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Use Algebra Tiles to Add Integers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use algebra tiles to explore integer addition.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

How many zero pairs are there on the workspace? three zero pairs

What steps did you take to simplify the expression? Sample answer: I removed any sets of zero pairs until there were none left.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Add Integers with the Same Sign

Objective

Students will understand that they can use a number line to add integers with the same sign.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to generate their own examples that demonstrate the rule for adding integers with the same sign, and use a number line if necessary.

As students discuss the *Talk About It!* question on Slide 4, encourage them to reason about how a number line illustrates that the sum of two negative integers will always be negative.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Give an example of adding integers with the same sign. Does your example reinforce the rules about the sign of the sum? Sample answer: $-2 + (-6) = -8$; This reinforces the rule because both addends are negative and the sum is negative.

(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

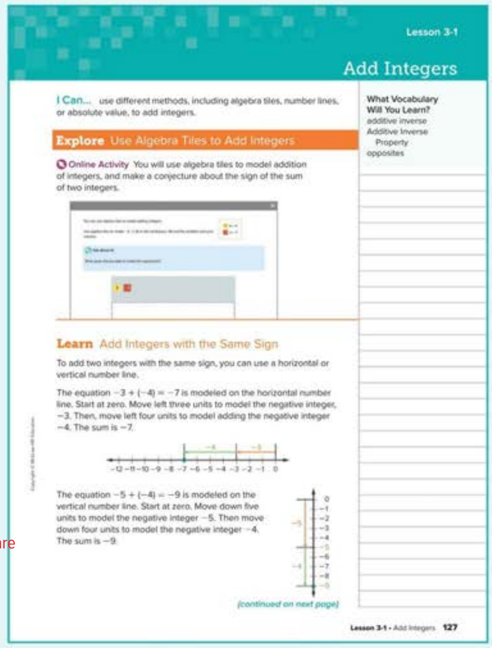
For students that may be struggling to understand how to add integers with the same sign, explain how they can view integers based on their position on the number line. The sum of two integers is their cumulative positive from 0. For each of the following sums, have students work with a partner to identify each of the addend's position on the number line with respect to 0. Then have them describe the location of the sum.

$-2 + (-7)$ 2 units left; 7 units left; 9 units left

$-1 + (-1)$ 1 unit left; 1 unit left; 2 units left

$-6 + (-14)$ 6 units left; 14 units left; 20 units left

$-3 + (-2)$ 3 units left; 2 units left; 5 units left



Lesson 3-1

Add Integers

I Can... use different methods, including algebra tiles, number lines, or absolute value, to add integers.

What Vocabulary Will You Learn?
additive inverse
Additive Inverse Property
opposites

Explore Use Algebra Tiles to Add Integers

Online Activity You will use algebra tiles to model addition of integers, and make a conjecture about the sign of the sum of two integers.

Learn Add Integers with the Same Sign

To add two integers with the same sign, you can use a horizontal or vertical number line.

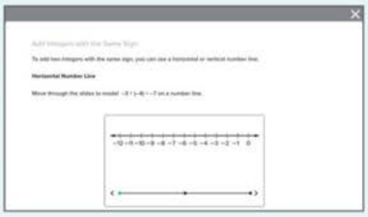
The equation $-3 + (-4) = -7$ is modeled on the horizontal number line. Start at zero. Move left three units to model the negative integer, -3. Then, move left four units to model adding the negative integer -4. The sum is -7.

The equation $-5 + (-4) = -9$ is modeled on the vertical number line. Start at zero. Move down five units to model the negative integer -5. Then move down four units to model the negative integer -4. The sum is -9.

(continued on next page)

Lesson 3-1 • Add Integers 127

Interactive Presentation



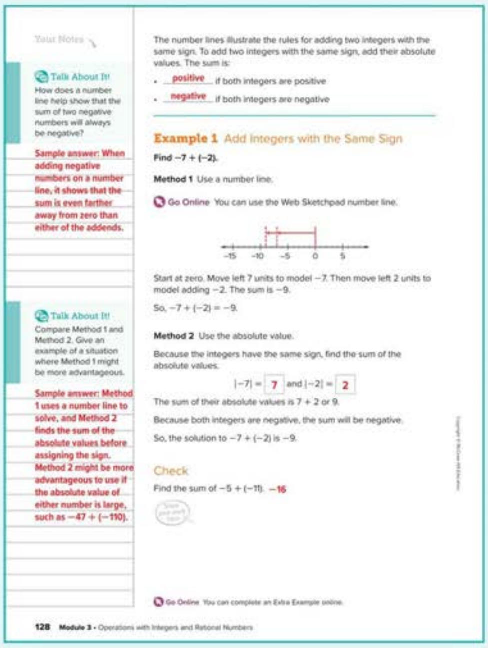
Learn, Add Integers with the Same Sign, Slide 1 of 4

CLICK

On Slide 1, students move through the slides to model $-3 + (-4) = -7$ on a horizontal number line.

CLICK

On Slide 2, students move through the slides to model $-5 + (-4) = -9$ on a vertical number line.



Your Notes

Talk About It!
How does a number line help show that the sum of two negative numbers will always be negative?

Sample answer: When adding negative numbers on a number line, it shows that the sum is even farther away from zero than either of the addends.

The number lines illustrate the rules for adding two integers with the same sign. To add two integers with the same sign, add their absolute values. The sum is:

- positive if both integers are positive
- negative if both integers are negative

Example 1 Add Integers with the Same Sign

Find $-7 + (-2)$.

Method 1 Use a number line.

Go Online You can use the Web Sketchpad number line.

Start at zero. Move left 7 units to model -7. Then move left 2 units to model adding -2. The sum is -9.

So, $-7 + (-2) = -9$.

Method 2 Use the absolute value.

Because the integers have the same sign, find the sum of the absolute values.

$|-7| = 7$ and $|-2| = 2$

The sum of their absolute values is $7 + 2$ or 9.

Because both integers are negative, the sum will be negative.

So, the solution to $-7 + (-2)$ is -9.

Talk About It!
Compare Method 1 and Method 2. Give an example of a situation where Method 1 might be more advantageous.

Sample answer: Method 1 uses a number line to solve, and Method 2 finds the sum of the absolute values before assigning the sign. Method 2 might be more advantageous to use if the absolute value of either number is large, such as $-47 + (-110)$.

Check

Find the sum of $-5 + (-11)$. -16

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 1, Add Integers with the Same Sign, Slide 2 of 5

WEB SKETCHPAD

On Slide 2, students use Web Sketchpad to see addends placed on a number line (Method 1).

CLICK

On Slide 3, students use absolute value to find the sum (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Add Integers with the Same Sign *(continued)*

Talk About It!

SLIDE 4

Mathematical Discourse

How does a number line help show that the sum of two negative numbers will always be negative? Sample answer: When adding negative numbers on the number line, it shows that the sum is even farther away from zero than either of the addends.

Example 1 Add Integers with the Same Sign

Objective

Students will add integers with the same sign.

Questions for Mathematical Discourse

SLIDE 2

AL Do both integers have the same sign? yes

AL What is the sign of both integers? negative

OL After pressing *Add*, how does the number line show us what the sign of the sum is? The number line shows that the sum of the two negative integers is negative.

BL If you were told that the sum of these numbers is positive, how could you justify why you knew the sum had to be negative? Sample answer: If you start at 0, and move left twice to indicate the two negative integers, the sum will still be to the left of 0, which is negative.

SLIDE 3

AL What are the absolute values of -7 and -2? 7 and 2

OL What will be the sign of the sum? Explain. Negative, because both integers are negative.

OL Which method do you prefer? See students' responses.

BL Without calculating, what will be the sign of the sum $-7 + (-2) + (-11)$? Explain. Then find the sum. negative; As long as all of the addends are negative, the sum will be negative. The sum is -20.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Add Integers with the Same Sign

Objective

Students will add integers with the same sign to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to correctly interpret their results within the context of the situation.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the integer that represents the amount of money Allie has at the end of the month

AL What do you know? She borrowed $139 for an ebook reader and $47 for apps, games, and movies.

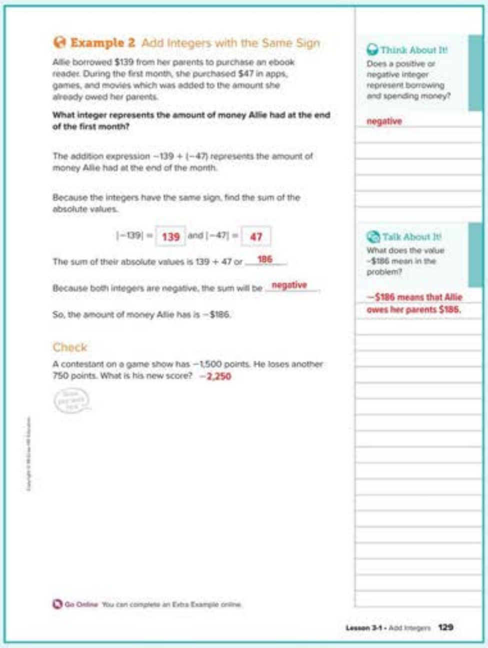
OL What expression represents the amount of money Allie had at the end of the month? $-139 + (-47)$

OL Without calculating, how do you know that the sum will be negative? The sum of two negative integers is negative.

BL Suppose Allie then returned $53 to her parents. Write an addition expression that represents the amount Allie had after this payment. $-139 + (-47) + 53$ or $-186 + 53$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Add Integers with the Same Sign

Allie borrowed $139 from her parents to purchase an ebook reader. During the first month, she purchased $47 in apps, games, and movies which was added to the amount she already owed her parents.

What integer represents the amount of money Allie had at the end of the first month?

The addition expression $-139 + (-47)$ represents the amount of money Allie had at the end of the month.

Because the integers have the same sign, find the sum of the absolute values.

$|-139| =$ 139 and $|-47| =$ 47

The sum of their absolute values is 139 + 47 or 186.

Because both integers are negative, the sum will be negative.

So, the amount of money Allie has is −$186.

Think About It! Does a positive or negative integer represent borrowing and spending money? negative

Talk About It! What does the value −$186 mean in the problem? −$186 means that Allie owes her parents $186.

Check

A contestant on a game show has −1,500 points. He loses another 750 points. What is his new score? −2,250

Go Online You can complete an Extra Example online.

Lesson 3-1 • Add Integers 129

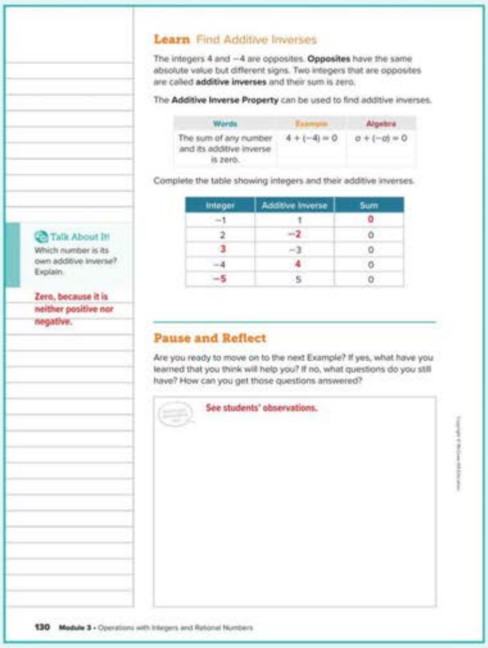
Interactive Presentation

Example 2, Add Integers with the Same Sign, Slide 2 of 4

CLICK On Slide 2, students move through the steps to solve the problem.

TYPE On Slide 2, students determine the solution.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Learn Find Additive Inverses

The integers 4 and −4 are opposites. **Opposites** have the same absolute value but different signs. Two integers that are opposites are called **additive inverses** and their sum is zero.

The **Additive Inverse Property** can be used to find additive inverses.

Words	Example	Algebra
The sum of any number and its additive inverse is zero.	$4 + (-4) = 0$	$a + (-a) = 0$

Complete the table showing integers and their additive inverses.

Integer	Additive Inverse	Sum
−1	1	0
2	−2	0
3	−3	0
−4	4	0
−5	5	0

Talk About It! Which number is its own additive inverse? Explain.

Zero, because it is neither positive nor negative.

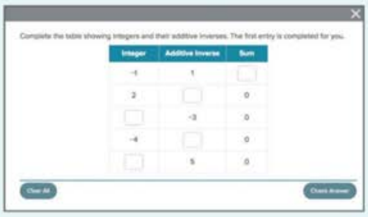
Pause and Reflect

Are you ready to move on to the next Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

See students' observations.

130 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Learn, Find Additive Inverses, Slide 2 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of the Additive Inverse Property.

TYPE

On Slide 2, students complete a table to show integers and their additive inverses.

Learn Find Additive Inverses

Objective

Students will understand that an integer and its opposite are called additive inverses, and their sum is zero.

MP Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 3, encourage them to attend to the meaning of the term *additive inverse* and clearly explain why the number they chose is its own additive inverse.

Teaching Notes

SLIDE 1

Students previously learned about integers and their opposites. Ask students why it makes sense that the sum of an integer and its opposite is zero. Have students select the *Words, Example,* and *Algebra* flashcards to view how the Additive Inverse Property can be described using these multiple representations.

Talk About It!

SLIDE 3

Mathematical Discourse

Which number is its own additive inverse? Explain. Zero, because it is neither positive nor negative.

DIFFERENTIATE

Language Development Activity ELL

To further students' understanding of additive inverses, have them work with a partner to explain why the terms *additive inverses* and *opposites* can be used interchangeably. Have them make a conjecture as to why there might be two terms that refer to the same concept.

Sample answer: Numbers that are opposites are on opposite sides of zero on the number line, which is why they are referred to as opposites. They also have opposite signs, + and −. Additive inverses are numbers that have a sum of zero, which is why the term additive is used to describe them. Additive inverses and opposites are the same. If two numbers are opposites (on opposite sides of zero on the number line), they will always be additive inverses (because their sum is zero).

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Example 3 Find Additive Inverses

Objective

Students will find the additive inverse of an integer in a real-world context.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question, encourage them to use clear and precise mathematical language, such as *additive inverses* and *opposites*.

Questions for Mathematical Discourse

SLIDE 2

AL What integer represents the elevation in feet above sea level at the beginning of the hiking trail? 150

AL What integer represents the elevation in feet above sea level at the end of the hiking trail? 0

OL How will you find the change in the elevation of the trail from beginning to end? Find the additive inverse of 150.

OL What is the additive inverse of 150? —150

BL Write and solve a real-world problem where you have to find the additive inverse. Sample answer: The temperature outside was —2 degrees Fahrenheit. The temperature is now 0 degrees Fahrenheit. What integer represents the change in temperature?; 2

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Add Integers with Different Signs

Objective

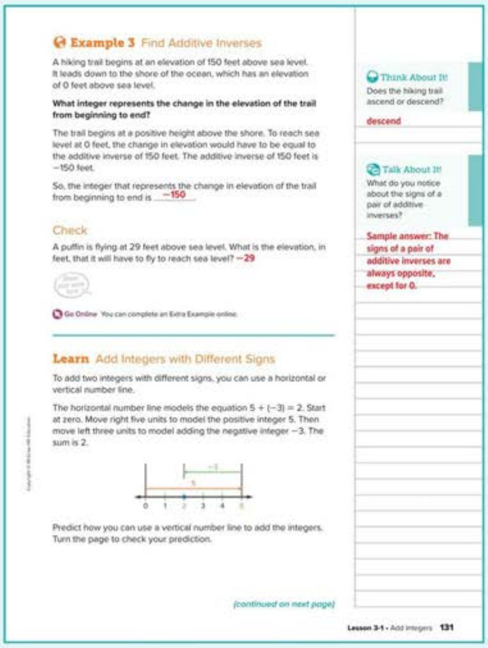
Students will understand that they can use a number line to add integers with different signs.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 3, encourage them to give an accurate example of adding integers with different signs that reinforces the rules about the sign of the sum.

Go Online to find additional teaching notes.

(continued on next page)



Example 3 Find Additive Inverses

A hiking trail begins at an elevation of 150 feet above sea level. It leads down to the shore of the ocean, which has an elevation of 0 feet above sea level.

What integer represents the change in the elevation of the trail from beginning to end?

The trail begins at a positive height above the shore. To reach sea level at 0 feet, the change in elevation would have to be equal to the additive inverse of 150 feet. The additive inverse of 150 feet is −150 feet.

So, the integer that represents the change in elevation of the trail from beginning to end is −150.

Think About It! Does the hiking trail ascend or descend? descend

Talk About It! What do you notice about the signs of a pair of additive inverses? Sample answer: The signs of a pair of additive inverses are always opposite, except for 0.

Check

A puffin is flying at 29 feet above sea level. What is the elevation, in feet, that it will have to fly to reach sea level? −29

Go Online You can complete an Extra Example online.

Learn Add Integers with Different Signs

To add two integers with different signs, you can use a horizontal or vertical number line.

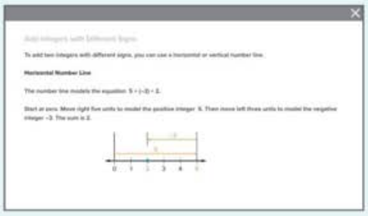
The horizontal number line models the equation $5 + (-3) = 2$. Start at zero. Move right five units to model the positive integer 5. Then move left three units to model adding the negative integer −3. The sum is 2.

Predict how you can use a vertical number line to add the integers. Turn the page to check your prediction.

(continued on next page)

Lesson 3-1 • Add Integers 131

Interactive Presentation



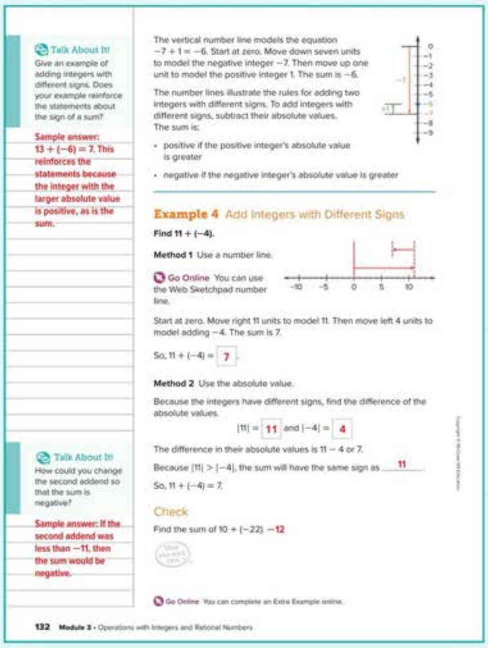
Learn, Add Integers with Different Signs, Slide 1 of 3

TYPE

On Slide 2 of Example 3, students determine the integer that represents the change in elevation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Talk About It!
Give an example of adding integers with different signs. Does your example reinforce the statements about the sign of a sum?

Sample answer: $13 + (-6) = 7$. This reinforces the statements because the integer with the larger absolute value is positive, as is the sum.

The vertical number line models the equation $-7 + 1 = -6$. Start at zero. Move down seven units to model the negative integer -7. Then move up one unit to model the positive integer 1. The sum is -6.

The number lines illustrate the rules for adding two integers with different signs. To add integers with different signs, subtract their absolute values. The sum is:

- positive if the positive integer's absolute value is greater
- negative if the negative integer's absolute value is greater

Example 4 Add Integers with Different Signs

Find $11 + (-4)$.

Method 1 Use a number line.

Go Online You can use the Web Sketchpad number line.

Start at zero. Move right 11 units to model 11. Then move left 4 units to model adding -4. The sum is 7.

So, $11 + (-4) =$ 7.

Method 2 Use the absolute value.

Because the integers have different signs, find the difference of the absolute values.

$|11| =$ 11 and $|-4| =$ 4

The difference in their absolute values is $11 - 4$ or 7.

Because $|11| > |-4|$, the sum will have the same sign as 11.

So, $11 + (-4) = 7$.

Talk About It!
How could you change the second addend so that the sum is negative?

Sample answer: If the second addend was less than -11, then the sum would be negative.

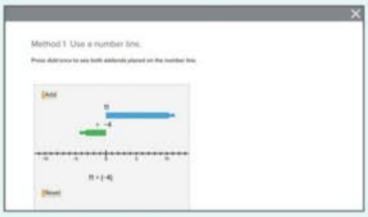
Check

Find the sum of $10 + (-22)$. -12

Go Online You can complete an Extra Example online.

132 **Module 3** • Operations with Integers and Rational Numbers

Interactive Presentation



Example 4, Add Integers with Different Signs, Slide 2 of 5

WEB SKETCHPAD

On Slide 2, students use Web Sketchpad to see addends placed on a number line (Method 1).

CLICK

On Slide 3, students use absolute value to find the sum (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Add Integers with Different Signs *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

Give an example of adding integers with different signs. Does your example reinforce the statements about the sign of a sum?
Sample answer: $13 + (-6) = 7$. This reinforces the statements because the integer with the greater absolute value, 13, is positive, as is the sum.

Example 4 Add Integers with Different Signs

Objective

Students will add integers with different signs.

Questions for Mathematical Discourse

SLIDE 2

AL Do the two integers have the same sign, or different signs? different signs

OL After pressing *Add*, explain why the first integer resulted in movement to the right, while the second integer resulted in movement to the left. Sample answer: The first integer is positive. The second integer is negative. Adding a negative integer results in movement to the left.

BL How could you alter one of the integers in the expression so that the sum is 0? Sample answer: Change the second integer to -11 so that the expression is $11 + (-11)$, which has a sum of 0.

SLIDE 3

AL Which integer has the greater absolute value? 11

OL Will the sum be positive or negative? Explain. Because the positive integer has the greater absolute value, the sum will be positive.

BL What is the greatest integer that can be added to 11 in order to make the sum negative? -12

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 5 Add Integers with Different Signs

Objective

Students will add integers with different signs to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to interpret the result of a positive integer in the context of the situation, and to use quantitative reasoning as they think of a possible change to the problem that would result in a positive integer solution.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the current depth of the whale

AL What was the original depth of the whale? 275 feet below the water's surface

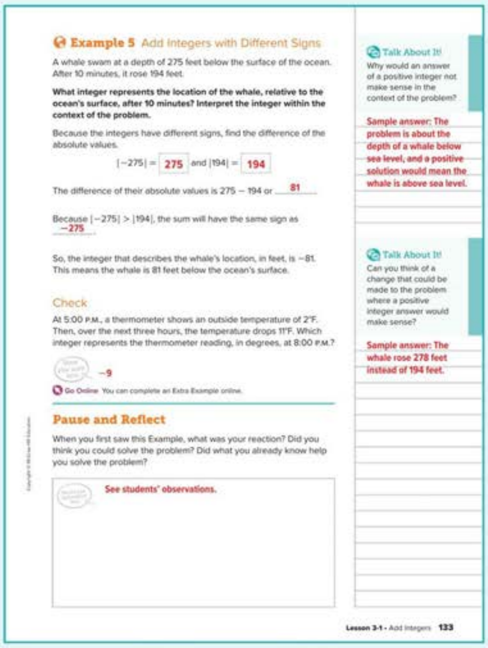
OL What integers are used to represent "a depth of 275 feet below the surface of the water?" and "rose 194 feet?" −275; 194

BL Will the whale reach the surface of the water? Explain. no; Sample answer: The whale was 275 feet below the surface and rose 194 feet. Because 194 < 275, the whale will not reach the surface of the water.

BL Suppose the whale jumped out of the water, at a height of 7 feet above the water's surface. How far did the whale travel, if it's original depth was 275 feet below the surface? 282 feet

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 5 Add Integers with Different Signs

A whale swam at a depth of 275 feet below the surface of the ocean. After 10 minutes, it rose 194 feet.

What integer represents the location of the whale, relative to the ocean's surface, after 10 minutes? Interpret the integer within the context of the problem.

Because the integers have different signs, find the difference of the absolute values.

$|-275| = 275$ and $|194| = 194$

The difference of their absolute values is 275 − 194 or 81

Because $|-275| > |194|$, the sum will have the same sign as −275

So, the integer that describes the whale's location, in feet, is −81. This means the whale is 81 feet below the ocean's surface.

Check

At 5:00 P.M., a thermometer shows an outside temperature of 2°F. Then, over the next three hours, the temperature drops 11°F. Which integer represents the thermometer reading, in degrees, at 8:00 P.M.?

−9

Go Online You can complete an Extra Example online.

Pause and Reflect

When you first saw this Example, what was your reaction? Did you think you could solve the problem? Did what you already know help you solve the problem?

See students' observations.

Talk About It!

Why would an answer of a positive integer not make sense in the context of the problem?

Sample answer: The problem is about the depth of a whale below sea level, and a positive solution would mean the whale is above sea level.

Talk About It!

Can you think of a change that could be made to the problem where a positive integer answer would make sense?

Sample answer: The whale rose 278 feet instead of 194 feet.

Lesson 3-1 • Add Integers 133

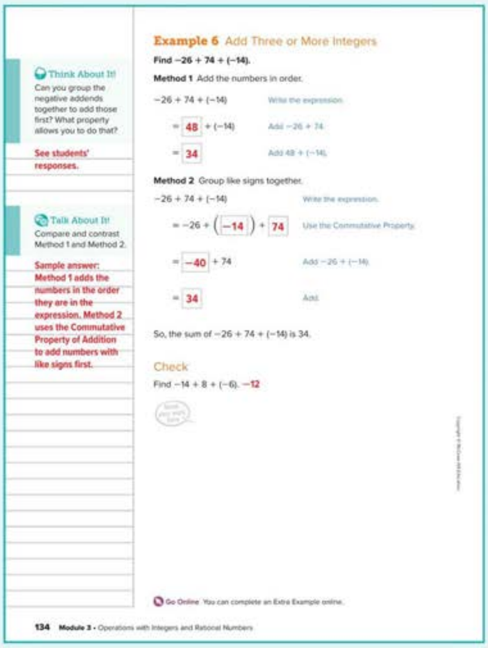
Interactive Presentation

Example 5, Add Integers with Different Signs, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the depth of the whale.

TYPE On Slide 2, students determine and interpret the sum.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Example 6 Add Three or More Integers

Find −26 + 74 + (−14).

Method 1 Add the numbers in order.

−26 + 74 + (−14) Write the expression.

= 48 + (−14) Add −26 + 74.

= 34 Add 48 + (−14).

Method 2 Group like signs together.

−26 + 74 + (−14) Write the expression.

= −26 + (−14) + 74 Use the Commutative Property.

= −40 + 74 Add −26 + (−14).

= 34 Add.

So, the sum of −26 + 74 + (−14) is 34.

Check

Find −14 + 8 + (−6). −12

Think About It! Can you group the negative addends together to add those first? What property allows you to do that? See students' responses.

Talk About It! Compare and contrast Method 1 and Method 2. Sample answer: Method 1 adds the numbers in the order they are in the expression. Method 2 uses the Commutative Property of Addition to add numbers with like signs first.

Go Online You can complete an Extra Example online.

134 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation

Example 6, Add Three or More Integers, Slide 3 of 5

CLICK

On Slide 2, students find the sum by adding the numbers in order (Method 1).

CLICK

On Slide 3, students find the sum by grouping like signs together (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 6 Add Three or More Integers

Objective

Students will add three or more integers.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of each of the methods used to find the sum of three or more integers.

6 Attend to Precision As student discuss the *Talk About It!* question on Slide 4, encourage students to use clear mathematical language as they explain the similarities and differences between the two methods.

Questions for Mathematical Discourse

SLIDE 2

AL Do all of the integers have the same sign? Explain. No; one integer is positive and the other two integers are negative.

OL After adding the first two integers, how do you know that the sum of 48 and —14 will be positive? The sign of the number with the larger absolute value, 48, is positive.

BL How could you mentally determine that the sum will be positive, without calculating? Sample answer: The absolute value of the sum of the two negative numbers, 40, is less than 74.

SLIDE 3

AL Which integers have like signs? –26 and –14

OL Which method do you prefer, to add the numbers in order or to group like signs together? Explain. See students' responses.

BL Consider the expression –12 + (–29) + (–8). How can you group the numbers together so that you can mentally find the sum? Sample answer: Group –12 and –8 together; I know their sum is –20. Then add –20 and –29, which is –49.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 7 Add Three or More Integers

Objective

Students will add three or more integers to solve a real-world problem.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to pay careful attention to whether each number should be represented as a positive or negative integer.

Questions for Mathematical Discourse

SLIDE 2

AL How many times did the roller coaster descend? How can this help you determine how many integers should be negative, and which one should be negative? The roller coaster descended once. This tells me there should be one negative integer, –80.

OL How do you know 43 is positive? Sample answer: The roller coaster started 43 feet above the ground, so 43 should be a positive integer.

BL If the roller coaster started at a height of 8 feet below the ground, how would the addition expression change? The first number would be −8, not 43.

SLIDE 3

AL What do you need to find? the height of the roller coaster at point B in relation to the ground

OL How could you mentally determine that the sum will be positive, without calculating? Sample answer: The sum of the three positive numbers, 43, 55, and 100, is greater than the absolute value of −80.

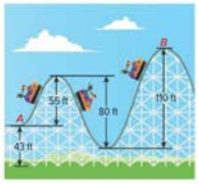
BL Write and solve a word problem involving a roller coaster that could be modeled by the expression $28 + 135 + (-90) + 22 + (-44)$. Sample answer: A roller coaster starts at a height of 28 feet above the ground. It ascends 135 feet, descends 90 feet, ascends 22 feet, and then descends 44 feet. Where is the roller coaster in relation to the ground?; 51 feet above the ground

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 7 Add Three or More Integers

A roller coaster starts at point *A*, 43 feet above the ground. It ascends 55 feet, descends 80 feet, then ascends 110 feet to point *B*.



What is the height of the roller coaster at point *B* in relation to the ground?

Ascending can be represented by a positive integer and descending can be represented by a negative integer. So, the addition expression $43 + 55 + (-80) + 110$ models the situation.

Simplify the expression.

$43 + 55 + (-80) + 110$	Write the expression.
$= 43 + 55 + \boxed{110} + (\boxed{-80})$	Commutative Property of Addition
$= \boxed{98} + 110 + (-80)$	Add 43 + 55.
$= \boxed{208} + (-80)$	Add 98 + 110.
$= \boxed{128}$	Add 208 + (−80).

So, point *B* is 128 feet above the ground.

Think About It! What addition expression could represent this problem?

See students' responses.

Talk About It! Describe another way you can solve the problem.

See students' responses

Lesson 3-1 • Add Integers 135

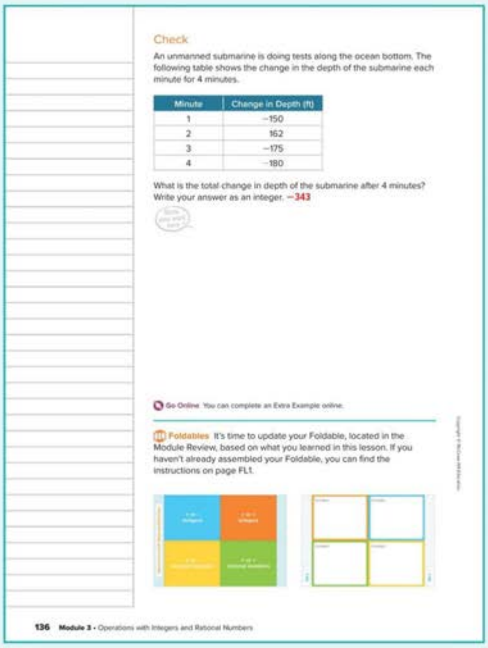
Interactive Presentation

Example 7, Add Three or More Integers, Slide 3 of 5

CLICK On Slide 2, students select the correct addition expression to model the situation.

CLICK On Slide 3, students move through the steps to find the sum.

CHECK Students complete the Check exercise online to determine if they are ready to move on.


Check

An unmanned submarine is doing tests along the ocean bottom. The following table shows the change in the depth of the submarine each minute for 4 minutes.

Minute	Change in Depth (ft)
1	−150
2	162
3	−175
4	−180

What is the total change in depth of the submarine after 4 minutes? Write your answer as an integer. −343

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

136 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation


Exit Ticket

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record the rules for adding integers. You may wish to have students share their Foldables with a partner to compare the information they recorded.

Essential Question Follow-Up

How are operations with rational numbers related to operations with integers?

In this lesson, students learned how to add integers with the same signs or different signs. Encourage them to work with a partner to compare and contrast adding integers to adding whole numbers. For example have them compare and contrast how they would simplify each of the expressions $-3 + (-4)$, $-3 + 4$, $3 + (-4)$, and $3 + 4$.

Exit Ticket

Refer to the Exit Ticket slide. Suppose the offense of your football team gains five yards on their first down and then loses eight yards on their second down. Find the number of yards your team has moved the ball on two plays. Write a mathematical argument that can be used to defend your solution. −3 yards; Sample answer: Write an addition sentence. A gain of five yards is represented by 5. A loss of eight yards is represented by −8. So, the ball has moved $5 + (-8)$ yards, or −3 yards on the first two plays.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7–15 odd, 17–20
- Extension: Balancing a Checkbook
- ALEKS Addition and Subtraction with Integers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–13, 15, 18
- Extension: Balancing a Checkbook
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–7
- ALEKS Plotting and Comparing Integers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Integers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	add integers with the same sign	1, 2
1	add integers with different signs	3, 4
1	add three or more integers	5, 6
2	add integers with the same sign to solve a real-world problem	7, 8
2	find the additive inverse of an integer in a real-world context	9, 10
2	practice adding integers with different signs to solve a real-world problem	11, 12
2	add three or more integers to solve a real-world problem	13
2	extend concepts learned in class to apply them in new contexts	14
3	solve application problems involving adding integers	15, 16
3	higher-order and critical thinking skills	17–20

Common Misconception

Students may assign the incorrect sign to an integer when solving the real-world problems. Review with students the words that represent negative integers such as *loss*, *descend*, and positive integers such as *gain*, *ascend*.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Add. (Examples 1, 4, and 6)

1. $-3 + (-8)$
-11

2. $-11 + (-13)$
-24

3. $9 + (-35)$
-26

4. $-28 + 14$
-14

5. $-22 + (-10) + 15$
-17

6. $18 + (-12) + 5$
11

7. Roger owes his father $15. He borrows another $25 from him. What integer represents the balance that he owes his father? (Example 2)
-40

8. A football team lost 14 yards on their first play then lost another 7 yards on the next play. What integer represents the total change in yards for the two plays? (Example 2)
-21

9. Kwan's beginning account balance was $20. His ending balance is $0. What integer represents the change in his account balance from beginning to end? (Example 3)
-20

10. Lucy's dog lost 6 pounds. How much weight does her dog need to gain in order to have a net change of 0 pounds? (Example 3)
6 pounds

11. The table shows Jewel's scores for the first 9 holes and the second 9 holes of her game of golf. What integer represents her score for the entire game? (Example 5)

Holes	Score
1–9	3 over par
10–18	4 under par

-1

12. At 4:00 A.M., the outside temperature was −28°F. By 4:00 P.M. that same day, it rose 38 degrees. What integer represents the temperature at 4:00 P.M.? (Example 5)
10

13. In 20 seconds, a roller coaster goes up a 100-meter hill, then down 72 meters, and then back up a 48-meter rise. How much higher or lower from the start of the ride is the coaster after the 20 seconds? (Example 7)
76 m higher

Test Practice

14. Open Response Joe opened a bank account with $80. He then withdrew $35 and deposited $115. What is his account balance after these transactions?
$160

Apply

15. The table shows the transactions for one week for Tasha and Jamal. Who has the greater account balance at the end of the week? How much greater?

Tasha; $1

Transactions	Tasha	Jamal
Initial Deposit	$250	$200
Withdrawals	$20	$60
Deposits	$65	$135
Debit Card Purchases	$46	$27

16. A hot air balloon rises 340 feet into the air. Then it descends 130 feet, goes up 80 feet, and then down another 45 feet. How many feet will the balloon need to travel to return to the ground? Represent this amount as an integer. Explain.

−245 ft; Add the integers: $340 + (-130) + 80 + (-45) = 245$. The integer is −245 because the balloon has to descend.

Higher-Order Thinking Problems

17. What value of x would result in the numerical value of zero for each expression?

a. $-10 + 11 + x$ **−1**

b. $7 + x + (-10)$ **3**

c. $x + 1 + (-1)$ **0**

18. **Find a Counterexample** Patrick stated that the sum of a positive integer and a negative integer is always negative. Find a counterexample that illustrates why this statement is not true.

Sample answer: $7 + (-3) = 4$; If the positive integer is greater than the negative integer, the sum will be positive.

19. Explain how you know that the sum of 2, 3, and −2 is positive without computing.

Sample answer: 2 and −2 are additive inverses and the sum of any number and its additive inverse is zero. The integer 3 is positive, so the sum will be positive.

20. **Create** Write and solve a real-world problem where you add three integers and the sum is negative.

Sample answer: Max owes his brother $110. He gives his brother $50 one week and then the next week he gives him $25. What integer represents the amount that he owes his brother? −$35

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students give a counterexample in order to support their reasoning in explaining why the statement is not true.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercise 16 Have students work in groups of 3–4. After completing Exercise 16, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Create your own higher-order thinking problem.

Use with Exercises 17-20 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Lesson 3-2

Subtract Integers

7.NS.A.1

LESSON GOAL

Students will solve problems subtracting integers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Algebra Tiles to Subtract Integers

Learn: Subtract Integers
Example 1: Subtract Integers
Example 2: Subtract Integers
Example 3: Subtract Expressions

Explore: Find Distance on a Number Line

Learn: Find the Distance Between Integers
Example 4: Find the Distance Between Integers
Example 5: Find the Distance Between Integers
Apply: The Solar System

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 15 of the *Language Development Handbook* to help your students build mathematical language related to subtraction of integers.

ELL You can use the tips and suggestions on page T15 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7. NS.A** by subtracting integers.
Standards for Mathematical Content: **7. NS.A.1, 7.NS.A.1.C, 7.NS.A.1.D,** *Also addresses 7.NS.A.3, 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students solved problems involving adding integers.
7.NS.A.1, 7.NS.A.1.B, 7.NS.A.1.D

Now
Students solve problems involving subtracting integers.
7.NS.A1.C

Next
Students will solve problems involving multiplying integers.
7.NS.A.2, 7.NS.A.2.A, 7.NS.A.2.C

Rigor

The Three Pillars of Rigor

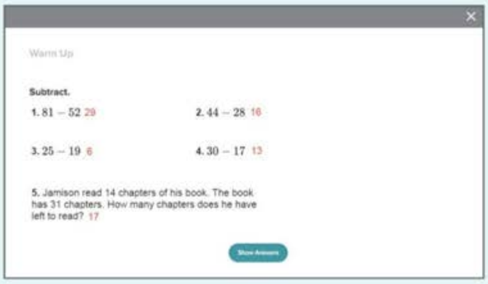
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of integers and subtraction to develop *understanding* of and build *fluency* in subtraction of integers. They will gain an understanding of finding the distance between between two integers. They *apply* their knowledge of finding the distance to real-world problems.

Mathematical Background

Addition and subtraction are inverse operations. To subtract an integer, add its additive inverse (opposite). To find the distance between two integers on a number line, find the absolute value of the difference between the two integers.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 1

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- subtracting whole numbers (Exercises 1–5)

Answers

1. 29
2. 16
3. 6
4. 13
5. 17

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about integers using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What are some synonyms for the term *absolute*? Make a conjecture as to what you think the *absolute value* of a number might be, based on what the term *absolute* means. Sample answer: Some synonyms are total, complete, universal, not in relation to other things. The absolute value of a number might mean the total value of the number.

Explore Use Algebra Tiles to Subtract Integers

Objective

Students will use algebra tiles to explore how to subtract integers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with algebra tiles representing 1 and −1. Throughout this activity, students will use the algebra tiles to subtract integers with the same sign and integers with different signs. They will see how the algebra tiles illustrate why subtracting integers is the same as adding the additive inverse.

Inquiry Question

How can you use algebra tiles to model integer subtraction?
Sample answer: By using tiles to represent positive and negative integers, integer subtraction can be modeled by taking away the number of tiles that represent the integer being subtracted. Sometimes it is necessary to add zero pairs to the workspace before taking away tiles.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

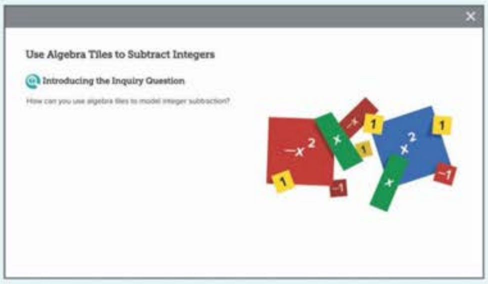
SLIDE 4

Mathematical Discourse

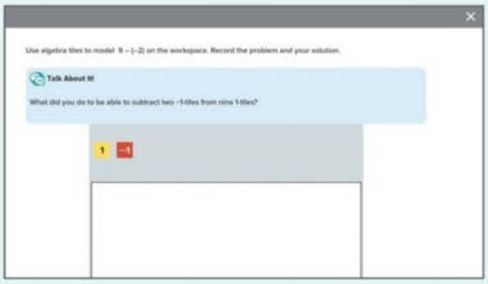
What did you do to be able to subtract two −1-tiles from nine 1-tiles?
Sample answer: I added enough zero pairs so that there were two negative 1-tiles to take away.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 9



Explore, Slide 4 of 9

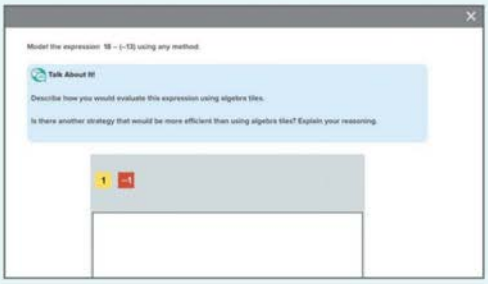
DRAG & DROP

Throughout the Explore, students drag algebra tiles to model subtracting integers.

WATCH

On Slide 3, students watch a video that explains how to subtract integers with algebra tiles.

Interactive Presentation



Explore, Slide 8 of 9

TYPE

On Slide 7, students describe the patterns they have observed.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Use Algebra Tiles to Subtract Integers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use algebra tiles to explore integer subtraction. The strategy of using algebra tiles helps build conceptual understanding for why and how subtraction of integers can be represented as addition of the additive inverse.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 8 are shown.

Talk About It!

SLIDE 8

Mathematical Discourse

Describe how you would evaluate this expression using algebra tiles. Sample answer: Model 18 by placing 18 positive 1-tiles on the workspace. I need to subtract 13 negative tiles, however there are no negative tiles on the workspace. Add 13 zero pairs to the workspace. Then I can remove 13 negative tiles from the workspace.

Is there another strategy that would be more efficient than using algebra tiles? Explain your reasoning. Sample answer: Yes, using algebra tiles is not necessarily efficient because there are so many tiles to add and subtract. It would be more efficient to evaluate the expression by adding the additive inverse.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Subtract Integers

Objective

Students will understand that they can use a number line to subtract integers.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to draw number lines and use mathematical reasoning to justify why the Commutative Property does not hold true for subtraction.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 4

Mathematical Discourse

The Commutative Property is true for addition. For example, $7 + 2 = 2 + 7$. Is the Commutative Property true for subtraction? Does $7 - 2 = 2 - 7$? Explain your reasoning using a number line. No, the Commutative Property does not hold true for subtraction. $7 - 2 = 5$, but $2 - 7 = -5$. See students' number lines.

DIFFERENTIATE

Reteaching Activity AL

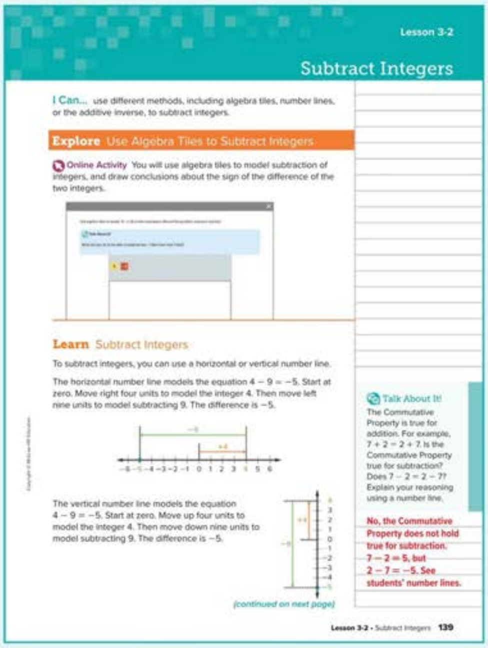
To help students better understand how to subtract integers, have them write each of the following subtraction expressions as an addition expression using an additive inverse.

$9 - (-6)$ $9 + 6$

$-6 - 3$ $-6 + (-3)$

$5 - 21$ $5 + (-21)$

$-4 - (-1)$ $-4 + 1$



Lesson 3-2

Subtract Integers

I Can... use different methods, including algebra tiles, number lines, or the additive inverse, to subtract integers.

Explore Use Algebra Tiles to Subtract Integers

Online Activity You will use algebra tiles to model subtraction of integers, and draw conclusions about the sign of the difference of the two integers.

Learn Subtract Integers

To subtract integers, you can use a horizontal or vertical number line.

The horizontal number line models the equation $4 - 9 = -5$. Start at zero. Move right four units to model the integer 4. Then move left nine units to model subtracting 9. The difference is -5.

The vertical number line models the equation $4 - 9 = -5$. Start at zero. Move up four units to model the integer 4. Then move down nine units to model subtracting 9. The difference is -5.

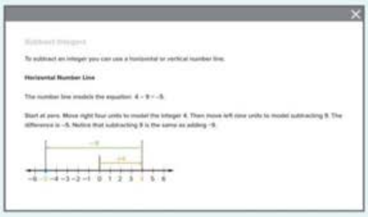
Talk About It! The Commutative Property is true for addition. For example, $7 + 2 = 2 + 7$. Is the Commutative Property true for subtraction? Does $7 - 2 = 2 - 7$? Explain your reasoning using a number line.

No, the Commutative Property does not hold true for subtraction. $7 - 2 = 5$, but $2 - 7 = -5$. See students' number lines.

(continued on next page)

Lesson 3-2 • Subtract Integers 139

Interactive Presentation



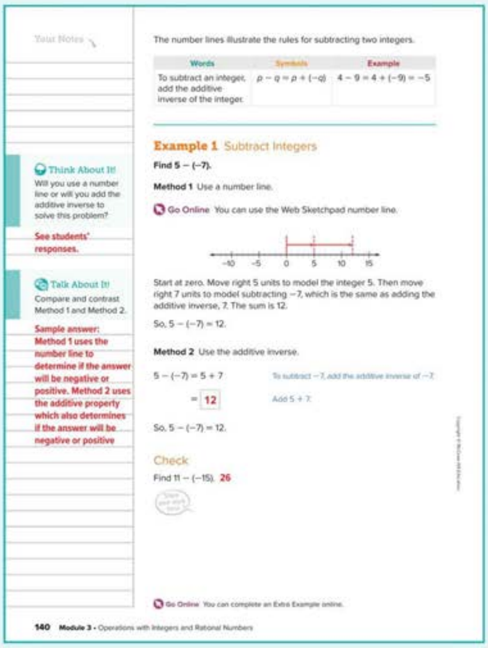
Learn, Subtract Integers, Slide 1 of 4

FLASHCARDS

On Slide 3, students use Flashcards to learn about rules for subtracting two integers.

CLICK

On Slide 3, students move through slides to see an example using the additive inverse.



Your Notes

The number lines illustrate the rules for subtracting two integers.

Words	Symbols	Example
To subtract an integer, add the additive inverse of the integer.	$p - q = p + (-q)$	$4 - 9 = 4 + (-9) = -5$

Example 1 Subtract Integers

Find $5 - (-7)$.

Method 1 Use a number line.

Go Online You can use the Web Sketchpad number line.

Start at zero. Move right 5 units to model the integer 5. Then move right 7 units to model subtracting −7, which is the same as adding the additive inverse, 7. The sum is 12.

So, $5 - (-7) = 12$.

Method 2 Use the additive inverse.

$5 - (-7) = 5 + 7$ To subtract −7, add the additive inverse of −7.

$= 12$ Add 5 + 7.

So, $5 - (-7) = 12$.

Think About It! Will you use a number line or will you add the additive inverse to solve this problem?

See students' responses.

Talk About It! Compare and contrast Method 1 and Method 2.

Sample answer: Method 1 uses the number line to determine if the answer will be negative or positive. Method 2 uses the additive property which also determines if the answer will be negative or positive

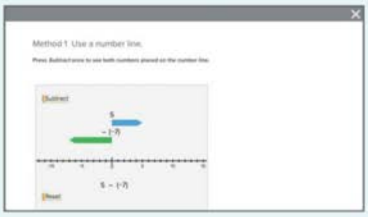
Check

Find $11 - (-15)$. **26**

Go Online You can complete an Extra Example online.

140 **Module 3 •** Operations with Integers and Rational Numbers

Interactive Presentation



Example 1, Subtract Integers, Slide 2 of 5

WEB SKETCHPAD

On Slide 2, students use Web Sketchpad to find the difference with a number line (Method 1).

CLICK

On Slide 3, students use the additive inverse to find the difference (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Subtract Integers

Objective

Students will subtract a negative integer from a positive integer.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use either method, a number line or the additive inverse, when subtracting two integers. As students discuss the *Talk About It!* question on Slide 4, encourage them to understand the benefits of each method and how they are related.

Questions for Mathematical Discourse

SLIDE 2

AL Are we subtracting a positive integer from 5, or a negative integer? We are subtracting a negative integer, −7, from 5.

OL After pressing *Subtract*, how does the number line illustrate how to subtract a negative integer? Sample answer: The number line shows that subtracting a negative integer is the same as adding the integer's additive inverse.

BL How would the number line change if the expression was 5 + (–7)? Sample answer: Instead of subtracting a negative number, we would add a negative number. Adding a negative number would move to the left on the number line, instead of to the right.

SLIDE 3

AL Of which integer do we find the additive inverse? Explain. We find the additive inverse of the second integer, –7, because that is the integer that is being subtracted.

OL Explain why it makes sense that the answer is positive. Sample answer: If I modeled this expression using algebra tiles, I would need to subtract 7 negative tiles from 5 positive tiles. To do so, I would need to add 7 zero pairs. After removing all 7 negative tiles, only positive tiles remain.

BL Compare and contrast the expressions 5 – (–7), 5 + (–7), and 5 – 7. Sample answer: Two of the expressions are subtraction expressions, 5 – (–7) and 5 – 7. The other expression, 5 + (–7) is an addition expression. In the first two expressions, the second integer is negative. In the last expression, the second integer is positive.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Subtract Integers

Objective

Students will subtract a negative integer from a negative integer.

Questions for Mathematical Discourse

SLIDE 2

AL What is the additive inverse of −17? 17

AL Rewrite the subtraction expression as an addition expression. −24 + 17

OL What other method could you use to find the difference? Sample answer: Use a number line.

BL If the first integer remained the same, what would the second integer need to be in order for the difference to be the least positive integer possible? −25

Example 3 Subtract Expressions

Objective

Students will evaluate an algebraic expression that involves subtracting integers.

Questions for Mathematical Discourse

SLIDE 1

AL What integer should replace x in the expression? −23

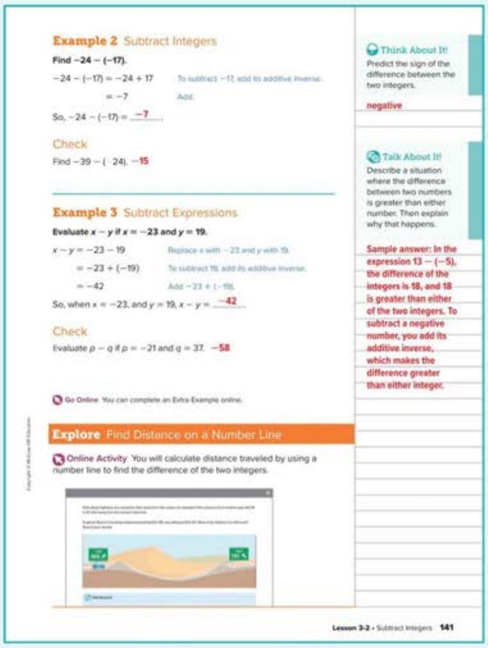
AL What integer should replace y in the expression? 19

OL Suppose a classmate substituted the values and wrote the expression −23 − (−19). How can you explain to them their error? Sample answer: The second integer is positive 19, not negative 19.

BL How would the answer change if the original expression was $y - x$? The answer would be 19 − (−23), which equals 42.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.


Example 2 Subtract Integers

Find −24 − (−17).

−24 − (−17) = −24 + 17 — To subtract −17, add its additive inverse.

= −7 — Add.

So, −24 − (−17) = −7.

Check

Find −39 − (−24). −15

Example 3 Subtract Expressions

Evaluate $x - y$ if $x = -23$ and $y = 19$.

$x - y = -23 - 19$ — Replace x with −23 and y with 19.

$= -23 + (-19)$ — To subtract 19, add its additive inverse.

$= -42$ — Add −23 + (−19).

So, when $x = -23$, and $y = 19$, $x - y =$ −42.

Check

Evaluate $p - q$ if $p = -21$ and $q = 37$. −58

Go Online You can complete an Extra Example online.

Explore Find Distance on a Number Line

Online Activity You will calculate distance traveled by using a number line to find the difference of the two integers.

Think About It! Predict the sign of the difference between the two integers.

negative

Talk About It! Describe a situation where the difference between two numbers is greater than either number. Then explain why that happens.

Sample answer: In the expression 13 − (−5), the difference of the integers is 18, and 18 is greater than either of the two integers. To subtract a negative number, you add its additive inverse, which makes the difference greater than either integer.

Lesson 3-2 • Subtract Integers 141

Interactive Presentation

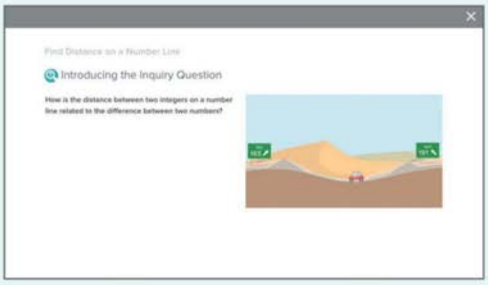
Example 2, Subtract Integers, Slide 2 of 4

CLICK On Slide 2 of Example 2, students move through the steps to find the difference.

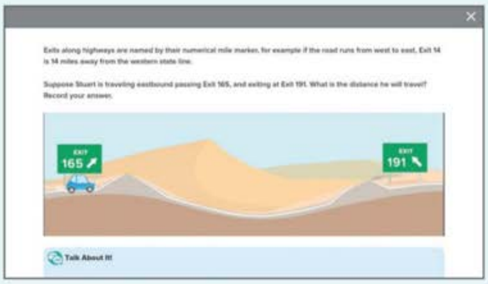
TYPE On Slide 1 of Example 3, students evaluate the expression.

CHECK Students complete the Check exercises online to determine if they are ready to move on.

Interactive Presentation



Explore, Slide 1 of 9



Explore, Slide 2 of 9

Explore Find Distance on a Number Line

Objective

Students will explore how the distance between integers on a number line is related to their difference.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a number line to find the distance that a car travels from one exit to another. Throughout this activity, students will write subtraction expressions to find the difference between two integers on a number line. They will compare these differences to the actual distances between the two numbers on the number line. They should note that, while the difference of a subtraction expression might be negative, the *distance* between those integers is always positive.

Inquiry Question

How is the distance between two integers on a number line related to the difference between the two numbers? Sample answer: The distance between two rational numbers is the absolute value of their difference. For example, the distance between −88 and −11 is 77 units.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Describe how you calculated the distance traveled. Sample answer: I found the difference 191 − 165. Stuart traveled 26 miles.

(continued on next page)

Explore Find Distance on a Number Line *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to explore the distance between two integers on a number line, and analyze how the distance compares to the difference of the subtraction expression.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

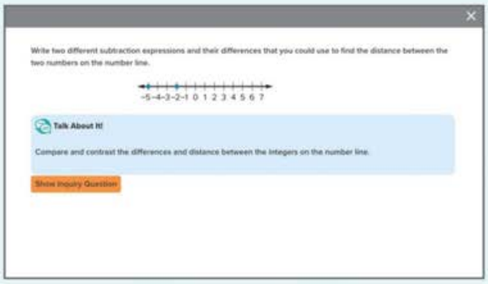
Talk About It!

SLIDE 6

Mathematical Discourse

Compare and contrast the differences and distance of the integers on the number line. Sample answer: The differences in the values of the expressions are opposites; $-2 - (-5) = 3$ and $-5 - (-2) = -3$. But the distance between the two integers is the same, 3 units.

Interactive Presentation



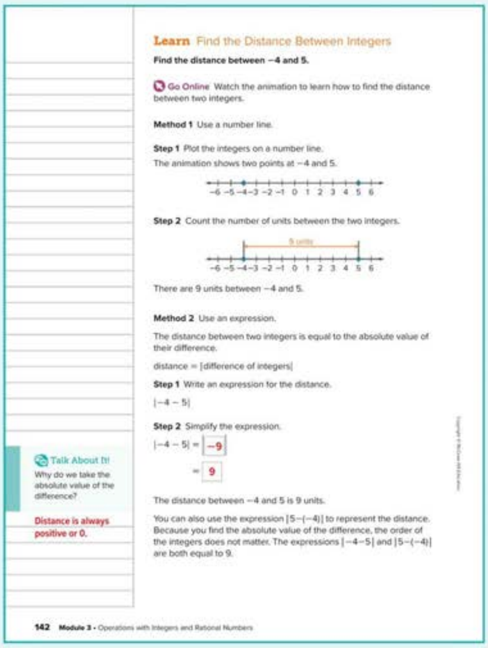
Explore, Slide 6 of 9

TYPE

On Slide 7, students type to make a conjecture about how the distance between two integers on a number line is related to their difference.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.



Learn Find the Distance Between Integers

Find the distance between −4 and 5.

Go Online Watch the animation to learn how to find the distance between two integers.

Method 1 Use a number line.

Step 1 Plot the integers on a number line.

The animation shows two points at −4 and 5.

Step 2 Count the number of units between the two integers.

There are 9 units between −4 and 5.

Method 2 Use an expression.

The distance between two integers is equal to the absolute value of their difference.

distance = |difference of integers|

Step 1 Write an expression for the distance.

$|-4-5|$

Step 2 Simplify the expression.

$|-4-5| = |-9|$

$= 9$

The distance between −4 and 5 is 9 units.

You can also use the expression $|5-(-4)|$ to represent the distance. Because you find the absolute value of the difference, the order of the integers does not matter. The expressions $|-4-5|$ and $|5-(-4)|$ are both equal to 9.

Talk About It! Why do we take the absolute value of the difference?

Distance is always positive or 0.

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Interactive Presentation

Learn, Find the Distance Between Integers, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that explains how to find the distance between two integers on a number line.

Learn Find the Distance Between Integers

Objective

Students will learn how to find the distance between two integers on a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of quantities and their relationships in problem situations. Students should recognize that distance is always positive.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to find the distance between two integers on a number line.

Talk About It!

SLIDE 2

Mathematical Discourse

Why do we take the absolute value of the difference? Distance is always positive or 0.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of distance between integers, have them find the integer(s) that satisfy each of the following descriptions.

8 units from 3 −5, 11

6 units from −2 −8, 4

10 units from −22 −32, −12

Example 4 Find the Distance Between Integers

Objective

Students will find the distance between two integers on a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the integers given in the example and the distance between them, whether they use a number line to find the distance or absolute value.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to communicate precisely the similarities and differences of the two methods.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the distance between –9 and 8

OL How many units are between the integers? 17 units

BL What is the difference of the expression –9 – 8? How does this compare to the distance between the integers? The difference is –17, but the distance between the integers is positive.

SLIDE 3

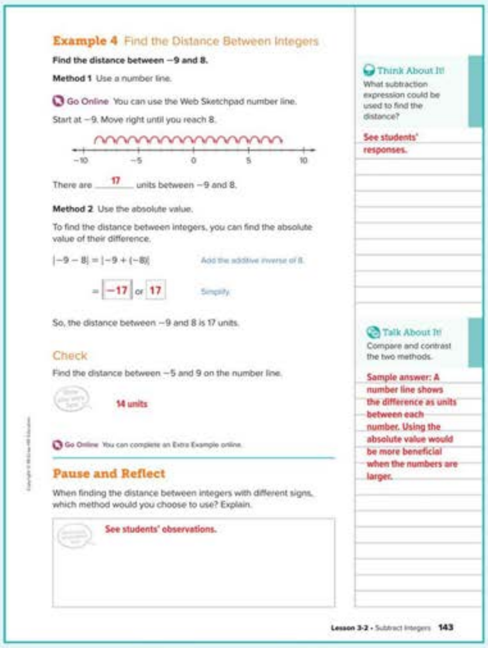
AL What is the absolute value of each integer? The absolute value of –9 is 9. The absolute value of 8 is 8.

OL Why do you need to find the absolute value of the difference? Distance cannot be negative.

BL Give an example of two integers, on opposite sides of zero, where the distance between them is 25? Sample answer: 15 and –10

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Find the Distance Between Integers

Find the distance between –9 and 8.

Method 1 Use a number line.

Go Online You can use the Web Sketchpad number line.

Start at –9. Move right until you reach 8.

There are 17 units between –9 and 8.

Method 2 Use the absolute value.

To find the distance between integers, you can find the absolute value of their difference.

$|-9 - 8| = |-9 + (-8)|$ Add the additive inverse of 8.

$= |-17|$ or 17 Simplify.

So, the distance between –9 and 8 is 17 units.

Check

Find the distance between –5 and 9 on the number line.

14 units

Go Online You can complete an Extra Example online.

Pause and Reflect

When finding the distance between integers with different signs, which method would you choose to use? Explain.

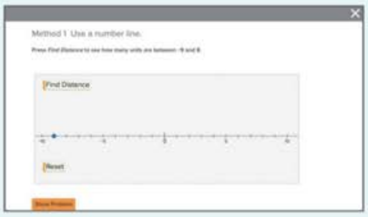
See students' observations.

Think About It! What subtraction expression could be used to find the distance? See students' responses.

Talk About It! Compare and contrast the two methods. Sample answer: A number line shows the difference as units between each number. Using the absolute value would be more beneficial when the numbers are larger.

Lesson 3-2 • Subtract Integers 143

Interactive Presentation



Example 4, Find the Distance Between Integers, Slide 2 of 5

WEB SKETCHPAD

On Slide 2, students use Web Sketchpad to find the distance with a number line (Method 1).

TYPE

On Slide 3, students use absolute value to find the distance (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Find the Distance Between Integers

The highest point in California is Mount Whitney with an elevation of 14,494 feet. The lowest point is Death Valley with an elevation of −282 feet.

What is the distance between the height of Mount Whitney and the depth of Death Valley?

$|14{,}494 - (-282)| = |14{,}494 + 282|$ To subtract −282, add its additive inverse.

$= |14{,}776|$ Add.

$= 14{,}776$ Find the absolute value.

So, the distance between the two points is 14,776 feet.

Think About It! Will the distance be greater or less than 14,494 feet? greater than

Talk About It! Is it reasonable to have a negative answer? Why or why not? No, because distance is always positive or 0.

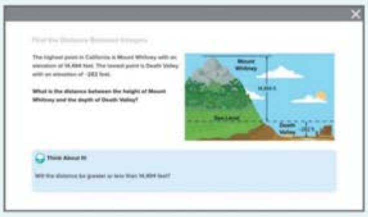
Check

The top of an iceberg is 55 feet above sea level, while the bottom is 385 feet below sea level. What is the distance between the top and bottom of the iceberg? 440 feet

Go Online You can complete an Extra Example online.

144 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Example 5, Find the Distance Between Integers, Slide 1 of 4

CLICK

On Slide 2, students move through the steps to find the absolute value of the difference between two integers.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Find the Distance Between Integers

Objective

Students will find the distance between two integers to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to use the mathematics they know, finding the distance between two integers, to solve the real-world problem and to make sure their answer makes sense in the context of the problem.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to think logically as they reason about whether a negative answer makes sense.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the distance between the elevations of Mount Whitney and Death Valley

AL What does it mean that the elevation of Death Valley is a negative integer? The elevation of Death Valley is below sea level.

OL Why do we find the absolute value of the difference? Distance cannot be negative.

BL Suppose a classmate stated that the distance between the elevations is 14,212 feet. How can you explain to them that their answer is not reasonable? Sample answer: Mount Whitney is above sea level and Death Valley is below sea level. The distance between them must be greater than either elevation.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply The Solar System

Objective

Students will come up with their own strategy to solve an application problem involving temperature of celestial objects.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the Write About It! prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

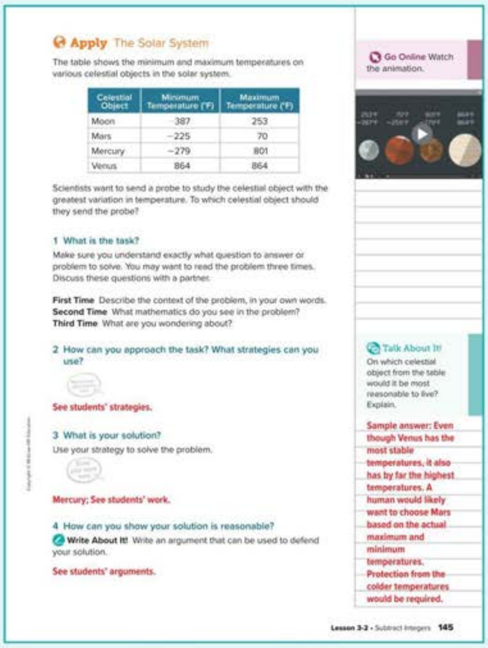
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does variation mean?
- What do you notice about Venus' temperatures?
- How might thinking about 0°F help you?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply The Solar System

The table shows the minimum and maximum temperatures on various celestial objects in the solar system.

Celestial Object	Minimum Temperature (°F)	Maximum Temperature (°F)
Moon	−387	253
Mars	−225	70
Mercury	−279	801
Venus	864	864

Scientists want to send a probe to study the celestial object with the greatest variation in temperature. To which celestial object should they send the probe?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Mercury; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It! On which celestial object from the table would it be most reasonable to live? Explain.

Sample answer: Even though Venus has the most stable temperatures, it also has by far the highest temperatures. A human would likely want to choose Mars based on the actual maximum and minimum temperatures. Protection from the colder temperatures would be required.

Lesson 3-2 • Subtract Integers 145

Interactive Presentation

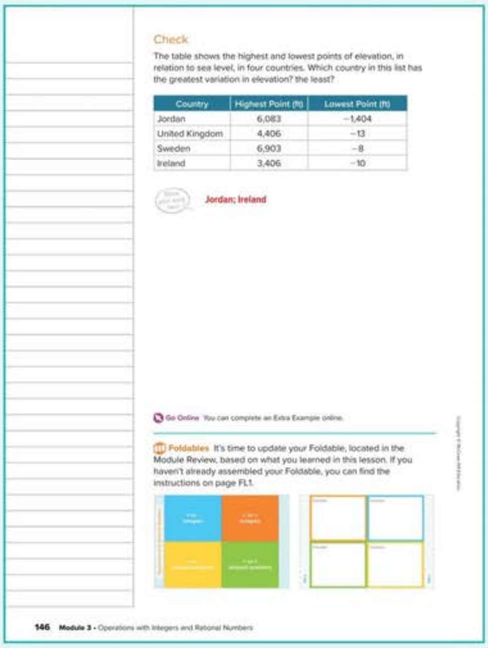
Apply, The Solar System

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows the highest and lowest points of elevation, in relation to sea level, in four countries. Which country in this list has the greatest variation in elevation? the least?

Country	Highest Point (ft)	Lowest Point (ft)
Jordan	6,083	−1,404
United Kingdom	4,406	−13
Sweden	6,903	−8
Ireland	3,406	−10

Jordan; Ireland

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

146 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of subtracting integers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with rational numbers related to operations with integers?

In this lesson, students learned how to subtract integers by adding the additive inverse. Encourage them to work with a partner to compare and contrast subtracting integers to subtracting whole numbers. For example, have them compare and contrast how they would simplify each of the expressions $-15 - 7$, $-15 - (-7)$, $15 - (-7)$, and $15 - 7$.

Exit Ticket

Refer to the Exit Ticket slide. New Orleans is 8 feet below sea level and Britton Hill is 345 feet above sea level. How far apart are the elevations? Explain how to find the distance between the elevations. Write a mathematical argument that can be used to defend your solution.

353 feet; Sample answer: Find the absolute value of the difference of the elevations; $|345 - (-8)| = 353$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 15, 17, 18–21
- ALEKS Addition and Subtraction with Integers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–14, 16, 19
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS Plotting and Comparing Integers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Integers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

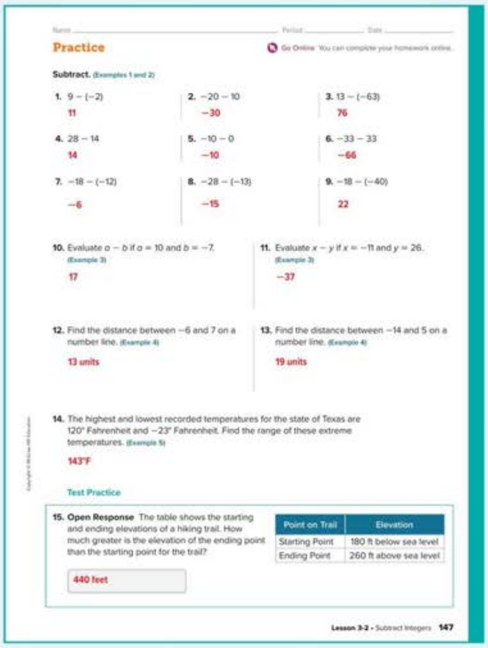
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	subtract integers	1–9
1	evaluate algebraic expressions involving subtraction	10, 11
1	find the distance between two integers on a number line	12, 13
2	find the distance between two integers to solve a real-world problem	14
2	extend concepts learned in class to apply them in new contexts	15
3	solve application problems that involve subtracting integers	16, 17
3	higher-order and critical thinking skills	18–21

Common Misconception

Students may have trouble identifying the sign of the difference when subtracting negative integers. In Exercise 7, students may recognize that the distance between –18 and –12 is 6, but fail to realize that subtracting –12 from –18 results in –6, not 6.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Subtract. (Examples 1 and 2)

1. 9 – (–2) **11**
2. –20 – 10 **–30**
3. 13 – (–63) **76**
4. 28 – 14 **14**
5. –10 – 0 **–10**
6. –33 – 33 **–66**
7. –18 – (–12) **–6**
8. –28 – (–13) **–15**
9. –18 – (–40) **22**

10. Evaluate $a - b$ if $a = 10$ and $b = -7$. (Example 3) **17**

11. Evaluate $x - y$ if $x = -11$ and $y = 26$. (Example 3) **–37**

12. Find the distance between –6 and 7 on a number line. (Example 4) **13 units**

13. Find the distance between –14 and 5 on a number line. (Example 4) **19 units**

14. The highest and lowest recorded temperatures for the state of Texas are 120° Fahrenheit and –23° Fahrenheit. Find the range of these extreme temperatures. (Example 5) **143°F**

Test Practice

15. **Open Response** The table shows the starting and ending elevations of a hiking trail. How much greater is the elevation of the ending point than the starting point for the trail?

Point on Trail	Elevation
Starting Point	180 ft below sea level
Ending Point	260 ft above sea level

440 feet

Lesson 3-2 • Subtract Integers 147

Apply *indicates multi-step problem

*16. The table shows the maximum and minimum account balances for three college students for one month. Giovanni claimed that he had the least variation (from maximum to minimum) in his account balance that month. Is he correct? Write a mathematical argument to justify your solution.

Giovanni is correct; Sample answer: Giovanni's variation is \$168 − \$15, or \$153. Jordan's variation is \$145 − (−\$25), or \$170. Elisa's variation is \$152 − (−\$10), or \$162.

Student	Maximum Balance (\$)	Minimum Balance (\$)
Jordan	145	−25
Giovanni	168	15
Elisa	152	−10

*17. The table shows the record high and record low temperatures for certain U.S. states. Which state in the list had the greatest variation in temperature? the least?

Utah; Nevada

State	Record High Temperature (°F)	Record Low Temperature (°F)
Alaska	100	−80
Idaho	118	−60
Nevada	125	−50
Utah	117	−69

Higher-Order Thinking Problems

18. MP **Use a Counterexample** Determine if each statement is *true* or *false*. If false, provide a counterexample.

a. Distance is always positive.
true

b. Change is always positive.
false; Change can be positive or negative. For example, the temperature dropping 2°F would be represented by a −2.

19. MP **Find the Error** A student is finding $4 - (-2)$. Find the student's mistake and correct it.

$$4 - (-2) = 4 - 2$$
$$= 2$$

The student incorrectly wrote 4 − 2 instead of 4 + 2. The correct solution is 6.

20. **Create** Write a subtraction expression with a positive and negative integer whose difference is negative. Then find the difference.

Sample answer: −3 − 2; −5

21. If you subtract two negative integers, will the difference *always*, *sometimes*, or *never* be negative? Explain using examples to justify your solution.

sometimes; Sample answer: For example, −10 − (−40) = 30 and −28 − (−13) = −15.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students use a counterexample if a statement is false.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 19, students find the error in another student's reasoning and correct it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 16–17 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 17 might be, "How do you find the variation in temperature?"

Listen and ask clarifying questions.
Use with Exercises 20–21 Have students work in pairs. Have students individually read Exercise 20 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 21.

Lesson 3-3

Multiply Integers

7.NS.A.2

LESSON GOAL

Students will solve problems multiplying integers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Algebra Tiles to Multiply Integers

Learn: Multiply Integers with Different Signs
Example 1: Multiply Integers with Different Signs
Example 2: Multiply Integers with Different Signs
Learn: Multiply Integers with the Same Sign
Example 3: Multiply Integers with the Same Sign
Example 4: Multiply Integers with the Same Sign
Example 5: Multiply Three or More Integers
Example 6: Multiply Three or More Integers
Learn: Use Properties to Multiply Integers
Apply: Agriculture

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Powers and Negatives		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 16 of the *Language Development Handbook* to help your students build mathematical language related to multiplcation of integers.

ELL You can use the tips and suggestions on page T16 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7. NS.A** by multiplying integers.
Standards for Mathematical Content: **7 .NS.A.2, 7.NS.A.2.A, 7.NS.A.2.C, 7.EE.B.3,** *Also addresses 7.NS.A.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students solved problems involving subtracting integers.
7.NS.A.1, 7.NS.A.1.C

Now
Students solve problems involving multiplying integers.
7.NS.A.2, 7.NS.A.2.A, 7.NS.A.2.C

Next
Students will solve problems involving dividing integers.
7.NS.A.2, 7.NS.A.2.B

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of the multiplication of whole numbers to develop *understanding* of multiplication of integers. They build *fluency* by multiplying two integers with different signs, multiplying two integers with the same signs, and multiplying groups of 3 or more integers.

Mathematical Background

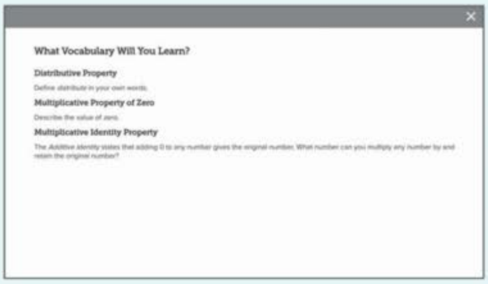
Multiplication can be expressed as repeated addition. By doing so, the following rules for multiplying integers can be developed.

- If the two integers have the same sign, the product is positive.
- If the two integers have different signs, the product is negative.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- multiplying whole numbers (Exercises 1–5)

Answers

1. 2,000 **4.** 39

2. 120 **5.** 72

3. 136

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about hair growth and hair loss.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Define *distribute* in your own words. Sample answer: Distribute means to divide, give out, or pass out, a group of items among several members.
- Describe the value of *zero*. Sample answer: Zero represents the absence of quantity, or nothing.
- The *Additive Identity* states that adding 0 to any number gives the original number. What number can you multiply any number by and retain the original number? Any number multiplied by 1 retains the original number.

Explore Use Algebra Tiles to Multiply Integers

Objective

Students will use algebra tiles to explore how to multiply integers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with algebra tiles representing 1 and −1. Throughout this activity, students will use the algebra tiles to model the product of two integers. Students will use their observations to make conjectures about multiplying integers when one or both of the integers are negative.

Inquiry Question

How can you determine the sign of the product of two integers? Sample answer: The sign of the product of two integers can be determined by modeling the multiplication with algebra tiles and observing the sign of the tiles in the end product.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

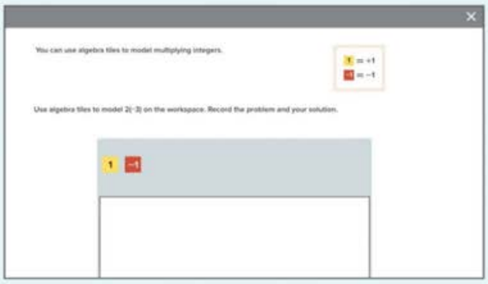
Mathematical Discourse

What does the expression 2(−3) mean? How can you model 2(−3) using algebra tiles? Sample answer: It means two sets of −3 tiles. I can model this by placing two sets of three negative 1-tiles on the workspace.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 2 of 8

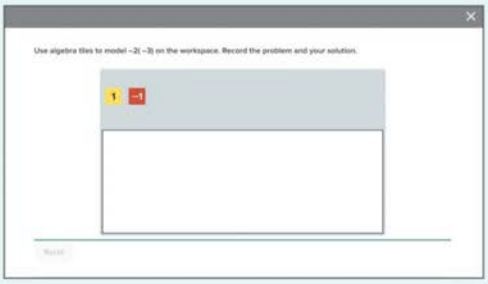
DRAG & DROP

Throughout the Explore, students drag algebra tiles to model integer multiplication.

WATCH

On Slide 3, students watch a video that explains how to multiply integers with algebra tiles.

Interactive Presentation



Explore, Slide 5 of 8

TYPE

On Slide 7, students make a conjecture about the product of two integers when one or both are negative.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Use Algebra Tiles to Multiply Integers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use algebra tiles to explore integer multiplication, and to make conjectures about multiplying integers when one or both of the integers are negative.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What steps did you take to simplify the expression? Sample answer: Added two sets of three zero pairs, and then removed two sets of three −1-tiles from the workspace.

How is this expression, −2(−3), different from the expression you just simplified, −2(3)? Sample answer: In the previous expression, only one integer was negative. In this expression, both integers are negative.

Learn Multiply Integers with Different Signs

Objective

Students will understand how a number line and repeated addition can be used to multiply integers with different signs.

Teaching Notes

SLIDE 1

Students will learn how to multiply integers with different signs using a number line. Students should understand how the number line illustrates that the product of the expression 3(−6) is a negative number, −18. Have students explain how the number line illustrates repeated addition of the negative addend, −6.

SLIDE 2

Have students select the *Words* and *Examples* flashcards to show how multiplying integers with different signs can be described using these multiple representations. When multiplying integers, students may have difficulty remembering how to determine the sign of the product. Encourage them to draw a number line or write the product as a repeated addition expression to verify that the product of two integers with different signs is negative.

DIFFERENTIATE

Reteaching Activity

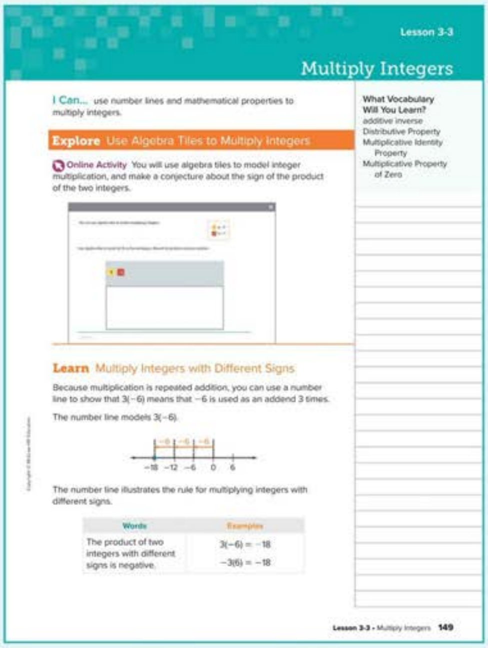
Some students may struggle to interpret a product as a number of groups. When multiplying two integers with different signs, explain that the positive integer can be viewed as a number of groups, just as in multiplication with whole numbers. The negative integer represents the number in each group. This can help students to more easily approach and make sense of multiplication problems. For each of the following, have students interpret the product using positive groups of negative integers.

3(−6) 3 groups of −6

4(−2) 4 groups of −2

−5(8) 8 groups of −5

−2(3) 3 groups of −2


Lesson 3-3

Multiply Integers

I Can... use number lines and mathematical properties to multiply integers.

What Vocabulary Will You Learn?
additive inverse
Distributive Property
Multiplicative Identity Property
Multiplicative Property of Zero

Explore Use Algebra Tiles to Multiply Integers

Online Activity You will use algebra tiles to model integer multiplication, and make a conjecture about the sign of the product of the two integers.

Learn Multiply Integers with Different Signs

Because multiplication is repeated addition, you can use a number line to show that 3(−6) means that −6 is used as an addend 3 times.

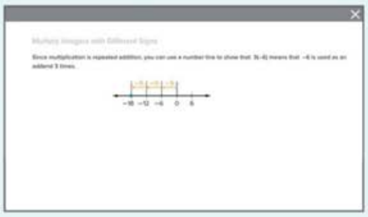
The number line models 3(−6).

The number line illustrates the rule for multiplying integers with different signs.

Words	Examples
The product of two integers with different signs is negative.	3(−6) = −18 −3(6) = −18

Lesson 3-3 • Multiply Integers 149

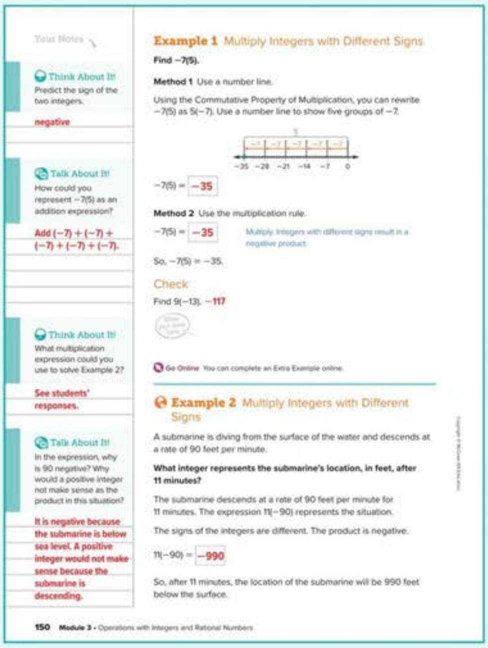
Interactive Presentation



Learn, Multiply Integers with Different Signs, Slide 1 of 2

FLASHCARDS

On Slide 2, students use Flashcards to view multiplying integers with different signs.



Your Notes

Think About It! Predict the sign of the two integers.

negative

Talk About It! How could you represent −7(5) as an addition expression?

Add (−7) + (−7) + (−7) + (−7) + (−7).

Think About It! What multiplication expression could you use to solve Example 2?

See students' responses.

Talk About It! In the expression, why is 90 negative? Why would a positive integer not make sense as the product in this situation?

It is negative because the submarine is below sea level. A positive integer would not make sense because the submarine is descending.

Example 1 Multiply Integers with Different Signs

Find −7(5).

Method 1 Use a number line.

Using the Commutative Property of Multiplication, you can rewrite −7(5) as 5(−7). Use a number line to show five groups of −7.

−35 −28 −21 −14 −7 0

−7(5) = −35

Method 2 Use the multiplication rule.

−7(5) = −35 Multiply integers with different signs result in a negative product.

So, −7(5) = −35.

Check

Find 9(−13). −117

Go Online You can complete an Extra Example online.

Example 2 Multiply Integers with Different Signs

A submarine is diving from the surface of the water and descends at a rate of 90 feet per minute.

What integer represents the submarine's location, in feet, after 11 minutes?

The submarine descends at a rate of 90 feet per minute for 11 minutes. The expression 11(−90) represents the situation.

The signs of the integers are different. The product is negative.

11(−90) = −990

So, after 11 minutes, the location of the submarine will be 990 feet below the surface.

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Interactive Presentation

Example 1, Multiply Integers with Different Signs, Slide 2 of 5

eTOOL

On Slide 2 of Example 1, students use the Number Line eTool to see 5 groups of −7.

CLICK

On Slide 2 of Example 2, students determine the sign of a product.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Multiply Integers with Different Signs

Objective

Students will multiply integers with different signs.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Encourage students to use the structure of the number line to illustrate the general rules for finding the product of two integers with different signs.

8 Look For and Express Regularity in Repeated Reasoning As students discuss the *Talk About It!* question on Slide 4, encourage them to understand how multiplication can be expressed as repeated addition.

Example 2 Multiply Integers with Different Signs

Objective

Students will multiply integers with different signs to solve a real-world problem.

Questions for Mathematical Discourse

SLIDE 2

AL What integer represents *descending at a rate of 90 feet per minute*? −90

AL For how long is the submarine descending? 11 minutes

OL Explain why using a number line or repeated addition is not the most efficient way to find this product. Sample answer: I would have to draw a number line out to almost −1,000, or I would have to add −90 eleven times.

BL After the submarine reached a depth of –900 feet, suppose it ascends 45 feet every 6 minutes for 1 hour. What will be the new depth of the submarine? How did you solve the problem? Sample answer: The submarine will be at −450 feet. Simplify the expression −900 + 45(10) since there are 10 sets of 6 minutes in 1 hour.

Go Online

- Find additional teaching notes, discussion questions, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Multiply Integers with the Same Sign

Objective

Students will use patterns to understand that the product of two integers with the same sign is positive.

Teaching the Mathematical Practices

8 Look For and Express Regularity in Repeating Reasoning
Encourage students to understand the pattern illustrated by the steps in the animation. As the first factor decreases by 1, the product increases by 3. Continuing this pattern to negative numbers shows that the product of two negative numbers is positive.

Go Online

Have students watch the animation on Slide 1. The animation illustrates how to multiply integers with the same sign.

Teaching Notes

SLIDE 1

You may wish to pause the animation after each step to be sure students understand the pattern. They previously learned that the product of a positive integer and a negative integer is negative, as illustrated by the sentence $(2)(-3) = -6$. As each new sentence is written, point out that the first factor decreases by 1 each time, and the product increases by 3 each time. By continuing the pattern to where the first factor is negative, students should see that the product is now positive.

SLIDE 2

Students will learn how to represent the rule for multiplying integers with the same sign using words and examples. You may wish to have a student volunteer select the flashcards and explain how the examples illustrate the words.

DIFFERENTIATE

Enrichment Activity BL

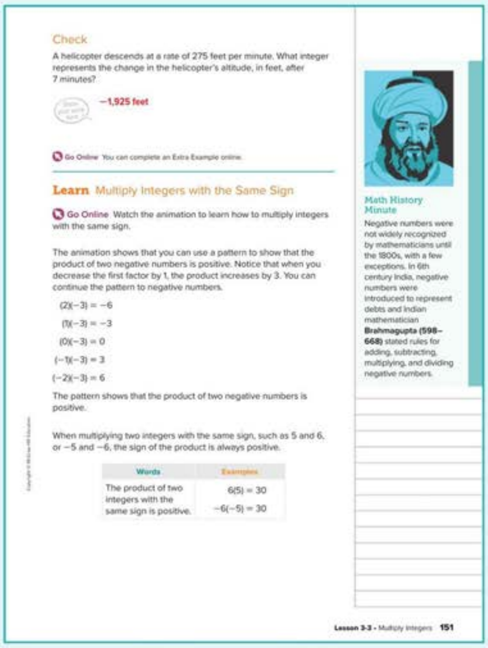
To challenge students' understanding of integer multiplication, have them find the missing factors for each of the following multiplication statements. Have them justify their reasoning. A sample justification for the first one is shown.

$\underline{-2}\,(3) = -6$ In order for the product of a positive integer and another integer to be negative, the other integer must be negative.

$\underline{-4}\,(-5) = 20$

$\underline{2}\,(-7) = -14$

$\underline{4}\,(6) = 24$



Check

A helicopter descends at a rate of 275 feet per minute. What integer represents the change in the helicopter's altitude, in feet, after 7 minutes?

−1,925 feet

Go Online You can complete an Extra Example online.

Learn Multiply Integers with the Same Sign

Go Online Watch the animation to learn how to multiply integers with the same sign.

The animation shows that you can use a pattern to show that the product of two negative numbers is positive. Notice that when you decrease the first factor by 1, the product increases by 3. You can continue the pattern to negative numbers.

$(2)(-3) = -6$
$(1)(-3) = -3$
$(0)(-3) = 0$
$(-1)(-3) = 3$
$(-2)(-3) = 6$

The pattern shows that the product of two negative numbers is positive.

When multiplying two integers with the same sign, such as 5 and 6, or −5 and −6, the sign of the product is always positive.

Words	Examples
The product of two integers with the same sign is positive.	$6(5) = 30$ $-6(-5) = 30$

Math History Minute

Negative numbers were not widely recognized by mathematicians until the 1800s, with a few exceptions. In 6th century India, negative numbers were introduced to represent debts and Indian mathematician **Brahmagupta (598–668)** stated rules for adding, subtracting, multiplying, and dividing negative numbers.

Lesson 3-3 • Multiply Integers 151

Interactive Presentation

Learn, Multiply Integers with the Same Sign, Slide 2 of 2

WATCH

On Slide 1, students watch an animation that illustrates the rule for multiplying two negative integers.

FLASHCARDS

On Slide 2, students use Flashcards to view the rule for multiplying integers with the same sign.

Example 3 Multiply Integers with the Same Sign

Find $-8(-9)$.

$-8(-9) = 72$ The signs of the integers are the same. The product is positive.

So, the product of $-8(-9)$ is 72.

Check

Find $-5(-13)$. 65

Example 4 Multiply Integers with the Same Sign

Evaluate xy if $x = -14$ and $y = -7$.

$xy = -14(-7)$ Replace x with -14 and y with -7.

$= 98$ The signs of the integers are the same. The product is positive.

So, the value of the expression is 98.

Check

Evaluate pq if $p = -16$ and $q = -7$. 112

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4, Multiply Integers with the Same Sign, Slide 1 of 2

CLICK

On Slide 1 of Example 3, students select from drop-down menus to find the product of two integers.

CLICK

On Slide 1 of Example 4, students move through the steps to evaluate the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 3 Multiply Integers with the Same Sign

Objective

Students will multiply integers with the same sign.

Questions for Mathematical Discourse

SLIDE 1

AL Do the integers have the same sign? yes

AL What is the sign of both integers? negative

OL Why is the product not negative? Sample answer: The integers have the same sign. The product of two integers with the same sign is positive.

BL Generate your own multiplication expression involving two negative numbers in which the product is 72. Sample answer: $-2(-36)$

Example 4 Multiply Integers with the Same Sign

Objective

Students will evaluate an algebraic expression that involves multiplying integers.

Questions for Mathematical Discourse

SLIDE 1

AL Do the values of x and y have the same sign? yes

AL What is the sign of both integers? negative

OL Is $xy = yx$? Explain how you know, without calculating. yes; Sample answer: The Commutative Property states that you can multiply two numbers in any order and the product remains the same.

BL Find $xy + 2y$. 84

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present the Extra Examples.

Example 5 Multiply Three or More Integers

Objective

Students will multiply three or more integers.

Questions for Mathematical Discourse

SLIDE 2

AL How many factors are there? three

AL Do all three integers have the same sign? yes

OL Why is the product of three negative integers negative? Sample answer: The product of the first two negative integers is positive. The product of the positive result and the remaining negative integer will be negative.

BL What do you think the sign of the product of four negative integers will be? five negative integers? *n* negative integers? Sample answer: The product of four negative integers will be positive. The product of five negative integers will be negative. The product of *n* negative integers will be positive if *n* is even, and negative if *n* is odd.

Example 6 Multiply Three or More Integers

Objective

Students will evaluate an algebraic expression that involves multiplying three or more integers.

Questions for Mathematical Discourse

SLIDE 1

AL What is the first step in finding the product? Substitute the values for the variables.

AL What is the next step? Evaluate the power.

OL Without calculating, how can you determine the sign of the final product? Sample answer: b^2 will be positive. The two remaining integers are negative, so their product will be positive. The product of a positive number and a positive number is positive.

BL Write and simplify a multiplication expression involving *a*, *b*, and *c*, in which the product is negative. Sample answer: a^3bc ; −40,500

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.

Example 5 Multiply Three or More Integers

Find −4(−7)(−2).

$-4(-7)(-2) = [-4(-7)](-2)$ Associative Property

$= 28(-2)$ Multiply (−4)(−7).

$= -56$ Multiply 28(−2).

So, the product of −4(−7)(−2) is −56.

Check

Find −3(−11)(−3). −99

Example 6 Multiply Three or More Integers

Evaluate ab^2c when $a = -5$, $b = 4$, and $c = -9$.

$ab^2c = -5(4)^2(-9)$ Substitute −5 for *a*, 4 for *b*, and −9 for *c*.

$= -5(16)(-9)$ Multiply 4 · 4.

$= [-5(16)](-9)$ Associative Property

$= -80(-9)$ Multiply −5(16).

$= 720$ Multiply −80(−9).

So, the value of the expression is 720.

Check

Evaluate *pqrs* if $p = -7$, $q = 15$, $r = 1$, and $s = -2$. 210

Think About It! What do you notice about the expression? See students' responses.

Talk About It! How can you tell, without multiplying, if the product will be positive or negative? Sample answer: When the number of negative factors is even, the product will be positive. When the number of negative factors is odd, the product will be negative.

Go Online You can complete an Extra Example online.

Lesson 3-3 • Multiply Integers 153

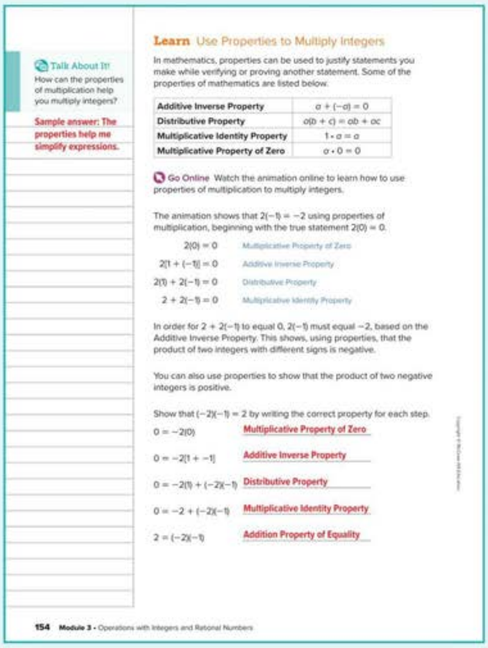
Interactive Presentation

Example 5, Multiply Three or More Integers, Slide 2 of 4

CLICK On Slide 2 of Example 5, students move through the steps to find the product.

TYPE On Slide 1 of Example 6, students evaluate the expression.

CHECK Students complete the Check exercises online to determine if they are ready to move on.



Talk About It! How can the properties of multiplication help you multiply integers?

Sample answer: The properties help me simplify expressions.

Learn Use Properties to Multiply Integers

In mathematics, properties can be used to justify statements you make while verifying or proving another statement. Some of the properties of mathematics are listed below.

Additive Inverse Property	$a + (-a) = 0$
Distributive Property	$a(b + c) = ab + ac$
Multiplicative Identity Property	$1 \cdot a = a$
Multiplicative Property of Zero	$a \cdot 0 = 0$

Go Online Watch the animation online to learn how to use properties of multiplication to multiply integers.

The animation shows that $2(-1) = -2$ using properties of multiplication, beginning with the true statement $2(0) = 0$.

$2(0) = 0$ Multiplicative Property of Zero

$2[1 + (-1)] = 0$ Additive Inverse Property

$2(1) + 2(-1) = 0$ Distributive Property

$2 + 2(-1) = 0$ Multiplicative Identity Property

In order for $2 + 2(-1)$ to equal 0, $2(-1)$ must equal -2, based on the Additive Inverse Property. This shows, using properties, that the product of two integers with different signs is negative.

You can also use properties to show that the product of two negative integers is positive.

Show that $(-2)(-1) = 2$ by writing the correct property for each step.

$0 = -2(0)$ Multiplicative Property of Zero

$0 = -2[1 + -1]$ Additive Inverse Property

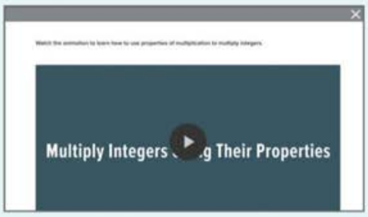
$0 = -2(1) + (-2)(-1)$ Distributive Property

$0 = -2 + (-2)(-1)$ Multiplicative Identity Property

$2 = (-2)(-1)$ Addition Property of Equality

154 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Learn, Use Properties to Multiply Integers, Slide 2 of 3

WATCH

On Slide 2, students watch an animation that explains how to use properties of multiplication to multiply integers.

CLICK

On Slide 3, students select from drop-down menus the correct properties to justify steps in the multiplication of two integers.

Learn Use Properties to Multiply Integers

Objective

Students will understand how the properties of operations can be applied to multiply integers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 1, encourage them to explain how knowing and flexibly using different properties of operations can help when multiplying integers.

Teaching Notes

SLIDE 2

The animation will show that the product of a positive integer and a negative integer is negative, using the properties of operations. You may wish to pause the animation after each step to ask students to explain their understanding of each step.

SLIDE 3

Students will identify properties of multiplication to justify each step. You may wish to have a student volunteer to identify each property and explain why that step illustrates that property.

Go Online

Have students watch the animation on Slide 2. The animation illustrates how to use properties of multiplication to multiply integers.

Talk About It!

SLIDE 1

Mathematical Discourse

How can the properties of multiplication help you multiply integers?

Sample answer: The properties help me simplify expressions.

Apply Agriculture

Objective

Students will come up with their own strategy to solve an application problem involving revenue, expenses, and savings for a farm.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

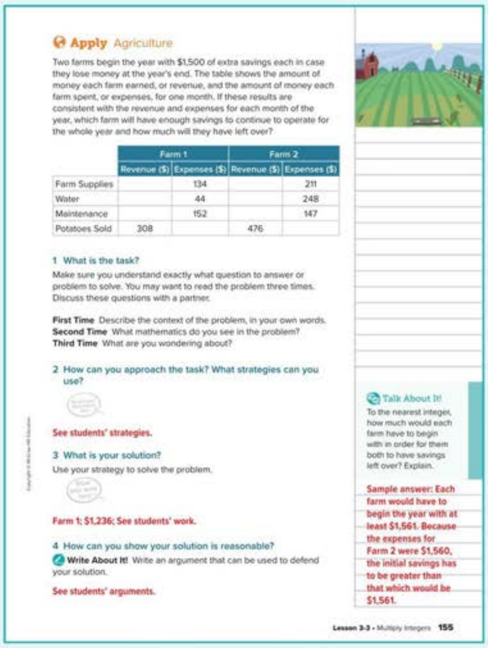
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What are savings?
- What do you notice about the revenue compared to the expenses?
- How might thinking about expenses in terms of negative integers help you?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Agriculture

Two farms begin the year with $1,500 of extra savings each in case they lose money at the year's end. The table shows the amount of money each farm earned, or revenue, and the amount of money each farm spent, or expenses, for one month. If these results are consistent with the revenue and expenses for each month of the year, which farm will have enough savings to continue to operate for the whole year and how much will they have left over?

	Farm 1		Farm 2	
	Revenue ($)	Expenses ($)	Revenue ($)	Expenses ($)
Farm Supplies		134		211
Water		44		248
Maintenance		152		147
Potatoes Sold	308		476	

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Farm 1; $1,236; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

To the nearest integer, how much would each farm have to begin with in order for them both to have savings left over? Explain.

Sample answer: Each farm would have to begin the year with at least $1,561. Because the expenses for Farm 2 were $1,560, the initial savings has to be greater than that which would be $1,561.

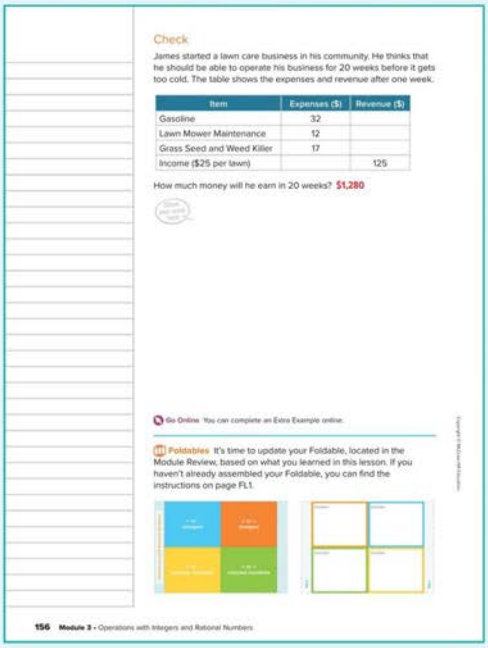
Lesson 3-3 • Multiply Integers 155

Interactive Presentation

Apply, Agriculture

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

James started a lawn care business in his community. He thinks that he should be able to operate his business for 20 weeks before it gets too cold. The table shows the expenses and revenue after one week.

Item	Expenses ($)	Revenue ($)
Gasoline	32	
Lawn Mower Maintenance	12	
Grass Seed and Weed Killer	17	
Income ($25 per lawn)		125

How much money will he earn in 20 weeks? $1,280

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

156 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of multiplying integers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with rational numbers related to operations with integers?

In this lesson, students learned how to multiply integers with the same sign or different signs. Encourage them to work with a partner to compare and contrast multiplying integers to multiplying whole numbers. For example, have them compare and contrast how they would simplify each of the expressions $-6(-8)$, $-6(8)$, $6(-8)$, and $6(8)$.

Exit Ticket

Refer to the Exit Ticket slide. Suppose a certain breed of dog loses about 75 hairs per day. What integer represents this change in the amount of hair per week? Write a mathematical argument that can be used to defend your solution. –525; Sample answer: W rite a multiplication sentence. A loss of 75 hairs per day is represented by the integer –75. There are 7 days in one week. So, the change in the amount of hair per week is –75(7), or –525.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 15, 17, 18–21
- Extension: Powers and Negatives
- ALEKS® Multiplication and Division with Integers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–14, 16, 18
- Extension: Powers and Negatives
- Personal Tutor
- Extra Examples 1–6
- ALEKS® Plotting and Comparing Integers

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS® Plotting and Comparing Integers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

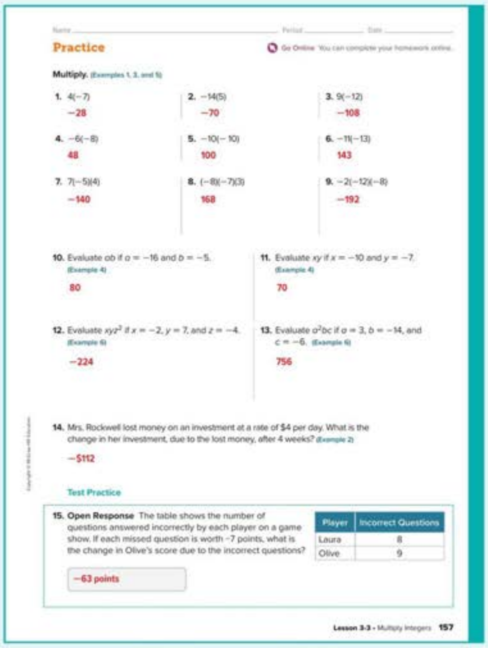
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	multiply integers	1–9
1	evaluate algebraic expressions involving multiplication of two integers	10, 11
1	evaluate algebraic expressions involving multiplication of three or more integers	12, 13
2	multiply integers with different signs to solve a real-world problem	14
2	extend concepts learned in class to apply them in new contexts	15
3	solve application problems involving multiplying integers	16, 17
3	higher-order and critical thinking skills	18–21

Common Misconception

When evaluating expressions with exponents, students may forget to apply the rules of multiplying negative integers. Demonstrate to students that $(-4)^2 = (-4) \cdot (-4) = 16$.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Multiply. (Examples 1, 3, and 5)

1. $4(-7)$ **-28**
2. $-14(5)$ **-70**
3. $9(-12)$ **-108**
4. $-6(-8)$ **48**
5. $-10(-10)$ **100**
6. $-11(-13)$ **143**
7. $7(-5)(4)$ **-140**
8. $(-8)(-7)(3)$ **168**
9. $-2(-12)(-8)$ **-192**

10. Evaluate ab if $a = -16$ and $b = -5$. (Example 4) **80**

11. Evaluate xy if $x = -10$ and $y = -7$. (Example 4) **70**

12. Evaluate xyz^2 if $x = -2$, $y = 7$, and $z = -4$. (Example 6) **-224**

13. Evaluate a^2bc if $a = 3$, $b = -14$, and $c = -6$. (Example 6) **756**

14. Mrs. Rockwell lost money on an investment at a rate of $4 per day. What is the change in her investment, due to the lost money, after 4 weeks? (Example 2) **$-\$112$**

Test Practice

15. **Open Response** The table shows the number of questions answered incorrectly by each player on a game show. If each missed question is worth −7 points, what is the change in Olive's score due to the incorrect questions?

Player	Incorrect Questions
Laura	8
Olive	9

−63 points

Lesson 3-3 • Multiply Integers 157

Apply *indicates multi-step problem

*16. Payton starts a lemonade stand for the summer. She thinks that she should be able to operate her business for 14 weeks. The table shows the expenses and revenue after 1 week. Based on this, how much money will Payton make during the 14 weeks?

Item	Expenses	Revenue
Cups	\$5	
Lemonade	\$6	
Ice	\$7	
Income		\$45

\$378

*17. Rakim's goal is to have at least \$500 in his checking account at the end of the year. The table shows his activity for the month of January. Will Rakim make his goal if the month of January's activity is representative of how much he will save and spend each month? Explain.

Transaction	Amount
Debit Card Purchases	\$500
ATM Withdraws	\$750
Deposits	\$1,300

yes; At the end of January, he had \$1,300 − \$1,250 or \$50 left in his account. \$50 × 12 = \$600 and \$600 > \$500.

Higher-Order Thinking Problems

18. **Reason Inductively** The product of two integers is −24. The difference between the two integers is 14. The sum of the two integers is 10. What are the two integers?

12 and −2

19. **Identify Structure** Name the property illustrated by the following.

a. $-x \cdot 1 = -x$

Multiplicative Identity Property

b. $x \cdot (-y) = (-y) \cdot x$

Commutative Property of Multiplication

20. If you multiply three negative integers, will the product *always*, *sometimes*, or *never* be negative. Explain.

always; Sample answer: The product of the first two negative integers is positive and the product of the positive integer and the third negative integer is negative. For example, −2(−3)(−4) = −24.

21. **Identify Structure** Name all the values of x if $6|x| = 48$.

−8 and 8

158 **Module 3** • Operations with Integers and Rational Numbers

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 18, students use several facts about two integers to find their values.

7 Look for and Make Use of Structure In Exercise 19, students name the property illustrated by each equation.
In Exercise 21, students use the structure of distance and absolute value to find all the values that satisfy an equation.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 16 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.
Use with Exercise 20 Have students work in groups of 3–4 to solve the problem in Exercise 20. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class.

Lesson 3-4

Divide Integers

7.NS.A.2.B

LESSON GOAL

Students will solve problems dividing integers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Algebra Tiles to Divide Integers

Learn: Divide Integers with Different Signs
Example 1: Divide Integers with Different Signs
Example 2: Divide Integers with Different Signs
Learn: Divide Integers with the Same Sign
Example 3: Divide Integers with the Same Sign
Example 4: Divide Integers with the Same Sign
Apply: Personal Finance

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Divide by Zero		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 17 of the *Language Development Handbook* to help your students build mathematical language related to division of integers.

ELL You can use the tips and suggestions on page T17 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7. NS.A** by dividing integers.
Standards for Mathematical Content: **7. NS.A.2, 7.NS.A.2.B, 7.NS.A.2.C,** *Also addresses 7.NS.A.1.D, 7.EE.B.3, 7.NS.A.3*
Standards for Mathematical Practice: **MP 1, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students solved problems involving multiplying integers.
7.NS.A.2, 7.NS.A.2.A, 7.NS.A.2.C

Now
Students solve problems involving dividing integers.
7.NS.A.2.B

Next
Students will solve problems by applying all operations to integers.
7.NS.A1.D, 7.NS.A.2.C, 7.NS.A.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

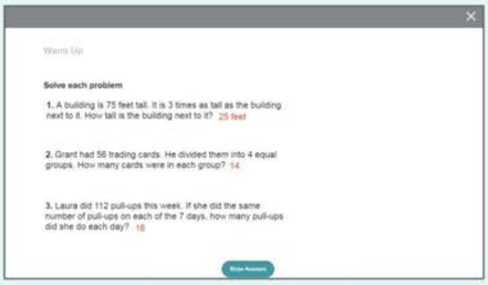
Conceptual Bridge In this lesson, students will draw on their knowledge of the division of whole numbers to develop *understanding* of division of integers. They build *fluency* by dividing two Integers with different signs and dividing two integers with the same signs.

Mathematical Background

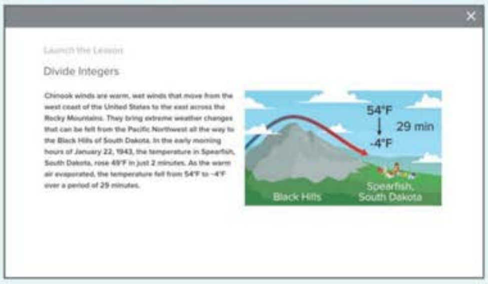
By relating division to multiplication, it can be shown that the rules for determining the sign of a quotient of two integers are the same as the rules for determining the sign of the product of two integers. The rules for dividing integers are shown.

- If the two integers have the same sign, the quotient is positive.
- If the two integers have different signs, the quotient is negative .

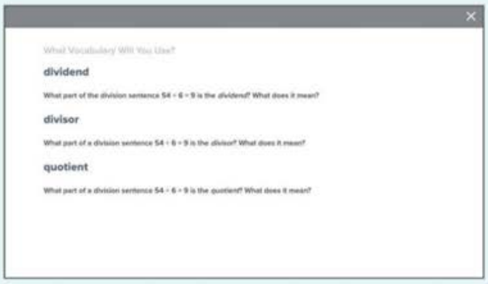
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- solving word problems involving dividing whole numbers (Exercises 1–3)

Answers

1. 25 feet
2. 14
3. 16

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about changes in temperatures as related to integers.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What part of the division sentence $54 \div 6 = 9$ is the *dividend*? What does it mean? 54; Sample answer: The dividend is the number that is being divided into equal groups. In this case, 54 is divided into 6 equal groups of 9.
- What part of a division sentence $54 \div 6 = 9$ is the *divisor*? What does it mean? 6; Sample answer: The divisor is the number of groups into which the dividend is divided. In this case, 54 is divided into 6 equal groups of 9.
- What part of a division sentence $54 \div 6 = 9$ is the *quotient*? What does it mean? 9; Sample answer: The quotient is the number in each group. It is the answer to the division problem. In this case, 54 is divided into 6 equal groups of 9.

Explore Use Algebra Tiles to Divide Integers

Objective

Students will use algebra tiles to explore how to divide integers.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with algebra tiles representing 1 and −1. Throughout this activity, students will use the algebra tiles to divide integers, and explain how the algebra tiles represent the dividend, divisor, and quotient.

Inquiry Question

How can you use algebra tiles to model integer division? Sample answer: The dividend represents the total number of algebra tiles. The divisor represents the number of equal groups into which the dividend is divided. The quotient represents the number of tiles in each group.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

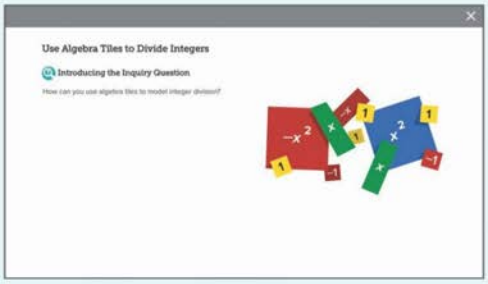
SLIDE 2

Mathematical Discourse

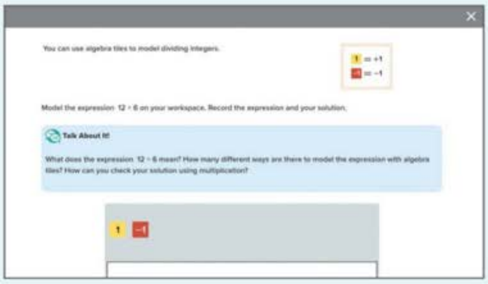
What does the expression 12 ÷ 6 mean? How many different ways are there to model the expression with algebra tiles? How can you check your solution using multiplication? Sample answer: The expression 12 ÷ 6 means *twelve tiles divided into six equal groups*. Using algebra tiles, I can model twelve 1-tiles and divide them into 6 equal groups. There are two tiles in each group, so the solution to the division problem is 2. I can use multiplication to check the solution, 2 • 6 = 12.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6



Explore, Slide 2 of 6

DRAG & DROP

Throughout the Explore, students drag algebra tiles to model integer division.

Interactive Presentation

Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Use Algebra Tiles to Divide Integers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use algebra tiles to explore integer division and explain how the algebra tiles represent the dividend, divisor, and quotient of a division problem.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Can you use algebra tiles to model the expression $15 \div (-3)$? Explain your reasoning. Can you find the quotient a different way?

Sample answer: No, if I have fifteen positive 1-tiles, I can't divide them into negative groups. I could use multiplication to find $-5 \cdot (-3)$, which is 15, so the solution is -5.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Divide Integers with Different Signs

Objective

Students will understand how they can use related multiplication sentences to determine how to divide integers with different signs.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 3, encourage them to use precise mathematical language to explain their reasoning.

Teaching Notes

SLIDE 1

Students will learn how to find the quotient of integers with different signs by using a related multiplication sentence. You may wish to have a student volunteer select each marker to see the related multiplication sentences for the division sentence, and determine what number multiplied by −3 results in 36.

SLIDE 2

You may wish to have a student volunteer select the *Words* and *Examples* flashcards to show how the rule for dividing integers with different signs can be described using these representations.

Talk About It!

SLIDE 3

Mathematical Discourse

What do you think is the quotient of −36 ÷ 3? Explain your reasoning. Sample answer: −36 ÷ 3 represents 36 negative 1-tiles divided into three groups. I also know that −12 • 3 is −36, so the quotient is −12.

DIFFERENTIATE

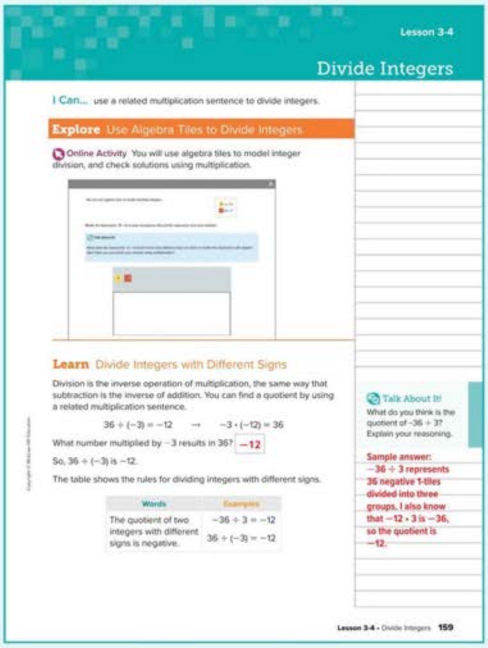
Reteaching Activity AL

For students that may be struggling to divide integers with different signs, remind them that they can check their work using multiplication. The product of two integers with different signs is negative. The product of two integers with the same sign is positive. For each of the following division sentences, have students write a related multiplication sentence that can be used to check the division for accuracy.

−12 ÷ 4 = −3 −3 • 4 = −12

15 ÷ (−3) = −5 −5 • (−3) = 15

42 ÷ (−2) = −21 −21 • (−2) = 42


Lesson 3-4

Divide Integers

I Can... use a related multiplication sentence to divide integers.

Explore Use Algebra Tiles to Divide Integers

Online Activity You will use algebra tiles to model integer division, and check solutions using multiplication.

Learn Divide Integers with Different Signs

Division is the inverse operation of multiplication, the same way that subtraction is the inverse of addition. You can find a quotient by using a related multiplication sentence.

36 ÷ (−3) = −12 → −3 • (−12) = 36

What number multiplied by −3 results in 36? −12

So, 36 ÷ (−3) is −12.

The table shows the rules for dividing integers with different signs.

Words	Examples
The quotient of two integers with different signs is negative.	−36 ÷ 3 = −12 36 ÷ (−3) = −12

Talk About It! What do you think is the quotient of −36 ÷ 3? Explain your reasoning.

Sample answer: −36 ÷ 3 represents 36 negative 1-tiles divided into three groups. I also know that −12 • 3 is −36, so the quotient is −12.

Lesson 3-4 • Divide Integers 159

Interactive Presentation

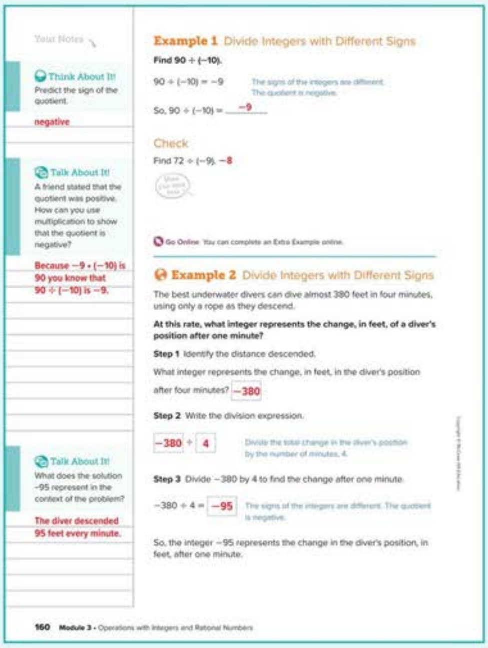
Learn, Divide Integers with Different Signs, Slide 2 of 3

CLICK

On Slide 1, students select markers to see related multiplication sentences.

FLASHCARDS

On Slide 2, students use Flashcards to learn about dividing integers with different signs.



Your Notes

Think About It!
Predict the sign of the quotient.

negative

Talk About It!
A friend stated that the quotient was positive. How can you use multiplication to show that the quotient is negative?

Because $-9 \cdot (-10)$ is 90 you know that $90 \div (-10)$ is -9.

Talk About It!
What does the solution −95 represent in the context of the problem?

The diver descended 95 feet every minute.

Example 1 Divide Integers with Different Signs

Find $90 \div (-10)$.

$90 \div (-10) = -9$ — The signs of the integers are different. The quotient is negative.

So, $90 \div (-10) =$ -9

Check

Find $72 \div (-9)$. -8

Go Online You can complete an Extra Example online.

Example 2 Divide Integers with Different Signs

The best underwater divers can dive almost 380 feet in four minutes, using only a rope as they descend.

At this rate, what integer represents the change, in feet, of a diver's position after one minute?

Step 1 Identify the distance descended.

What integer represents the change, in feet, in the diver's position after four minutes? -380

Step 2 Write the division expression.

$-380 \div 4$ — Divide the total change in the diver's position by the number of minutes, 4.

Step 3 Divide −380 by 4 to find the change after one minute.

$-380 \div 4 = -95$ — The signs of the integers are different. The quotient is negative.

So, the integer −95 represents the change in the diver's position, in feet, after one minute.

160 **Module 3** • Operations with Integers and Rational Numbers

Interactive Presentation

Example 2, Divide Integers with Different Signs, Slide 4 of 6

CLICK

On Slide 2 of Example 1, students select from drop-down menus to determine the sign of a quotient.

TYPE

On Slide 4 of Example 2, students determine and interpret the quotient.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Divide Integers with Different Signs

Objective

Students will divide integers with different signs.

Questions for Mathematical Discourse

SLIDE 2

AL Identify the dividend and divisor. The dividend is 90. The divisor is −10.

OL Without calculating, how do you know that the quotient will be negative? The integers have different signs. The quotient of two integers with different signs is negative.

OL How can you check your answer? Use multiplication. $-9(-10) = 90$

BL Generate a division expression in which the dividend and quotient are both negative. Sample answer: $-45 \div 5$ which has a quotient of -9

Example 2 Divide Integers with Different Signs

Objective

Students will divide integers with different signs to solve a real-world problem.

Questions for Mathematical Discourse

SLIDE 2

AL Would the diver's descent be better represented as a positive integer or a negative integer? *Descending* is better represented as a negative integer.

OL What integer represents the change, in feet, in the diver's position after four minutes? -380

BL Assuming the same rate, what integer would represent the change, in feet, in the diver's position after eight minutes? Explain. -760; Sample answer: The diver would have descended another 380 feet, $-380 + (-380) = -760$

Go Online

- Find additional teaching notes, discussion questions, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** 3 APPLICATION

Learn Divide Integers with the Same Sign

Objective

Students will understand how they can use related multiplication sentences to determine how to divide integers with the same sign.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use precise mathematical language to compare and contrast multiplying and dividing integers.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

How are division and multiplication of integers similar? How are they different? Sample answer: The same rules apply for the sign of the solution, and you can use algebra tiles to model the operations. When dividing, you are not able to use algebra tiles if the divisor is negative.

Example 3 Divide Integers with the Same Sign

Objective

Students will divide integers with the same sign.

Questions for Mathematical Discourse

SLIDE 1

AL Do the dividend and divisor have the same sign? yes

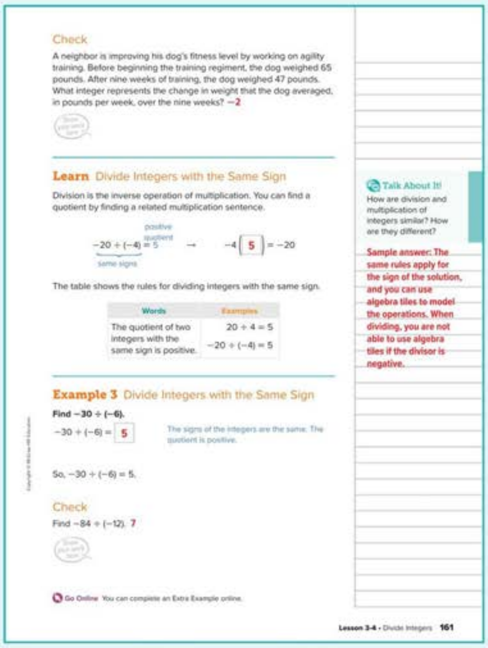
AL What is true about the quotient of a division problem when both integers have the same sign? The quotient is positive.

OL How can you use multiplication to check your answer? Multiply 5 by −6. The answer is −30, which is the dividend, so my answer is correct.

BL How could you alter the problem so that the quotient is negative? Sample answer: Change −30 to 30.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

A neighbor is improving his dog's fitness level by working on agility training. Before beginning the training regiment, the dog weighed 65 pounds. After nine weeks of training, the dog weighed 47 pounds. What integer represents the change in weight that the dog averaged, in pounds per week, over the nine weeks? −2

Learn Divide Integers with the Same Sign

Division is the inverse operation of multiplication. You can find a quotient by finding a related multiplication sentence.

$-20 \div (-4) = 5$ (same signs; positive quotient) → $-4\,[5] = -20$

The table shows the rules for dividing integers with the same sign.

Words	Examples
The quotient of two integers with the same sign is positive.	$20 \div 4 = 5$ $-20 \div (-4) = 5$

Talk About It! How are division and multiplication of integers similar? How are they different?

Sample answer: The same rules apply for the sign of the solution, and you can use algebra tiles to model the operations. When dividing, you are not able to use algebra tiles if the divisor is negative.

Example 3 Divide Integers with the Same Sign

Find $-30 \div (-6)$.

$-30 \div (-6) = [5]$ The signs of the integers are the same. The quotient is positive.

So, $-30 \div (-6) = 5$.

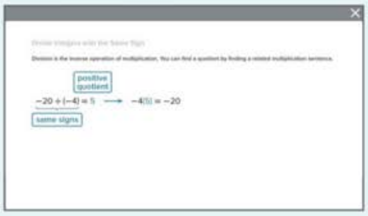
Check

Find $-84 \div (-12)$. 7

Go Online You can complete an Extra Example online.

Lesson 3-4 • Divide Integers 161

Interactive Presentation



Learn, Divide Integers with the Same Sign, Slide 1 of 3

FLASHCARDS

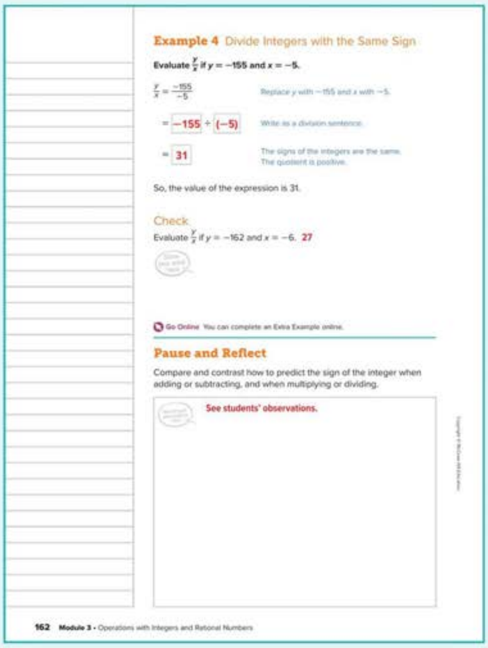
On Slide 2 of the Learn, students use Flashcards to learn about dividing integers with the same sign.

CLICK

On Slide 1 of Example 3, students select from drop-down menus to determine the sign of a quotient.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.



Example 4 Divide Integers with the Same Sign

Evaluate $\frac{y}{x}$ if $y = -155$ and $x = -5$.

$\frac{y}{x} = \frac{-155}{-5}$ Replace y with −155 and x with −5.

$= -155 \div (-5)$ Write as a division sentence.

$= 31$ The signs of the integers are the same. The quotient is positive.

So, the value of the expression is 31.

Check

Evaluate $\frac{y}{x}$ if $y = -162$ and $x = -6$. 27

Go Online You can complete an Extra Example online.

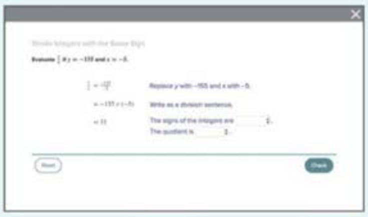
Pause and Reflect

Compare and contrast how to predict the sign of the integer when adding or subtracting, and when multiplying or dividing.

See students' observations.

162 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Example 4, Divide Integers with the Same Sign, Slide 1 of 2

CLICK

On Slide 1, students select from drop-down menus to determine the sign of the quotient.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Divide Integers with the Same Sign

Objective

Students will evaluate an algebraic expression involving division of integers.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to follow the general rules for finding the quotient of integers with the same signs and to pay careful attention to the signs of the dividend, divisor, and quotient.

Questions for Mathematical Discourse

SLIDE 1

AL Do the values of x and y have the same sign? yes

AL What is the sign of both integers? negative

OL Without calculating, how do you know the quotient will be positive? The quotient of two integers with the same sign is positive.

BL Find $\frac{-y}{x}$. −31

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of integer division, have them find the quotient for each of the following division problems.

$\frac{-24 \div (-4)}{-3}$ −2

$\frac{84 \div (-3)}{-7}$ 4

Apply Personal Finance

Objective

Students will come up with their own strategy to solve an application problem involving managing a bank account.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the starting balance?
- How will a withdrawal affect the account?
- How can you find the net amount saved for each week?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Personal Finance

Natalie had $165 in her bank account at the beginning of the summer. Over the next 10 weeks, she worked at a summer camp and added $160 to her savings each week, while spending only $40 per week. Once she gets back to school, she plans to spend $105 per week. For how many weeks can she make withdrawals until her balance is $0?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

13 weeks; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

If Natalie wants to be able to withdrawal $105 for 15 weeks, how much can she spend each week during the summer?

$19

Lesson 3-4 • Divide Integers 163

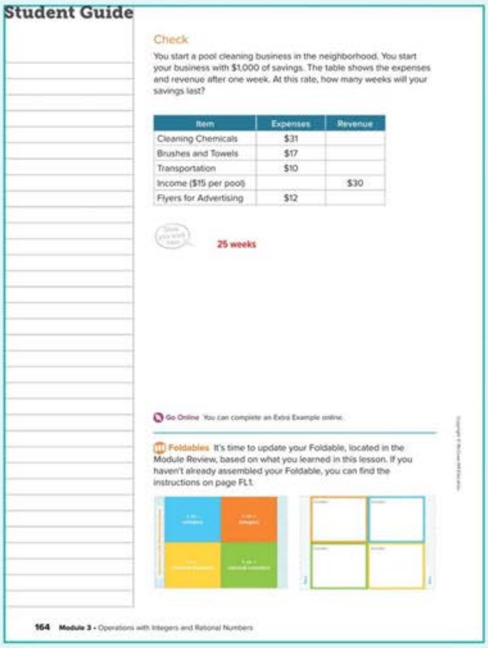
Interactive Presentation

Apply, Personal Finance

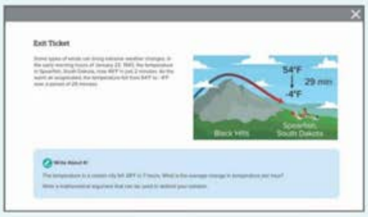
CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Student Guide



Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of dividing integers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with rational numbers related to operations with integers?

In this lesson, students learned how to divide integers with the same signs or different signs. Encourage them to work with a partner to compare and contrast dividing integers to diving whole numbers. For example, have them compare and contrast how they would simplify each of the expressions −48 ÷ −12, −48 ÷ 12, 48 ÷ −12, and 48 ÷ 12.

Exit Ticket

Refer to the Exit Ticket slide. The temperature in a certain city fell 28°F in 7 hours. What is the average change in temperature per hour? Write a mathematical argument that can be used to defend your solution. −4 degrees Fahrenheit per hour; Sample answer: Write the division expression −28 ÷ 7. Then divide the integers, −28 ÷ 7 = −4.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 13–15, 17–20
- Extension: Divide by Zero
- ALEKS Multiplication and Division with Integers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–13, 15, 17, 18
- Extension: Divide by Zero
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Plotting and Comparing Integers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Plotting and Comparing Integers

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

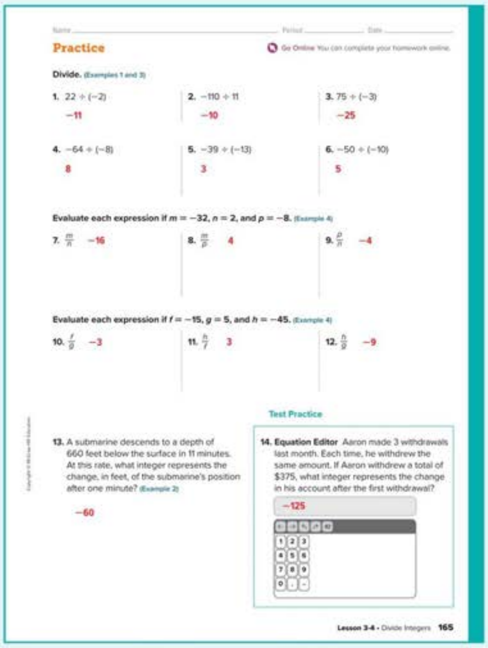
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	divide integers with different signs	1–3
1	divide integers with the same sign	4–6
1	evaluate algebraic expressions involving division of integers	7–12
2	divide integers with different signs to solve real-world problems	13
2	extend concepts learned in class to apply them in new contexts	14
3	solve application problems that involve dividing integers	15, 16
3	higher-order and critical thinking skills	17–20

Common Misconception

Some students may incorrectly find the sign when dividing two integers. They may incorrectly think that the signs of quotients are determined in the opposite way as the signs of products, since multiplication and division are inverse operations. In Exercises 1-6, students may write the correct quotient values with the incorrect signs. Explain to students that they can and should use multiplication to check their work.


Name _______ Period _______ Date _______

Practice

Go Online You can complete your homework online.

Divide. (Examples 1 and 3)

1. $22 \div (-2)$ **−11**
2. $-110 \div 11$ **−10**
3. $75 \div (-3)$ **−25**
4. $-64 \div (-8)$ **8**
5. $-39 \div (-13)$ **3**
6. $-50 \div (-10)$ **5**

Evaluate each expression if $m = -32$, $n = 2$, and $p = -8$. (Example 4)

7. $\frac{m}{n}$ **−16**
8. $\frac{m}{p}$ **4**
9. $\frac{p}{n}$ **−4**

Evaluate each expression if $f = -15$, $g = 5$, and $h = -45$. (Example 4)

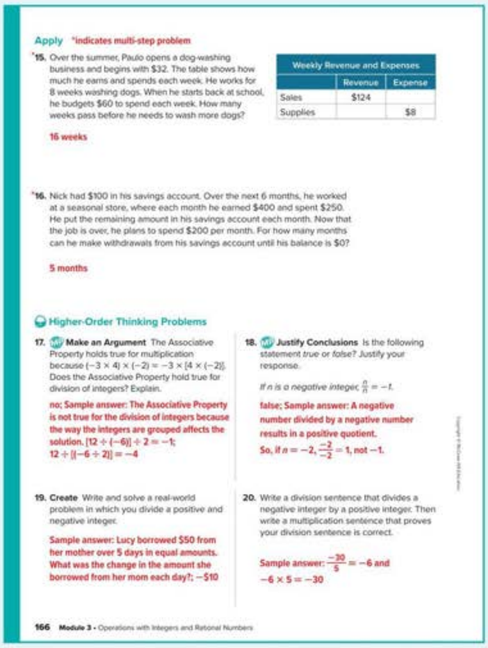
10. $\frac{f}{g}$ **−3**
11. $\frac{h}{f}$ **3**
12. $\frac{h}{g}$ **−9**

13. A submarine descends to a depth of 660 feet below the surface in 11 minutes. At this rate, what integer represents the change, in feet, of the submarine's position after one minute? (Example 2) **−60**

Test Practice

14. **Equation Editor** Aaron made 3 withdrawals last month. Each time, he withdrew the same amount. If Aaron withdrew a total of $375, what integer represents the change in his account after the first withdrawal? **−125**

Lesson 3-4 • Divide Integers 165



Apply *indicates multi-step problem

*15. Over the summer, Paulo opens a dog-washing business and begins with $32. The table shows how much he earns and spends each week. He works for 8 weeks washing dogs. When he starts back at school, he budgets $60 to spend each week. How many weeks pass before he needs to wash more dogs?

Weekly Revenue and Expenses		
	Revenue	Expense
Sales	$124	
Supplies		$8

16 weeks

*16. Nick had $100 in his savings account. Over the next 6 months, he worked at a seasonal store, where each month he earned $400 and spent $250. He put the remaining amount in his savings account each month. Now that the job is over, he plans to spend $200 per month. For how many months can he make withdrawals from his savings account until his balance is $0?

5 months

Higher-Order Thinking Problems

17. **Make an Argument** The Associative Property holds true for multiplication because $(-3 \times 4) \times (-2) = -3 \times [4 \times (-2)]$. Does the Associative Property hold true for division of integers? Explain.

no; Sample answer: The Associative Property is not true for the division of integers because the way the integers are grouped affects the solution. $[12 \div (-6)] \div 2 = -1$; $12 \div [(-6 \div 2)] = -4$

18. **Justify Conclusions** Is the following statement *true* or *false*? Justify your response.

If n is a negative integer, $\frac{n}{n} = -1$.

false; Sample answer: A negative number divided by a negative number results in a positive quotient. So, if $n = -2$, $\frac{-2}{-2} = 1$, not -1.

19. **Create** Write and solve a real-world problem in which you divide a positive and negative integer.

Sample answer: Lucy borrowed $50 from her mother over 5 days in equal amounts. What was the change in the amount she borrowed from her mom each day?; −$10

20. Write a division sentence that divides a negative integer by a positive integer. Then write a multiplication sentence that proves your division sentence is correct.

Sample answer: $\frac{-30}{5} = -6$ and $-6 \times 5 = -30$

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students make an argument for whether or not the Associative Property holds true for division of integers.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students determine if a statement is true or false and justify their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercises 15–16 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 15 might be, “Paulo spends $64 over 8 weeks on supplies.” An example of a false statement might be, “The amount of revenue is less than the amount of expenses.” Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Create your own higher-order thinking problem.
Use with Exercises 17–18 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other’s work, and discuss and resolve any differences.

Lesson 3-5

Apply Integer Operations

7.NS.A.2, 7.NS.A.3

LESSON GOAL

Students will solve problems by applying all operations to integers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Example 1: Order of Integer Operations
Example 2: Order of Integer Operations
Example 3: Order of Integer Operations
Example 4: Order of Integer Operations

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 18 of the *Language Development Handbook* to help your students build mathematical language related to integer operations.

ELL You can use the tips and suggestions on page T18 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7. NS.A** by solving problems by applying all operations to integers.
Standards for Mathematical Content: **7 .NS.A.1, 7.NS.A.1.D, 7.NS.A.2, 7.NS.A.2.C, 7.NS.A.3, 7.EE.B.3**
Standards for Mathematical Practice: **M P3, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students solved problems involving dividing integers.
7.NS.A.2, 7.NS.A.2.B

Now
Students solve problems by applying all operations to integers.
7.NS.A1.D, 7.NS.A.2.C, 7.NS.A.3

Next
Students will identify terminating and repeating decimals, and use long division to convert rational numbers to decimals.
7.NS.A.2.B, 7.NS.A.2.D, 8.NS.A.1

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

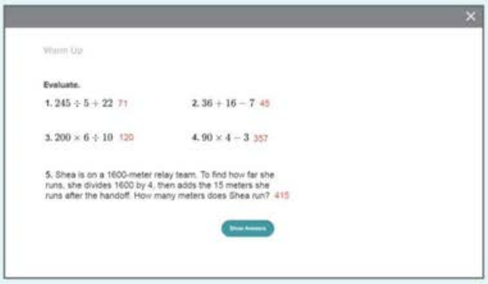
Conceptual Bridge In this lesson, students will draw on their knowledge of operations with integers and the order of operations to develop *fluency* in applying the order of operations to integers. They will *apply* their knowledge of integers operations in the order of operations to solve real-world problems.

Mathematical Background

The *order of operations* indicates that mathematical operations must be performed in this order.

1. Simplify expressions inside grouping symbols, such as parentheses.
2. Find the values of all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- evaluating expressions using the order of operations with whole numbers (Exercises 1–4)
- solving real-world problems by writing and evaluating expressions involving whole number operations (Exercise 5)

Answers

1. 71 **3.** 120 **5.** 415

2. 45 **4.** 357

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using the order of integer operations with temperature change.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Summarize the *order of operations*. Sample answer: The order of operations states to perform the operations inside grouping symbols, such as parentheses and brackets, first. Then evaluate any exponents. Next, multiply and divide in order from left to right. Finally, add and subtract in order from left to right.

Example 1 Order of Integer Operations

Objective

Students will apply the order of operations to evaluate a numerical expression involving integers.

Questions for Mathematical Discourse

SLIDE 2

AL Identify the two operations in this expression. multiplication and addition

AL Which operation should you perform first? multiplication

OL Is the product of –4 and 3 positive or negative? negative

OL Will the sum of the product and –7 be positive or negative? Explain. negative; The product is –12. The sum of –12 and –7 will be negative, because the sum of two negative integers is negative.

BL Write a different expression involving two integer operations, in which the final result is negative. Then simplify the expression. Sample answer: –5(3) + 2; –13

Example 2 Order of Integer Operations

Objective

Students will apply the order of operations to evaluate a numerical expression involving integers.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the operations involved in this expression. Three integers are multiplied. Then –8 is subtracted from that product.

AL Which operation(s) should be performed first? multiplication

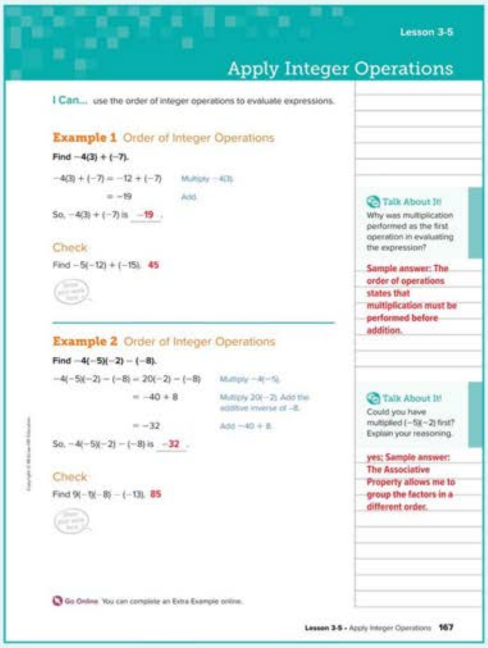
OL Without calculating, how do you know what the sign of the product of the three integers will be? The product will be negative because there are three negative integers.

OL Why do we add 8? Subtracting –8 is the same as adding +8.

BL Describe one change you could make to the expression so that the final result is a positive number. Sample answer: Change –8 to –41.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.



Lesson 3-5

Apply Integer Operations

I Can... use the order of integer operations to evaluate expressions.

Example 1 Order of Integer Operations

Find $-4(3) + (-7)$.

$-4(3) + (-7) = -12 + (-7)$ Multiply $-4(3)$.

$= -19$ Add.

So, $-4(3) + (-7)$ is $\underline{-19}$.

Check

Find $-5(-12) + (-15)$. 45

Example 2 Order of Integer Operations

Find $-4(-5)(-2) - (-8)$.

$-4(-5)(-2) - (-8) = 20(-2) - (-8)$ Multiply $-4(-5)$.

$= -40 + 8$ Multiply $20(-2)$. Add the additive inverse of -8.

$= -32$ Add $-40 + 8$.

So, $-4(-5)(-2) - (-8)$ is $\underline{-32}$.

Check

Find $9(-1)(-8) - (-13)$. 85

Talk About It! Why was multiplication performed as the first operation in evaluating the expression?

Sample answer: The order of operations states that multiplication must be performed before addition.

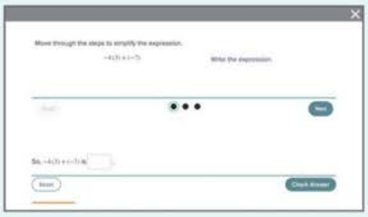
Talk About It! Could you have multiplied $(-5)(-2)$ first? Explain your reasoning.

yes; Sample answer: The Associative Property allows me to group the factors in a different order.

Go Online You can complete an Extra Example online.

Lesson 3-5 • Apply Integer Operations 167

Interactive Presentation

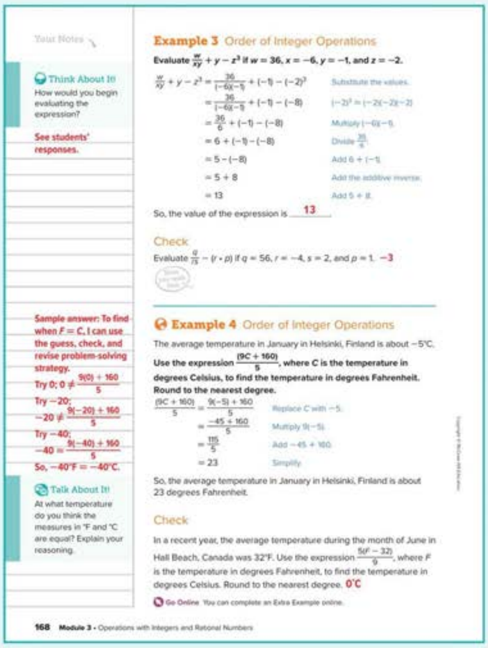


Example 1, Order of Integer Operations, Slide 2 of 4

CLICK On Slide 2 of Example 1, students move through the steps to evaluate an expression.

TYPE On Slide 2 of Example 2, students evaluate the expression.

CHECK Students complete the Check exercises online to determine if they are ready to move on.



Your Notes

Think About It!
How would you begin evaluating the expression?

See students' responses.

Example 3 Order of Integer Operations

Evaluate $\frac{w}{xy} + y - z^3$ if $w = 36$, $x = -6$, $y = -1$, and $z = -2$.

$\frac{w}{xy} + y - z^3 = \frac{36}{(-6)(-1)} + (-1) - (-2)^3$	Substitute the values.
$= \frac{36}{(-6)(-1)} + (-1) - (-8)$	$(-2)^3 = (-2)(-2)(-2)$
$= \frac{36}{6} + (-1) - (-8)$	Multiply $(-6)(-1)$.
$= 6 + (-1) - (-8)$	Divide $\frac{36}{6}$.
$= 5 - (-8)$	Add $6 + (-1)$.
$= 5 + 8$	Add the additive inverse.
$= 13$	Add $5 + 8$.

So, the value of the expression is 13.

Check

Evaluate $\frac{q}{rs} - (r \cdot p)$ if $q = 56$, $r = -4$, $s = 2$, and $p = 1$. -3

Sample answer: To find when $F = C$, I can use the guess, check, and revise problem-solving strategy.
Try 0; $0 \neq \frac{9(0) + 160}{5}$
Try −20; $-20 \neq \frac{9(-20) + 160}{5}$
Try −40; $-40 = \frac{9(-40) + 160}{5}$
So, −40°F = −40°C.

Talk About It!
At what temperature do you think the measures in °F and °C are equal? Explain your reasoning.

Example 4 Order of Integer Operations

The average temperature in January in Helsinki, Finland is about −5°C.

Use the expression $\frac{(9C + 160)}{5}$, where C is the temperature in degrees Celsius, to find the temperature in degrees Fahrenheit. Round to the nearest degree.

$\frac{(9C + 160)}{5} = \frac{9(-5) + 160}{5}$	Replace C with −5.
$= \frac{-45 + 160}{5}$	Multiply 9(−5).
$= \frac{115}{5}$	Add −45 + 160.
$= 23$	Simplify.

So, the average temperature in January in Helsinki, Finland is about 23 degrees Fahrenheit.

Check

In a recent year, the average temperature during the month of June in Hall Beach, Canada was 32°F. Use the expression $\frac{5(F - 32)}{9}$, where F is the temperature in degrees Fahrenheit, to find the temperature in degrees Celsius. Round to the nearest degree. 0°C

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3, Order of Integer Operations, Slide 1 of 2

TYPE

On Slide 1 of Example 3, students evaluate the expression.

TYPE

On Slide 2 of Example 4, students interpret the evaluated expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Order of Integer Operations

Objective

Students will evaluate an algebraic expression involving the four operations with integers.

Questions for Mathematical Discourse

SLIDE 1

AL What is the first step in evaluating the expression? Substitute the values for the variables.

AL What integer should replace *w* in the expression? *x*? *y*? *z*? 36; −6; −1; −2

OL According to the order of operations, what should you do first? Evaluate $(-2)^3$.

OL What should you do after you evaluate the power? Multiply −6 and −1.

BL Suppose a classmate found $\frac{w}{xy}$ first, then added *y* and subtracted *z*, and then raised the final result to the third power. What would their result be? Why is it incorrect? 343; Sample answer: They did not follow the order of operations. They raised most of the entire expression to the third power, not just *z*.

Example 4 Order of Integer Operations

Objective

Students will solve a real-world problem involving operations with integers.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the equivalent temperature in degrees Fahrenheit of −5 degrees Celsius

AL What is the first step? Replace *C* with −5.

OL What operation should be performed first? Multiply 9 by −5.

BL What is the corresponding temperature in degrees Fahrenheit for a temperature of 1 degree Celsius? Does this mean that 2 degrees Celsius equals 2(33.8), or 67.6 degrees Fahrenheit? Justify your response. 33.8 degrees Fahrenheit; No, the relationship is not proportional, so this is not a unit rate. Two degrees Celsius would be equal to 35.6 degrees Fahrenheit.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Exit Ticket

Refer to the Exit Ticket slide. The temperature rose 14°F from Monday to Tuesday. The temperature fell 6°F from Tuesday to Wednesday. The temperature fell 11°F from Wednesday to Thursday. Find the average change in temperature per day from Monday to Thursday. Write a mathematical argument that can be used to defend your solution.
–1°F; Sample answer: Write an expression to represent this situation. A temperature rising 14°F is represented by 14. A temperature falling 6°F is represented by –6. A temperature falling 11°F is represented by –11. Add the three changes in temperature and divide by 3 to find the average change in temperature from Monday to Thursday.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

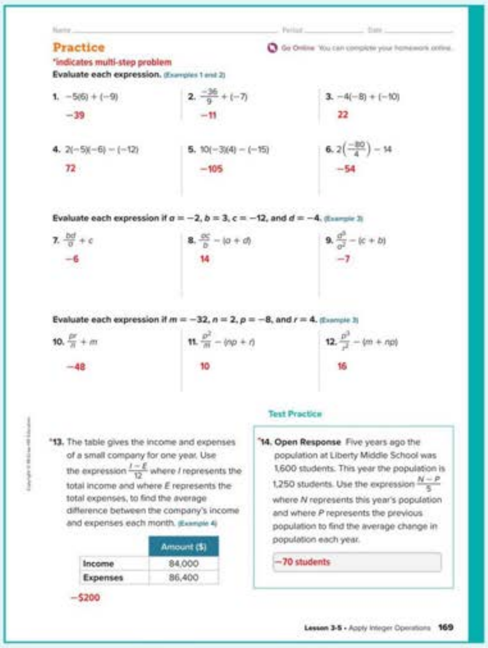
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	apply the order of operations to evaluate numerical expressions involving integers	1–6
1	evaluate algebraic expressions involving the four operations with integers	7–12
2	solve real-world problems involving operations with integers	13, 14
3	solve application problems that involve applying integer operations	15, 16
3	higher-order and critical thinking skills	17–20

Common Misconception

Students may have difficulty substituting the values of the variables when there are negative integers. Encourage student to use parentheses when replacing a variable with an integer.



Name ____________ Period ______ Date ______

Practice

Go Online You can complete your homework online.

*indicates multi-step problem

Evaluate each expression. (Examples 1 and 2)

1. $-5(6) + (-9)$ **−39**
2. $\frac{-36}{9} + (-7)$ **−11**
3. $-4(-8) + (-10)$ **22**
4. $2(-5)(-6) - (-12)$ **72**
5. $10(-3)(4) - (-15)$ **−105**
6. $2\left(\frac{-80}{4}\right) - 14$ **−54**

Evaluate each expression if $a = -2$, $b = 3$, $c = -12$, and $d = -4$. (Example 3)

7. $\frac{bd}{a} + c$ **−6**
8. $\frac{ac}{b} - (a + d)$ **14**
9. $\frac{d^3}{a^2} - (c + b)$ **−7**

Evaluate each expression if $m = -32$, $n = 2$, $p = -8$, and $r = 4$. (Example 3)

10. $\frac{pr}{n} + m$ **−48**
11. $\frac{p^2}{m} - (np + r)$ **10**
12. $\frac{p^3}{r^2} - (m + np)$ **16**

Test Practice

*13. The table gives the income and expenses of a small company for one year. Use the expression $\frac{I - E}{12}$ where I represents the total income and where E represents the total expenses, to find the average difference between the company's income and expenses each month. (Example 4)

	Amount ($)
Income	84,000
Expenses	86,400

−$200

*14. **Open Response** Five years ago the population at Liberty Middle School was 1,600 students. This year the population is 1,250 students. Use the expression $\frac{N - P}{5}$ where N represents this year's population and where P represents the previous population to find the average change in population each year.

−70 students

Lesson 3-5 • Apply Integer Operations 169

Interactive Presentation

Exit Ticket

Apply

*15. The table shows the extreme temperatures for different U.S. cities in degrees Fahrenheit. Use the expression $\frac{5(F-32)}{9}$, where F represents the temperature in degrees Fahrenheit to convert each temperature to degrees Celsius. Which city had the greatest difference in temperature extremes in degrees Celsius? Which city had the least? Round to the nearest degree. Explain.

City	Low Extreme (°F)	High Extreme (°F)
Chicago	−27	104
Nashville	−17	107
Oklahoma City	−8	110

Chicago; Oklahoma City; The difference between the extremes for Chicago is 40°C − (−33°C) or 77°C. The difference between the extremes for Nashville is 42°C − (−27°C) or 69°C. The difference between the extremes for Oklahoma City is 43°C − (−22°C) or 65°C. 77 > 69 > 65

*16. The table shows the yearly initial and ending balance for each sibling in a family for their account balance with their parents. Use the expression $\frac{I-E}{12}$ where I represents the initial balance and where E represents the ending balance, to find the average difference between the initial balance and ending balance each month. Which sibling had the greatest monthly change?

Sibling	Initial Balance	Ending Balance
Alma	$226	−$50
Laurel	$200	−$64
Wes	$290	$2

Wes

Higher-Order Thinking Problems

17. MP **Find the Error** A student solved the problem shown below. Find the student's mistake and correct it.

Find $-3(-6)(-4) - (-9)$.

$$(18)(-4) - (-9) = (-72) - (-9)$$
$$= -81$$

The student subtracted 9 instead of adding its additive inverse. The correct answer should be −63.

18. **Create** Write and solve a real-world problem in which you perform more than one operation with integers.

Sample answer: The average temperature in Iceland is about −10°C. Use the expression $\frac{9C+160}{5}$, where C is the temperature in degrees Celsius, to find the temperature in degrees Fahrenheit. Round to the nearest degree; 14°F

19. When simplifying $5(-2)(9) - 3$, a student first subtracted 3 from 9. Is this the correct first step? Explain.

no; According to the order of operations, multiplication should be performed from left to right before subtraction.

20. MP **Identify Structure** When simplifying $-7(3)(-10) + (-4)$, can you multiply 3 and −10 first and get the same result as multiplying −7 and 3 first? Explain.

yes; Sample answer: The Associative Property of Multiplication states that the order in which numbers are grouped does not change the product. So, $-7(3)(-10) = 210$ and $-10(3)(-7) = 210$.

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MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students find the error in a student's reasoning and correct it.

7 Look for and Make Use of Structure In Exercise 20, students use the structure of an expression to determine if operations can be performed in different orders.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Solve the problem another way.

Use with Exercises 15–16 Have students work in groups of 3–4. After completing Exercise 15, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 16.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 13, 15, 17–20
- ALEKS® Exponents and Order of Operations

IF students score 66-89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–13, 16–18
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Plotting and Comparing Integers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Plotting and Comparing Integers

Lesson 3-6

Rational Numbers

7.NS.A.2

LESSON GOAL

Students will identify terminating and repeating decimals, and use long division to convert rational numbers to decimals.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Rational Numbers

Explore: Rational Numbers Written as Decimals

Learn: Rational Numbers Written as Decimals
Example 1: Write Fractions as Decimals
Example 2: Write Fractions as Decimals
Learn: Write Repeating Decimals as Fractions
Example 3: Write Repeating Decimals as Fractions
Example 4: Write Repeating Decimals as Mixed Numbers
Apply: Crafting

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Repeating Decimals and Equivalence: Why Does 0.999... Equal 1?, Special Fraction-Decimal Equivalents		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 19 of the *Language Development Handbook* to help your students build mathematical language related to rational numbers.

ELL You can use the tips and suggestions on page T19 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System

Major Cluster(s): In this lesson, students address major cluster **7 .NS.A** by converting a rational number to a decimal using long division and identifying terminating or repeating decimals.

Supporting Cluster(s): In this lesson, students address the supporting cluster **8.NS.A** by converting rational numbers between decimal and fraction forms.

Standards for Mathematical Content: 7. NS.A.2, 7.NS.A.2.B, 7.NS.A.2.D, 8.NS.A.1, *Also addresses 7.NS.A.3, 7.EE.B.3*

Standards for Mathematical Practice: MP 1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Coherence

Vertical Alignment

Previous
Students solved problems by applying all operations to integers.
7.NS.A.3

Now
Students identify terminating and repeating decimals, and use long division to convert rational numbers to decimals.
7.NS.A.2.B, 7.NS.A.2.D, 8.NS.A1

Next
Students will add and subtract rational numbers.
7.NS.A.1, 7.NS.A.1.C

Rigor

The Three Pillars of Rigor

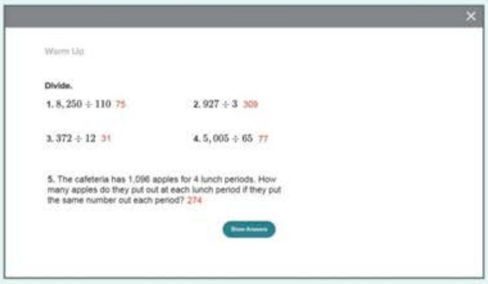
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of rational numbers to develop the *understanding* that rational numbers can be represented as fractions or decimals. They will use this understanding to build *fluency* in writing rational numbers as either terminating or repeating fractions.

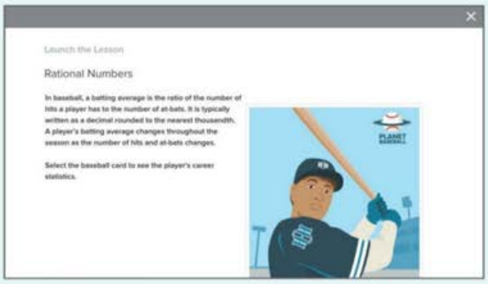
Mathematical Background

A *rational number* is any number that can be written as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Every rational number, or fraction, can be written as a repeating decimal. Some *repeating decimals* repeat zeros, such as 0.25000.... Repeating decimals in which zeros repeat are also called *terminating decimals*, because they can be written without the repeating zeros. For decimals in which non-zero numbers repeat, use bar notation to show that some or all of the digits of the decimal repeat infinitely.

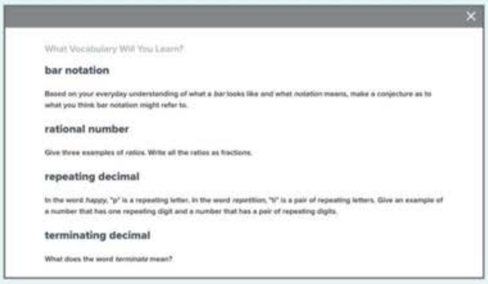
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- dividing whole numbers (Exercises 1–5)

Answers

1. 75	**4.** 77
2. 309	**5.** 274
3. 31	

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about finding batting average as a decimal.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- Based on your everyday understanding of what a *bar* looks like and what *notation* means, make a conjecture as to what you think bar notation might refer to. Sample answer: Bar notation might refer to a way of writing a number (notation) that includes a bar.
- The word *ratio* makes up part of the term *rational*. How can you use your knowledge of ratios to help understand what a rational number might be? Sample answer: A *ratio* is the comparison of two quantities by division. So, a rational number might be a number that can be written as a ratio.

Explore Rational Numbers Written as Decimals

Objective

Students will use Web Sketchpad to explore how to convert a rational number to a decimal.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch called the color calculator. The color calculator displays patterns in the digits of the decimal form of any fraction. Throughout this activity, students will use the color calculator to display the decimal form of several fractions. They will use the patterns in the digits of the decimal form to make conjectures about which fractions eventually repeat zeros and which fractions repeat nonzero digits, when written as decimals.

Inquiry Question

What are the patterns in the decimal form of a rational number? Sample answer: If the prime factorization of the denominator contains only twos, only fives, or a combination of twos and fives, the decimal will eventually repeat zeros. Otherwise, the decimal will repeat nonzero digits.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

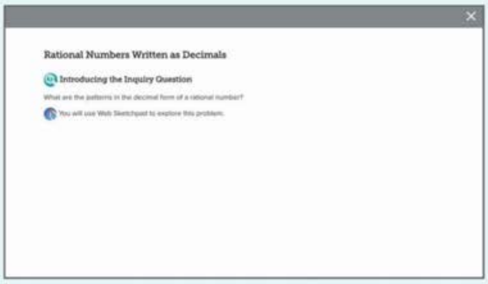
SLIDE 3

Mathematical Discourse

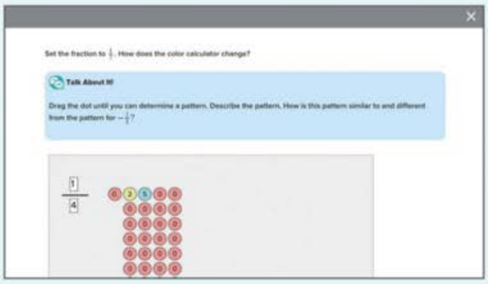
Drag the dot until you can determine a pattern. Describe the pattern. How is the pattern similar to and different from the pattern for $-\frac{1}{3}$?
Sample answer: Both decimals have repeating digits. In the pattern for $-\frac{1}{3}$, only one digit repeats and all the digits are colored green. But for $\frac{1}{7}$, a group of six digits repeats, so there are six different colors that repeat in order.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 7

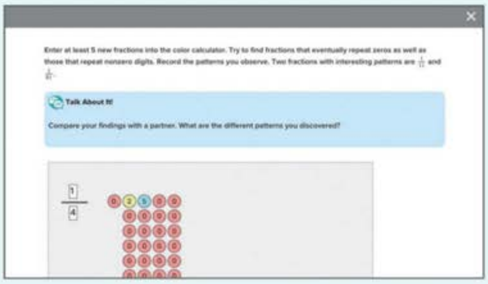


Explore, Slide 3 of7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the patterns in the decimal forms of rational numbers.

Interactive Presentation



Explore, Slide 4 of 7

DRAG & DROP

On Slide 6, students drag to sort fractions by decimal patterns.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Rational Numbers Written as Decimals *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use the color calculator sketch to help them gain insight into the decimal form of a rational number.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Name three different fractions that eventually repeat zeros. Then name three fractions that will repeat nonzero digits. Sample answer: Some examples of fractions that eventually repeat zeros are $\frac{1}{2}, -\frac{3}{4}, \frac{7}{10}$… . Some examples of fractions that repeat nonzero digits are $\frac{1}{6}, \frac{3}{13}, \frac{8}{15}$… .

Learn Rational Numbers

Objective

Students will learn how to identify rational numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to pause and consider the meaning of the negative sign and how its placement in the numerator or denominator of a fraction may or may not affect the value of the fraction.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, they should use clear and precise mathematical language in their responses.

Teaching Notes

SLIDE 1

Select the cards and ask students what they notice is true for all the rational numbers. Students should notice that all the rational numbers can be written as fractions. Remind them that the denominator cannot be zero because division by zero is undefined. Have students explain why the sign of a number does not determine whether or not the number is rational. Encourage students to give examples of other integers, fractions, decimals, and percents that are rational numbers.

Talk About It!

SLIDE 2

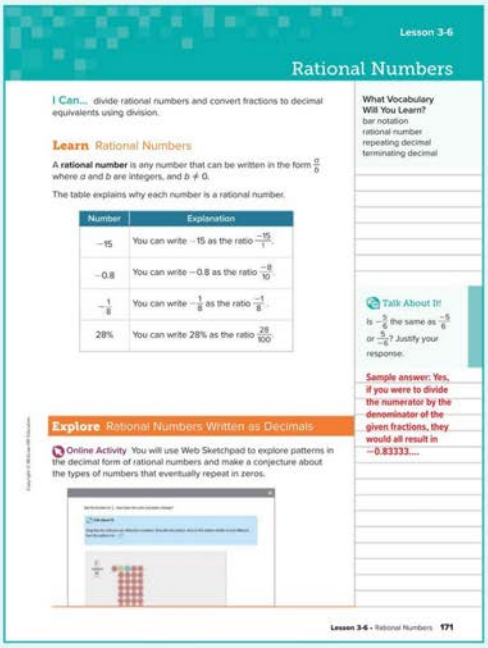
Mathematical Discourse

Is $-\frac{5}{6}$ the same as $\frac{-5}{6}$ or $\frac{5}{-6}$? Justify your response. Sample answer: Yes, if you were to divide the numerator by the denominator of the given fractions, they would all result in −0.83333....

DIFFERENTIATE

Language Development Activity ELL

To support students' understanding of rational numbers, have them work with a partner to generate at least 4 different numbers (integer, decimal, fraction, and percent) and explain why each one is a rational number according to the definition of a rational number. At least two of their selected numbers should be negative.



Interactive Presentation

Learn, Rational Numbers, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to find out why certain numbers are rational.

Your Notes

Math History Minute

Katherine Johnson (1918-) was a research mathematician who was hired to be a "human computer" by the National Advisory Committee for Aeronautics, later known as NASA. While computers had been programmed to control the movements of astronaut John Glenn's space capsule in 1962, Glenn asked Katherine to perform the same calculations by hand to confirm that the computer's calculations were correct.

Learn Rational Numbers Written as Decimals

Any fraction can be expressed as a decimal by dividing the numerator by the denominator. The decimal form of a rational number either terminates in 0s or eventually repeats. **Repeating decimals** are decimals in which 1 or more digits repeat and can be represented using bar notation. In **bar notation**, a bar is drawn only over the digit(s) that repeat.

The following decimals are written in bar notation.

$-0.44444... = -0.\overline{4}$ The digit 4 repeats.

$2.4343... = 2.\overline{43}$ The digits 43 repeat.

Complete the table by writing each decimal using bar notation.

Decimal	Bar Notation
0.11111....	$0.\overline{1}$
0.61111....	$0.6\overline{1}$
0.616161....	$0.\overline{61}$
6.160000....	$6.16\overline{0}$
6.1611611611....	$6.\overline{161}$

Every decimal can be considered a repeating decimal. Decimals with a repeating digit of zero are also called **terminating decimals**, because the repeating zeros in a terminating decimal are usually truncated, or dropped. For example, the terminating decimal $0.25\overline{0}$ is written as 0.25. The decimal $0.25\overline{0}$ can be considered repeating because the digit 0 repeats.

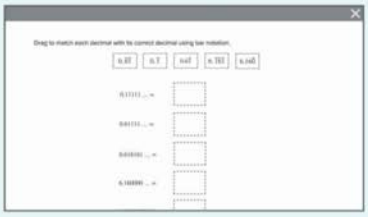
Pause and Reflect

Are you ready to move on to the Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

See students' observations.

172 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Learn, Rational Numbers Written as Decimals, Slide 2 of 3

CLICK

On Slide 1, students move a slider to see decimals written using bar notation.

DRAG & DROP

On Slide 2, students drag to match each number with its correct bar notation.

Learn Rational Numbers Written as Decimals

Objective

Students will understand that the decimal form of a rational number either terminates in 0s or eventually repeats.

MP Teaching the Mathematical Practices

7 Look for and Make Use of Structure S tudents should analyze the structure of each decimal represented using bar notation and the structure of each decimal represented without bar notation in order to complete the drag and drop activity on Slide 2.

Teaching Notes

SLIDE 1

Be sure students understand that the decimal form of a rational number either terminates in 0s or eventually repeats. Drag the slider to see two repeating decimals written using bar notation. You may wish to ask students to generate other examples of repeating decimals.

SLIDE 2

Prior to completing the drag and drop activity, you may wish to ask students to identify the digits that repeat in each decimal. This will help them determine where the bar is placed. Be sure they understand that the bar is placed only over the digits that repeat.

SLIDE 3

Point out that a terminating decimal can be considered a repeating decimal. Students should note that a terminating decimal's repeating digit is zero and it is usually not written using bar notation.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of bar notation, have them write the expanded form of the each of the following decimals.

$0.\overline{181}$ 0.181181181...

$8.1\overline{4411}$ 8.14411411411...

$-1.\overline{5758}$ −1.575857585758...

$-0.70\overline{1232}$ −0.70123212321232...

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 1 Write Fractions as Decimals

Objective

Students will use long division to convert a fraction to a decimal and determine if the decimal is terminating.

Teaching the Mathematical Practices

8 Look For and Express Regularity in Repeated Reasoning
Encourage students to look for repeating calculations when writing fractions as decimals, in order to determine if the decimal is terminating.

Questions for Mathematical Discourse

SLIDE 2

AL Why do we divide? Sample answer: The fraction bar is another way of showing division.

AL Which number will be the dividend? the numerator, 1

AL Which number will be the divisor? the denominator, 40

OL How do you write $\frac{1}{40}$ as a decimal? Use long division to divide the numerator of 1 by the denominator of 40.

OL What is the first step? Sample answer: Divide 1 by 40, write the result above the bar, and find the remainder.

BL Evaluate $\frac{1}{40} + \frac{1}{8}$. Write the answer as a decimal. 0.15

SLIDE 3

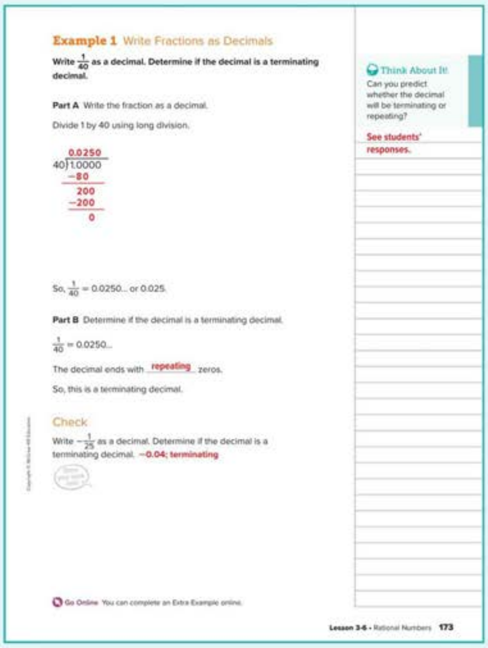
AL Why did we stop dividing? The remainder was 0.

OL Is the fraction terminating? Explain. Yes; the decimal ends with repeating zeros.

BL Determine if $\frac{1}{40} + \frac{1}{8}$ is a terminating decimal. yes

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Write Fractions as Decimals

Write $\frac{1}{40}$ as a decimal. Determine if the decimal is a terminating decimal.

Part A Write the fraction as a decimal.

Divide 1 by 40 using long division.

$$\begin{array}{r} 0.0250 \\ 40\overline{)1.0000} \\ -80 \\ \hline 200 \\ -200 \\ \hline 0 \end{array}$$

So, $\frac{1}{40}$ = 0.0250... or 0.025.

Part B Determine if the decimal is a terminating decimal.

$\frac{1}{40}$ = 0.0250...

The decimal ends with repeating zeros.

So, this is a terminating decimal.

Check

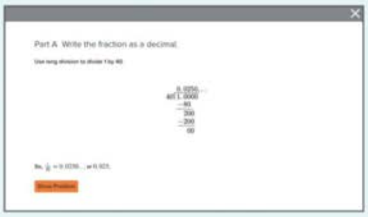
Write $-\frac{1}{25}$ as a decimal. Determine if the decimal is a terminating decimal. −0.04; terminating

Think About It! Can you predict whether the decimal will be terminating or repeating? See students' responses.

Go Online You can complete an Extra Example online.

Lesson 3-6 • Rational Numbers 173

Interactive Presentation



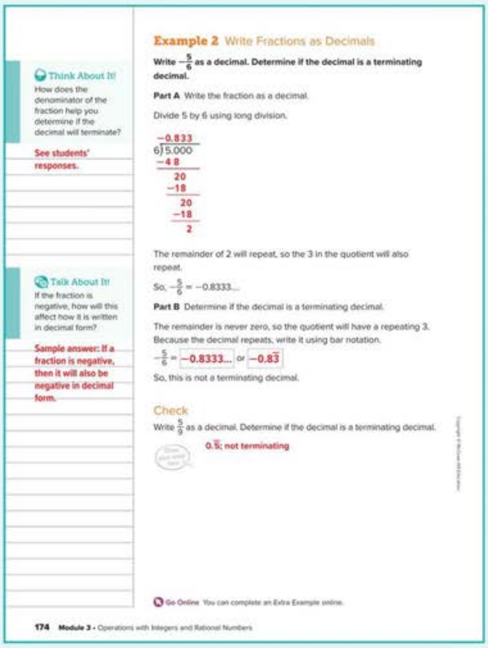
Example 1, Write Fractions as Decimals, Slide 2 of 4

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.NS.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION



Think About It! How does the denominator of the fraction help you determine if the decimal will terminate?

See students' responses.

Talk About It! If the fraction is negative, how will this affect how it is written in decimal form?

Sample answer: If a fraction is negative, then it will also be negative in decimal form.

Example 2 Write Fractions as Decimals

Write $-\frac{5}{6}$ as a decimal. Determine if the decimal is a terminating decimal.

Part A Write the fraction as a decimal.

Divide 5 by 6 using long division.

$$\begin{array}{r} -0.833 \\ 6\overline{)5.000} \\ -4\,8 \\ \hline 20 \\ -18 \\ \hline 20 \\ -18 \\ \hline 2 \end{array}$$

The remainder of 2 will repeat, so the 3 in the quotient will also repeat.

So, $-\frac{5}{6} = -0.8333...$

Part B Determine if the decimal is a terminating decimal.

The remainder is never zero, so the quotient will have a repeating 3. Because the decimal repeats, write it using bar notation.

$-\frac{5}{6} = -0.8333...$ or $-0.8\overline{3}$

So, this is not a terminating decimal.

Check

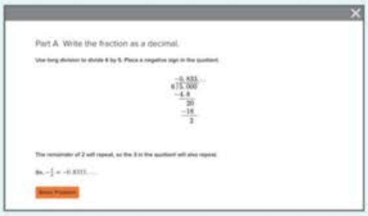
Write $\frac{5}{9}$ as a decimal. Determine if the decimal is a terminating decimal.

$0.\overline{5}$; not terminating

Go Online You can complete an Extra Example online.

174 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Example 2, Write Fractions as Decimals, Slide 2 of 5

CLICK

On Slide 3, students select from a drop-down menu whether or not a decimal is terminating.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Write Fractions as Decimals

Objective

Students will use long division to convert a fraction to a decimal and determine if the decimal is terminating.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 4, encourage them to reason that if a fraction is negative, then its decimal form is also negative.

8 Look for and Express Regularity in Repeated Reasoning Encourage students to look for repeating calculations when writing fractions as decimals in order to determine if the decimal is terminating.

Questions for Mathematical Discourse

SLIDE 2

AL How would you express –5 divided by 6 using long division? Use long division to divide 5 by 6. Then place a negative sign in the quotient.

OL Will the division when dividing 5 by 6 ever end? Why or why not? no; Sample answer; The digit 3 repeats, so there will always be a remainder.

BL Give two other examples of a fraction in which the remainder will always repeat. Sample answer: $\frac{1}{3}, \frac{2}{11}$

SLIDE 3

AL How can you indicate that a decimal repeats? Use bar notation over the repeating number(s).

AL Is the fraction terminating? no

OL Which digit repeats in this example? 3 Where do you place the bar? over the 3

OL Why is bar notation helpful when writing fractions as decimals? Sample answer: It clearly indicates the digit or group of digits that repeats.

BL How do you know that the decimal will repeat nonzero digits without dividing? Sample answer: The prime factorization of the denominator of the simplified fraction does not contain only twos, only fives, or a combination of twos and fives, so the decimal repeats nonzero digits.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Write Repeating Decimals as Fractions

Objective

Students will learn how to convert repeating, non-terminating decimals into fractions.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* questions, encourage them to use mathematical reasoning to create a plausible argument for why the repeating decimal $0.\overline{4}$ is not equal to either of the fractions. Ask students how they can use and extend this reasoning to determine if $0.\overline{4}$ is a rational number.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates writing a repeating, non-terminating decimal as a fraction.
- Find sample answers for the *Talk About It!* questions.

Example 3 Write Repeating Decimals as Fractions

Objective

Students will write repeating, non-terminating decimals as fractions in simplest form.

Questions for Mathematical Discourse

SLIDE 2

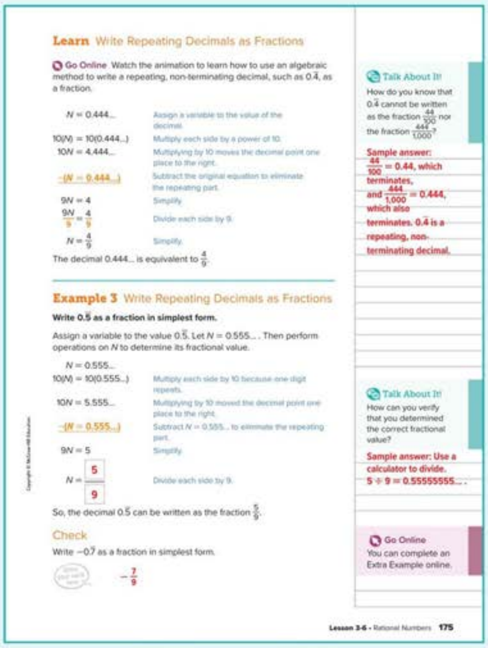
AL What does the bar notation in $0.\overline{5}$ mean? It means that the digit 5 repeats forever.

OL What is the purpose of assigning the decimal to a variable and multiplying it by a power of 10? Sample answer: to be able to use subtraction afterwards to eliminate the repeating part of the decimal

BL What would you have multiplied N by if N were 0.505050...? Explain. 100; Sample answer: Multiply by ten to the power of the number of repeating digits.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Write Repeating Decimals as Fractions

Go Online Watch the animation to learn how to use an algebraic method to write a repeating, non-terminating decimal, such as $0.\overline{4}$, as a fraction.

$N = 0.444...$	Assign a variable to the value of the decimal.
$10(N) = 10(0.444...)$	Multiply each side by a power of 10.
$10N = 4.444...$	Multiplying by 10 moves the decimal point one place to the right.
$-(N = 0.444...)$	Subtract the original equation to eliminate the repeating part.
$9N = 4$	Simplify.
$\frac{9N}{9} = \frac{4}{9}$	Divide each side by 9.
$N = \frac{4}{9}$	Simplify.

The decimal 0.444... is equivalent to $\frac{4}{9}$.

Talk About It! How do you know that $0.\overline{4}$ cannot be written as the fraction $\frac{44}{100}$ nor the fraction $\frac{444}{1,000}$?

Sample answer: $\frac{44}{100} = 0.44$, which terminates, and $\frac{444}{1,000} = 0.444$, which also terminates. $0.\overline{4}$ is a repeating, non-terminating decimal.

Example 3 Write Repeating Decimals as Fractions

Write $0.\overline{5}$ as a fraction in simplest form.

Assign a variable to the value $0.\overline{5}$. Let $N = 0.555...$. Then perform operations on N to determine its fractional value.

$N = 0.555...$	
$10(N) = 10(0.555...)$	Multiply each side by 10 because one digit repeats.
$10N = 5.555...$	Multiplying by 10 moved the decimal point one place to the right.
$-(N = 0.555...)$	Subtract $N = 0.555...$ to eliminate the repeating part.
$9N = 5$	Simplify.
$N = \frac{5}{9}$	Divide each side by 9.

So, the decimal $0.\overline{5}$ can be written as the fraction $\frac{5}{9}$.

Talk About It! How can you verify that you determined the correct fractional value?

Sample answer: Use a calculator to divide. $5 \div 9 = 0.55555555...$.

Check

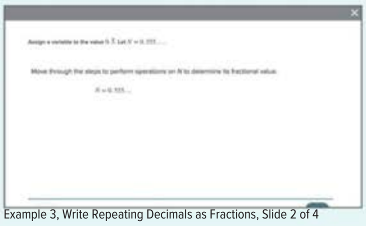
Write $-0.\overline{7}$ as a fraction in simplest form.

$-\frac{7}{9}$

Go Online You can complete an Extra Example online.

Lesson 3-6 • Rational Numbers 175

Interactive Presentation



Example 3, Write Repeating Decimals as Fractions, Slide 2 of 4

TYPE

On Slide 2 of Example 3, students enter the equivalent fraction for the decimal.

CLICK

On Slide 2 of Example 3, students move through the steps to write the decimal as a fraction.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Think About It!
How many digits repeat?
two

Talk About It!
In the second line of the solution, why do you multiply by 100 instead of 10?
Sample answer: Because two digits repeat.

Example 4 Write Repeating Decimals as Mixed Numbers

Write $2.\overline{18}$ as a mixed number in simplest form.

Assign a variable to the value $2.\overline{18}$. Let $N = 2.181818...$. Then perform operations on N to determine its fractional value.

$N = 2.181818...$

$100(N) = 100(2.181818...)$

$100N = 218.181818...$

$-(N = 2.181818...)$

$99N = 216$

$N = \frac{216}{99}$

$N = 2\frac{2}{11}$

So, the decimal $2.\overline{18}$ can be written as $2\frac{2}{11}$.

Check

Write $1.\overline{42}$ as a mixed number in simplest form.

$1\frac{14}{33}$

Go Online You can complete an Extra Example online.

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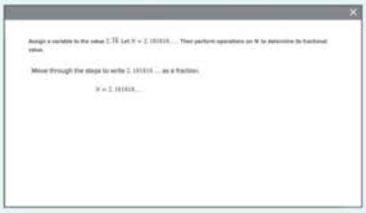
Interactive Presentation



Example 4, Write Repeating Decimals as Mixed Numbers, Slide 2 of 4

TYPE

On Slide 2, students enter the mixed number in simplest form.

CLICK

On Slide 2, students move through the steps to write a decimal as a mixed number.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Write Repeating Decimals as Mixed Numbers

Objective

Students will write repeating, non-terminating decimals as mixed numbers in simplest form.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to explain to themselves the purpose behind assigning the decimal to a variable and multiplying it by a power of 10. Students should understand why multiplying by that power of 10 will eventually eliminate the decimal portion and why that is beneficial when writing the decimal as a fraction.

While discussing the *Talk About It!* question on Slide 3, encourage students to make a connection between the number of digits that repeat in a decimal, and the power of 10 that is multiplied by both sides of the equation.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the decimal to identify the number of digits that repeat.

Questions for Mathematical Discourse

SLIDE 2

AL Identify the repeating digit(s). 1 and 8

OL Explain why we multiply by 100. Sample answer: There are two digits that repeat, so multiply by 10^2, which is 100.

OL Explain why we subtract 2.181818... . Sample answer: to eliminate the repeating part of the decimal

BL If $2.\overline{18} = 2\frac{2}{11}$, make a conjecture as to the fractional form of the decimal $2.\overline{09}$. Explain your reasoning. Sample answer: $0.\overline{09}$ is half of $0.\overline{18}$. So, the fractional form of $0.\overline{09}$ will be half of $\frac{2}{11}$, or $\frac{1}{11}$. Adding the whole number 2 to make it a mixed number means the fractional form of $2.\overline{09}$ will be $2\frac{1}{11}$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Crafting

Objective

Students will come up with their own strategy to solve an application problem involving the sizes of different signs at a craft show.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What types of numbers do you see in the problem?
- How do you compare numbers written in different forms?
- Would you rather change fractions to decimals, or decimals to fractions?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Crafting

Two vendors at a craft show are selling signs. The lengths of signs offered by Signs & More and Wood Works are shown in the table. Each sign has the same price. Henry wants to buy the longest of each kind of sign. From which vendor should he buy each style of sign?

	Signs & More	Wood Works
rounded corner sign	$16\frac{1}{8}$ in.	16.25 in.
square corner sign	$16\frac{1}{3}$ in.	16.3 in.

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

rounded corner sign: Wood Works; square corner sign: Signs and More; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.
See students' arguments.

Talk About It!
Did you convert any of the numbers to a different form in order to solve the problem? Explain.

Sample answer: Yes, in order to compare the measurements, it is helpful to convert the mixed numbers to decimals.

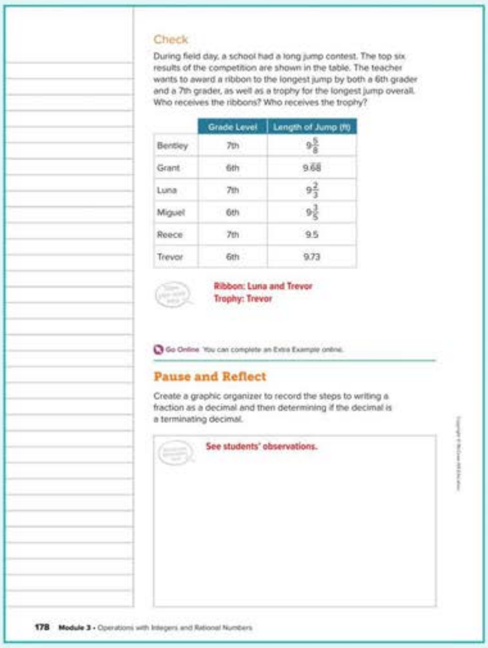
Lesson 3-6 • Rational Numbers 177

Interactive Presentation

Apply, Crafting

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

During field day, a school had a long jump contest. The top six results of the competition are shown in the table. The teacher wants to award a ribbon to the longest jump by both a 6th grader and a 7th grader, as well as a trophy for the longest jump overall. Who receives the ribbons? Who receives the trophy?

	Grade Level	Length of Jump (ft)
Bentley	7th	$9\frac{5}{8}$
Grant	6th	$9.\overline{68}$
Luna	7th	$9\frac{2}{3}$
Miguel	6th	$9\frac{3}{5}$
Reece	7th	9.5
Trevor	6th	9.73

Ribbon: Luna and Trevor
Trophy: Trevor

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer to record the steps to writing a fraction as a decimal and then determining if the decimal is a terminating decimal.

See students' observations.

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Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Suppose a baseball player had *a* hits in *b* at-bats. Explain how to find the baseball player's batting average.

Sample answer: Divide the number of hits, *a*, by the number of at-bats, *b*. Then round the decimal to the nearest thousandth.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 11, 13–17
- Extension: Repeating Decimals and Equivalence: Why Does 0.999... Equal 1?, Special Fraction-Decimal Equivalents
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 66-89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–11, 13, 14, 16
- Extension: Repeating Decimals and Equivalence: Why Does 0.999... = 1?, Special Fraction-Decimal Equivalents
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Venn Diagrams and Sets of Rational Numbers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

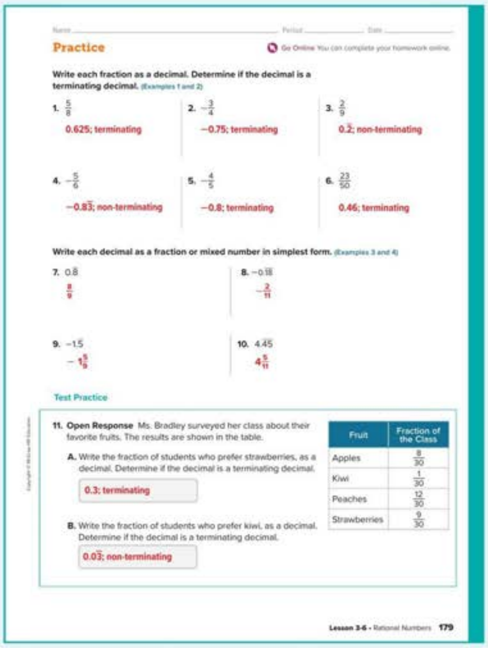
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use long division to convert a fraction to a decimal and determine if the decimal is terminating	1–6
1	write repeating decimals as fractions and mixed numbers	7–10
2	extend concepts learned in class to apply them in new contexts	11
3	solve application problems involving rational numbers	12, 13
3	higher-order and critical thinking skills	14–17

Common Misconception

Some students may incorrectly think that a repeating decimal is one in which only one value is repeated rather than a sequence of values. A student with this misconception would consider decimals of the form 0.2222... repeating but not 0.232323... .



Name ______ Period ______ Date ______

Practice Go Online You can complete your homework online.

Write each fraction as a decimal. Determine if the decimal is a terminating decimal. (Examples 1 and 2)

1. $\frac{5}{8}$ 0.625; terminating
2. $-\frac{3}{4}$ −0.75; terminating
3. $\frac{2}{9}$ $0.\overline{2}$; non-terminating
4. $-\frac{5}{6}$ $-0.8\overline{3}$; non-terminating
5. $-\frac{4}{5}$ −0.8; terminating
6. $\frac{23}{50}$ 0.46; terminating

Write each decimal as a fraction or mixed number in simplest form. (Examples 3 and 4)

7. $0.\overline{8}$ $\frac{8}{9}$
8. $-0.\overline{18}$ $-\frac{2}{11}$
9. $-1.\overline{5}$ $-1\frac{5}{9}$
10. $4.\overline{45}$ $4\frac{5}{11}$

Test Practice

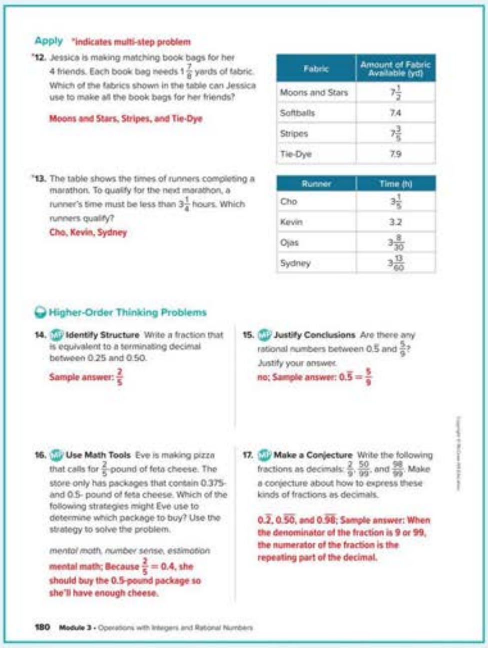
11. **Open Response** Ms. Bradley surveyed her class about their favorite fruits. The results are shown in the table.

A. Write the fraction of students who prefer strawberries, as a decimal. Determine if the decimal is a terminating decimal.
0.3; terminating

B. Write the fraction of students who prefer kiwi, as a decimal. Determine if the decimal is a terminating decimal.
$0.0\overline{3}$; non-terminating

Fruit	Fraction of the Class
Apples	$\frac{8}{30}$
Kiwi	$\frac{1}{30}$
Peaches	$\frac{12}{30}$
Strawberries	$\frac{9}{30}$

Lesson 3-6 • Rational Numbers 179



Apply *indicates multi-step problem

*12. Jessica is making matching book bags for her 4 friends. Each book bag needs $1\frac{7}{8}$ yards of fabric. Which of the fabrics shown in the table can Jessica use to make all the book bags for her friends?

Moons and Stars, Stripes, and Tie-Dye

Fabric	Amount of Fabric Available (yd)
Moons and Stars	$7\frac{1}{2}$
Softballs	7.4
Stripes	$7\frac{3}{5}$
Tie-Dye	7.9

*13. The table shows the times of runners completing a marathon. To qualify for the next marathon, a runner's time must be less than $3\frac{1}{4}$ hours. Which runners qualify?

Cho, Kevin, Sydney

Runner	Time (h)
Cho	$3\frac{1}{5}$
Kevin	3.2
Ojas	$3\frac{8}{30}$
Sydney	$3\frac{13}{60}$

Higher-Order Thinking Problems

14. **Identify Structure** Write a fraction that is equivalent to a terminating decimal between 0.25 and 0.50.

Sample answer: $\frac{2}{5}$

15. **Justify Conclusions** Are there any rational numbers between $0.\overline{5}$ and $\frac{5}{9}$? Justify your answer.

no; Sample answer: $0.\overline{5} = \frac{5}{9}$

16. **Use Math Tools** Eve is making pizza that calls for $\frac{2}{5}$-pound of feta cheese. The store only has packages that contain 0.375- and 0.5- pound of feta cheese. Which of the following strategies might Eve use to determine which package to buy? Use the strategy to solve the problem.

mental math, number sense, estimation

mental math; Because $\frac{2}{5} = 0.4$, she should buy the 0.5-pound package so she'll have enough cheese.

17. **Make a Conjecture** Write the following fractions as decimals: $\frac{2}{9}$, $\frac{50}{99}$, and $\frac{98}{99}$. Make a conjecture about how to express these kinds of fractions as decimals.

$0.\overline{2}$, $0.\overline{50}$, and $0.\overline{98}$; Sample answer: When the denominator of the fraction is 9 or 99, the numerator of the fraction is the repeating part of the decimal.

180 Module 3 • Operations with Integers and Rational Numbers

Teaching the Mathematical Practices

7 Look For and Make Use of Structure In E xercise 14, students write a fraction with an equivalent terminating decimal between 0.25 and 0.50.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 15, students determine if any rational numbers lie between $0.\overline{5}$ and $\frac{5}{9}$ and justify their conclusions.

5 Use Appropriate Tools Strategically In Exercise 16, students determine which strategy helps them solve a problem.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students make a conjecture about how to express certain kinds of fractions as decimals.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 12 Have students work together to prepare a brief demonstration that illustrates why this problem is an application problem. For example, before she can compare the total amount of fabric she needs to the available fabric, she must multiply the amount of fabric each book bag needs by the number of book bags she will be making. Have each pair or group of students present their response to the class.

Be sure everyone understands.

Use with Exercises 14–15 Have students work in groups of 3–4 to solve the problem in Exercise 14. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 15.

Lesson 3-7

Add and Subtract Rational Numbers

7.NS.A.1

LESSON GOAL

Students will demonstrate application of the additive inverse, and an understanding of addition and subtraction of rational numbers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Rational Numbers and Additive Inverses
Examples 1-2: Find Additive Inverses
Learn: Add Rational Numbers
Examples 3-4: Add Rational Numbers
Learn: Add Rational Numbers
Example 5: Add Rational Numbers
Learn: Subtract Rational Numbers
Examples 6-7: Subtract Rational Numbers
Example 8: Evaluate Expressions
Apply: Animal Care

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 20 of the *Language Development Handbook* to help your students build mathematical language related to addition and subtraction of rational numbers.

ELL You can use the tips and suggestions on page T20 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min: 0.5 day
45 min: 1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7. NS.A** by finding the additive inverse, adding, and subtracting rational numbers.
Standards for Mathematical Content: **7 .NS.A.1, 7.NS.A.1.A, 7.NS.A.1.B, 7.NS.A.1.C, 7.NS.A.1.D, 7.EE.B.3,** *Also addresses 7.NS.A.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students used long division to convert rational numbers to decimals.
7.NS.A.2.D

Now
Students demonstrate application of the additive inverse and an understanding of addition and subtraction of rational numbers.
7.NS.A.1, 7.NS.A.1.B, 7.NS.A.1.C

Next
Students will multiply and divide rational numbers.
7.NS.A.2, 7.NS.A.3

Rigor

The Three Pillars of Rigor

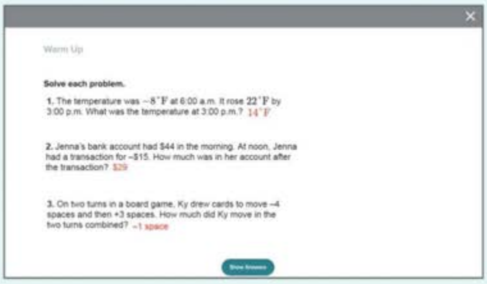
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of rational numbers and integers to develop the *understanding* of adding and subtracting rational numbers. Students will use their understanding to gain *fluency* in adding and subtracting rational numbers. Students also *apply* their understanding of adding and subtracting rational numbers to real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- adding integers (Exercises 1–3)

Answers

1. 14°F
2. $29
3. −1 space

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about finding changes in temperature.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does the term *inverse* mean? Sample answer: An inverse is the opposite or reverse of something.
- Give two examples of *opposites* in everyday language. Sample answer: on and off; up and down

Learn Rational Numbers and Additive Inverses

Objective

Students will learn how to find the additive inverse of a rational number.

Teaching Notes

SLIDE 2

Use the animation to help students see how the sum of additive inverses is equal to zero. Ask students to write in words the steps shown in the animation. They should note to start at zero, move $\frac{3}{4}$ unit to the left to show $-\frac{3}{4}$, then move $\frac{3}{4}$ unit to the right to show $+\frac{3}{4}$.

Go Online

- Find additional teaching notes.
- Have students watch the brief animation on Slide 2. The animation illustrates how the sum of additive inverses is equal to zero.

Example 1 Find Additive Inverses

Objective

Students will find the additive inverse of rational numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to understand the relationship between a number and its additive inverse, specifically in how that relationship can be shown on a number line.

Questions for Mathematical Discourse

SLIDE 1

AL How can you find the opposite of a number on a number line? Sample answer: Find the points that are the same distance from zero, but on the opposite sides of zero.

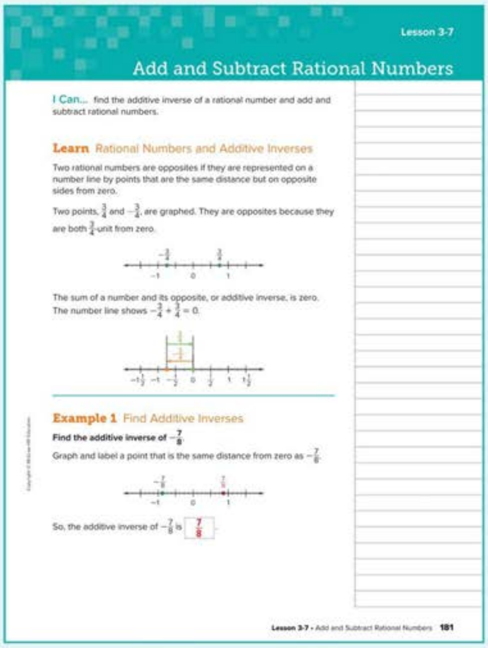
OL What is the opposite of $\frac{7}{8}$? $-\frac{7}{8}$

OL What is the sum of $\frac{7}{8}$ and $-\frac{7}{8}$? 0

BL Write $-\frac{7}{8}$ as a decimal and find its additive inverse. -0.875; The additive inverse is 0.875.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Lesson 3-7

Add and Subtract Rational Numbers

I Can... find the additive inverse of a rational number and add and subtract rational numbers.

Learn Rational Numbers and Additive Inverses

Two rational numbers are opposites if they are represented on a number line by points that are the same distance but on opposite sides from zero.

Two points, $\frac{3}{4}$ and $-\frac{3}{4}$, are graphed. They are opposites because they are both $\frac{3}{4}$-unit from zero.

The sum of a number and its opposite, or additive inverse, is zero. The number line shows $-\frac{3}{4} + \frac{3}{4} = 0$.

Example 1 Find Additive Inverses

Find the additive inverse of $-\frac{7}{8}$.

Graph and label a point that is the same distance from zero as $-\frac{7}{8}$.

So, the additive inverse of $-\frac{7}{8}$ is $\frac{7}{8}$.

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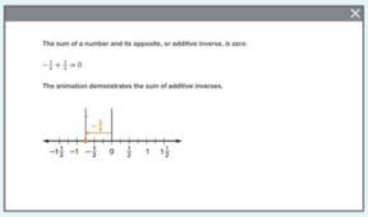
Interactive Presentation



Learn, Rational Numbers and Additive Inverses, Slide 2 of 2

WATCH On Slide 2 of the Learn, students watch a brief animation that illustrates the sum of additive inverses.

CLICK On Slide 1 of Example 1, students select buttons to graph a fraction and its additive inverse.

CHECK Students complete the Check exercise online to determine if they are ready to move on.


Check

Find the additive inverse of $-\frac{1}{8}$

Go Online You can complete an Extra Example online.

Think About It! Should Annalise's babysitting and part-time job money be represented with positive or negative numbers?

positive

Example 2 Find Additive Inverses

Annalise earned $36.82 at her part time job, and she earned $18.50 babysitting.

Find the total amount she earned, the additive inverse, and describe a situation so that Annalise ends the week with zero dollars.

Part A Find the total amount she earned.

If p is the amount of money Annalise earned at her part time job, b is the amount of money Annalise earned while babysitting, and t is the total amount of money earned, then

$t = p + b$

$= \$36.82 + \$18.50$

$= \$55.32$

Part B Find the additive inverse.

Annalise ended the week with $0. What number could you add to $55.32 that would result in a sum of $0?

−$55.32

Talk About It! How do you know that losing $55.32, donating $55.32, and spending $55.32 all represent a negative value?

Sample answer: Each of those examples represent money being taken away, or subtracted from the total amount.

Part C Describe a situation so Annalise ends the week with zero dollars.

Circle the situations that represent −$55.32.

losing $55.32	getting a gift of $55.32
finding $55.32	earning $55.32
donating $55.32	spending $55.32

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Interactive Presentation

Example 2, Find Additive Inverses, Slide 4 of 6

TYPE

On Slide 2, students enter a missing value to complete a sum.

DRAG & DROP

On Slide 4, students drag to sort situations that represent −$55.32.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Find Additive Inverses

Objective

Students will find the additive inverse of rational numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of the three given phrases and why they can each be represented by a negative quantity.

Encourage students to apply the mathematics they know, such as finding the additive inverse of the total amount Annalise earned, to generate a real-world problem in which Annalise will end the week with zero dollars.

Questions for Mathematical Discourse

SLIDE 2

AL Annalise earned two amounts. What are those amounts? $36.82 and $18.50

AL How can you find the total amount Annalise earned? Sample answer: Add the amount she earned at her part time job and the amount she earned babysitting.

OL What is the total amount Annalise earned? $55.32

BL Describe a situation that can be represented by a negative amount. Sample answer: losing something

SLIDE 4

AL With what amount of money does Annalise need to end the week? $0

OL What could Annalisa do to end with $0? Sample answer: She could spend $55.32.

OL What other phrases beside these three can be represented by a negative amount? Sample answers: paying, lending

BL Give an example of a real-world problem that involves finding the additive inverse of a number. Sample answer: Nadine earned $*x* babysitting. Of the money she earned, she spent $20 on jeans and $10 on a shirt. She has $0 remaining. How much money did she earn?

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Add Rational Numbers

Objective

Students will understand that they can apply what they know about adding fractions, decimals, and integers to the set of rational numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the bar notation and what it means for the quantity $3.1\overline{6}$ to be an addend in an addition problem with the negative number −2.5.

Teaching Notes

SLIDE 1

Be sure students understand that to add rational numbers, they can apply the same rules they used for adding fractions, decimals, and integers. Present the chart and have students explain whether the decimal or fraction form should be used when adding rational numbers in different forms. Student should note that if the decimal ends in repeating digits that are zeros, or it terminates, they could use either form; if the decimal ends in nonzero repeating digits, they should use the fraction form.

Talk About It!

SLIDE 2

Mathematical Discourse

Why would it be difficult to add −2.5 and $3.1\overline{6}$? Sample answer: $3.1\overline{6}$ is a repeating decimal, and it never ends. It would be hard to add them because you wouldn't be able to write out all of the digits of $3.1\overline{6}$.

DIFFERENTIATE

Reteaching Activity AL

Some students may struggle in deciding when it might be more efficient to write numbers as decimals or fractions when adding rational numbers. Have them work with a partner to generate several examples of rational number addition expressions in which it is more efficient to write the addends as fractions. Have them explain their reasoning. Then have them generate several examples in which it is more efficient to write the addends as decimals, and explain their reasoning. Have them compare their examples with another pair of students.

Check

Zoey spent $12 on a video and $25.82 on a poster at the music store. Find the total amount she spent, the additive inverse, and describe a situation so that Zoey ends the week with zero dollars.

Part A What was the total amount? $37.82

Part B What is the additive inverse of the total? −$37.82

Part C Which describes a situation so that Zoey ends the week with zero dollars?

● Zoey earned $37.82 at her lemonade stand.
Ⓑ Zoey spent $37.82 on dinner and a movie with friends.
Ⓒ Zoey and a friend split the cost of a video game that cost $37.82.
Ⓓ Zoey got a $35 gift card for her birthday.

Go Online You can complete an Extra Example online.

Learn Add Rational Numbers

The rules that apply to adding fractions and decimals also apply to rational numbers. The rules for adding integers also apply to positive and negative rational numbers.

Use the chart to see some strategies for how to add rational numbers written in different forms.

Words	Example
Terminating Decimals $\left(-\frac{1}{4}, \frac{3}{8}, \frac{7}{10}\ldots\right)$	
If the fractions are decimals that terminate, use decimals or fractions to add.	$-\frac{1}{5} + 0.8 = -\frac{1}{5} + \frac{4}{5}$ or $-\frac{1}{5} + 0.8 = -0.2 + 0.8$
Non-Terminating Decimals $\left(-\frac{1}{9}, \frac{2}{3}, \frac{11}{15}\ldots\right)$	
If the fractions are decimals that repeat nonzero digits, use fractions to add.	$\frac{1}{3} + (-0.25) = \frac{1}{3} + \left(-\frac{1}{4}\right)$

When a fraction is negative, the sign may be applied to the fraction, the numerator, or the denominator.

$$-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$$

When you are adding two fractions with negative signs, the sign is usually applied to the numerator.

Talk About It! Why would it be difficult to add −2.5 and $3.1\overline{6}$?

Sample answer: $3.1\overline{6}$ is a repeating decimal, and it never ends. It would be hard to add them because you would not be able to write out all of the digits of $3.1\overline{6}$.

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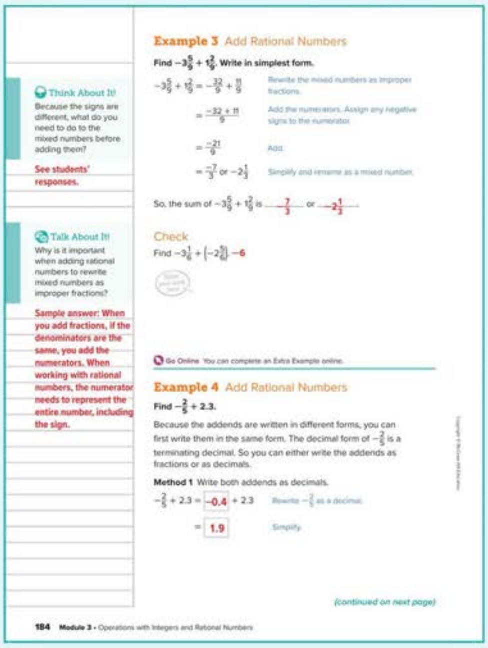
Interactive Presentation

Learn, Add Rational Numbers, Slide 1 of 2



Think About It! Because the signs are different, what do you need to do to the mixed numbers before adding them?

See students' responses.

Talk About It! Why is it important when adding rational numbers to rewrite mixed numbers as improper fractions?

Sample answer: When you add fractions, if the denominators are the same, you add the numerators. When working with rational numbers, the numerator needs to represent the entire number, including the sign.

Example 3 Add Rational Numbers

Find $-3\frac{5}{9}+1\frac{2}{9}$. Write in simplest form.

$-3\frac{5}{9}+1\frac{2}{9}=-\frac{32}{9}+\frac{11}{9}$ Rewrite the mixed numbers as improper fractions.

$=\frac{-32+11}{9}$ Add the numerators. Assign any negative signs to the numerator.

$=\frac{-21}{9}$ Add.

$=\frac{-7}{3}$ or $-2\frac{1}{3}$ Simplify and rename as a mixed number.

So, the sum of $-3\frac{5}{9}+1\frac{2}{9}$ is $-\frac{7}{3}$ or $-2\frac{1}{3}$.

Check

Find $-3\frac{1}{6}+\left(-2\frac{5}{6}\right)$. -6

Go Online You can complete an Extra Example online.

Example 4 Add Rational Numbers

Find $-\frac{2}{5}+2.3$.

Because the addends are written in different forms, you can first write them in the same form. The decimal form of $-\frac{2}{5}$ is a terminating decimal. So you can either write the addends as fractions or as decimals.

Method 1 Write both addends as decimals.

$-\frac{2}{5}+2.3=-0.4+2.3$ Rewrite $-\frac{2}{5}$ as a decimal.

$=1.9$ Simplify.

(continued on next page)

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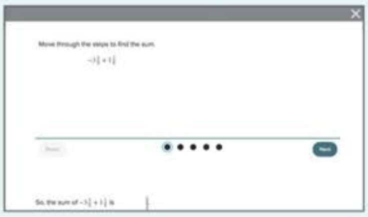
Interactive Presentation



Example 3, Add Rational Numbers, Slide 2 of 4

CLICK

On Slide 2 of Example 3, students move through steps to find a sum.

TYPE

On Slide 1 of Example 4, students enter a missing value to indicate a sum.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 3 Add Rational Numbers

Objective

Students will add rational numbers written as like fractions and mixed numbers.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the denominators? They are alike.

AL Why is it helpful to write the mixed numbers as improper fractions? Sample answer: It helps make the addition calculations easier, because we can add the numerators since the denominators are alike.

OL What is the sign of the sum? negative How do you know this is correct? The absolute value of $-3\frac{5}{9}$ is greater than the absolute value of $1\frac{2}{9}$ and the sign of the sum is the same as the number with the greater absolute value.

BL How could you add the mixed numbers using another method? Sample answer: The signs are different, so subtract the absolute values of the whole numbers and the fractions separately. $3-1=2$ and $\frac{5}{9}-\frac{2}{9}=\frac{1}{3}$. The answer is negative, $-2\frac{1}{3}$, because the absolute value of $-3\frac{5}{9}$ is greater than that of $1\frac{2}{9}$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Add Rational Numbers

Objective

Students will add rational numbers written in different forms.

Questions for Mathematical Discourse

SLIDE 2

AL What type of number is the first addend? second addend? a fraction; a decimal

AL Why would we rewrite the fraction as a decimal? Sample answer: To add numbers, it is helpful for them to be in the same form. Sometimes adding decimals can be easier than adding fractions.

OL Why can you write the first addend as a decimal? The fraction $-\frac{2}{5}$, when written as a decimal, is a terminating decimal.

BL Find $-\frac{2}{5}+2.3+\frac{4}{3}$. Write the sum as a decimal. Then write the sum as a fraction or mixed number, in simplest form. $3.2\overline{3}$; $3\frac{7}{30}$

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 4 Add Rational Numbers *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL What is 2.3 in words? two and three tenths

AL What is 0.3 written as a fraction? $\frac{3}{10}$

OL What is the least common denominator of $\frac{2}{5}$ and $\frac{3}{10}$? 10

BL Compare and contrast both methods for adding rational numbers written in different form. See students' responses.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Add Rational Numbers

Objective

Students will learn how to use the properties of operations to add three or more rational numbers.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure A s students discuss the *Talk About It!* question, encourage them to analyze the structure of the addition expression in order to identify each term as a fraction or decimal, prior to writing the terms all in the same form.

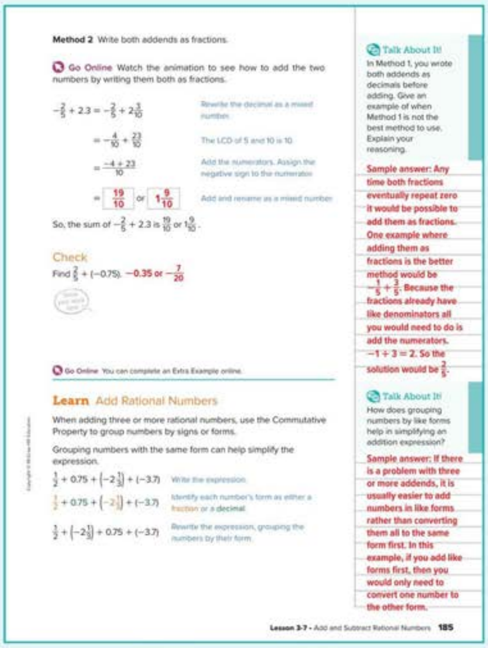
Go Online to find additional teaching notes.

Talk About It!

SLIDE 1

Mathematical Discourse

How does grouping numbers by like forms help in simplifying an addition expression? Sample answer: If there is a problem with three or more addends, it is usually easier to add numbers in like forms rather than converting them all to the same form first. In this example, if you add like forms first, then you would need to convert only one number to the other form.



Method 2 Write both addends as fractions.

Go Online Watch the animation to see how to add the two numbers by writing them both as fractions.

$-\frac{2}{5} + 2.3 = -\frac{2}{5} + 2\frac{3}{10}$ Rewrite the decimal as a mixed number.

$= -\frac{4}{10} + \frac{23}{10}$ The LCD of 5 and 10 is 10.

$= \frac{-4 + 23}{10}$ Add the numerators. Assign the negative sign to the numerator.

$= \frac{19}{10}$ or $1\frac{9}{10}$ Add and rename as a mixed number.

So, the sum of $-\frac{2}{5} + 2.3$ is $\frac{19}{10}$ or $1\frac{9}{10}$.

Check

Find $\frac{2}{5} + (-0.75)$. -0.35 or $-\frac{7}{20}$

Go Online You can complete an Extra Example online.

Learn Add Rational Numbers

When adding three or more rational numbers, use the Commutative Property to group numbers by signs or forms.

Grouping numbers with the same form can help simplify the expression.

$\frac{1}{2} + 0.75 + \left(-2\frac{1}{3}\right) + (-3.7)$ Write the expression.

$\frac{1}{2} + 0.75 + \left(-2\frac{1}{3}\right) + (-3.7)$ Identify each number's form as either a fraction or a decimal.

$\frac{1}{2} + \left(-2\frac{1}{3}\right) + 0.75 + (-3.7)$ Rewrite the expression, grouping the numbers by their form.

Talk About It! In Method 1, you wrote both addends as decimals before adding. Give an example of when Method 1 is not the best method to use. Explain your reasoning.

Sample answer: Any time both fractions eventually repeat zero it would be possible to add them as fractions. One example where adding them as fractions is the better method would be $-\frac{1}{5} + \frac{3}{5}$. Because the fractions already have like denominators all you would need to do is add the numerators. $-1 + 3 = 2$. So the solution would be $\frac{2}{5}$.

Talk About It! How does grouping numbers by like forms help in simplifying an addition expression?

Sample answer: If there is a problem with three or more addends, it is usually easier to add numbers in like forms rather than converting them all to the same form first. In this example, if you add like forms first, then you would need to convert one number to the other form.

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Interactive Presentation

Example 4, Add Rational Numbers, Slide 2 of 5

WATCH

On Slide 3, of the Example students watch an animation that explains how to write both addends as fractions (Method 2).

CLICK

In the Learn, students move through the steps to see how to add three or more rational numbers.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Add Rational Numbers

The lowest recorded elevation in the contiguous United States, 282 feet below sea level, is in Death Valley. Suppose you began a hike in the parking area near Badwater Basin at 266 feet below sea level. The stopping points of the hike and the elevation traveled are shown in the table.

What is the elevation of your stopping point?

Stop	Elevation Traveled (ft)
1	-12.5
2	$26\frac{3}{5}$
3	$-3\frac{1}{2}$
4	397.3

Because the denominators of the mixed numbers are 2 and 5, it is easy to convert those to decimals. Write all of the numbers as decimals.

$-266 + (-12.5) + 26\frac{3}{5} + \left(-3\frac{1}{2}\right) + 397.3$ Write the expression.

$= -266 + (-12.5) + 26.6 + (-3.5) + 397.3$ Write fractions as decimals.

$= -266 + (-12.5) + (-3.5) + 26.6 + 397.3$ Commutative Property

$= [-266 + (-12.5) + (-3.5)] + [26.6 + 397.3]$ Associative Property

$= -282 + 423.9$ Simplify.

$= 141.9$ Add.

So, your elevation at the end of the hike is 141.9 feet above sea level.

Think About It!
How will you represent the starting point of the hike in the expression?

See students' responses.

Talk About It!
If the starting point was a rational number, such as $-266\frac{1}{9}$, would it have changed how you found the sum?

Sample answer: If the starting point was a number that repeated non-zero digits, then any numbers written as decimals would have to first be written as fractions before adding.

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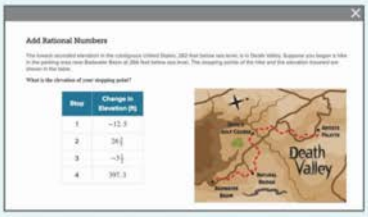
Interactive Presentation



Example 5, Add Rational Numbers, Slide 1 of 5

CLICK

On Slide 3, students move through the steps to find a sum.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.NS.A.1

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 5 Add Rational Numbers

Objective

Students will add four rational numbers written in different forms to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage students to compare how starting with a rational number that has a decimal form that does not terminate would result in the need for a different strategy to find the sum.

6 Attend to Precision Encourage students to perform the calculations efficiently and accurately to find the sum, paying careful attention to the sign of the sum.

Questions for Mathematical Discourse

SLIDE 2

AL What is the starting elevation? −266 feet

OL How many stops were there? What does this tell you about the number of addends in the expression? 4; There will be 5 addends, one that represents the starting elevation and 4 additional addends that represent the stops.

BL You decided to end your hike after Stop 3. What is the elevation of your stopping point? −255.4 feet

SLIDE 3

AL Identify the form(s) of the numbers in the expression. There is one integer, two decimals, and two mixed numbers.

OL Will it be more efficient to express the numbers in decimal or fraction form? Explain. Sample answer: decimal form; the denominators of the mixed numbers are 2 and 5, which can easily be converted to decimals.

BL Suppose a classmate mentally grouped −12.5 and $-3\frac{1}{2}$ together and found their sum to be −16. How does thinking this way help simplify the calculations in the expression? Sample answer: If there is a way to mentally group some of the numbers together to eliminate fractions or decimals, it can often be easier to add integers.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Subtract Rational Numbers

Objective

Students will learn how to subtract rational numbers.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to use what they know about adding rational numbers and the relationship between addition and subtraction to help them remember that subtracting rational numbers follows the same guidelines of adding the additive inverse.

Teaching Notes

SLIDE 1

Play the animation for the class. Students will learn different methods for subtracting rational numbers expressed in different forms.

Go Online to have students watch the animation on Slide 1. The animation illustrates different methods for subtracting rational numbers expressed in different forms.

(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

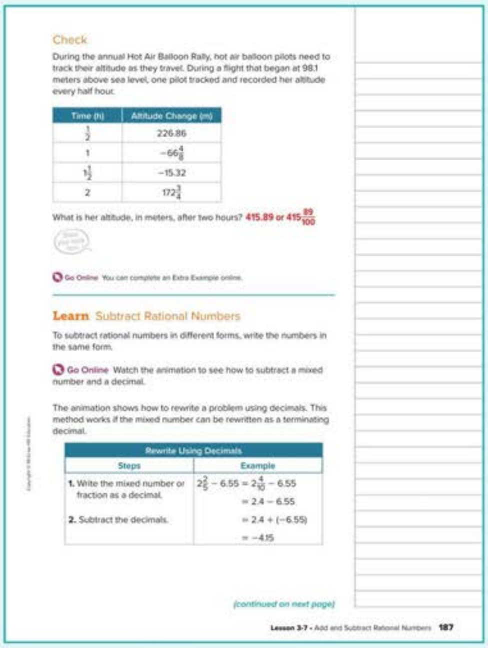
For students that may be struggling to understand how to subtract rational numbers, have them practice writing each of the following fractions and mixed numbers in decimal form.

$\frac{4}{5}$ 0.8

$2\frac{7}{8}$ 2.875

$8\frac{3}{16}$ 8.1875

$\frac{17}{20}$ 0.85



Check

During the annual Hot Air Balloon Rally, hot air balloon pilots need to track their altitude as they travel. During a flight that began at 98.1 meters above sea level, one pilot tracked and recorded her altitude every half hour.

Time (h)	Altitude Change (m)
$\frac{1}{2}$	226.86
1	$-66\frac{4}{8}$
$1\frac{1}{2}$	−15.32
2	$172\frac{3}{4}$

What is her altitude, in meters, after two hours? 415.89 or $415\frac{89}{100}$

Go Online You can complete an Extra Example online.

Learn Subtract Rational Numbers

To subtract rational numbers in different forms, write the numbers in the same form.

Go Online Watch the animation to see how to subtract a mixed number and a decimal.

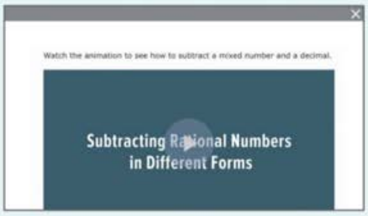
The animation shows how to rewrite a problem using decimals. This method works if the mixed number can be rewritten as a terminating decimal.

Rewrite Using Decimals	
Steps	**Example**
1. Write the mixed number or fraction as a decimal.	$2\frac{2}{5} - 6.55 = 2\frac{4}{10} - 6.55$ $= 2.4 - 6.55$
2. Subtract the decimals.	$= 2.4 + (-6.55)$ $= -4.15$

(continued on next page)

Lesson 3-7 • Add and Subtract Rational Numbers 187

Interactive Presentation



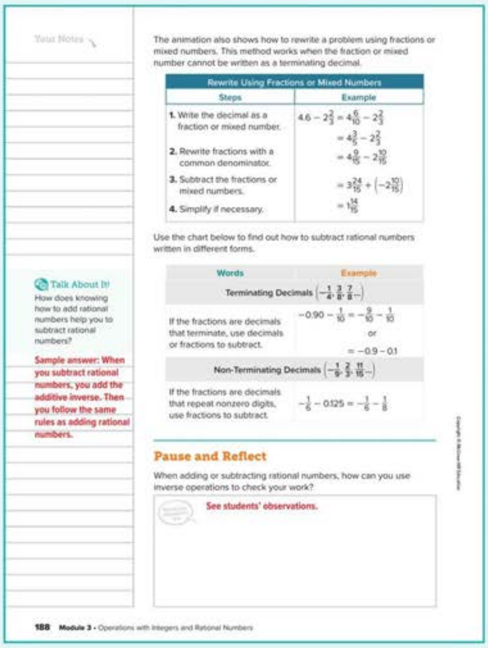
Learn, Subtract Rational Numbers, Slide 1 of 3

WATCH

On Slide 1, students watch an animation that explains howto subtract a mixed number and a decimal.

7.NS.A.1

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION


Your Notes

The animation also shows how to rewrite a problem using fractions or mixed numbers. This method works when the fraction or mixed number cannot be written as a terminating decimal.

Rewrite Using Fractions or Mixed Numbers	
Steps	**Example**
1. Write the decimal as a fraction or mixed number.	$4.6 - 2\frac{2}{3} = 4\frac{6}{10} - 2\frac{2}{3}$ $= 4\frac{3}{5} - 2\frac{2}{3}$
2. Rewrite fractions with a common denominator.	$= 4\frac{9}{15} - 2\frac{10}{15}$
3. Subtract the fractions or mixed numbers.	$= 3\frac{24}{15} + \left(-2\frac{10}{15}\right)$
4. Simplify if necessary.	$= 1\frac{14}{15}$

Use the chart below to find out how to subtract rational numbers written in different forms.

Words	Example
Terminating Decimals $\left(-\frac{1}{4}, \frac{3}{8}, \frac{7}{8} \ldots\right)$	
If the fractions are decimals that terminate, use decimals or fractions to subtract.	$-0.90 - \frac{1}{10} = -\frac{9}{10} - \frac{1}{10}$ or $= -0.9 - 0.1$
Non-Terminating Decimals $\left(-\frac{1}{9}, \frac{2}{3}, \frac{11}{15} \ldots\right)$	
If the fractions are decimals that repeat nonzero digits, use fractions to subtract.	$-\frac{1}{6} - 0.125 = -\frac{1}{6} - \frac{1}{8}$

Talk About It!
How does knowing how to add rational numbers help you to subtract rational numbers?

Sample answer: When you subtract rational numbers, you add the additive inverse. Then you follow the same rules as adding rational numbers.

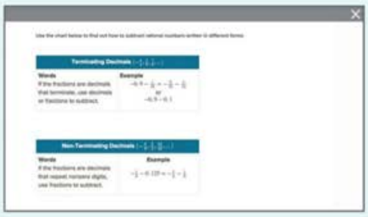
Pause and Reflect

When adding or subtracting rational numbers, how can you use inverse operations to check your work?

See students' observations.

188 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Learn, Subtract Rational Numbers, Slide 2 of 3

Learn Subtract Rational Numbers *(continued)*

Teaching Notes

SLIDE 2

Students will learn how to subtract rational numbers in different forms. They should understand the reasons behind why it may be more efficient to use decimals when the numbers can be written as terminating decimals. Ask students why it may be more efficient to use fractions when the numbers can be written as decimals that repeat nonzero digits.

Talk About It!

SLIDE 3

Mathematical Discourse

How does knowing how to add rational numbers help you to subtract rational numbers? Sample answer: When you subtract rational numbers, you add the additive inverse. Then you follow the same rules as adding rational numbers.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of rational number subtraction, have them work with a partner to fill in the blanks for each of the following subtraction statements. This will help prepare them for solving equations with rational numbers in a future module.

10 – 6.534 = 3.466

–6.217 – (–1.3) = –4.917

–0.331 – (–1.04) = 0.709

Example 6 Subtract Rational Numbers

Objective

Students will subtract rational numbers written as decimals.

Questions for Mathematical Discourse

SLIDE 1

AL How do we rewrite subtraction? Add the additive inverse.

AL What is the additive inverse of –6.7? 6.7

OL Rewrite the subtraction expression as an addition expression. $-3.27 + 6.7$

OL Will the answer be positive or negative? Explain. positive; When rewritten as an addition problem, 6.7 has the greater absolute value.

BL What would you have to add to the expression for the answer to be 0? Sample answer: I would have to add the additive inverse of 3.43.

Example 7 Subtract Rational Numbers

Objective

Students will subtract rational numbers written as unlike fractions and mixed numbers.

Questions for Mathematical Discourse

SLIDE 1

AL Are the denominators like or unlike? unlike

AL What do you need to do before subtracting unlike denominators? Determine the LCD and rewrite as equivalent fractions.

AL What is the LCD of the denominators? 9

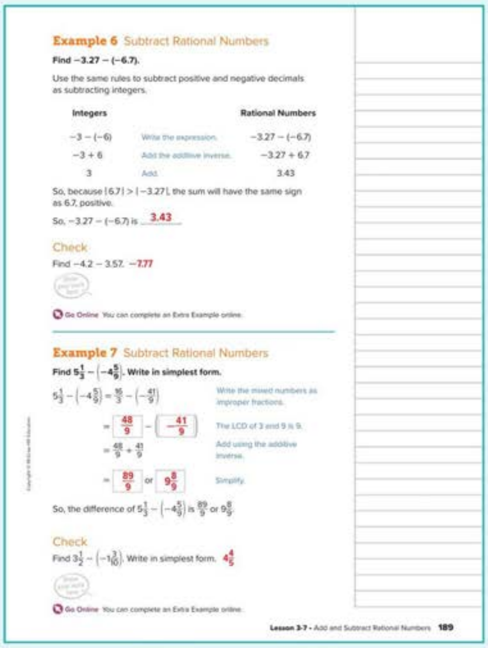
OL Before subtracting, should you rewrite the numbers as decimals? Explain. no; the fractions expressed as decimals repeat non-zero digits.

OL How do you know whether the sign of the difference will be positive or negative? The difference will be positive because when the expression is rewritten as addition of the additive inverse, both addends are positive.

BL How could you write the expression as an addition expression with repeating decimals? $5.\overline{3} + 4.\overline{5} = 9.\overline{8}$

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 6 Subtract Rational Numbers

Find $-3.27 - (-6.7)$.

Use the same rules to subtract positive and negative decimals as subtracting integers.

Integers		Rational Numbers
$-3 - (-6)$	Write the expression.	$-3.27 - (-6.7)$
$-3 + 6$	Add the additive inverse.	$-3.27 + 6.7$
3	Add.	3.43

So, because $|6.7| > |-3.27|$, the sum will have the same sign as 6.7, positive.

So, $-3.27 - (-6.7)$ is 3.43.

Check

Find $-4.2 - 3.57$. -7.77

Go Online You can complete an Extra Example online.

Example 7 Subtract Rational Numbers

Find $5\frac{1}{3} - \left(-4\frac{5}{9}\right)$. Write in simplest form.

$5\frac{1}{3} - \left(-4\frac{5}{9}\right) = \frac{16}{3} - \left(-\frac{41}{9}\right)$ Write the mixed numbers as improper fractions.

$= \frac{48}{9} - \left(-\frac{41}{9}\right)$ The LCD of 3 and 9 is 9.

$= \frac{48}{9} + \frac{41}{9}$ Add using the additive inverse.

$= \frac{89}{9}$ or $9\frac{8}{9}$ Simplify.

So, the difference of $5\frac{1}{3} - \left(-4\frac{5}{9}\right)$ is $\frac{89}{9}$ or $9\frac{8}{9}$.

Check

Find $3\frac{1}{2} - \left(-1\frac{3}{10}\right)$. Write in simplest form. $4\frac{4}{5}$

Go Online You can complete an Extra Example online.

Lesson 3-7 • Add and Subtract Rational Numbers 189

Interactive Presentation

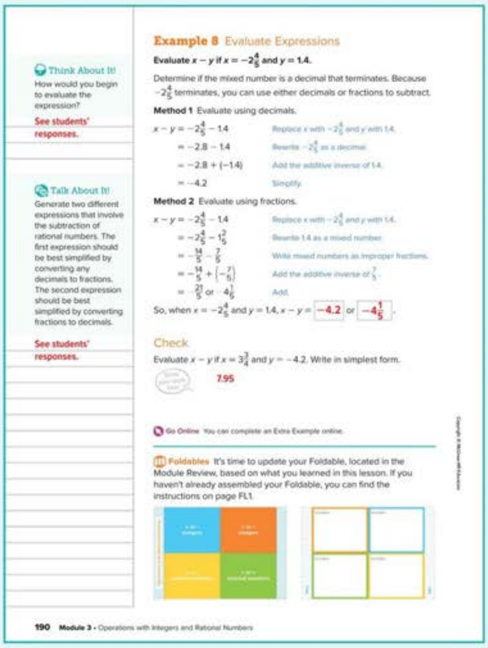
Example 7, Subtract Rational Numbers, Slide 1 of 2

CLICK

On Slide 1, of Example 7 students will move through the steps to find a difference.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.



Example 8 Evaluate Expressions

Evaluate $x - y$ if $x = -2\frac{4}{5}$ and $y = 1.4$.

Determine if the mixed number is a decimal that terminates. Because $-2\frac{4}{5}$ terminates, you can use either decimals or fractions to subtract.

Method 1 Evaluate using decimals.

$x - y = -2\frac{4}{5} - 1.4$	Replace x with $-2\frac{4}{5}$ and y with 1.4.
$= -2.8 - 1.4$	Rewrite $-2\frac{4}{5}$ as a decimal.
$= -2.8 + (-1.4)$	Add the additive inverse of 1.4.
$= -4.2$	Simplify.

Method 2 Evaluate using fractions.

$x - y = -2\frac{4}{5} - 1.4$	Replace x with $-2\frac{4}{5}$ and y with 1.4.
$= -2\frac{4}{5} - 1\frac{2}{5}$	Rewrite 1.4 as a mixed number.
$= -\frac{14}{5} - \frac{7}{5}$	Write mixed numbers as improper fractions.
$= -\frac{14}{5} + \left(-\frac{7}{5}\right)$	Add the additive inverse of $\frac{7}{5}$.
$= -\frac{21}{5}$ or $-4\frac{1}{5}$	Add.

So, when $x = -2\frac{4}{5}$ and $y = 1.4$, $x - y =$ -4.2 or $-4\frac{1}{5}$.

Think About It! How would you begin to evaluate the expression? See students' responses.

Talk About It! Generate two different expressions that involve the subtraction of rational numbers. The first expression should be best simplified by converting any decimals to fractions. The second expression should be best simplified by converting fractions to decimals. See students' responses.

Check

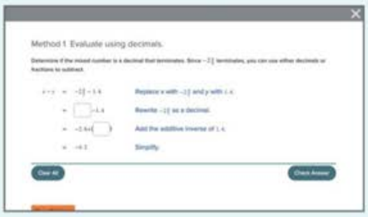
Evaluate $x - y$ if $x = 3\frac{3}{4}$ and $y = -4.2$. Write in simplest form. 7.95

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

190 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Example 8, Evaluate Expressions, Slide 2 of 5

TYPE

On Slide 2, students evaluate the expression using decimals (Method 1).

CLICK

On Slide 3, students evaluate the expression using fractions (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 8 Evaluate Expressions

Objective

Students will evaluate an algebraic expression involving subtraction of rational numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enco urage students to make sense of the quantities given in the expression, and understand why they can use either method (writing the numbers as fractions, or writing the numbers as decimals) in order to find the difference. Students should be able to reason that the fraction, when expressed in decimal form, will terminate (repeat zeros), so either method is efficient.

Questions for Mathematical Discourse

SLIDE 2

AL What is the value of x? $-2\frac{4}{5}$ What is the value of y? 1.4

AL How do you subtract numbers in different forms? Express the numbers in the same form.

OL Why do we rewrite the fraction as a decimal? Sample answer: Its denominator is 5, so its decimal form will terminate.

OL How would you express the value of x as a decimal? -2.8

BL If $z = -1\frac{3}{4}$, what would be the value of $x - y + z$? -5.95

SLIDE 3

AL How is this method different from the previous method? In the previous method, we rewrote the fraction as a decimal. In this method, we are rewriting the decimal as a fraction.

OL How is the decimal written as a fraction? Write 1.4 as $1\frac{4}{10}$ and then simplify.

BL If $z = -2\frac{2}{5}$, find $x - y - z$. Express as a mixed number. $-1\frac{4}{5}$

Go Online

- Find additional teaching notes, T eaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information about adding and subtracting rational numbers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Apply Animal Care

Objective

Students will come up with their own strategy to solve an application problem involving the change in weight of a cat.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you find the change in the cat's weight each month?
- How do you add numbers written in different forms?
- Would you rather change fractions to decimals, or decimals to fractions?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Robert Daly/Caiaimage/Getty Images

Apply Animal Care

A veterinarian measures the changes in a cat's weight over four months. If the cat weighed 17.25 pounds at its first visit, what is the cat's weight after its last visit?

Month	Change from Previous Month (lb)
1	-0.5
2	$-2\frac{1}{5}$
3	$-\frac{3}{10}$
4	0.35

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

14.6 pounds; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Why is it more efficient in this problem to write each of the values as a decimal before adding?

Sample answer: The majority of the values in the expression are written as decimals, so it is more efficient to write the fractions and mixed numbers in decimal form to find the sum.

Lesson 3-7 • Add and Subtract Rational Numbers 191

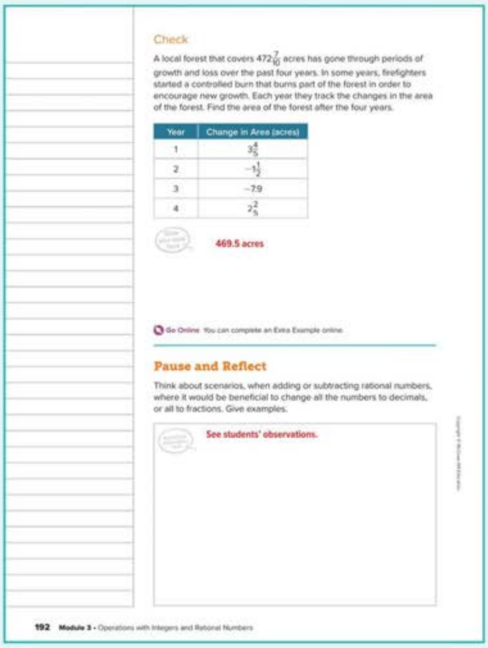
Interactive Presentation

Apply, Animal Care

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY **3 APPLICATION**


Check

A local forest that covers $472\frac{7}{10}$ acres has gone through periods of growth and loss over the past four years. In some years, firefighters started a controlled burn that burns part of the forest in order to encourage new growth. Each year they track the changes in the area of the forest. Find the area of the forest after the four years.

Year	Change in Area (acres)
1	$3\frac{4}{5}$
2	$-1\frac{1}{2}$
3	−7.9
4	$2\frac{2}{5}$

469.5 acres

Go Online You can complete an Extra Example online.

Pause and Reflect

Think about scenarios, when adding or subtracting rational numbers, where it would be beneficial to change all the numbers to decimals, or all to fractions. Give examples.

See students' observations.

192 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How are operations with rational numbers related to operations with integers?

In this lesson, students learned how to add and subtract rational numbers. Encourage them to work with a partner to compare and contrast adding and subtracting rational numbers to adding and subtracting integers.

Exit Ticket

Refer to the Exit Ticket slide. Suppose the temperature rose from −2.3°F to 8.6°F . Explain how to find the change in temperature. Then find the change. Sample answer: Subtract −2.3°F from 8.6°F by adding the opposite of −2.3°F; 10.9°F.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 16–22
- Extension: Arithmetic Sequences, Series and Arithmetic Sequences
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 66-89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–15, 17, 21
- Extension: Arithmetic Sequences, Series and Arithmetic Sequences
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–8
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Venn Diagrams and Sets of Rational Numbers

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

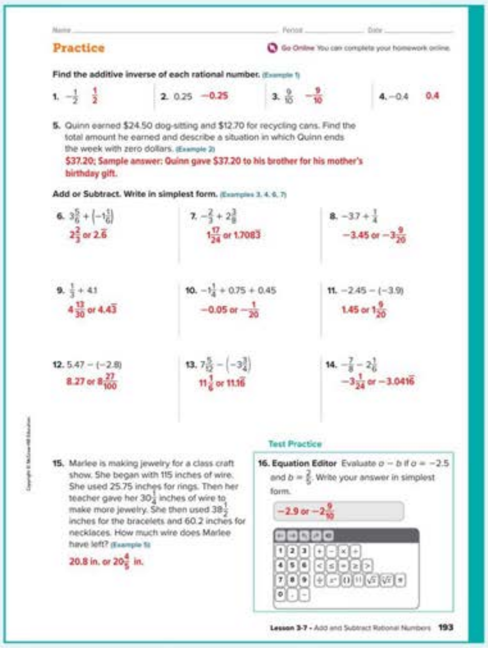
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the additive inverse of rational numbers	1–4
2	find the additive inverse of rational numbers	5
1	add rational numbers	6–10
1	subtract rational numbers written as decimals	11–14
2	solve a real-world problem involving addition of rational numbers	15
2	extend concepts learned in class to apply them in new contexts	16
3	solve application problems involving adding and subtracting rational numbers	17, 18
3	higher-order and critical thinking skills	19, 22

Common Misconception

When adding rational numbers where one or more of the numbers are negative, students may determine the incorrect sum. Encourage students to express each mixed number as an improper fraction, making sure to keep the negative sign if the mixed number is negative. Then, after finding common denominators, they should include the negative sign with the numerator when finding the sum.


Name ____ Period ____ Date ____

Practice Go Online You can complete your homework online.

Find the additive inverse of each rational number. (Example 1)

1. $-\frac{1}{2}$ **$\frac{1}{2}$**
2. 0.25 **−0.25**
3. $\frac{9}{10}$ **$-\frac{9}{10}$**
4. −0.4 **0.4**

5. Quinn earned \$24.50 dog-sitting and \$12.70 for recycling cans. Find the total amount he earned and describe a situation in which Quinn ends the week with zero dollars. (Example 2)
\$37.20; Sample answer: Quinn gave \$37.20 to his brother for his mother's birthday gift.

Add or Subtract. Write in simplest form. (Examples 3, 4, 6, 7)

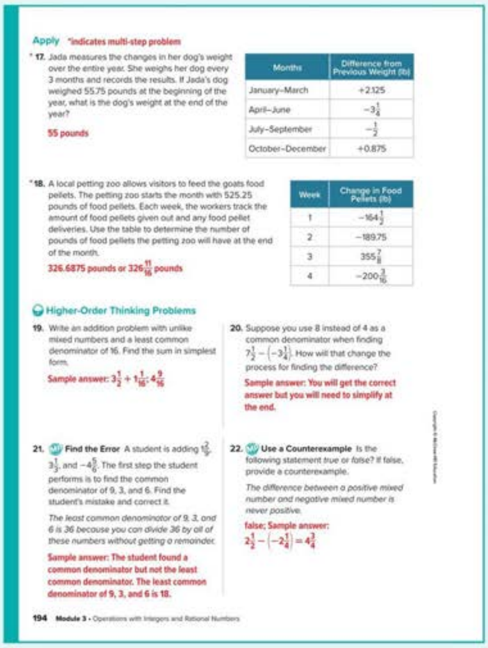
6. $3\frac{5}{6} + \left(-1\frac{1}{6}\right)$ **$2\frac{2}{3}$ or $2.\overline{6}$**
7. $-\frac{2}{3} + 2\frac{3}{8}$ **$1\frac{17}{24}$ or $1.708\overline{3}$**
8. $-3.7 + \frac{1}{4}$ **-3.45 or $-3\frac{9}{20}$**
9. $\frac{1}{3} + 4.1$ **$4\frac{13}{30}$ or $4.4\overline{3}$**
10. $-1\frac{1}{4} + 0.75 + 0.45$ **-0.05 or $-\frac{1}{20}$**
11. $-2.45 - (-3.9)$ **1.45 or $1\frac{9}{20}$**
12. $5.47 - (-2.8)$ **8.27 or $8\frac{27}{100}$**
13. $7\frac{5}{12} - \left(-3\frac{3}{4}\right)$ **$11\frac{1}{6}$ or $11.1\overline{6}$**
14. $-\frac{7}{8} - 2\frac{1}{6}$ **$-3\frac{1}{24}$ or $-3.041\overline{6}$**

15. Marlee is making jewelry for a class craft show. She began with 115 inches of wire. She used 25.75 inches for rings. Then her teacher gave her $30\frac{1}{4}$ inches of wire to make more jewelry. She then used $38\frac{1}{2}$ inches for the bracelets and 60.2 inches for necklaces. How much wire does Marlee have left? (Example 5)
20.8 in. or $20\frac{4}{5}$ in.

Test Practice

16. **Equation Editor** Evaluate $a - b$ if $a = -2.5$ and $b = \frac{2}{5}$. Write your answer in simplest form.
-2.9 or $-2\frac{9}{10}$

Lesson 3-7 • Add and Subtract Rational Numbers 193



Apply *indicates multi-step problem

*17. Jada measures the changes in her dog's weight over the entire year. She weighs her dog every 3 months and records the results. If Jada's dog weighed 55.75 pounds at the beginning of the year, what is the dog's weight at the end of the year?

Months	Difference from Previous Weight (lb)
January–March	+2.125
April–June	$-3\frac{1}{4}$
July–September	$-\frac{1}{2}$
October–December	+0.875

55 pounds

*18. A local petting zoo allows visitors to feed the goats food pellets. The petting zoo starts the month with 525.25 pounds of food pellets. Each week, the workers track the amount of food pellets given out and any food pellet deliveries. Use the table to determine the number of pounds of food pellets the petting zoo will have at the end of the month.

Week	Change in Food Pellets (lb)
1	$-164\frac{1}{2}$
2	−189.75
3	$355\frac{7}{8}$
4	$-200\frac{3}{16}$

326.6875 pounds or $326\frac{11}{16}$ pounds

Higher-Order Thinking Problems

19. Write an addition problem with unlike mixed numbers and a least common denominator of 16. Find the sum in simplest form.

Sample answer: $3\frac{1}{2} + 1\frac{1}{16}$; $4\frac{9}{16}$

20. Suppose you use 8 instead of 4 as a common denominator when finding $7\frac{1}{2} - \left(-3\frac{1}{4}\right)$. How will that change the process for finding the difference?

Sample answer: You will get the correct answer but you will need to simplify at the end.

21. MP Find the Error A student is adding $1\frac{2}{9}$, $3\frac{1}{3}$, and $-4\frac{5}{6}$. The first step the student performs is to find the common denominator of 9, 3, and 6. Find the student's mistake and correct it.

The least common denominator of 9, 3, and 6 is 36 because you can divide 36 by all of these numbers without getting a remainder.

Sample answer: The student found a common denominator but not the least common denominator. The least common denominator of 9, 3, and 6 is 18.

22. MP Use a Counterexample Is the following statement true or false? If false, provide a counterexample.

The difference between a positive mixed number and negative mixed number is never positive.

false; Sample answer: $2\frac{1}{2} - \left(-2\frac{1}{4}\right) = 4\frac{3}{4}$

194 Module 3 • Operations with Integers and Rational Numbers

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 21, students find the error in a student's reasoning.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 22, students determine if a statement is true or false and use a counterexample.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercises 17–18 Have students work in groups of 3–4. After completing Exercise 17, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 18.

Listen and ask clarifying questions.

Use with Exercises 19 and 21 Have students work in pairs. Have students individually read Exercise 19 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 21.

Lesson 3-8

Multiply and Divide Rational Numbers

7.NS.A.2, 7.NS.A.3

LESSON GOAL

Students will apply understanding of multiplication and division to rational numbers, and use the order of operations to solve real-world problems.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Multiply Rational Numbers
Examples 1-2: Multiply Rational Numbers
Learn: Multiply Rational Numbers
Examples 3-5: Multiply Rational Numbers
Learn: Divide Rational Numbers
Examples 6-7: Divide Rational Numbers
Learn: Divide Rational Numbers
Example 8: Divide Rational Numbers
Apply: Temperature

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 21 of the *Language Development Handbook* to help your students build mathematical language related to multiplication and division of rational numbers.

ELL You can use the tips and suggestions on page T21 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7.NS.A** by multiplying and dividing rational numbers.
Standards for Mathematical Content: **7.NS.A.2, 7.NS.A.2.A, 7.NS.A.2.B, 7.NS.A.2.C, 7.NS.A.3,** *Also Addresses 7.NS.A.1.D, 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students added and subtracted rational numbers.
7.NS.A.1, 7.NS.A.1.B, 7.NS.A.1.C

Now
Students apply understanding of multiplication and division to rational numbers, and use the order of operations to solve real-world problems.
7.NS.A.2, 7.NS.A.3

Next
Students will apply rational number operations.
7.NS.A.2

Rigor

The Three Pillars of Rigor

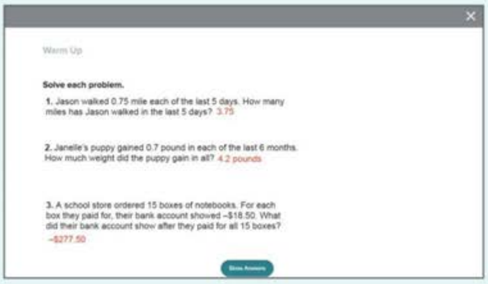
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of rational numbers and integers to develop the *understanding* of multiplying and dividing rational numbers. Students will use their understanding to gain *fluency* in multiplying and dividing rational numbers.

Mathematical Background

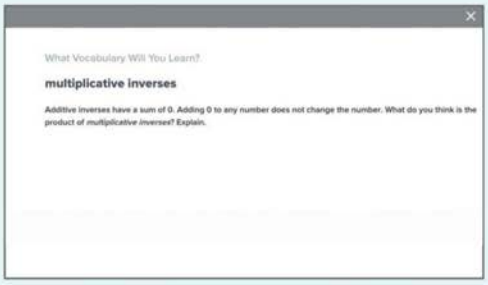
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- multiplying whole numbers and decimals (Exercises 1–3)

Answers

1. 3.75

2. 4.2 pounds

3. −$277.50

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about multiplying quantities in a recipe to serve more people.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Additive inverses have a sum of 0. Adding 0 to any number does not change the number. What do you think is the product of *multiplicative inverses?* Explain. Sample answer: The product of multiplicative inverses is 1 because multiplying any number by 1 does not change the number.

Learn Multiply Rational Numbers

Objective

Students will understand that they can apply what they know about multiplying fractions, decimals, and integers to the set of rational numbers.

Go Online to find additional teaching notes.

Example 1 Multiply Rational Numbers

Objective

Students will multiply rational numbers written as fractions.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to calculate accurately and efficiently, using their knowledge of multiplying fractions and paying attention to the sign of the product.

Questions for Mathematical Discourse

SLIDE 1

AL Will the product be positive or negative? Explain. positive; both factors have the same sign

OL Why do you divide 3 and 9 by their GCF, 3? Sample answer: in order to simplify the fractions prior to multiplying; it makes the multiplication calculations easier

BL Does it change the product if you do not simplify before multiplying? Explain. No, you can simplify the product and obtain the same answer.

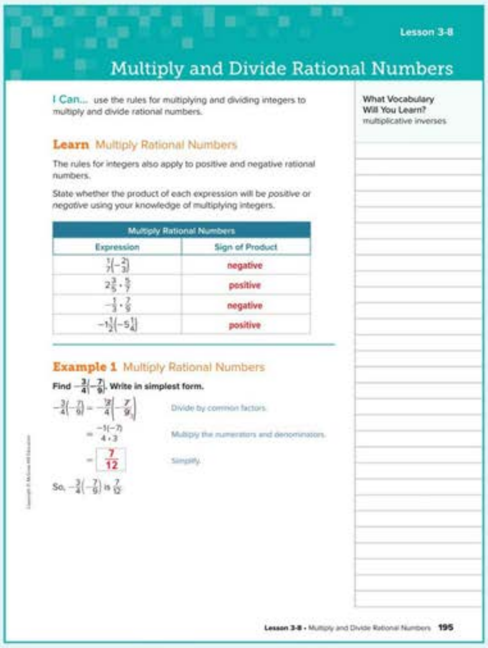
Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Reteaching Activity AL

Some students may struggle to determine the sign of the product when working with rational numbers. Remind them that the same rules that apply to multiplying integers apply to rational numbers. Have them review the rules for multiplying integers, and if necessary, look back at the lessons on multiplying integers they have previously completed. It may benefit them to create a graphic organizer or flashcards that show the rules, with examples, so they can apply them to working with rational numbers.



Lesson 3-8

Multiply and Divide Rational Numbers

I Can... use the rules for multiplying and dividing integers to multiply and divide rational numbers.

What Vocabulary Will You Learn?
multiplicative inverses

Learn Multiply Rational Numbers

The rules for integers also apply to positive and negative rational numbers.

State whether the product of each expression will be *positive* or *negative* using your knowledge of multiplying integers.

Multiply Rational Numbers	
Expression	Sign of Product
$\frac{1}{7}\left(-\frac{2}{3}\right)$	negative
$2\frac{3}{5}\cdot\frac{5}{7}$	positive
$-\frac{1}{3}\cdot\frac{7}{9}$	negative
$-1\frac{1}{2}\left(-5\frac{1}{4}\right)$	positive

Example 1 Multiply Rational Numbers

Find $-\frac{3}{4}\left(-\frac{7}{9}\right)$. Write in simplest form.

$-\frac{3}{4}\left(-\frac{7}{9}\right) = -\frac{\overset{1}{\cancel{3}}}{4}\left(-\frac{7}{\underset{3}{\cancel{9}}}\right)$ Divide by common factors.

$= \frac{-1(-7)}{4 \cdot 3}$ Multiply the numerators and denominators.

$= \frac{7}{12}$ Simplify.

So, $-\frac{3}{4}\left(-\frac{7}{9}\right)$ is $\frac{7}{12}$.

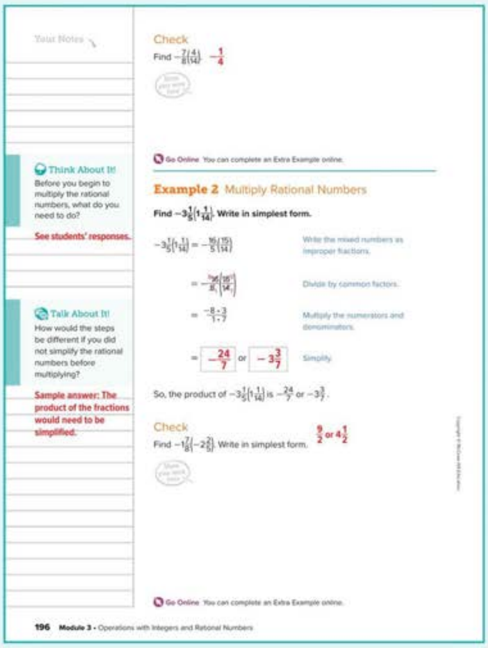
Lesson 3-8 • Multiply and Divide Rational Numbers 195

Interactive Presentation

Learn, Multiply Rational Numbers

CLICK

On Slide 1 of Example 1, students move through the steps to find a product.



Interactive Presentation

Example 2, Multiply Rational Numbers, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the product.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Multiply Rational Numbers

Objective

Students will multiply rational numbers written as mixed numbers.

MP Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to calculate accurately and efficiently, using their knowledge of multiplying mixed numbers, and paying attention to the sign of the product. Students should be able to explain why the mixed numbers need to be written as improper fractions.

As students discuss the *Talk About It!* question on Slide 3, encourage them to explain how the steps would be different by using clear and precise mathematical language, such as *simplify* and *product*.

Questions for Mathematical Discourse

SLIDE 2

AL Will the product be positive or negative? Explain. negative; The signs of the factors are different.

OL Why do we write the mixed numbers as improper fractions first? Sample answer: In order to use the rules for multiplying fractions, the numbers need to be written as improper fractions.

OL Why do we divide 14 and 16 by their GCF, 2? to simplify before multiplying

BL Write and solve a multiplication problem with two mixed numbers whose product is negative. Sample answer: Find $-2\frac{3}{5}\left(1\frac{1}{2}\right)$; $-3\frac{9}{10}$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Multiply Rational Numbers

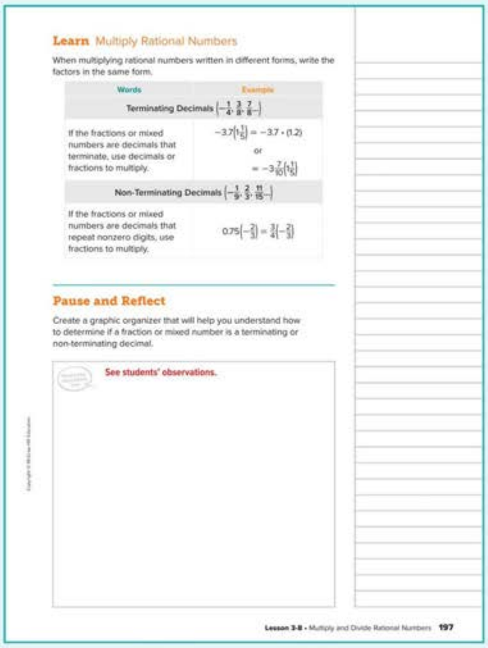
Objective

Students will understand when it is more efficient to write numbers as fractions or decimals when multiplying rational numbers written in different forms.

Teaching Notes

SLIDE 1

Ask students to recall the procedures and rules they used to add and subtract rational numbers in different forms. Students should recall writing the rational numbers in the same form, and determining in which form to write them (fractions or decimals) in order for the calculation process to be more efficient. Ask students to explain whether they can use the same procedures for multiplying rational numbers in different forms. Have them explain their reasoning.



Learn Multiply Rational Numbers

When multiplying rational numbers written in different forms, write the factors in the same form.

Words	Example
Terminating Decimals $\left(-\frac{1}{4}, \frac{3}{8}, \frac{7}{8} ...\right)$	
If the fractions or mixed numbers are decimals that terminate, use decimals or fractions to multiply.	$-3.7\left(1\frac{1}{5}\right) = -3.7 \cdot (1.2)$ or $= -3\frac{7}{10}\left(1\frac{1}{5}\right)$
Non-Terminating Decimals $\left(-\frac{1}{9}, \frac{2}{3}, \frac{11}{15} ...\right)$	
If the fractions or mixed numbers are decimals that repeat nonzero digits, use fractions to multiply.	$0.75\left(-\frac{2}{3}\right) = \frac{3}{4}\left(-\frac{2}{3}\right)$

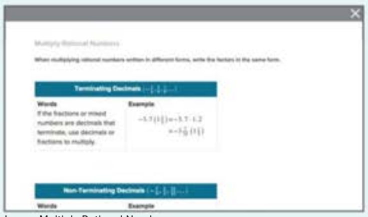
Pause and Reflect

Create a graphic organizer that will help you understand how to determine if a fraction or mixed number is a terminating or non-terminating decimal.

See students' observations.

Lesson 3-8 • Multiply and Divide Rational Numbers 197

Interactive Presentation

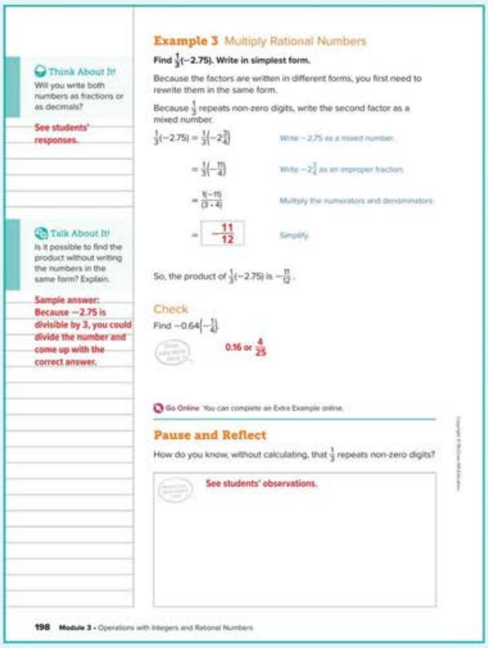


Learn, Multiply Rational Numbers

DIFFERENTIATE

Language Development Activity ELL

To further students' understanding of multiplying rational numbers and support their use of correct mathematical terminology, have them make a conjecture as to why it might be more efficient to write $0.75\left(-\frac{2}{3}\right)$ as fractions instead of decimals in order to find the product. Encourage them to generate their own examples and explain if it is more efficient to write the numbers as fractions or decimals.



Think About It! Will you write both numbers as fractions or as decimals?

See students' responses.

Talk About It! Is it possible to find the product without writing the numbers in the same form? Explain.

Sample answer: Because −2.75 is divisible by 3, you could divide the number and come up with the correct answer.

Example 3 Multiply Rational Numbers

Find $\frac{1}{3}(-2.75)$. Write in simplest form.

Because the factors are written in different forms, you first need to rewrite them in the same form.

Because $\frac{1}{3}$ repeats non-zero digits, write the second factor as a mixed number.

$\frac{1}{3}(-2.75) = \frac{1}{3}\left(-2\frac{3}{4}\right)$ Write −2.75 as a mixed number.

$= \frac{1}{3}\left(-\frac{11}{4}\right)$ Write $-2\frac{3}{4}$ as an improper fraction.

$= \frac{1(-11)}{(3 \cdot 4)}$ Multiply the numerators and denominators.

$= -\frac{11}{12}$ Simplify.

So, the product of $\frac{1}{3}(-2.75)$ is $-\frac{11}{12}$.

Check

Find $-0.64\left(-\frac{1}{4}\right)$.

0.16 or $\frac{4}{25}$

Go Online You can complete an Extra Example online.

Pause and Reflect

How do you know, without calculating, that $\frac{1}{3}$ repeats non-zero digits?

See students' observations.

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Interactive Presentation

Example 3, Multiply Rational Numbers, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the product.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Multiply Rational Numbers

Objective

Students will multiply rational numbers written in different forms.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them As students discuss the *Talk About It!* question on Slide 3, encourage them to consider alternative approaches to finding the product and explain the correspondences between the approaches. Students should notice that multiplying a number by $\frac{1}{3}$ is the same as dividing that number by 3.

7 Look for and Make Use of Structure Encourage students to analyze the structure of each factor, noting that because the denominator of the first factor is 3, its decimal form will have repeating non-zero digits. So, it will be easier to perform the calculations if both numbers are written as fractions or mixed numbers.

Questions for Mathematical Discourse

SLIDE 2

AL What type of number is the first factor? second factor? a fraction; a decimal

AL What should be the first step to finding the product? Express both numbers in the same form.

OL Why do we write −2.75 as a fraction? Sample answer: The fraction $\frac{1}{3}$ repeats non-zero digits, so the calculations will be more difficult if we write it as a decimal. It is easier to perform the calculations by writing the second factor as a fraction.

OL What is the fraction form of −2.75? $-2\frac{3}{4}$

BL What is $\frac{1}{5}$ of the product? How did you find it? $-\frac{11}{60}$; Sample answer: Multiply the product by $\frac{1}{5}$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 4 Multiply Rational Numbers

Objective

Students will evaluate an algebraic expression involving multiplication of rational numbers.

Teaching the Mathematical Practices

6 Attend to Precision S tudents should pay careful attention to the sign of each factor, noting that when there are two positive factors and one negative factor, the product will be negative.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the algebraic expression by noting that it contains three factors, all of which are fractions or mixed numbers. So, it will be easier to perform the calculations if both numbers are written as fractions or mixed numbers.

Questions for Mathematical Discourse

SLIDE 1

AL What is the value of *a*? *b*? $1\frac{3}{7}$; $-\frac{4}{9}$

AL How many factors are there? 3

OL Are the factors expressed in the same form? Explain. Y es, all of the factors are fractions or mixed numbers.

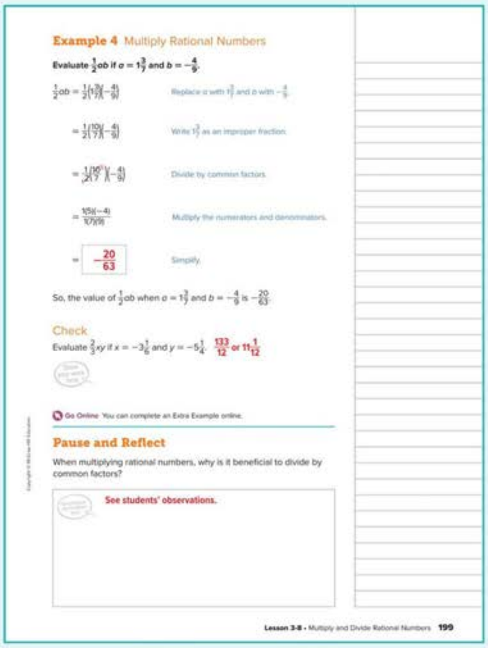
OL Will the product be positive or negative? Explain. negative; Two of the factors are positive, so their product will be positive. The third factor is negative, and the product of a positive and a negative is negative.

BL Does it change the product if you simplify before multiplying three rational numbers? Explain. no; Sample answer: You get the same answer whether you simplify before multiplying or at the end.

BL Evaluate 0.75*ab*. Explain the steps you used. $-\frac{10}{21}$; Sample answer: The product *ab* is a fraction with repeating non-zero digits, so I wrote 0.75 as $\frac{3}{4}$. Then I multiplied the three fractions.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Multiply Rational Numbers

Evaluate $\frac{1}{2}ab$ if $a = 1\frac{3}{7}$ and $b = -\frac{4}{9}$.

$\frac{1}{2}ab = \frac{1}{2}\left(1\frac{3}{7}\right)\left(-\frac{4}{9}\right)$ Replace *a* with $1\frac{3}{7}$ and *b* with $-\frac{4}{9}$.

$= \frac{1}{2}\left(\frac{10}{7}\right)\left(-\frac{4}{9}\right)$ Write $1\frac{3}{7}$ as an improper fraction.

$= \frac{1}{2}\left(\frac{10}{7}\right)\left(-\frac{4}{9}\right)$ Divide by common factors.

$= \frac{1(5)(-4)}{1(7)(9)}$ Multiply the numerators and denominators.

$= -\frac{20}{63}$ Simplify.

So, the value of $\frac{1}{2}ab$ when $a = 1\frac{3}{7}$ and $b = -\frac{4}{9}$ is $-\frac{20}{63}$.

Check

Evaluate $\frac{2}{3}xy$ if $x = -3\frac{1}{6}$ and $y = -5\frac{1}{4}$. $\frac{133}{12}$ or $11\frac{1}{12}$

Go Online You can complete an Extra Example online.

Pause and Reflect

When multiplying rational numbers, why is it beneficial to divide by common factors?

See students' observations.

Lesson 3-8 • Multiply and Divide Rational Numbers 199

Interactive Presentation

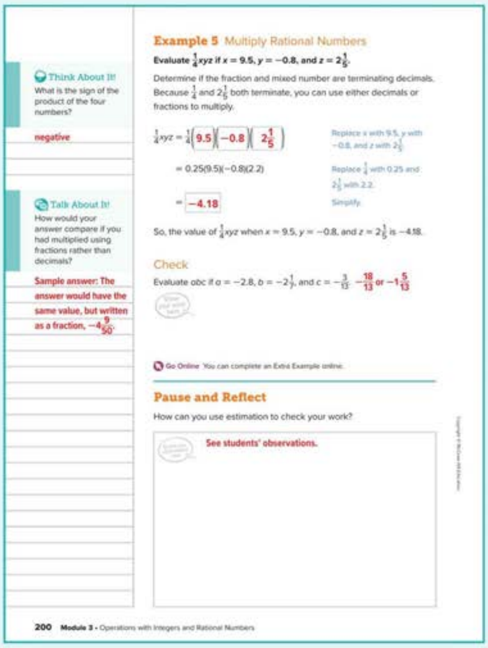
Example 4, Multiply Rational Numbers, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to evaluate the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 5 Multiply Rational Numbers

Evaluate $\frac{1}{4}xyz$ if $x = 9.5$, $y = -0.8$, and $z = 2\frac{1}{5}$.

Determine if the fraction and mixed number are terminating decimals. Because $\frac{1}{4}$ and $2\frac{1}{5}$ both terminate, you can use either decimals or fractions to multiply.

$\frac{1}{4}xyz = \frac{1}{4}(9.5)(-0.8)(2\frac{1}{5})$ — Replace x with 9.5, y with −0.8, and z with $2\frac{1}{5}$.

$= 0.25(9.5)(-0.8)(2.2)$ — Replace $\frac{1}{4}$ with 0.25 and $2\frac{1}{5}$ with 2.2.

$= -4.18$ — Simplify.

So, the value of $\frac{1}{4}xyz$ when $x = 9.5$, $y = -0.8$, and $z = 2\frac{1}{5}$ is −4.18.

Think About It! What is the sign of the product of the four numbers? negative

Talk About It! How would your answer compare if you had multiplied using fractions rather than decimals? Sample answer: The answer would have the same value, but written as a fraction, $-4\frac{9}{50}$.

Check

Evaluate abc if $a = -2.8$, $b = -2\frac{1}{7}$, and $c = -\frac{3}{13}$. $-\frac{18}{13}$ or $-1\frac{5}{13}$

Go Online You can complete an Extra Example online.

Pause and Reflect

How can you use estimation to check your work?

See students' observations.

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Interactive Presentation

Example 5, Multiply Rational Numbers, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to evaluate the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Multiply Rational Numbers

Objective

Students will evaluate an algebraic expression involving different forms of rational numbers.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them As students discuss the *Talk About It!* on Slide 3, encourage them to consider alternative approaches to finding the product, and explain the relationship between the product written as a fraction and the product written as a decimal.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the algebraic expression by noting that it contains four factors, some of which are decimals, and others fractions or mixed numbers. Because the fractions terminate (repeat zeros), it will be easier to perform the calculations if all of the numbers are written as decimals.

Questions for Mathematical Discourse

SLIDE 2

AL How many factors are there? 4

AL Are the factors expressed in the same form? No, two are decimals and two are fractions or mixed numbers.

OL Should we write the numbers all as fractions or all as decimals? Explain. Sample answer: decimals; The fraction and mixed number both are terminating decimals, so the calculations will be easier as decimals.

OL How can you check your answer? Sample answer: Use estimation. Round each number to the nearest whole number and then multiply by $\frac{1}{4}$.

BL If $\frac{1}{4}xyz = -4.18$, what is the value of xyz? Describe two different ways to find this value. −16.72; Sample answer: Multiply −4.18 by 4 since −4.18 represents *one-fourth of xyz*. Another way to find this value is to evaluate *xyz* by replacing the variables with each given value.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Divide Rational Numbers

Objective

Students will understand that they can apply what they know about dividing fractions, decimals, and integers to the set of rational numbers.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Have students watch the animation on Slide 1. The animation illustrates that dividing by a fraction is the same as multiplying by the fraction's multiplicative inverse.

Talk About It!

SLIDE 2

Mathematical Discourse

Why is the reciprocal of a fraction, with 1 in the numerator, an integer? Sample answer: Every integer can be written as a fraction with a denominator of 1. When a fraction's numerator is 1, its reciprocal will have a denominator of 1, making it an integer.

Example 6 Divide Rational Numbers

Objective

Students will divide rational numbers written as fractions.

Questions for Mathematical Discourse

SLIDE 1

AL Will the quotient be positive or negative? Explain. negative; The signs of the rational numbers are different.

AL What is the multiplicative inverse of $\frac{1}{9}$? Why do we need to find it? 9; Sample answer: To divide by a fraction, multiply by its multiplicative inverse.

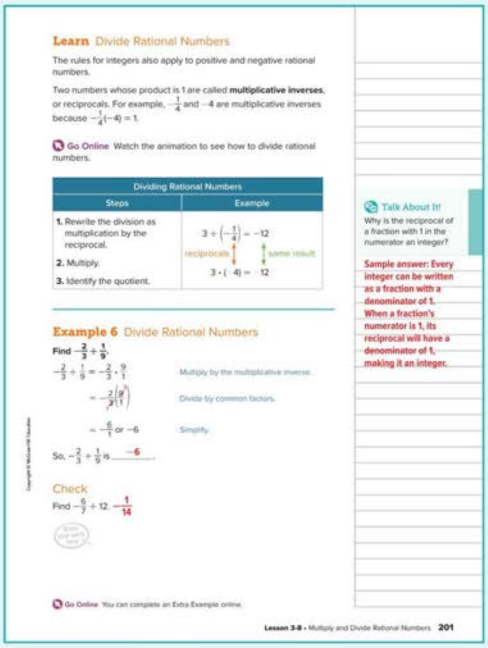
OL How is $-\frac{2}{3} \div \frac{1}{9}$ rewritten as a multiplication problem? $-\frac{2}{3} \cdot \frac{9}{1}$

OL Why do we divide 3 and 9 by their GCF? to simplify before multiplying

BL Simplify $-\frac{27}{4} \div \frac{9}{16}$. -12

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Divide Rational Numbers

The rules for integers also apply to positive and negative rational numbers.

Two numbers whose product is 1 are called **multiplicative inverses**, or reciprocals. For example, $-\frac{1}{4}$ and -4 are multiplicative inverses because $-\frac{1}{4}(-4) = 1$.

Go Online Watch the animation to see how to divide rational numbers.

Dividing Rational Numbers	
Steps	**Example**
1. Rewrite the division as multiplication by the reciprocal. 2. Multiply. 3. Identify the quotient.	$3 \div \left(-\frac{1}{4}\right) = -12$ reciprocals ↕ same result ↕ $3 \cdot (-4) = -12$

Talk About It! Why is the reciprocal of a fraction with 1 in the numerator an integer?

Sample answer: Every integer can be written as a fraction with a denominator of 1. When a fraction's numerator is 1, its reciprocal will have a denominator of 1, making it an integer.

Example 6 Divide Rational Numbers

Find $-\frac{2}{3} \div \frac{1}{9}$.

$-\frac{2}{3} \div \frac{1}{9} = -\frac{2}{3} \cdot \frac{9}{1}$ Multiply by the multiplicative inverse.

$= -\frac{2}{3}\left(\frac{9}{1}\right)$ Divide by common factors.

$= -\frac{6}{1}$ or -6 Simplify.

So, $-\frac{2}{3} \div \frac{1}{9}$ is -6.

Check

Find $-\frac{6}{7} \div 12$. $-\frac{1}{14}$

Go Online You can complete an Extra Example online.

Lesson 3-8 • Multiply and Divide Rational Numbers 201

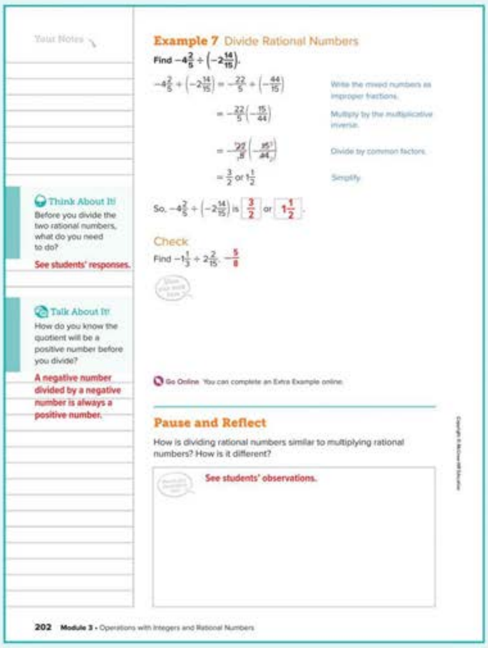
Interactive Presentation

Learn, Divide Rational Numbers, Slide 1 of 2

WATCH On Slide 1 of the Learn, students watch an animation that explains how to divide rational numbers.

TYPE On Slide 1 of Example 1, students enter a missing value to find the quotient.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Your Notes

Example 7 Divide Rational Numbers

Find $-4\frac{2}{5} \div \left(-2\frac{14}{15}\right)$.

$-4\frac{2}{5} \div \left(-2\frac{14}{15}\right) = -\frac{22}{5} \div \left(-\frac{44}{15}\right)$ Write the mixed numbers as improper fractions.

$= -\frac{22}{5}\left(-\frac{15}{44}\right)$ Multiply by the multiplicative inverse.

$= -\frac{\cancel{22}^{1}}{\cancel{5}_{1}}\left(-\frac{\cancel{15}^{3}}{\cancel{44}_{2}}\right)$ Divide by common factors.

$= \frac{3}{2}$ or $1\frac{1}{2}$ Simplify.

So, $-4\frac{2}{5} \div \left(-2\frac{14}{15}\right)$ is $\frac{3}{2}$ or $1\frac{1}{2}$.

Think About It! Before you divide the two rational numbers, what do you need to do? See students' responses.

Check

Find $-1\frac{1}{3} \div 2\frac{2}{15}$. $-\frac{5}{8}$

Talk About It! How do you know the quotient will be a positive number before you divide? A negative number divided by a negative number is always a positive number.

Go Online You can complete an Extra Example online.

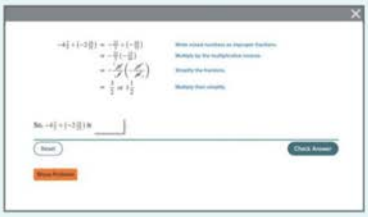
Pause and Reflect

How is dividing rational numbers similar to multiplying rational numbers? How is it different?

See students' observations.

202 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Example 7, Divide Rational Numbers, Slide 2 of 4

TYPE

On Slide 2, students enter a missing value to find the quotient.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 7 Divide Rational Numbers

Objective

Students will divide rational numbers written as mixed numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to be able to understand and explain how the rules for determining the sign of the quotient when dividing integers apply to dividing rational numbers.

6 Attend to Precision Encourage students to calculate accurately and efficiently, and be able to explain why the mixed numbers should be written as improper fractions. Students should adhere to the rules for writing division as multiplication of the multiplicative inverse, and pay attention to the sign of the quotient.

Questions for Mathematical Discourse

SLIDE 2

AL Will the quotient be positive or negative? Explain. positive; The signs of the rational numbers are the same.

AL Why do we rewrite both mixed numbers as improper fractions? in order to multiply the numerators and multiply the denominators

OL How do we rewrite $-\frac{22}{5} \div \left(-\frac{44}{15}\right)$ as a multiplication expression? $-\frac{22}{5}\left(-\frac{15}{44}\right)$

OL Explain how to simplify the fractions before multiplying. Divide 22 and 44 by their GCF, 22. Divide 5 and 15 by their GCF, 5.

BL Find $1.375 \div -\left(2\frac{1}{8}\right)$. $-\frac{11}{17}$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Divide Rational Numbers

Objective

Students will understand when it is more efficient to write numbers as fractions or decimals when dividing rational numbers written in different forms.

Teaching Notes

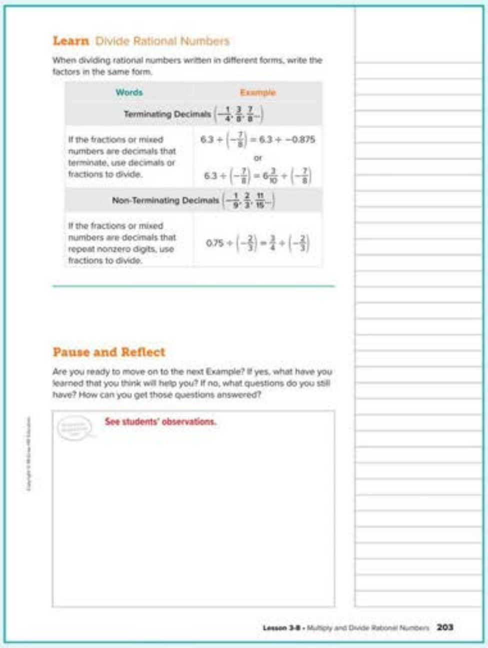
SLIDE 1

Be sure that students understand and can be able to explain why it is often easier to write all of the numbers as decimals before computing, if the fractions or mixed numbers would have decimal forms that terminate (repeat zeros). The calculations are often easier since the decimals terminate. If the fractions or mixed numbers would have decimal forms that repeat non-zero digits, it is often easier to write all of the numbers as fractions. Be sure students can explain why; the calculations could be more difficult if the numbers were written as deciamls, due to the non-zero repeating digits.

DIFFERENTIATE

Reteaching Activity L

Some students may struggle in deciding when it might be more efficient to write numbers as decimals or fractions when dividing rational numbers. Have them work with a partner to generate several examples of rational number division expressions in which it is more efficient to write the numbers as fractions. Have them explain their reasoning. Then have them generate several examples in which it is more efficient to write the numbers as decimals, and explain their reasoning. Have them compare their examples with another pair of students.



Learn Divide Rational Numbers

When dividing rational numbers written in different forms, write the factors in the same form.

Words	Example
Terminating Decimals $\left(-\frac{1}{4}, \frac{3}{8}, \frac{7}{8} \ldots\right)$	
If the fractions or mixed numbers are decimals that terminate, use decimals or fractions to divide.	$6.3 \div \left(-\frac{7}{8}\right) = 6.3 \div -0.875$ or $6.3 \div \left(-\frac{7}{8}\right) = 6\frac{3}{10} \div \left(-\frac{7}{8}\right)$
Non-Terminating Decimals $\left(-\frac{1}{9}, \frac{2}{3}, \frac{11}{15} \ldots\right)$	
If the fractions or mixed numbers are decimals that repeat nonzero digits, use fractions to divide.	$0.75 \div \left(-\frac{2}{3}\right) = \frac{3}{4} \div \left(-\frac{2}{3}\right)$

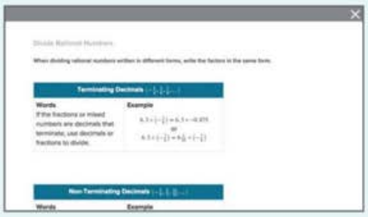
Pause and Reflect

Are you ready to move on to the next Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

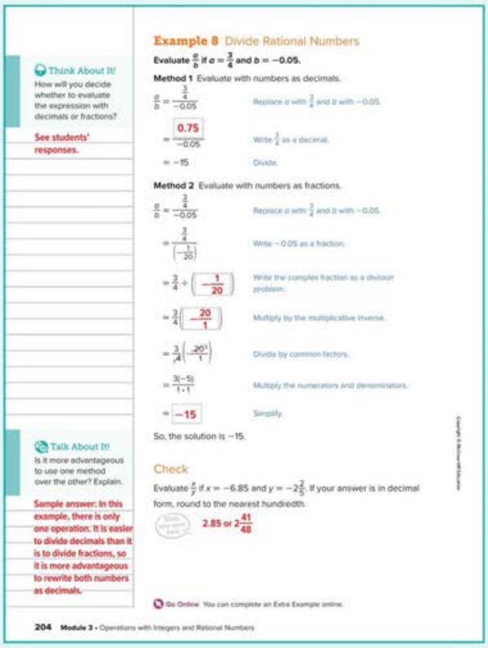
See students' observations.

Lesson 3-8 • Multiply and Divide Rational Numbers 203

Interactive Presentation



Learn, Divide Rational Numbers



Think About It! How will you decide whether to evaluate the expression with decimals or fractions?

See students' responses.

Example 8 Divide Rational Numbers

Evaluate $\frac{a}{b}$ if $a = \frac{3}{4}$ and $b = -0.05$.

Method 1 Evaluate with numbers as decimals.

$\frac{a}{b} = \frac{\frac{3}{4}}{-0.05}$ Replace a with $\frac{3}{4}$ and b with -0.05.

$= \frac{0.75}{-0.05}$ Write $\frac{3}{4}$ as a decimal.

$= -15$ Divide.

Method 2 Evaluate with numbers as fractions.

$\frac{a}{b} = \frac{\frac{3}{4}}{-0.05}$ Replace a with $\frac{3}{4}$ and b with -0.05.

$= \frac{\frac{3}{4}}{\left(-\frac{1}{20}\right)}$ Write -0.05 as a fraction.

$= \frac{3}{4} \div \left(-\frac{1}{20}\right)$ Write the complex fraction as a division problem.

$= \frac{3}{4}\left(-\frac{20}{1}\right)$ Multiply by the multiplicative inverse.

$= \frac{3}{4}\left(-\frac{20}{1}\right)$ Divide by common factors.

$= \frac{3(-5)}{1 \cdot 1}$ Multiply the numerators and denominators.

$= -15$ Simplify.

So, the solution is −15.

Talk About It! Is it more advantageous to use one method over the other? Explain.

Sample answer: In this example, there is only one operation. It is easier to divide decimals than it is to divide fractions, so it is more advantageous to rewrite both numbers as decimals.

Check

Evaluate $\frac{x}{y}$ if $x = -6.85$ and $y = -2\frac{2}{5}$. If your answer is in decimal form, round to the nearest hundredth.

2.85 or $2\frac{41}{48}$

Go Online You can complete an Extra Example online.

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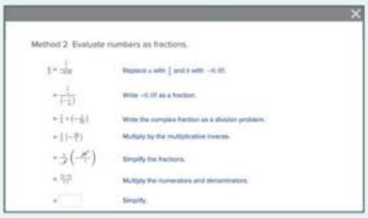
Interactive Presentation



Example 8, Divide Rational Numbers, Slide 3 of 5

TYPE

On Slide 2, students evaluate the numbers as decimals (Method 1).

TYPE

On Slide 3, students evaluate the numbers as fractions (Method 2).

CHECK

Complete the Check exercise online to determine if students are ready to move on.

Example 8 Divide Rational Numbers

Objective

Students will evaluate an algebraic expression involving division of rational numbers.

MP Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them Encourage students to consider both methods (writing the numbers as decimals, and writing the numbers as fractions) as valid methods and explain the correspondencies between them.

6 Attend to Precision Students should be able to fluently perform all of the calculations necessary to evaluate the expression, paying attention to the specific values of each variable and the sign of the quotient.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that this is a division problem? Any fraction is division of the numerator by the denominator.

OL Are the numbers expressed in the same form? No, one is a fraction and one is a decimal.

OL How can you write the problem in a different form? $\frac{3}{4} \div (-0.05)$

BL If $c = -\frac{1}{8}$, what is $\frac{\frac{a}{b}}{c}$? 120

SLIDE 3

AL How can you write −0.05 as a fraction in simplest form? $-\frac{5}{100}$, which is $-\frac{1}{20}$ in simplest form

AL Will the quotient be positive or negative? Explain. negative; The rational numbers have different signs.

OL Describe the difference between Method 1 and Method 2. Sample answer: In Method 1, we write both numbers as decimals. In Method 2, we write both numbers as fractions. Both methods yield the correct answer.

BL If $\frac{a}{b} = -15$, describe two ways to find $\frac{1}{5} \cdot \frac{a}{b}$. Sample answer: *One-fifth* of −15 is the same as dividing −15 by 5, so the solution would be −3. Another way is to simplify $\frac{1}{5} \cdot \frac{a}{b}$ as $\frac{a}{5b}$ and evaluate the expression at the specific values of the variables *a* and *b*.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Temperature

Objective

Students will come up with their own strategy to solve an application problem involving a city's change in temperature overnight.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does it mean to find $\frac{2}{3}$ of a number?
- How do you know if the temperature increased or decreased?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Temperature

A sign shows the temperature in Badger, Minnesota, at 10 P.M. is −11.5°F. At 4 A.M., the temperature changed by $\frac{2}{3}$ of the current value. What is the final temperature?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$-19\frac{1}{6}$ degrees; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Is the final temperature warmer or colder than the original temperature? Explain.

colder; Sample answer: The temperature changed by a negative value since two-thirds of −11.5 is negative. That means that the temperature is decreasing and colder than the original temperature.

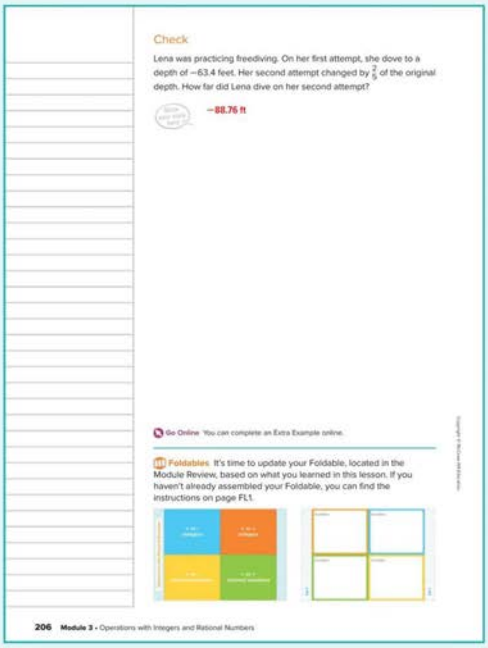
Lesson 3-8 • Multiply and Divide Rational Numbers 205

Interactive Presentation

Apply, Temperature

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Lena was practicing freediving. On her first attempt, she dove to a depth of −63.4 feet. Her second attempt changed by $\frac{7}{5}$ of the original depth. How far did Lena dive on her second attempt?

−88.76 ft

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information about multiplying and dividing rational numbers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with rational numbers related to operations with integers?

In this lesson, students learned how to multiply and divide rational numbers. Encourage them to work with a partner to compare and contrast multiplying and dividing rational numbers to multiplying and dividing integers.

Exit Ticket

Refer to the Exit Ticket slide. If you want to triple the recipe, explain how to find the amount of cooked pasta you will need. Sample answer: Write $3\frac{3}{4}$ as an improper fraction. Multiply the improper fraction by 3, because you want to triple the recipe. Multiply the numerators and denominators. Then simplify. You will need $11\frac{1}{4}$ cups of cooked pasta.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 14, 16, 18–21
- Extension: Geometric Sequences, Series and Geometric Sequences
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 66-89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–14, 17, 18
- Extension: Geometric Sequences, Series and Geometric Sequences
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–8
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Venn Diagrams and Sets of Rational Numbers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

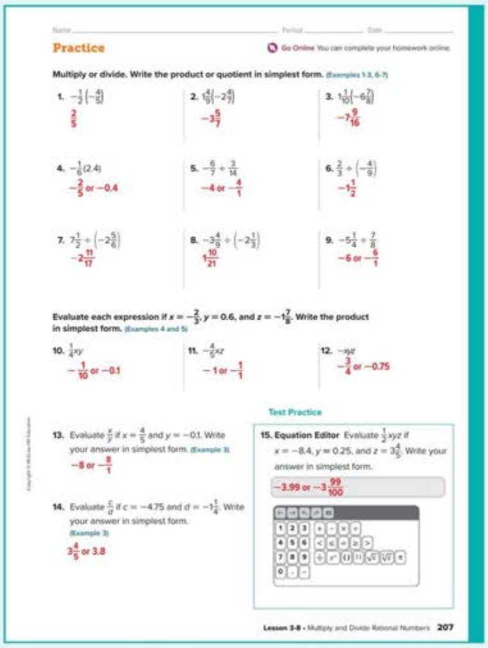
Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	multiply rational numbers	1–4
1	divide rational numbers	5–9
2	evaluate algebraic expressions involving multiplication of rational numbers	10–12
2	evaluate algebraic expressions involving division of rational numbers	13,14
2	extend concepts learned in class to apply them in new contexts	15
3	solve application problems that involve multiplying rational numbers	16, 17
3	higher-order and critical thinking skills	18–21

Common Misconception

Students may attempt to multiply two mixed numbers by multiplying the whole number parts and the fractional parts separately. This method can work when adding mixed numbers, but it results in an incorrect product when multiplying. Ask students to explain why it works for adding mixed numbers, but does not work for multiplying mixed numbers. A mixed number can be thought of as an addition expression. For example, $1\frac{1}{2}$ is really $1 + \frac{1}{2}$. When adding two mixed numbers, the Associative Property allows for addition in any order. So, the whole number parts can be added separately and the fractional parts can be added separately. However, when multiplying two mixed numbers, such as $1\frac{1}{2}\left(2\frac{1}{4}\right)$, the expression is really $\left(1 + \frac{1}{2}\right)\left(2 + \frac{1}{4}\right)$, or $\left(1\frac{1}{2}\right)\left(2 + \frac{1}{4}\right)$. Students could apply the Distributive Property to multiply $1\frac{1}{2}$ by 2 and $1\frac{1}{2}$ by $\frac{1}{4}$, and then find the sum.

When dividing mixed numbers, students may attempt to find the common denominator before multiplying by the multiplicative inverse. Explain to students that doing so may make dividing by common factors more difficult.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Multiply or divide. Write the product or quotient in simplest form. (Examples 1-3, 6-7)

1. $-\frac{1}{2}\left(-\frac{4}{5}\right)$ $\frac{2}{5}$
2. $1\frac{4}{9}\left(-2\frac{4}{7}\right)$ $-3\frac{5}{7}$
3. $1\frac{1}{10}\left(-6\frac{7}{8}\right)$ $-7\frac{9}{16}$
4. $-\frac{1}{6}(2.4)$ $-\frac{2}{5}$ or -0.4
5. $-\frac{6}{7} \div \frac{3}{14}$ -4 or $-\frac{4}{1}$
6. $\frac{2}{3} \div \left(-\frac{4}{9}\right)$ $-1\frac{1}{2}$
7. $7\frac{1}{2} \div \left(-2\frac{5}{6}\right)$ $-2\frac{11}{17}$
8. $-3\frac{4}{9} \div \left(-2\frac{1}{3}\right)$ $1\frac{10}{21}$
9. $-5\frac{1}{4} \div \frac{7}{8}$ -6 or $-\frac{6}{1}$

Evaluate each expression if $x = -\frac{2}{3}$, $y = 0.6$, and $z = -1\frac{7}{8}$. Write the product in simplest form. (Examples 4 and 5)

10. $\frac{1}{4}xy$ $-\frac{1}{10}$ or -0.1
11. $-\frac{4}{5}xz$ -1 or $-\frac{1}{1}$
12. $-xyz$ $-\frac{3}{4}$ or -0.75

Test Practice

13. Evaluate $\frac{x}{y}$ if $x = \frac{4}{5}$ and $y = -0.1$. Write your answer in simplest form. (Example 3) -8 or $-\frac{8}{1}$
14. Evaluate $\frac{c}{d}$ if $c = -4.75$ and $d = -1\frac{1}{4}$. Write your answer in simplest form. (Example 3) $3\frac{4}{5}$ or 3.8
15. **Equation Editor** Evaluate $\frac{1}{2}xyz$ if $x = -8.4$, $y = 0.25$, and $z = 3\frac{4}{5}$. Write your answer in simplest form. -3.99 or $-3\frac{99}{100}$

Lesson 3-8 • Multiply and Divide Rational Numbers **207**

7.NS.A.2, 7.NS.A.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Apply *indicates multi-step problem

*16. The table shows the change in the value of Rudo's stocks one day. The next day, the value of the All-Plus stock dropped $\frac{1}{4}$ of the amount it changed from the previous day. What was the total change in the All-Plus stock? Round to the nearest cent.

−$3.13

Stock	Change
All-Plus	−$2.50
True-Fit	−$3.75

*17. The photography club is selling hot chocolate at soccer games to raise money for new cameras. The table shows their profit per game for the first five games. Based on the average profit per game, how much total money can the club expect to earn by the end of the 10-game season?

$135.30

Game	Profit ($)
1	−12.50
2	−10.15
3	18.65
4	25.90
5	45.75

Higher-Order Thinking Problems

18. **Find the Error** A student is finding $-\frac{6}{7} \div \left(-\frac{5}{6}\right)$. Find the student's mistake and correct it.

$-\frac{6}{7} \div \left(-\frac{5}{6}\right) = -\frac{7}{6} \cdot \left(-\frac{5}{6}\right)$
$= \frac{35}{36}$

Sample answer: The student multiplied by the reciprocal of the dividend instead of the divisor. The correct work is $-\frac{6}{7} \cdot \left(-\frac{6}{5}\right)$ or $1\frac{1}{35}$.

19. **Use a Counterexample** Is the following statement *true* or *false*? If false, provide a counterexample.

The product of a fraction between 0 and 1 and a whole number or mixed number is never less than the whole number or mixed number.

false; Sample answer: $\frac{1}{2}\left(2\frac{1}{4}\right) = 1\frac{1}{8}$ and $1\frac{1}{8} < 2\frac{1}{4}$

20. **Persevere with Problems** Find the missing fraction for each problem.

a. $-\frac{6}{11} \cdot \left(\frac{x}{y}\right) = -\frac{1}{11}$

$\frac{1}{6}$

b. $\frac{a}{b} \div \left(-\frac{2}{3}\right) = 1$

$-\frac{2}{3}$

21. **Make an Argument** Which is greater, $20 \cdot \frac{3}{4}$ or $20 \div \frac{3}{4}$? Explain.

$20 \div \frac{3}{4}$; Multiplying 20 by a number less than 1 will result in a number less than 20. Dividing 20 by a number less than 1 will result in a number greater than 20.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students find the error in another student's reasoning.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 19, students determine if a statement is true or false and provide a counterexample.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 20, students use number sense to determine missing fractions.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 21, students make an argument about comparing two numerical expressions.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.

Use with Exercises 16–17 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 16 might be, "How could you find one-fourth of a decimal?"

Clearly explain your strategy.

Use with Exercise 20 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would estimate the product, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Lesson 3-9

Apply Rational Number Operations

7.NS.A.1, 7.NS.A.2

LESSON GOAL

Students will apply understanding of the four operations with rational numbers to evaluate mathematical expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Apply Rational Number Operations
Example 1: Apply Rational Number Operations
Example 2: Apply Rational Number Operations
Apply: Food

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 22 of the *Language Development Handbook* to help your students build mathematical language related to rational number operations.

ELL You can use the tips and suggestions on page T22 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7. NS.A** by adding, subtracting, multiplying, and dividing rational numbers.
Standards for Mathematical Content: **7 .NS.A.1, 7.NS.A.1.D, 7.NS.A.2, 7.NS.A.2.C, 7.NS.A.3,** *Also addresses, 7.EE.A.2, 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students multiplied and divided rational numbers.
7.NS.A.2, 7.NS.A.3

Now
Students apply understanding of the four operations with rational numbers and order of operations to evaluate mathematical expressions.
7.NS.A.3

Next
Students develop and use the Laws of Exponents to evaluate, simplify, and perform computations with expressions with powers.
8.EE.A.1, 8.EE.A.3, 8.EE.A.4

Rigor

The Three Pillars of Rigor

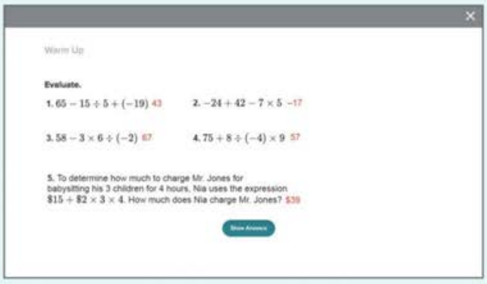
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of rational numbers and integers to develop an *understanding* of applying the order of operations to rational numbers. They *apply* their understanding of rational number operations to solve real-world problems.

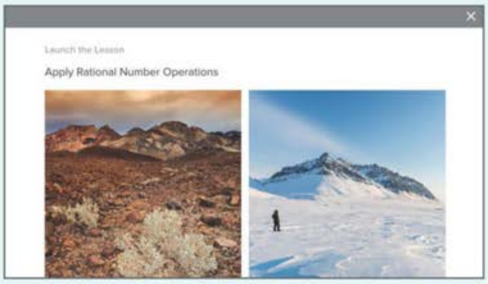
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

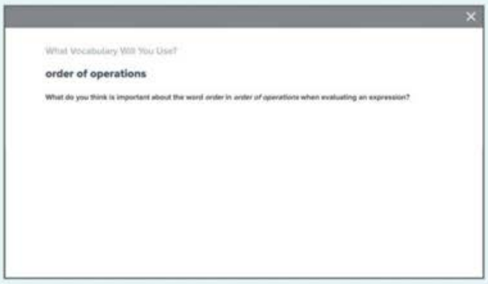
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- using the order of operations to evaluate expressions involving integers (Exercises 1–5)

Answers

1. 43 **4.** 57
2. −17 **5.** $39
3. 67

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using rational number operations to find differences in extreme temperatures.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What do you think is important about the word *order* in *order of operations* when evaluating an expression? Sample answer: The word *order* implies that the order in which you evaluate an expression is important and can affect the outcome of the answer.

Learn Apply Rational Number Operations

Objective

Students will understand how to apply the properties of operations to evaluate expressions involving rational numbers.

Go Online to find additional teaching notes.

Example 1 Apply Rational Number Operations

Objective

Students will use the properties of operations to evaluate expressions involving different forms of rational numbers.

Questions for Mathematical Discourse

SLIDE 2

AL What is the first step in evaluating this expression? Replace the variables with their values.

OL What is the expression after replacing the values? $\frac{5}{6} \cdot \left(-\frac{4}{5}\right) + 0.75 - \frac{1}{3}$

BL Find $ab + c - d$ if c is *one third* of its original value and d is 4 *times as great* as its original value. $-\frac{21}{12}$

SLIDE 3

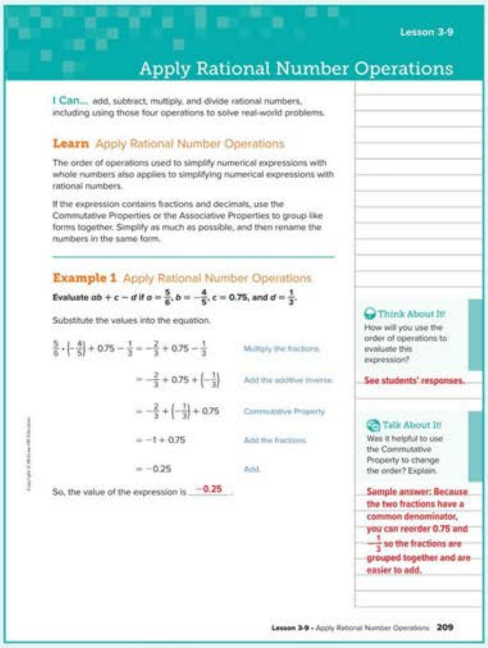
AL According to the order of operations, which operation should we perform first? Multiply the fractions.

OL Why do we reorder the terms using the Commutative Property, instead of adding and subtracting in order from left to right? Sample answer: Because the two fractions have the same denominator, it is easier to add them first, and then add the decimal. The Commutative Property allows us to do that.

BL Explain why it is unnecessary to write all of the numbers as fractions when evaluating this expression. Sample answer: By analyzing the two fractions, I know their sum is –1 using mental math. Because –1 is not a fraction, it is not necessary to write the other number, 0.75, as a fraction.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Lesson 3-9

Apply Rational Number Operations

I Can... add, subtract, multiply, and divide rational numbers, including using those four operations to solve real-world problems.

Learn Apply Rational Number Operations

The order of operations used to simplify numerical expressions with whole numbers also applies to simplifying numerical expressions with rational numbers.

If the expression contains fractions and decimals, use the Commutative Properties or the Associative Properties to group like forms together. Simplify as much as possible, and then rename the numbers in the same form.

Example 1 Apply Rational Number Operations

Evaluate $ab + c - d$ if $a = \frac{5}{6}$, $b = -\frac{4}{5}$, $c = 0.75$, and $d = \frac{1}{3}$.

Substitute the values into the equation.

$\frac{5}{6} \cdot \left(-\frac{4}{5}\right) + 0.75 - \frac{1}{3} = -\frac{2}{3} + 0.75 - \frac{1}{3}$ Multiply the fractions.

$= -\frac{2}{3} + 0.75 + \left(-\frac{1}{3}\right)$ Add the additive inverse.

$= -\frac{2}{3} + \left(-\frac{1}{3}\right) + 0.75$ Commutative Property

$= -1 + 0.75$ Add the fractions.

$= -0.25$ Add.

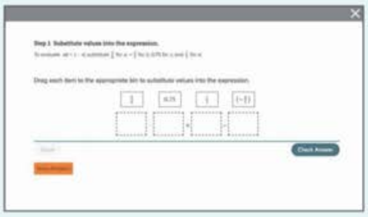
So, the value of the expression is -0.25.

Think About It! How will you use the order of operations to evaluate this expression? See students' responses.

Talk About It! Was it helpful to use the Commutative Property to change the order? Explain. Sample answer: Because the two fractions have a common denominator, you can reorder 0.75 and $-\frac{1}{3}$ so the fractions are grouped together and are easier to add.

Lesson 3-9 • Apply Rational Number Operations 209

Interactive Presentation



Example 1, Apply Rational Number Operations, Slide 2 of 5

DRAG & DROP

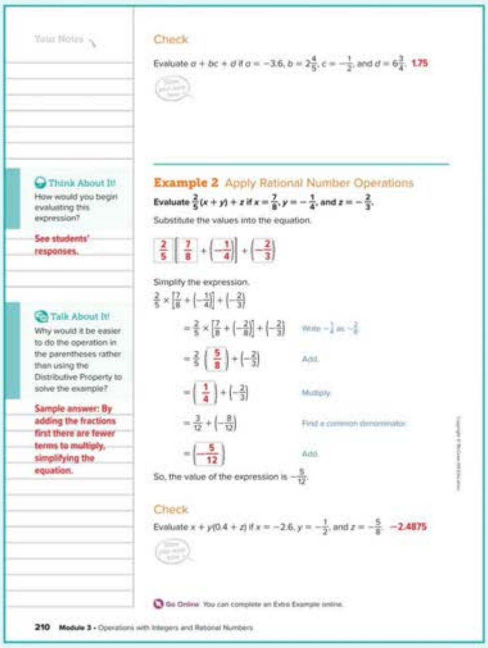
On Slide 2 of Example 1, students drag to substitute values into an expression.

TYPE

On Slide 3 of Example 1, students evaluate the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Your Notes

Check

Evaluate $a + bc + d$ if $a = -3.6$, $b = 2\frac{4}{5}$, $c = -\frac{1}{2}$, and $d = 6\frac{3}{4}$. 1.75

Think About It!
How would you begin evaluating this expression?

See students' responses.

Example 2 Apply Rational Number Operations

Evaluate $\frac{2}{5}(x + y) + z$ if $x = \frac{7}{8}$, $y = -\frac{1}{4}$, and $z = -\frac{2}{3}$.

Substitute the values into the equation.

$\frac{2}{5}\left[\frac{7}{8} + \left(-\frac{1}{4}\right)\right] + \left(-\frac{2}{3}\right)$

Simplify the expression.

$\frac{2}{5} \times \left[\frac{7}{8} + \left(-\frac{1}{4}\right)\right] + \left(-\frac{2}{3}\right)$

$= \frac{2}{5} \times \left[\frac{7}{8} + \left(-\frac{2}{8}\right)\right] + \left(-\frac{2}{3}\right)$ Write $-\frac{1}{4}$ as $-\frac{2}{8}$.

$= \frac{2}{5}\left(\frac{5}{8}\right) + \left(-\frac{2}{3}\right)$ Add.

$= \left(\frac{1}{4}\right) + \left(-\frac{2}{3}\right)$ Multiply.

$= \frac{3}{12} + \left(-\frac{8}{12}\right)$ Find a common denominator.

$= \left(-\frac{5}{12}\right)$ Add.

So, the value of the expression is $-\frac{5}{12}$.

Talk About It!
Why would it be easier to do the operation in the parentheses rather than using the Distributive Property to solve the example?

Sample answer: By adding the fractions first there are fewer terms to multiply, simplifying the equation.

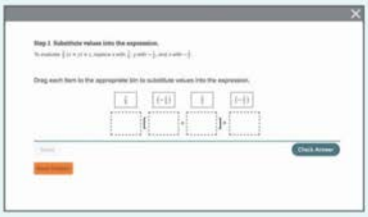
Check

Evaluate $x + y(0.4 + z)$ if $x = -2.6$, $y = -\frac{1}{2}$, and $z = -\frac{5}{8}$. −2.4875

Go Online You can complete an Extra Example online.

210 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Example 2, Apply Rational Number Operations, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag to substitute values into an expression.

TYPE

On Slide 3, students enter missing values to evaluate the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Apply Rational Number Operations

Objective

Students will use the properties of operations to evaluate expressions involving different forms of rational numbers.

MP Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them Encourage students to plan a solution pathway prior to jumping into a solution attempt. Students should notice that, in this particular expression, it is easier to perform the operations inside the parentheses rather than first using the Distributive Property to remove the parentheses.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to use clear and precise mathematical language in their explanations.

Questions for Mathematical Discourse

SLIDE 2

AL Which numbers should be placed inside the parentheses? $\frac{7}{8}$ and $-\frac{1}{4}$

OL A classmate wrote the expression as $\frac{2}{5}\left(\frac{7}{8} - \frac{1}{4}\right) - \frac{2}{3}$. Is this an equivalent expression? Explain. yes; Sample answer: The addition of a negative number is the same as subtracting a positive number.

BL How would the expression change if x was negative? $\frac{2}{5}\left[-\frac{7}{8} + \left(-\frac{1}{4}\right)\right] + \left(-\frac{2}{3}\right)$

SLIDE 3

AL Which operation(s) do we need to perform first? the operations inside the parentheses

OL Why did we not write $\frac{2}{5}$ with the common denominator? Sample answer: It is not being added with the other fractions. It is being multiplied. You only need a common denominator when adding or subtracting.

BL A classmate wrote the expression as 0.4(0.875 − 0.25) − 0.66. Describe their mistake. Sample answer: The fraction $-\frac{2}{3}$ is not equivalent to −0.66. It is equivalent to $-0.\overline{6}$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Food

Objective

Students will come up with their own strategy to solve an application problem that involves changing amounts in a recipe.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

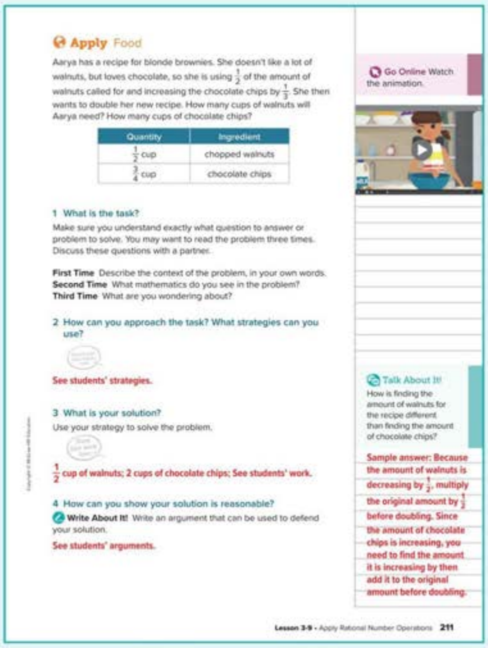
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you calculate the change in walnuts?
- How do you calculate the change in chocolate chips?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Food

Aarya has a recipe for blonde brownies. She doesn't like a lot of walnuts, but loves chocolate, so she is using $\frac{1}{2}$ of the amount of walnuts called for and increasing the chocolate chips by $\frac{1}{3}$. She then wants to double her new recipe. How many cups of walnuts will Aarya need? How many cups of chocolate chips?

Quantity	Ingredient
$\frac{1}{2}$ cup	chopped walnuts
$\frac{3}{4}$ cup	chocolate chips

Go Online Watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$\frac{1}{2}$ cup of walnuts; 2 cups of chocolate chips; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

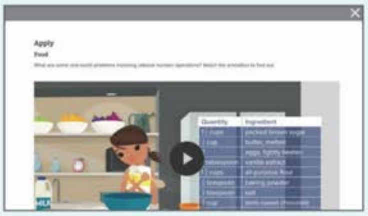
See students' arguments.

Talk About It! How is finding the amount of walnuts for the recipe different than finding the amount of chocolate chips?

Sample answer: Because the amount of walnuts is decreasing by $\frac{1}{2}$, multiply the original amount by $\frac{1}{2}$ before doubling. Since the amount of chocolate chips is increasing, you need to find the amount it is increasing by then add it to the original amount before doubling.

Lesson 3-9 • Apply Rational Number Operations **211**

Interactive Presentation



Apply, Food

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Adel City is enclosing a skate park with fencing. The skate park is $31\frac{1}{3}$ yards wide and $24\frac{2}{3}$ yards long. Fencing is sold in 8-foot sections and costs $67.99 per section. How much will it cost to fence in the entire skate park? $2,855.58

Go Online You can complete an Extra Example online.

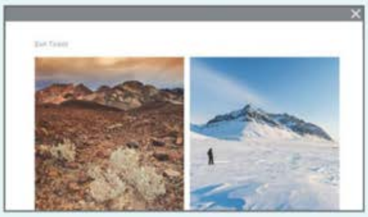
Pause and Reflect

Why is it helpful to use parentheses when substituting values for variables?

See students' observations.

212 Module 3 • Operations with Integers and Rational Numbers

Interactive Presentation



Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. To convert Celsius temperatures to Fahrenheit, you can use the formula $F = \frac{9}{5}C + 32$. Find the difference between the two temperatures. Describe the steps you used.

213.84 degrees; Sample answer: Convert –62.1°C to °F by multiplying –62.1 by $\frac{9}{5}$. Then add 32. So, –62.1°C = –79.78°F. Then subtract to find the difference between –79.78°F and 134.06°F, which is 213.84°F.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 13, 15–19
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–11, 14, 16
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Venn Diagrams and Sets of Rational Numbers

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

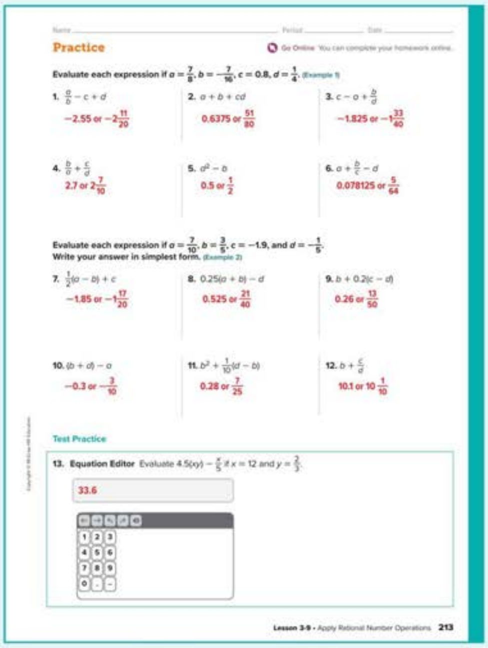
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use the properties of operations to evaluate expressions involving rational numbers	1–6
1	use the properties of operations to evaluate expressions involving different forms of rational numbers	7–12
2	extend concepts learned in class to apply them in new contexts	13
3	solve application problems that involve applying rational number operations	14, 15
3	higher-order and critical thinking skills	16–19

Common Misconception

Some students may consider converting all numbers to either a fraction or decimal before solving. Explain to students that it may not always be necessary if they are able to use properties of operations such as the Commutative Property.



Practice

Evaluate each expression if $a = \frac{7}{8}$, $b = -\frac{7}{16}$, $c = 0.8$, $d = \frac{1}{4}$. (Example 1)

1. $\frac{a}{b} - c + d$ -2.55 or $-2\frac{11}{20}$
2. $a + b + cd$ 0.6375 or $\frac{51}{80}$
3. $c - a + \frac{b}{d}$ -1.825 or $-1\frac{33}{40}$
4. $\frac{b}{a} + \frac{c}{d}$ 2.7 or $2\frac{7}{10}$
5. $a^2 - b$ 0.5 or $\frac{1}{2}$
6. $a + \frac{b}{c} - d$ 0.078125 or $\frac{5}{64}$

Evaluate each expression if $a = \frac{7}{10}$, $b = \frac{3}{5}$, $c = -1.9$, and $d = -\frac{1}{5}$. Write your answer in simplest form. (Example 2)

7. $\frac{1}{2}(a - b) + c$ -1.85 or $-1\frac{17}{20}$
8. $0.25(a + b) - d$ 0.525 or $\frac{21}{40}$
9. $b + 0.2(c - d)$ 0.26 or $\frac{13}{50}$
10. $(b + d) - a$ -0.3 or $-\frac{3}{10}$
11. $b^2 + \frac{1}{10}(d - b)$ 0.28 or $\frac{7}{25}$
12. $b + \frac{c}{d}$ 10.1 or $10\frac{1}{10}$

Test Practice

13. **Equation Editor** Evaluate $4.5(xy) - \frac{x}{5}$ if $x = 12$ and $y = \frac{2}{3}$.

33.6

Lesson 3-9 • Apply Rational Number Operations 213

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Apply *indicates multi-step problem

*14. Jake is enclosing his vegetable garden with fencing. The table shows the dimensions of his rectangular garden. Fencing is sold in 2.5-foot sections and costs $25.99 per section. How much will it cost to fence in the entire garden?

Garden Dimension	Measurement (yards)
Length	$14\frac{2}{3}$
Width	$10\frac{1}{3}$

$1,559.40

*15. Maya has a recipe for blueberry chocolate chip muffins. She doesn't like a lot of oats, but loves blueberries, so she is using $\frac{3}{4}$ of the amount of oats called for and increasing the amount of blueberries by $\frac{1}{4}$ of the original amount. She then wants to double her new recipe. How many cups of oats will Maya need? How many cups of blueberries?

Ingredient	Amount (cups)
Blueberries	$\frac{3}{4}$
Chocolate Chips	1
Oats	$\frac{1}{2}$
White Sugar	1

$\frac{3}{4}$ cup oats; $1\frac{7}{8}$ cups of blueberries

Higher-Order Thinking Problems

16. **Justify Conclusions** A student says that the Commutative Property and Associative Property cannot be used to simplify expressions with rational numbers. Is the student correct? Justify your answer.

no; Sample answer: The Commutative Properties and the Associative Properties can be used to simplify expressions without affecting the sum or product. These properties only work with addition and multiplication.

17. **Persevere with Problems** There are 120 bouncy balls in a display bin. Of the bouncy balls in the bin, one-half are priced at $0.25 each, one-fifth are priced at $0.75, and the remaining bouncy balls are priced at $1.50. What is the total value of the bouncy balls in the bin? Write a numerical expression that can be used to solve the problem. Then solve it.

$\left(\frac{1}{2} \times 120 \times 0.25\right) + \left(\frac{1}{5} \times 120 \times 0.75\right) + \left(\frac{3}{10} \times 120 \times 1.50\right)$; $87

18. A preschool teacher has 17.5 feet of yarn for her students to make necklaces to raise money for a local animal shelter. Each necklace requires 14 inches of yarn. If the necklaces sell for $2.50, how much money will they raise if they sell all the necklaces? Explain how you solved.

$37.50; First, find the number of inches of yarn the teacher has: $17.5 \times 12 = 210$ inches. Then find the number of necklaces the students can make: 210 inches $\div$ 14 inches $= 15$ necklaces. Multiply the number of necklaces by how much each necklace sells for: $15 \times \$2.50 = \$37.50$.

19. **Create** Write and solve a multi-step real-world problem where you apply rational number operations.

Sample answer: A homeowner is enclosing his rectangular property with fencing. The rectangular property is $25\frac{1}{2}$ yards wide and $30\frac{2}{3}$ yards long. Fencing is sold in 8-foot sections and costs $45.50 per section. How much will it cost to fence in the rectangular property?; $1,911

214 Module 3 • Operations with Integers and Rational Numbers

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students analyze a student's statement to determine if it is correct and justify their reasoning.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 17, students use multiple steps to write a numerical expression and find a total value.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercises 14–15 Have students work in groups of 3–4 to solve the problem in Exercise 14. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 15.

Interview a student.

Use with Exercises 16–17 Have pairs of students interview each other as they complete this problem. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 16 might be, "What is the difference between the Commutative Property and the Associative Property?"

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of applying the rules for operations with integers rational numbers. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

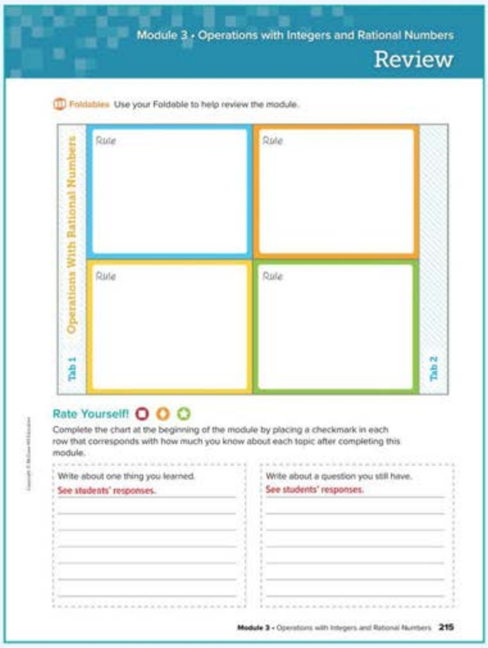
Assessment Resources

Put It All Together 1: Lessons 3-1 through 3-5
Put It All Together 2: Lessons 3-6 through 3-8
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **The Number System**.

- Integers
- Fractions and Decimals



Module 3 • Operations with Integers and Rational Numbers

Review

Foldables Use your Foldable to help review the module.

Rule	Rule
Rule	Rule

Operations With Rational Numbers

Tab 1

Tab 2

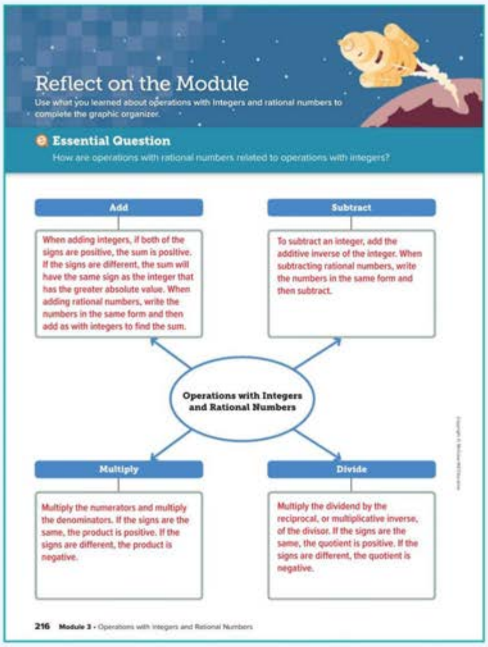
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 3 • Operations with Integers and Rational Numbers 215



Reflect on the Module

Use what you learned about operations with integers and rational numbers to complete the graphic organizer.

Essential Question

How are operations with rational numbers related to operations with integers?

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How are operations with rational numbers related to operations with integers? See students' graphic organizers.

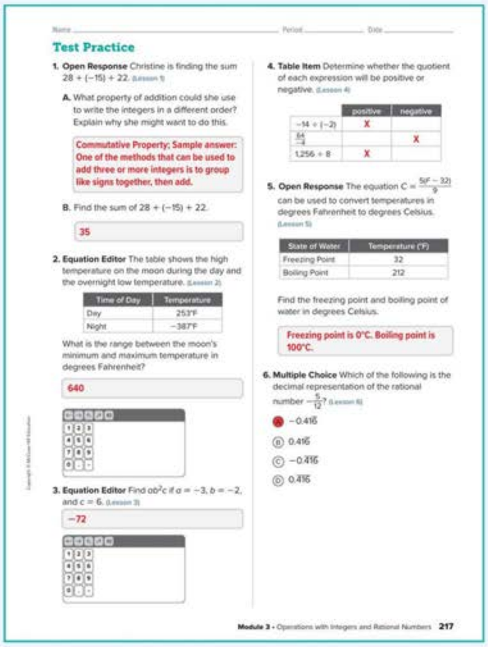
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–11 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	6, 7, 9
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	2, 3, 8, 10
Table Item	Students complete a table.	4
Open Response	Students construct their own response in the area provided.	1, 5, 7, 11

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.NS.A.1	3-1, 3-2, 3-5, 3-7, 3-9	1, 2, 5, 7, 8, 11
7.NS.A.1.A	3-1, 3-7	1, 7
7.NS.A.1.B	3-1, 3-7	1, 7
7.NS.A.1.C	3-2, 3-7	2, 8
7.NS.A.1.D	3-1, 3-2, 3-5, 3-7, 3-9	1, 2, 5, 7, 8, 11
7.NS.A.2	3-3, 3-5, 3-6, 3-7, 3-8, 3-9	3, 5, 6, 9-11
7.NS.A.2.A	3-3, 3-8	3, 9
7.NS.A.2.B	3-4, 3-6, 3-7, 3-8	4, 6, 7, 10
7.NS.A.2.C	3-3, 3-4, 3-5, 3-8, 3-9	3-5, 9-11
7.NS.A.2.D	3-6	6
7.NS.A.3	3-5, 3-8, 3-9	5, 9-11
7.EE.B.3	3-1, 3-3, 3-5, 3-7	1, 3, 5, 7, 8


Name ______ Period ______ Date ______

Test Practice

1. **Open Response** Christine is finding the sum $28 + (-15) + 22$. (Lesson 1)

   A. What property of addition could she use to write the integers in a different order? Explain why she might want to do this.

   Commutative Property; Sample answer: One of the methods that can be used to add three or more integers is to group like signs together, then add.

   B. Find the sum of $28 + (-15) + 22$.

   35

2. **Equation Editor** The table shows the high temperature on the moon during the day and the overnight low temperature. (Lesson 2)

Time of Day	Temperature
Day	253°F
Night	−387°F

   What is the range between the moon's minimum and maximum temperature in degrees Fahrenheit?

   640

3. **Equation Editor** Find ab^2c if $a = -3$, $b = -2$, and $c = 6$. (Lesson 3)

   −72

4. **Table Item** Determine whether the quotient of each expression will be positive or negative. (Lesson 4)

	positive	negative
$-14 \div (-2)$	X	
$\frac{64}{-4}$		X
$1{,}256 \div 8$	X	

5. **Open Response** The equation $C = \frac{5(F - 32)}{9}$ can be used to convert temperatures in degrees Fahrenheit to degrees Celsius. (Lesson 5)

State of Water	Temperature (°F)
Freezing Point	32
Boiling Point	212

   Find the freezing point and boiling point of water in degrees Celsius.

   Freezing point is 0°C. Boiling point is 100°C.

6. **Multiple Choice** Which of the following is the decimal representation of the rational number $-\frac{5}{12}$? (Lesson 6)

   (A) $-0.41\overline{6}$
   (B) $0.41\overline{6}$
   (C) $-0.\overline{416}$
   (D) $0.\overline{416}$

Module 3 • Operations with Integers and Rational Numbers 217

7. Open Response The manager of Happy Puppy dog boarding company begins the month with 85.4 pounds of dog food and tracks the weekly use, along with the supply delivery received in the middle of the month. (Lesson 7)

Week	Weekly Dog Food Use (lb)
1	−28.4
2	$-21\frac{5}{8}$
3	$97\frac{3}{4}$
4	−30.25

How much dog food does the company have on hand at the end of the month?

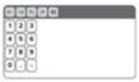
102.875 or $102\frac{7}{8}$ pounds

8. Equation Editor At the beginning of the day, Meredith has 85.4 pounds of apples to sell at a farmer's market. The table shows the hourly change in the amount of apples she has after the first several hours. (Lesson 7)

Hour	Change in Apples (lb)
1	−8.5
2	$-12\frac{2}{3}$
3	$-16\frac{1}{5}$
4	−10.2

After 4 hours, about how many pounds of apples does Meredith have left?

37.83



9. Multiple Choice What is the result when $-\frac{7}{8}$ is multiplied by −3.5? (Lesson 8)

- (A) $3\frac{1}{16}$
- (B) $2\frac{11}{16}$
- (C) $-2\frac{11}{16}$
- (D) $-3\frac{1}{16}$

10. Equation Editor Find $-\frac{11}{12} \div \left(-\frac{2}{3}\right)$. Express your answer as a fraction or mixed number in simplest form. (Lesson 8)

$1\frac{3}{8}$

11. Open Response Nathaniel has 132 DVDs in his collection. Each DVD case has the dimensions shown. He plans to buy shelves for the collection that are 31.5 inches wide and cost $15.75 each. How much will it cost Nathaniel to shelve his DVD collection? Explain how you found your answer. (Lesson 9)

$47.25; Sample answer: I divided 31.5 by the thickness of the DVD case, 0.6 to get 52.5. So, each shelf can hold at most 52 DVDs. Then I divided 132 by 52 to get approximately 2.5. Therefore, I know that I need 3 shelves. Then I multiplied 3 by the cost per shelf, $15.75.

Module 4

Exponents and Scientific Notation

Module Goal

Develop and use the Laws of Exponents to evaluate, simplify, and perform computations with expressions with powers.

Focus

Domain: Expressions and Equations

Major Cluster(s):

8.EE.A W ork with radicals and integer exponents.

Standards for Mathematical Content:

8.EE.A.1 K now and apply the properties of integer exponents to generate equivalent numerical expressions.

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Used scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Also addresses 8.EE.A.3.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- fluently multiply rational numbers

Use the Module Pretest to diagnose students' readiness for this module. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students added, subtracted, multiplied, and divided integers and rational numbers.
7.NS.A.1, 7.NS.A.2

Now
Students develop and use the Laws of Exponents to evaluate, simplify, and perform computations with expressions with powers.
8.EE.A.1, 8.EE.A.3, 8.EE.A.4

Next
Students will learn about the real number system by studying rational and irrational numbers. **8.NS.A.1, 8.NS.A.2, 8.EE.A.2**

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of exponents to develop *understanding* of the properties of exponents and scientific notation. They use this understanding to build *fluency* with simplifying algebraic expressions involving powers and computing with scientific notation. They *apply* their fluency to solve multi-step real-world problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

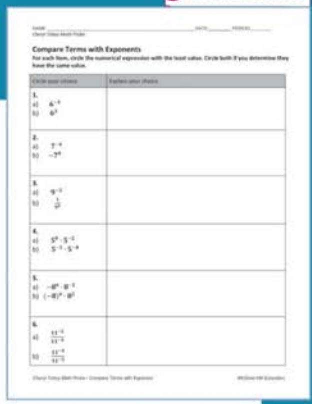
Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
4-1	Powers and Exponents	Foundational for 8.EE.A.1	1	0.5
4-2	Multiply and Divide Monomials	8.EE.A.1	2	1
4-3	Powers of Monomials	8.EE.A.1	1	0.5
Put It All Together 1: Lessons 4-2 and 4-3			0.5	0.25
4-4	Zero and Negative Exponents	8.EE.A.1	2	1
Put It All Together 2: Lessons 4-2 through 4-4			0.5	0.25
4-5	Scientific Notation	8.EE.A.3, 8.EE.A.4	2	1
4-6	Compute with Scientific Notation	8.EE.A.3, 8.EE.A.4, *Also addresses 8.EE.A.1*	1	0.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	13	6.5

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe
Compare Terms with Exponents

Module Resource



Analyze the Probe

Review the probe prior to assigning it to your students. In this probe, students will identify the numerical expression that has the least value, and explain their choice.

Targeted Concept Understand the meaning of positive and negative exponents as distinct from positive and negative integers.

Targeted Misconceptions

- Students may incorrectly pair *any* two negative signs together as positive.
- Students may count the number of negative signs to incorrectly use the total number of negative signs as an indicator of how "negative" or how small the value is.

Assign the probe after Lesson 4.

Correct Answers: 1. a; **2.** b; **3.** b; **4.** b; **5.** a; **6.** b

Collect and Assess Student Work

If the student selects...	**Then** the student likely...
4. a **5.** b **6.** a	incorrectly pairs *any* two negative signs together as positive. **Example:** In item 4, choice b, student pairs together two negatives to make a positive. They then view choice a as negative, and choice b as positive, viewing choice a as having the least value.
1. a* **2.** both **3.** a **4.** b* **5.** a* **6.** both *Items 1, 4, and 5 are the correct answer; check students' explanations for misconception.	counts the number of negative signs to incorrectly use the total number of negative signs as an indicator of how "negative" or small the value is. **Example:** For item 5, the student sees two negative signs in choice a, and only one in choice b. They then believe that since choice a has more negative signs, it has the least value.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign and/or revisit the following resources.

- ALEKS Exponents, Polynomials, and Radicals
- Lesson 4, Examples 1–5
- Lesson 2, Examples 1–7
- Lesson 3, Examples 1–4
- Lesson 4, Examples 1–5

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

![IGNITE!]

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

Why are exponents useful when working with very large or very small numbers? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

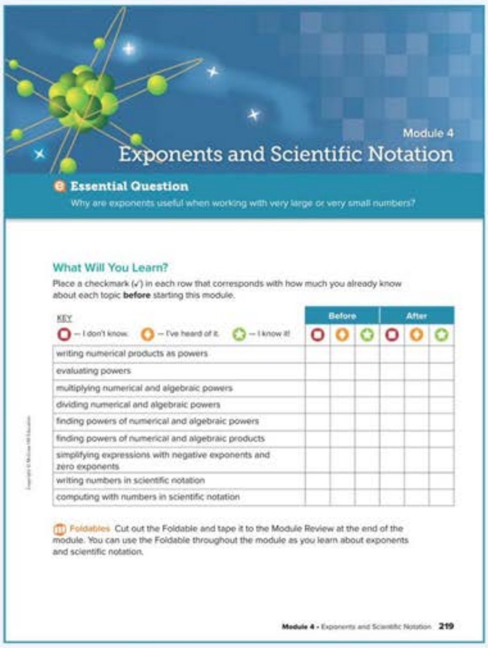
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of stars, galaxies, and the mass of an atom to introduce the idea of exponents and scientific notation. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 4

Exponents and Scientific Notation

Essential Question

Why are exponents useful when working with very large or very small numbers?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know	I've heard of it	I know it!	I don't know	I've heard of it	I know it!
writing numerical products as powers						
evaluating powers						
multiplying numerical and algebraic powers						
dividing numerical and algebraic powers						
finding powers of numerical and algebraic powers						
finding powers of numerical and algebraic products						
simplifying expressions with negative exponents and zero exponents						
writing numbers in scientific notation						
computing with numbers in scientific notation						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about exponents and scientific notation.

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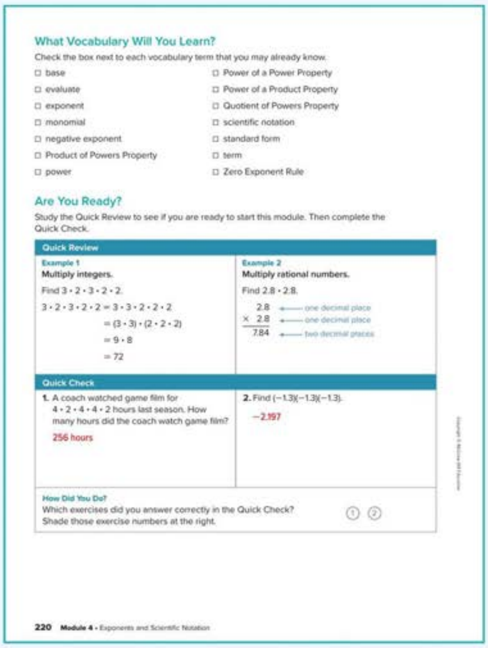
Interactive Student Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ base	☐ Power of a Power Property
☐ evaluate	☐ Power of a Product Property
☐ exponent	☐ Quotient of Powers Property
☐ monomial	☐ scientific notation
☐ negative exponent	☐ standard form
☐ Product of Powers Property	☐ term
☐ power	☐ Zero Exponent Rule

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Multiply integers.
Find $3 \cdot 2 \cdot 3 \cdot 2 \cdot 2$.

$$\begin{aligned} 3 \cdot 2 \cdot 3 \cdot 2 \cdot 2 &= 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \\ &= (3 \cdot 3) \cdot (2 \cdot 2 \cdot 2) \\ &= 9 \cdot 8 \\ &= 72 \end{aligned}$$

Example 2
Multiply rational numbers.
Find $2.8 \cdot 2.8$.

$$\begin{array}{r} 2.8 \\ \times\ 2.8 \\ \hline 7.84 \end{array}$$

2.8 ← one decimal place
2.8 ← one decimal place
7.84 ← two decimal places

Quick Check

1. A coach watched game film for $4 \cdot 2 \cdot 4 \cdot 4 \cdot 2$ hours last season. How many hours did the coach watch game film? 256 hours
2. Find $(-1.3)(-1.3)(-1.3)$. -2.197

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

220 **Module 4** • Exponents and Scientific Notation

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define Scientific notation is a way of expressing a number as the product of a factor and an integer power of 10. The factor must be greater than or equal to 1 and less than 10.

Example The distance from Earth to the moon is about 238,900 miles. In scientific notation, this distance is 2.389×10^5 miles.

Ask The equatorial circumference of Earth is about 24,901 miles. Write this distance in scientific notation. 2.4901×10^4 miles

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- understanding properties of operations
- understanding coefficients
- understanding integer operations
- finding products of powers, simplifying powers

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Exponents, Polynomials, and Radicals** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Collaborative Risk Taking

Some students may be averse to taking risks during math class, such as sharing an idea, strategy, or solution. They may worry about their grades or scores on tests, or some might feel less confident solving math problems, especially in front of their peers. Create a classroom environment where it is safe for students to take risks, including setting norms for how students will engage in classroom conversations. Encourage students to view mistakes as part of the path to success.

How Can I Apply It?

In the **Practice** section of each lesson, **Collaborative Practice** tips are provided for several exercises in the Teacher Edition. Assign those exercises and encourage students to take risks together as they solve problems, try new solution paths, and discuss their strategies.

When assigning the **application problems**, have students look for alternative approaches. Encourage them to view their solution process as one of refinement, as needed. They may try different paths, monitor their progress, and change course if necessary. This is part of the natural process of problem solving.

Lesson 4-1

Powers and Exponents

Foundational for 8.EE.A.1

LESSON GOAL

Students will write and evaluate expressions involving powers and exponents.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Exponents

Learn: Write Products as Powers
Example 1: Write Numerical Products as Powers
Example 2: Write Algebraic Products as Powers
Learn: Negative Bases and Parentheses
Learn: Evaluate Powers
Example 3: Evaluate Numerical Expressions
Example 4: Evaluate Algebraic Expressions
Example 5: Evaluate Algebraic Expressions
Apply: Mammals

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 23 of the *Language Development Handbook* to help your students build mathematical language related to powers and exponents.

ELL You can use the tips and suggestions on page T23 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.A** by writing and evaluating expressions involving powers and exponents.
Standards for Mathematical Content: **Foundational for 8 .EE.A.1**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students used the order of operations to evaluate expressions without exponents.
7.NS.A.1.D, 7.NS.A.2.C, 7.EE.A.1

Now
Students write and evaluate expressions involving powers and exponents.
Foundational for 8.EE.A.1

Next
Students will use the Laws of Exponents to simplify expressions involving products and quotients of monomials.
8.EE.A.1

Rigor

The Three Pillars of Rigor

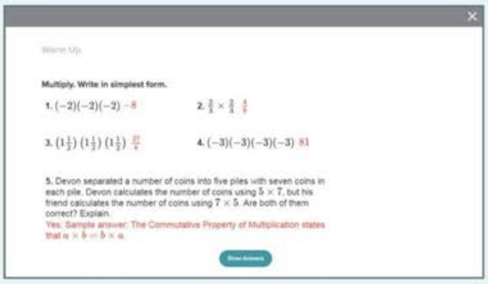
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of exponents (gained in Grade 6), and operations with rational numbers to build *fluency* with evaluating numeric and algebraic expressions involving powers and rational numbers.

Mathematical Background

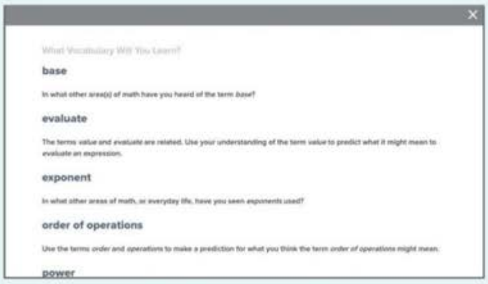
A *power* is a product of repeated factors using an exponent and a base. The *base* is the factor. The *exponent* tells how many times the base is used as a factor. According to the order of operations, evaluate any powers before performing other operations.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying rational numbers (Exercises 1 – 4)
- understanding properties of operations (Exercise 5)

Answers

1. -8
2. $\frac{4}{9}$
3. $\frac{27}{8}$
4. 81
5. Yes; Sample answer: The Commutative Property of Multiplication states that $a \times b = b \times a$.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about data storage capacity as based on powers of 2.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to facilitate a class discussion.

Ask:

- In what other area(s) of math have you heard of the term *base*? Sample answers: the base of a parallelogram, the base of a triangle, the base of a rectangular prism
- The terms *value* and *evaluate* are related. Use your understanding of the term *value* to predict what it might mean to *evaluate* an expression. Sample answer: To evaluate an expression might mean to find its numerical value.
- In what other areas of math, or everyday life, have you seen *e xponents* used? Sample answer: The formulas for the area of a square and volume of a cube use exponents. The units for area are in square units, such as square feet or square inches.
- Use the terms *order* and *operations* to mak e a prediction for what you think the term *order of operations* might mean. Sample answer: The order of operations might be the order in which operations, such as addition, subtraction, multiplication, and division, should be performed.
- What does *power* mean in everyday life ? Sample answer: strength, authority, influence

Explore Exponents

Objective

Students will explore how to write repeated multiplication using exponents.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with Emily's savings plan. She doubles the amount of money she puts into the piggy bank each week. Throughout this activity, students will use repeated multiplication and exponents to find the number of weeks it takes Emily to reach her savings goal.

Inquiry Question

How can you write repeated multiplication in a different way? Sample answer: I can use exponents where the number in the exponent represents the number of times the base appears as a factor.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

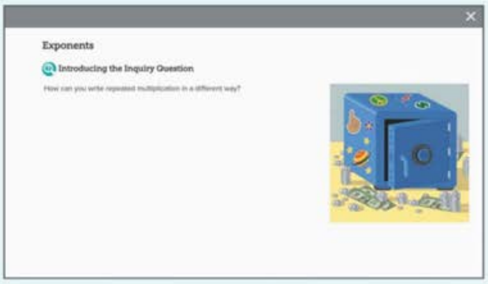
Mathematical Discourse

How many 2s are multiplied to find her weekly savings in week 4? In week 5? Sample answer: There are four 2s multiplied to find her weekly savings in week 4 and five 2s multiplied in week 5.

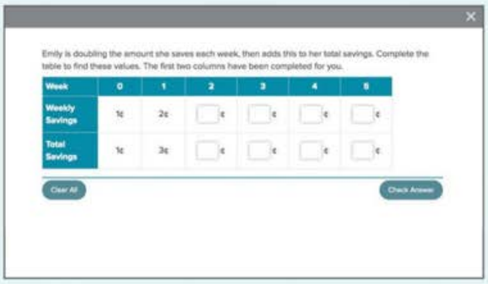
Is Emily close to her goal of $80 after 5 weeks? Explain. Sample answer: No, she is not close to her goal of $80 because after 5 weeks she has only made a total of 63 cents.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6

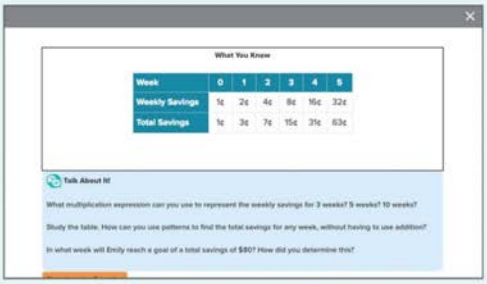


Explore, Slide 3 of 6

TYPE

On Slide 3, students enter missing values in the table displaying how much money Emily saved over 5 weeks.

Interactive Presentation



Explore, Slide 4 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and can view a sample answer.

Explore Exponents *(continued)*

Teaching the Mathematical Practices

8 Look for and Express Regularity in Repeated Reasoning
Encourage students to identify the patterns in their calculations of the amount Emily saved each week, and the total amount saved. Students should notice that the weekly savings can be represented by repeated multiplication of the factor 2, and that writing these amounts as exponents can be more efficient than writing a multiplication expression.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What multiplication expression can you use to represent the weekly savings for 3 weeks? 5 weeks? 10 weeks? Sample answer: For 3 weeks, you can use the expression 2 • 2 • 2. For 5 weeks, you can use the expression 2 • 2 • 2 • 2 • 2. For 10 weeks, you can use the expression 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2 • 2.

Study the table. How can you use patterns to find the total savings for any week, without having to use addition? Sample answer: You can find the next week's savings, then subtract 1¢.

In what week will Emily reach a goal of a total savings of $80? How did you determine this? Sample answer: Emily will reach a goal of a total savings of $80 in week 12. See students' explanations.

Learn Write Products as Powers

Objective

Students will learn how to write products as powers.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Enc ourage students to analyze the structure of the power in order to break it up into its parts. Some students may confuse the terms *power* and *exponent*. Encourage them to understand that the power consists of the base and the exponent.

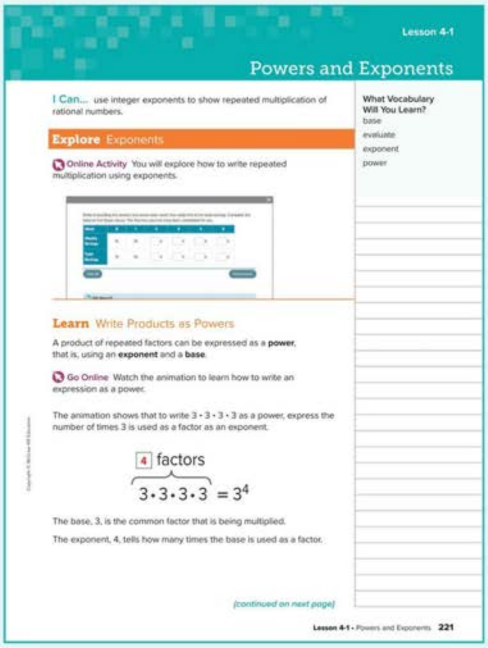
Go Online to have students watch the animation from Slide 1. The animation illustrates writing an expression as a power.

Teaching Notes

SLIDE 1

Students will learn the definitions of *power*, *exponent*, and *base*. Play the animation for the class. Students will learn how to write a repeated multiplication expression as a power. Point out to students that in the first expression, $3 \cdot 3 \cdot 3 \cdot 3$, there is one repeated factor 3. In the second expression, $2 \cdot 2 \cdot 2 \cdot 2 \cdot (-4) \cdot (-4) \cdot (-4)$, there are two repeated factors, 2, and (–4).

(continued on next page)



Lesson 4-1

Powers and Exponents

I Can... use integer exponents to show repeated multiplication of rational numbers.

What Vocabulary Will You Learn?
base
evaluate
exponent
power

Explore Exponents

Online Activity You will explore how to write repeated multiplication using exponents.

Learn Write Products as Powers

A product of repeated factors can be expressed as a **power**, that is, using an **exponent** and a **base**.

Go Online Watch the animation to learn how to write an expression as a power.

The animation shows that to write $3 \cdot 3 \cdot 3 \cdot 3$ as a power, express the number of times 3 is used as a factor as an exponent.

4 factors

$3 \cdot 3 \cdot 3 \cdot 3 = 3^4$

The base, 3, is the common factor that is being multiplied.

The exponent, 4, tells how many times the base is used as a factor.

(continued on next page)

Lesson 4-1 • Powers and Exponents 221

Interactive Presentation

Learn, Products as Powers, Slide 1 of 3

WATCH

On Slide 1, students watch an animation that illustrates how to write an expression as a power.

DIFFERENTIATE

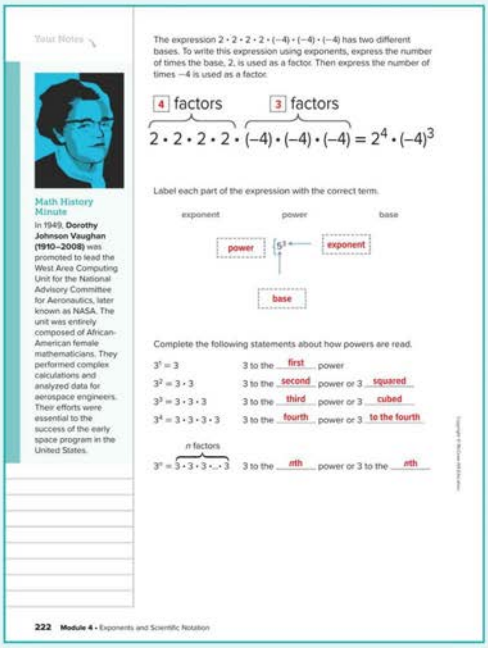
Reteaching Activity AL

If any of your students have difficulty writing expressions as powers, have them write the base and then count, using dots above each of the repeated factors, to determine the exponent. For example, with $4 \times 4 \times 4$, students would write the base as 4, and then place a dot above each of the repeated factors, and then count the dots, 3. They would then write the expression as 4^3. Have students practice with the following expressions.

$2 \cdot 2 \cdot 2 \cdot 2$ 2^4

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ 7^6

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ 3^5



Math History Minute

In 1949, **Dorothy Johnson Vaughan (1910–2008)** was promoted to lead the West Area Computing Unit for the National Advisory Committee for Aeronautics, later known as NASA. The unit was entirely composed of African-American female mathematicians. They performed complex calculations and analyzed data for aerospace engineers. Their efforts were essential to the success of the early space program in the United States.

The expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot (-4) \cdot (-4) \cdot (-4)$ has two different bases. To write this expression using exponents, express the number of times the base, 2, is used as a factor. Then express the number of times −4 is used as a factor.

$$\underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ factors}} \cdot \underbrace{(-4) \cdot (-4) \cdot (-4)}_{3 \text{ factors}} = 2^4 \cdot (-4)^3$$

Label each part of the expression with the correct term.

exponent power base

5^3: power, exponent, base

Complete the following statements about how powers are read.

$3^1 = 3$ 3 to the first power

$3^2 = 3 \cdot 3$ 3 to the second power or 3 squared

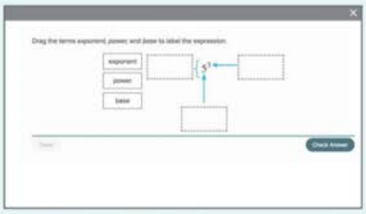
$3^3 = 3 \cdot 3 \cdot 3$ 3 to the third power or 3 cubed

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$ 3 to the fourth power or 3 to the fourth

$3^n = \underbrace{3 \cdot 3 \cdot 3 \cdot \ldots \cdot 3}_{n \text{ factors}}$ 3 to the nth power or 3 to the nth

222 Module 4 • Exponents and Scientific Notation

Interactive Presentation



Learn, Write Products as Powers, Slide 2 of 3

DRAG & DROP

On Slide 2, students practice academic vocabulary by dragging the terms to their corresponding parts on the expression.

Learn Write Products as Powers *(continued)*

Teaching Notes

SLIDE 2

Students will become more familiar with the terms *base*, *exponent*, and *power*. Ŷou may wish to have student volunteers come up to the board to drag each term to its appropriate bin. If students need more practice, have them generate their own powers and trade with a partner to have each student identify the power, base, and exponent in each expression.

SLIDE 3

Have students move through the slides to learn how to read the powers. After seeing the first example, have students make a conjecture as to how other powers might be read before moving to each slide. Point out that there are different ways to read powers of 2 and 3.

Example 1 Write Numerical Products as Powers

Objective

Students will write numerical repeated multiplication expressions as powers.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 3, encourage them to think about possible reasons behind the developed notation for exponents.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to describe any advantages using clear and precise mathematical language.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the given expression in order to determine the repeated factors and relate this to the bases and exponents in the simplified expression.

Questions for Mathematical Discourse

SLIDE 2

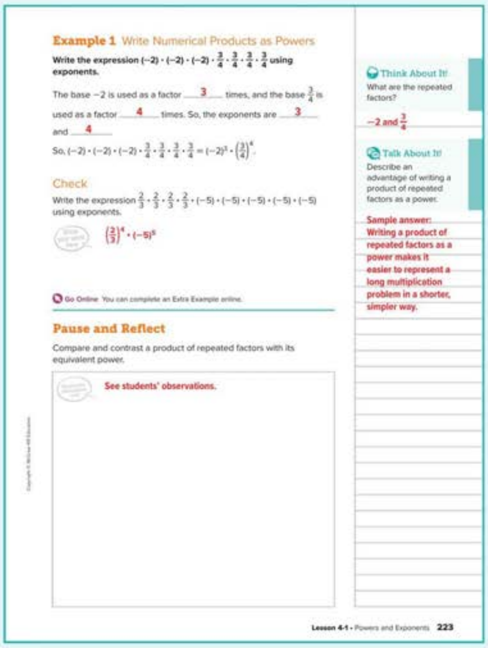
AL What are the bases that appear in the given expression? -2 and $\frac{3}{4}$

OL How many factors of each base are present? There are three factors of the base -2 and four factors of the base $\frac{3}{4}$.

BL If a power has a negative base, which exponents result in a negative value? Explain. odd exponents; Sample answer: The product of an even number of negative factors is positive, and the product of an odd number of negative factors is negative.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Write Numerical Products as Powers

Write the expression $(-2)\cdot(-2)\cdot(-2)\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$ using exponents.

The base -2 is used as a factor __3__ times, and the base $\frac{3}{4}$ is used as a factor __4__ times. So, the exponents are __3__ and __4__.

So, $(-2)\cdot(-2)\cdot(-2)\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}=(-2)^3\cdot\left(\frac{3}{4}\right)^4$.

Check

Write the expression $\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot\frac{2}{3}\cdot(-5)\cdot(-5)\cdot(-5)\cdot(-5)\cdot(-5)$ using exponents.

$\left(\frac{2}{3}\right)^4\cdot(-5)^5$

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare and contrast a product of repeated factors with its equivalent power.

See students' observations.

Think About It! What are the repeated factors?

-2 and $\frac{3}{4}$

Talk About It! Describe an advantage of writing a product of repeated factors as a power.

Sample answer: Writing a product of repeated factors as a power makes it easier to represent a long multiplication problem in a shorter, simpler way.

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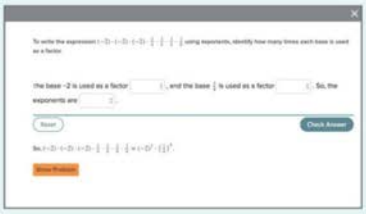
Interactive Presentation



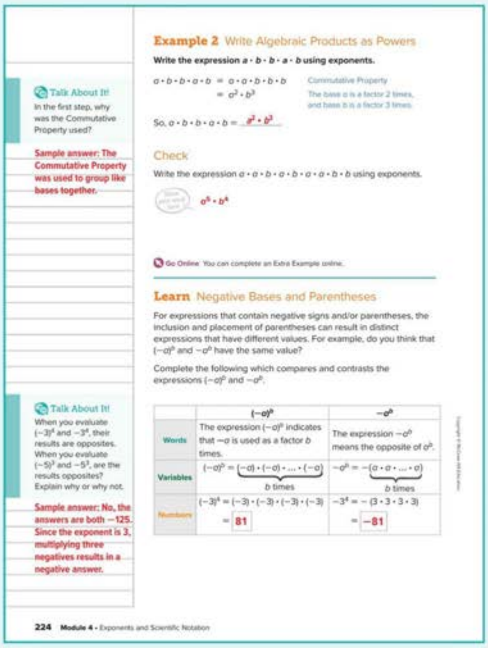
Example 1, Write Numerical Products as Powers, Slide 2 of 4

CLICK

On Slide 2, students determine the number of times each base is used as a factor.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 2 Write Algebraic Products as Powers

Write the expression $a \cdot b \cdot b \cdot a \cdot b$ using exponents.

$a \cdot b \cdot b \cdot a \cdot b = a \cdot a \cdot b \cdot b \cdot b$ Commutative Property

$= a^2 \cdot b^3$ The base a is a factor 2 times, and base b is a factor 3 times.

So, $a \cdot b \cdot b \cdot a \cdot b$ = $a^2 \cdot b^3$

Talk About It! In the first step, why was the Commutative Property used?

Sample answer: The Commutative Property was used to group like bases together.

Check

Write the expression $a \cdot a \cdot b \cdot a \cdot b \cdot a \cdot a \cdot b \cdot b$ using exponents.

$a^5 \cdot b^4$

Go Online You can complete an Extra Example online.

Learn Negative Bases and Parentheses

For expressions that contain negative signs and/or parentheses, the inclusion and placement of parentheses can result in distinct expressions that have different values. For example, do you think that $(-a)^b$ and $-a^b$ have the same value?

Complete the following which compares and contrasts the expressions $(-a)^b$ and $-a^b$.

	$(-a)^b$	$-a^b$
Words	The expression $(-a)^b$ indicates that $-a$ is used as a factor b times.	The expression $-a^b$ means the opposite of a^b.
Variables	$(-a)^b = \underbrace{(-a) \cdot (-a) \cdot \ldots \cdot (-a)}_{b \text{ times}}$	$-a^b = -\underbrace{(a \cdot a \cdot \ldots \cdot a)}_{b \text{ times}}$
Numbers	$(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$	$-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

Talk About It! When you evaluate $(-3)^4$ and -3^4, their results are opposites. When you evaluate $(-5)^3$ and -5^3, are the results opposites? Explain why or why not.

Sample answer: No, the answers are both −125. Since the exponent is 3, multiplying three negatives results in a negative answer.

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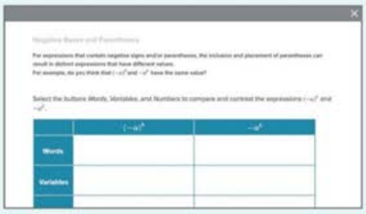
Interactive Presentation



Learn, Negative Bases and Parentheses, Slide 1 of 2

CLICK

On Slide 1 of the Learn, students compare and contrast two different expressions.

TYPE

On Slide 2 of Example 2, students enter the missing value to write the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Write Algebraic Products as Powers

Objective

Students will write algebraic repeated multiplication expressions as powers.

Questions for Mathematical Discourse

SLIDE 2

AL What does the Commutative Property state? Sample answer: For multiplication, the Commutative Property states that you can multiply two numbers in any order.

OL How does the exponent of each power relate to the factors of that power in the product? Sample answer: The exponent of each power is equal to the number of factors of that power in the product.

BL How would you write the expression $a \cdot c \cdot b \cdot b \cdot c \cdot a \cdot b \cdot a \cdot a$ using exponents? $a^4 \cdot b^3 \cdot c^2$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Negative Bases and Parentheses

Objective

Students will learn what the inclusion and placement of parentheses around a negative base indicates about the value of the power.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to compare and contrast the structure of each expression and use their understanding of the order of operations to explain the results.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

When you evaluate $(-3)^4$ and -3^4, their results are opposites. When you evaluate $(-5)^3$ and -5^3, are the results opposites? Explain why or why not. Sample answer: No, the answers are both –125. Since the exponent is 3, multiplying three negatives results in a negative answer.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Evaluate Powers

Objective

Students will learn how to evaluate an expression that contains a power.

Go Online to find additional teaching notes.

Example 3 Evaluate Numerical Expressions

Objective

Students will evaluate numerical expressions that contain powers.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to carefully analyze the structure of the expression in order to determine how to evaluate it. Students should see that the expression consists of two powers, each of which can be written as repeated multiplication. As students discuss the *Talk About It!* question on Slide 3, encourage them to analyze the structure of each expression, paying careful attention to the placement and inclusion of the parentheses, in order to determine whether or not they are equivalent.

Questions for Mathematical Discourse

SLIDE 2

AL What are the bases and exponents in the expression? The base −2 has the exponent 3, and the base 3.5 has the exponent 2.

OL How can the expression be rewritten using repeated multiplication? (−2)(−2)(−2) + (3.5)(3.5)

BL Is the equation $a^p + b^p = (a + b)^p$ true or false? Explain your reasoning. false; Sample answer: Use a counterexample, such as if $a = 1$, $b = 2$, and $p = 2$. The value of $a^p + b^p$ is $1^2 + 2^2 = 1 \cdot 1 + 2 \cdot 2 = 5$. The value of $(a + b)^p$ is $(1 + 2)^2 = 3^2 = 9$. The values are not equal, so the equation is false.

Go Online

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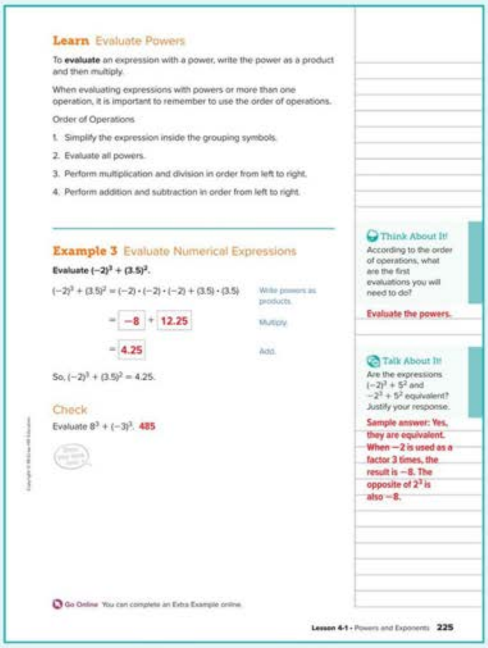
- View performance reports of the Checks.
- Assign or present an Extra Example.


Learn Evaluate Powers

To **evaluate** an expression with a power, write the power as a product and then multiply.

When evaluating expressions with powers or more than one operation, it is important to remember to use the order of operations.

Order of Operations

1. Simplify the expression inside the grouping symbols.
2. Evaluate all powers.
3. Perform multiplication and division in order from left to right.
4. Perform addition and subtraction in order from left to right.

Example 3 Evaluate Numerical Expressions

Evaluate $(-2)^3 + (3.5)^2$.

$(-2)^3 + (3.5)^2 = (-2) \cdot (-2) \cdot (-2) + (3.5) \cdot (3.5)$ Write powers as products.

$= -8 + 12.25$ Multiply.

$= 4.25$ Add.

So, $(-2)^3 + (3.5)^2 = 4.25$.

Check

Evaluate $8^3 + (-3)^3$. 485

Think About It! According to the order of operations, what are the first evaluations you will need to do? Evaluate the powers.

Talk About It! Are the expressions $(-2)^3 + 5^2$ and $-2^3 + 5^2$ equivalent? Justify your response. Sample answer: Yes, they are equivalent. When −2 is used as a factor 3 times, the result is −8. The opposite of 2^3 is also −8.

Go Online You can complete an Extra Example online.

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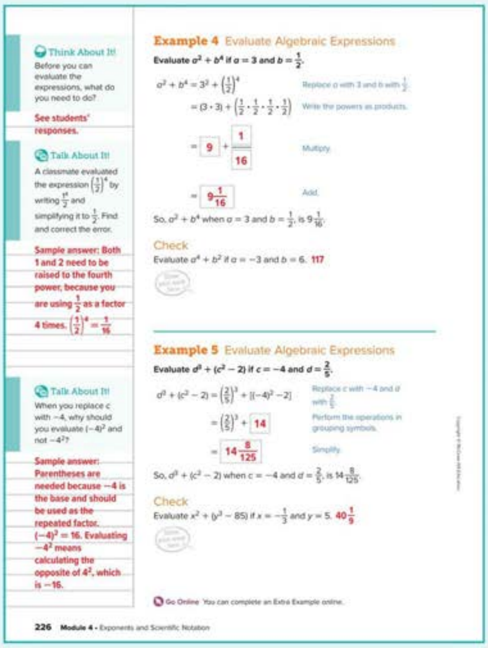
Interactive Presentation

Example 3, Evaluate Numerical Expressions, Slide 2 of 4

TYPE On Slide 2 of Example 3, students enter the missing value.

CLICK On Slide 2 of Example 3, students move through the steps to evaluate the numerical expression.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It! Before you can evaluate the expressions, what do you need to do?

See students' responses.

Talk About It! A classmate evaluated the expression $\left(\frac{1}{2}\right)^4$ by writing $\frac{1^4}{2}$ and simplifying it to $\frac{1}{2}$. Find and correct the error.

Sample answer: Both 1 and 2 need to be raised to the fourth power, because you are using $\frac{1}{2}$ as a factor 4 times. $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

Example 4 Evaluate Algebraic Expressions

Evaluate $a^2 + b^4$ if $a = 3$ and $b = \frac{1}{2}$.

$a^2 + b^4 = 3^2 + \left(\frac{1}{2}\right)^4$ Replace a with 3 and b with $\frac{1}{2}$.

$= (3 \cdot 3) + \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}\right)$ Write the powers as products.

$= 9 + \frac{1}{16}$ Multiply.

$= 9\frac{1}{16}$ Add.

So, $a^2 + b^4$ when $a = 3$ and $b = \frac{1}{2}$, is $9\frac{1}{16}$.

Check

Evaluate $a^4 + b^2$ if $a = -3$ and $b = 6$. 117

Example 5 Evaluate Algebraic Expressions

Evaluate $d^3 + (c^2 - 2)$ if $c = -4$ and $d = \frac{2}{5}$.

$d^3 + (c^2 - 2) = \left(\frac{2}{5}\right)^3 + [(-4)^2 - 2]$ Replace c with -4 and d with $\frac{2}{5}$.

$= \left(\frac{2}{5}\right)^3 + 14$ Perform the operations in grouping symbols.

$= 14\frac{8}{125}$ Simplify.

So, $d^3 + (c^2 - 2)$ when $c = -4$ and $d = \frac{2}{5}$, is $14\frac{8}{125}$.

Check

Evaluate $x^2 + (y^3 - 85)$ if $x = -\frac{1}{3}$ and $y = 5$. $40\frac{1}{9}$

Talk About It! When you replace c with -4, why should you evaluate $(-4)^2$ and not -4^2?

Sample answer: Parentheses are needed because -4 is the base and should be used as the repeated factor. $(-4)^2 = 16$. Evaluating -4^2 means calculating the opposite of 4^2, which is -16.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 5, Evaluate Algebraic Expressions, Slide 2 of 4

CLICK On Slide 2 of Example 4, student move through the steps to evaluate the expression.

CLICK On Slide 2 of Example 5, students move through the steps to evaluate the expression.

CHECK Students complete the Check exercises online to determine if they are ready to move on.

Example 4 Evaluate Algebraic Expressions

Objective

Students will evaluate algebraic expressions that contain powers.

Questions for Mathematical Discourse

SLIDE 2

AL How can the expression be rewritten using the values of *a* and *b*?

$3^2 + \left(\frac{1}{2}\right)^4$

OL How can the expression be rewritten using the values of *a* and *b* and repeated multiplication? $3 \cdot 3 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

BL How can you show that a fraction raised to a power is equal to the numerator raised to the power, divided by the denominator raised to that power: $\left(\frac{x}{y}\right)^p = \frac{x^p}{y^p}$? Sample answer: Use the fraction $\frac{2}{3}$ as an example, and simplify.

$$\left(\frac{2}{3}\right)^5 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{2^5}{3^5}$$

Example 5 Evaluate Algebraic Expressions

Objective

Students will evaluate algebraic expressions that contain powers.

Questions for Mathematical Discourse

SLIDE 2

AL How can the expression be rewritten using the values of *c* and *d*?

$\left(\frac{2}{5}\right)^3 + [(-4)^2 - 2)]$

OL How can the expression be rewritten using the values of *c* and *d* without exponents? $\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} + [(-4)(-4) - 2]$

BL A classmate stated that the value of $c - 2$ should be found before applying the exponent 2 because the parentheses represent a grouping, which should be evaluated before exponents based on the order of operations. Explain why your classmate is incorrect. Sample answer: The expression within the parentheses should be evaluated first, but that expression itself must be evaluated using the order of operations. This means that the value of *c* must be squared before subtracting 2.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | **3 APPLICATION**

Apply Mammals

Objective

Students will come up with their own strategy to solve an application problem involving the average weights of two mammals.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

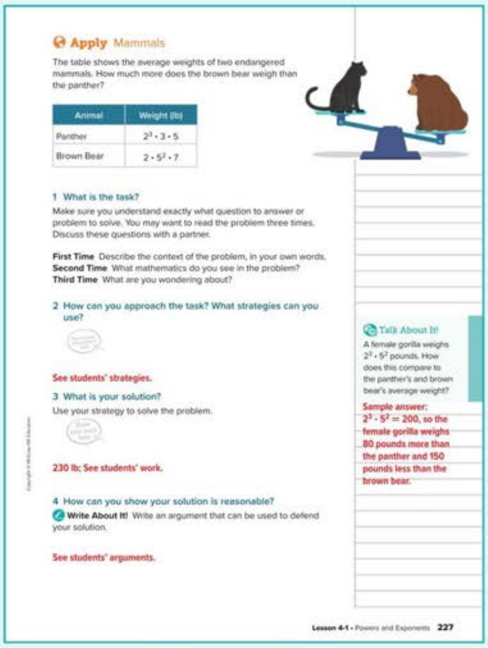
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does *how much more* mean?
- How are the average weights represented in the table?
- How can you use the order of operations to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Mammals

The table shows the average weights of two endangered mammals. How much more does the brown bear weigh than the panther?

Animal	Weight (lb)
Panther	$2^3 \cdot 3 \cdot 5$
Brown Bear	$2 \cdot 5^2 \cdot 7$

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

230 lb; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
A female gorilla weighs $2^3 \cdot 5^2$ pounds. How does this compare to the panther's and brown bear's average weight?

Sample answer: $2^3 \cdot 5^2 = 200$, so the female gorilla weighs 80 pounds more than the panther and 150 pounds less than the brown bear.

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Interactive Presentation

Apply, Mammals

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Interstate 90 stretches almost $2^4 \cdot 7^2 \cdot 4$ miles across the United States. Interstate 70 stretches almost $2^3 \cdot 5^2 \cdot 11$ miles across the United States. How much longer is Interstate 90 than Interstate 70?

936 miles

Go Online You can complete an Extra Example online.

Pause and Reflect

How well do you understand the concepts from today's lesson? What questions do you still have? How can you get those questions answered?

See students' observations.

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Interactive Presentation

Exit Ticket

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | **3 APPLICATION**

Exit Ticket

Refer to the Exit Ticket slide. A specific computer model is made to have 4 gigabytes of RAM. Find the number of bytes of RAM the computer has. Write a mathematical argument that can be used to defend your solution. 2^{32} bytes; Sample answer: 4 can be written as 2^2, so the total number of bytes is $2^2 \times 2^{30}$. This expression represents the product of 32 factors of 2.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11 odd, 12–15
- ALEKS® Exponents and Order of Operations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 10, 13, 14
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Exponents and Order of Operations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS® Exponents and Scientific Notation

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

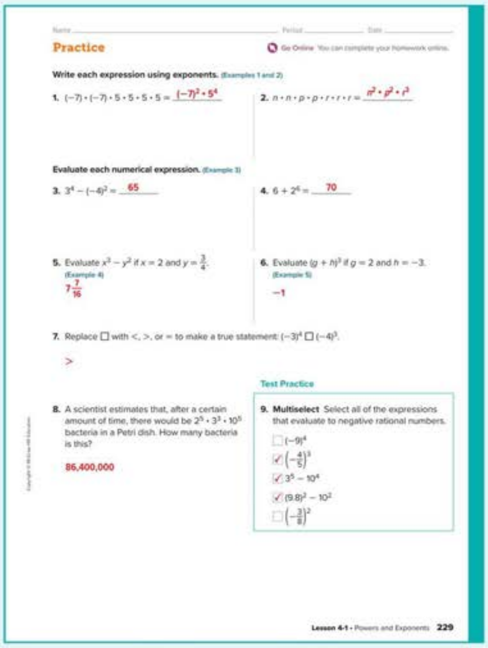
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	write numerical and algebraic multiplication expressions as powers	1, 2
1	evaluate numerical expressions that contain powers	3, 4
2	evaluate algebraic expressions that contain powers	5, 6
2	extend concepts learned in class to apply them in new contexts	7–9
3	solve application problems involving powers and exponents	10, 11
3	higher-order and critical thinking skills	12–15

Common Misconception

Some students may incorrectly evaluate numerical expressions that involve exponents. Remind students that the base is the common factor that is being multiplied and the exponent tells how many times the base is used as a factor. In Exercise 3, 3^4 and $(-4)^2$ must be simplified before they are subtracted. Remind students that to simplify 3^4, they must use 3 as a factor 4 times. To simplify $(-4)^2$, they must use −4 as a factor twice.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Write each expression using exponents. (Examples 1 and 2)

1. $(-7)\cdot(-7)\cdot 5\cdot 5\cdot 5\cdot 5 =$ $(-7)^2\cdot 5^4$

2. $n\cdot n\cdot p\cdot p\cdot r\cdot r\cdot r =$ $n^2\cdot p^2\cdot r^3$

Evaluate each numerical expression. (Example 3)

3. $3^4-(-4)^2 =$ 65

4. $6+2^6 =$ 70

5. Evaluate x^3-y^2 if $x=2$ and $y=\frac{3}{4}$. (Example 4)
$7\frac{7}{16}$

6. Evaluate $(g+h)^3$ if $g=2$ and $h=-3$. (Example 5)
−1

7. Replace □ with <, >, or = to make a true statement: $(-3)^4$ □ $(-4)^3$.
>

Test Practice

8. A scientist estimates that, after a certain amount of time, there would be $2^5\cdot 3^3\cdot 10^5$ bacteria in a Petri dish. How many bacteria is this?
86,400,000

9. **Multiselect** Select all of the expressions that evaluate to negative rational numbers.
- [ ] $(-9)^4$
- [x] $\left(-\frac{4}{5}\right)^3$
- [x] 3^5-10^4
- [x] $(9.8)^2-10^2$
- [ ] $\left(-\frac{3}{8}\right)^2$

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Apply *indicates multi-step problem

*10. The table shows the approximate number of species of each type of tree. How many more species of palm tree are there than maple tree?

2,872 species

Tree Type	Number of Species
Palm	$2^3 \cdot 3 \cdot 5^3$
Maple	2^7

*11. The table shows the approximate number of lakes in two different states. How many more lakes does Florida have than Minnesota?

8,200 lakes

State	Number of Lakes
Florida	$3 \cdot 10^4$
Minnesota	$2^3 \cdot 5^2 \cdot 109$

Higher-Order Thinking Problems

12. **Identify Structure** Without evaluating, explain why $(-8.4)^5$ is less than 2^2.

Sample answer: Even though the exponent is greater in $(-8.4)^5$, the base is multiplied five times. Multiplying a negative number five times results in a negative product. Any negative number is less than 2^2 or 4.

13. **Find the Error** A student evaluated the expression $(x^2 - y)^3$ if $x = -4$ and $y = 7$. Find his mistake and correct it.

$$\begin{aligned}(x^2 - y)^3 &= (-4^2 - 7)^3\\ &= (-16 - 7)^3\\ &= (-23)^3\\ &= -12{,}167\end{aligned}$$

Sample answer: He substituted −4 for *x*, but should have taken −4 to the second power, not just 4. Placing parentheses around −4 would have helped him to take the correct value to the second power. The correct value of the expression is 729.

14. Determine if the statement is *true* or *false*. Justify your response.

The expressions $(-x)^y$ and $-x^y$ have the same value.

false; Sample answer: The base, $-x$, is used as a factor *y* times. The expression $-x^y$, means the opposite of x^y.

15. A student finds that $\left(\frac{1}{4}\right)^2 = \left(\frac{1}{2}\right)^4$ and concludes, then, that $\left(\frac{1}{3}\right)^2 = \left(\frac{1}{2}\right)^3$. Is this reasoning correct? Explain.

It is incorrect. Because $4^2 = 2^4$, the first set of expressions are equal. This is not the case for $\left(\frac{1}{3}\right)^2$ and $\left(\frac{1}{2}\right)^3$ because $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ and $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Ex ercise 13, students will find the mistake in the problem and correct it. Encourage students to determine the error by analyzing the worked-out solution and explain how they could fix it.

7 Look for and Make Use of Structure In Exercise 12, students will analyze the given power and explain how they can use their knowledge of positive and negative bases to determine whether the power will be less than or greater than the other given power.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 10–11 After completing the application problems, have students write their own real-world problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Be sure everyone understands.
Use with Exercises 14–15 Have students work in groups of 3–4 to solve the problem in Exercise 14. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 15.

Lesson 4-2

Multiply and Divide Monomials

8.EE.A.1

LESSON GOAL

Students will use Laws of Exponents to multiply and divide monomials.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Product of Powers

Learn: Monomials
Learn: Product of Powers
Example 1: Multiply Numerical Powers
Example 2: Multiply Algebraic Powers
Example 3: Multiply Monomials

Explore: Quotient of Powers

Learn: Quotient of Powers
Example 4: Divide Algebraic Powers
Example 5: Divide Powers
Example 6: Divide Numerical Powers
Example 7: Divide Monomials
Apply: Computer Science

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 24 of the *Language Development Handbook* to help your students build mathematical language related to multiplying and dividing monomials.

ELL You can use the tips and suggestions on page T24 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.A** by writing and evaluating expressions involving powers and exponents.
Standards for Mathematical Content: **8.E E.A.1**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students wrote and evaluated expressions involving powers and exponents.
Foundational for 8.EE.A.1

Now
Students use the Laws of Exponents to simplify expressions involving products and quotients of monomials.
8.EE.A.1

Next
Students will use the Laws of Exponents to find powers of monomials.
8.EE.A.1

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

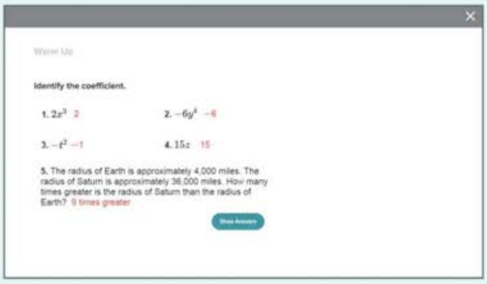
Conceptual Bridge In this lesson, students develop *understanding* of the Product of Powers and Quotient of Powers properties. They come to understand that when simplifying a product of powers with the same base, they can add the exponents, and when simplifying a quotient of powers with the same base they subtract the exponents. They build *fluency* with using these properties by simplifying numeric and algebraic expressions.

Mathematical Background

The Laws of Exponents include:

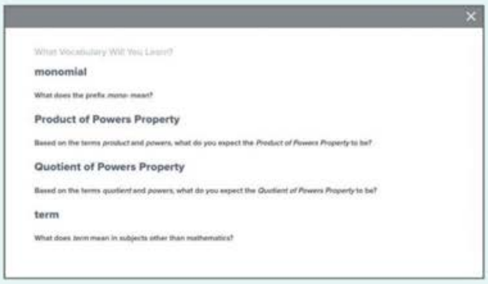
- *Product of Powers*: To multiply powers with the same base, add their exponents.
- *Quotient of Powers*: To divide powers with the same base, subtract their exponents.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding coefficients (Exercises 1–4)
- understanding integer operations (Exercise 5)

Answers

1. 2

2. −6

3. −1

4. 15

5. 9 times greater

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the speeds and distances of comets.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does the prefix *mono-* mean? Sample answer: one
- Based on the terms *product* and *powers*, what do you e xpect the *Product of Powers Property* to be? Sample answer: A rule for finding the product when two powers are multiplied.
- Based on the terms *quotient* and *powers,* what do you e xpect the *Quotient of Powers Property* to be? Sample answer: A rule for finding the quotient when one power is divided by another power.
- What does *term* mean in subjects other than mathematics ? Sample answer: a description for a word, a time period, or a condition for an agreement.

Explore Products of Powers

Objective

Students will use Web Sketchpad to explore how to simplify a product of powers with like bases.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with powers and products of powers and will be able to see visual representations of the values of all of them based on the value of the base. Throughout this activity, students will compare values and make and test their conjectures about the product of powers.

Inquiry Question

How can you simplify a product of powers with like bases?
Sample answer: A product of powers with like bases can be simplified by adding the exponents.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

What do you notice about the expressions of bars with the same length?
Sample answer: The exponents in each bar of the same length have the same sum.

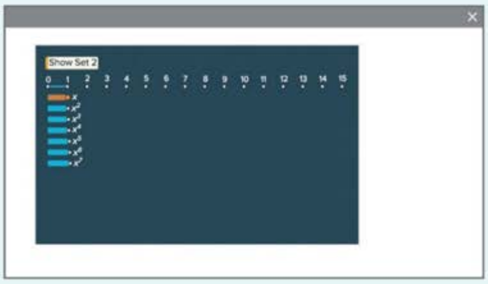
Do the three new bars match any existing bars? If so, which one(s)?
Yes. The three new bars match the existing bar labeled x^6.

What do you notice about the exponents in the three new bars, compared to the exponent(s) in the existing bar(s) of the same length?
Sample answer: The sum of exponents in each of the three new bars is equivalent to the exponent in the existing bar of the same length.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 3 of 6

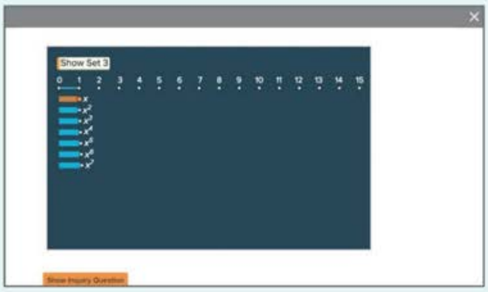
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationship between the lengths of the bars that represent each expression, and their corresponding exponents.

TYPE

On Slide 4, students make a conjecture about how they could simplify a product of powers with like bases.

Interactive Presentation



Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and can view a sample answer.

Explore Products of Powers *(continued)*

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Enc ourage students to examine how the lengths of the bars representing the expressions compare to the structure of the expressions, paying particular attention to the exponents.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Does your conjecture hold true with Set 3? Explain. Sample answer: Yes. The bars in Set 3 are the same length as the x^7 bar. The exponents in these expressions all have a sum of 7.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Monomials

Objective

Students will understand what a monomial is and how to identify one.

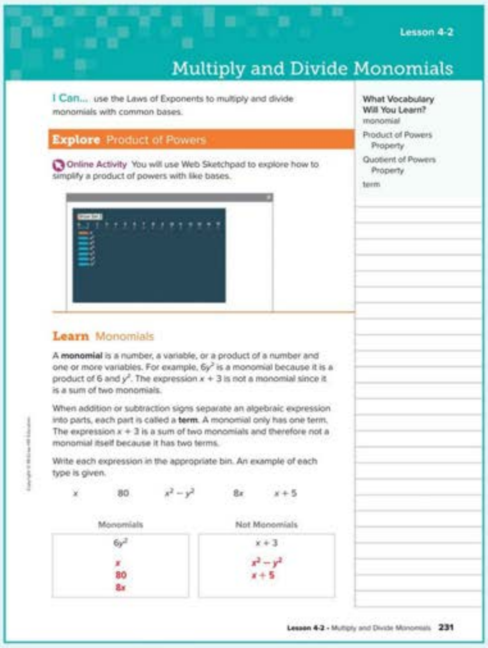
Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to use the definition of *monomial* in sorting each expression into the appropriate category.

Teaching Notes

SLIDE 1

Students will learn the definitions for the terms *monomial* and *term*. You may wish to have student volunteers come up to the board to drag each expression to its appropriate bin. Ask students why each expression is, or is not, a monomial.


Lesson 4-2

Multiply and Divide Monomials

I Can... use the Laws of Exponents to multiply and divide monomials with common bases.

What Vocabulary Will You Learn?
monomial
Product of Powers Property
Quotient of Powers Property
term

Explore Product of Powers

Online Activity You will use Web Sketchpad to explore how to simplify a product of powers with like bases.

Learn Monomials

A **monomial** is a number, a variable, or a product of a number and one or more variables. For example, $6y^2$ is a monomial because it is a product of 6 and y^2. The expression $x + 3$ is not a monomial since it is a sum of two monomials.

When addition or subtraction signs separate an algebraic expression into parts, each part is called a **term**. A monomial only has one term. The expression $x + 3$ is a sum of two monomials and therefore not a monomial itself because it has two terms.

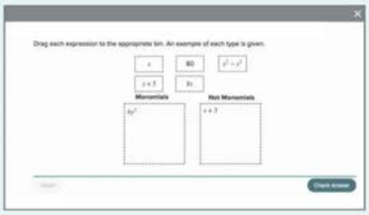
Write each expression in the appropriate bin. An example of each type is given.

x 80 $x^2 - y^2$ $8x$ $x + 5$

Monomials	Not Monomials
$6y^2$	$x + 3$
x	$x^2 - y^2$
80	$x + 5$
$8x$	

Lesson 4-2 • Multiply and Divide Monomials 231

Interactive Presentation



Learn, Monomials, Slide 1 of 1

DRAG & DROP

On Slide 1, students drag to sort expressions as to whether or not they are monomials.

DIFFERENTIATE

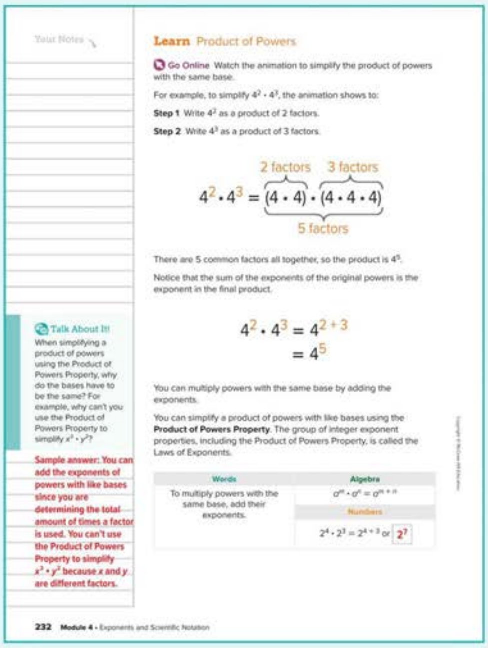
Language Development Activity ELL

If any of your students have difficulty in determining whether or not an expression in a monomial, have them create a flow chart for assistance. A sample flow chart could include the questions such as *does the expression include a plus or minus sign, is the expression a number, a variable, or a product of a number and one or more variables,* etc. Have them use their flow charts to determine if the following expressions are monomials. Ask them to clearly state why or why not, using the word *term* in their responses.

$y + 6$ no y yes $8y$ yes $x + 34$ no $7y^3$ yes

8.EE.A.1

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION


Your Notes

Learn Product of Powers

Go Online Watch the animation to simplify the product of powers with the same base.

For example, to simplify $4^2 \cdot 4^3$, the animation shows to:

Step 1 Write 4^2 as a product of 2 factors.

Step 2 Write 4^3 as a product of 3 factors.

$$4^2 \cdot 4^3 = \underbrace{\overbrace{(4 \cdot 4)}^{\text{2 factors}} \cdot \overbrace{(4 \cdot 4 \cdot 4)}^{\text{3 factors}}}_{\text{5 factors}}$$

There are 5 common factors all together, so the product is 4^5.

Notice that the sum of the exponents of the original powers is the exponent in the final product.

$$4^2 \cdot 4^3 = 4^{2+3} = 4^5$$

You can multiply powers with the same base by adding the exponents.

You can simplify a product of powers with like bases using the **Product of Powers Property**. The group of integer exponent properties, including the Product of Powers Property, is called the Laws of Exponents.

Words	Algebra
To multiply powers with the same base, add their exponents.	$a^m \cdot a^n = a^{m+n}$
	Numbers
	$2^4 \cdot 2^3 = 2^{4+3}$ or 2^7

Talk About It! When simplifying a product of powers using the Product of Powers Property, why do the bases have to be the same? For example, why can't you use the Product of Powers Property to simplify $x^3 \cdot y^2$?

Sample answer: You can add the exponents of powers with like bases since you are determining the total amount of times a factor is used. You can't use the Product of Powers Property to simplify $x^3 \cdot y^2$ because x and y are different factors.

232 Module 4 • Exponents and Scientific Notation

Interactive Presentation

Learn, Product of Powers, Slide 2 of 3

WATCH

On Slide 1, students watch an animation that shows what happens when two powers with the same base are multiplied.

FLASHCARDS

On Slide 2, students use Flashcards to view multiple representations of the Product of Powers Property.

Learn Product of Powers

Objective

Students will understand how the Product of Powers Property can be applied to simplify a product of powers with the same base.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* questions on Slide 3, encourage them to use reasoning to make sense of how powers and exponents represent repeated multiplication of the same base, which is why the bases must be the same in order to apply the Product of Powers Property.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to simplify a product of powers.

Talk About It!

SLIDE 3

Mathematical Discourse

When simplifying a product of powers using the Product of Powers Property, why do the bases have to be the same? For example, why can't you use the Product of Powers Property to simplify $x^{3\,2}$ y ?

Sample answer: You can add the exponents when simplifying a product of powers with like bases since you are determining the total amount of times a factor is used. If the factors are different, you cannot add their exponents. You can't use the Product of Powers Property to simplify $x^{3\,2}$ y because x and y are different factors.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding in determining whether or not they can apply the Product of Powers Property to simplify a product of powers, use the following activity to reinforce the concept. Ask students to determine if the following products can be simplified using the Product of Powers Property, reminding them that the property can only be applied if the powers have like bases. Have students support their answer with a logical explanation. See students' explanations.

$2^{3\,2}$ 3 no $\quad$ $4^{4\,3}$4 yes $\quad$ $7^{1\,7}$1 no $\quad$ $3^{2\,4}$3 yes

Example 1 Multiply Numerical Powers

Objective

Students will use the Product of Powers Property to multiply numerical powers.

Questions for Mathematical Discourse

SLIDE 2

AL What does the exponent indicate about a power? Sample answer: the number of times the base appears as a factor

OL Why can we write 5 as 5^1? Sample answer: Any number to the power of 1 is the original number.

OL How can you determine the total number of factors of the base 5? Sample answer: The exponent 4 indicates that 5^4 is equal to 4 factors of 5. With the additional factor of 5 in the expression, there are 5 factors of 5 in $5^4 \cdot 5$.

BL How could you simplify $5^4 \cdot 25$? Sample answer: 25 is equal to 5^2, so $5^4 \cdot 25 = 5^4 \cdot 5^2 = 5^{4+2} = 5^6$.

Example 2 Multiply Algebraic Powers

Objective

Students will use the Product of Powers Property to multiply algebraic powers.

Questions for Mathematical Discourse

SLIDE 2

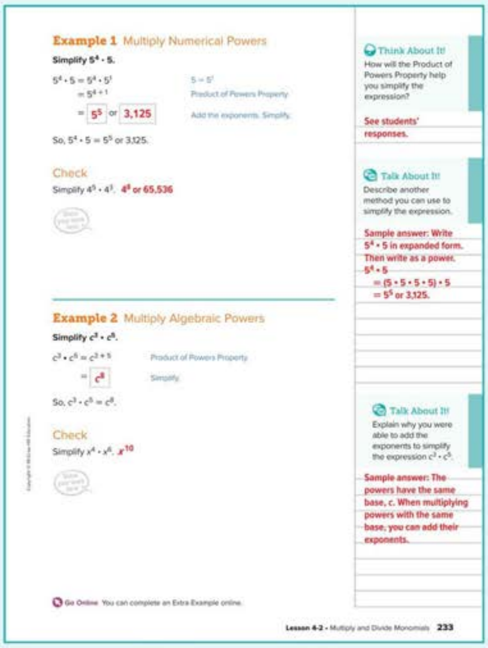
AL How many repeated factors of the base c are there in the expression altogether? 8

OL Explain why the Product of Powers Property can be used with variable bases, just as it can be used with numerical bases. Sample answer: Powers with variable bases can be written as repeated multiplication, just as with numerical bases. The exponent on the product will be the sum of the exponents of the factors, as long as the bases are the same.

BL What is another product of powers that is equivalent to $c^3 \cdot c^5$? Sample answer: $c^2 \cdot c^6$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Multiply Numerical Powers

Simplify $5^4 \cdot 5$.

$5^4 \cdot 5 = 5^4 \cdot 5^1$ — $5 = 5^1$

$= 5^{4+1}$ — Product of Powers Property

$= 5^5$ or 3,125 — Add the exponents. Simplify.

So, $5^4 \cdot 5 = 5^5$ or 3,125.

Check

Simplify $4^5 \cdot 4^3$. 4^8 or 65,536

Think About It! How will the Product of Powers Property help you simplify the expression? See students' responses.

Talk About It! Describe another method you can use to simplify the expression. Sample answer: Write $5^4 \cdot 5$ in expanded form. Then write as a power. $5^4 \cdot 5 = (5 \cdot 5 \cdot 5 \cdot 5) \cdot 5 = 5^5$ or 3,125.

Example 2 Multiply Algebraic Powers

Simplify $c^3 \cdot c^5$.

$c^3 \cdot c^5 = c^{3+5}$ — Product of Powers Property

$= c^8$ — Simplify.

So, $c^3 \cdot c^5 = c^8$.

Check

Simplify $x^4 \cdot x^6$. x^{10}

Talk About It! Explain why you were able to add the exponents to simplify the expression $c^3 \cdot c^5$. Sample answer: The powers have the same base, c. When multiplying powers with the same base, you can add their exponents.

Go Online You can complete an Extra Example online.

Lesson 4-2 • Multiply and Divide Monomials 233

Interactive Presentation

Example 1, Multiply Numerical Powers, Slide 2 of 4

TYPE

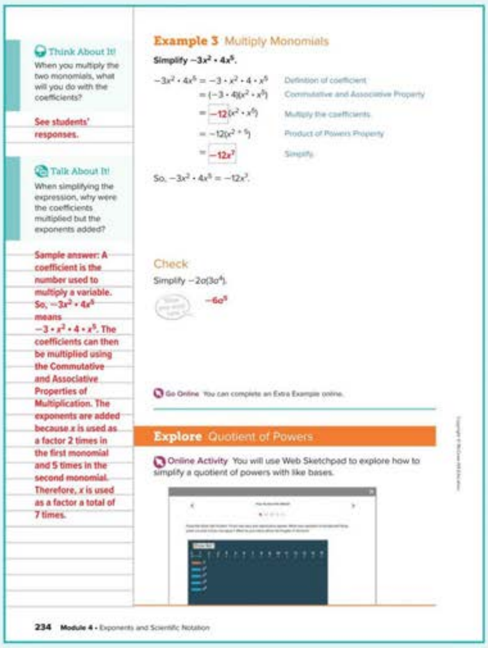
On Slide 2 of Example 1 and Slide 2 of Example 2, students enter the simplified expression.

CLICK

On Slide 2 of Example 1 and Slide 2 of Example 2, students move through the steps to simplify the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.



Think About It!
When you multiply the two monomials, what will you do with the coefficients?

See students' responses.

Talk About It!
When simplifying the expression, why were the coefficients multiplied but the exponents added?

Sample answer: A coefficient is the number used to multiply a variable. So, $-3x^2 \cdot 4x^5$ means $-3 \cdot x^2 \cdot 4 \cdot x^5$. The coefficients can then be multiplied using the Commutative and Associative Properties of Multiplication. The exponents are added because x is used as a factor 2 times in the first monomial and 5 times in the second monomial. Therefore, x is used as a factor a total of 7 times.

Example 3 Multiply Monomials

Simplify $-3x^2 \cdot 4x^5$.

$-3x^2 \cdot 4x^5$	$= -3 \cdot x^2 \cdot 4 \cdot x^5$	Definition of coefficient
	$= (-3 \cdot 4)(x^2 \cdot x^5)$	Commutative and Associative Property
	$= -12(x^2 \cdot x^5)$	Multiply the coefficients.
	$= -12(x^{2+5})$	Product of Powers Property
	$= -12x^7$	Simplify.

So, $-3x^2 \cdot 4x^5 = -12x^7$.

Check

Simplify $-2a(3a^4)$.

$-6a^5$

Go Online You can complete an Extra Example online.

Explore Quotient of Powers

Online Activity You will use Web Sketchpad to explore how to simplify a quotient of powers with like bases.

234 Module 4 • Exponents and Scientific Notation

Interactive Presentation

Example 3, Multiply Monomials, Slide 2 of 4

TYPE

On Slide 2, students enter the simplified expression.

CLICK

On Slide 2, students move through the steps to simplify the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Multiply Monomials

Objective

Students will use the Product of Powers Property to multiply monomials.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the coefficients given in the expression and be able to use reasoning to understand that the coefficients are multiplied, yet the exponents are added. As students discuss the *Talk About It!* question on Slide 3, encourage them to understand what coefficients and powers each indicate in a multiplication expression in order to be able to explain why they are treated differently when simplifying the expression.

6 Attend to Precision Encourage students to use academic vocabulary, such as the Product of Powers Property, to explain how to simplify the expression.

Questions for Mathematical Discourse

SLIDE 2

AL What are the coefficients in the expression? The coefficients are -3 and 4.

OL How can you find the coefficient of the simplified expression? Multiply the coefficients of the two monomials.

OL Why do you multiply the coefficients, but add the exponents on the bases? Sample answer: This is a multiplication expression, so coefficients are multiplied. When multiplying powers, the exponents are added according to the Product of Powers Property.

BL What would change if the original expression was $-3x^2 \cdot 4y^5$? Explain. Sample answer: The coefficients would still be multiplied, but the powers could not be combined since the bases are not the same. The simplified expression would be $-12x^2y^5$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Quotient of Powers

Objective

Students will use Web Sketchpad to explore how to simplify a quotient of powers with like bases.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with powers and quotients of powers and will be able to see visual representations of the values of all of them based on the value of the base. Throughout this activity, students will compare values and make and test their conjectures about the quotient of powers.

Inquiry Question

How can you simplify a quotient of powers with like bases?
Sample answer: Simplify a quotient of powers with like bases by subtracting the exponents.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

What do you notice about the expressions of bars with the same length?
Sample answer: The exponents in each bar of the same length have the same difference.

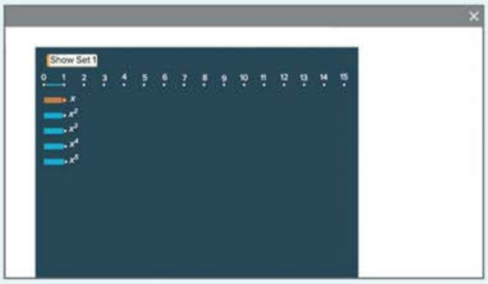
Do the three new bars match any existing bars? If so, which one(s)?
Yes. The three new bars match the x^2 bar.

What do you notice about the exponents in the three new bars, compared to the exponent(s) in the existing bar(s) of the same length?
Sample answer: The difference between the exponents in each of the three new bars is equivalent to the exponent in the existing bar of the same length.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationship between the lengths of the bars that represent each expression, and their corresponding exponents.

Interactive Presentation

Explore, Slide 5 of 6

WEB SKETCHPAD

On Slide 5 of the Explore, students use Web Sketchpad to test their conjecture.

TYPE

On Slide 6, students respond to the Inquiry Question and can view a sample answer.

Explore Quotient of Powers *(continued)*

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to examine how the lengths of the bars representing the expressions compare to the structure of the expressions, paying particular attention to the exponents.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Does your conjecture hold true with Set 3? Explain. Sample answer: Yes. The bars in Set 3 are the same length as the x^3 bar. The exponents in these expressions all have a difference of 3.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Quotient of Powers

Objective

Students will understand how the Quotient of Powers Property can be applied to simplify the quotient of powers with the same base.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the quantities given in the division expression and to be able to reason why the Quotient of Powers Property cannot be applied when the bases are not the same.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to simplify a product of powers.

Talk About It!

SLIDE 3

Mathematical Discourse

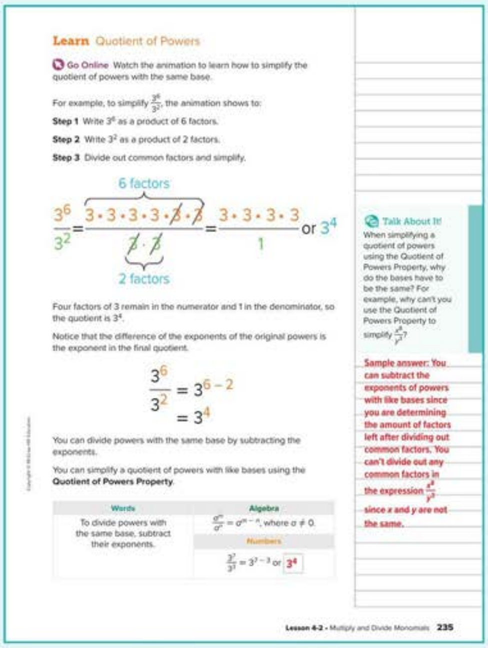
When simplifying a quotient of powers using the Quotient of Powers Property, why do the bases have to be the same? For example, why can't you use the Quotient of Powers Property to simplify $\frac{x^8}{y^3}$? Sample answer: You can subtract the exponents when simplifying a quotient of powers with like bases since you are determining the number of factors left after dividing out common factors. If the factors are different, you cannot subtract their exponents. You can't divide out any common factors in the expression $\frac{x^8}{y^3}$ since x and y are not the same.

DIFFERENTIATE

Reteaching Activity AL

If any of your students have difficulty in determining whether or not they can apply the Quotient of Powers Property to simplify a quotient of powers, use the following activity to reinforce the concept. Ask students to determine if the following quotients can be simplified using the Quotient of Powers Property, reminding them that the property can only be applied if the powers have like bases. Have students support their answer with a logical explanation.
See students' explanations.

$\frac{2^3}{3^2}$ no　$\frac{4^4}{4^3}$ yes　$\frac{7^1}{1^7}$ no　$\frac{3^8}{3^4}$ yes



Learn Quotient of Powers

Go Online Watch the animation to learn how to simplify the quotient of powers with the same base.

For example, to simplify $\frac{3^6}{3^2}$, the animation shows to:

Step 1 Write 3^6 as a product of 6 factors.

Step 2 Write 3^2 as a product of 2 factors.

Step 3 Divide out common factors and simplify.

$$\frac{3^6}{3^2} = \frac{\overbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot \cancel{3} \cdot \cancel{3}}^{6\text{ factors}}}{\underbrace{\cancel{3} \cdot \cancel{3}}_{2\text{ factors}}} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{1} \text{ or } 3^4$$

Four factors of 3 remain in the numerator and 1 in the denominator, so the quotient is 3^4.

Notice that the difference of the exponents of the original powers is the exponent in the final quotient.

$$\frac{3^6}{3^2} = 3^{6-2} = 3^4$$

You can divide powers with the same base by subtracting the exponents.

You can simplify a quotient of powers with like bases using the **Quotient of Powers Property**.

Words	Algebra
To divide powers with the same base, subtract their exponents.	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$
	Numbers
	$\frac{3^7}{3^3} = 3^{7-3}$ or 3^4

Talk About It! When simplifying a quotient of powers using the Quotient of Powers Property, why do the bases have to be the same? For example, why can't you use the Quotient of Powers Property to simplify $\frac{x^8}{y^3}$?

Sample answer: You can subtract the exponents of powers with like bases since you are determining the amount of factors left after dividing out common factors. You can't divide out any common factors in the expression $\frac{x^8}{y^3}$ since x and y are not the same.

Lesson 4-2 • Multiply and Divide Monomials 235

Interactive Presentation

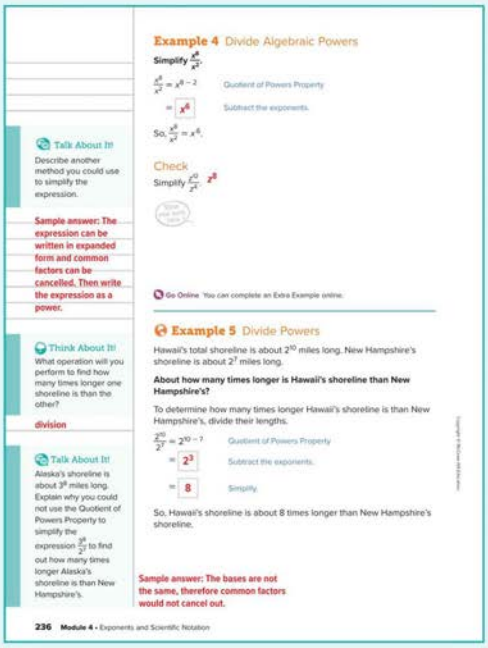
Learn, Quotient of Powers, Slide 2 of 3

WATCH

On Slide 1, students watch an animation that shows what happens when two powers with the same base are divided.

FLASHCARDS

On Slide 2, students use Flashcards to view multiple representations of the Quotient of Powers Property.



Example 4 Divide Algebraic Powers

Simplify $\frac{x^8}{x^2}$.

$\frac{x^8}{x^2} = x^{8-2}$ Quotient of Powers Property

$= x^6$ Subtract the exponents.

So, $\frac{x^8}{x^2} = x^6$.

Check

Simplify $\frac{z^{10}}{z^2}$. z^8

Go Online You can complete an Extra Example online.

Talk About It! Describe another method you could use to simplify the expression.

Sample answer: The expression can be written in expanded form and common factors can be cancelled. Then write the expression as a power.

Example 5 Divide Powers

Hawaii's total shoreline is about 2^{10} miles long. New Hampshire's shoreline is about 2^7 miles long.

About how many times longer is Hawaii's shoreline than New Hampshire's?

To determine how many times longer Hawaii's shoreline is than New Hampshire's, divide their lengths.

$\frac{2^{10}}{2^7} = 2^{10-7}$ Quotient of Powers Property

$= 2^3$ Subtract the exponents.

$= 8$ Simplify.

So, Hawaii's shoreline is about 8 times longer than New Hampshire's shoreline.

Think About It! What operation will you perform to find how many times longer one shoreline is than the other?

division

Talk About It! Alaska's shoreline is about 3^8 miles long. Explain why you could not use the Quotient of Powers Property to simplify the expression $\frac{3^8}{2^7}$ to find out how many times longer Alaska's shoreline is than New Hampshire's.

Sample answer: The bases are not the same, therefore common factors would not cancel out.

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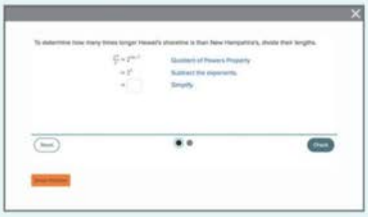
Interactive Presentation



Example 5, Divide Powers, Slide 2 of 4

TYPE

On Slide 2 of Example 5, students enter the value of expression.

CLICK

On Slide 2 of Example 4, students move through the steps to simplify the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 4 Divide Algebraic Powers

Objective

Students will use the Quotient of Powers Property to divide algebraic powers.

Questions for Mathematical Discourse

SLIDE 2

AL Write the numerator and denominator as repeated factors of *x*. $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$

OL Explain why the Quotient of Powers Property makes sense. Sample answer: There are 8 factors of *x* in the numerator and 2 in the denominator. Canceling 2 factors of each results in $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{1}$, which is x^6. This is the same as subtracting the exponents, $8 - 2 = 6$.

BL How do you think you might be able to use repeated factors to simplify the expression $\frac{x^2}{x^8}$? Sample answer: Write the expression as $\frac{x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$. Canceling 2 factors of the numerator and denominator results in 6 factors of *x* remaining in the denominator and 1 in the numerator. The result would be $\frac{1}{x^6}$.

Example 5 Divide Powers

Objective

Students will apply the Quotient of Powers Property to divide numerical powers in order to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to reason that the Quotient of Powers Property can only be applied to powers of the same base.

6 Attend to Precision Students should be able to apply the Quotient of Powers Property efficiently and accurately to simplify the expression and solve the problem. As students discuss the *Talk About It!* question, they should use clear and precise mathematical language in their explanations.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 6 Divide Numerical Powers

Objective

Students will use the Quotient of Powers Property to divide numerical powers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to use reasoning to understand how to write the fraction as a product of fractions with the same base.

6 Attend to Precision Students should use the definition of the Quotient of Powers Property to subtract the exponents of like bases in order to simplify the expression. As students discuss the *Talk About It!* question on Slide 3, encourage them to understand the conditions under which the Quotient of Powers Property applies, and be able to communicate those conditions clearly and precisely in their response.

Questions for Mathematical Discourse

SLIDE 2

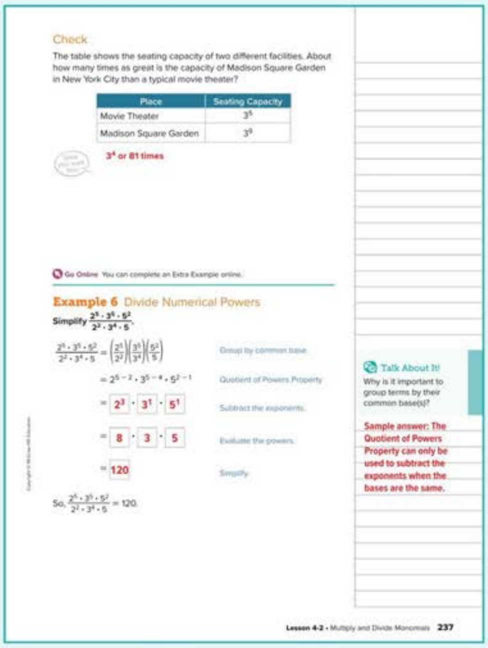
AL What bases will appear in the simplified expression? 2, 3, and 5

OL How can you write the fraction so that the Quotient of Powers Property can be applied? Sample answer: Write the fraction as a product of three fractions, one with powers of base 2, one with powers of base 3, and one with powers of base 5.

BL A classmate states that $\frac{2^5 \cdot 3^5 \cdot 5^2}{2^2 \cdot 3^4 \cdot 5}$ simplifies to $2^3 \cdot 3 \cdot 5^2$. Describe the error that may have been made. Sample answer: When simplifying the powers with the base of 5, the classmate may have thought that the exponent on 5 is 0 rather than 1.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

The table shows the seating capacity of two different facilities. About how many times as great is the capacity of Madison Square Garden in New York City than a typical movie theater?

Place	Seating Capacity
Movie Theater	3^5
Madison Square Garden	3^9

3^4 or 81 times

Go Online You can complete an Extra Example online.

Example 6 Divide Numerical Powers

Simplify $\frac{2^5 \cdot 3^5 \cdot 5^2}{2^2 \cdot 3^4 \cdot 5}$.

$\frac{2^5 \cdot 3^5 \cdot 5^2}{2^2 \cdot 3^4 \cdot 5} = \left(\frac{2^5}{2^2}\right)\left(\frac{3^5}{3^4}\right)\left(\frac{5^2}{5}\right)$ Group by common base.

$= 2^{5-2} \cdot 3^{5-4} \cdot 5^{2-1}$ Quotient of Powers Property

$= 2^3 \cdot 3^1 \cdot 5^1$ Subtract the exponents.

$= 8 \cdot 3 \cdot 5$ Evaluate the powers.

$= 120$ Simplify.

So, $\frac{2^5 \cdot 3^5 \cdot 5^2}{2^2 \cdot 3^4 \cdot 5} = 120$.

Talk About It! Why is it important to group terms by their common base(s)?

Sample answer: The Quotient of Powers Property can only be used to subtract the exponents when the bases are the same.

Lesson 4-2 • Multiply and Divide Monomials 237

Interactive Presentation

Example 6, Divide Numerical Powers, Slide 2 of 4

TYPE

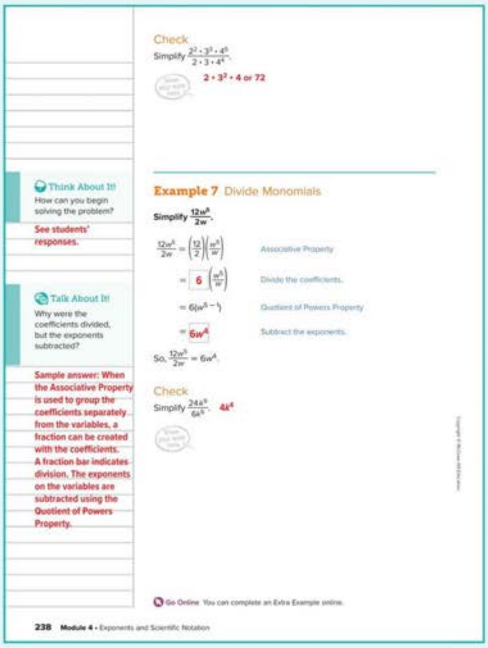
On Slide 2, students enter the value of the expression.

CLICK

On Slide 2, students move through the steps to simplify the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

Simplify $\frac{2^2 \cdot 3^3 \cdot 4^5}{2 \cdot 3 \cdot 4^4}$. $2 \cdot 3^2 \cdot 4$ or 72

Think About It! How can you begin solving the problem? See students' responses.

Example 7 Divide Monomials

Simplify $\frac{12w^5}{2w}$.

$\frac{12w^5}{2w} = \left(\frac{12}{2}\right)\left(\frac{w^5}{w}\right)$ Associative Property

$= 6\left(\frac{w^5}{w}\right)$ Divide the coefficients.

$= 6(w^{5-1})$ Quotient of Powers Property

$= 6w^4$ Subtract the exponents.

So, $\frac{12w^5}{2w} = 6w^4$.

Talk About It! Why were the coefficients divided, but the exponents subtracted?

Sample answer: When the Associative Property is used to group the coefficients separately from the variables, a fraction can be created with the coefficients. A fraction bar indicates division. The exponents on the variables are subtracted using the Quotient of Powers Property.

Check

Simplify $\frac{24x^9}{6x^5}$. $4x^4$

Go Online You can complete an Extra Example online.

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Example 7 Divide Monomials

Objective

Students will use the Quotient of Powers Property to divide monomials.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the coefficients given in the expression and be able to use reasoning to understand that the coefficients are divided, yet the exponents are subtracted. As students discuss the *Talk About It!* question on Slide 3, encourage them to understand what coefficients and powers each indicate in a division expression in order to be able to explain why they are treated differently when simplifying the expression.

6 Attend to Precision Encourage students to use academic vocabulary, such as of the Quotient of Powers Property, to explain how to simplify the expression.

Questions for Mathematical Discourse

SLIDE 2

AL How can you simplify $\frac{12}{2}$? Divide 12 by 2 to obtain 6.

OL How can you apply the Quotient of Powers Property? Sample answer: The Quotient of Powers Property can be used to simplify $\frac{w^5}{w}$ as w^{5-1}, or w^4.

BL A classmate simplifies the expression $\frac{12w^5}{2w}$ as $6w^5$. Describe the error that may have been made. Sample answer: The classmate may have divided the exponents instead of subtracting them.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Interactive Presentation


Example 7, Divide Monomials, Slide 2 of 4

TYPE On Slide 2, students enter the simplified expression.

CLICK On Slide 2, students move through the steps to simplify the expression.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Apply Computer Science

Objective

Students will come up with their own strategy to solve an application problem involving the processing speed of computers.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What do you know about the processing speed of each computer?
- How can you use the Product of Powers Property in this problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Computer Science

The processing speed of a certain computer is 10^{12} instructions per second. A second computer has a processing speed that is 10^5 times as fast as the first computer. A third computer has a processing speed of 10^{15} instructions per second. Which computer has the fastest processing speed?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

Computer 2; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
How did understanding the Laws of Exponents help you solve the problem?

See students' responses.

Lesson 4-2 • Multiply and Divide Monomials 239

Interactive Presentation

Apply, Computer Science

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Bolivia has a population of 10^7 people, while the population of Aruba is 10^5 people. The Philippines has a population that is 10^3 times greater than Aruba's population. Which statement is true about the population of the three countries?

(A) Aruba has the least population.

(B) Boliva has the greatest population.

(C) The Phillipines has the least population.

(D) Bolivia and the Philippines have the same population.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of multiplying and dividing powers with the same base. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

Why are exponents useful when working with very large or very small numbers?

In this lesson, students learned how to multiply and divide terms with exponents when the bases are the same. Encourage them to work with a partner to compare and contrast evaluating an expression like $4^4 \cdot 4^6$ using the Product of Powers and by evaluating 4^4 and 4^6 first, and then multiplying. Have them state which method they prefer and explain why they chose that method.

Exit Ticket

Refer to the Exit Ticket slide. Suppose a comet is traveling 6^7 miles per hour. How long does it take for the comet to travel 6^{12} miles? Explain your reasoning. 6^5 hours; Sample answer: $\frac{6^{12}\text{mi}}{6^7\text{mph}} = 6^{12-7}$ or 6^5 hours

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7–11 odd, 12–15
- Extension: Extension Resources
- ALEKS® Product, Power, and Quotient Rules

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 11, 13, 15
- Extension: Extension Resources
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–7
- ALEKS® Exponents and Order of Operations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Exponents and Scientific Notation

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

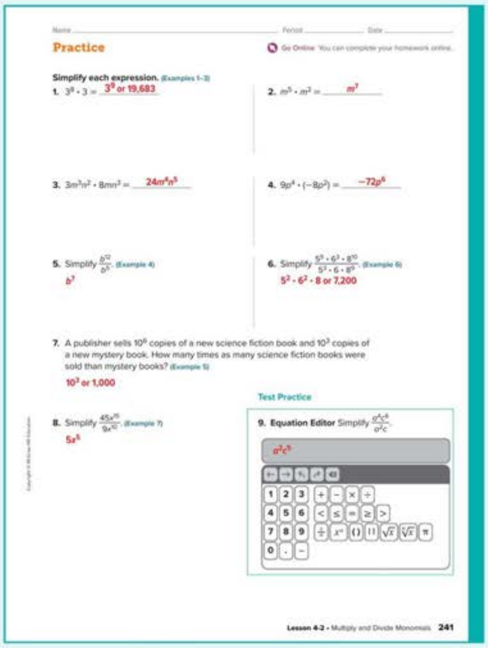
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use the Product of Powers Property to multiply numerical powers and algebraic powers	1, 2
1	use the Product of Powers Property to multiply monomials	3, 4
1	use the Quotient of Powers Property to divide algebraic and numerical powers	5, 6
2	use the Quotient of Powers Property to divide numerical powers from a real-world problem	7
1	use the Quotient of Powers Property to divide monomials	8
2	extend concepts learned in class to apply them in new contexts	9
3	solve application problems involving multiplying monomials	10, 11
3	higher-order and critical thinking skills	12–15

Common Misconception

Students may incorrectly assign an exponent of 0 to a number that does not have an exponent. Remind students that a number written without an exponent actually can be written with an exponent of 1. In Exercise 1, students may incorrectly write 3 as 3^0. Remind them that the number 3 means that 3 is being used as a factor *once*, not zero times. Encourage students to rewrite the problem using $3 = 3^1$.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Simplify each expression. (Examples 1–3)

1. $3^8 \cdot 3 =$ 3^9 or 19,683
2. $m^5 \cdot m^2 =$ m^7
3. $3m^3n^2 \cdot 8mn^3 =$ $24m^4n^5$
4. $9p^4 \cdot (-8p^2) =$ $-72p^6$
5. Simplify $\frac{b^{12}}{b^5}$. (Example 4)
b^7
6. Simplify $\frac{5^5 \cdot 6^3 \cdot 8^{10}}{5^3 \cdot 6 \cdot 8^9}$. (Example 6)
$5^2 \cdot 6^2 \cdot 8$ or 7,200
7. A publisher sells 10^6 copies of a new science fiction book and 10^3 copies of a new mystery book. How many times as many science fiction books were sold than mystery books? (Example 5)
10^3 or 1,000
8. Simplify $\frac{45x^{15}}{9x^{10}}$. (Example 7)
$5x^5$

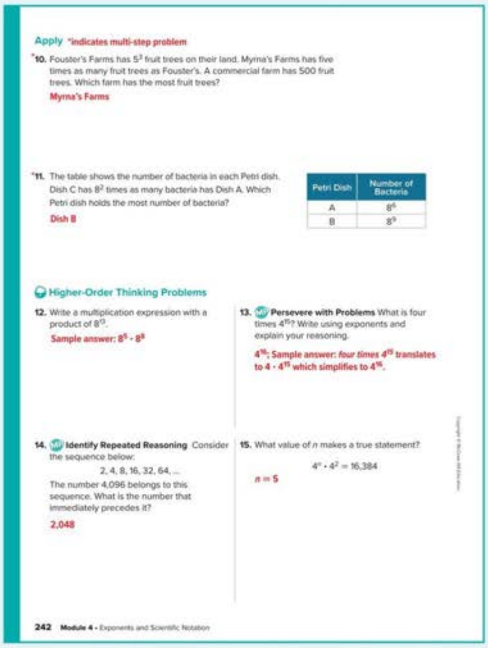
Test Practice

9. **Equation Editor** Simplify $\frac{a^4c^6}{a^2c}$.
a^2c^5

Lesson 4-2 • Multiply and Divide Monomials 241

8.EE.A.1

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Apply *indicates multi-step problem

*10. Fouster's Farms has 5^3 fruit trees on their land. Myrna's Farms has five times as many fruit trees as Fouster's. A commercial farm has 500 fruit trees. Which farm has the most fruit trees?

Myrna's Farms

*11. The table shows the number of bacteria in each Petri dish. Dish C has 8^2 times as many bacteria has Dish A. Which Petri dish holds the most number of bacteria?

Petri Dish	Number of Bacteria
A	8^6
B	8^9

Dish B

Higher-Order Thinking Problems

12. Write a multiplication expression with a product of 8^{13}.

Sample answer: $8^5 \cdot 8^8$

13. **Persevere with Problems** What is four times 4^{15}? Write using exponents and explain your reasoning.

4^{16}; Sample answer: *four times* 4^{15} translates to $4 \cdot 4^{15}$ which simplifies to 4^{16}.

14. **Identify Repeated Reasoning** Consider the sequence below:

2, 4, 8, 16, 32, 64, ...

The number 4,096 belongs to this sequence. What is the number that immediately precedes it?

2,048

15. What value of n makes a true statement?

$$4^n \cdot 4^2 = 16{,}384$$

$n = 5$

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Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 13, students will determine what the problem is asking for and then apply what they know about powers and the Product of Powers Property. Students should use the expression written in words to write a similar numerical expression before simplifying.

8 Look for and Express Regularity in Repeated Reasoning In Exercise 14, students will assess the given sequence to determine a pattern. Students should recognize that the numbers in the sequence could be written in a different form, using powers, so that it is easier to see the repeated expression that is being used and to determine the number that precedes 4,096.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.
Use with Exercises 10–11 Have students work in pairs. Have students individually read Exercise 10 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 11.

Clearly explain your strategy.
Use with Exercise 14 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would find the number immediately preceding 4,096, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Lesson 4-3

Powers of Monomials

8.EE.A.1

LESSON GOAL

Students will use Laws of Exponents to find powers of monomials.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Power of a Power

Learn: Power of a Power
Example 1: Power of a Power
Example 2: Power of a Power
Learn: Power of a Product
Example 3: Power of a Product
Example 4: Power of a Product
Apply: Geometry

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Powers of Multiple Powers		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 25 of the *Language Development Handbook* to help your students build mathematical language related to powers of monomials.

ELL You can use the tips and suggestions on page T25 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster by **8.EE.A** writing and evaluating expressions involving powers and exponents.
Standards for Mathematical Content: **8.E E.A.1**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students used the Laws of Exponents to simplify expressions involving products and quotients of monomials.
8.EE.A.1

Now
Students use the Laws of Exponents to find powers of monomials.
8.EE.A.1

Next
Students will simplify expressions that have zero and negative exponents.
8.EE.A.1

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

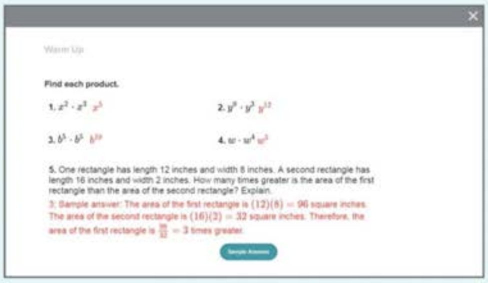
Conceptual Bridge In this lesson, students draw on their knowledge of exponents and the Power of Products Property to develop *understanding* of the Power of a Power Property. They learn how to raise a power to a power and build *fluency* with using this property to simplify numeric and algebraic expressions.

Mathematical Background

The Laws of Exponents include the Power of a Power Property and the Power of a Product Property:

- *Power of a Power*: To find the power of a power, multiply the exponents.
- *Power of a Product*: To find the power of a product, find the power of each factor and multiply.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding products of powers (Exercises 1–4)
- understanding integer operations (Exercise 5)

Answers

1. x^5

2. y^{12}

3. b^{10}

4. w^5

5. 3; Sample answer: The area of the first rectangle is (12)(8) = 96 square inches. The area of the second rectangle is (16)(2) = 32 square inches. Therefore, the area of the first rectangle is $\frac{96}{32} = 3$ times greater.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the large area of murals.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What do you think a *power of a power* is? Given an example. Sample answer: The power of a power might be a power raised to another power, such as $(x^2)^3$.
- What do you think a *power of a product* is ? Give an example. Sample answer: The power of a product might be a product that is raised to a power, such as $(x \cdot y)^3$.

Explore Power of a Power

Objective

Students will explore how to simplify a power of a power.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a volume problem in which a power of a power must be calculated. Throughout this activity, students will use repeated multiplication to evaluate a power of a power, and use the result to make and test a conjecture about how to find a power of a power.

Inquiry Question

How can you simplify a power raised to another power? Sample answer: To simplify a power raised to another power, multiply the exponents. When the powers are expanded, the number of repeated factors is equivalent to the product of the exponents.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

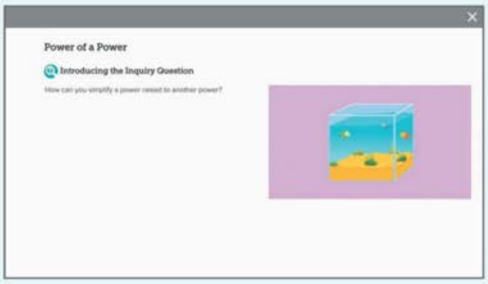
SLIDE 4

Mathematical Discourse

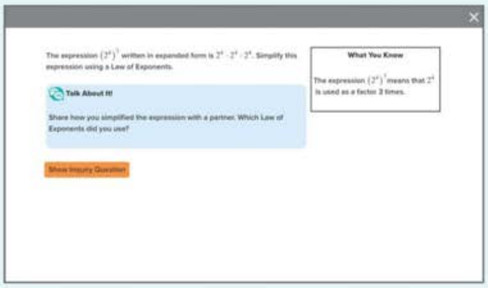
Share how you simplified the expression with a partner. Which Law of Exponents did you use? Sample answer: You can add the exponents in the expression $2^{4}{}^{4}\cdot{}^{4}2 \cdot 2$ and the result is 2^{12}. The Law of Exponents used is the Product of Powers Property.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8

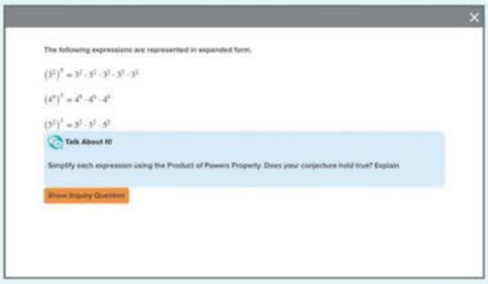


Explore, Slide 4 of 8

WATCH

On Slide 2, students watch an animation that presents the volume problem they will explore in this activity.

Interactive Presentation



Explore, Slide 7 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and can view a sample answer.

Explore Power of a Power *(continued)*

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Enc ourage students to identify the structure of expressions presented in the activity, and rewrite them in expanded form in order to apply the Product of Powers Property.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Simplify each expression using the Product of Powers Property. Does your conjecture hold true? Explain. Sample answer: Yes. When using the Product of Powers Property to add the exponents, the resulting exponent is the product of the two original exponents.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Power of a Power

Objective

Students will understand how the Power of a Power Property can be applied to simplify powers of powers.

Teaching Notes

SLIDE 1

Students will learn how the power of a power can be simplified using the Product of Powers Property. Have them move through the steps to simplify $(6^4)^3$. You may wish to have students work with a partner to consider multiple methods for simplifying this expression and have them discuss how these methods are related. For example, a student could write $(6 \cdot 6 \cdot 6 \cdot 6)$ as a factor three times and determine that altogether 6 is used as a factor 12 times.

SLIDE 2

Have students use the *Words*, *Algebra*, and *Number* Flashcards to learn about how the Power of a Power Property can be expressed using these multiple representations. Have students discuss with a partner how they can use the Product of Powers Property to simplify a power of a power, if they happen to forget the Power of a Power Property, or to check their work.

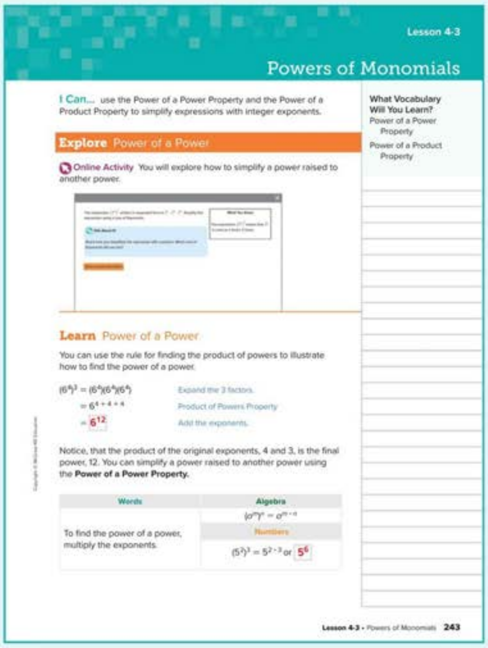
DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of the Power of a Power Property, display the following and allow students to determine whether the Power of a Power Property is used correctly. If the property is shown incorrectly, encourage students to explain how it could be fixed.

$(2^3)^2 = 2^{3+2} = 2^5$ incorrect; The exponents need to be multiplied, not added.

$(3^2)^4 = 3^{2 \cdot 4} = 3^8$ correct


Lesson 4-3

Powers of Monomials

I Can... use the Power of a Power Property and the Power of a Product Property to simplify expressions with integer exponents.

What Vocabulary Will You Learn?
Power of a Power Property
Power of a Product Property

Explore Power of a Power

Online Activity You will explore how to simplify a power raised to another power.

Learn Power of a Power

You can use the rule for finding the product of powers to illustrate how to find the power of a power.

$(6^4)^3 = (6^4)(6^4)(6^4)$ Expand the 3 factors.
$= 6^{4+4+4}$ Product of Powers Property
$= 6^{12}$ Add the exponents.

Notice, that the product of the original exponents, 4 and 3, is the final power, 12. You can simplify a power raised to another power using the **Power of a Power Property.**

Words	Algebra
To find the power of a power, multiply the exponents.	$(a^m)^n = a^{m \cdot n}$
	Numbers
	$(5^2)^3 = 5^{2 \cdot 3}$ or 5^6

Lesson 4-3 • Powers of Monomials 243

Interactive Presentation

Learn, Power of a Power, Slide 2 of 2

FLASHCARDS

On Slide 2, students use Flashcards to view multiple representations of the Power of a Power Property.

CLICK

On Slide 1, students move through the steps to simplify a power of a power.

Interactive Presentation

Example 1, Power of a Power, Slide 2 of 4

TYPE

On Slide 2 of Example 1 and Slide 1 of Example 2, students enter the simplified expression.

CLICK

On Slide 2 of Example 1 and Slide 1 of Example 2, students move through the steps to simplify the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 1 Power of a Power

Objective

Students will apply the Power of a Power Property to expressions with numerical bases.

Questions for Mathematical Discourse

SLIDE 2

AL When finding the power of a power, do you add, subtract, multiply, or divide the exponents? multiply

OL Explain why the Power of a Power Property makes sense, using this expression as an example. Sample answer: I can write $(8^6)^3$ as a product of repeated factors, $8^6 \cdot 8^6 \cdot 8^6$. The base is 8, and I can add the exponents to obtain a result of 8^{18}, which is the same result as multiplying the exponents in the original expression.

BL Simplify the expression $8^2 \cdot (8^6)^3$. Describe the steps you used. 8^{20}; Sample answer: First simplify $(8^6)^3$ as 8^{18} using the Power of a Power Property. Then use the Product of Powers Property to simplify the expression $8^2 \cdot 8^{18} = 8^{20}$.

Example 2 Power of a Power

Objective

Students will apply the Power of a Power Property to expressions with algebraic bases.

Questions for Mathematical Discourse

SLIDE 1

AL What property applies to this expression? Power of a Power Property

OL How can you simplify the expression by expanding it? Sample answer: Expand $(k^7)^5$ to obtain $k^7 \cdot k^7 \cdot k^7 \cdot k^7 \cdot k^7$. So, $k^{7+7+7+7+7} = k^{35}$.

BL If the value of k were negative, would the value of the expression be negative or positive? Explain. Sample answer: The expression would be negative. A negative number raised to an odd power, 35, is negative.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Power of a Product

Objective

Students will understand how the Power of a Product Property can be applied to find the powers of products.

Go Online to find additional teaching notes.

Example 3 Power of a Product

Objective

Students will use the Power of a Product Property to simplify monomials.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the coefficient of p^3 given in the expression and understand how it should be treated differently than the exponents.

6 Attend to Precision Students should be able to clearly explain why the coefficient is raised to the fourth power, but the exponents are multiplied. As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language to explain that an exponent outside of parentheses is applied to each factor within the parentheses.

Questions for Mathematical Discourse

SLIDE 2

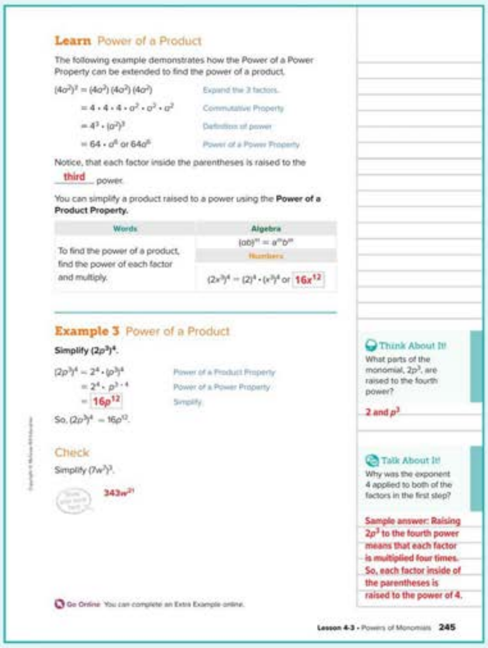
AL The expression inside the parentheses is the product of which factors? 2 and p^3

OL How can you simplify the expression using a different method? Sample answer: Expand to obtain $2p^3 \cdot 2p^3 \cdot 2p^3 \cdot 2p^3$. Multiply the coefficients, and use the Product of Powers Property to add the exponents.

BL A classmate simplifies $(2p^3)^4$ as $8p^7$. Describe the error that was made. Sample answer: The classmate multiplied 2 by the exponent 4, when 2 should have been raised the fourth power instead and added the exponents 3 and 4 instead of multiplying them.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Power of a Product

The following example demonstrates how the Power of a Power Property can be extended to find the power of a product.

$(4a^2)^3 = (4a^2)(4a^2)(4a^2)$ Expand the 3 factors.

$= 4 \cdot 4 \cdot 4 \cdot a^2 \cdot a^2 \cdot a^2$ Commutative Property

$= 4^3 \cdot (a^2)^3$ Definition of power

$= 64 \cdot a^6$ or $64a^6$ Power of a Power Property

Notice, that each factor inside the parentheses is raised to the third power.

You can simplify a product raised to a power using the **Power of a Product Property.**

Words	Algebra
To find the power of a product, find the power of each factor and multiply.	$(ab)^m = a^m b^m$
	Numbers
	$(2x^3)^4 = (2)^4 \cdot (x^3)^4$ or $16x^{12}$

Example 3 Power of a Product

Simplify $(2p^3)^4$.

$(2p^3)^4 = 2^4 \cdot (p^3)^4$ Power of a Product Property

$= 2^4 \cdot p^{3 \cdot 4}$ Power of a Power Property

$= 16p^{12}$ Simplify

So, $(2p^3)^4 = 16p^{12}$.

Think About It! What parts of the monomial, $2p^3$, are raised to the fourth power?

2 and p^3

Talk About It! Why was the exponent 4 applied to both of the factors in the first step?

Sample answer: Raising $2p^3$ to the fourth power means that each factor is multiplied four times. So, each factor inside of the parentheses is raised to the power of 4.

Check

Simplify $(7w^7)^3$.

$343w^{21}$

Go Online You can complete an Extra Example online.

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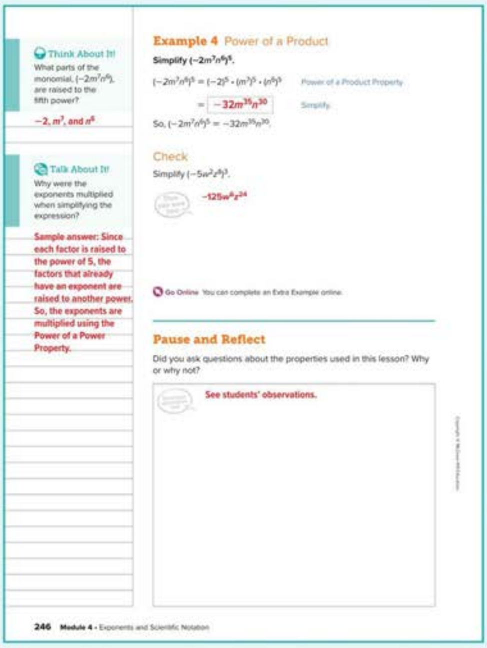
Interactive Presentation

Learn, Power of a Product, Slide 2 of 2

FLASHCARDS On Slide 2 of the Learn, students use Flashcards to view multiple representations of the Power of a Product Property.

CLICK On Slide 2 of Example 3, students move through the steps to simplify the expression.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Interactive Presentation

Example 4, Power of a Product, Slide 2 of 4

TYPE

On Slide 2, students enter the simplified expression.

CLICK

On Slide 2, students move through the steps to simplify the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Power of a Product

Objective

Students will use the Power of a Product Property to simplify monomials.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language, such as *exponents* and the *Power of a Product Property*, in their explanations.

Questions for Mathematical Discourse

SLIDE 2

AL What is the "base" that is raised to the 5th power? $-2m^7n^6$

AL How many factors are inside the parentheses? Identify them. 3 factors; -2; m^7; n^6

OL Explain why this is a power of a product. The product of -2, m^7, and n^6 is raised to the fifth power.

BL How can you simplify the expression if the exponent of n was k instead of 6? Sample answer: The Power of a Product Property can still be applied; $(-2m^7n^k)^5 = (-2)^5(m^7)^5(n^k)^5 = -32m^{35}n^{5k}$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Students may confuse the properties *Product of Powers*, *Power of a Power*, and *Power of a Product* because the names are similar. Have students create a graphic organizer or table, similar to the one shown, to compare and contrast these properties. Encourage them to make sense of the name of the property to help them remember what it means and how to distinguish it from the other properties involving powers and products. A sample table is shown.

	Product of Powers	Power of a Power	Power of a Product
Terms Included	product, powers	power	power, product
In My Own Words	when two or more powers with the same base are multiplied	when a power is raised to another power	when a product of two or more expressions is raised to a power
Example	$3^5 \cdot 3^2 = 3^7$	$(3^5)^2 = 3^{10}$	$(3^5 \cdot x^3)^2 = 3^{10} \cdot x^6$

Apply Geometry

Objective

Students will come up with their own strategy to solve an application problem involving the area of squares.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you find the area of a square?
- What is the relationship between the area of the floor and the area of each tile?
- How can you use the Laws of Exponents to help solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Geometry

A square floor has a side length of $8x^3y^2$ units. A square tile has a side length of xy units. How many tiles will it take to cover the floor?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$64x^4y^2$ tiles; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It! To find the number of tiles needed to cover the floor, why do you need to divide the areas?

Sample answer: The area of the floor is equal to the number of tiles needed to cover the floor multiplied by the area of one of the tiles. Since you know the area of the floor and the area of each tile, you can work backward and divide to find the number of tiles needed.

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Interactive Presentation

Apply, Geometry

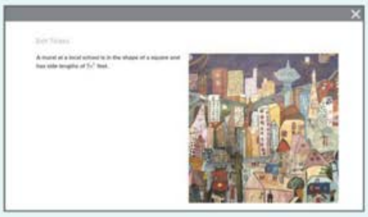
WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of the Power of a Power and the Power of a Product Properties. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Write a simplified expression that represents the area of the mural. Show the steps you used.

$(5x^3)^2 = 5^2x^6 = 25x^6$ square feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11, 13–16
- Extension: Powers of Multiple Powers
- ALEKS Product, Power, and Quotient Rules

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 11, 13, 15
- Extension: Powers of Multiple Powers
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Exponents and Order of Operations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Exponents and Order of Operations

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

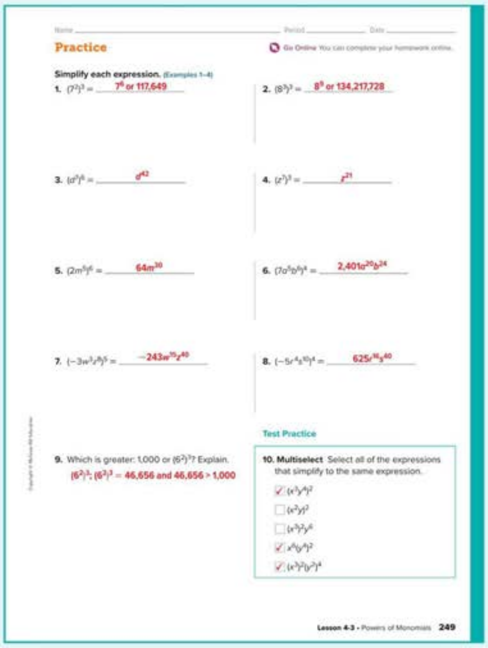
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	apply the Power of a Power Property to expressions with numerical bases	1, 2
1	apply the Power of a Power Property to expressions with algebraic bases	3, 4
1	use the Power of a Product Property to simplify monomials	5–8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving powers of monomials	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may incorrectly use the Power of a Product Property when simplifying expressions. In Exercise 7, students may not apply the power of 5 to the base of –3 when simplifying. Remind students to rewrite the expression using the Power of a Product Property, making sure that the exponent outside the parentheses is applied to each numeric and algebraic factor inside the parentheses.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Simplify each expression. (Examples 1–4)

1. $(7^2)^3 =$ 7^6 or 117,649
2. $(8^3)^3 =$ 8^9 or 134,217,728
3. $(d^7)^6 =$ d^{42}
4. $(z^7)^3 =$ z^{21}
5. $(2m^5)^6 =$ $64m^{30}$
6. $(7a^5b^6)^4 =$ $2{,}401a^{20}b^{24}$
7. $(-3w^7z^8)^5 =$ $-243w^{35}z^{40}$
8. $(-5r^4s^{10})^4 =$ $625r^{16}s^{40}$

9. Which is greater: 1,000 or $(6^2)^3$? Explain.
$(6^2)^3$; $(6^2)^3 = 46{,}656$ and $46{,}656 > 1{,}000$

Test Practice

10. **Multiselect** Select all of the expressions that simplify to the same expression.
- [x] $(x^3y^4)^2$
- [ ] $(x^2y)^2$
- [ ] $(x^3)^2y^6$
- [x] $x^6(y^4)^2$
- [x] $(x^3)^2(y^2)^4$

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Apply *indicates multi-step problem

*11. A square floor has a side length of $7x^5y^6$ units. A square tile has a side length of x^2y units. How many tiles will it take to cover the floor?
$49x^6y^{10}$ tiles

*12. A cube has a side length of $3x^6$ units. A smaller cube has a side length of x^2 units. How many smaller cubes will fit in the larger cube?
$27x^{12}$ cubes

Higher-Order Thinking Problems

13. Make an argument for why $(4^2)^4 = (4^4)^2$.
Sample answer: Using the Power of a Power law of exponents, both expressions simplify to 4^8. Multiplication is commutative, so $2 \cdot 4$ is the same as $4 \cdot 2$.

14. **Persevere with Problems** Describe all the positive integers that would make $\left[\left(\frac{1}{2}\right)^4\right]^n$ less than $\left(\frac{1}{2}\right)^7$. Explain your reasoning.
$n \geq 2$; Sample answer: When fractions are raised to a positive power, their value decreases. To determine a number less than $\left(\frac{1}{2}\right)^7$, the exponent must be 8 or greater. So, $4 \cdot n$ must be 8 or greater, by the Power of a Power property. So, $n \geq 2$.

15. Without computing, determine which number is greater, $[(-4)^4]^5$ or $-[(4^{12})^3]$. Explain your reasoning.
$[(-4)^4]^5$; Sample answer: Using the Power of a Power property, $[(-4)^4]^5$ simplifies to $(-4)^{20}$, a positive number. The expression $-[(4^{12})^3]$ simplifies to $-(4^{36})$, a negative number.

16. **Identify Structure** Charlotte states that $(4^3)^3$ can be rewritten as 2^{18}. Explain how she is correct.
Sample answer: $(4^3)^3$ can be simplified as 4^9. The expression 4^9 can be rewritten as $(2 \cdot 2)^9$. The Power of a Product Property states that $(2 \cdot 2)^9 = 2^9 \cdot 2^9$. Finally, $2^9 \cdot 2^9 = 2^{18}$.

250 **Module 4** • Exponents and Scientific Notation

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 14, students must identify what the problem is asking and then determine what information in the problem will be used. Students should notice that the base of both expressions is the same. They should investigate what a power does to a fraction and then apply this reasoning when determining what positive integers would make this inequality true.

7 Look for and Make Use of Structure In Exercise 16, students should hypothesize how the given expression could be rewritten as 2^{18}. Encourage students to identify the base in 2^{18} and determine how 4^9 can be written as an expression involving exponents and bases of 2 using the Power of a Product Property. Students should recognize the base of 2 is repeatedly used to write an equivalent power.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 12 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Interview a student.
Use with Exercises 14–15 Have pairs of students interview each other as they complete these problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 14 might be, "If $n = 1$, will $\left[\left(\frac{1}{2}\right)^4\right]^n$ be less than $\left(\frac{1}{2}\right)^7$? Why or why not?"

Lesson 4-4

Zero and Negative Exponents

8.EE.A.1

LESSON GOAL

Students will simplify expressions that have zero and negative exponents.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Exponents of Zero

Learn: Exponents of Zero
Example 1: Exponents of Zero

Explore: Negative Exponents

Learn: Negative Exponents
Examples 2-5: Negative Exponents
Apply: Measurement

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 26 of the *Language Development Handbook* to help your students build mathematical language related to zero and negative exponents.

ELL You can use the tips and suggestions on page T26 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.A** by writing and evaluating expressions involving powers and exponents.
Standards for Mathematical Content: **8.E E.A.1**
Standards for Mathematical Practice: **MP 1, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students used the Laws of Exponents to find powers of monomials.
8.EE.A.1

Now
Students simplify expressions that have zero and negative exponents.
8.EE.A.1

Next
Students will use scientific notation to write large and small numbers.
8.EE.A.3, 8.EE.A.4

Rigor

The Three Pillars of Rigor

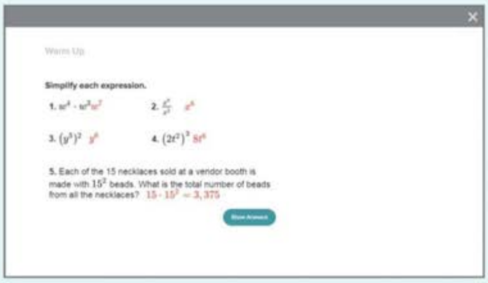
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to expand their *understanding* of exponents by simplifying numeric and algebraic expressions with zero and negative exponents. They use division and the properties of exponents to learn that any nonzero number to the zero power is 1, and that negative exponents are the result of repeated division. They build *fluency* with these properties by simplifying numeric and algebraic expressions.

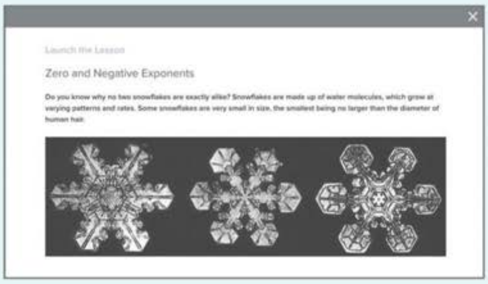
Mathematical Background

By definition, any nonzero number raised to the zero power is 1, and any nonzero number raised to a negative n power is the multiplicative inverse of its nth power.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- simplifying powers (Exercises 1–5)

Answers

1. w^7
2. x^6
3. y^6
4. $8t^6$
5. $15 \cdot 15^2 = 3,375$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the small size of snowflakes.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What do you think a *negative exponent* might mean? Sample answer: It might mean an exponent that is a negative number, such as x^{-2}.
- What do you think a *zero e xponent* might mean? Sample answer: It might mean an exponent of zero, such as n^0.

National Oceanic and Atmospheric Administration/Department of Commerce/Wilson Bentley photographer

Explore Exponents of Zero

Objective

Students will explore how to simplify expressions with exponents of zero.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with quotients of identical powers. Throughout this activity, students will compute quotients of identical powers using the Quotient of Powers Property and repeated multiplication in order to identify a relationship, and what it means for a number to have an exponent of zero.

Inquiry Question

What does it mean when a number has an exponent of zero? Sample answer: When you raise a nonzero number to a zero exponent, the value is one.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Simplify the expression $\frac{2^3}{2^3}$ using the Quotient of Powers Property. What is the result? The result is 2^0.

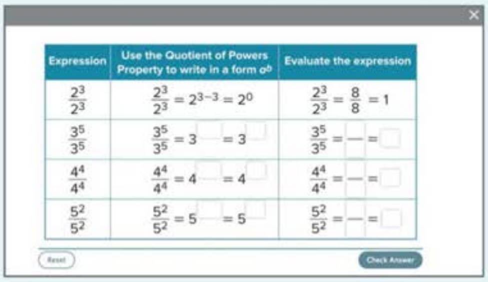
Evaluate the original expression by writing each power as repeated multiplication. What is the result? The result is $\frac{8}{8}$ or 1.

Compare and contrast the results. What do you notice? Sample answer: Using the Quotient of Powers Property, the result is an exponent of 0. Evaluating the expression gives you a result of 1.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6



Expression	Use the Quotient of Powers Property to write in a form a^b	Evaluate the expression
$\frac{2^3}{2^3}$	$\frac{2^3}{2^3} = 2^{3-3} = 2^0$	$\frac{2^3}{2^3} = \frac{8}{8} = 1$
$\frac{3^5}{3^5}$	$\frac{3^5}{3^5} = 3^{\square - \square} = 3^{\square}$	$\frac{3^5}{3^5} = \frac{\square}{\square} = \square$
$\frac{4^4}{4^4}$	$\frac{4^4}{4^4} = 4^{\square - \square} = 4^{\square}$	$\frac{4^4}{4^4} = \frac{\square}{\square} = \square$
$\frac{5^2}{5^2}$	$\frac{5^2}{5^2} = 5^{\square - \square} = 5^{\square}$	$\frac{5^2}{5^2} = \frac{\square}{\square} = \square$

Reset Check Answer

Explore, Slide 3 of 6

TYPE

On Slide 3, students complete a table in order to investigate patterns and similarities between expressions.

Interactive Presentation

Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and can view a sample answer.

Explore Exponents of Zero *(continued)*

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others Encourage students to draw a valid conclusion about what it might mean for a number to have an exponent of zero. Have them listen and critique the conclusions of other students, and explain why those may be valid or invalid conclusions.

8 Look for and Express Regularity in Repeated Reasoning Encourage students to investigate the patterns and relationships they encounter during the Explore activity.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Without calculating, what do you think 7^0, 9^0, and 15^0 simplify to? Justify your response. Sample answer: All three simplify to 1 because expressions that contain an exponent of 0 when simplified, also simplify to 1.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Exponents of Zero

Objective

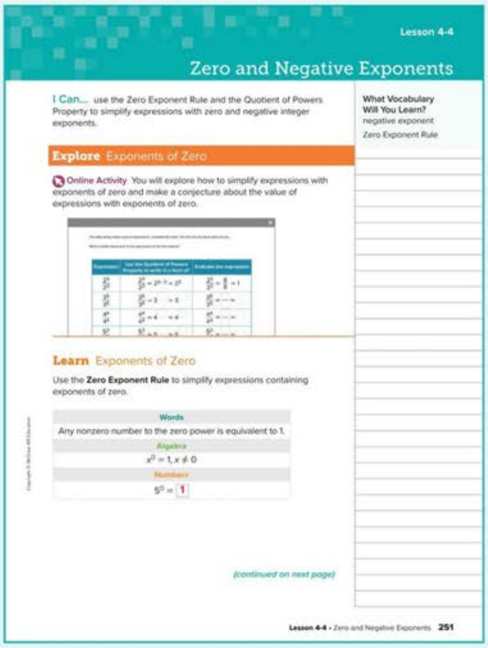
Students will understand how the Zero Exponent Rule can be applied to simplify expressions that contain exponents of zero.

Teaching Notes

SLIDE 1

Have students view the *Words, Algebra*, and *Numbers* Flashcards to see how the Zero Exponent Rule can be expressed in these multiple representations. You may wish to have students work with a partner to create a logical argument that explains why any nonzero number raised to the zero power is equivalent to 1.

(continued on next page)


Lesson 4-4

Zero and Negative Exponents

I Can... use the Zero Exponent Rule and the Quotient of Powers Property to simplify expressions with zero and negative integer exponents.

What Vocabulary Will You Learn?
negative exponent
Zero Exponent Rule

Explore Exponents of Zero

Online Activity You will explore how to simplify expressions with exponents of zero and make a conjecture about the value of expressions with exponents of zero.

Learn Exponents of Zero

Use the **Zero Exponent Rule** to simplify expressions containing exponents of zero.

Words
Any nonzero number to the zero power is equivalent to 1.
Algebra
$x^0 = 1, x \neq 0$
Numbers
$5^0 = 1$

(continued on next page)

Lesson 4-4 • Zero and Negative Exponents 251

Interactive Presentation

Learn, Exponents of Zero, Slide 1 of 3

FLASHCARDS

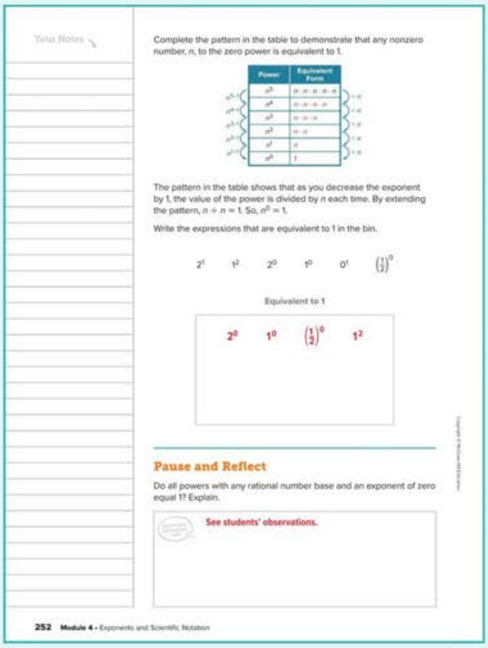
On Slide 1, students use Flashcards to view multiple representations of the Zero Exponent Rule.

DIFFERENTIATE

Enrichment Activity

To further students' understanding of what it means for a number to have an exponent of zero, have them work with a partner to prepare a brief presentation that illustrates the Zero Exponent Rule, and why it is true. Encourage them to use the patterns and relationships they discovered in the Explore activity, but have them generate their own expressions. Have each pair of students present their argument to another pair, or to the whole class. Some students may be uncomfortable speaking in front of others. Encourage them to make appropriate eye contact, and articulate their thoughts clearly and loudly enough for others to hear.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION



Your Notes

Complete the pattern in the table to demonstrate that any nonzero number, n, to the zero power is equivalent to 1.

Power	Equivalent Form
n^5	$n \cdot n \cdot n \cdot n \cdot n$
n^4	$n \cdot n \cdot n \cdot n$
n^3	$n \cdot n \cdot n$
n^2	$n \cdot n$
n^1	n
n^0	1

The pattern in the table shows that as you decrease the exponent by 1, the value of the power is divided by n each time. By extending the pattern, $n \div n = 1$. So, $n^0 = 1$.

Write the expressions that are equivalent to 1 in the bin.

2^1 1^2 2^0 1^0 0^1 $\left(\frac{1}{2}\right)^0$

Equivalent to 1

2^0 1^0 $\left(\frac{1}{2}\right)^0$ 1^2

Pause and Reflect

Do all powers with any rational number base and an exponent of zero equal 1? Explain.

See students' observations.

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Learn Exponents of Zero *(continued)*

Teaching Notes

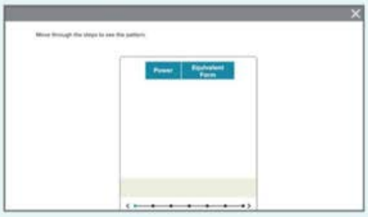
SLIDE 2

Have students study the table to investigate the pattern between the repeated division and the value of the exponent. Before moving to each next row, have students make a conjecture as to what the next power will be, with each successive division. Have them explain why n divided by n is equivalent to 1, and what that means for the value of the exponent.

SLIDE 3

Students will learn to identify powers equal to 1 using a drag and drop activity. You may wish to have student volunteers come up to the board to identify each power that is equal to 1. Have students explain why each expression is, or is not, equivalent to 1. You may wish to have them generate their own expressions and identify whether each one is equivalent to 1.

Interactive Presentation



Learn, Exponents of Zero, Slide 2 of 3

CLICK

On Slide 2, students click through the steps to see the pattern that occurs when you decrease the exponent by 1.

DRAG & DROP

On Slide 3, students drag to sort the expressions as to whether they are equivalent to 1 or not.

Example 1 Exponents of Zero

Objective

Students will simplify expressions with exponents of zero.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to accurately and efficiently find the value of an expression containing an exponent of zero. Have them cite the name of the property that allows them to simplify the expression, the Zero Exponent Rule.

Questions for Mathematical Discourse

SLIDE 1

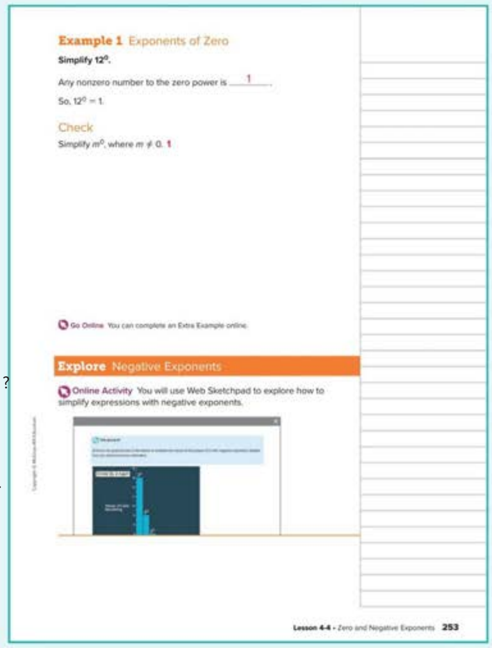
AL What does an exponent indicate about a power? Sample answer: the number of factors of the base

OL How many factors of 12 would be in the expanded form of 12^0? Explain. 0; Sample answer: The exponent indicates the number of factors. Since the exponent is 0, there are no factors of 12 in the expanded form.

BL Simplify $(-12)^0$. Does it have a different value than 12^0? Explain. 1; no; Sample answer: The simplified form of $(-12)^0$ is 1. Any nonzero base raised to the zero power is equal to 1.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Exponents of Zero

Simplify 12^0.

Any nonzero number to the zero power is 1.

So, $12^0 = 1$.

Check

Simplify m^0, where $m \neq 0$. 1

Go Online You can complete an Extra Example online.

Explore Negative Exponents

Online Activity You will use Web Sketchpad to explore how to simplify expressions with negative exponents.

Lesson 4-4 • Zero and Negative Exponents 253

Interactive Presentation

Example 1, Exponents of Zero, Slide 1 of 2

TYPE

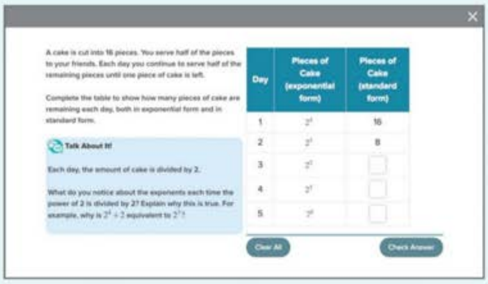
On Slide 1, students enter the missing values to simplify the expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Interactive Presentation

Explore, Slide 1 of 7



Explore, Slide 2 of 7

WEB SKETCHPAD

Throughout the Explore, students will use Web Sketchpad to explore the descending progression of values for the powers of 2.

TYPE

On Slide 2, students complete a table to show the amount of cake left after a certain number of days.

Explore Negative Exponents

Objective

Students will use Web Sketchpad to explore how to simplify expressions with negative exponents.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a scenario in which they must repeatedly divide a power of 2 by 2, eventually encountering negative exponents. Throughout this activity, students will explore patterns of decreasing powers in both exponential and standard forms.

Inquiry Question

How can you simplify an expression with a negative exponent? Sample answer: To simplify an expression with a negative exponent, you need to write it as a fraction, with one in the numerator and the positive power in the denominator. For example, $2^{-1} = \frac{1}{2^1}$ or $\frac{1}{2}$.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Each day, the amount of cake is divided by 2.

What do you notice about the exponents each time the power of 2 is divided by 2? Explain why this is true. For example, why is $2^4 \div 2$ equivalent to 2^3? Sample answer: Dividing by 2 decreases the power by one each time. This is true because of the Quotient of Powers Property. The expression $2^n = 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2$, where 2 appears as a factor n times. If you divide this by 2, you cancel the last factor, leaving one less factor of 2.

(continued on next page)

Explore Negative Exponents *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use a table to help organize the information in order to make a conjecture about what a negative exponent might mean.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

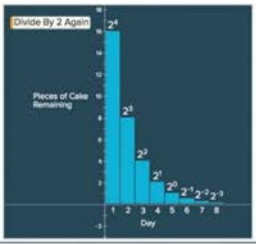
Talk About It!

SLIDE 5

Mathematical Discourse

Examine the graphed bars in the sketch to estimate the values of the powers of 2 with negative exponents. Explain how you determined your estimation. Sample answer: The values of the powers of 2 with negative exponents will be less than one but greater than 0. I can use the heights of the bars in the sketch to determine this.

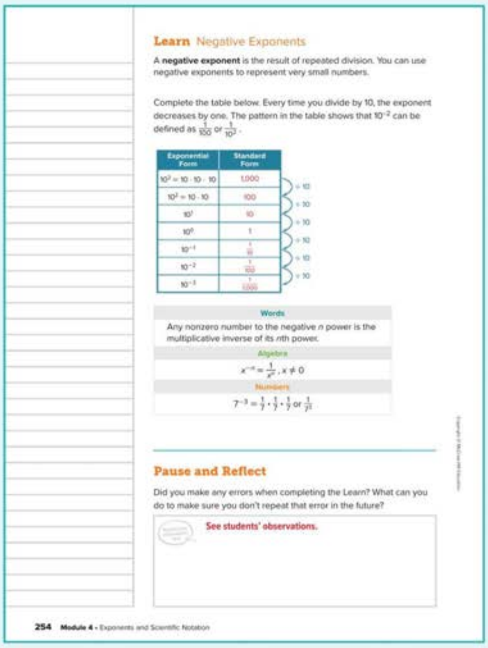
Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 6, students complete a table to show the amount of cake left after a certain number of days. On Slide 7, students will respond to the Inquiry Question and can view a sample answer.



Learn Negative Exponents

A **negative exponent** is the result of repeated division. You can use negative exponents to represent very small numbers.

Complete the table below. Every time you divide by 10, the exponent decreases by one. The pattern in the table shows that 10^{-2} can be defined as $\frac{1}{100}$ or $\frac{1}{10^2}$.

Exponential Form	Standard Form
$10^3 = 10 \cdot 10 \cdot 10$	1,000
$10^2 = 10 \cdot 10$	100
10^1	10
10^0	1
10^{-1}	$\frac{1}{10}$
10^{-2}	$\frac{1}{100}$
10^{-3}	$\frac{1}{1,000}$

÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10

Words

Any nonzero number to the negative n power is the multiplicative inverse of its nth power.

Algebra

$x^{-n} = \frac{1}{x^n}, x \neq 0$

Numbers

$7^{-3} = \frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7}$ or $\frac{1}{7^3}$

Pause and Reflect

Did you make any errors when completing the Learn? What can you do to make sure you don't repeat that error in the future?

See students' observations.

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Interactive Presentation

Learn, Negative Exponents, Slide 1 of 2

CLICK

On Slide 1, students investigate the pattern that occurs when powers of 10 are divided.

FLASHCARDS

On Slide 2, students use Flashcards to view multiple representations of negative exponents.

Learn Negative Exponents

Objective

Students will understand the relationship between negative exponents and repeated division.

Teaching Notes

SLIDE 1

Have students study the table to investigate the pattern between the repeated division and the value of the exponent. Before moving to each next row, have students make a conjecture as to what the next power will be, with each successive division.

SLIDE 2

Have students use the *Words, Algebra*, and *Numbers* Flashcards to view the concept of negative exponents expressed in these multiple representations. You may wish to have them generate their own expressions that involve negative exponents, and have them explain how they can interpret that value by writing their expressions using only positive exponents.

Example 2 Negative Exponents

Objective

Students will express powers with negative exponents using positive exponents.

Questions for Mathematical Discourse

SLIDE 2

AL Identify the base and exponent of the power. The base is 6 and the exponent is −3.

OL Is 6^{-3} less than or greater than 6^3? than 6? than 0? Explain. 6^{-3} is less than 6^3 and less than 6, but greater than 0; Sample answer: 6^{-3} represents a very small, but positive number.

OL How does 6^{-3} relate to 6^3? $6^{-3} = \frac{1}{6^3}$; They are reciprocals.

OL What is the product of 6^{-3} and 6^3? Explain. $(6^{-3})(6^3) = 6^0$, or 1

BL How can you write the expression $(-6)^{-3}$ using a positive exponent? $\frac{1}{(-6)^3}$

Example 3 Negative Exponents

Objective

Students will express fractions with powers in the denominator using negative exponents.

Questions for Mathematical Discourse

SLIDE 1

AL What do you notice about the numerator? What do you notice about the exponent in the denominator? The numerator is 1. The exponent in the denominator is positive.

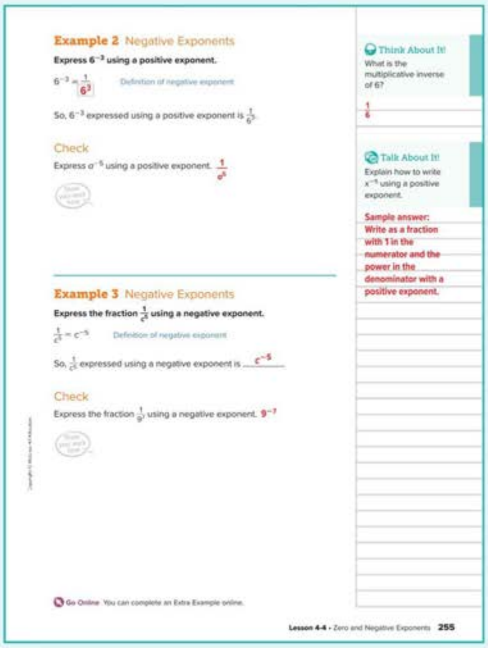
OL What is the relationship between c^5 and c^{-5}? They are reciprocals.

OL How can you use this fact that c^5 and c^{-5} are reciprocals to demonstrate that $\frac{1}{c^5}$ is equal to c^{-5}? Sample answer: Since $c^{-5} = \frac{1}{c^5}$, and $\frac{1}{c^5}$ and c^5 are also reciprocals, this means that $\frac{1}{c^5} = c^{-5}$.

BL Express $\frac{\frac{1}{c^5}}{c^7}$ using a negative exponent. c^{-12}

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Interactive Presentation

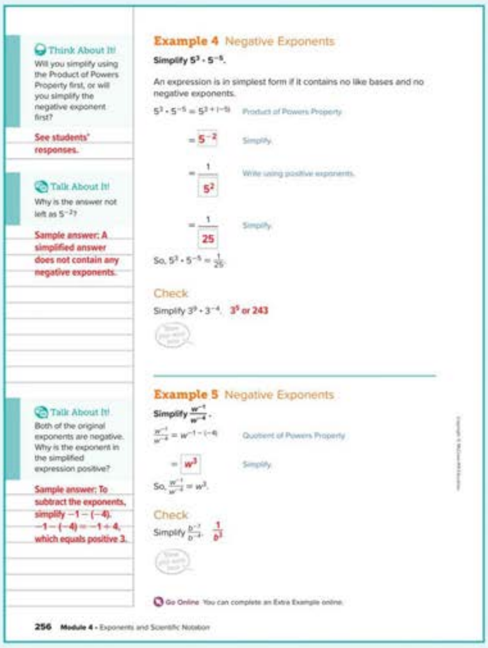
Example 2, Negative Exponents, Slide 2 of 4

TYPE

On Slide 2 of Example 2 and Slide 1 of Example 3, students enter the missing value to complete the sentence.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.



Think About It! Will you simplify using the Product of Powers Property first, or will you simplify the negative exponent first?

See students' responses.

Talk About It! Why is the answer not left as 5^{-2}?

Sample answer: A simplified answer does not contain any negative exponents.

Example 4 Negative Exponents

Simplify $5^3 \cdot 5^{-5}$.

An expression is in simplest form if it contains no like bases and no negative exponents.

$5^3 \cdot 5^{-5} = 5^{3+(-5)}$ Product of Powers Property

$= 5^{-2}$ Simplify.

$= \frac{1}{5^2}$ Write using positive exponents.

$= \frac{1}{25}$ Simplify.

So, $5^3 \cdot 5^{-5} = \frac{1}{25}$.

Check

Simplify $3^9 \cdot 3^{-4}$. 3^5 or 243

Talk About It! Both of the original exponents are negative. Why is the exponent in the simplified expression positive?

Sample answer: To subtract the exponents, simplify $-1 - (-4)$. $-1 - (-4) = -1 + 4$, which equals positive 3.

Example 5 Negative Exponents

Simplify $\frac{w^{-1}}{w^{-4}}$.

$\frac{w^{-1}}{w^{-4}} = w^{-1-(-4)}$ Quotient of Powers Property

$= w^3$ Simplify.

So, $\frac{w^{-1}}{w^{-4}} = w^3$.

Check

Simplify $\frac{b^{-7}}{b^{-4}}$. $\frac{1}{b^3}$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4, Negative Exponents, Slide 2 of 4

CLICK

On Slide 2 of Example 4 and Slide 2 of Example 5, students move through the steps to simplify the expression.

TYPE

On Slide 2 of Example 4 and Slide 2 of Example 5, students enter the missing value to simplify the expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 4 Negative Exponents

Objective

Students will simplify a product of powers with negative exponents.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to clearly explain what it means for an expression to be in simplest form.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the expression, noting that it involves the product of two powers, and that one of the powers has a negative exponent.

Questions for Mathematical Discourse

SLIDE 2

AL Describe in words what the expression represents. Sample answer: the product of two powers

OL Explain why the Product of Powers Property can be used. The bases are the same.

BL The product of 5^3 and what power results in 1? Explain. 5^{-3}; Sample answer: The result is equal to 5^0. Since the exponents are added, the other power must be 5^{-3}.

Example 5 Negative Exponents

Objective

Students will simplify a quotient of powers with negative exponents.

Questions for Mathematical Discourse

SLIDE 2

AL To divide powers with the same base, do you add or subtract the exponents? subtract

OL What is $-1 - (-4)$? 3

BL How can you simplify $\left(\frac{w^{-1}}{w^{-4}}\right)^{-1}$? Sample answer: The exponent -1 can be applied to the numerator and denominator separately. Then use the Power of a Power Property and the Quotient of Powers Property to simplify.

$$\left(\frac{w^{-1}}{w^{-4}}\right)^{-1} = \frac{(w^{-1})^{-1}}{(w^{-4})^{-1}} = \frac{w^1}{w^4} = w^{1-4} = w^{-3} = \frac{1}{w^3}$$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Measurement

Objective

Students will come up with their own strategy to solve an application problem that involves comparing diameters.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does *how many times larger* mean?
- What are some examples of powers of 10?
- How can you use the definition of negative exponents to solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Measurement

One strand of human hair is about 0.001 inch in diameter. The diameter of a certain wire for jewelry is 10^{-2} inch. How many times larger is the diameter of the wire than a strand of hair? Write the decimal as a power of 10.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

10 times larger; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

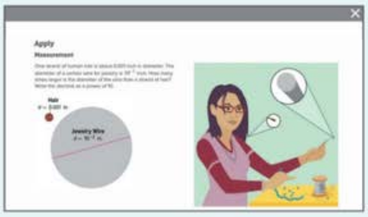
See students' arguments.

Talk About It! Why might writing 0.001 as a fraction be helpful?

Sample answer: To compare the diameters, the values need to be in the same form. In order to write 0.001 using a negative exponent, it is first helpful to write it as a fraction, with 1 in the numerator.

Lesson 4-4 • Zero and Negative Exponents 257

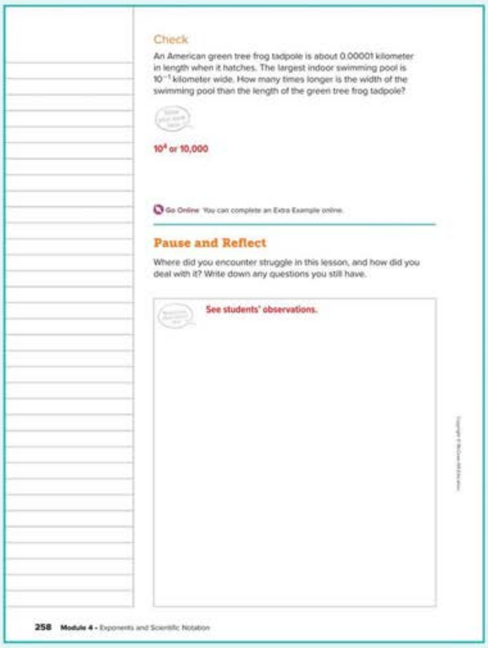
Interactive Presentation



Apply, Measurement

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

An American green tree frog tadpole is about 0.00001 kilometer in length when it hatches. The largest indoor swimming pool is 10^{-1} kilometer wide. How many times longer is the width of the swimming pool than the length of the green tree frog tadpole?

10^4 or 10,000

Go Online You can complete an Extra Example online.

Pause and Reflect

Where did you encounter struggle in this lesson, and how did you deal with it? Write down any questions you still have.

See students' observations.

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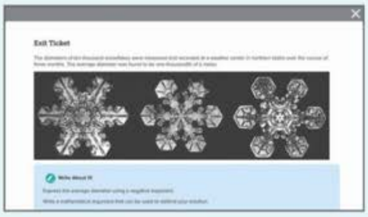
Interactive Presentation



Exit Ticket

Essential Question Follow-Up

Why are exponents useful when working with very large or very small numbers?

In this lesson, students learned that negative exponents are the result of repeated division and how to use the Product of Powers property to simplify expressions with negative exponents. Encourage them to discuss with a partner how a negative exponent can represent a very small number, and why using exponents are useful when multiplying or dividing small numbers with the same base. For example, why using negative exponents makes it easier to simplify $6^{-5} \div 6^{-3}$?

Exit Ticket

Refer to the Exit Ticket slide. Express the average diameter using a negative exponent. Write a mathematical argument that can be used to defend your solution. 10^{-3} meters; Sample answer: One thousandth can be expressed as $\frac{1}{1{,}000}$ or $\frac{1}{10^3}$. So, $\frac{1}{10^3} = 10^{-3}$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks **BL**
THEN assign:

- Practice, Exercises 11, 13, 15–18
- Extension: Rewrite Powers to Have the Same Base
- ALEKS® Negative Exponents

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–10, 13, 15
- Extension: Rewrite Powers to Have the Same Base
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Product, Power, and Quotient Rules

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Product, Power, and Quotient Rules

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

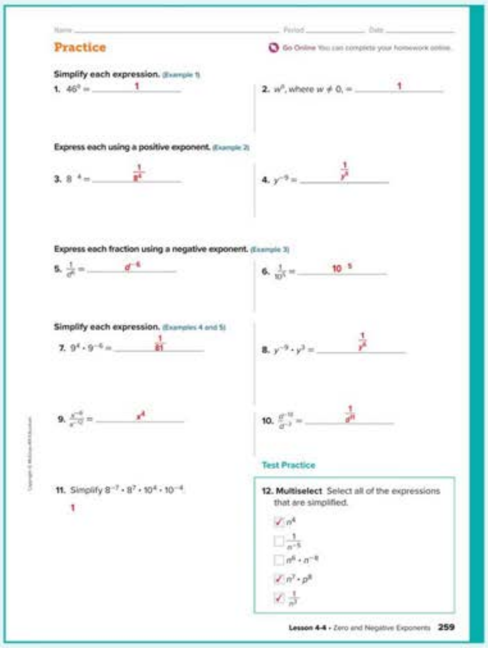
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	simplify expressions with exponents of zero	1, 2
1	express powers with negative exponents using positive exponents	3, 4
1	express fractions with powers in the denominator using negative exponents	5, 6
1	simplify products of powers and quotients of powers with negative exponents	7–10
2	extend concepts learned in class to apply them in new contexts	11, 12
3	solve application problems involving negative exponents	13, 14
3	higher-order and critical thinking skills	15–18

Common Misconception

Some students may not fully simplify an expression that contains negative exponents. Students may apply the Product of Powers Property correctly, but forget to rewrite the expression using positive exponents. In Exercise 7, students may write 9^{-2} as the final answer. Remind students that an expression is in simplest form if it contains no like bases and no negative exponents. Encourage students to explain how they could write 9^{-2} with a positive exponent.


Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Simplify each expression. (Example 1)

1. $46^0 =$ 1

2. w^0, where $w \neq 0$, = 1

Express each using a positive exponent. (Example 2)

3. $8^{-4} =$ $\frac{1}{8^4}$

4. $y^{-9} =$ $\frac{1}{y^9}$

Express each fraction using a negative exponent. (Example 3)

5. $\frac{1}{d^6} =$ d^{-6}

6. $\frac{1}{10^5} =$ 10^{-5}

Simplify each expression. (Examples 4 and 5)

7. $9^4 \cdot 9^{-6} =$ $\frac{1}{81}$

8. $y^{-9} \cdot y^3 =$ $\frac{1}{y^6}$

9. $\frac{x^{-6}}{x^{-10}} =$ x^4

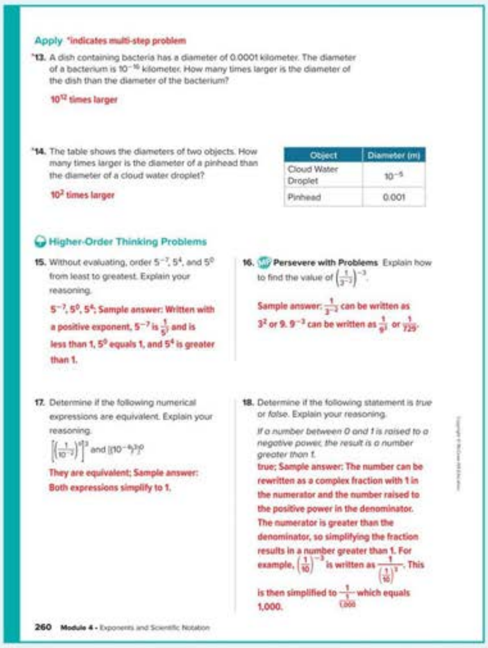
10. $\frac{d^{-10}}{d^{-1}} =$ $\frac{1}{d^9}$

11. Simplify $8^{-7} \cdot 8^7 \cdot 10^4 \cdot 10^{-4}$.
1

Test Practice

12. **Multiselect** Select all of the expressions that are simplified.
- [x] n^4
- [ ] $\frac{1}{n^{-5}}$
- [ ] $n^6 \cdot n^{-8}$
- [x] $n^7 \cdot p^8$
- [x] $\frac{1}{n^3}$

Lesson 4-4 • Zero and Negative Exponents 259



Apply *indicates multi-step problem

*13. A dish containing bacteria has a diameter of 0.0001 kilometer. The diameter of a bacterium is 10^{-16} kilometer. How many times larger is the diameter of the dish than the diameter of the bacterium?

10^{12} times larger

*14. The table shows the diameters of two objects. How many times larger is the diameter of a pinhead than the diameter of a cloud water droplet?

Object	Diameter (m)
Cloud Water Droplet	10^{-5}
Pinhead	0.001

10^{2} times larger

Higher-Order Thinking Problems

15. Without evaluating, order 5^{-7}, 5^4, and 5^0 from least to greatest. Explain your reasoning.

5^{-7}, 5^0, 5^4; Sample answer: Written with a positive exponent, 5^{-7} is $\frac{1}{5^7}$ and is less than 1, 5^0 equals 1, and 5^4 is greater than 1.

16. **Persevere with Problems** Explain how to find the value of $\left(\frac{1}{3^{-2}}\right)^{-3}$.

Sample answer: $\frac{1}{3^{-2}}$ can be written as 3^2 or 9. 9^{-3} can be written as $\frac{1}{9^3}$ or $\frac{1}{729}$.

17. Determine if the following numerical expressions are equivalent. Explain your reasoning.

$\left[\left(\frac{1}{10^{-2}}\right)^0\right]^2$ and $[(10^{-4})^3]^0$

They are equivalent; Sample answer: Both expressions simplify to 1.

18. Determine if the following statement is *true* or *false*. Explain your reasoning.

If a number between 0 and 1 is raised to a negative power, the result is a number greater than 1.

true; Sample answer: The number can be rewritten as a complex fraction with 1 in the numerator and the number raised to the positive power in the denominator. The numerator is greater than the denominator, so simplifying the fraction results in a number greater than 1. For example, $\left(\frac{1}{10}\right)^{-3}$ is written as $\frac{1}{\left(\frac{1}{10}\right)^3}$. This is then simplified to $\frac{1}{\frac{1}{1{,}000}}$ which equals 1,000.

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Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 16, students will determine what steps and properties will need to be used to find the value of the given expression, and then explain how the expression can be simplified in a concise manner. Encourage students to check each step of their answer.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.
Use with Exercise 13–14 Have students work in groups of 3–4 to solve the problem in Exercise 13. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 14.

Create your own higher-order thinking problem.
Use with Exercises 15–18 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Lesson 4-5

Scientific Notation

8.EE.A.3, 8.EE.A.4

LESSON GOAL

Students will write numbers in scientific notation.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Scientific Notation

Learn: Scientific Notation
Examples 1-2: Write Numbers in Standard Form
Learn: Scientific Notation and Technology
Example 3: Scientific Notation and Technology
Learn: Write Numbers in Scientific Notation
Examples 4-5: Write Numbers in Scientific Notation
Learn: Use Scientific Notation
Example 6: Choose Units of Appropriate Size
Example 7: Estimate with Scientific Notation
Apply: Travel

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	L B	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 27 of the *Language Development Handbook* to help your students build mathematical language related to scientific notation.

ELL You can use the tips and suggestions on page T27 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.A** by writing and evaluating expressions involving powers and exponents.
Standards for Mathematical Content: **8.E E.A.3, 8.EE.A.4**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students simplified expressions that have zero and negative exponents.
8.EE.A.1

Now
Students use scientific notation to write large and small numbers.
8.EE.A.3, 8.EE.A.4

Next
Students will compute with numbers written in scientific notation.
8.EE.A.3, 8.EE.A.4

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

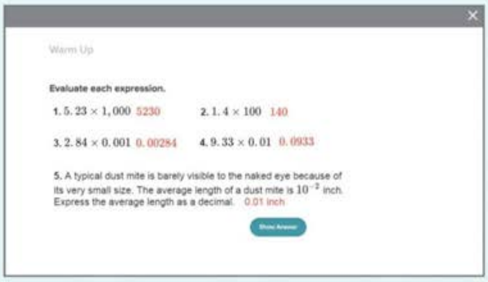
Conceptual Bridge In this lesson, students develop *understanding* of scientific notation. They learn that scientific notation can be used to represent very large or very small numbers. They build *fluency* with scientific notation by converting very large or very small numbers between standard form and scientific notation, and *apply* scientific notation to real-world problems.

Mathematical Background

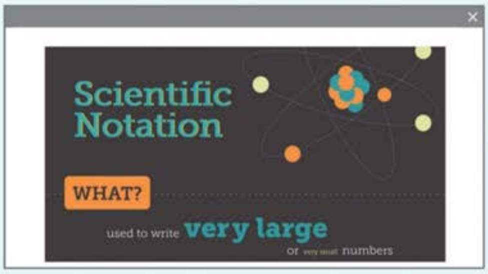
It is often helpful to express very large numbers such as 5,000,000,000 and very small numbers such as 0.000034 in *scientific notation*, where a number is written as the product of a factor (greater than or equal to 1 and less than 10) and an integer power of 10.

When the exponent of the power of 10 is positive, the number is greater than 1. When the exponent of the power of 10 is negative, the number is between 0 and 1.

Interactive Presentation



Warm Up



Launch the Lesson

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying by powers of 10 (Exercises 1–4)
- writing powers of 10 using exponents (Exercise 5)

Answers

1. 5,230

2. 140

3. 0.00284

4. 0.0933

5. 0.01 inch

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using scientific notation to express very large or very small numbers.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What might the term *notation* mean? What might the term *scientific* mean? Sample answer: Notation is a way to express numbers or concepts in writing. Scientific is an adjective that means relating to science or something that is systematic or methodical.
- What are some synonyms for the term *standard* used in everyday life ? Sample answer: normal, typical, established

Explore Scientific Notation

Objective

Students will explore how to write very large and very small numbers using scientific notation.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with several very large and very small numbers. Throughout this activity, students will determine the best way to write very large or very small numbers in a standardized way.

Inquiry Question

How can you write very large or very small numbers in a different way? Sample answer: Write the number as a product of a factor and an integer power of 10.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

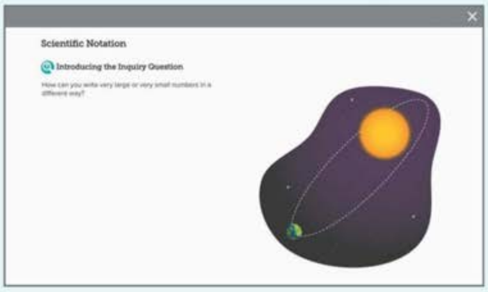
SLIDE 2

Mathematical Discourse

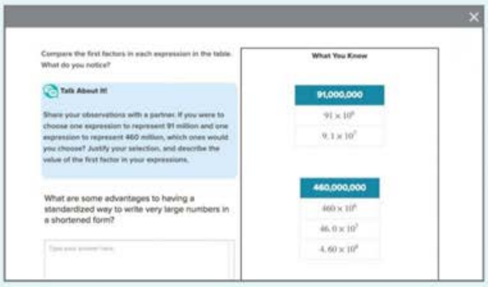
Consider the minimum distance, 91 million miles. How many different ways can you think of to write this number? How could you write this number as a product of 91 and another number? Sample answer: I could also write this number as 91,000,000. I could write this number as a product of 91 and 1,000,000.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 11

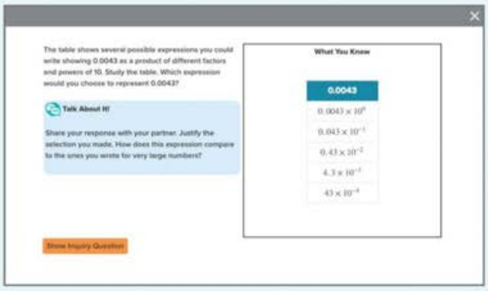


Explore, Slide 7 of 11

TYPE

On Slide 7, students respond to the question about the advantages of having a standardized way to write very large numbers in a shortened form.

Interactive Presentation



Explore, Slide 10 of 11

TYPE

On Slide 11, students respond to the Inquiry Question and can view a sample answer.

Explore Scientific Notation *(continued)*

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Enc ourage students to use the structure of powers and exponents to help simplify the process of writing very large or very small numbers.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 10 is shown.

Talk About It!

SLIDE 10

Mathematical Discourse

Share your response with your partner. Justify the selection you made. How does this expression compare to the ones you wrote for very large numbers? Students might say 43×10^{-4} or 4.3×10^{-3} since they are the "shortest" expressions.

Learn Scientific Notation

Objective

Students will understand how scientific notation can be used to write very large or very small numbers in a compact way.

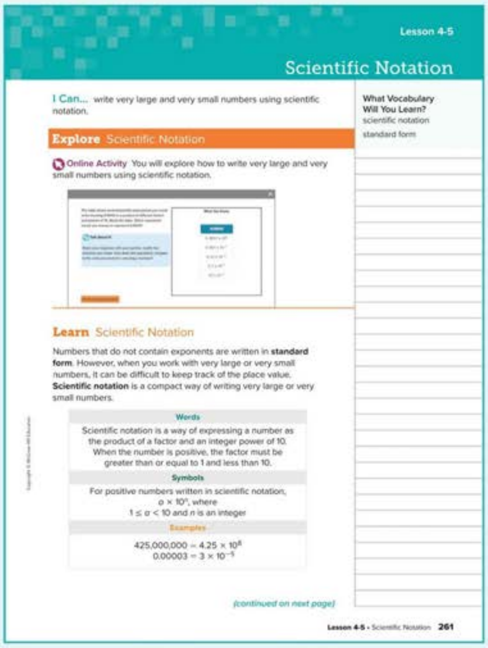
Teaching Notes

SLIDE 1

Have students study the *Words, Symbols*, and *Examples* Flashcards to learn about how scientific notation can be expressed in these multiple representations. You may wish to facilitate a class discussion on why the factor must be greater than or equal to 1 and less than 10. Have students discuss why this convention is important. They should be able to understand that if this convention was not established, it could be difficult to compare numbers such as 4.25×10^8 and 42.5×10^7. While both expressions represent the same number, only the first one is written in scientific notation.

SLIDE 2

Students will be asked to determine or not whether each number in the list is written in scientific notation. You may wish to have student volunteers come up to the board to select the response for each number. Ask students to clarify what the requirements are for a number to be written in scientific notation.



Lesson 4-5

Scientific Notation

I Can... write very large and very small numbers using scientific notation.

What Vocabulary Will You Learn?
scientific notation
standard form

Explore Scientific Notation

Online Activity You will explore how to write very large and very small numbers using scientific notation.

Learn Scientific Notation

Numbers that do not contain exponents are written in **standard form**. However, when you work with very large or very small numbers, it can be difficult to keep track of the place value. **Scientific notation** is a compact way of writing very large or very small numbers.

Words

Scientific notation is a way of expressing a number as the product of a factor and an integer power of 10. When the number is positive, the factor must be greater than or equal to 1 and less than 10.

Symbols

For positive numbers written in scientific notation, $a \times 10^n$, where $1 \le a < 10$ and n is an integer

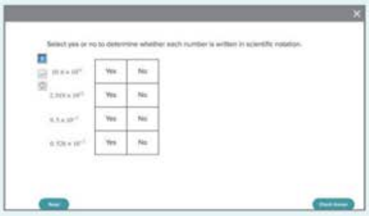
Examples

$425{,}000{,}000 = 4.25 \times 10^8$
$0.00003 = 3 \times 10^{-5}$

(continued on next page)

Lesson 4-5 • Scientific Notation 261

Interactive Presentation



Learn, Scientific Notation, Slide 2 of 2

FLASHCARDS

On Slide 1, students use Flashcards to learn about scientific notation.

CLICK

On Slide 2, students select yes or no to determine whether each number is written in scientific notation.

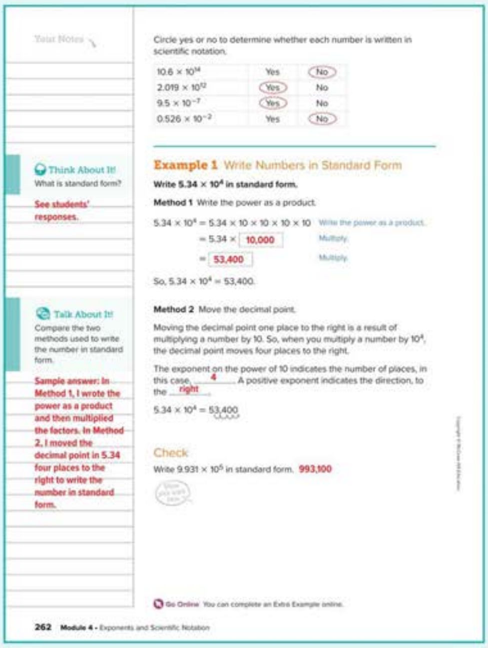
DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of scientific notation, have pairs of students use the Internet or another source to research real-world examples of how scientific notation is used in the fields of STEM (Science, Technology, Engineering, and Mathematics). Have them create a list of examples of very large and very small numbers they encounter, and have them explain why it is helpful to record these numbers using scientific notation. Have them present their examples and explanations to another pair of students, or to the entire class.

8.EE.A.3, 8.EE.A.4

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION


Your Notes

Circle yes or no to determine whether each number is written in scientific notation.

10.6×10^{14}	Yes	(No)
2.019×10^{12}	(Yes)	No
9.5×10^{-7}	(Yes)	No
0.526×10^{-2}	Yes	(No)

Think About It! What is standard form? See students' responses.

Example 1 Write Numbers in Standard Form

Write 5.34×10^4 in standard form.

Method 1 Write the power as a product.

$5.34 \times 10^4 = 5.34 \times 10 \times 10 \times 10 \times 10$ Write the power as a product.

$= 5.34 \times$ 10,000 Multiply.

$=$ 53,400 Multiply.

So, $5.34 \times 10^4 = 53{,}400$.

Method 2 Move the decimal point.

Moving the decimal point one place to the right is a result of multiplying a number by 10. So, when you multiply a number by 10^4, the decimal point moves four places to the right.

The exponent on the power of 10 indicates the number of places, in this case, 4. A positive exponent indicates the direction, to the right.

$5.34 \times 10^4 = 53{,}400$

Talk About It! Compare the two methods used to write the number in standard form. Sample answer: In Method 1, I wrote the power as a product and then multiplied the factors. In Method 2, I moved the decimal point in 5.34 four places to the right to write the number in standard form.

Check

Write 9.931×10^5 in standard form. 993,100

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 1, Write Numbers in Standard Form, Slide 2 of 5

TYPE

On Slide 2, students enter the number in standard form.

CLICK

On Slide 2, students move through the steps to write the number in standard form.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Write Numbers in Standard Form

Objective

Students will write large numbers that are written in scientific notation in standard form.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to consider the advantages of the two different methods of writing numbers in standard form from scientific notation, and to make a case for each one.

Questions for Mathematical Discourse

SLIDE 2

AL What does the exponent 4 signify? Sample answer: There are 4 factors of 10 in the product.

OL What is the value of 10^4? 10,000

OL Without calculating, explain whether 5.34×10^4 is less than or greater than 5.34. greater than; Sample answer: The exponent on the power of 10 is positive, so 5.34 is being multiplied by a large number (10,000).

BL Explain why it makes sense that there are two zeros in the answer 53,400. Sample answer: 5.34 is multiplied by four factors of 10. To multiply 5.34 by two factors of 10 would result in the number 534. Multiplying by an additional two factors of 10 will add two zeros at the end of 534.

SLIDE 3

AL What happens to the decimal point when you multiply 5.34 by 10? when you divide 5.34 by 10? Sample answer: The decimal point moves one place to the right when multiplying by 10 and one place to the left when dividing by 10.

OL How many places will the decimal point move when multiplying by 10^4? In which direction will it move? 4 places to the right

BL Without performing any calculations, how can you determine how many times greater is 5.34×10^8 than 5.34×10^4? Explain. Sample answer: You can count the number of factors of 10. 5.34×10^8 has 4 more factors of 10 than 5.34×10^4, so 5.34×10^8 is 10^4 times greater than 5.34×10^4.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Write Numbers in Standard Form

Objective

Students will write small numbers that are written in scientific notation in standard form.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enourage students to make sense of the two different approaches for writing a number, expressed in scientific notation, in standard form. Students should be able to understand the correspondences between the two methods.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to consider the advantages of the two different methods of writing numbers in standard form from scientific notation, and to make a case for each one.

6 Attend to Precision Encourage students to use precision when multiplying by negative powers of 10 and/or moving the decimal point the precise number of places in the correct direction.

Questions for Mathematical Discourse

SLIDE 2

AL How do you write 10^{-3} with a positive exponent? $\frac{1}{10^3}$

OL What does the exponent –3 signify on the power of 10? Sample answer: There are 3 factors of $\frac{1}{10}$ in the product.

BL Describe how you can use division, instead of multiplication, to write the number in standard form. Sample answer: Divide 3.27 by 10 three times.

SLIDE 3

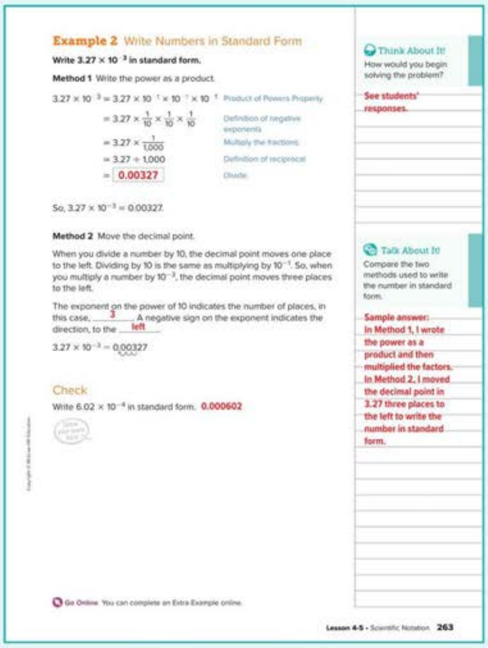
AL What happens to the decimal point when you multiply 3.27 by 10? When you divide 3.27 by 10? Sample answer: The decimal point moves one place to the right when multiplying by 10 and one place to the left when dividing by 10.

OL How many places will the decimal point move when multiplying by 10^{-3}? In which direction will it move? 3 places to the left

BL Without performing any calculations, how can you determine how many times greater 3.27×10^6 is than 3.27×10^{-3}? Explain. Sample answer: You can count the number of extra factors of 10. 3.27×10^6 has 9 more factors of 10 than 3.27×10^{-3}, so 3.27×10^6 is 10^9 times greater than 3.27×10^{-3}.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Write Numbers in Standard Form

Write 3.27×10^{-3} in standard form.

Method 1 Write the power as a product.

$3.27 \times 10^{-3} = 3.27 \times 10^{-1} \times 10^{-1} \times 10^{-1}$ Product of Powers Property

$= 3.27 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$ Definition of negative exponents

$= 3.27 \times \frac{1}{1{,}000}$ Multiply the fractions.

$= 3.27 \div 1{,}000$ Definition of reciprocal

$= 0.00327$ Divide.

So, $3.27 \times 10^{-3} = 0.00327$.

Method 2 Move the decimal point.

When you divide a number by 10, the decimal point moves one place to the left. Dividing by 10 is the same as multiplying by 10^{-1}. So, when you multiply a number by 10^{-3}, the decimal point moves three places to the left.

The exponent on the power of 10 indicates the number of places, in this case, 3. A negative sign on the exponent indicates the direction, to the left.

$3.27 \times 10^{-3} = 0.00327$

Check

Write 6.02×10^{-4} in standard form. 0.000602

Think About It! How would you begin solving the problem? See students' responses.

Talk About It! Compare the two methods used to write the number in standard form. Sample answer: In Method 1, I wrote the power as a product and then multiplied the factors. In Method 2, I moved the decimal point in 3.27 three places to the left to write the number in standard form.

Go Online You can complete an Extra Example online.

Lesson 4-5 • Scientific Notation 263

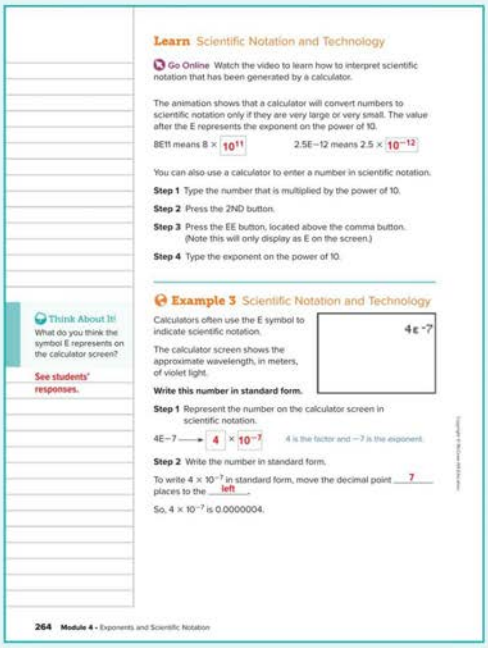
Interactive Presentation

Example 2, Write Numbers in Standard From, Slide 2 of 5

TYPE On Slide 2, students enter the number in standard form.

CLICK On Slide 2, students move through the steps to write the number in standard form.

CHECK Students complete the Check exercise online to determine if they are ready to move on.


Learn Scientific Notation and Technology

Go Online Watch the video to learn how to interpret scientific notation that has been generated by a calculator.

The animation shows that a calculator will convert numbers to scientific notation only if they are very large or very small. The value after the E represents the exponent on the power of 10.

8E11 means 8 × 10^{11} 2.5E−12 means 2.5 × 10^{-12}

You can also use a calculator to enter a number in scientific notation.

Step 1 Type the number that is multiplied by the power of 10.

Step 2 Press the 2ND button.

Step 3 Press the EE button, located above the comma button. (Note this will only display as E on the screen.)

Step 4 Type the exponent on the power of 10.

Example 3 Scientific Notation and Technology

Calculators often use the E symbol to indicate scientific notation.

The calculator screen shows the approximate wavelength, in meters, of violet light.

Write this number in standard form.

Step 1 Represent the number on the calculator screen in scientific notation.

4E−7 → 4 × 10^{-7} 4 is the factor and −7 is the exponent.

Step 2 Write the number in standard form.

To write 4 × 10^{-7} in standard form, move the decimal point 7 places to the left.

So, 4 × 10^{-7} is 0.0000004.

Think About It! What do you think the symbol E represents on the calculator screen?

See students' responses.

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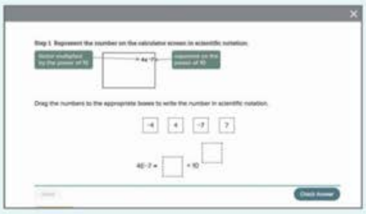
Interactive Presentation



Example 3, Scientific Notation and Technology, Slide 2 of 4

WATCH

On Slide 1 of the Learn, students watch a video to learn about scientific notation on a calculator.

DRAG & DROP

On Slide 2 of Example 3, students drag the numbers to the appropriate boxes to express the number in scientific notation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING   2 FLUENCY   3 APPLICATION

Learn Scientific Notation and Technology

Objective

Students will learn how to interpret scientific notation generated by a calculator.

Go Online

- Find additional teaching notes.
- Have students watch the video on Slide 1. The video illustrates how to interpret scientific notation that has been generated by a calculator.

Example 3 Scientific Notation and Technology

Objective

Students will write numbers written in scientific notation generated by technology in standard form.

Questions for Mathematical Discourse

SLIDE 2

AL Which number in 4E−7 represents the exponent on the power of 10? −7

OL What does 4E−7 mean in scientific notation? 4×10^{-7}

BL Is 4E−7 equivalent to −4E7? Explain your reasoning. no; Sample answer: The notation using *E* does not represent multiplication as if 4, E, and −7 are individual factors. The 4 represents a factor and the −7 represents the exponent on the power of 10.

SLIDE 3

AL What is standard form? Sample answer: A number in standard form shows no operations or exponents.

OL What does the exponent indicate about the number of places to move the decimal point? In which direction should it be moved? Sample answer: The exponent −7 indicates that the decimal point should be moved 7 places to the left.

BL A classmate writes 4×10^{7} in standard form as −40,000,000. What was the mistake? Sample answer: The classmate most likely thought that the exponent indicated whether or not the number was positive or negative and by how many factors of 10 to multiply.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Write Numbers in Scientific Notation

Objective

Students will learn how to write numbers in scientific notation.

Teaching Notes

SLIDE 1

Have students study the table to learn how to determine the sign of the exponent of a number in scientific notation, based on the number's standard form. Be sure they can explain why it makes sense that, if the power of 10 is negative, then the number is between 0 and 1. Some students may incorrectly assume that if the power of 10 is negative, then the number is negative. If the power of 10 is negative, then the factor is actually being divided by powers of 10 (multiplied by inverse powers of 10).

Talk About It!

SLIDE 2

Mathematical Discourse

If the number in standard form is greater than or equal to 1 and less than 10, what is the exponent on the power of 10 when the number is written in scientific notation? Explain. 0; Sample answer: Because the number is already greater than or equal to 1 and less than 10, you do not move the decimal point and the exponent on the power of 10 is zero. For example, $3.1 = 3.1 \times 10^0$.

DIFFERENTIATE

Reteaching Activity AL

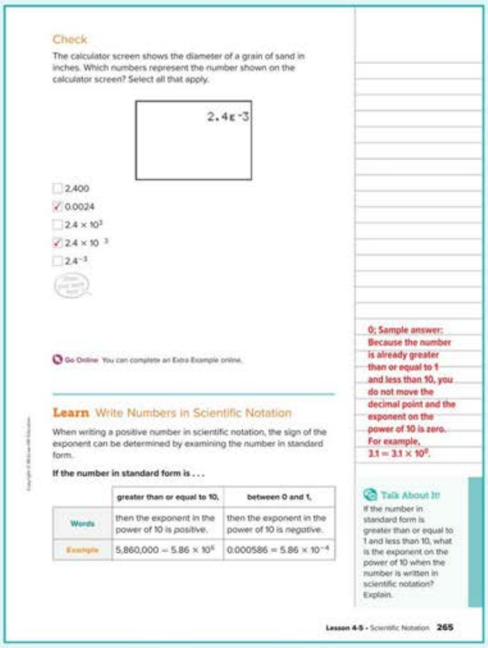
If any of your students are struggling to determine the sign of the exponent on the power of 10, have them work with a partner to create their own flashcards. Have them generate several numbers in standard form, one on the front of each flashcard. They should have a variety of very large numbers and very small numbers. On the back of each flashcard, have them determine whether the sign of the exponent on the power of 10 (when the number is written in scientific notation) is positive or negative. Then have them write the number in scientific notation on the back as a check. Encourage them to use their flashcards for study and practice. You may also wish to have them exchange their flashcards with a partner. Some example numbers are shown below.

1,223,500 positive

0.00034 negative

6,000,000,000 positive

0.0000000088 negative



Check

The calculator screen shows the diameter of a grain of sand in inches. Which numbers represent the number shown on the calculator screen? Select all that apply.

2.4E-3

- [ ] 2,400
- [x] 0.0024
- [ ] 2.4×10^3
- [x] 2.4×10^{-3}
- [ ] 2.4^{-3}

Go Online You can complete an Extra Example online.

Learn Write Numbers in Scientific Notation

When writing a positive number in scientific notation, the sign of the exponent can be determined by examining the number in standard form.

If the number in standard form is . . .

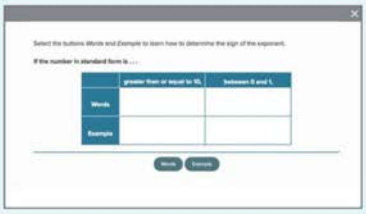
	greater than or equal to 10,	between 0 and 1,
Words	then the exponent in the power of 10 is *positive*.	then the exponent in the power of 10 is *negative*.
Example	$5,860,000 = 5.86 \times 10^6$	$0.000586 = 5.86 \times 10^{-4}$

0; Sample answer: Because the number is already greater than or equal to 1 and less than 10, you do not move the decimal point and the exponent on the power of 10 is zero. For example, $3.1 = 3.1 \times 10^0$.

Talk About It! If the number in standard form is greater than or equal to 1 and less than 10, what is the exponent on the power of 10 when the number is written in scientific notation? Explain.

Lesson 4-5 • Scientific Notation 265

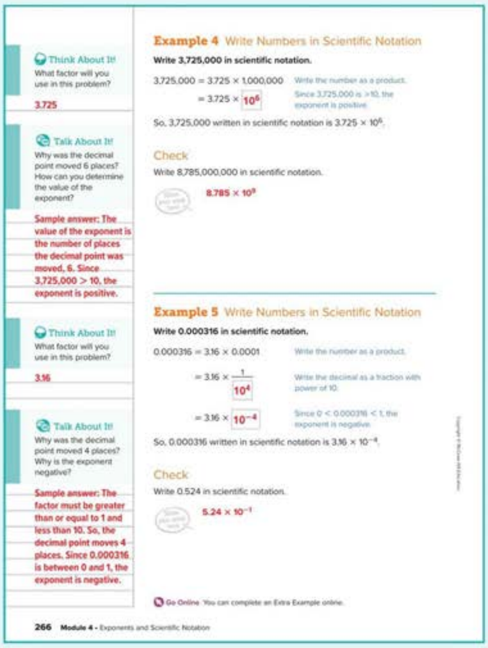
Interactive Presentation



Learn, Write Numbers in Scientific Notation, Slide 1 of 2

CLICK

On Slide 1, students select to view an example of how to determine the sign of the exponent.



Think About It! What factor will you use in this problem? 3.725

Talk About It! Why was the decimal point moved 6 places? How can you determine the value of the exponent? Sample answer: The value of the exponent is the number of places the decimal point was moved, 6. Since 3,725,000 > 10, the exponent is positive.

Example 4 Write Numbers in Scientific Notation

Write 3,725,000 in scientific notation.

$3{,}725{,}000 = 3.725 \times 1{,}000{,}000$ Write the number as a product.

$= 3.725 \times 10^6$ Since 3,725,000 is >10, the exponent is positive.

So, 3,725,000 written in scientific notation is 3.725×10^6.

Check

Write 8,785,000,000 in scientific notation. 8.785×10^9

Think About It! What factor will you use in this problem? 3.16

Talk About It! Why was the decimal point moved 4 places? Why is the exponent negative? Sample answer: The factor must be greater than or equal to 1 and less than 10. So, the decimal point moves 4 places. Since 0.000316 is between 0 and 1, the exponent is negative.

Example 5 Write Numbers in Scientific Notation

Write 0.000316 in scientific notation.

$0.000316 = 3.16 \times 0.0001$ Write the number as a product.

$= 3.16 \times \frac{1}{10^4}$ Write the decimal as a fraction with power of 10.

$= 3.16 \times 10^{-4}$ Since 0 < 0.000316 < 1, the exponent is negative.

So, 0.000316 written in scientific notation is 3.16×10^{-4}.

Check

Write 0.524 in scientific notation. 5.24×10^{-1}

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4, Write Numbers in Scientific Notation, Slide 2 of 4

TYPE On Slide 2 of Example 4 and Slide 2 of Example 5, students enter the number in scientific notation.

CLICK On Slide 2 of Example 4 and Slide 2 of Example 5, students move through the steps to write the number in scientific notation.

CHECK Students complete the Check exercises online to determine if they are ready to move on.

Example 4 Write Numbers in Scientific Notation

Objective

Students will write large numbers that are written in standard form in scientific notation.

Questions for Mathematical Discourse

SLIDE 2

AL Where should you place the decimal point in the first factor? Why? between the digits 3 and 7, so that the first factor is between 1 and 10

AL How do you know that the exponent on the power of 10 will be positive? The number 3,725,000 is greater than 10.

OL Explain how to determine the exponent on the power of 10. Sample answer: The decimal point was moved 6 times, so the exponent is 6.

OL A classmate wrote the number as 37.25×10^5. Describe the error that was made. Sample answer: The first factor must be a number between 1 and 10. It should have been 3.725, not 37.25. This will increase the power of 10 by 1.

BL How can you write $1{,}000 \times 3{,}752{,}000$ in scientific notation? Explain. Sample answer: 1,000 is equal to 10^3. Since 3,725,000 is written as 3.725×10^6 in scientific notation, $1{,}000 \times 3{,}752{,}000$ is written as 3.725×10^9.

Example 5 Write Numbers in Scientific Notation

Objective

Students will write small numbers that are written in standard form in scientific notation.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to use clear and precise mathematical language to explain the relationship between moving the decimal point a certain number of places and the value of the exponent on the power of 10. Students should be able to clearly articulate why the exponent on the power of 10 is negative in this example.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Use Scientific Notation

Objective

Students will learn about choosing units of appropriate size and estimating with scientific notation.

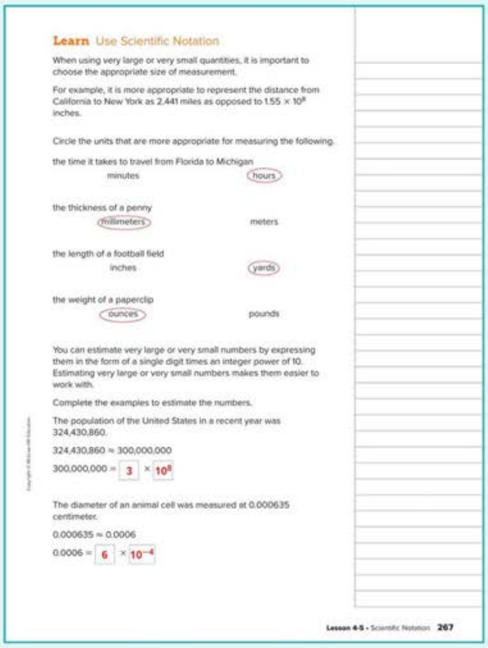
Teaching Notes

SLIDE 1

Students will learn how to determine appropriate units of measurement for certain contexts. You may wish to have student volunteers come to the board to select the correct units for each context.

SLIDE 2

Have students select the cards to see how very small and very large numbers can be estimated using powers of 10.



Learn Use Scientific Notation

When using very large or very small quantities, it is important to choose the appropriate size of measurement.

For example, it is more appropriate to represent the distance from California to New York as 2,441 miles as opposed to 1.55×10^8 inches.

Circle the units that are more appropriate for measuring the following.

the time it takes to travel from Florida to Michigan
minutes — (hours)

the thickness of a penny
(millimeters) — meters

the length of a football field
inches — (yards)

the weight of a paperclip
(ounces) — pounds

You can estimate very large or very small numbers by expressing them in the form of a single digit times an integer power of 10. Estimating very large or very small numbers makes them easier to work with.

Complete the examples to estimate the numbers.

The population of the United States in a recent year was 324,430,860.

$324{,}430{,}860 \approx 300{,}000{,}000$

$300{,}000{,}000 = \boxed{3} \times \boxed{10^8}$

The diameter of an animal cell was measured at 0.000635 centimeter.

$0.000635 \approx 0.0006$

$0.0006 = \boxed{6} \times \boxed{10^{-4}}$

Lesson 4-5 • Scientific Notation 267

Interactive Presentation

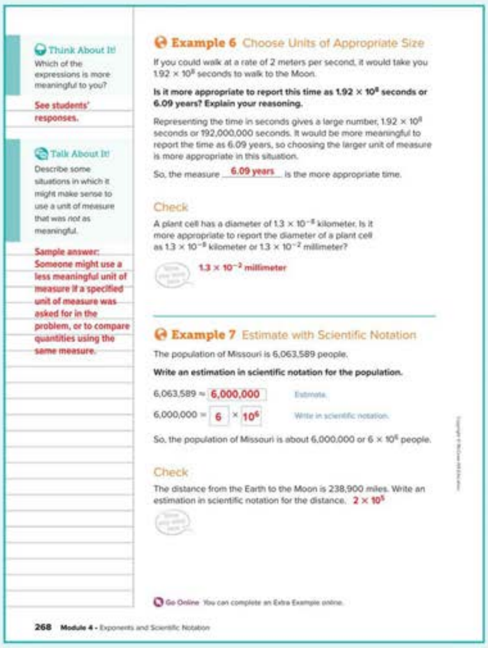
Learn, Use Scientific Notation, Slide 1 of 2

CLICK

On Slide 1, students select the appropriate unit of measurement for the given scenarios.

FLASHCARDS

On Slide 2, students use the Flashcards to see an estimation of a very large number and a very small number.



Think About It! Which of the expressions is more meaningful to you?

See students' responses.

Talk About It! Describe some situations in which it might make sense to use a unit of measure that was not as meaningful.

Sample answer: Someone might use a less meaningful unit of measure if a specified unit of measure was asked for in the problem, or to compare quantities using the same measure.

Example 6 Choose Units of Appropriate Size

If you could walk at a rate of 2 meters per second, it would take you 1.92×10^8 seconds to walk to the Moon.

Is it more appropriate to report this time as 1.92×10^8 seconds or 6.09 years? Explain your reasoning.

Representing the time in seconds gives a large number, 1.92×10^8 seconds or 192,000,000 seconds. It would be more meaningful to report the time as 6.09 years, so choosing the larger unit of measure is more appropriate in this situation.

So, the measure 6.09 years is the more appropriate time.

Check

A plant cell has a diameter of 1.3×10^{-8} kilometer. Is it more appropriate to report the diameter of a plant cell as 1.3×10^{-8} kilometer or 1.3×10^{-2} millimeter?

1.3×10^{-2} millimeter

Example 7 Estimate with Scientific Notation

The population of Missouri is 6,063,589 people.

Write an estimation in scientific notation for the population.

6,063,589 ≈ 6,000,000 Estimate.

6,000,000 = 6 × 10^6 Write in scientific notation.

So, the population of Missouri is about 6,000,000 or 6×10^6 people.

Check

The distance from the Earth to the Moon is 238,900 miles. Write an estimation in scientific notation for the distance. 2×10^5

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 6, Choose Units of Appropriate Size, Slide 2 of 4

TYPE

On Slide 1 of Example 7, students enter an estimation, in scientific notation, for a population.

TYPE

On Slide 2 of Example 6, students enter the appropriate measurement.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 6 Choose Units of Appropriate Size

Objective

Students will choose the appropriate units for measurements of large and small quantities.

Questions for Mathematical Discourse

SLIDE 2

AL Is the number of seconds it takes to walk to the Moon a large quantity or a small quantity? large

OL Which number makes the most sense to communicate the amount of travel time, 1.92×10^8 seconds or 6.09 years? Explain. Sample answer: 6.09 years is easier to read, write, understand, and communicate the amount of time it takes to walk to the Moon.

BL What unit of time measurement might be most appropriate to describe the time it takes for light to travel to or from the Moon? Explain. Sample answer: Light travels very quickly, so it would likely take seconds or minutes to travel to or from the Moon.

Example 7 Estimate with Scientific Notation

Objective

Students will estimate very large or very small quantities using scientific notation.

Questions for Mathematical Discourse

SLIDE 1

AL How might you round the population of Missouri, 6,063,589, to a number that is easy to communicate? Sample answer: 6,000,000 or six million

OL How can you write one million people using a power of 10? $10^6 = 1,000,000$

OL How can you write six million people in scientific notation? 6×10^6

BL How might you estimate the population using scientific notation if it were 6,572,912 people? about 6.6×10^6

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Travel

Objective

Students will come up with their own strategy to solve an application problem that involves comparing the number of international visitors.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does percent of change mean?
- How are the number of visitors represented in the table?
- Do you think the percent of change for any country is negative? Why or why not?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Travel

The table shows the approximate number of international visitors to the United States, from several countries, for July and August of a recent year. Which country had the greatest percent of change in the approximate number of visitors from July to August?

Country	Visitors (Jul.)	Visitors (Aug.)
Canada	2.09×10^6	2.43×10^6
Mexico	1.55×10^6	1.63×10^6
United Kingdom	4.2×10^5	4.9×10^5
Japan	3.05×10^5	3.9×10^5

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Japan; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! What method did you use to solve the problem? Explain why you chose that method.

See students' responses.

Lesson 4-5 • Scientific Notation 269

Interactive Presentation

Apply, Travel

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The approximate attendance and average ticket price for four Major League baseball teams for a recent year is shown in the table.

Team	Attendance	Ticket Price ($)
Los Angeles Angels	3,020,000	32.70
Miami Marlins	1.71×10^6	28.31
Pittsburgh Pirates	2,450,000	29.96
St. Louis Cardinals	3.44×10^6	34.20

Which team had the most revenue from ticket sales?

St. Louis Cardinals

Go Online You can complete an Extra Example online.

Pause and Reflect

What have you learned about scientific notation in this lesson? Include a real-world example of scientific notation in your summary.

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

Why are exponents useful when working with very large or very small numbers?

In this lesson, students learned how to write very large or very small numbers using scientific notation. Encourage them to brainstorm two real-world situations in which someone would need to use scientific notation to describe a very large number and a very small number. For example, an astronomer might use scientific notation when discussing the speed of light, or a scientist might use scientific notation when describing a molecule.

Exit Ticket

Refer to the Exit Ticket slide. A powerful microscope is used to measure the diameter of a specific red blood cell. The diameter is 0.000006834 meter. Express this measurement in scientific notation.

6.834×10^{-6} meter

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7–13 odd, 14–16
- ALEKS Scientific Notation

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 11, 13, 14
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–7
- ALEKS Exponents and Order of Operations, Negative Exponents

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Exponents and Order of Operations, Negative Exponents

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

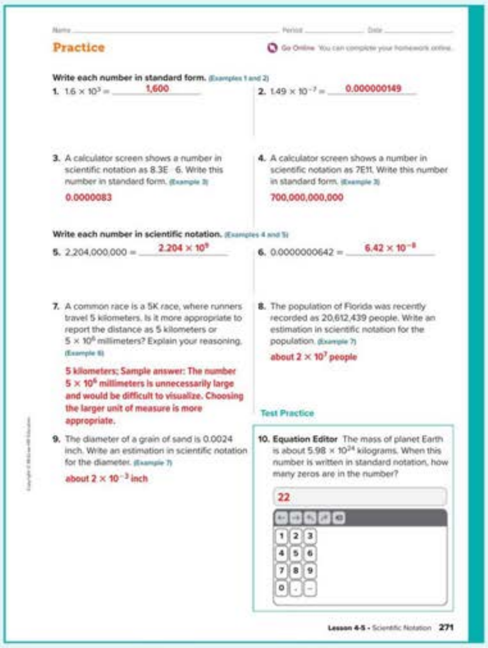
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	write large and small numbers written in scientific notation in standard form	1, 2
1	write numbers written in scientific notation generated by technology in standard form	3, 4
1	write large and small numbers written in standard form in scientific notation	5, 6
2	choose the appropriate units for measurements of large and small quantities	7
2	estimate quantities using scientific notation	8, 9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving scientific notation	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may incorrectly write a number in scientific notation in standard form by moving the decimal point in the wrong direction. Students may think that when a number written in scientific notation has a negative exponent, the decimal point is moved to the right. In Exercise 2, students may write their answer as 14,900,000. Encourage them to reason about place value as they move the decimal point in either direction. Remind students that a negative sign on the exponent indicates moving the decimal to the left because a negative power of 10 as a factor indicates the product of 1.49 and 10^{-7} will be less than 1.49.



Name ______ Period ______ Date ______

Practice Go Online You can complete your homework online.

Write each number in standard form. (Examples 1 and 2)

1. 1.6×10^3 = **1,600**
2. 1.49×10^{-7} = **0.000000149**
3. A calculator screen shows a number in scientific notation as 8.3E −6. Write this number in standard form. (Example 3) **0.0000083**
4. A calculator screen shows a number in scientific notation as 7E11. Write this number in standard form. (Example 3) **700,000,000,000**

Write each number in scientific notation. (Examples 4 and 5)

5. 2,204,000,000 = **2.204×10^9**
6. 0.0000000642 = **6.42×10^{-8}**
7. A common race is a 5K race, where runners travel 5 kilometers. Is it more appropriate to report the distance as 5 kilometers or 5×10^6 millimeters? Explain your reasoning. (Example 6) **5 kilometers; Sample answer: The number 5×10^6 millimeters is unnecessarily large and would be difficult to visualize. Choosing the larger unit of measure is more appropriate.**
8. The population of Florida was recently recorded as 20,612,439 people. Write an estimation in scientific notation for the population. (Example 7) **about 2×10^7 people**
9. The diameter of a grain of sand is 0.0024 inch. Write an estimation in scientific notation for the diameter. (Example 7) **about 2×10^{-3} inch**

Test Practice

10. **Equation Editor** The mass of planet Earth is about 5.98×10^{24} kilograms. When this number is written in standard notation, how many zeros are in the number? **22**

Lesson 4-5 • Scientific Notation 271

3 REFLECT AND PRACTICE

8.EE.A.3, 8.EE.A.4

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Apply *indicates multi-step problem

*11. The table shows the approximate number of insects in each group. What is the combined population of the four insect groups?

Insect	Population
Honeybees	7.3498×10^4
Ants	6.822×10^3
Termites	9.8×10^5
Aphids	2.9502×10^4

1,089,822 insects

*12. The table shows the approximate population and land area for several states. Which state has the least population density?

State	Population	Land Area (mi^2)
Arizona	6.83×10^6	113,594
Colorado	5.46×10^6	103,642
Kentucky	4.43×10^6	39,486
Minnesota	5.45×10^6	79,627

Colorado

Higher-Order Thinking Problems

13. Simplify the following expression and write in scientific notation.

$$\frac{(0.00045)(500{,}000)}{0.0015}$$

1.5×10^5

14. **Be Precise** The amount of time spent doing homework is 2.7×10^3 seconds. Choose a more appropriate measurement to report the amount of time spent doing homework. Justify your answer.

Sample answer: The value 2,700 is too large in relation to doing homework. Because 2.7×10^3 seconds is equal to 45 minutes, expressing the amount of time in minutes is more appropriate.

15. **Find the Error** Katrina states that 3.5×10^4 is greater than 2.1×10^6 because $3.5 > 2.1$. Explain her mistake and correct it.

Sample answer: Katrina did not take into account place value and the powers of ten. The number 3.5×10^4, in standard form, is 35,000. The number 2.1×10^6 is 2,100,000. Therefore, 2.1×10^6 is greater.

16. Give a number in scientific notation that is between the two numbers on a number line.

7.1×10^3 and 71,000,000

Sample answer: 7.1×10^5

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Teaching the Mathematical Practices

6 Attend to Precision In E xercise 14, students will attend to precision when converting the scientific notation to standard form and determining the appropriate unit of measurement to use. Students should recognize that 2,700 seconds is not a common way that we express time. Encourage students to convert the time to a measurement that is more appropriate.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 15, students will explain the student's mistake and correct it. Encourage students to determine the error and explain how they could fix it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercises 11–12 Have students work in groups of 3–4. After completing Exercise 11, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 12.

Listen and ask clarifying questions.

Use with Exercises 15–16 Have students work in pairs. Have students individually read Exercise 15 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 16.

Lesson 4-6

Compute with Scientific Notation

8.EE.A.3, 8.EE.A.4

LESSON GOAL

Students will compute with numbers written in scientific notation.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Multiply and Divide with Scientific Notation
Example 1: Multiply with Scientific Notation
Example 2: Divide with Scientific Notation
Learn: Add and Subtract with Scientific Notation
Example 3: Add or Subtract with Scientific Notation
Apply: Population

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Raise a Number in Scientific Notation to a Power		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 28 of the *Language Development Handbook* to help your students build mathematical language related to computation with scientific notation.

ELL You can use the tips and suggestions on page T28 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.A** by writing and evaluating expressions involving powers and exponents.
Standards for Mathematical Content: **8.E E.A.3, 8.EE.A.4,** *Also addresses 8.EE.A.1*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students used scientific notation to write large and small numbers.
8.EE.A.3, 8.EE.A.4

Now
Students compute with numbers written in scientific notation.
8.EE.A.3, 8.EE.A.4

Next
Students will learn about the real number system by studying rational and irrational numbers.
8.NS.A.1, 8.NS.A.2, 8.EE.A.2

Rigor

The Three Pillars of Rigor

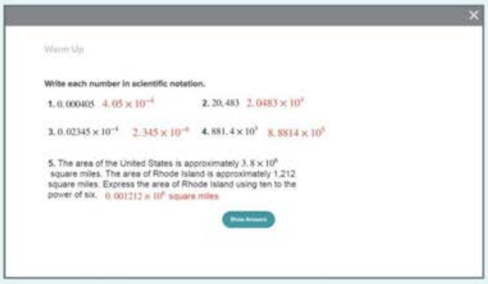
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to expand their *understanding* of scientific notation by adding, subtracting, multiplying, and dividing numbers written in scientific notation and standard form. They come to understand that in order to perform the operations, they need to draw on their knowledge of place value and exponents. They build *fluency* with performing the operations, and *apply* that fluency to solve real-world problems.

Mathematical Background

When working with all operations in *scientific notation*, it is often helpful to write all of the numbers in the same form, scientific notation or standard form, before computing.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- expressing numbers in scientific notation (Exercises 1–4)
- expressing numbers with any power of 10 (Exercise 5)

Answers

1. 4.05×10^{-4}
2. 2.0483×10^{4}
3. 2.345×10^{-6}
4. 8.8814×10^{5}
5. 0.001212×10^{6} square miles

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the differences in volume between Lake Superior and the Pacific Ocean.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What is an example of a number written in scientific notation?
  Sample answer: 3.512×10^{-8}

Learn Multiply and Divide with Scientific Notation

Objective

Students will learn how to multiply and divide with numbers written in scientific notation.

Go Online to find additional teaching notes.

Example 1 Multiply with Scientific Notation

Objective

Students will multiply numbers written in scientific notation.

Questions for Mathematical Discourse

SLIDE 2

AL How can you write 1 billion in standard form? 1,000,000,000

OL What number should the exponent be for 1 billion to be written as a power of 10? 9

BL Why is it useful to write the number of acres in scientific notation? Sample answer: If both numbers are in scientific notation, the Product of Powers Property can be used to find the product.

SLIDE 3

AL What properties allow you to multiply the factors separately from the powers? Commutative and Associative Properties

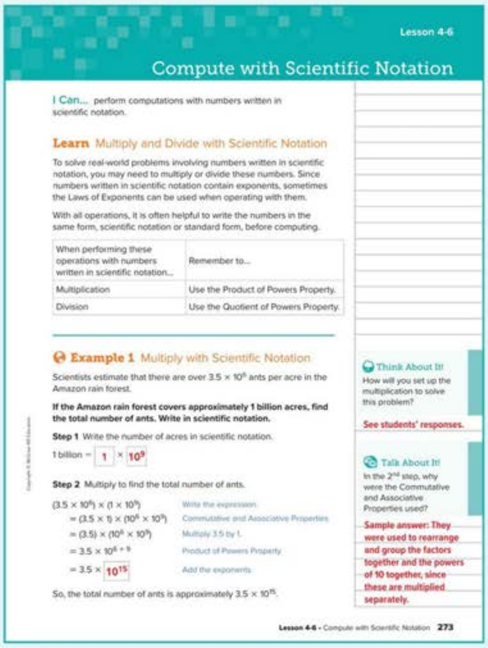
OL What is the result of multiplying the factors? Of multiplying the powers? $3.5 \times 1 = 3.5$, $10^6 \times 10^9 = 10^{15}$

OL What property can you use to multiply the powers? What does it allow you to do? Product of Powers Property; It allows me to add the exponents since the bases are the same.

BL A classmate states that the total number of ants in the rainforest is 4.5×10^{15}. What mistake was likely made? Sample answer: The classmate likely added both of the exponents and the factors rather than multiplying the factors and adding the exponents.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-6

Compute with Scientific Notation

I Can... perform computations with numbers written in scientific notation.

Learn Multiply and Divide with Scientific Notation

To solve real-world problems involving numbers written in scientific notation, you may need to multiply or divide these numbers. Since numbers written in scientific notation contain exponents, sometimes the Laws of Exponents can be used when operating with them.

With all operations, it is often helpful to write the numbers in the same form, scientific notation or standard form, before computing.

When performing these operations with numbers written in scientific notation...	Remember to...
Multiplication	Use the Product of Powers Property.
Division	Use the Quotient of Powers Property.

Example 1 Multiply with Scientific Notation

Scientists estimate that there are over 3.5×10^6 ants per acre in the Amazon rain forest.

If the Amazon rain forest covers approximately 1 billion acres, find the total number of ants. Write in scientific notation.

Step 1 Write the number of acres in scientific notation.

1 billion = 1×10^9

Step 2 Multiply to find the total number of ants.

$(3.5 \times 10^6) \times (1 \times 10^9)$ Write the expression.

$= (3.5 \times 1) \times (10^6 \times 10^9)$ Commutative and Associative Properties

$= (3.5) \times (10^6 \times 10^9)$ Multiply 3.5 by 1.

$= 3.5 \times 10^{6+9}$ Product of Powers Property

$= 3.5 \times 10^{15}$ Add the exponents.

So, the total number of ants is approximately 3.5×10^{15}.

Think About It! How will you set up the multiplication to solve this problem? See students' responses.

Talk About It! In the 2nd step, why were the Commutative and Associative Properties used? Sample answer: They were used to rearrange and group the factors together and the powers of 10 together, since these are multiplied separately.

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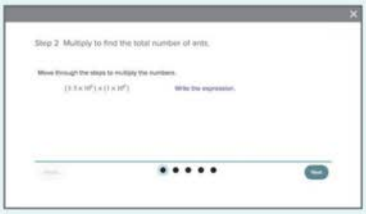
Interactive Presentation

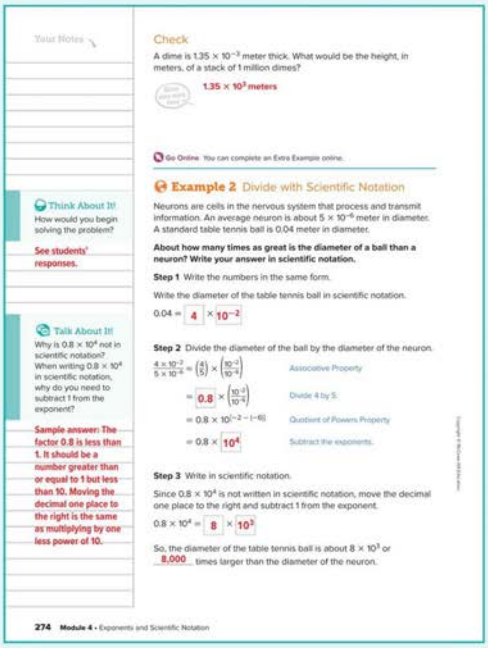


Example 1, Multiply with Scientific Notation, Slide 3 of 5

TYPE On Slide 2 of Example 1, students enter the number of acres in scientific notation.

CLICK On Slide 3 of Example 1, students move through the steps to multiply the numbers.

CHECK Students complete the Check exercise online to determine if they are ready to move on.


Your Notes

Check

A dime is 1.35×10^{-3} meter thick. What would be the height, in meters, of a stack of 1 million dimes?

1.35×10^3 meters

Go Online You can complete an Extra Example online.

Think About It! How would you begin solving the problem?

See students' responses.

Example 2 Divide with Scientific Notation

Neurons are cells in the nervous system that process and transmit information. An average neuron is about 5×10^{-6} meter in diameter. A standard table tennis ball is 0.04 meter in diameter.

About how many times as great is the diameter of a ball than a neuron? Write your answer in scientific notation.

Step 1 Write the numbers in the same form.

Write the diameter of the table tennis ball in scientific notation.

$0.04 = 4 \times 10^{-2}$

Step 2 Divide the diameter of the ball by the diameter of the neuron.

$\frac{4 \times 10^{-2}}{5 \times 10^{-6}} = \left(\frac{4}{5}\right) \times \left(\frac{10^{-2}}{10^{-6}}\right)$ Associative Property

$= 0.8 \times \left(\frac{10^{-2}}{10^{-6}}\right)$ Divide 4 by 5.

$= 0.8 \times 10^{(-2-(-6))}$ Quotient of Powers Property

$= 0.8 \times 10^4$ Subtract the exponents.

Talk About It! Why is 0.8×10^4 not in scientific notation? When writing 0.8×10^4 in scientific notation, why do you need to subtract 1 from the exponent?

Sample answer: The factor 0.8 is less than 1. It should be a number greater than or equal to 1 but less than 10. Moving the decimal one place to the right is the same as multiplying by one less power of 10.

Step 3 Write in scientific notation.

Since 0.8×10^4 is not written in scientific notation, move the decimal one place to the right and subtract 1 from the exponent.

$0.8 \times 10^4 = 8 \times 10^3$

So, the diameter of the table tennis ball is about 8×10^3 or 8,000 times larger than the diameter of the neuron.

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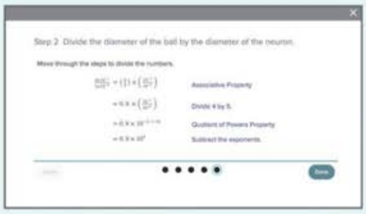
Interactive Presentation



Example 2, Divide with Scientific Notation, Slide 3 of 6

TYPE

On Slide 2, students enter the diameter of the ball in scientific notation.

CLICK

On Slide 3, students move through the steps to divide with scientific notation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Divide with Scientific Notation

Objective

Students will divide numbers written in scientific notation.

Questions for Mathematical Discourse

SLIDE 2

AL What operation should you use to find how many times greater the diameter of the ball is than the diameter of a neuron? division

OL How can you write the diameter of the table tennis ball in scientific notation? 4×10^{-2} meter

BL One meter is equal to 100 centimeters. How could you write each diameter using centimeters? Explain. Sample answer: Multiply each diameter in meters by 100 to find the diameter in centimeters. The diameter of the neuron is $(5 \times 10^{-6}) \times 10^2 = 5 \times 10^{-4}$ centimeter, and the diameter of the table tennis ball is $0.04 \times 100 =$ 4 centimeters.

SLIDE 3

AL Which quantity should be divided by the other? Divide the table tennis ball diameter by the diameter of the neuron.

OL What is the result of dividing the two factors? Of dividing the two powers? $\frac{4}{5} = 0.8$ and $\frac{10^{-2}}{10^{-6}} = 10^{-2-(-6)} = 10^4$

BL A classmate states that the diameter of the table tennis ball is 1.25×10^{-4} times greater than the diameter of the neuron. What mistake was likely made? Sample answer: The classmate likely divided the diameter of the neuron by the diameter of the ball instead of the other way around.

SLIDE 4

AL What values can the factor of a number written in scientific notation have? Sample answer: The factor must be between 1 and 10.

OL Moving the decimal point one place to the right is equivalent to multiplying by what number? 10

BL A classmate writes 0.8×10^4 in scientific notation as 8×10^5. What mistake was likely made? Explain. Sample answer: The classmate multiplied 0.8 by 10 to get 8, but instead of reducing the exponent of 10 by 1 to counteract the change, they added 1 to the exponent.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Add and Subtract with Scientific Notation

Objective

Students will learn how to add and subtract with numbers written in scientific notation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to think about the structure of scientific notation and how it relates to place value when adding or subtracting numbers.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to clearly articulate that the Product of Powers Property can only be applied when multiplying expressions.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Explain why you cannot add the exponents on the powers of 10, when adding the expressions. Sample answer: I cannot add the exponents since I am adding the expressions. The Product of Powers Property can only be used when multiplying expressions.

DIFFERENTIATE

Reteaching Activity AL

If your students are having a hard time remembering how to write numbers, written in scientific notation, with the same powers of 10, encourage them to make a flow chart like the following to use when determining how to move the decimal point and whether to increase or decrease the exponent.

Do you need to increase the exponent? If so, move the decimal point to the left one space and increase the exponent by one.

Do you need to decrease the exponent? If so, move the decimal point to the right one space and decrease the exponent by one.

Have them practice by writing the following pairs of numbers in scientific notation with the same power of 10.

1.04×10^7, 8.62×10^5 Sample answer: 104×10^5, 8.62×10^5

5.234×10^2, 5.234×10^5 Sample answer: 5.234×10^2, 5234×10^2

Check

In a recent year, the population of China was about 1.3×10^9. According to the census data, the population of the United States was about 308,745,538. About how many times greater was the population of China than the population of the United States?

4 times greater

Go Online You can complete an Extra Example online.

Learn Add and Subtract with Scientific Notation

To add or subtract numbers written in scientific notation, it is helpful to rewrite the numbers so that the exponents on each power of 10 have the same value.

To add 3.6×10^4 to the numbers shown, each number needs to be rewritten so the exponents on each power of 10 are the same. Rewrite each number so that it has an exponent of 4.

$1.43 \times 10^3 = 0.143 \times 10^4$

$1.43 \times 10^5 = 14.3 \times 10^4$

$1.43 \times 10^2 = 0.0143 \times 10^4$

Once numbers written in scientific notation have the same exponent on the power of 10, you can use the Distributive Property to add or subtract.

$(4.36 \times 10^5) + (2.09 \times 10^6)$
$= (4.36 \times 10^5) + (20.9 \times 10^5)$ Rewrite with the same power of 10.
$= (4.36 + 20.9) \times 10^5$ Distributive Property
$= (25.26) \times 10^5$ Add 4.36 and 20.9.
$= 2.526 \times 10^6$ Rewrite in scientific notation.

You can also write each number in standard form before computing or to check your work.

Talk About It! Explain why you cannot add the exponents on the powers of 10, when adding the expressions.

Sample answer: I cannot add the exponents since I am adding the expressions. The Product of Powers Property can only be used when multiplying expressions.

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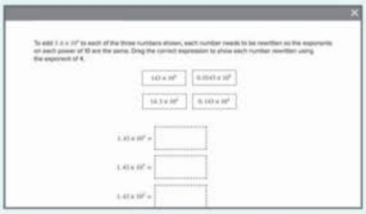
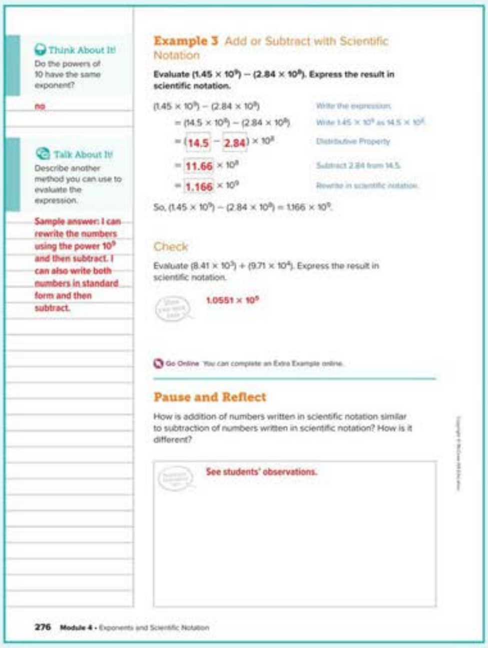
Interactive Presentation



Learn, Add and Subtract with Scientific Notation, Slide 1 of 3

DRAG & DROP

On Slide 1, students drag the correct expression to show each number rewritten using the correct exponent.



Think About It!
Do the powers of 10 have the same exponent?
no

Talk About It!
Describe another method you can use to evaluate the expression.
Sample answer: I can rewrite the numbers using the power 10^9 and then subtract. I can also write both numbers in standard form and then subtract.

Example 3 Add or Subtract with Scientific Notation

Evaluate $(1.45 \times 10^9) - (2.84 \times 10^8)$. Express the result in scientific notation.

$(1.45 \times 10^9) - (2.84 \times 10^8)$ — Write the expression.
$= (14.5 \times 10^8) - (2.84 \times 10^8)$ — Write 1.45×10^9 as 14.5×10^8.
$= (14.5 - 2.84) \times 10^8$ — Distributive Property
$= 11.66 \times 10^8$ — Subtract 2.84 from 14.5.
$= 1.166 \times 10^9$ — Rewrite in scientific notation.

So, $(1.45 \times 10^9) - (2.84 \times 10^8) = 1.166 \times 10^9$.

Check
Evaluate $(8.41 \times 10^3) + (9.71 \times 10^4)$. Express the result in scientific notation.
1.0551×10^5

Go Online You can complete an Extra Example online.

Pause and Reflect
How is addition of numbers written in scientific notation similar to subtraction of numbers written in scientific notation? How is it different?
See students' observations.

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Interactive Presentation

Example 3, Add or Subtract with Scientific Notation, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to subtract expressions written in scientific notation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Add or Subtract with Scientific Notation

Objective

Students will add and subtract numbers written in scientific notation.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to think about the different ways that a number can be written using powers of 10 or standard form.

6 Attend to Precision Students should be able to use precision in rewriting one of the numbers so that both expressions have the same power of 10, in order to be able to subtract the expressions.

7 Look for and Make Use of Structure Encourage students to analyze the structure of each expression, noting that the exponents on the powers of 10 are different.

Questions for Mathematical Discourse

SLIDE 2

AL What must be true about the powers of 10 before you can add or subtract two numbers in scientific notation? Sample answer: The powers of 10 must have equivalent exponents.

OL How can you rewrite one of the numbers so that they have the same power of 10? Explain. Sample answer: The number 1.45×10^9 can be rewritten as 14.5×10^8, so that each number has the same power of 10.

BL How could you write 1.45×10^9 and 258 so that they have the same power of 10? Sample answer: You can write each number using 10^2. $1.45 \times 10^9 = 14{,}500{,}000 \times 10^2$ and $258 = 2.58 \times 10^2$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Population

Objective

Students will come up with their own strategy to solve an application problem that involves comparing populations.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How are the populations represented?
- What is a reasonable estimate for the world population?
- How can you use the Laws of Exponents to help solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Population

In a recent year, the world population was about 7,289,332,000. The population of the United States was about 3×10^8. About how many times larger was the world population than the population of the United States?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

about 23 times larger; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Why might it be helpful to round the world population?

Sample answer: Rounding the world population to the nearest billion allows the value to be written in scientific notation using a whole number.

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Interactive Presentation

Apply, Population

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

In 2005, 8.1×10^{10} text messages were sent in the United States. In 2010, the number of annual text messages had risen to 1,810,000,000,000. About how many times as great was the number of text messages in 2010 than 2005?

$\frac{2 \times 10^{12}}{8 \times 10^{10}} \approx 25$ times

Go Online You can complete an Extra Example online.

Pause and Reflect

Have you ever wondered when you might use the concepts you learn in math class? What are some everyday scenarios in which you might use what you learned today?

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

Why are exponents useful when working with very large or very small numbers?

In this lesson, students learned how to perform the four operations with numbers written in scientific notation. Encourage them to work with a partner to compare and contrast evaluating an expression like 0.000025 • 3,560,000 by first multiplying the two numbers in standard form, and then multiplying them using scientific notation. Have them state which method they prefer and explain why they chose that method.

Exit Ticket

Refer to the Exit Ticket slide. About how many times larger is the Pacific Ocean than Lake Superior? Write a mathematical argument that can be used to defend your solution. about 54,000 times; Sample answer: $\frac{6.6 \times 10^8}{1.22 \times 10^4} \approx 5.4 \times 10^4$

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 9–12
- Extension: Raise a Number in Scientific Notation to a Power
- ALEKS Scientific Notation

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 7, 9, 11
- Extension: Raise a Number in Scientific Notation to a Power
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Scientific Notation

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Scientific Notation

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

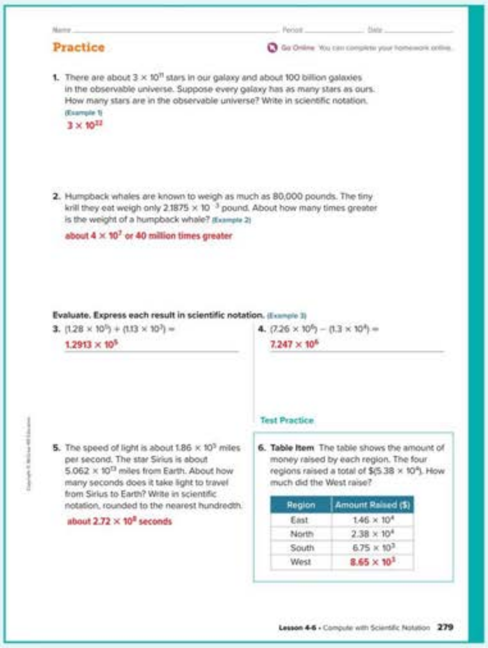
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	multiply numbers written in scientific notation	1
1	divide numbers written in scientific notation	2
1	add and subtract numbers written in scientific notation	3, 4
2	extend concepts learned in class to apply them in new contexts	5, 6
3	solve application problems involving computations with scientific notation	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

Some students may incorrectly add or subtract with scientific notation. Remind students that in order to add numbers expressed in scientific notation, they must write each number with the same power of 10 before applying the Distributive Property and adding or subtracting the decimals. Encourage students to think about the powers of 10 indicating whether or not they can combine like terms. Have them view expressions with the same power of 10 as like terms. For example, in Exercise 3, they cannot add 1.28 and 1.13 because the powers of 10 are not the same.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. There are about 3×10^{11} stars in our galaxy and about 100 billion galaxies in the observable universe. Suppose every galaxy has as many stars as ours. How many stars are in the observable universe? Write in scientific notation. (Example 1)
   3×10^{22}

2. Humpback whales are known to weigh as much as 80,000 pounds. The tiny krill they eat weigh only 2.1875×10^{-3} pound. About how many times greater is the weight of a humpback whale? (Example 2)
   about 4×10^{7} or 40 million times greater

Evaluate. Express each result in scientific notation. (Example 3)

3. $(1.28 \times 10^{5}) + (1.13 \times 10^{3}) =$
   1.2913×10^{5}

4. $(7.26 \times 10^{6}) - (1.3 \times 10^{4}) =$
   7.247×10^{6}

Test Practice

5. The speed of light is about 1.86×10^{5} miles per second. The star Sirius is about 5.062×10^{13} miles from Earth. About how many seconds does it take light to travel from Sirius to Earth? Write in scientific notation, rounded to the nearest hundredth.
   about 2.72×10^{8} seconds

6. **Table Item** The table shows the amount of money raised by each region. The four regions raised a total of $\$(5.38 \times 10^{4})$. How much did the West raise?

Region	Amount Raised ($)
East	1.46×10^{4}
North	2.38×10^{4}
South	6.75×10^{3}
West	8.65×10^{3}

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3 REFLECT AND PRACTICE

8.EE.A.3, 8.EE.A.4

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Apply *indicates multi-step problem

*7. A bacterium was found to have a mass of 2×10^{-12} gram. After 30 hours, one bacterium was replaced by a population of 480,000,000 bacteria. What is the mass of the population of bacteria after 30 hours? Write your answer in scientific notation.

9.6×10^{-4} gram

*8. At the beginning of the business day, a bank's vault held \$575,900. By the end of the day, \$$(3.5 \times 10^3)$ had been added to the vault. How much money did the bank's vault hold at the end of the business day? Write your answer in standard form.

\$579,400

Higher-Order Thinking Problems

9. Explain how the Product of Powers and Quotient of Powers Properties help you to multiply and divide numbers in scientific notation.

Sample answer: Each number has a factor that is a power of 10. Since the bases are the same, these properties can be applied to multiply or divide the powers of 10.

10. **Persevere with Problems** An Olympic-sized swimming pool holds 6.6043×10^5 gallons of water. A standard garden hose can deliver 9 gallons of water per minute. If the garden hose was filling the pool 24 hours per day, how many days would it take to fill the Olympic-sized pool? Explain your reasoning.

51 days; Sample answer: 6.6043×10^5 gallons at 9 gallons per minute is about 73,381 minutes. 73,381 minutes ≈ 1,223 hours ≈ 51 days.

11. **Find the Error** Miguel found the quotient $\frac{5.78 \times 10^5}{0.000002}$ as 0.289. Find his mistake and correct it.

Sample answer: He found 5.78 ÷ 2 as 2.89, but incorrectly found the power of ten. The numerator is 10^5 and the denominator is 10^{-6}. When dividing, the resulting power of ten is 10^{11}. The correct answer should be 2.89×10^{11}.

12. **Be Precise** A *googol* is the number 1 followed by 100 zeros. Earth's mass is 5.972×10^{24} kilograms. How many Earths are needed to have a total mass of 1 googol kilograms?

1.67×10^{75}

280 **Module 4** • Exponents and Scientific Notation

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 10, students will analyze the problem to determine what is being asked. They then can plan a strategy that can be implemented to find how many days it would take to fill the pool.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students will find the mistake in the problem and correct it. Encourage students to determine the error and explain how they could fix it.

6 Attend to Precision In Exercise 12, students will use precision when dividing 1 googol kilograms by the mass of Earth to determine how many Earths are needed to have a total mass of 1 googol kilograms.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 8 Have students work together to prepare a brief demonstration that illustrates why this problem may require multiple steps to solve. For example, before they can find the total amount the bank's vault held at the end of the day, they must first represent the amounts in the same form, either scientific notation or standard form. Have each pair or group of students present their response to the class.

Solve the problem another way.

Use with Exercise 12 Have students work in groups of 3–4. After completing Exercise 12, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include descriptions and examples of the Laws of Exponents. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

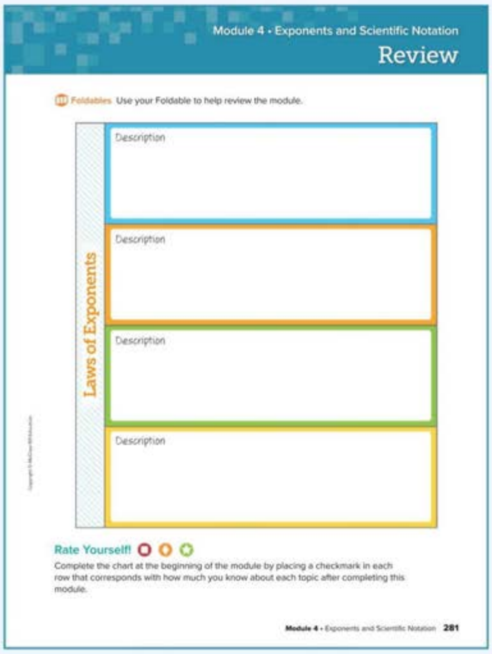
Assessment Resources

Put It All Together 1: Lessons 4-2 and 4-3
Put It All Together 2: Lessons 4-2 through 4-4
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Expressions and Equations**.

- Apply Properties of Integer Exponents
- Represent and Evaluate Expressions with Very Large and Very Small Numbers
- Perform Operations with Numbers Written in Scientific Notation


Module 4 • Exponents and Scientific Notation

Review

Foldables Use your Foldable to help review the module.

Laws of Exponents

Description

Description

Description

Description

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

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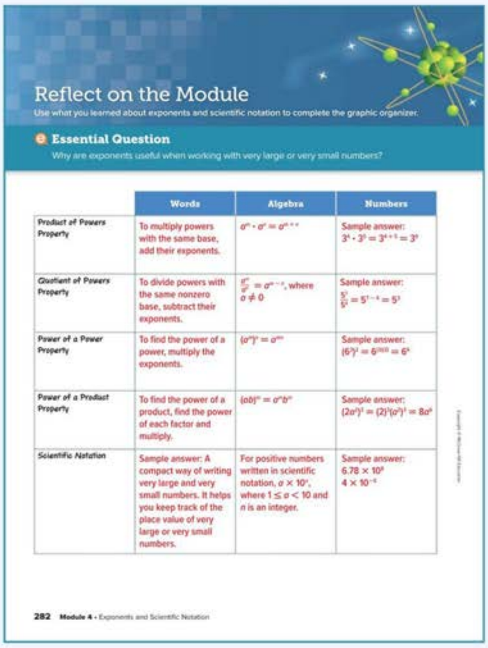

Reflect on the Module

Use what you learned about exponents and scientific notation to complete the graphic organizer.

Essential Question

Why are exponents useful when working with very large or very small numbers?

	Words	Algebra	Numbers
Product of Powers Property	To multiply powers with the same base, add their exponents.	$a^m \cdot a^n = a^{m+n}$	Sample answer: $3^4 \cdot 3^5 = 3^{4+5} = 3^9$
Quotient of Powers Property	To divide powers with the same nonzero base, subtract their exponents.	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$	Sample answer: $\frac{5^7}{5^4} = 5^{7-4} = 5^3$
Power of a Power Property	To find the power of a power, multiply the exponents.	$(a^m)^n = a^{mn}$	Sample answer: $(6^3)^2 = 6^{(3)(2)} = 6^6$
Power of a Product Property	To find the power of a product, find the power of each factor and multiply.	$(ab)^m = a^m b^m$	Sample answer: $(2a^3)^3 = (2)^3(a^3)^3 = 8a^9$
Scientific Notation	Sample answer: A compact way of writing very large and very small numbers. It helps you keep track of the place value of very large or very small numbers.	For positive numbers written in scientific notation, $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.	Sample answer: 6.78×10^8 4×10^{-6}

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

Why are exponents useful when working with very large or very small numbers? See students' graphic organizers.

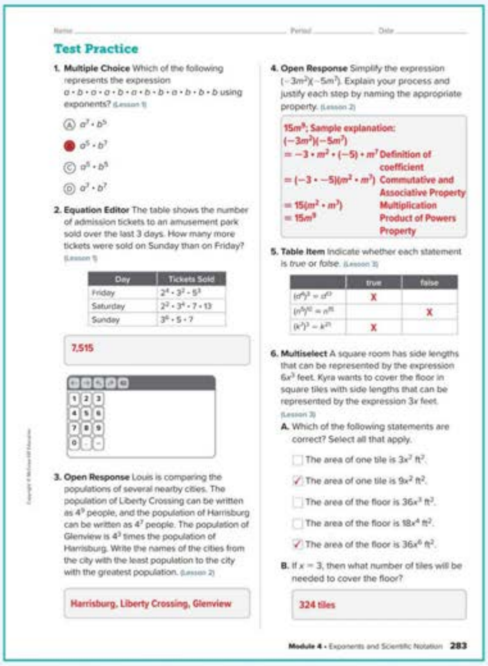
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–14 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 9, 12
Multiselect	Multiple answers may be correct. Students must select all correct answers.	6
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	2, 10, 14
Table Item	Students complete a table.	5, 8
Open Response	Students construct their own response in the area provided.	3, 4, 7, 11, 13

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
Foundational for 8.EE.A.1	4-1	1, 2
8.EE.A.1	4-2, 4-3, 4-4	3, 4, 5, 6, 7, 8, 9
8.EE.A.3	4-5, 4-6	10, 11
8.EE.A.4	4-5, 4-6	12, 13, 14



Name ______ Period ______ Date ______

Test Practice

1. **Multiple Choice** Which of the following represents the expression $a \cdot b \cdot a \cdot a \cdot b \cdot a \cdot b \cdot b \cdot a \cdot b \cdot b \cdot b$ using exponents? (Lesson 1)

   Ⓐ $a^7 \cdot b^5$

   ● $a^5 \cdot b^7$

   Ⓒ $a^5 \cdot b^5$

   Ⓓ $a^7 \cdot b^7$

2. **Equation Editor** The table shows the number of admission tickets to an amusement park sold over the last 3 days. How many more tickets were sold on Sunday than on Friday? (Lesson 1)

Day	Tickets Sold
Friday	$2^4 \cdot 3^2 \cdot 5^3$
Saturday	$2^2 \cdot 3^4 \cdot 7 \cdot 13$
Sunday	$3^6 \cdot 5 \cdot 7$

7,515

3. **Open Response** Louis is comparing the populations of several nearby cities. The population of Liberty Crossing can be written as 4^9 people, and the population of Harrisburg can be written as 4^7 people. The population of Glenview is 4^3 times the population of Harrisburg. Write the names of the cities from the city with the least population to the city with the greatest population. (Lesson 2)

   Harrisburg, Liberty Crossing, Glenview

4. **Open Response** Simplify the expression $(-3m^2)(-5m^7)$. Explain your process and justify each step by naming the appropriate property. (Lesson 2)

   $15m^9$; Sample explanation:

   $(-3m^2)(-5m^7)$

   $= -3 \cdot m^2 \cdot (-5) \cdot m^7$ Definition of coefficient

   $= (-3 \cdot -5)(m^2 \cdot m^7)$ Commutative and Associative Property

   $= 15(m^2 \cdot m^7)$ Multiplication

   $= 15m^9$ Product of Powers Property

5. **Table Item** Indicate whether each statement is *true* or *false*. (Lesson 3)

	true	false
$(a^6)^3 = a^{18}$	X	
$(n^5)^{10} = n^{15}$		X
$(k^7)^3 = k^{21}$	X	

6. **Multiselect** A square room has side lengths that can be represented by the expression $6x^3$ feet. Kyra wants to cover the floor in square tiles with side lengths that can be represented by the expression $3x$ feet. (Lesson 3)

   A. Which of the following statements are correct? Select all that apply.

   ☐ The area of one tile is $3x^2$ ft².

   ☑ The area of one tile is $9x^2$ ft².

   ☐ The area of the floor is $36x^3$ ft².

   ☐ The area of the floor is $18x^4$ ft².

   ☑ The area of the floor is $36x^6$ ft².

   B. If $x = 3$, then what number of tiles will be needed to cover the floor?

   324 tiles

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7. Open Response Express the fraction $\frac{1}{6^4}$ using a negative exponent.

6^{-4}

8. Table Item Place an X in the correct column to indicate if each statement is *sometimes*, *always*, or *never* true.

	sometimes	always	never
$4^0 = 1$		X	
$n^0 = 1$	X		
$z^0 = 1, z \neq 0$		X	

9. Multiple Choice Which expression represents $w^{-7} \cdot w^3$ written in simplest form?

(A) $\frac{1}{w^4}$ (selected)

(B) w^{-4}

(C) $\frac{1}{w^{-4}}$

(D) w^4

10. Equation Editor Write 33,800,000 in scientific notation.

3.38×10^7

11. Open Response Write 7.2×10^{-3} in standard form.

0.0072

12. Multiple Choice Which of the following is most appropriate to describe the average distance from Earth to the Sun?

(A) 1.5×10^{14} mm

(B) 1.5×10^{13} cm

(C) 1.5×10^{11} m

(D) 1.5×10^8 km (selected)

13. Open Response Evaluate the expression $(7.2 \times 10^7) + (5.5 \times 10^6)$. Express the result in scientific notation.

7.75×10^7

14. Equation Editor Ryan downloaded a video game to his computer. The size of the file was 2.4×10^5 kilobytes. After the game was installed, he downloaded a patch file for the game that was 48,000 kilobytes in size.

A. What is the size of the game in standard form?

240,000

B. How many times larger was the game than the patch file?

5

Module 5
Real Numbers

Module Goal
Learn about the real number system by identifying, calculating, and estimating irrational numbers and comparing them to rational numbers.

Focus
Domain: The Number System
Major Cluster(s): **8.EE.A** W ork with radicals and integer exponents.
Supporting Cluster(s): **8.NS.A** K now that there are numbers that are not rational, and approximate them by rational numbers.
Standards for Mathematical Content:
8.NS.A.1 K now that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
Also addresses 8.EE.A.2.
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover
Students need to have a thorough understanding of the prerequisite skills required for this module.

- write equivalent forms of fractions, decimals, and percents
- find powers

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence
Vertical Alignment

Previous
Students studied the set of rational numbers.
6.NS.C.6, 7.NS.A.2.D

Now
Students learn about the real number system by identifying, calculating, and estimating irrational numbers and comparing them to rational numbers.
8.NS.A.1, 8.NS.A.2, 8.EE.A.2

Next
Students will study and use the properties of rational and irrational numbers.
HS.RN.B.3

Rigor
The Three Pillars of Rigor
In this module, students draw on their knowledge of the set of rational numbers to develop *understanding* of the set of real numbers. They use this understanding to build *fluency* with determining if numbers are rational or irrational, finding roots of perfect squares and cubes, and estimating roots of numbers. They *apply* their fluency to solve multi-step real-world problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

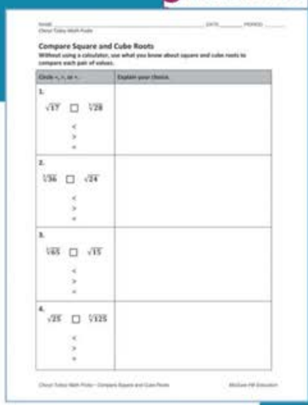
Suggested Pacing

Lesson		Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
5-1	Roots	8.EE.A.2	2	1
5-2	Real Numbers	8.NS.A.1, 8.EE.A.2	1	0.5
Put It All Together 1: Lessons 5-1 and 5-2			0.5	0.25
5-3	Estimate Irrational Numbers	8.NS.A.2, *Also addresses 8.EE.A.2*	2	1
5-4	Compare and Order Real Numbers	8.NS.A.1, 8.NS.A.2	1	0.5
Put It All Together 2: Lessons 5-3 and 5-4			0.5	0.25
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	10	5

![CHERYL TOBEY MATH PROBES]

Formative Assessment Math Probe
Compare Square and Cube Roots

Module Resource


Compare Square and Cube Roots

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students use what they know about square and cube roots to compare pairs of values.

Targeted Concept Understand the difference between square roots and cube roots, and what it means to compare their values.

Targeted Misconceptions

- Students may think that the index (number outside the radical sign) is the divisor of the radicand (number inside the radical sign).
- Students may confuse square root with cube root, and find square roots for cube root expressions.
- Students may not use benchmark numbers (perfect squares and cubes) to estimate the value of a square root and cube root and therefore have difficulty comparing the two values.

Assign the probe after Lesson 4.

Correct Answers:
1. >; **2.** <; **3.** >; **4.** =

Collect and Assess Student Answers

If the student selects....	**Then** the student likely...
1. < **2.** = **3.** > **4.** <	assumes the index is the divisor of the radicand and divides the radicand by the index. **Example:** For item 4, the student incorrectly compares 25 ÷ 2 to 125 ÷ 3.
1. < **2.** > **3.** > **4.** < Note that Item 3 is the correct answer. Check the students' explanations for misconceptions.	misinterprets cube root as square root. **Example:** For item 1, the student estimates the square root of 17 and the square root of 28, and then compares.
Various other incorrect responses	uses an incorrect method for determining the values of the roots.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign and/or revisit the following resources.

- ALEKS® Exponents, Polynomials, and Radicals
- Lesson 1, Examples 1–8; Lesson 3, Examples 1–4; Lesson 4, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

![IGNITE!]

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

Why do we classify numbers? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of grocery shopping, filling a gas tank, recipes, temperature, and circumference to introduce the idea of classifying numbers. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.

Module 5

Real Numbers

Essential Question

Why do we classify numbers?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
finding square roots and cube roots						
identifying real numbers						
describing sets of real numbers						
estimating irrational numbers						
comparing and ordering real numbers						
graphing real numbers on a number line						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about real numbers.

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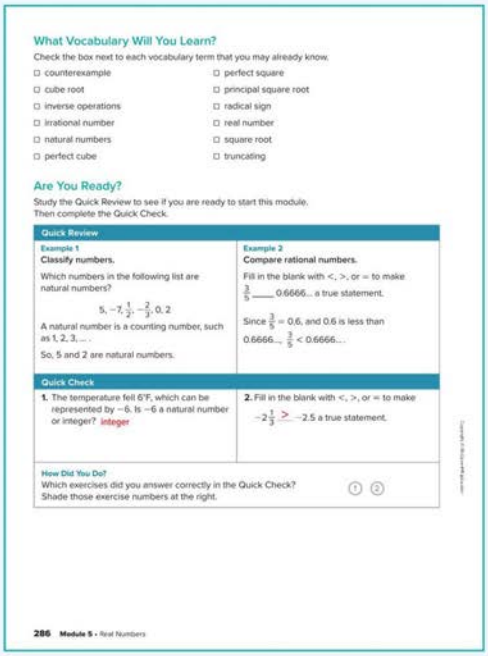
Interactive Presentation

Schroptschop/Vetta/Getty Images



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ counterexample
- ☐ cube root
- ☐ inverse operations
- ☐ irrational number
- ☐ natural numbers
- ☐ perfect cube
- ☐ perfect square
- ☐ principal square root
- ☐ radical sign
- ☐ real number
- ☐ square root
- ☐ truncating

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review	
Example 1 **Classify numbers.** Which numbers in the following list are natural numbers? $5, -7, \frac{1}{2}, -\frac{2}{3}, 0, 2$ A natural number is a counting number, such as 1, 2, 3, ... So, 5 and 2 are natural numbers.	Example 2 **Compare rational numbers.** Fill in the blank with <, >, or = to make $\frac{3}{5}$ ____ 0.6666... a true statement. Since $\frac{3}{5} = 0.6$, and 0.6 is less than 0.6666..., $\frac{3}{5} < 0.6666...$
Quick Check	
1. The temperature fell 6°F, which can be represented by −6. Is −6 a natural number or integer? integer	2. Fill in the blank with <, >, or = to make $-2\frac{1}{3}$ > −2.5 a true statement.
How Did You Do? Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②	

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine.

Define A **square root** of a number is one of its two equal factors.

Example The square root of 9 is 3 because $3 \cdot 3 = 9$.

Ask What is the square root of 81?

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- dividing whole numbers
- solving one-step equations
- computing with powers, square roots, and cube roots
- identifying rational number sets (natural, whole, integers)
- graphing on a number line
- writing numbers in decimal notation

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Exponents, Polynomials, and Radicals** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Mistakes = Learning

When anyone makes a mistake and goes on to learn from it, that person can actually build new connections in his or her brain as he or she determines a new path or process that can be used toward a solution to the problem.

How Can I Apply It?

Have students complete the **Checks** after each Example, either digitally or in their *Interactive Student Edition*, as a form of student-centered formative assessment. Encourage them to analyze any mistakes they might have made and determine what they could do to self correct.

ALEKS is a great tool not only to individualize learning for each student, but also to help students understand that making mistakes and trying new problems will help them to learn and grow long term. Have students keep track of their ALEKS Pie Chart to view their progress.

Lesson 5-1

Roots

8.EE.A.2

LESSON GOAL

Students will find square and cube roots.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Find Square Roots Using a Square Model

Learn: Square Roots
Examples 1-3: Find Positive, Negative, and Both Square Roots
Example 4: Square Roots of Negative Numbers
Learn: Use Square Roots to Solve Equations
Example 5: Use Square Roots to Solve Equations
Learn: Cube Roots
Examples 6-7: Cube Roots of Positive and Negative Numbers
Example 8: Use Cube Roots to Solve Equations
Apply: Bulletin Boards

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

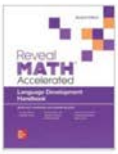
View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 29 of the *Language Development Handbook* to help your students build mathematical language related to square and cube roots.

ELL You can use the tips and suggestions on page T29 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.A** by finding square and cube roots.
Standards for Mathematical Content: **8.E E.A.2**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students wrote and solved one-step equations.
6.EE.B.7

Now
Students find square and cube roots.
8.EE.A.2

Next
Students will identify and describe sets of numbers in the real number system.
8.NS.A.1, 8.EE.A.2

Rigor

The Three Pillars of Rigor

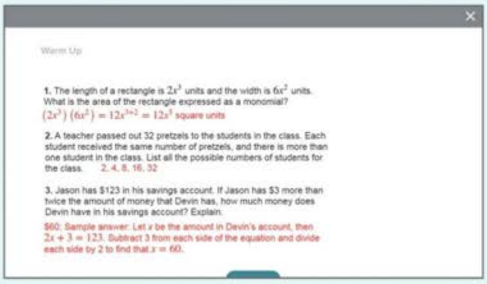
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of exponents to develop *understanding* of the roots of perfect squares and perfect cubes. They come to understand that a positive number has two possible square roots, but every number has only one cube root. They build *fluency* with finding the roots of perfect squares and cubes, and use them to solve equations.

Mathematical Background

Go online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- computing with powers (Exercise 1)
- finding factors (Exercise 2)
- solving equations (Exercise 3)

1–3. See Warm Up slide online for correct answers.

Launch the Lesson, Slide 1 of 2

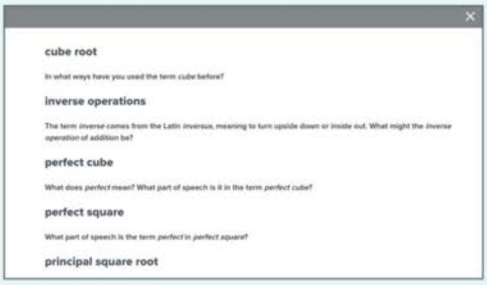
Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the Great Pyramid of Giza and its base side lengths.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.



What Vocabulary Will You Learn?

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions available online.

Ask:

- In what ways have you used the term cube before? Sample answer: when studying 3D shapes and exponents
- The term *inverse* comes from the L atin *inversus*, meaning to turn upside down or inside out. What might the *inverse operation* of addition be? Sample answer: subtraction because it is the opposite of addition
- What does *perfect* mean ? What part of speech is it in the term *perfect cube*? Sample answer: ideal, or without flaw; It is an adjective.
- What part of speech is the term *perfect* in *perfect square*? It is an adjective.

Explore Find Square Roots Using a Square Model

Objective
Students will use Web Sketchpad to explore how to use square models to find square roots.

Ideas for Use
Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity
Students will be presented with squares, along with their areas and side lengths. Throughout this activity, students will be asked to identify the relationship between the side length of a square and its area, and how a square root can illustrate this relationship.

Inquiry Question
What does the square root of a number mean? Sample answer: The square root of a number is the value that gives that number when multiplied by itself. It is the inverse of squaring a number.

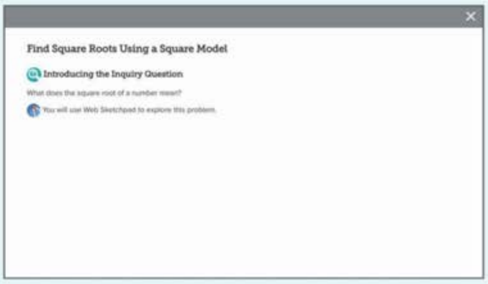
Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!
SLIDE 4

Mathematical Discourse
Does your conjecture about the relationship between the side length of the square and area of the square hold true for all of the given side lengths in the table? Explain. Sample answer: Yes, multiplying the side of the square by itself gives you the area of the square.

(continued on next page)

Interactive Presentation



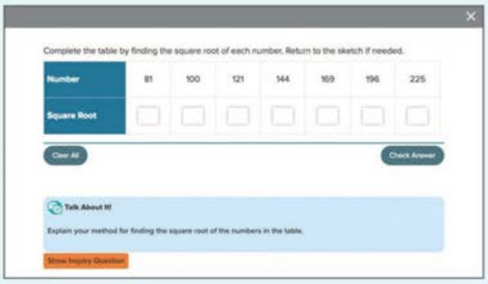
Explore, Slide 1 of 7

Explore, Slide 4 of 7

WEB SKETCHPAD
Throughout the Explore, students use Web Sketchpad to explore the relationship between the area of a square and its side length.

TYPE
On Slide 3, students enter the missing information in the table.

Interactive Presentation



Explore, Slide 6 of 7

TYPE

On Slide 6, students complete a table of the square roots of perfect squares.

TYPE

On Slide 7, students respond to the Inquiry Question and can view a sample answer.

Explore Find Square Roots Using a Square Model *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use the Web Sketchpad virtual dot paper to help them gain insight into the meaning of square roots.

8 Look for and Express Regularity in Repeated Reasoning Students should think about the patterns they observed throughout the Explore to make a generalization as to what it means to find the square root of a number.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Explain your method for finding the square root of the numbers in the table. Sample answer: I found the value, when multiplied by itself, was the given number.

Learn Square Roots

Objective

Students will understand what it means for a number to be a square root, and what it means for a number to be a perfect square.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discussing the *Talk About It!* question on Slide 2, encourage students to make sense of the meanings of the terms *perfect square* and *square root*, and understand why some numbers are not perfect squares.

6 Attend to Precision While discussing the *Talk About It!* question on Slide 4, encourage students to use precise mathematical notation when representing the square root of 16 in different ways.

Teaching Notes

SLIDE 1

Students will select the flashcards to learn about square roots using words, symbols, and an example.

Talk About It!

SLIDE 2

Mathematical Discourse

Why do you think the term *perfect square* uses the adjective *perfect*? What are some numbers that aren't perfect squares? Explain why. Sample answer: because the square root of a perfect square is a whole number; 5, 12, and 30 are not perfect squares, because their square roots are not whole numbers.

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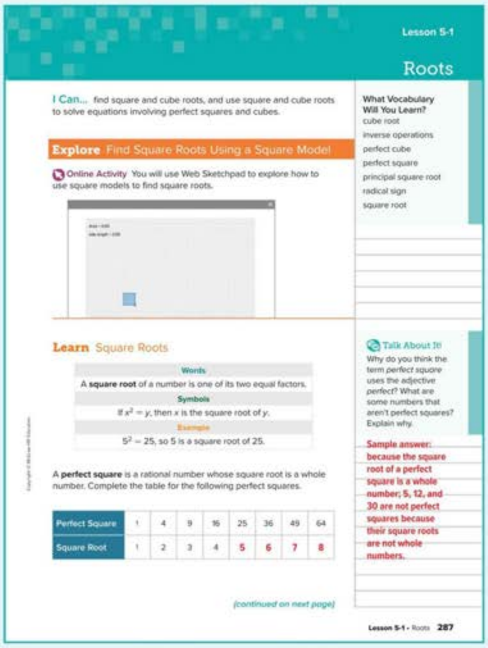


Lesson 5-1

Roots

I Can... find square and cube roots, and use square and cube roots to solve equations involving perfect squares and cubes.

What Vocabulary Will You Learn?
cube root
inverse operations
perfect cube
perfect square
principal square root
radical sign
square root

Explore Find Square Roots Using a Square Model

Online Activity You will use Web Sketchpad to explore how to use square models to find square roots.

Learn Square Roots

Words
A **square root** of a number is one of its two equal factors.
Symbols
If $x^2 = y$, then x is the square root of y.
Example
$5^2 = 25$, so 5 is a square root of 25.

A **perfect square** is a rational number whose square root is a whole number. Complete the table for the following perfect squares.

Perfect Square	1	4	9	16	25	36	49	64
Square Root	1	2	3	4	5	6	7	8

Talk About It! Why do you think the term *perfect square* uses the adjective *perfect*? What are some numbers that aren't perfect squares? Explain why.

Sample answer: because the square root of a perfect square is a whole number; 5, 12, and 30 are not perfect squares because their square roots are not whole numbers.

(continued on next page)

Lesson 5-1 • Roots 287

Interactive Presentation

Learn, Square Roots, Slide 1 of 4

FLASHCARDS

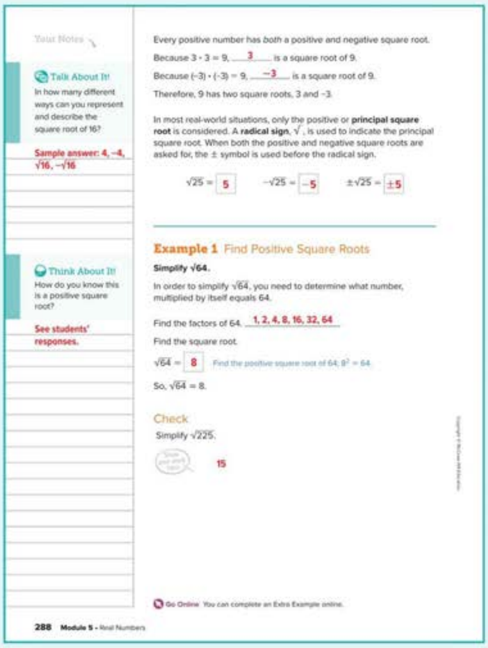
On Slide 1, students use Flashcards to view multiple representations of square roots.

TYPE

On Slide 2, students complete the table to find the perfect squares.

CLICK

On Slide 3, students select the buttons to see an example of a positive and negative square root.



Your Notes

Talk About It! In how many different ways can you represent and describe the square root of 16?

Sample answer: 4, −4, $\sqrt{16}$, $-\sqrt{16}$

Think About It! How do you know this is a positive square root?

See students' responses.

Every positive number has *both* a positive and negative square root.

Because $3 \cdot 3 = 9$, 3 is a square root of 9.

Because $(-3) \cdot (-3) = 9$, −3 is a square root of 9.

Therefore, 9 has two square roots, 3 and −3.

In most real-world situations, only the positive or **principal square root** is considered. A **radical sign**, $\sqrt{\ }$, is used to indicate the principal square root. When both the positive and negative square roots are asked for, the ± symbol is used before the radical sign.

$\sqrt{25} = 5$ $-\sqrt{25} = -5$ $\pm\sqrt{25} = \pm 5$

Example 1 Find Positive Square Roots

Simplify $\sqrt{64}$.

In order to simplify $\sqrt{64}$, you need to determine what number, multiplied by itself equals 64.

Find the factors of 64. 1, 2, 4, 8, 16, 32, 64

Find the square root.

$\sqrt{64} = 8$ Find the positive square root of 64; $8^2 = 64$

So, $\sqrt{64} = 8$.

Check

Simplify $\sqrt{225}$. 15

Go Online You can complete an Extra Example online.

288 Module 5 • Real Numbers

Interactive Presentation

Example 1, Find Positive Square Roots, Slide 2 of 3

CLICK

On Slide 2, students select the factors of 64.

TYPE

On Slide 2, students enter the value of the square root.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Square Roots *(continued)*

Teaching Notes

SLIDE 3

Students will learn about positive and negative square roots and that the *radical sign* is used to indicate the *principal*, or positive, square root of a number.

Talk About It!

SLIDE 4

Mathematical Discourse

In how many different ways can you represent and describe the square root of 16? Sample answer: 4, −4, $\sqrt{16}$, $-\sqrt{16}$

Example 1 Find Positive Square Roots

Objective

Students will find the positive square root of a number.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Enc ourage students to pause and consider the meaning of the radical sign. Students should be able to interpret the radical sign as the principal (positive) square root.

Questions for Mathematical Discourse

SLIDE 2

AL Of the factors of 64, which one is the square root of 64? Explain why. 8; It is the only factor that when multiplied by itself, yields a product of 64.

OL The square root of 64 is an integer. What does this mean about 64? 64 is a perfect square.

BL Is −8 also a solution to this problem? Explain. no; Sample answer: The radical sign indicated that we were to find the principal square root, which is the positive square root.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find Both Square Roots

Objective

Students will find the positive and negative square root of a number.

Questions for Mathematical Discourse

SLIDE 2

AL What number, multiplied by itself, equals 121? 11

OL How do you know that you are finding both the positive and negative square roots of 121? The $\pm$ in front of the radical sign indicates both positive and negative square roots.

BL If you were asked to find the square root of 2.25, what whole number would you find the square root of first? 225

SLIDE 3

AL Why is the placement of the decimal point important when indicating the square root? Sample answer: 11 • 11 = 121; This is much greater than 1.21, so the decimal point should be placed differently to indicate the square root of 1.21.

OL Can you eliminate 0.11 or 1.1 as the square root of 1.21 just by considering the sizes of the numbers? Explain. Yes; Sample answer: Squaring 0.11 gives a number less than 1, so it cannot be the square root of 1.21.

BL Find the square root of 0.0121. Explain your method. 0.11; Sample answer: Find the square root of 121, and place the decimal point appropriately in the root in order for the product to be 0.0121.

Example 3 Find Negative Square Roots

Objective

Students will find the negative square root of a number.

Questions for Mathematical Discourse

SLIDE 1

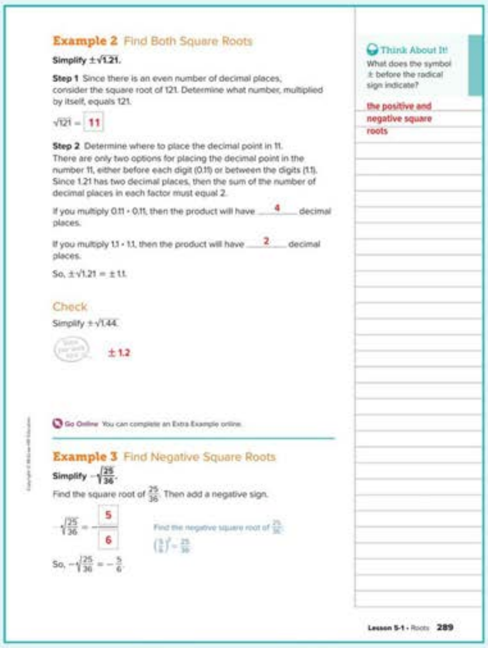
AL What is the square root of 25? the square root of 36? $\sqrt{25} = 5$, $\sqrt{36} = 6$

OL How can you write $-\sqrt{\frac{25}{36}}$ using two separate radical signs? $-\frac{\sqrt{25}}{\sqrt{36}}$

BL Consider the expression $\sqrt{0.81}$. How can you write $\sqrt{0.81}$ as the square root of a fraction where the numerator and denominator are both perfect squares? $\sqrt{\frac{81}{100}}$

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Find Both Square Roots

Simplify $\pm\sqrt{1.21}$.

Step 1 Since there is an even number of decimal places, consider the square root of 121. Determine what number, multiplied by itself, equals 121.

$\sqrt{121} = 11$

Step 2 Determine where to place the decimal point in 11. There are only two options for placing the decimal point in the number 11, either before each digit (0.11) or between the digits (1.1). Since 1.21 has two decimal places, then the sum of the number of decimal places in each factor must equal 2.

If you multiply 0.11 • 0.11, then the product will have 4 decimal places.

If you multiply 1.1 • 1.1, then the product will have 2 decimal places.

So, $\pm\sqrt{1.21} = \pm 1.1$.

Check

Simplify $\pm\sqrt{1.44}$.

± 1.2

Go Online You can complete an Extra Example online.

Example 3 Find Negative Square Roots

Simplify $-\sqrt{\frac{25}{36}}$.

Find the square root of $\frac{25}{36}$. Then add a negative sign.

$-\sqrt{\frac{25}{36}} = -\frac{5}{6}$

Find the negative square root of $\frac{25}{36}$. $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$

So, $-\sqrt{\frac{25}{36}} = -\frac{5}{6}$.

Think About It! What does the symbol $\pm$ before the radical sign indicate?

the positive and negative square roots

Lesson 5-1 • Roots 289

Interactive Presentation

Example 3, Find Negative Square Roots, Slide 1 of 2

TYPE

On Slide 1 of Example 3, students enter the value of the square root.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Check

Simplify $-\sqrt{\frac{49}{64}}$.

$-\frac{7}{8}$

Think About It! Can you think of a number when multiplied by itself, equals −16? Why or why not?

See students' responses.

Example 4 Square Roots of Negative Numbers

Simplify $\sqrt{-16}$ using rational numbers. If the expression cannot be simplified, explain why.

In order to simplify $\sqrt{-16}$, you need to determine what number, multiplied by itself equals −16.

There is no rational number square root of −16 because no number times itself is equal to −16.

So, $\sqrt{-16}$ cannot be simplified and has no rational number solution.

Talk About It! What is the difference between $\sqrt{-16}$ and $-\sqrt{16}$?

Sample answer: $\sqrt{-16}$ is the square root of negative 16. $-\sqrt{16}$ is the negative square root of 16. The first expression has no rational number square root, but the second expression is equal to −4.

Check

Simplify $\sqrt{-81}$ using rational numbers. If the expression cannot be simplified, explain why.

The expression cannot be simplified. There is no rational square root because no number times itself is equal to −81.

Go Online You can complete an Extra Example online.

290 Module 5 • Real Numbers

Interactive Presentation

Example 4, Square Roots of Negative Numbers, Slide 2 of 4

CLICK

On Slide 2, students select the phrase that best explains why there is no rational number square root of −16.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Square Roots of Negative Numbers

Objective

Students will determine that there is no rational number square root of negative numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should be able to reason that there is no rational number that when multiplied by itself, equals a negative number.

6 Attend to Precision While discussing the *Talk About It!* question on Slide 3, encourage students to use clear and precise mathematical language to explain the differences in the two notations. They should be able to understand and explain the difference between the phrases *the square root of negative sixteen* and *the negative square root of sixteen*.

7 Look for and Make Use of Structure Encourage students to pause and consider the meaning of the negative sign and how its placement *inside* the radical sign indicates that there is no rational number solution.

Questions for Mathematical Discourse

SLIDE 2

AL What is $\sqrt{16}$? 4

OL Use examples to explain why there is no rational number, that when multiplied by itself, equals −16? Sample answer: $4^2 = 16$ and $(-4)^2 = 16$. There is no number that when squared yields a product of −16.

OL Use the rules for multiplying rational numbers to explain why there is no rational number, that when multiplied by itself, equals a negative number. Sample answer: The product of two positive numbers is positive, and the product of two negative numbers is positive. In order for the product of two numbers to be negative, the numbers cannot have the same sign.

BL Find the value of $\sqrt{\frac{-16}{-25}}$. $\frac{4}{5}$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Use Square Roots to Solve Equations

Objective

Students will learn how to solve equations of the form $x^2 = p$.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Find sample answers for the *Talk About It!* questions.

Example 5 Use Square Roots to Solve Equations

Objective

Students will solve equations of the form $x^2 = p$.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discussing the *Talk About It!* questions on Slide 3, encourage students to make sense of why both the negative and positive square roots must be considered solutions to the equation.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the equation and the relationship between the terms t^2 and 169. Students should understand that by taking the square root of each side, they must consider both the positive and negative square root.

Questions for Mathematical Discourse

SLIDE 2

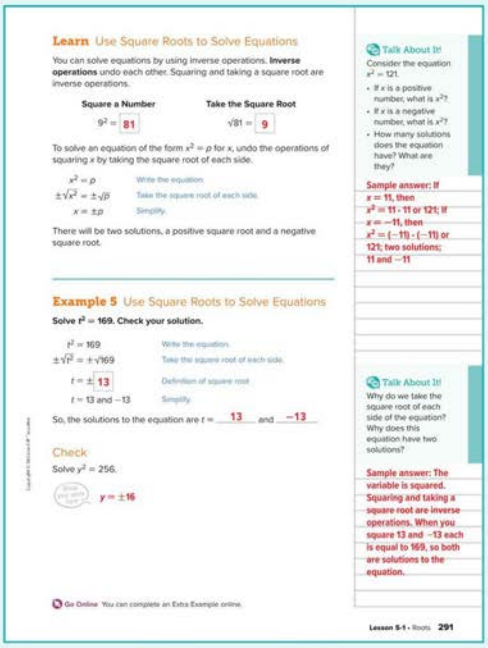
AL What is the inverse operation of squaring a number? taking the square root

OL Why isn't the equation equivalent to $\sqrt{t^2} = \sqrt{169}$? Sample answer: This would only include the positive square root of 169. But the negative square root is also a solution.

BL Can you solve $t^2 = -169$? Explain. Sample answer: This equation has no rational number solution, because there is no rational number equal to $\sqrt{-169}$.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Use Square Roots to Solve Equations

You can solve equations by using inverse operations. **Inverse operations** undo each other. Squaring and taking a square root are inverse operations.

Square a Number	Take the Square Root
$9^2 =$ 81	$\sqrt{81} =$ 9

To solve an equation of the form $x^2 = p$ for x, undo the operations of squaring x by taking the square root of each side.

$x^2 = p$ Write the equation.
$\pm\sqrt{x^2} = \pm\sqrt{p}$ Take the square root of each side.
$x = \pm p$ Simplify.

There will be two solutions, a positive square root and a negative square root.

Talk About It! Consider the equation $x^2 = 121$.
- If x is a positive number, what is x^2?
- If x is a negative number, what is x^2?
- How many solutions does the equation have? What are they?

Sample answer: If $x = 11$, then $x^2 = 11 \cdot 11$ or 121; If $x = -11$, then $x^2 = (-11) \cdot (-11)$ or 121; two solutions; 11 and −11

Example 5 Use Square Roots to Solve Equations

Solve $t^2 = 169$. Check your solution.

$t^2 = 169$ Write the equation.
$\pm\sqrt{t^2} = \pm\sqrt{169}$ Take the square root of each side.
$t = \pm$ 13 Definition of square root
$t = 13$ and -13 Simplify.

So, the solutions to the equation are $t =$ 13 and −13

Check

Solve $y^2 = 256$.

$y = \pm 16$

Talk About It! Why do we take the square root of each side of the equation? Why does this equation have two solutions?

Sample answer: The variable is squared. Squaring and taking a square root are inverse operations. When you square 13 and −13 each is equal to 169, so both are solutions to the equation.

Go Online You can complete an Extra Example online.

Lesson 5-1 • Roots 291

Interactive Presentation

Learn, Use Square Roots to Solve Equations, Slide 1 of 3

FLASHCARDS

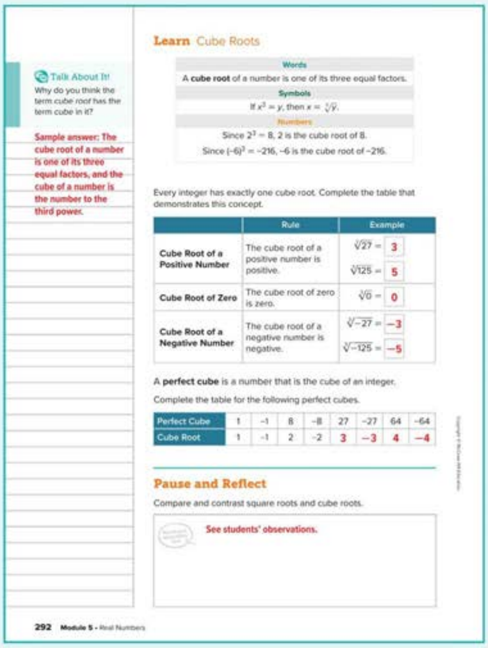
On Slide 1 of the Learn, students use Flashcards to view how squares and square roots are related.

CLICK

On Slide 2 of Example 5, students will move through the steps to solve the equation. They will also select the correct answer.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Cube Roots

Words
A **cube root** of a number is one of its three equal factors.
Symbols
If $x^3 = y$, then $x = \sqrt[3]{y}$.
Numbers
Since $2^3 = 8$, 2 is the cube root of 8.
Since $(-6)^3 = -216$, −6 is the cube root of −216.

Talk About It!
Why do you think the term cube root has the term cube in it?

Sample answer: The cube root of a number is one of its three equal factors, and the cube of a number is the number to the third power.

Every integer has exactly one cube root. Complete the table that demonstrates this concept.

	Rule	Example
Cube Root of a Positive Number	The cube root of a positive number is positive.	$\sqrt[3]{27} =$ 3 $\sqrt[3]{125} =$ 5
Cube Root of Zero	The cube root of zero is zero.	$\sqrt[3]{0} =$ 0
Cube Root of a Negative Number	The cube root of a negative number is negative.	$\sqrt[3]{-27} =$ −3 $\sqrt[3]{-125} =$ −5

A **perfect cube** is a number that is the cube of an integer.

Complete the table for the following perfect cubes.

Perfect Cube	1	−1	8	−8	27	−27	64	−64
Cube Root	1	−1	2	−2	3	−3	4	−4

Pause and Reflect

Compare and contrast square roots and cube roots.

See students' observations.

292 Module 5 • Real Numbers

Interactive Presentation

Learn, Cube Roots, Slide 1 of 4

FLASHCARDS

On Slide 1, students use Flashcards to learn about cube roots.

Learn Cube Roots

Objective

Students will understand what it means for a number to be a cube root, and what it means for a number to be a perfect cube.

MP Teaching the Mathematical Practices

6 Attend to Precision While discussing the *Talk About It!* question on Slide 4, encourage students to use clear and precise mathematical language, such as *cube root, cube, factor, power,* and/or *exponent* in their explanations.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 4

Mathematical Discourse

Why do you think the term *cube root* has the term *cube* in it?

Sample answer: The cube root of a number is one of its three equal factors, and the cube of a number is the number to the third power.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are having difficulty understanding the similarities and differences between square roots and cube roots of negative numbers, have them work with a partner to create a chart like the one below. Then have them use their understanding of operations with signed numbers to explain why each of the following statements makes sense.

Number	Square	Cube
−3	9	−27
−2	4	−8
−1	1	−1
0	0	0
1	1	1
2	4	8
3	9	27

The square of any number is always positive, regardless of whether the number is positive or negative. The product of two positive numbers is positive. The product of two negative numbers is positive.

There is no rational number that is a square root of a negative number. Multiplying any two numbers with the same sign will always be positive, never negative.

The cube of a positive number is positive, while the cube of a negative number is negative. The product of three positive numbers is positive. The product of three negative numbers is negative.

Example 6 Cube Roots of Positive Numbers

Objective

Students will find the cube root of a positive number.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the expression, noting how the radical sign indicates a cube root (versus a square root). Students should pay attention to the positive sign inside the radical sign and what this indicates about the sign of the cube root.

Questions for Mathematical Discourse

SLIDE 1

AL What number multiplied three times equals 125? 5

OL Is the cube root of 125 positive or negative? Explain. positive; Sample answer: $5^3 = 125$ and $(-5)^3 = -125$. Since the given number is 125, not −125, the cube root is positive.

BL Is $\sqrt[3]{125}$ less than or greater than $\sqrt{125}$? Explain. less than; Sample answer: The cube root of 125 is 5. The square root of 125 must be greater than 11 since $11^2 = 121$.

Example 7 Cube Roots of Negative Numbers

Objective

Students will find the cube root of a negative number.

Questions for Mathematical Discourse

SLIDE 2

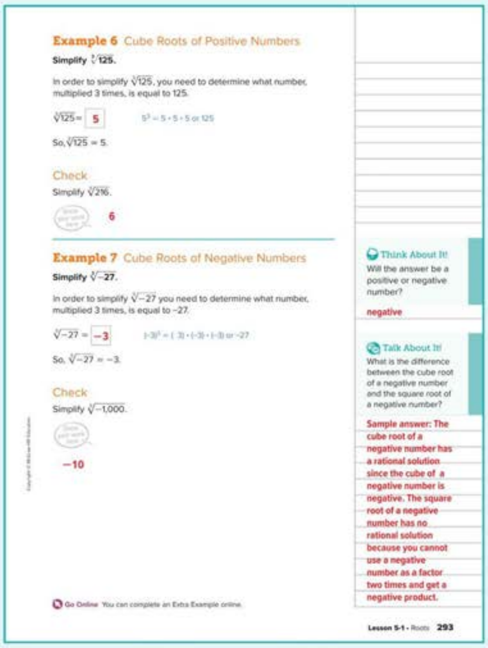
AL What number multiplied three times equals −27? −3

OL Is the cube root of −27 positive or negative? Explain. negative; Sample answer: $(-3)^3 = -27$ and $3^3 = 27$. Since the given number is −27, not 27, the cube root is negative.

BL Explain how you could find $\sqrt[5]{-32}$. Sample answer: Find the number that when multiplied five times, equals −32. That number is −2.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 6 Cube Roots of Positive Numbers

Simplify $\sqrt[3]{125}$.

In order to simplify $\sqrt[3]{125}$, you need to determine what number, multiplied 3 times, is equal to 125.

$\sqrt[3]{125} = 5$ $5^3 = 5 \cdot 5 \cdot 5$ or 125

So, $\sqrt[3]{125} = 5$.

Check

Simplify $\sqrt[3]{216}$.

6

Example 7 Cube Roots of Negative Numbers

Simplify $\sqrt[3]{-27}$.

In order to simplify $\sqrt[3]{-27}$ you need to determine what number, multiplied 3 times, is equal to −27.

$\sqrt[3]{-27} = -3$ $(-3)^3 = (-3) \cdot (-3) \cdot (-3)$ or −27

So, $\sqrt[3]{-27} = -3$.

Check

Simplify $\sqrt[3]{-1{,}000}$.

−10

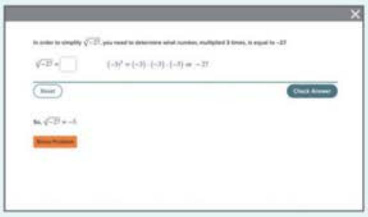
Go Online You can complete an Extra Example online.

Think About It! Will the answer be a positive or negative number? negative

Talk About It! What is the difference between the cube root of a negative number and the square root of a negative number? Sample answer: The cube root of a negative number has a rational solution since the cube of a negative number is negative. The square root of a negative number has no rational solution because you cannot use a negative number as a factor two times and get a negative product.

Lesson 5-1 • Roots 293

Interactive Presentation



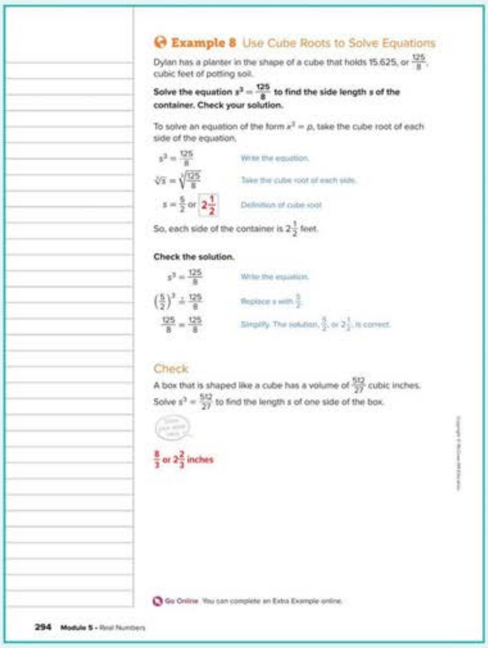
Example 7, Cube Roots of Negative Numbers, Slide 2 of 4

TYPE

On Slide 2 of Example 7, students enter the value of the cube root.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.



Example 8 Use Cube Roots to Solve Equations

Dylan has a planter in the shape of a cube that holds 15.625, or $\frac{125}{8}$, cubic feet of potting soil.

Solve the equation $s^3 = \frac{125}{8}$ to find the side length s of the container. Check your solution.

To solve an equation of the form $x^3 = p$, take the cube root of each side of the equation.

$s^3 = \frac{125}{8}$ Write the equation.

$\sqrt[3]{s^3} = \sqrt[3]{\frac{125}{8}}$ Take the cube root of each side.

$s = \frac{5}{2}$ or $2\frac{1}{2}$ Definition of cube root

So, each side of the container is $2\frac{1}{2}$ feet.

Check the solution.

$s^3 = \frac{125}{8}$ Write the equation.

$\left(\frac{5}{2}\right)^3 \stackrel{?}{=} \frac{125}{8}$ Replace s with $\frac{5}{2}$.

$\frac{125}{8} = \frac{125}{8}$ Simplify. The solution, $\frac{5}{2}$, or $2\frac{1}{2}$, is correct.

Check

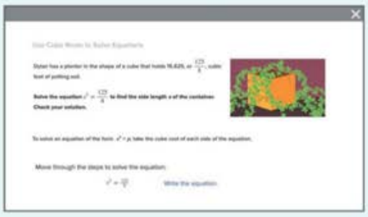
A box that is shaped like a cube has a volume of $\frac{512}{27}$ cubic inches. Solve $s^3 = \frac{512}{27}$ to find the length s of one side of the box.

$\frac{8}{3}$ or $2\frac{2}{3}$ inches

Go Online You can complete an Extra Example online.

294 Module 5 • Real Numbers

Interactive Presentation



Example 8, Use Cube Roots to Solve Equations, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to solve the equation.

TYPE

On Slide 1, students enter the correct solution.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 8 Use Cube Roots to Solve Equations

Objective

Students will solve equations of the form $x^3 = p$.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure Encourage students to analyze the structure of the equation and the inverse relationship between cubing a number and taking the cube root of a number.

Questions for Mathematical Discourse

SLIDE 1

AL What does s represent? the side length of the container

OL Why do we take the cube root of each side of the equation? To undo the operation of cubing the side length s, take the cube root. Cubing a number and taking the cube root are inverse operations.

OL How can you check your answer? Sample answer: I can check that the volume of a cube with side length $2\frac{1}{2}$ feet is $\frac{125}{8}$ cubic feet by finding $\left(\frac{5}{2}\right)^3 = \frac{125}{8}$.

BL If the side length of the container was 2.5 feet, how would the original equation be altered? $s^3 = 15.625$

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Bulletin Boards

Objective

Students will come up with their own strategy to solve an application problem involving a bulletin board display.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

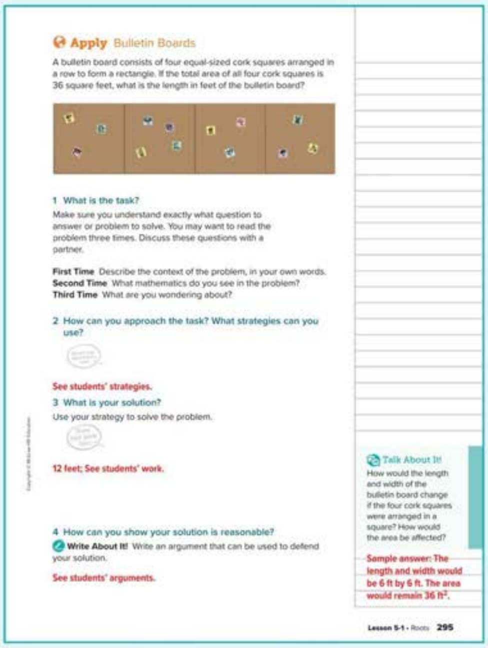
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you find the area of one bulletin board?
- What do you know about the lengths of the sides of a square?
- How can you solve a problem involving perfect squares?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Bulletin Boards

A bulletin board consists of four equal-sized cork squares arranged in a row to form a rectangle. If the total area of all four cork squares is 36 square feet, what is the length in feet of the bulletin board?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

12 feet; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
How would the length and width of the bulletin board change if the four cork squares were arranged in a square? How would the area be affected?

Sample answer: The length and width would be 6 ft by 6 ft. The area would remain 36 ft^2.

Lesson 5-1 • Roots 295

Interactive Presentation

Apply, Bulletin Boards

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A set of windows consists of three equal-sized squares arranged in a row to form a rectangle. If the total area of all three windows is 108 square feet, what is the length, in feet, of the windows?

18 feet

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that will help you study the vocabulary and concepts from this lesson.

See students' observations.

296 Module 5 • Real Numbers

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Find the approximate length of each side of the square base in the actual Great Pyramid of Giza. Then find the length of each side of the square base in its replica. Write a mathematical argument that can be used to defend your solution. The length of each side of the square base in the actual Great Pyramid of Giza is about 750 feet. The length of each side of the square base in its replica is 1.2 feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11, 13–16
- Extension: n th Roots, Simplify Radicals
- ALEKS Square Roots and Irrational Numbers, Higher Roots and Nonlinear Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 11, 13, 16
- Extension: n th Roots, Simplify Radicals
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 –8
- ALEKS Square Roots and Irrational Numbers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Square Roots and Irrational Numbers

Ablestock/Alamy Stock Photo

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

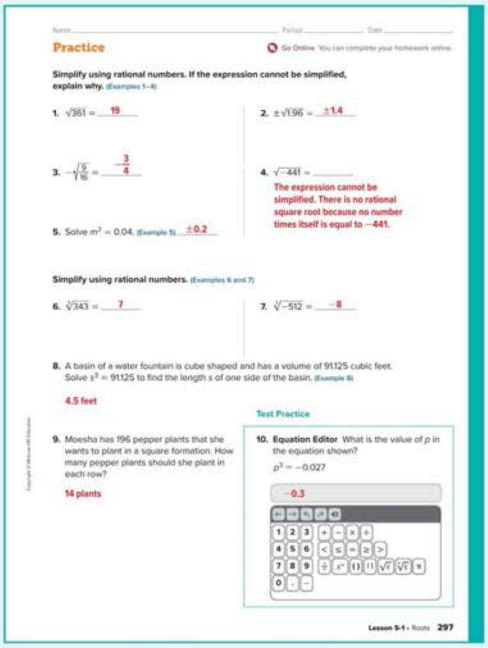
Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	determine the positive and negative square roots of numbers	1–4
1	solve equations of the form $x^2 = p$	5
1	find the cube root of positive and negative numbers	6, 7
2	solve equations of the form $x^3 = p$	8
2	extend concepts learned in class to apply them in new contexts	9, 10
3	solve application problems involving square roots	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Students often think that the square root of a number is equal to both the positive and negative roots, because there are two unique roots associated with the radicand. Teach students to use precise mathematical language to accurately identify which root they are referencing. The radical sign denotes the positive root. A negative sign before the radical sign denotes the negative root.

Similarly, some students may place a negative sign inside of the square root to indicate the negative root of the radicand. Remind students that the definition of a square root is a value that can be multiplied by itself to produce the radicand. Demonstrate to students that there is not a number that, when multiplied by itself, will produce a negative number. Therefore, to indicate the negative root, the negative sign must be placed before the radical sign.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Simplify using rational numbers. If the expression cannot be simplified, explain why. (Examples 1–4)

1. $\sqrt{361} =$ 19

2. $\pm\sqrt{1.96} =$ ±1.4

3. $-\sqrt{\frac{9}{16}} =$ $-\frac{3}{4}$

4. $\sqrt{-441} =$ ______ The expression cannot be simplified. There is no rational square root because no number times itself is equal to −441.

5. Solve $m^2 = 0.04$. (Example 5) ±0.2

Simplify using rational numbers. (Examples 6 and 7)

6. $\sqrt[3]{343} =$ 7

7. $\sqrt[3]{-512} =$ −8

8. A basin of a water fountain is cube shaped and has a volume of 91.125 cubic feet. Solve $s^3 = 91.125$ to find the length s of one side of the basin. (Example 8)

4.5 feet

9. Moesha has 196 pepper plants that she wants to plant in a square formation. How many pepper plants should she plant in each row?

14 plants

Test Practice

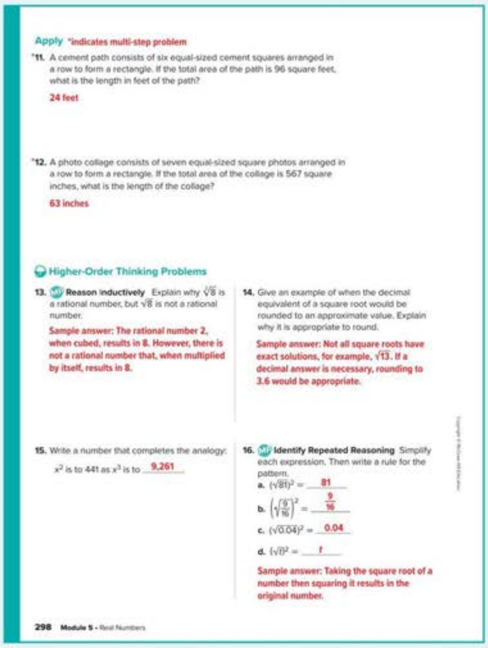
10. **Equation Editor** What is the value of p in the equation shown?

$p^3 = -0.027$

−0.3

Lesson 5-1 • Roots 297

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*11. A cement path consists of six equal-sized cement squares arranged in a row to form a rectangle. If the total area of the path is 96 square feet, what is the length in feet of the path?

24 feet

*12. A photo collage consists of seven equal-sized square photos arranged in a row to form a rectangle. If the total area of the collage is 567 square inches, what is the length of the collage?

63 inches

Higher-Order Thinking Problems

13. **Reason Inductively** Explain why $\sqrt[3]{8}$ is a rational number, but $\sqrt{8}$ is not a rational number.

Sample answer: The rational number 2, when cubed, results in 8. However, there is not a rational number that, when multiplied by itself, results in 8.

14. Give an example of when the decimal equivalent of a square root would be rounded to an approximate value. Explain why it is appropriate to round.

Sample answer: Not all square roots have exact solutions, for example, $\sqrt{13}$. If a decimal answer is necessary, rounding to 3.6 would be appropriate.

15. Write a number that completes the analogy: x^2 is to 441 as x^3 is to **9,261**

16. **Identify Repeated Reasoning** Simplify each expression. Then write a rule for the pattern.

a. $(\sqrt{81})^2 =$ **81**

b. $\left(\sqrt{\frac{9}{16}}\right)^2 =$ **$\frac{9}{16}$**

c. $(\sqrt{0.04})^2 =$ **0.04**

d. $(\sqrt{f})^2 =$ **f**

Sample answer: Taking the square root of a number then squaring it results in the original number.

298 Module 5 • Real Numbers

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students will reason why the square root of 8 is not a rational number but the cube root of 8 is a rational number.

8 Look for and Express Regularity in Repeated Reasoning In Exercise 16, students will study the expressions and determine a rule for the pattern. Students should focus on a rule that is easily identifiable among all four expressions.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 11–12 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Be sure everyone understands.
Use with Exercises 15–16 Have students work in groups of 3–4 to solve the problem in Exercise 15. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 16.

Lesson 5-2

Real Numbers

8.NS.A.1, 8.EE.A.2

LESSON GOAL

Students will identify and describe sets of numbers in the real number system.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Real Numbers

Learn: Real Numbers
Example 1: Identify Real Numbers
Example 2: Classify Real Numbers
Example 3: Classify Real Numbers
Example 4: Classify Real Numbers
Learn: Describe Sets of Real Numbers
Example 5: Describe Sets of Real Numbers
Example 6: Describe Sets of Real Numbers

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Imaginary Numbers		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 30 of the *Language Development Handbook* to help your students build mathematical language related to the real number system.

ELL You can use the tips and suggestions on page T30 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address the major cluster **8.EE.A** and the supporting cluster **8.NS.A** by identifying and describing sets of numbers in the real number system.
Standards for Mathematical Content: **8.NS. A.1, 8.EE.A.2**
Standards for Mathematical Practice: **MP 2, MP3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found square and cube roots.
8.EE.A.2

Now
Students identify and describe sets of numbers in the real number system.
8.NS.A.1, 8.EE.A.2

Next
Students will estimate irrational numbers.
8.NS.A.2

Rigor

The Three Pillars of Rigor

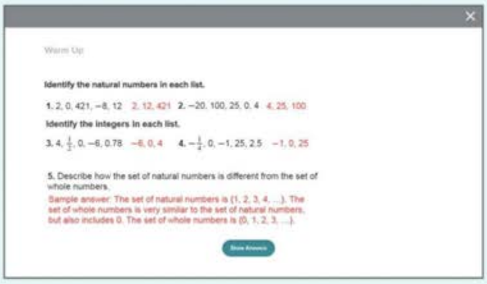
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of the set of rational numbers to develop *understanding* of irrational numbers and the set of real numbers. They learn how the different sets of numbers (natural, whole, ...) are related, and that all numbers that can be graphed on a number line are real numbers. They build *fluency* with classifying numbers into different subsets of real numbers.

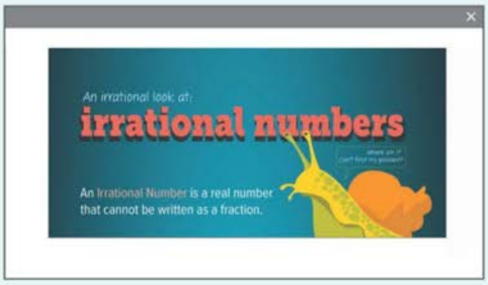
Mathematical Background

Go online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- identifying rational number sets (natural numbers, whole numbers, integers) (Exercises 1–5)

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about what it means for a number to be irrational.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The prefix *counter-* comes from the Latin term *contra*, which means *against*, or *contrary to*. What do you think a *counterexample* might be? Sample answer: A counterexample might be an example that is against, or contrary to, a certain argument.
- The prefix *ir-* means not. What are some other words that begin with the prefix *ir-*? Make a prediction for what you think an *irrational number* might be. Sample answer: irresponsible, irreplaceable, irrevocable; An irrational number might be a number that is not rational.
- Use the Internet or another source to look up the definition of *real number* in mathematics. Then give an example of a real number. Sample answer: A real number is a number that can be found on the number line, such as 2.5 or π.

Explore Real Numbers

Objective

Students will use number lines to explore the set of real numbers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with various rational and irrational numbers to classify or graph on a number line. Throughout this activity, students will use decimals to graph numbers on a number line and use decimal expansions to classify numbers as rational or irrational.

Inquiry Question

What different types of numbers can be found on the number line? Sample answer: Fractions, integers, square roots, and decimals, including those that terminate or repeat eventually and those that never repeat nor terminate, can be graphed on the number line.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

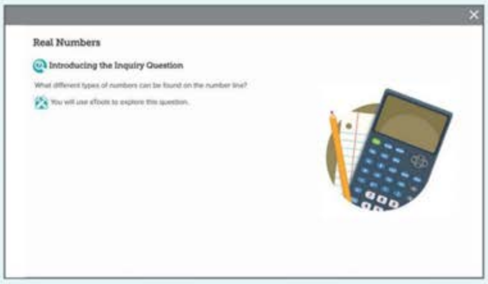
SLIDE 2

Mathematical Discourse

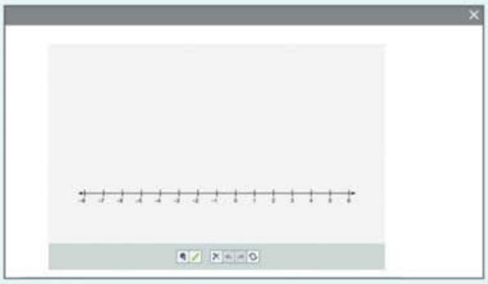
What method did you use to graph $\sqrt{25}$ on the number line?
Sample answer: I first simplified the square root; $\sqrt{25} = 5$.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 5

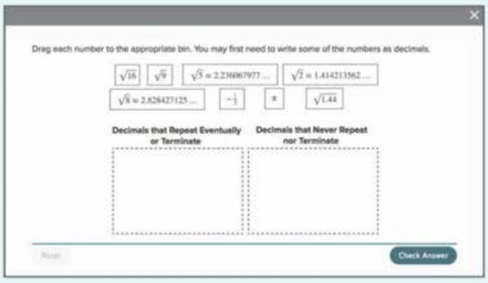


Explore, Slide 2 of 5

eTOOLS

Throughout the Explore, students use the Number Line eTool to graph a set of integers on a number line.

Interactive Presentation



Explore, Slide 4 of 5

DRAG AND DROP

On Slide 4, students drag to sort each number as to whether its decimal form repeats eventually or never repeats.

TYPE

On Slide 5, students respond to the Inquiry Question and can view a sample answer.

Explore Real Numbers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Number Line eTool to explore and examine real numbers.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Describe the numbers in the *Decimals that Never Repeat nor Terminate* bin. Sample answer: The decimals do not terminate nor repeat. Some are square roots of non-perfect squares.

Do you think that the numbers in this bin can be graphed on the number line? Why or why not? Sample answer: Yes; the approximate location of the numbers can be graphed on the number line. For example, since $\sqrt{2} \approx 1.414213562...$, then $\sqrt{2}$ can be graphed between 1 and 2 on the number line.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Real Numbers

Objective

Students will understand that the set of real numbers are numbers that can be found on the number line.

Teaching the Mathematical Practices

6 Attend to Precision While discus sing the *Talk About It!* questions on Slide 3, encourage students to understand and be able to apply the definitions of rational and irrational numbers in order to explain why the given numbers are irrational.

Go Online to have students watch the animation on Slide 1. The animation illustrates real numbers.

Teaching Notes

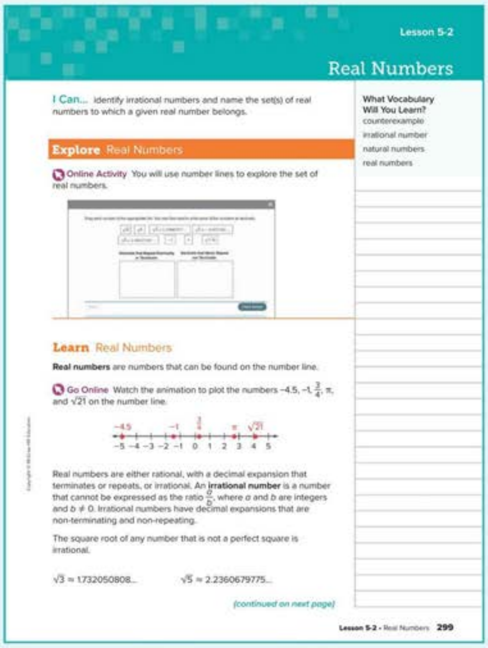
SLIDE 1

Play the animation for the class. For each real number given, you may wish to pause the animation and ask students where that number is located on the number line, and how they determined that location.

SLIDE 2

Students will learn about *irrational numbers* and view a Venn diagram to learn about the classification of real numbers as rational numbers, irrational numbers, integers, whole numbers, and natural numbers. The set of real numbers includes both rational and irrational numbers. Have students select each button on the Venn diagram to include examples of each type of number. You may wish to have the class discuss the location of each type of number.

(continued on next page)


Lesson 5-2

Real Numbers

I Can... identify irrational numbers and name the set(s) of real numbers to which a given real number belongs.

What Vocabulary Will You Learn?
counterexample
irrational number
natural numbers
real numbers

Explore Real Numbers

Online Activity You will use number lines to explore the set of real numbers.

Learn Real Numbers

Real numbers are numbers that can be found on the number line.

Go Online Watch the animation to plot the numbers $-4.5, -1, \frac{3}{4}, \pi,$ and $\sqrt{21}$ on the number line.

Real numbers are either rational, with a decimal expansion that terminates or repeats, or irrational. An **irrational number** is a number that cannot be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Irrational numbers have decimal expansions that are non-terminating and non-repeating.

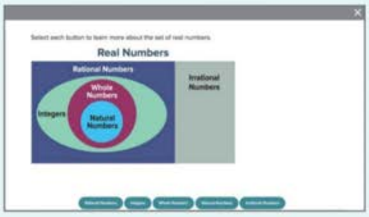
The square root of any number that is not a perfect square is irrational.

$\sqrt{3} \approx 1.732050808...$ $\sqrt{5} \approx 2.2360679775...$

(continued on next page)

Lesson 5-2 • Real Numbers 299

Interactive Presentation



Learn, Real Numbers, Slide 2 of 3

WATCH

On Slide 1, students watch an animation to learn about the set of real numbers.

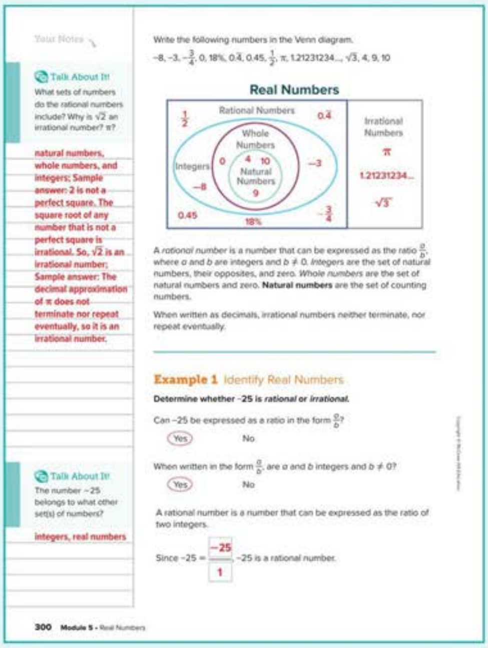
CLICK

On Slide 2, students select each button to view examples of real numbers.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of the set of real numbers, have them generate two additional numbers that can be placed into each category on the Venn diagram. They should be prepared to defend why they placed their chosen numbers into each category.


Your Notes

Talk About It!
What sets of numbers do the rational numbers include? Why is $\sqrt{2}$ an irrational number? π?

natural numbers, whole numbers, and integers; Sample answer: 2 is not a perfect square. The square root of any number that is not a perfect square is irrational. So, $\sqrt{2}$ is an irrational number; Sample answer: The decimal approximation of π does not terminate nor repeat eventually, so it is an irrational number.

Write the following numbers in the Venn diagram.
$-8, -3, -\frac{3}{4}, 0, 18\%, 0.\bar{4}, 0.45, \frac{1}{2}, \pi, 1.21231234..., \sqrt{3}, 4, 9, 10$

A *rational number* is a number that can be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$. *Integers* are the set of natural numbers, their opposites, and zero. *Whole numbers* are the set of natural numbers and zero. **Natural numbers** are the set of counting numbers.

When written as decimals, irrational numbers neither terminate, nor repeat eventually.

Example 1 Identify Real Numbers

Determine whether −25 is *rational* or *irrational*.

Can −25 be expressed as a ratio in the form $\frac{a}{b}$?
(Yes) No

When written in the form $\frac{a}{b}$, are a and b integers and $b \neq 0$?
(Yes) No

A rational number is a number that can be expressed as the ratio of two integers.

Since $-25 = \frac{-25}{1}$, −25 is a rational number.

Talk About It!
The number −25 belongs to what other set(s) of numbers?

integers, real numbers

300 Module 5 • Real Numbers

Interactive Presentation

Example 1, Identify Real Numbers, Slide 2 of 4

CLICK

On Slide 2 of Example 1, students respond to a series of questions to determine if the number is rational or irrational.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Real Numbers *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

What sets of numbers do the rational numbers include? Why is $\sqrt{2}$ an irrational number? π? natural numbers, whole numbers, and integers; Sample answer: 2 is not a perfect square. The square root of any number that is not a perfect square is irrational. So, $\sqrt{2}$ is an irrational number; Sample answer: The decimal approximation of π does not terminate nor repeat eventually, so it is an irrational number.

Example 1 Identify Real Numbers

Objective

Students will determine whether a number is rational or irrational.

MP Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to use clear and precise mathematical language to classify the number −25 as *rational* or *irrational*. While discussing the *Talk About It!* question on Slide 3, encourage students to use the precise mathematical terminology when identifying the sets of numbers to which −25 belongs.

Questions for Mathematical Discourse

SLIDE 2

AL Can you write −25 as the ratio of two integers? Explain. yes; Sample answer: Write −25 as the numerator, and 1 as the denominator.

OL Are all integers rational numbers? Explain. yes; Sample answer: Every integer can be written as a fraction with a denominator of 1.

BL Explain why −25 is not a whole number. The set of whole numbers includes zero and positive integers, not negative integers.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Classify Real Numbers

Objective

Students will identify the real number set(s) to which a decimal or fraction belongs.

Teaching the Mathematical Practices

7 Look for and Make Use of Structure While discussing the *Talk About It!* question on Slide 3, encourage students to analyze the structure of the decimal in order to write it another way, such as using bar notation.

Questions for Mathematical Discourse

SLIDE 2

AL Explain whether the decimal terminates (repeats zeros) or repeats non-zero digits. The decimal repeats the non-zero digits 2 and 5.

OL Explain why the number is rational, but not an integer. Sample answer: The number can be written as the fraction $\frac{25}{99}$. It is not an integer because the denominator of the fraction is not 1.

BL A classmate states that any number with a repeating decimal is rational. Use reasoning to explain why this is correct. Sample answer: Any number with a repeating decimal can be written as a ratio of two integers.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Students may only select one real number set when a number belongs to more than one number set. Provide pairs of students with blank Venn diagrams, similar to the one shown on page 92. Give students the following numbers. Ask them to write each number inside its appropriate section. Then have them list the other sections in which that number is also located. They may have trouble seeing that a smaller section is a subset of a larger section. Have them classify each number by using the sentence structures below. Then have them generate 3-4 of their own numbers and trade with another pair of students to correctly classify each number.

12 The number 12 is a natural number. All natural numbers are whole numbers, so 12 is also a whole number. All whole numbers are integers, so 12 is also an integer. All integers are rational numbers, so 12 is also a rational number.

−4 The number −4 is an integer. All integers are rational numbers, so −4 is also a rational number.

Check

Determine whether $-\frac{2}{9}$ is *rational* or *irrational*.

rational

Example 2 Classify Real Numbers

Name all the sets of numbers to which the real number 0.2525... belongs.

The set of real numbers include natural numbers, whole numbers, integers, rational numbers, and irrational numbers.

Can 0.2525... be expressed as a ratio in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$?

(Yes) No

Is 0.2525... from the set of integers {..., −3, −2, −1, 0, 1, 2, 3, ...}?

Yes (No)

So, 0.2525... is a rational number because it is equivalent to $\frac{25}{99}$.

Talk About It! What is another way to write 0.2525...?

Sample answer: Using bar notation: $0.\overline{25}$

Check

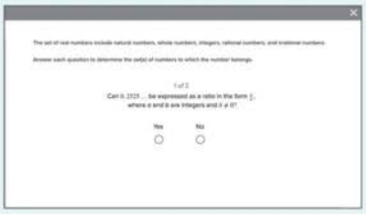
Name all the sets of numbers to which the real number $\frac{21}{\sqrt{4}}$ belongs.

rational

Go Online You can complete an Extra Example online.

Lesson 5-2 • Real Numbers 301

Interactive Presentation



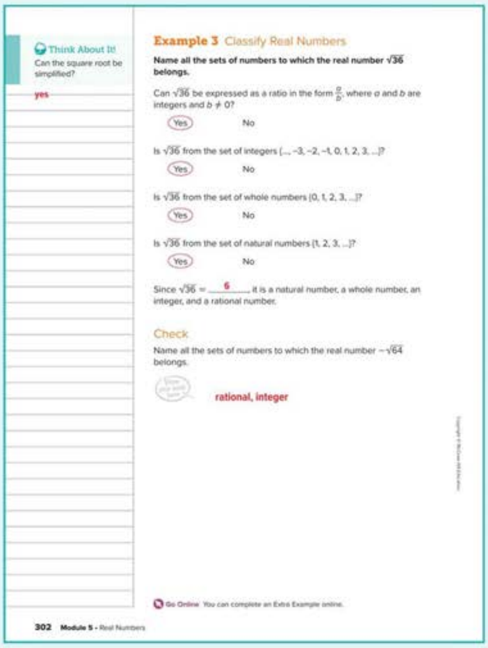
Example 2, Classify Real Numbers, Slide 2 of 4

CLICK

On Slide 2, students respond to a series of questions to classify the number.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! Can the square root be simplified?

yes

Example 3 Classify Real Numbers

Name all the sets of numbers to which the real number $\sqrt{36}$ belongs.

Can $\sqrt{36}$ be expressed as a ratio in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$?

(Yes) No

Is $\sqrt{36}$ from the set of integers {..., −3, −2, −1, 0, 1, 2, 3, ...}?

(Yes) No

Is $\sqrt{36}$ from the set of whole numbers {0, 1, 2, 3, ...}?

(Yes) No

Is $\sqrt{36}$ from the set of natural numbers {1, 2, 3, ...}?

(Yes) No

Since $\sqrt{36}$ = 6, it is a natural number, a whole number, an integer, and a rational number.

Check

Name all the sets of numbers to which the real number $-\sqrt{64}$ belongs.

rational, integer

Go Online You can complete an Extra Example online.

302 Module 5 • Real Numbers

Interactive Presentation

Example 3, Classify Real Numbers, Slide 2 of 3

CLICK

On Slide 2, students respond to a series of questions to classify the number.

TYPE

On Slide 2, students enter the correct value of the radical.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Classify Real Numbers

Objective

Students will identify the real number set(s) to which a square root of a perfect square belongs.

Teaching the Mathematical Practices

6 Attend to Precision Students should use the correct terminology to name the number $\sqrt{36}$ as a natural number, a whole number, an integer, a rational number, and a real number.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the square root, noting that it can be simplified.

Questions for Mathematical Discourse

SLIDE 2

AL Simplify $\sqrt{36}$. 6

OL Identify all the sets of numbers to which 6 belongs. natural numbers, whole numbers, integers, rational numbers, and real numbers

OL Explain why any natural number is also a whole number, an integer, a rational number, and a real number. Sample answer: The set of natural numbers is a subset of all of these other sets of numbers, so any natural number will also belong to these sets of numbers.

BL How would $-\sqrt{36}$ be classified differently? $-\sqrt{36} = -6$, so it belongs to the set of integers, rational numbers, and real numbers. It is neither a whole nor a natural number.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Classify Real Numbers

Objective

Students will identify the real number set(s) to which an irrational number belongs.

Teaching the Mathematical Practices

6 Attend to Precision Students should use the correct terminology to name the number $-\sqrt{7}$ as an irrational number.

7 Look for and Make Use of Structure Encourage students to analyze the structure of the square root, noting that it cannot be simplified. While discussing the *Talk About It!* question on Slide 3, encourage students to analyze the structure of the expression $-\sqrt{7}$ in order to determine why the number is irrational.

Questions for Mathematical Discourse

SLIDE 2

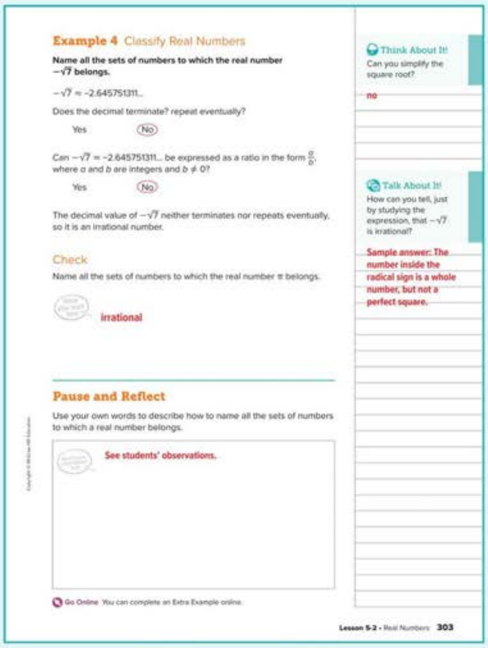
AL Can you simplify $\sqrt{7}$? no

OL Is $-\sqrt{7}$ rational or irrational? irrational

BL Would $\sqrt{7}$ be classified any differently than $-\sqrt{7}$? Explain. no; Sample answer: $\sqrt{7}$ and $-\sqrt{7}$ are both irrational because each number cannot be written as the ratio of two integers.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Classify Real Numbers

Name all the sets of numbers to which the real number $-\sqrt{7}$ belongs.

$-\sqrt{7} \approx -2.645751311...$

Does the decimal terminate? repeat eventually?

Yes (No)

Can $-\sqrt{7} = -2.645751311...$ be expressed as a ratio in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$?

Yes (No)

The decimal value of $-\sqrt{7}$ neither terminates nor repeats eventually, so it is an irrational number.

Check

Name all the sets of numbers to which the real number π belongs.

irrational

Pause and Reflect

Use your own words to describe how to name all the sets of numbers to which a real number belongs.

See students' observations.

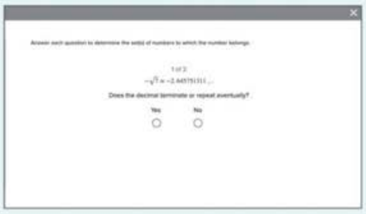
Go Online You can complete an Extra Example online.

Think About It! Can you simplify the square root? no

Talk About It! How can you tell, just by studying the expression, that $-\sqrt{7}$ is irrational? Sample answer: The number inside the radical sign is a whole number, but not a perfect square.

Lesson 5-2 • Real Numbers 303

Interactive Presentation



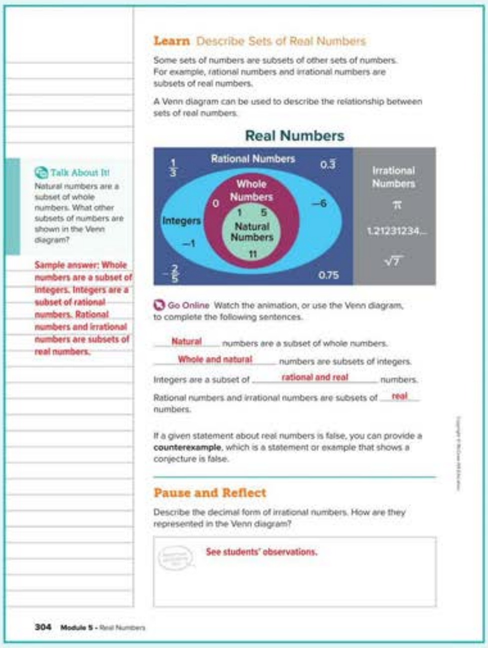
Example 4, Classify Real Numbers, Slide 2 of 4

CLICK

On Slide 2, students respond to a series of questions to classify the number.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Describe Sets of Real Numbers

Some sets of numbers are subsets of other sets of numbers. For example, rational numbers and irrational numbers are subsets of real numbers.

A Venn diagram can be used to describe the relationship between sets of real numbers.

Talk About It! Natural numbers are a subset of whole numbers. What other subsets of numbers are shown in the Venn diagram?

Sample answer: Whole numbers are a subset of integers. Integers are a subset of rational numbers. Rational numbers and irrational numbers are subsets of real numbers.

Go Online Watch the animation, or use the Venn diagram, to complete the following sentences.

Natural numbers are a subset of whole numbers.

Whole and natural numbers are subsets of integers.

Integers are a subset of rational and real numbers.

Rational numbers and irrational numbers are subsets of real numbers.

If a given statement about real numbers is false, you can provide a **counterexample**, which is a statement or example that shows a conjecture is false.

Pause and Reflect

Describe the decimal form of irrational numbers. How are they represented in the Venn diagram?

See students' observations.

304 Module 5 • Real Numbers

Interactive Presentation

Learn, Describe Sets of Real Numbers, Slide 2 of 2

WATCH

On Slide 1, students watch the animation to see how a Venn diagram can be used to describe the relationship between sets of real numbers.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Describe Sets of Real Numbers

Objective

Students will learn how to describe the relationship between sets of real numbers.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discussing the *Talk About It!* question on Slide 2, encourage students to make sense of the sets of numbers and their relationships to one another that are shown in the Venn diagram.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates sets of real numbers.

Talk About It!

SLIDE 2

Mathematical Discourse

Natural numbers are a subset of whole numbers. What other subsets of numbers are shown in the Venn diagram? Sample answer: Whole numbers are a subset of integers. Integers are a subset of rational numbers. Rational numbers and irrational numbers are subsets of real numbers.

DIFFERENTIATE

Reteaching Activity AL

If any of your students have difficulty in determining what subsets are, have students discuss the concept of subsets using a real-world example, such as the students in the classroom. Each of the following may be subsets of the larger set of students in the classroom – female students, male students, students wearing T-shirts, students wearing blue, left-handed students, etc. Have students generate other subsets that are possible. Then have them explain whether or not there is any overlap between the subsets. For example, are there any female students who are wearing blue T-shirts?

Example 5 Describe Sets of Real Numbers

Objective

Students will describe the relationship between sets of real numbers.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others Students should be able to generate a counterexample supporting their claim. While discussing the *Talk About It!* question on Slide 4, encourage students to use mathematical reasoning to provide additional counterexamples showing the statement is false.

Questions for Mathematical Discourse

SLIDE 2

AL How can you use the Venn diagram to determine whether there are rational numbers that are not integers? Sample answer: The oval representing integers is completely inside the rectangle representing rational numbers. There is space inside the rectangle representing rational numbers that is not inside the oval representing integers.

OL Explain why it makes sense that every integer is a rational number, but not every rational number is an integer. Sample answer: Every integer can be written as the ratio/fraction of two integers, where the denominator is 1. There are some rational numbers that do not have denominators of 1, so those would not be integers.

BL Do integers and irrational numbers belong to the same subset? Explain. yes; Sample answer: Both integers and irrational numbers are subsets of real numbers.

SLIDE 3

AL What is a counterexample? Sample answer: an example that is used to show that a statement is false

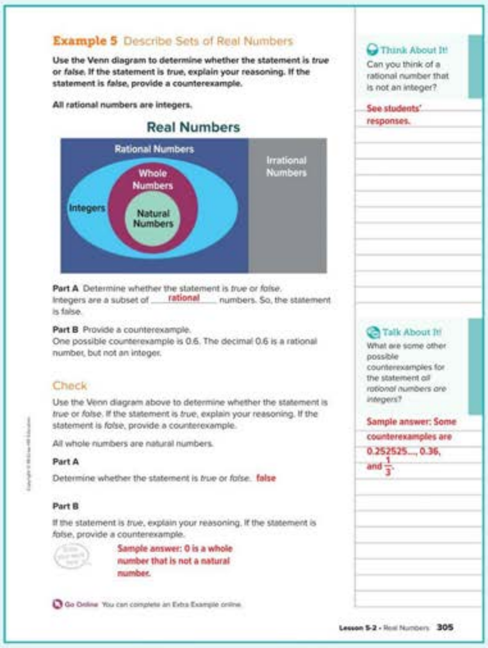
OL What kind of number would every counterexample have to be, in this situation? a rational number that is not an integer

OL Explain why 0.6 is a valid counterexample. It is a rational number, but not an integer.

BL Explain why $\frac{246}{2}$ cannot be used as a counterexample. $\frac{246}{2} = 123$, so it is an integer.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 5 Describe Sets of Real Numbers

Use the Venn diagram to determine whether the statement is *true* or *false*. If the statement is *true*, explain your reasoning. If the statement is *false*, provide a counterexample.

All rational numbers are integers.

Part A Determine whether the statement is *true* or *false*.
Integers are a subset of rational numbers. So, the statement is false.

Part B Provide a counterexample.
One possible counterexample is 0.6. The decimal 0.6 is a rational number, but not an integer.

Check

Use the Venn diagram above to determine whether the statement is *true* or *false*. If the statement is *true*, explain your reasoning. If the statement is *false*, provide a counterexample.

All whole numbers are natural numbers.

Part A

Determine whether the statement is *true* or *false*. false

Part B

If the statement is *true*, explain your reasoning. If the statement is *false*, provide a counterexample.

Sample answer: 0 is a whole number that is not a natural number.

Go Online You can complete an Extra Example online.

Think About It! Can you think of a rational number that is not an integer?

See students' responses.

Talk About It! What are some other possible counterexamples for the statement *all rational numbers are integers?*

Sample answer: Some counterexamples are 0.252525..., 0.36, and $\frac{1}{3}$.

Lesson 5-2 • Real Numbers 305

Interactive Presentation

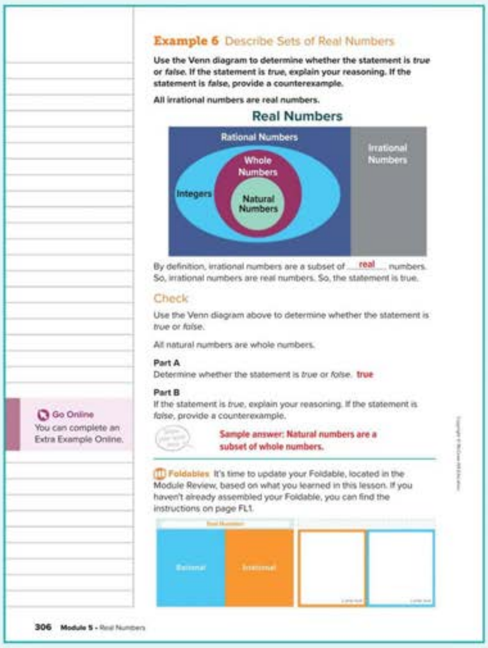
Example 5, Describe Sets of Real Numbers, Slide 2 of 5

CLICK

On Slide 2, students determine whether the statement is true or false.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 6 Describe Sets of Real Numbers

Use the Venn diagram to determine whether the statement is *true* or *false*. If the statement is *true*, explain your reasoning. If the statement is *false*, provide a counterexample.

All irrational numbers are real numbers.

By definition, irrational numbers are a subset of real numbers. So, irrational numbers are real numbers. So, the statement is true.

Check

Use the Venn diagram above to determine whether the statement is *true* or *false*.

All natural numbers are whole numbers.

Part A
Determine whether the statement is *true* or *false*. true

Part B
If the statement is *true*, explain your reasoning. If the statement is *false*, provide a counterexample.

Sample answer: Natural numbers are a subset of whole numbers.

Go Online You can complete an Extra Example Online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

306 Module 5 • Real Numbers

Interactive Presentation

Example 6, Describe Sets of Real Numbers

CLICK On Slide 1, students determine whether the statement is true or false.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 6 Describe Sets of Real Numbers

Objective

Students will describe the relationship between sets of real numbers.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to understand the relationship between the sets of rational numbers and real numbers in order to make a case for why the given statement is true.

3 Construct Viable Arguments and Critique the Reasoning of Others Students should be able to explain their reasoning to support their claim.

Questions for Mathematical Discourse

SLIDE 1

AL How are the sections in the Venn diagram representing irrational numbers and real numbers related? The section representing irrational numbers is included entirely within the section representing real numbers.

OL Is the set of irrational numbers a subset of the real numbers? What does this mean? yes; Sample answer: If A is a subset of B, then every element of A is an element of B. This means that every irrational number is also a real number.

BL Is the statement *all real numbers are irrational numbers* true? Explain. no; Sample answer: The number 4 is a counterexample because it is a real number and not an irrational number.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could write about irrational numbers. You may wish to have students share their Foldables with a partner to compare the information they recorded.

Essential Question Follow-Up

Why do we classify numbers?

In this lesson, students learned about the different classifications of numbers that make up the set of real numbers. Encourage them to discuss with a partner when they would need to use a certain classification of number as the answer to a problem. For example, they would use a whole number to describe the number of buses needed to transport a given number of students.

Exit Ticket

Refer to the Exit Ticket slide. What type of number is the circumference of Earth, 7,926π miles? Write a mathematical argument that can be used to defend your solution. Sample answer: The circumference is an irrational number because it cannot be expressed as a ratio of two integers, meaning that it is not rational.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

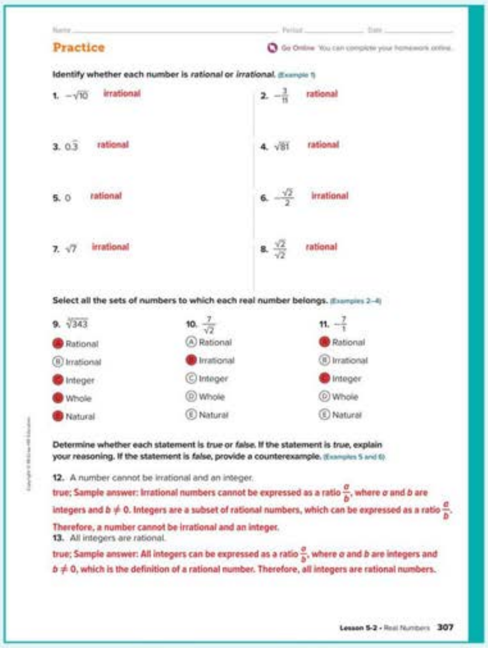
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	determine whether a number is rational or irrational	1–8
2	identify the real number sets to which decimals, fractions, square roots of perfect squares, and irrational numbers belong	9–11
2	describe sets of real numbers	12, 13
2	extend concepts learned in class to apply them in new contexts	14
3	higher-order and critical thinking skills	15–18

Common Misconception

Some students may incorrectly identify rational and irrational numbers. Encourage students to replicate the Venn diagram on page 92 to use when completing Exercises 1–8. Remind them to adhere to the definitions of rational and irrational numbers. If a number *cannot* be written as the ratio of two integers, then the number is not rational. In other words, it is irrational.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Identify whether each number is *rational* or *irrational*. (Example 1)

1. $-\sqrt{10}$ irrational
2. $-\frac{3}{11}$ rational
3. $0.\overline{3}$ rational
4. $\sqrt{81}$ rational
5. 0 rational
6. $-\frac{\sqrt{2}}{2}$ irrational
7. $\sqrt{7}$ irrational
8. $\frac{\sqrt{2}}{\sqrt{2}}$ rational

Select all the sets of numbers to which each real number belongs. (Examples 2–4)

9. $\sqrt[3]{343}$
(A) Rational (B) Irrational (C) Integer (D) Whole (E) Natural — selected: Rational, Integer, Whole, Natural

10. $\frac{7}{\sqrt{2}}$
(A) Rational (B) Irrational (C) Integer (D) Whole (E) Natural — selected: Irrational

11. $-\frac{7}{1}$
(A) Rational (B) Irrational (C) Integer (D) Whole (E) Natural — selected: Rational, Integer

Determine whether each statement is *true* or *false*. If the statement is true, explain your reasoning. If the statement is false, provide a counterexample. (Examples 5 and 6)

12. A number cannot be irrational and an integer.
true; Sample answer: Irrational numbers cannot be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$. Integers are a subset of rational numbers, which can be expressed as a ratio $\frac{a}{b}$. Therefore, a number cannot be irrational and an integer.

13. All integers are rational.
true; Sample answer: All integers can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$, which is the definition of a rational number. Therefore, all integers are rational numbers.

Lesson 5-2 • Real Numbers 307

Interactive Presentation

Exit Ticket

8.NS.A.1, 8.EE.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Test Practice

14. **Multiselect** Select the numbers that are part of the set of rational numbers.

- [x] $-\frac{11}{\sqrt{9}}$
- [ ] $\frac{1}{\sqrt{2}}$
- [ ] $\sqrt{-16}$
- [x] $\frac{\sqrt[3]{-4{,}096}}{\sqrt{16}}$
- [ ] $\sqrt[3]{4}$
- [x] 0.333...

Higher-Order Thinking Problems

15. **Use Math Tools** Explain how you could use a calculator to determine if $\sqrt{8}$ expressed as a decimal ever terminates.
Sample answer: I would use a calculator to find $\sqrt{8}$. The calculator shows 2.82842712474619. Then I would multiply that answer by itself, without using the x^2 button. If the solution is 8, then it is a terminating decimal. If the solution is not 8, then I know that the original solution was rounded by the calculator, and it is not a terminating decimal.

Justify Conclusions Determine whether each statement is *true* or *false*. If the statement is false, give a counterexample or explain your reasoning.

16. The product of a non-zero rational number and an irrational number is rational.
false; Sample answer: $2(\sqrt{2}) = 2.8284271...$

17. Expressing $\sqrt{2}$ as the ratio $\frac{\sqrt{2}}{1}$ means that $\sqrt{2}$ is a rational number.
false; The definition of a rational number is a number expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$. In the ratio $\frac{\sqrt{2}}{1}$, $\sqrt{2}$ is not an integer, therefore, it does not satisfy the definition of a rational number.

18. The product of two irrational numbers is irrational.
false; Sample answer: $(\sqrt{2})(\sqrt{2}) = 2$

308 Module 5 • Real Numbers

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically In E xercise 15, students will explain how they could use a calculator to determine if $\sqrt{8}$ expressed as a decimal ever terminates. Students should mention squaring the result of $\sqrt{8}$ to determine whether the decimal terminates or not.

3 Reason Abstractly and Quantitatively In Exercises 16-18, students should provide a counterexample or explanation to support their answer.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercises 16–18 After completing the exercises, have students write two true statements about rational or irrational numbers and one false statement. An example of a true statement might be, "The sum of two rational numbers is rational." An example of a false statement might be, "All square roots are irrational." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. For each false statement, have them generate a counterexample. Have them discuss and resolve any differences.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9–13 odd, 15–18
- Extension: Imaginary Numbers
- ALEKS Square Roots and Irrational Numbers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–13, 17
- Extension: Imaginary Numbers
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 –6
- ALEKS Venn Diagrams and Sets of Rational Numbers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Venn Diagrams and Sets of Rational Numbers

Lesson 5-3

Estimate Irrational Numbers

8.NS.A.2

LESSON GOAL

Students will estimate irrational numbers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Roots of Non-Perfect Squares

Learn: Estimate Irrational Numbers Using a Number Line
Example 1: Estimate Square Roots to the Nearest Integer
Example 2: Estimate Square Roots to the Nearest Tenth
Example 3: Estimate Cube Roots to the Nearest Integer
Learn: Estimate Irrational Numbers by Truncating
Example 4: Estimate by Truncating
Apply: Golden Rectangle

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 31 of the *Language Development Handbook* to help your students build mathematical language related to estimating irrational numbers.

ELL You can use the tips and suggestions on page T31 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: The Number System
Supporting Cluster(s): In this lesson, students address the supporting cluster **8.NS.A** by approximating irrational numbers using rational numbers.
Standards for Mathematical Content: **8.NS. A.2,** *Also addresses 8.EE.A.2*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students identified and described sets of numbers in the real number system.
8.NS.A.1, 8.EE.A.2

Now
Students estimate irrational numbers.
8.NS.A.2

Next
Students will compare and order numbers in the real number system.
8.NS.A.1, 8.NS.A.2

Rigor

The Three Pillars of Rigor

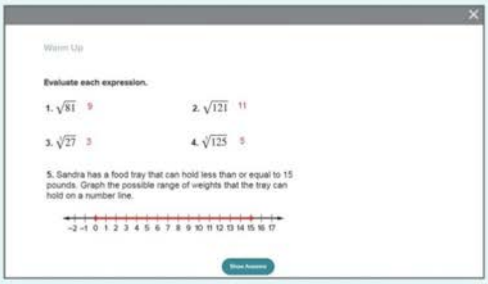
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to expand their *understanding* of the real number system by estimating the value of irrational numbers. They learn to estimate irrational roots by informal interpolation or by truncation to build *fluency* with estimating irrational numbers. They *apply* estimation to solve real-world problems involving irrational numbers.

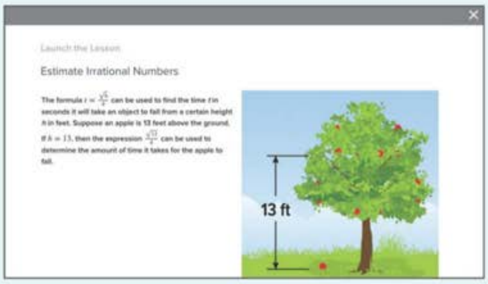
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- finding roots of perfect squares and roots of perfect cubes (Exercises 1–4)
- graphing on a number line (Exercise 5)

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the amount of time it will take for an object to fall to the ground, using estimation of irrational numbers.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The *verb truncate* means to shorten an object by cutting off the top or the end. What do you think it might mean to *truncate* the decimal 3.4555...? Sample answer: It might mean to shorten the decimal to just include the whole number part, 3.

Explore Roots of Non-Perfect Squares

Objective

Students will use Web Sketchpad to explore how to find the square root of a non-perfect square.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with two squares with integer-valued diagonals and be asked to find the side lengths of the squares. Throughout this activity, students will use square models to estimate the side lengths of the squares, which are irrational numbers.

Inquiry Question

How does a square model help you find the square root of a non-perfect square? Sample answer: The side length of the square is the square root of the area of the square. Place the side on a number line and estimate the length.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

How can you use your knowledge of triangles to find the area of the square? Sample answer: Four triangles that are the same shape and size are formed by the segments. Each triangle has a base of 3 units and a height of 3 units. So, the area of one triangle is $A = \frac{1}{2}(3)(3)$ or 4.5 square units. Multiply the area of the triangle by 4 to find the area of the square: $4.5(4) = 18$ square units.

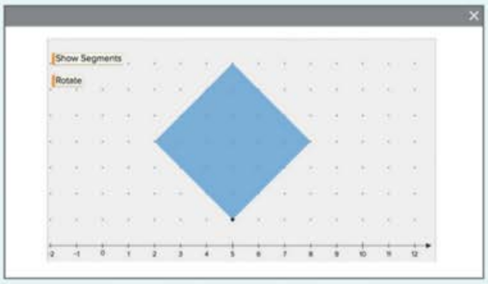
How can you find the side length of the square, given its area? Sample answer: I can find the side length of a square by taking the square root of the area.

What is the length of one side, written as a square root? $\sqrt{18}$

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6

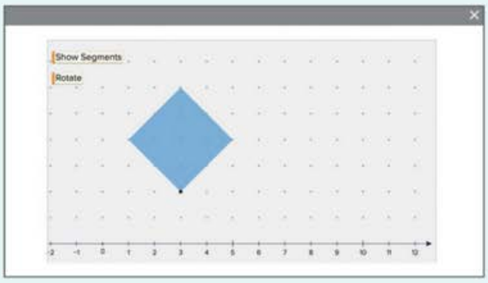


Explore, Slide 3 of 6

WEB SKETCHPAD

Throughout the Explore, student will use Web Sketchpad to explore how to find the square root of a non-perfect square.

Interactive Presentation



Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and can view a sample answer.

Explore Roots of Non-Perfect Squares *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore and examine how to find the square root of a non-perfect square.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

How does this square compare to the previous square? Sample answer: The area of this square is 8 square units, which is less than the area of the previous square. The side length of this square is about 2.8 units long, which is less than the side length of the previous square.

Can you use this same method to find the side length of any square? Sample answer: Yes; place the bottom edge of the square on the number line, with the point on zero, then find the side length of the square. If the side length of the square is between two whole numbers, then the side length will be an estimate.

Learn Estimate Irrational Numbers Using a Number Line

Objective

Students will learn how to estimate irrational numbers using a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage students to consider the relationships among the numbers 4, 8 and 9 in order to identify the relationships among the numbers $\sqrt{4}$, $\sqrt{8}$, and $\sqrt{9}$.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates estimating an irrational number on a number line.

Talk About It!

SLIDE 2

Mathematical Discourse

How do you know that $\sqrt{8}$ is between 2 and 3? How do you know that $\sqrt{8}$ is closer to 3 than 2? Sample answer: $\sqrt{8}$ is between $\sqrt{4}$ and $\sqrt{9}$, since 8 is between 4 and 9. 4 and 9 are the perfect squares above and below 8. Since $\sqrt{4} = 2$ and $\sqrt{9} = 3$, then $\sqrt{8}$ is between 2 and 3. Sample answer: $\sqrt{8}$ is closer to $\sqrt{9}$ than $\sqrt{4}$, since 8 is closer to 9 than 4. Since $\sqrt{9} = 3$ and $\sqrt{4} = 2$, then $\sqrt{8}$ is closer to 3 than 2.

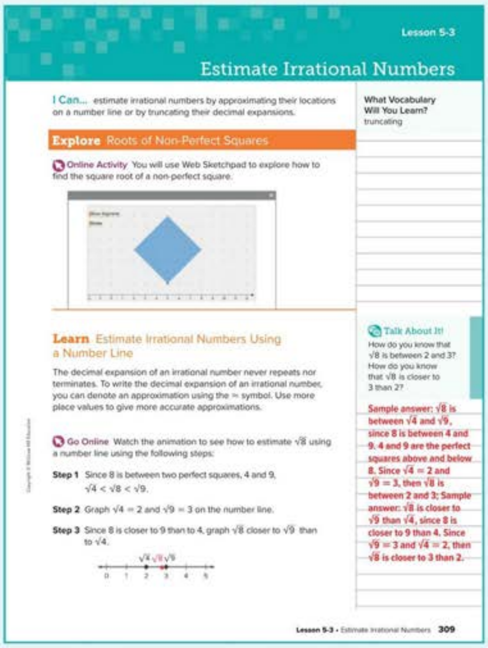
DIFFERENTIATE

Reteaching Activity AL

If any of your students have difficulty approximating the location of irrational numbers on the number line, they may struggle with identifying the two integers between which the number lies. Have them create a list of perfect squares to use as a reference. Have them work with a partner to describe the approximate location of each of the following irrational numbers by having them follow these steps.

1. Find the two integers between which the rational number lies. Write these integers as the square roots of perfect squares, as this will help them compare the numbers under the radicand more easily.
2. Determine the closer number by comparing the numbers under the radicand.
3. Simplify the square roots of perfect squares.

$\sqrt{11}$ $\sqrt{9}$ and $\sqrt{16}$; $\sqrt{11}$ is closer to $\sqrt{9}$; $\sqrt{11}$ is between 3 and 4, and closer to 3.
$\sqrt{17}$ $\sqrt{16}$ and $\sqrt{25}$; $\sqrt{17}$ is closer to $\sqrt{16}$; $\sqrt{17}$ is between 4 and 5, and closer to 4.
$\sqrt{61}$ $\sqrt{49}$ and $\sqrt{64}$; $\sqrt{61}$ is closer to $\sqrt{64}$; $\sqrt{61}$ is between 7 and 8, and closer to 8.


Lesson 5-3

Estimate Irrational Numbers

I Can... estimate irrational numbers by approximating their locations on a number line or by truncating their decimal expansions.

What Vocabulary Will You Learn?
truncating

Explore Roots of Non-Perfect Squares

Online Activity You will use Web Sketchpad to explore how to find the square root of a non-perfect square.

Learn Estimate Irrational Numbers Using a Number Line

The decimal expansion of an irrational number never repeats nor terminates. To write the decimal expansion of an irrational number, you can denote an approximation using the ≈ symbol. Use more place values to give more accurate approximations.

Go Online Watch the animation to see how to estimate $\sqrt{8}$ using a number line using the following steps:

Step 1 Since 8 is between two perfect squares, 4 and 9, $\sqrt{4} < \sqrt{8} < \sqrt{9}$.

Step 2 Graph $\sqrt{4} = 2$ and $\sqrt{9} = 3$ on the number line.

Step 3 Since 8 is closer to 9 than to 4, graph $\sqrt{8}$ closer to $\sqrt{9}$ than to $\sqrt{4}$.

Talk About It!
How do you know that $\sqrt{8}$ is between 2 and 3? How do you know that $\sqrt{8}$ is closer to 3 than 2?

Sample answer: $\sqrt{8}$ is between $\sqrt{4}$ and $\sqrt{9}$, since 8 is between 4 and 9. 4 and 9 are the perfect squares above and below 8. Since $\sqrt{4} = 2$ and $\sqrt{9} = 3$, then $\sqrt{8}$ is between 2 and 3; Sample answer: $\sqrt{8}$ is closer to $\sqrt{9}$ than $\sqrt{4}$, since 8 is closer to 9 than 4. Since $\sqrt{9} = 3$ and $\sqrt{4} = 2$, then $\sqrt{8}$ is closer to 3 than 2.

Lesson 5-3 • Estimate Irrational Numbers 309

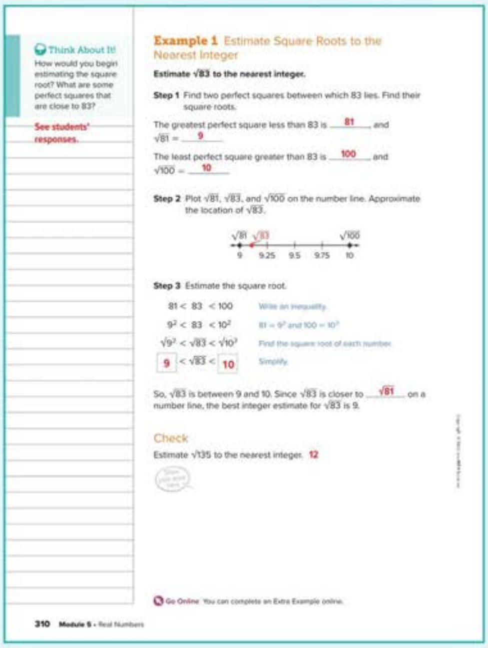
Interactive Presentation

Learn, Estimate Irrational Numbers on a Number Line, Slide 1 of 2

WATCH

On Slide 1, student watch the animation to see how to estimate $\sqrt{8}$ using a number line.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION


Think About It! How would you begin estimating the square root? What are some perfect squares that are close to 83? See students' responses.

Example 1 Estimate Square Roots to the Nearest Integer

Estimate $\sqrt{83}$ to the nearest integer.

Step 1 Find two perfect squares between which 83 lies. Find their square roots.

The greatest perfect square less than 83 is 81 and $\sqrt{81}$ = 9

The least perfect square greater than 83 is 100 and $\sqrt{100}$ = 10

Step 2 Plot $\sqrt{81}$, $\sqrt{83}$, and $\sqrt{100}$ on the number line. Approximate the location of $\sqrt{83}$.

Step 3 Estimate the square root.

$81 < 83 < 100$ Write an inequality.

$9^2 < 83 < 10^2$ $81 = 9^2$ and $100 = 10^2$

$\sqrt{9^2} < \sqrt{83} < \sqrt{10^2}$ Find the square root of each number.

$9 < \sqrt{83} < 10$ Simplify.

So, $\sqrt{83}$ is between 9 and 10. Since $\sqrt{83}$ is closer to $\sqrt{81}$ on a number line, the best integer estimate for $\sqrt{83}$ is 9.

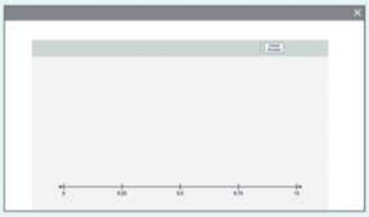
Check

Estimate $\sqrt{135}$ to the nearest integer. 12

Go Online You can complete an Extra Example online.

310 Module 5 • Real Numbers

Interactive Presentation



Example 1, Estimate Square Roots to the Nearest Integer, Slide 3 of 5

TYPE

On Slide 2, students enter values to complete the sentences.

eTOOLS

On Slide 3, students use the Number Line eTool to plot radicals on the number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Estimate Square Roots to the Nearest Integer

Objective

Students will estimate square roots to the nearest integer.

Questions for Mathematical Discourse

SLIDE 2

AL How can you identify perfect squares near 83? Sample answer: Find the squares of whole numbers to determine which ones are close to 83.

OL What are the two perfect squares between which 83 lies? $9^2 = 81$ and $10^2 = 100$

OL What are the square roots of these perfect squares? 9 and 10

BL Make a prediction as to whether $\sqrt{83}$ is closer to $\sqrt{81}$ or $\sqrt{100}$. Explain. Sample answer: Since 83 is closer to 81 than it is to 100, $\sqrt{83}$ is closer to $\sqrt{81}$ than it is to $\sqrt{100}$.

SLIDE 3

AL What is the value of $\sqrt{81}$? 9

OL How does the value of $\sqrt{83}$ compare to the values of $\sqrt{81}$ and $\sqrt{100}$? Sample answer: $\sqrt{83}$ is between $\sqrt{81}$ and $\sqrt{100}$, but closer to $\sqrt{83}$.

BL How could you test if $\sqrt{83}$ is less than or greater than 9.1? 9.2? Sample answer: Find the square of each number. Since $(9.1)^2 = 82.81$, and $(9.2)^2 = 84.64$, $\sqrt{83}$ is greater than 9.1, and less than 9.2

SLIDE 4

AL What does the inequality $81 < 83 < 100$ mean? Sample answer: 83 is between 81 and 100.

OL How do you know that $\sqrt{83}$ is closer to 9 than it is to 10? Sample answer: Since 83 is closer to 81 (the square of 9), than it is to 100 (the square of 10), $\sqrt{83}$ is closer to 9 than it is to 10.

BL Is $\sqrt{90}$ closer to 9 than it is to 10? $\sqrt{91}$? $\sqrt{90}$ is closer to 9, and $\sqrt{91}$ is closer to 10.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Estimate Square Roots to the Nearest Tenth

Objective

Students will estimate square roots to the nearest tenth.

Questions for Mathematical Discourse

SLIDE 2

AL Between which two integers does $\sqrt{83}$ lie? 9 and 10

OL How can you determine to which integer $\sqrt{83}$ is closest? Sample answer: Compare 83 to the perfect squares 81 and 100. Since 83 is much closer to 81 than to 100, $\sqrt{83}$ is closer to $\sqrt{81}$, which is 9.

BL Describe another strategy to determine which integer is closest to $\sqrt{83}$. Sample answer: $\sqrt{83}$ is between 9 and 10 and $(9.5)^2 = 90.25$. Since 83 is less than $(9.5)^2$, $\sqrt{83}$ is closer to 9 than to 10.

SLIDE 3

AL Why do we use question marks above the inequality symbols? Sample answer: Until we simplify, we are not sure that the inequality is true.

OL Why is it better to start with 9 and 9.1 instead of 9.4 and 9.5? Sample answer: Since 83 is very close to 81, it is likely that $\sqrt{83}$ is very close to 9.

BL Now that you know the inequality is not true, would it be better to choose the interval between 9.1 and 9.2, or 9.3 and 9.4? Explain. Sample answer: The inequality is not true, but 83 is close to 82.81, so it is better to choose the next interval between 9.1 and 9.2.

SLIDE 4

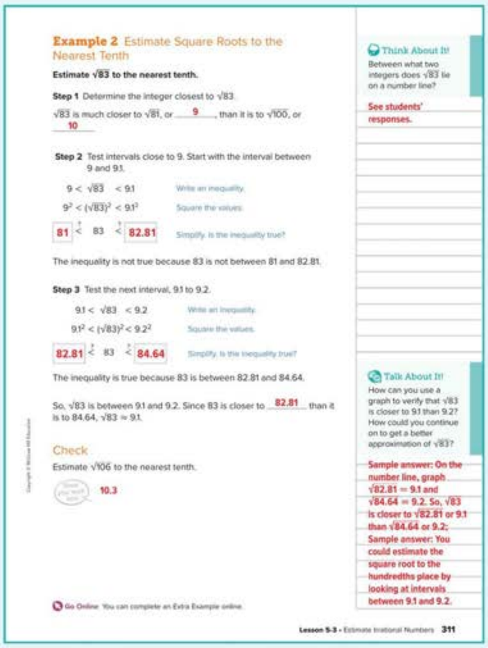
AL Why do we need to test the next interval between 9.1 and 9.2? Sample answer: When we tested the interval between 9 and 9.1, the inequality was not true.

OL Why is squaring 9.1 and 9.2 helpful in order to determine if $\sqrt{83}$ is in the interval between 9.1 and 9.2? Sample answer: Squaring the numbers allows us to compare them to 83 rather than comparing 9.1 and 9.2 to $\sqrt{83}$.

BL List some examples of intervals you can use to approximate $\sqrt{83}$ to the nearest hundredth. Sample answer: 9.11 to 9.12, 9.13 to 9.14

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Estimate Square Roots to the Nearest Tenth

Estimate $\sqrt{83}$ to the nearest tenth.

Step 1 Determine the integer closest to $\sqrt{83}$.

$\sqrt{83}$ is much closer to $\sqrt{81}$, or 9, than it is to $\sqrt{100}$, or 10.

Step 2 Test intervals close to 9. Start with the interval between 9 and 9.1.

$9 < \sqrt{83} < 9.1$ Write an inequality.

$9^2 < (\sqrt{83})^2 < 9.1^2$ Square the values.

$81 \overset{?}{<} 83 \overset{?}{<} 82.81$ Simplify. Is the inequality true?

The inequality is not true because 83 is not between 81 and 82.81.

Step 3 Test the next interval, 9.1 to 9.2.

$9.1 < \sqrt{83} < 9.2$ Write an inequality.

$9.1^2 < (\sqrt{83})^2 < 9.2^2$ Square the values.

$82.81 \overset{?}{<} 83 \overset{?}{<} 84.64$ Simplify. Is the inequality true?

The inequality is true because 83 is between 82.81 and 84.64.

So, $\sqrt{83}$ is between 9.1 and 9.2. Since 83 is closer to 82.81 than it is to 84.64, $\sqrt{83} \approx 9.1$.

Check

Estimate $\sqrt{106}$ to the nearest tenth.

10.3

Go Online You can complete an Extra Example online.

Think About It! Between what two integers does $\sqrt{83}$ lie on a number line?

See students' responses.

Talk About It! How can you use a graph to verify that $\sqrt{83}$ is closer to 9.1 than 9.2? How could you continue on to get a better approximation of $\sqrt{83}$?

Sample answer: On the number line, graph $\sqrt{82.81} = 9.1$ and $\sqrt{84.64} = 9.2$. So, $\sqrt{83}$ is closer to $\sqrt{82.81}$ or 9.1 than $\sqrt{84.64}$ or 9.2; Sample answer: You could estimate the square root to the hundredths place by looking at intervals between 9.1 and 9.2.

Lesson 5-3 • Estimate Irrational Numbers 311

Interactive Presentation

Example 2, Estimate Square Roots to the Nearest Tenth, Slide 4 of 6

TYPE

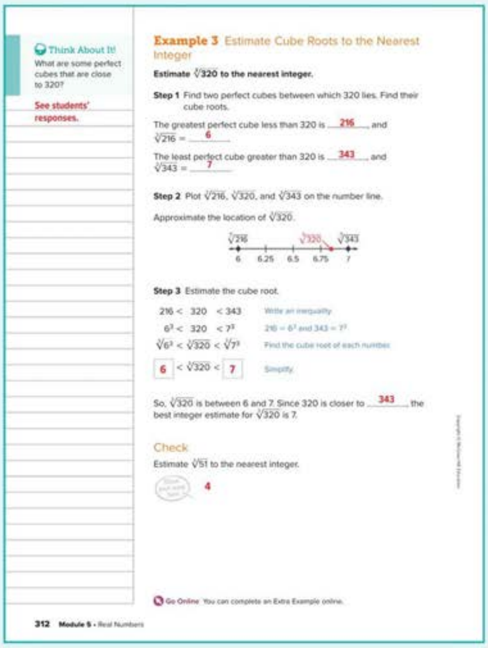
On Slide 2, students determine the integer closest to $\sqrt{83}$.

CLICK

On Slides 3 and 4, students test appropriate intervals.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! What are some perfect cubes that are close to 320?

See students' responses.

Example 3 Estimate Cube Roots to the Nearest Integer

Estimate $\sqrt[3]{320}$ to the nearest integer.

Step 1 Find two perfect cubes between which 320 lies. Find their cube roots.

The greatest perfect cube less than 320 is 216 and $\sqrt[3]{216}$ = 6

The least perfect cube greater than 320 is 343 and $\sqrt[3]{343}$ = 7

Step 2 Plot $\sqrt[3]{216}$, $\sqrt[3]{320}$, and $\sqrt[3]{343}$ on the number line.

Approximate the location of $\sqrt[3]{320}$.

Step 3 Estimate the cube root.

$216 < 320 < 343$ Write an inequality.

$6^3 < 320 < 7^3$ $216 = 6^3$ and $343 = 7^3$

$\sqrt[3]{6^3} < \sqrt[3]{320} < \sqrt[3]{7^3}$ Find the cube root of each number.

$6 < \sqrt[3]{320} < 7$ Simplify.

So, $\sqrt[3]{320}$ is between 6 and 7. Since 320 is closer to 343 the best integer estimate for $\sqrt[3]{320}$ is 7.

Check

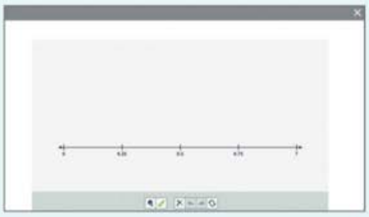
Estimate $\sqrt[3]{51}$ to the nearest integer.

4

Go Online You can complete an Extra Example online.

312 Module 5 • Real Numbers

Interactive Presentation



Example 3, Estimate Cube Roots to the Nearest Integer, Slide 3 of 5

TYPE

On Slide 2, students enter values to complete the sentences.

eTOOLS

On Slide 3, students use the Number Line eTool to plot radicals on the number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Estimate Cube Roots to the Nearest Integer

Objective

Students will estimate cube roots to the nearest integer.

Questions for Mathematical Discourse

SLIDE 2

AL What is a perfect cube? Sample answer: a number that is the cube of an integer

OL How can you find perfect cubes between which 320 lies? Sample answer: I can choose integers, and cube them. Continue until I find integers that have cubes that are near 320.

BL Explain how you know the cube root of 320 is closer to 7 than 6. Sample answer: 320 is closer to 343 than it is to 216.

SLIDE 3

AL Which cube root has the lesser value? What is its value? $\sqrt[3]{216} = 6$

OL Where is the value of $\sqrt[3]{320}$ on the number line compared to $\sqrt[3]{216}$ and $\sqrt[3]{343}$? It is to the right of $\sqrt[3]{216}$ and to the left of $\sqrt[3]{343}$.

BL How can you determine whether $\sqrt[3]{320}$ is to the right or to the left of 6.5? Sample answer: I can cube 6.5. If the result is less than 320, then 6.5 is less than $\sqrt[3]{320}$. If the result is greater than 320, then 6.5 is greater than $\sqrt[3]{320}$.

SLIDE 4

AL Why can you write the inequality using 216 and 343? Sample answer: 216 and 343 are perfect cubes, so their cube roots are integers.

OL How can you determine whether $\sqrt[3]{320}$ is closer to 6 or to 7? Sample answer: Since 320 is closer to 343 than it is to 216, $\sqrt[3]{320}$ is closer to 7 than it is to 6.

BL If you continued this method to the nearest hundredth, thousandth, and beyond, would you ever find a terminating or repeating decimal that is exactly equal to $\sqrt[3]{320}$? Explain. no; Sample answer: $\sqrt[3]{320}$ is an irrational number, so the decimal will neither terminate, nor repeat.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Estimate Irrational Numbers by Truncating

Objective

Students will learn how to estimate irrational numbers by truncating decimal expansions.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* questions on Slide 2, encourage students to understand and be able to clearly explain each method (rounding and truncating). They should use clear and precise mathematical language when describing the difference between rounding and truncating.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

How could you continue truncating $\sqrt{12}$ to get better approximations? Sample answer: Drop the digits after the ten-thousandths place. $\sqrt{12} \approx$ 3.464~~101615~~

If you *rounded* $\sqrt{12}$ to the nearest tenth, what would be the approximation? 3.5

If you *truncated* $\sqrt{12}$ to the nearest tenth, what would be the approximation? 3.4

What is the difference between truncating and rounding? Sample answer: Truncating simply drops all of the digits after a decimal place without rounding. Rounding looks at the decimal place to the right of a digit and then rounds up or down.

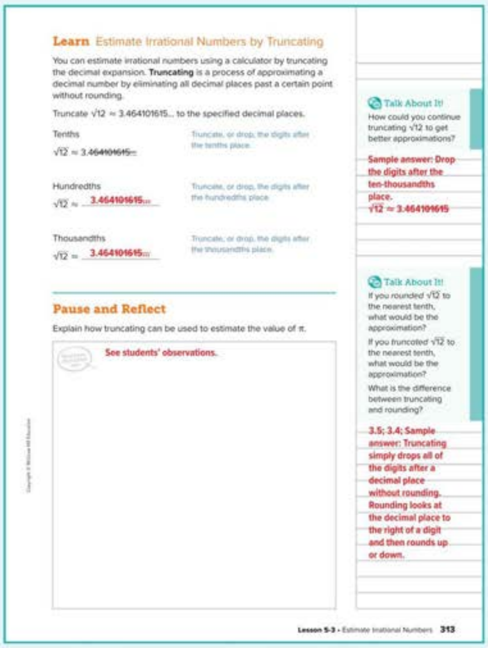
DIFFERENTIATE

Language Development Activity ELL

Students may not be familiar with the term *truncate*. In everyday use, to truncate an object is to shorten it by cutting off a part of it. Provide students with examples of the term truncate in everyday use or ask them to think of examples. One example is shown.

- A speaker needed to truncate the end of her presentation because it was too long.

In mathematics, to truncate a decimal expansion means to eliminate all decimal places after a certain point. Ask students to explain the difference between truncating the decimal expansion of the number $\sqrt{5}$ after the hundredths place and rounding to the hundredths place. Sample answer: $\sqrt{5} \approx$ 2.236. Truncating after the hundredths place results in the value 2.23. Rounding to the nearest hundredths results in the value 2.24.



Learn Estimate Irrational Numbers by Truncating

You can estimate irrational numbers using a calculator by truncating the decimal expansion. **Truncating** is a process of approximating a decimal number by eliminating all decimal places past a certain point without rounding.

Truncate $\sqrt{12} \approx$ 3.464101615... to the specified decimal places.

Tenths — Truncate, or drop, the digits after the tenths place.
$\sqrt{12} \approx$ 3.~~464101615...~~

Hundredths — Truncate, or drop, the digits after the hundredths place.
$\sqrt{12} \approx$ 3.46~~4101615...~~

Thousandths — Truncate, or drop, the digits after the thousandths place.
$\sqrt{12} \approx$ 3.464~~101615...~~

Pause and Reflect

Explain how truncating can be used to estimate the value of π.

See students' observations.

Talk About It!
How could you continue truncating $\sqrt{12}$ to get better approximations?
Sample answer: Drop the digits after the ten-thousandths place. $\sqrt{12} \approx$ 3.464~~101615~~

Talk About It!
If you *rounded* $\sqrt{12}$ to the nearest tenth, what would be the approximation?
If you *truncated* $\sqrt{12}$ to the nearest tenth, what would be the approximation?
What is the difference between truncating and rounding?
3.5; 3.4; Sample answer: Truncating simply drops all of the digits after a decimal place without rounding. Rounding looks at the decimal place to the right of a digit and then rounds up or down.

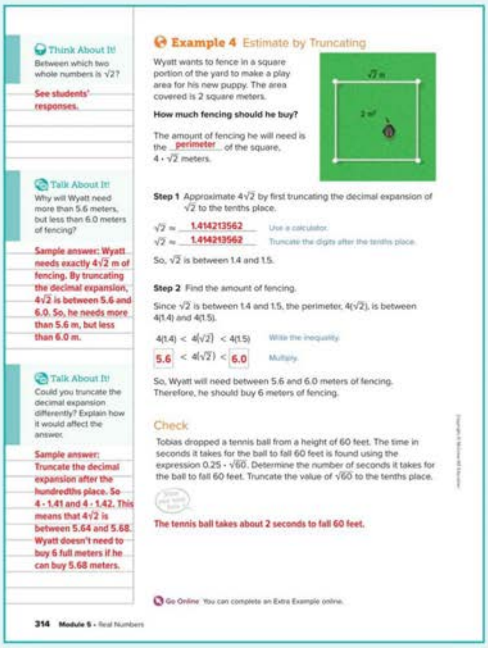
Lesson 5-3 • Estimate Irrational Numbers 313

Interactive Presentation

Learn, Estimate Irrational Numbers by Truncating, Slide 1 of 2

CLICK

On Slide 1, students select each button to truncate the decimal.



Think About It!
Between which two whole numbers is $\sqrt{2}$?

See students' responses.

Talk About It!
Why will Wyatt need more than 5.6 meters, but less than 6.0 meters of fencing?

Sample answer: Wyatt needs exactly $4\sqrt{2}$ m of fencing. By truncating the decimal expansion, $4\sqrt{2}$ is between 5.6 and 6.0. So, he needs more than 5.6 m, but less than 6.0 m.

Talk About It!
Could you truncate the decimal expansion differently? Explain how it would affect the answer.

Sample answer: Truncate the decimal expansion after the hundredths place. So $4 \cdot 1.41$ and $4 \cdot 1.42$. This means that $4\sqrt{2}$ is between 5.64 and 5.68. Wyatt doesn't need to buy 6 full meters if he can buy 5.68 meters.

Example 4 Estimate by Truncating

Wyatt wants to fence in a square portion of the yard to make a play area for his new puppy. The area covered is 2 square meters.

How much fencing should he buy?

The amount of fencing he will need is the perimeter of the square.
$4 \cdot \sqrt{2}$ meters.

Step 1 Approximate $4\sqrt{2}$ by first truncating the decimal expansion of $\sqrt{2}$ to the tenths place.

$\sqrt{2} \approx$ 1.414213562 Use a calculator.
$\sqrt{2} \approx$ 1.414213562 Truncate the digits after the tenths place.

So, $\sqrt{2}$ is between 1.4 and 1.5.

Step 2 Find the amount of fencing.

Since $\sqrt{2}$ is between 1.4 and 1.5, the perimeter, $4(\sqrt{2})$, is between 4(1.4) and 4(1.5).

$4(1.4) < 4(\sqrt{2}) < 4(1.5)$ Write the inequality.
$5.6 < 4(\sqrt{2}) < 6.0$ Multiply.

So, Wyatt will need between 5.6 and 6.0 meters of fencing. Therefore, he should buy 6 meters of fencing.

Check

Tobias dropped a tennis ball from a height of 60 feet. The time in seconds it takes for the ball to fall 60 feet is found using the expression $0.25 \cdot \sqrt{60}$. Determine the number of seconds it takes for the ball to fall 60 feet. Truncate the value of $\sqrt{60}$ to the tenths place.

The tennis ball takes about 2 seconds to fall 60 feet.

Go Online You can complete an Extra Example online.

314 Module 5 • Real Numbers

Interactive Presentation

Example 4, Estimate by Truncating, Slide 1 of 5

CLICK

On Slide 3, students determine the amount of fencing.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 4 Estimate by Truncating

Objective

Students will solve problems that involve estimating irrational numbers by truncating decimal expansions.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discussing the *Talk About It!* questions on Slide 4, encourage students to make sense of the inequality in order to determine that Wyatt needs more than 5.6 meters, but less than 6.0 meters.

6 Attend to Precision Ask students to explain each step as they determine the amount of fencing that is needed, encouraging them to use correct mathematical terminology, such as *truncating, inequality, and perimeter*. While discussing the *Talk About It!* questions on Slide 4, students should be able to use clear and precise mathematical language to explain how truncating the decimal expansion differently might affect the answer.

Questions for Mathematical Discourse

SLIDE 2

AL Why is truncating $\sqrt{2}$ to the tenths place helpful? Sample answer: The value is truncated so that a simpler number can be used in computations.

OL In this case, how does truncating to the tenths place compare to rounding to the tenths place? Sample answer: Both yield a result of 1.4.

BL How could you have approximated $\sqrt{2}$ without a calculator? Sample answer: Test intervals between 1 and 2.

SLIDE 3

AL Why is $4\sqrt{2}$ between 4(1.4) and 4(1.5)? Sample answer: because $\sqrt{2}$ is between 1.4 and 1.5, and we need to multiply the value by 4.

OL Why should Wyatt buy 6.0 meters of fencing instead of 5.6? Sample answer: He will need more than 5.6 meters, but less than 6.0 meters.

BL How would the inequality change if Wyatt wanted to have three fenced areas of this size? $16.8 < 3 \cdot 4\sqrt{2} < 18.0$

Go Online

- Find additional teaching notes and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Golden Rectangle

Objective

Students will come up with their own strategy to solve an application problem involving the golden rectangle.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage students to pause and consider how using a more precise approximation of $\sqrt{5}$ might affect the estimate of the ratio.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

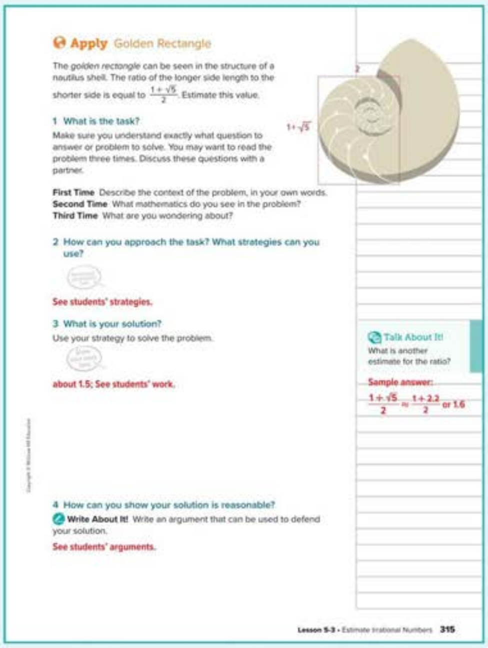
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What operation should you do first?
- To what perfect square is $\sqrt{5}$ closest?
- What is the best integer estimate for $\sqrt{5}$?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Golden Rectangle

The golden *rectangle* can be seen in the structure of a nautilus shell. The ratio of the longer side length to the shorter side is equal to $\frac{1+\sqrt{5}}{2}$. Estimate this value.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

about 1.5; See students' work.

Talk About It! What is another estimate for the ratio?

Sample answer: $\frac{1+\sqrt{5}}{2} \approx \frac{1+2.2}{2}$ or 1.6

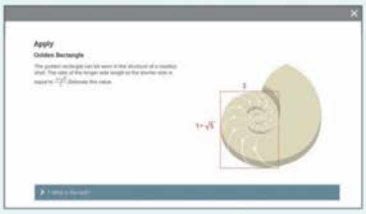
4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Lesson 5-3 • Estimate Irrational Numbers 315

Interactive Presentation



Apply, Golden Rectangle

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

In Little League, the bases are squares with sides of 14 inches. The expression $\sqrt{s^2 + s^2}$ represents the distance diagonally across a square of side length s. Estimate the diagonal distance across a base to the nearest inch.

20 inches

Go Online You can complete an Extra Example online.

Math History Minute

Hindu mathematician **Bhāskara II (1114–1185)** is considered by many to have been the leading mathematician of the 12th century. One of his many accomplishments was to produce several approximations of the number π, including $\frac{22}{7}$, which is still used today.

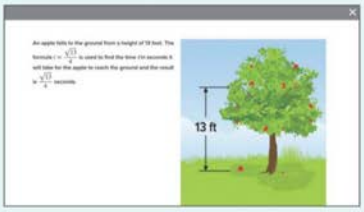
Pause and Reflect

Compare and contrast estimating rational numbers on a number line with estimating irrational numbers on a number line.

See students' observations.

316 Module 5 • Real Numbers

Interactive Presentation



Exit Ticket

Essential Question Follow-Up

Why do we classify numbers?

In this lesson, students learned how to estimate the value of an irrational number. Encourage them to discuss with a partner when an irrational number is an acceptable answer to a problem, and when it would not make sense for an irrational number to be the final answer. For example, you could find the area of a circle to be exactly 5π square feet, but if you wanted to know how much paint you would need to cover the inside of the circle, you would need to round that answer to about 16 square feet.

Exit Ticket

Refer to the Exit Ticket slide. Estimate the value of $\sqrt{13}$ to the nearest tenth. Then use that estimate to find the approximate value of $\frac{\sqrt{13}}{4}$. What does this value mean within the context of the problem? 3.6; 0.9; It will take approximately 0.9 second for the apple to reach the ground from a height of 13 feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 9–13 odd, 14–17
- ALEKS® Square Roots and Irrational Numbers

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–9, 13, 15
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Venn Diagrams and Sets of Rational Numbers

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Venn Diagrams and Sets of Rational Numbers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

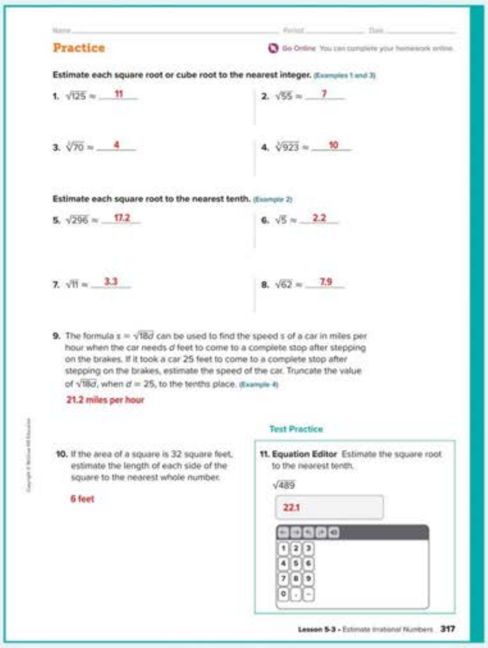
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	estimate square roots and cube roots to the nearest integer	1–4
1	estimate square roots to the nearest tenth	5–8
2	estimate irrational numbers by truncating decimal expansions	9
2	extend concepts learned in class to apply them in new contexts	10, 11
3	solve application problems involving estimating irrational numbers	12, 13
3	higher-order and critical thinking skills	14–17

Common Misconception

Some students may incorrectly determine the two perfect squares betwen which a square root lies. Remind students what a perfect square is. Some students may find it beneficial to create a list of perfect squares and their square roots to use when working through the exercises.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Estimate each square root or cube root to the nearest integer. (Examples 1 and 3)

1. $\sqrt{125} \approx$ 11
2. $\sqrt{55} \approx$ 7
3. $\sqrt[3]{70} \approx$ 4
4. $\sqrt[3]{923} \approx$ 10

Estimate each square root to the nearest tenth. (Example 2)

5. $\sqrt{296} \approx$ 17.2
6. $\sqrt{5} \approx$ 2.2
7. $\sqrt{11} \approx$ 3.3
8. $\sqrt{62} \approx$ 7.9

9. The formula $s = \sqrt{18d}$ can be used to find the speed s of a car in miles per hour when the car needs d feet to come to a complete stop after stepping on the brakes. If it took a car 25 feet to come to a complete stop after stepping on the brakes, estimate the speed of the car. Truncate the value of $\sqrt{18d}$, when $d = 25$, to the tenths place. (Example 4)

21.2 miles per hour

Test Practice

10. If the area of a square is 32 square feet, estimate the length of each side of the square to the nearest whole number.

6 feet

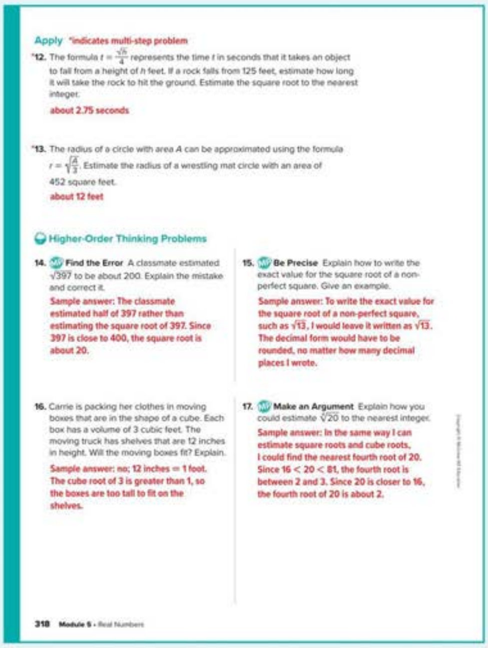
11. **Equation Editor** Estimate the square root to the nearest tenth.

$\sqrt{489}$

22.1

Lesson 5-3 • Estimate Irrational Numbers 317

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*12. The formula $t = \frac{\sqrt{h}}{4}$ represents the time t in seconds that it takes an object to fall from a height of h feet. If a rock falls from 125 feet, estimate how long it will take the rock to hit the ground. Estimate the square root to the nearest integer.

about 2.75 seconds

*13. The radius of a circle with area A can be approximated using the formula $r = \sqrt{\frac{A}{3}}$. Estimate the radius of a wrestling mat circle with an area of 452 square feet.

about 12 feet

Higher-Order Thinking Problems

14. **Find the Error** A classmate estimated $\sqrt{397}$ to be about 200. Explain the mistake and correct it.

Sample answer: The classmate estimated half of 397 rather than estimating the square root of 397. Since 397 is close to 400, the square root is about 20.

15. **Be Precise** Explain how to write the exact value for the square root of a non-perfect square. Give an example.

Sample answer: To write the exact value for the square root of a non-perfect square, such as $\sqrt{13}$, I would leave it written as $\sqrt{13}$. The decimal form would have to be rounded, no matter how many decimal places I wrote.

16. Carrie is packing her clothes in moving boxes that are in the shape of a cube. Each box has a volume of 3 cubic feet. The moving truck has shelves that are 12 inches in height. Will the moving boxes fit? Explain.

Sample answer: no; 12 inches = 1 foot. The cube root of 3 is greater than 1, so the boxes are too tall to fit on the shelves.

17. **Make an Argument** Explain how you could estimate $\sqrt[4]{20}$ to the nearest integer.

Sample answer: In the same way I can estimate square roots and cube roots, I could find the nearest fourth root of 20. Since 16 < 20 < 81, the fourth root is between 2 and 3. Since 20 is closer to 16, the fourth root of 20 is about 2.

318 Module 5 • Real Numbers

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will identify the mistake the classmate made and then find the correct answer. Students should be able to support their answer with a logical explanation.

In Exercise 17, students will construct a viable argument as to how they could estimate $\sqrt[4]{20}$ to the nearest integer. Students should use the same strategy they use for estimating square and cube roots.

6 Attend to Precision In Exercise 15, students will explain how to write the exact value for the square root of a non-perfect square. Students should use proper terminology and give a logical explanation to support their answer.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 12–13 Have students work in pairs. Have students individually read Exercise 12 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 13.

Solve the problem another way.

Use with Exercise 16 Have students work in groups of 3-4. After completing Exercise 16, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Lesson 5-4

Compare and Order Real Numbers

8.NS.A.2

LESSON GOAL

Students will compare and order numbers in the real number system.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Compare and Order Real Numbers
Example 1: Compare Real Numbers
Example 2: Compare Real Numbers
Example 3: Order Real Numbers
Example 4: Use Real Numbers
Apply: Line of Sight

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Sums and Products of Real Numbers		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 32 of the *Language Development Handbook* to help your students build mathematical language related to comparing and ordering real numbers.

ELL You can use the tips and suggestions on page T32 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: The Number System
Supporting Cluster(s): In this lesson, students address the supporting cluster **8.NS.A** by comparing and ordering numbersin the real number system.
Standards for Mathematical Content: **8.NS.A. 1, 8.NS.A.2**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6**

Coherence

Vertical Alignment

Previous
Students estimated irrational numbers.
8.NS.A.2

Now
Students compare and order numbers in the real number system.
8.NS.A.1, 8.NS.A.2

Next
Students will study and use properties of rational and irrational numbers.
HS.RN.B.3

Rigor

The Three Pillars of Rigor

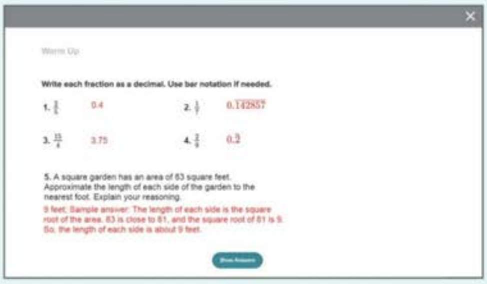
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of the set of real numbers. They learn that when comparing or ordering real numbers, it may be easier to write them as decimals or decimal approximations and graph them on a number line. They build *fluency* with comparing and ordering real numbers, and *apply* it to real-world problems.

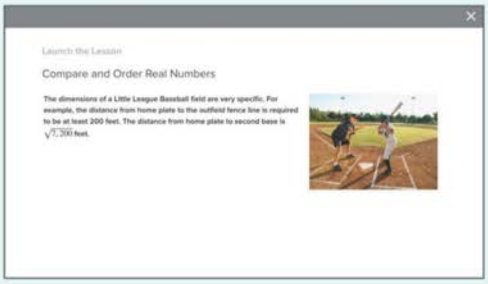
Mathematical Background

In order to compare real numbers, it is often helpful to write the numbers in decimal notation. By doing so, the numbers can easily be compared and ordered using place-value digits. To compare irrational numbers, use an estimation technique to a precision accurate enough to avoid error.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing numbers in decimal notation (Exercises 1–4)
- estimating irrational numbers and understanding square roots (Exercise 5)

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the dimensions of a Little League Baseball field, written in different notations.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What sets of numbers make up the set of real numbers? Sample answer: irrational numbers and rational numbers, including integers, whole numbers, and natural numbers

Learn Compare and Order Real Numbers

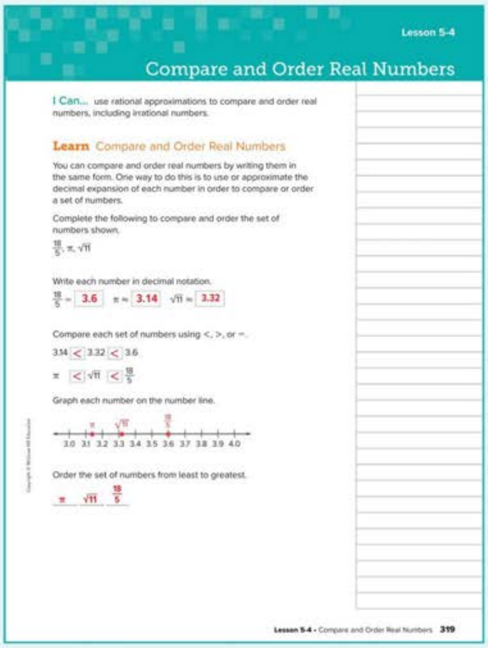
Objective

Students will learn how to compare and order real numbers.

Teaching Notes

SLIDE 1

Students will learn about ordering and comparing numbers using decimal notation. You may wish to ask students to list several different notations or forms of numbers (fractions, decimals, integers, square or cube roots, etc.) and why it can be challenging to compare numbers expressed in different forms.



Lesson 5-4

Compare and Order Real Numbers

I Can... use rational approximations to compare and order real numbers, including irrational numbers.

Learn Compare and Order Real Numbers

You can compare and order real numbers by writing them in the same form. One way to do this is to use or approximate the decimal expansion of each number in order to compare or order a set of numbers.

Complete the following to compare and order the set of numbers shown.

$\frac{18}{5}$, π, $\sqrt{11}$

Write each number in decimal notation.

$\frac{18}{5}$ = 3.6 $\pi \approx$ 3.14 $\sqrt{11} \approx$ 3.32

Compare each set of numbers using <, >, or =.

3.14 < 3.32 < 3.6

π < $\sqrt{11}$ < $\frac{18}{5}$

Graph each number on the number line.

Order the set of numbers from least to greatest.

π, $\sqrt{11}$, $\frac{18}{5}$

Lesson 5-4 • Compare and Order Real Numbers 319

DIFFERENTIATE

Reteaching Activity AL

If any of your students have difficulty ordering the set of numbers presented in the Learn, have them work with a partner to respond to the following questions.

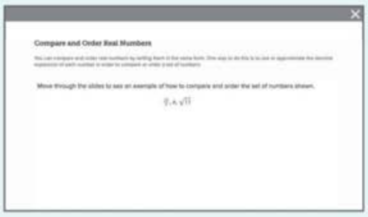
By studying the numbers $\frac{18}{5}$, π, and $\sqrt{11}$, how do you know that the number line can be drawn to start at 3? Sample answer: $\frac{18}{5}$ is greater than 3 because $\frac{15}{5} = 3$ and $18 > 15$, π is greater than 3, and $\sqrt{11}$ is greater than 3 because $3^2 = 9$ and $11 > 9$.

How can you determine where to place $\frac{18}{5}$ on the number line? Sample answer: $\frac{18}{5} = 3.6$, so place the location at 3.6.

How can you determine where to place π on the number line? Sample answer: $\pi \approx 3.14$, so place the location a little less than halfway between 3.1 and 3.2.

How can you determine where to place $\sqrt{11}$ on the number line? Sample answer: $\sqrt{11}$ is between $(3.3)^2$ and $(3.4)^2$ and closer to $(3.3)^2$, so place $\sqrt{11}$ a little after $(3.3)^2$.

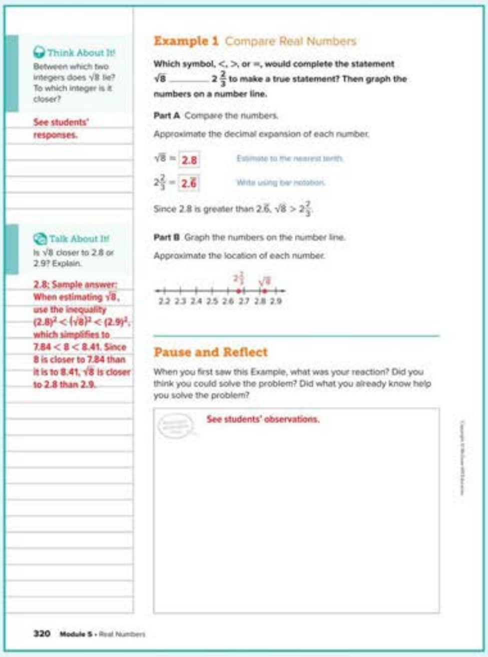
Interactive Presentation



Learn, Compare and Order Real Numbers

CLICK

On Slide 1, students move through the steps to see how to compare and order the set of numbers shown.



Think About It! Between which two integers does $\sqrt{8}$ lie? To which integer is it closer?

See students' responses.

Talk About It! Is $\sqrt{8}$ closer to 2.8 or 2.9? Explain.

2.8; Sample answer: When estimating $\sqrt{8}$, use the inequality $(2.8)^2 < (\sqrt{8})^2 < (2.9)^2$, which simplifies to $7.84 < 8 < 8.41$. Since 8 is closer to 7.84 than it is to 8.41, $\sqrt{8}$ is closer to 2.8 than 2.9.

Example 1 Compare Real Numbers

Which symbol, <, >, or =, would complete the statement $\sqrt{8}$ ______ $2\frac{2}{3}$ to make a true statement? Then graph the numbers on a number line.

Part A Compare the numbers.

Approximate the decimal expansion of each number.

$\sqrt{8} \approx$ 2.8 Estimate to the nearest tenth.

$2\frac{2}{3} =$ $2.\overline{6}$ Write using bar notation.

Since 2.8 is greater than $2.\overline{6}$, $\sqrt{8} > 2\frac{2}{3}$.

Part B Graph the numbers on the number line.

Approximate the location of each number.

Pause and Reflect

When you first saw this Example, what was your reaction? Did you think you could solve the problem? Did what you already know help you solve the problem?

See students' observations.

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Interactive Presentation

Example 1, Compare Real Numbers, Slide 3 of 5

CLICK

On Slide 2, students select the correct symbol.

eTOOLS

On Slide 3, students use the Number Line eTool to graph numbers on a number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Compare Real Numbers

Objective

Students will compare two positive real numbers and graph the numbers on a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage them to consider how they can square numbers in order to test how close they are to the square root of a number.

6 Attend to Precision Encourage students to approximate the value of each number accurately and efficiently, in order to compare the numbers using place value.

Questions for Mathematical Discourse

SLIDE 2

AL Why is it useful to write both numbers in decimal notation?
Sample answer: Numbers in decimal notation can be easily compared using place value.

OL How can you determine which number is greater?
Sample answer: The tenths place of $\sqrt{8}$ is 8 and the tenths place of $2\frac{2}{3}$ is 6, or 7 if rounded. Since the numbers are equal until the tenths place and $\sqrt{8}$ has a greater number in the tenths place, $\sqrt{8}$ is greater than $2\frac{2}{3}$.

BL A classmate claims that $\sqrt{8} = 2\frac{2}{3}$ after using truncation. How can you use reasoning to show that he or she is incorrect?
Sample answer: 8 is not a perfect square, so $\sqrt{8}$ is irrational; $2\frac{2}{3}$ is rational. The numbers cannot be equal.

SLIDE 3

AL How should $\sqrt{8}$ be positioned on the number line compared to $2\frac{2}{3}$? Sample answer: to the right of $2\frac{2}{3}$

OL What is the value of $2\frac{2}{3}$ to the nearest hundredth? 2.67

BL How could you better approximate the location of $\sqrt{8}$ on the number line? Sample answer: Use an approximation to the nearest hundredth.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Check

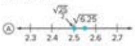
Which symbol, <, >, or =, would complete the statement $\frac{\sqrt{25}}{2}$ ______ $\sqrt{6.25}$ to make a true statement? Then graph the numbers on a number line.

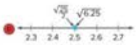
Part A

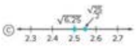
Write the symbol, <, >, or =, that makes $\frac{\sqrt{25}}{2}$ __=__ $\sqrt{6.25}$ a true statement.

Part B

Which of the following is the correct graph of $\frac{\sqrt{25}}{2}$ and $\sqrt{6.25}$ on a number line?

Ⓐ 

Ⓑ 

Ⓒ 

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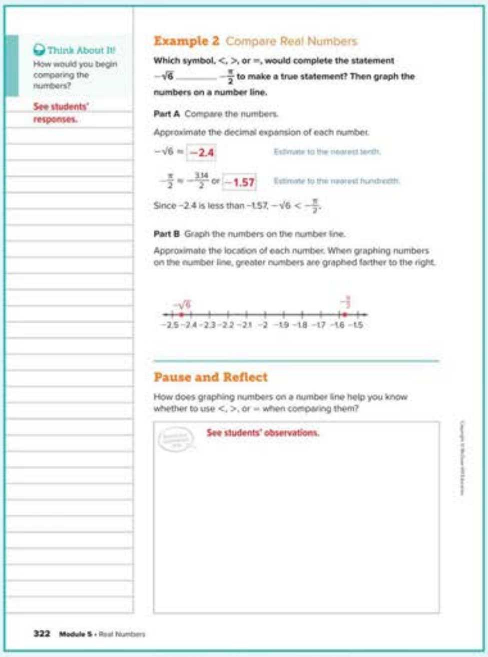
Go Online You can complete an Extra Example online.

8.NS.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION



Think About It!
How would you begin comparing the numbers?
See students' responses.

Example 2 Compare Real Numbers

Which symbol, <, >, or =, would complete the statement $-\sqrt{6}$ ______ $-\frac{\pi}{2}$ to make a true statement? Then graph the numbers on a number line.

Part A Compare the numbers.

Approximate the decimal expansion of each number.

$-\sqrt{6} \approx -2.4$ Estimate to the nearest tenth.

$-\frac{\pi}{2} \approx -\frac{3.14}{2}$ or -1.57 Estimate to the nearest hundredth.

Since −2.4 is less than −1.57, $-\sqrt{6} < -\frac{\pi}{2}$.

Part B Graph the numbers on the number line.

Approximate the location of each number. When graphing numbers on the number line, greater numbers are graphed farther to the right.

Pause and Reflect

How does graphing numbers on a number line help you know whether to use <, >, or = when comparing them?

See students' observations.

322 Module 5 • Real Numbers

Interactive Presentation

Example 2, Compare Real Numbers, Slide 3 of 4

TYPE

On Slide 2, students enter the numbers in decimal notation.

eTOOLS

On Slide 3, students use the Number Line eTool to graph the number on the number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Compare Real Numbers

Objective

Students will compare two negative real numbers and graph the numbers on a number line.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to approximate the value of each number accurately and efficiently, in order to compare the numbers using place value. Students should pay careful attention to the negative signs and the decimal approximations when comparing the values and graphing them on the number line.

Questions for Mathematical Discourse

SLIDE 2

AL To the nearest tenth, what is $-\sqrt{6}$? −2.4

AL To the nearest hundredth, what is $-\frac{\pi}{2}$? −1.57

OL Why is it unnecessary to round both numbers to the nearest hundredth? Sample answer: −2.4 is less than −1.57 without needing to further compare decimal places.

BL A classmate states that $-\sqrt{6} > -\frac{\pi}{2}$ because 2.4 is greater than 1.57. Describe the error that was made. Sample answer: The classmate compared the positive numbers 2.4 and 1.57 instead of the negative numbers −2.4 and −1.57.

SLIDE 3

AL How will the position of $-\sqrt{6}$ compare to the position of $-\frac{\pi}{2}$ on the number line? $-\sqrt{6}$ is to the left of $-\frac{\pi}{2}$.

OL How do you know that $-\frac{\pi}{2}$ is between −1.5 and −1.6, but closer to −1.6? Sample answer: $-\frac{\pi}{2}$ can be approximated as −1.57, which is between −1.5 and −1.6, but closer to −1.6.

BL Approximately how far apart on the number line are $-\sqrt{6}$ and $-\frac{\pi}{2}$? Sample answer: a little more than 0.8 unit

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

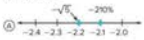
Which symbol, <, >, or =, would complete the statement $-\sqrt{5}$ ________ −210% to make a true statement? Then graph the numbers on a number line.

Part A

Write the symbol, <, >, or =, that makes $-\sqrt{5}$ __<__ −210% a true statement.

Part B

Which of the following is the correct graph of $-\sqrt{5}$ and −210% on a number line?

Ⓐ 

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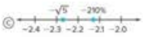
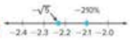
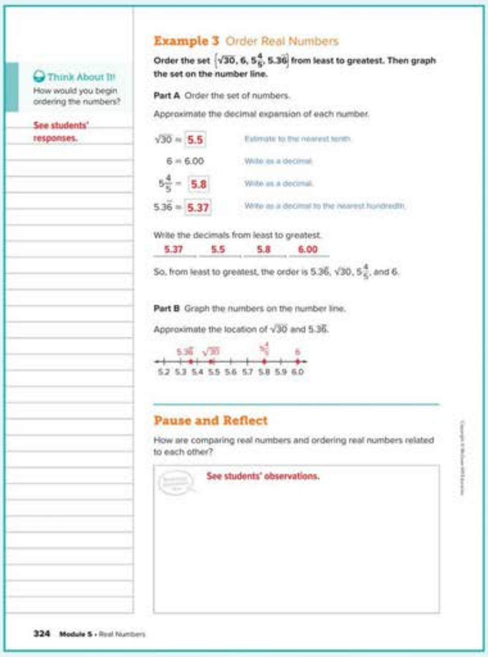
Ⓒ 

● 

Go Online You can complete an Extra Example online.

Lesson 5-4 • Compare and Order Real Numbers 323



Think About It! How would you begin ordering the numbers?

See students' responses.

Example 3 Order Real Numbers

Order the set $\{\sqrt{30}, 6, 5\frac{4}{5}, 5.3\overline{6}\}$ from least to greatest. Then graph the set on the number line.

Part A Order the set of numbers.

Approximate the decimal expansion of each number.

$\sqrt{30} \approx$ 5.5 — Estimate to the nearest tenth.

$6 = 6.00$ — Write as a decimal.

$5\frac{4}{5} =$ 5.8 — Write as a decimal.

$5.3\overline{6} \approx$ 5.37 — Write as a decimal to the nearest hundredth.

Write the decimals from least to greatest.

5.37, 5.5, 5.8, 6.00

So, from least to greatest, the order is $5.3\overline{6}$, $\sqrt{30}$, $5\frac{4}{5}$, and 6.

Part B Graph the numbers on the number line.

Approximate the location of $\sqrt{30}$ and $5.3\overline{6}$.

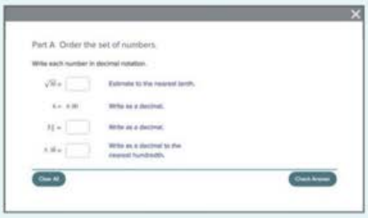
Pause and Reflect

How are comparing real numbers and ordering real numbers related to each other?

See students' observations.

324 Module 5 • Real Numbers

Interactive Presentation



Example 3, Order Real Numbers, Slide 2 of 4

DRAG & DROP

On Slide 2, student drag the numbers to order the set from least to greatest.

eTOOLS

On Slide 3, students use the Number Line eTool to graph the numbers on the number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Order Real Numbers

Objective

Students will order a set of real numbers and graph the numbers on a number line.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to appro ximate or find the decimal value of each number accurately and efficiently, in order to order the numbers from least to greatest, using place value. Students should pay careful attention when graphing the numbers on the number line.

Questions for Mathematical Discourse

SLIDE 2

AL What does the bar in $5.3\overline{6}$ mean? The digit 6 is repeated forever $5.3\overline{6} = 5.36666...$

OL How does 6 compare to the rest of the numbers? It is greater than all of the other numbers.

OL How do you know that 6 is greater than $\sqrt{30}$? $6^2 = 36$, so $\sqrt{30}$ must be less than 6.

BL If the set contained $5.\overline{3}$ instead of $5.3\overline{6}$, would the order change? Explain. no; Sample answer: $5.3\overline{6}$ was the least number in the original set, and $5.\overline{3}$ is less than $5.3\overline{6}$.

SLIDE 3

AL Which number will be the farthest right on the number line? 6

OL Why are the locations of $\sqrt{30}$ and $5.3\overline{6}$ approximated, but the location of $5\frac{4}{5}$ is exact? Sample answer: $5\frac{4}{5} = 5.8$, so it does not need to be approximated.

BL Between which two numbers of this set of numbers will lie $\frac{11}{2}$? between $\sqrt{30}$ and $5\frac{4}{5}$

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Check

Order the set $\{\sqrt{3}, \frac{\pi}{2}, 160\%, 1.3\overline{5}\}$ from least to greatest. Then graph the set on the number line.

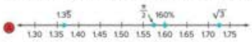
Part A

Order of the set from least to greatest.

$\{1.3\overline{5}, \frac{\pi}{2}, 160\%, \sqrt{3}\}$

Part B

Which of the following is the correct graph of the set of numbers?

Ⓐ 

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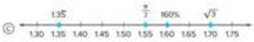
Ⓒ 

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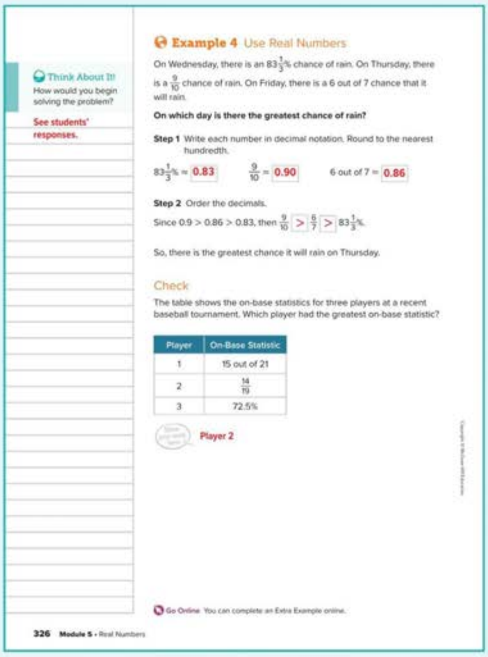
Go Online You can complete an Extra Example online.

8.NS.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Example 4 Use Real Numbers

On Wednesday, there is an $83\frac{1}{3}\%$ chance of rain. On Thursday, there is a $\frac{9}{10}$ chance of rain. On Friday, there is a 6 out of 7 chance that it will rain.

On which day is there the greatest chance of rain?

Think About It! How would you begin solving the problem? See students' responses.

Step 1 Write each number in decimal notation. Round to the nearest hundredth.

$83\frac{1}{3}\% \approx$ 0.83 $\frac{9}{10} =$ 0.90 6 out of 7 ≈ 0.86

Step 2 Order the decimals.

Since 0.9 > 0.86 > 0.83, then $\frac{9}{10} > \frac{6}{7} > 83\frac{1}{3}\%$.

So, there is the greatest chance it will rain on Thursday.

Check

The table shows the on-base statistics for three players at a recent baseball tournament. Which player had the greatest on-base statistic?

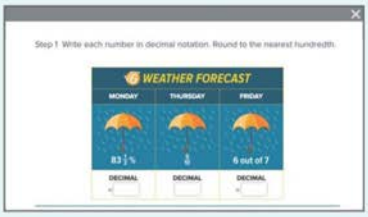
Player	On-Base Statistic
1	15 out of 21
2	$\frac{14}{19}$
3	72.5%

Player 2

Go Online You can complete an Extra Example online.

326 Module 5 • Real Numbers

Interactive Presentation



Example 4, Use Real Numbers, Slide 2 of 4

TYPE

On Slide 2, students enter each number in decimal notation.

CLICK

On Slide 3, students determine on which day there is the greatest chance of rain.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Use Real Numbers

Objective

Students will solve problems that involve ordering real numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the quantities given in the problem in order to determine on which day there is the greatest chance of rain.

6 Attend to Precision Students should approximate or find the decimal value of each number accurately and efficiently, in order to compare them to solve the problem.

Questions for Mathematical Discourse

SLIDE 2

AL Write *6 out of 7* as a fraction and as a decimal to the nearest hundredth. $\frac{6}{7}$; 0.86

OL Write the value of $83\frac{1}{3}\%$ as a decimal to the nearest hundredth. 0.83

BL A classmate writes the decimal form of $83\frac{1}{3}\%$ as 83.333.... What mistake did he or she likely make? Sample answer: The classmate did not notice that the number is a percent. He or she must divide 83.333... by 100 to find the decimal value.

SLIDE 3

AL Which has the greatest value: $83\frac{1}{3}\%$, $\frac{9}{10}$, or 6 out of 7? $\frac{9}{10}$

OL Could you have rounded to the nearest tenth in order to solve the problem? Explain. no; Sample answer: If approximations to the nearest tenth were used, the chance of rain on Thursday and Friday would be the same, even though there is actually a greater chance of rain on Thursday.

BL Which form do you prefer to use when talking about the chance of rain: percent, fraction, or decimal? See students' preferences.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Line of Sight

Objective

Students will come up with their own strategy to solve an application problem involving the line of sight from atop a building.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

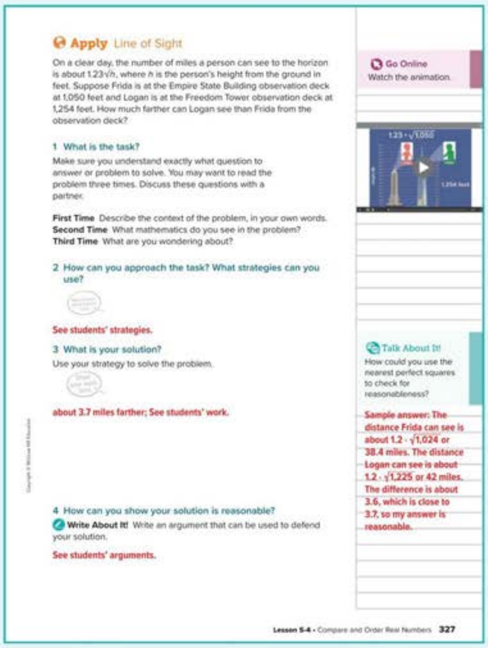
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- To what perfect square is $\sqrt{1,050}$ closest?
- To what perfect square is $\sqrt{1,254}$ closest?
- How can you use these approximations to help solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Line of Sight

On a clear day, the number of miles a person can see to the horizon is about $1.23\sqrt{h}$, where h is the person's height from the ground in feet. Suppose Frida is at the Empire State Building observation deck at 1,050 feet and Logan is at the Freedom Tower observation deck at 1,254 feet. How much farther can Logan see than Frida from the observation deck?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

about 3.7 miles farther; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

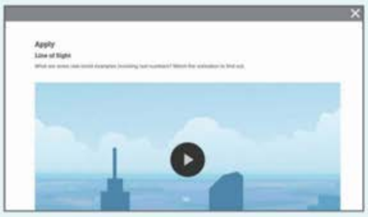
Go Online Watch the animation.

Talk About It! How could you use the nearest perfect squares to check for reasonableness?

Sample answer: The distance Frida can see is about $1.2 \cdot \sqrt{1,024}$ or 38.4 miles. The distance Logan can see is about $1.2 \cdot \sqrt{1,225}$ or 42 miles. The difference is about 3.6, which is close to 3.7, so my answer is reasonable.

Lesson 5-4 • Compare and Order Real Numbers 327

Interactive Presentation



Apply, Line of Sight

WATCH

Students watch an animation that introduces the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The time in seconds that it takes an object to fall d feet can be found using the expression $\frac{\sqrt{d}}{4}$. Suppose Aiden drops a tennis ball from a height of 50 feet at the same time Mason drops a similar tennis ball from a height of 20 feet. How much longer will it take Aiden's tennis ball to reach the ground than Mason's tennis ball? Round to the nearest hundredth.

0.65 second

Go Online You can complete an Extra Example online.

Pause and Reflect

Review the Examples from this module. Which one did you find most challenging? What are the steps you would take to solve a problem of this type?

See students' observations.

328 Module 5 • Real Numbers

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Which outfield fence is farther from home plate? Write a mathematical argument that can be used to defend your solution. Sample answer: The right outfield fence is farther from home plate. $\sqrt{40,200}$ is approximately equal to 200, so the distance from home plate to the left outfield fence is less than the distance to the right outfield fence, 205 feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 7, 9, 10–13
- Extension: Sums and Products of Real Numbers
- ALEKS® Square Roots and Irrational Numbers

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–6, 9–11
- Extension: Sums and Products of Real Numbers
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Venn Diagrams and Sets of Rational Numbers

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ALEKS® Venn Diagrams and Sets of Rational Numbers

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

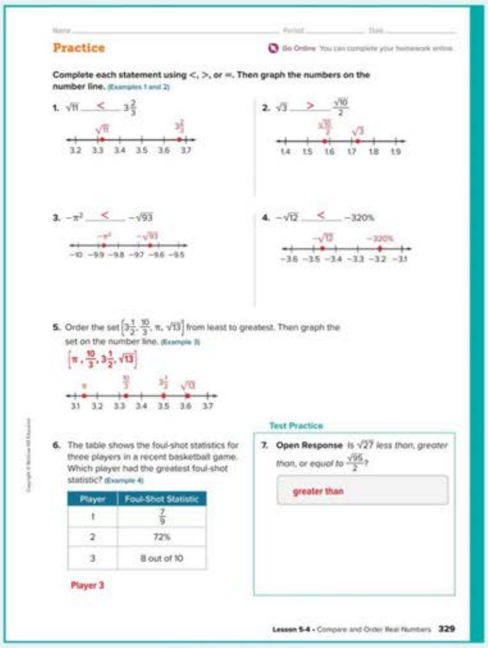
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	compare two positive or two negative numbers and graph the numbers on a number line	1–4
1	order a set of real numbers and graph the numbers on a number line	5
2	extend concepts learned in class to apply them in new contexts	6, 7
3	solve application problems involving comparing and ordering real numbers	8, 9
3	higher-order and critical thinking skills	10–13

Common Misconception

Students may compare fractions without considering the order of operations implied by the fraction bar. For example, when evaluating $\frac{\sqrt{8}}{2}$, students might divide 8 by 2, causing them to incorrectly conclude $\frac{\sqrt{8}}{2} = \sqrt{4}$ and therefore $\frac{\sqrt{8}}{2} > 1.5$. Similarly, they may focus on the radicand, 8, and decide that $\frac{\sqrt{8}}{2} > \frac{3}{2}$, because 8 is much greater than 3. Remind students that fractions are division expressions and that roots and exponents must be evaluated before dividing.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Complete each statement using <, >, or =. Then graph the numbers on the number line. (Examples 1 and 2)

1. $\sqrt{11}$ __<__ $3\frac{2}{3}$

2. $\sqrt{3}$ __>__ $\frac{\sqrt{10}}{2}$

3. $-\pi^2$ __<__ $-\sqrt{93}$

4. $-\sqrt{12}$ __<__ -320%

5. Order the set $\left\{3\frac{1}{2}, \frac{10}{3}, \pi, \sqrt{13}\right\}$ from least to greatest. Then graph the set on the number line. (Example 3)
$\left\{\pi, \frac{10}{3}, 3\frac{1}{2}, \sqrt{13}\right\}$

6. The table shows the foul-shot statistics for three players in a recent basketball game. Which player had the greatest foul-shot statistic? (Example 4)

Player	Foul-Shot Statistic
1	$\frac{7}{9}$
2	72%
3	8 out of 10

Player 3

Test Practice

7. **Open Response** Is $\sqrt{27}$ less than, greater than, or equal to $\frac{\sqrt{95}}{2}$?

greater than

Lesson 5-4 • Compare and Order Real Numbers 329

3 REFLECT AND PRACTICE

Apply *indicates multi-step problem

*8. The radius of a circle can be approximated using the expression $\sqrt{\frac{A}{3}}$. A circular kiddie swimming pool has an area of about 28 square feet. An inflatable full-size circular pool has an area of about 113 square feet. How much greater is the radius of the full-size pool than the radius of the kiddie pool? Round to the nearest whole number.

3 feet

*9. The time in seconds that it takes an object to fall d feet can found using the expression $\frac{\sqrt{d}}{4}$. In an egg drop contest, Clara successfully dropped her egg container from a height of 35 feet, while Vladimir successfully dropped his egg container from a height of 23 feet. How much longer did it take Clara's egg to reach the floor than Vladimir's egg? Round to the nearest tenth.

0.3 second

Higher-Order Thinking Problems

10. **Which One Doesn't Belong?** Identify the number that does not belong in the group. Explain your reasoning.

$-23.\overline{2}$ $\quad -23\frac{1}{5}$ $\quad -\sqrt{23}$ $\quad -23.2$

$-\sqrt{23}$; Sample answer: $-\sqrt{23}$ is approximately -4.80, while all of the other numbers are approximately -23. On a number line, $-\sqrt{23}$ is the closest to zero.

11. **MP Justify Conclusions** Which number is greater, 3.14 or π? Justify your answer.

π; Sample answer: 3.14 can be extended to 3.14000... The number π written as a decimal is 3.141.... and $3.141 > 3.140$.

12. **MP Find the Error** Kendra states that $\sqrt{3} > 2$ because 3 is greater than 2. Explain Kendra's mistake and correct it.

Sample answer: Kendra did not estimate the square root of 3. Since $\sqrt{3}$ is approximately 1.73 and 1.73 is less than 2, $\sqrt{3} < 2$.

13. Identify two numbers, one rational and one irrational, that are between 1.6 and 1.8. Write an inequality to compare the two numbers.

Sample answer: 1.7 and $\sqrt{3}$; $1.7 < \sqrt{3}$

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8.NS.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students will determine which number is greater, 3.14 or π. Students should be able to justify their answer with a logical explanation. In Exercise 12, students will find Kendra's mistake and then correct it. Students should use an explanation that explains what Kendra did wrong and how to correct it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 8 Have students work together to prepare a brief demonstration that illustrates why this problem may require multiple steps to solve. For example, before they can find the difference in the radii, they must first approximate the radius of each pool. Have each pair or group of students present their response to the class.

Create your own higher-order thinking problem.
Use with Exercises 10–13 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of real numbers written as decimals, fractions, and roots. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

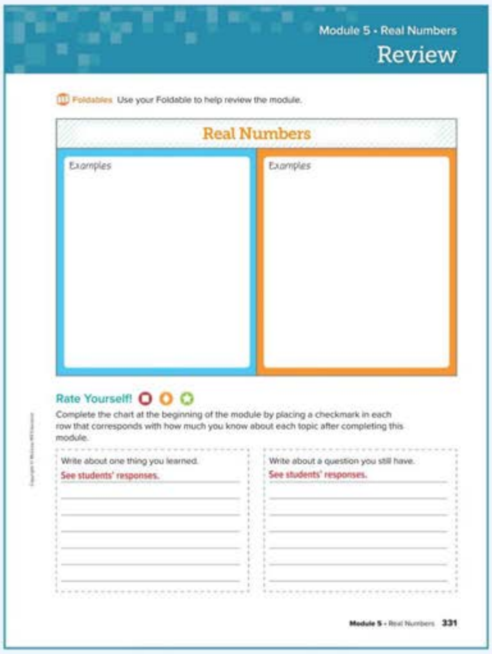
Assessment Resources

Put It All Together 1: Lessons 5-1 and 5-2
Put It All Together 2: Lessons 5-3 and 5-4
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **The Number System.**

- Convert Between Fractions and Decimals
- Identify Rational and Irrational Numbers
- Approximate Irrational Numbers



Module 5 • Real Numbers

Review

Foldables Use your Foldable to help review the module.

Real Numbers

Examples | Examples

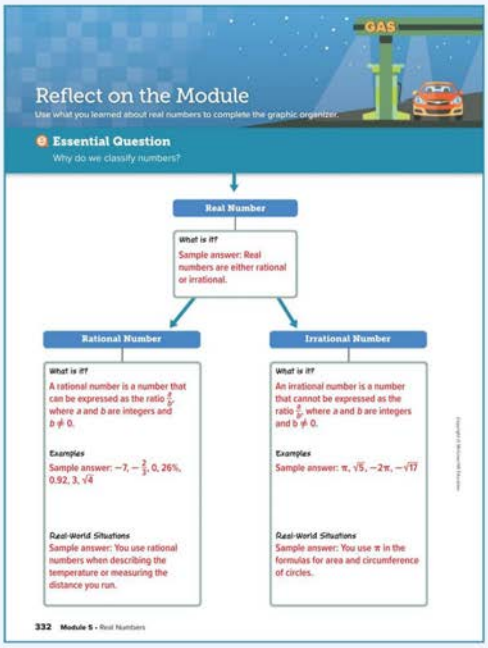
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 5 • Real Numbers 331



Reflect on the Module

Use what you learned about real numbers to complete the graphic organizer.

Essential Question

Why do we classify numbers?

Real Number

What is it?
Sample answer: Real numbers are either rational or irrational.

Rational Number

What is it?
A rational number is a number that can be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples
Sample answer: $-7, -\frac{2}{3}, 0, 26\%, 0.92, 3, \sqrt{4}$

Real-World Situations
Sample answer: You use rational numbers when describing the temperature or measuring the distance you run.

Irrational Number

What is it?
An irrational number is a number that cannot be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples
Sample answer: $\pi, \sqrt{5}, -2\pi, -\sqrt{17}$

Real-World Situations
Sample answer: You use π in the formulas for area and circumference of circles.

332 Module 5 • Real Numbers

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

Why do we classify numbers? See students' graphic organizers.

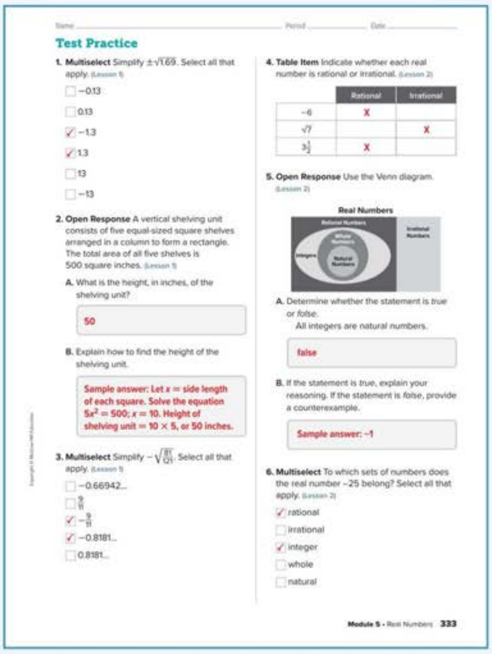
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–12 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiselect	Multiple answers may be correct. Students must select all correct answers.	1, 3, 6
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	8, 9
Table Item	Students complete a table.	4, 7, 10
Open Response	Students construct their own response in the area provided.	2, 5, 11, 12

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
8.NS.A.1	2-2, 2-4	4, 5, 6
8.NS.A.2	2-3, 2-4	7, 8, 9, 10, 11, 12
8.EE.A.2	2-1, 2-2	1, 2, 3


Name ______ Period ______ Date ______

Test Practice

1. **Multiselect** Simplify $\pm\sqrt{1.69}$. Select all that apply. (Lesson 1)
   - [ ] −0.13
   - [ ] 0.13
   - [x] −1.3
   - [x] 1.3
   - [ ] 13
   - [ ] −13

2. **Open Response** A vertical shelving unit consists of five equal-sized square shelves arranged in a column to form a rectangle. The total area of all five shelves is 500 square inches. (Lesson 1)

   A. What is the height, in inches, of the shelving unit?

   50

   B. Explain how to find the height of the shelving unit.

   Sample answer: Let x = side length of each square. Solve the equation $5x^2 = 500$; $x = 10$. Height of shelving unit = 10 × 5, or 50 inches.

3. **Multiselect** Simplify $-\sqrt{\frac{81}{121}}$. Select all that apply. (Lesson 1)
   - [ ] −0.66942...
   - [ ] $\frac{9}{11}$
   - [x] $-\frac{9}{11}$
   - [x] −0.8181...
   - [ ] 0.8181...

4. **Table Item** Indicate whether each real number is rational or irrational. (Lesson 2)

	Rational	Irrational
−6	X	
$\sqrt{7}$		X
$3\frac{1}{2}$	X	

5. **Open Response** Use the Venn diagram. (Lesson 2)

   A. Determine whether the statement is *true* or *false*.
   All integers are natural numbers.

   false

   B. If the statement is *true*, explain your reasoning. If the statement is *false*, provide a counterexample.

   Sample answer: −1

6. **Multiselect** To which sets of numbers does the real number −25 belong? Select all that apply. (Lesson 2)
   - [x] rational
   - [ ] irrational
   - [x] integer
   - [ ] whole
   - [ ] natural

Module 5 • Real Numbers 333

7. Table Item Indicate the integer to which each square root is closest on a number line. (Lesson 3)

	7	8	9
$\sqrt{70}$		X	
$\sqrt{79}$			X
$\sqrt{88}$			X
$\sqrt{52}$	X		
$\sqrt{60}$		X	
$\sqrt{47}$	X		
$\sqrt{65}$		X	

8. Equation Editor A shipping box, in the shape of a cube, has a volume of 2,300 cubic inches. Estimate the length of the side of the shipping box to the nearest integer. (Lesson 3)

13

9. Open Response Winston wants to put trim board around his square shaped windows. Each window has an area of 3 square feet. Estimate the perimeter of each window. Approximate your answer by truncating the decimal expansion to the hundredths place.

6.92 ft

10. Table Item Indicate which symbol makes each statement correct.

	<	>	=
$\sqrt[3]{24}$ ___ $\frac{\sqrt{400}}{6}$	X		
$-7\frac{5}{8}$ ___ $-\sqrt{55}$	X		
$-\sqrt{0.25}$ ___ $-\frac{1}{2}$			X

11. Open Response Consider the real numbers $-2\frac{1}{5}$ and $-\sqrt{7}$. (Lesson 4)

A. Compare the numbers. Use <, >, or =.
$-2\frac{1}{5}$ ___ $-\sqrt{7}$

>

B. Graph $-\sqrt{7}$ and $-2\frac{1}{5}$ on the number line.

12. Open Response The table shows the number of aces per serving attempts for three players at a recent volleyball tournament. Order the players from least aces per serving attempt to greatest aces per serving attempt. (Lesson 4)

Player	Number of Aces
Angela	9 out of 22
Jaylin	$22\frac{2}{5}$%
Mya	$\frac{3}{10}$

Jaylin, Mya, Angela

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Module 6

Algebraic Expressions

Module Goal

Use properties of operations to simplify algebraic expressions.

Focus

Domain: Expressions and Equations

Major Cluster(s):

7.EE.A Use pr operties of operations to generate equivalent expressions.

Standards for Mathematical Content:

7.EE.A.1 A pply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- fluently perform the four operations with rational numbers
- apply the Order of Operations to numerical expressions involving rational numbers
- evaluate simple algebraic expressions

Use the Module Pretest to diagnose students' readiness for this module. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students added, subtracted, multiplied, and divided integers and rational numbers. **7.NS.A.3**

Now
Students use properties of operations to simplify algebraic expressions. **7.EE.A.1, 7.EE.A.2**

Next
Students will apply the use of expressions to write and solve equations and formulas. **7.EE.B.4**

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of operations with algebraic expressions, greatest common factors and the distributive property (all gained in grade 6) to gain an *understanding* of simplifying algebraic expessions which includes distributing integers across algebraic expressions, adding and subtracting algebraic expressions, combining like terms, and factoring algebraic expressions.

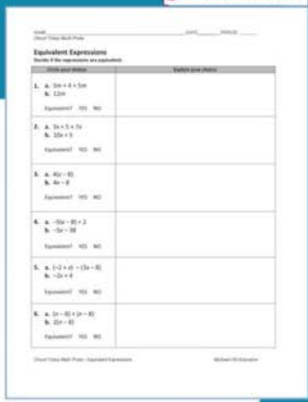
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
6-1	Simplify Algebraic Expressions	7.EE.A.1, 7.EE.A.2	1	0.5
6-2	Add Linear Expressions	7.EE.A.1	1	0.5
6-3	Subtract Linear Expressions	7.EE.A.1	1	0.5
Put It All Together 1: Lessons 6-1 through 6-3			0.5	0.25
6-4	Factor Linear Expressions	7.EE.A.1	1	0.5
6-5	Combine Operations with Linear Expressions	7.EE.A.1	1	0.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	8.5	4.25

Formative Assessment Math Probe
Equivalent Expressions

Module Resource



Correct Answers: **1.** No **2.** Yes **3.** No **4.** No **5.** No **6.** Yes

Analyze the Probe

Review the probe prior to assigning it to your students.
In this probe, students will determine if each pair of expressions is equivalent.

Targeted Concept Expressions can look different but still be equivalent. Strategies such as combining like terms, factoring, and distribution can be used to determine whether expressions are equivalent.

Targeted Misconceptions

- Students may fail to recognize the Distributive Property or apply the property incorrectly.
- Students may factor incorrectly or factor only part of an algebraic expression.
- Students may lack understanding of "like terms".

Assign the probe after Lesson 5.

Collect and Assess Student Work

If the student selects...	Then the student likely...
1. Yes with various other No selections	incorrectly combined unlike terms.
3. Yes, **4.** Yes, **5.** Yes, **6.** No	did not distribute to each term or factored only part of the expression.
Various incorrect choices.	incorrectly calculated operations with signed numbers.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Whole Numbers and Integers, Fractions, Decimals
- Lesson 1, Examples 1–6
- Lesson 2, Examples 1–2
- Lesson 3, Examples 1–3
- Lesson 4, Examples 1–5
- Lesson 5, Examples 1–3

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

![IGNITE!]

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

Why is it beneficial to rewrite expressions in different forms?
See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

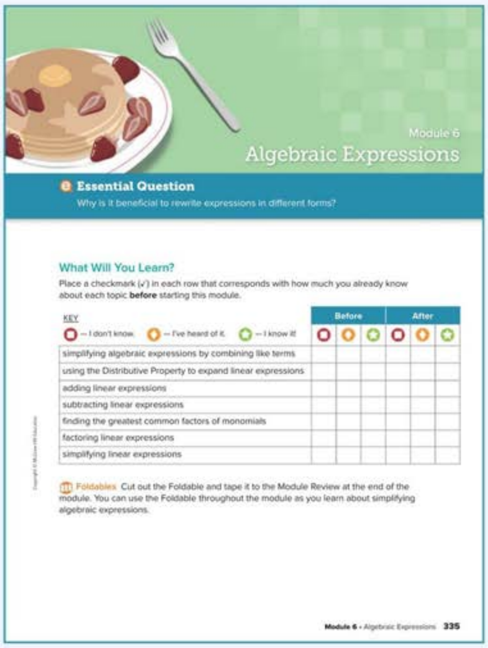
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of tipping a server at a restaurant and the cost of a cell phone plan to introduce the idea of simplifying algebraic expressions. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.


Module 6

Algebraic Expressions

Essential Question

Why is it beneficial to rewrite expressions in different forms?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

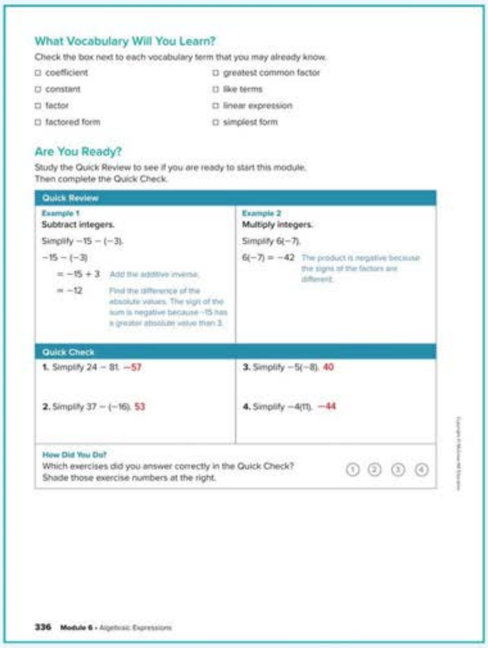
KEY: ○ — I don't know. ○ — I've heard of it. ○ — I know it!

	Before			After		
	○	○	○	○	○	○
simplifying algebraic expressions by combining like terms						
using the Distributive Property to expand linear expressions						
adding linear expressions						
subtracting linear expressions						
finding the greatest common factors of monomials						
factoring linear expressions						
simplifying linear expressions						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about simplifying algebraic expressions.

Module 6 • Algebraic Expressions 335

Interactive Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ coefficient
- ☐ constant
- ☐ factor
- ☐ factored form
- ☐ greatest common factor
- ☐ like terms
- ☐ linear expression
- ☐ simplest form

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1	Example 2
Subtract integers. Simplify $-15 - (-3)$. $-15 - (-3)$ $= -15 + 3$ Add the additive inverse. $= -12$ Find the difference of the absolute values. The sign of the sum is negative because −15 has a greater absolute value than 3.	**Multiply integers.** Simplify $6(-7)$. $6(-7) = -42$ The product is negative because the signs of the factors are different.

Quick Check

1. Simplify $24 - 81$. -57	3. Simplify $-5(-8)$. 40
2. Simplify $37 - (-16)$. 53	4. Simplify $-4(11)$. -44

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ② ③ ④

336 **Module 6** • Algebraic Expressions

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define A **coefficient** is the numerical factor of a term that contains a variable.

Example In the term $\frac{1}{2}b$, $\frac{1}{2}$ is the coefficient.

Ask What is the coefficient in the equation $\frac{2}{5}t = \frac{12}{15}$? $\frac{2}{5}$

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- writing and evaluating expressions using the order of operations
- adding and subtracting rational numbers
- finding the greatest common factor of two numbers
- multiplying and dividing rational numbers

ALEKS®

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Whole Numbers and Integers, Fractions,** and **Decimals** topics – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

"Not Yet" Doesn't Mean "Never"

Students with a growth mindset understand that just because they haven't yet found a solution, that does not mean they won't find one with additional effort and reasoning. It can take time and continued effort to reason through different strategies that can be used to solve a problem.

How Can I Apply It?

Assign students the **Formative Assessment Math Probes** that are available for each module. Have them complete the probe before starting the module, and then again at the specified lesson within the module, or at the end of the module so that they can see their progress.

Lesson 6-1

Simplify Algebraic Expressions

7.EE.A.1, 7.EE.A.2

LESSON GOAL

Students will simplify algebraic expressions by combining like terms and using the Distributive Property.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Simplify Algebraic Expressions

Learn: Like Terms
Learn: Combine Like Terms
Examples 1–3: Combine Like Terms
Learn: Expand Linear Expressions
Example 4: Distribute Over Addition
Example 5: Distribute Over Subtraction
Example 6: Distribute Negative Numbers
Apply: Geometry

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: The FOIL Method		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 33 of the *Language Development Handbook* to help your students build mathematical language related to simplifying algebraic expressions.

ELL You can use the tips and suggestions on page T33 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7. EE.A** by simplifying algebraic expressions by combining like terms and using the Distributive Property.
Standards for Mathematical Content: **7 .EE.A.1, 7.EE.A.2**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students added, subtracted, multiplied, and divided integers and rational numbers.
7.NS.A.3

Now
Students simplify algebraic expressions by combining like terms and using the Distributive Property.
7.EE.A.1, 7.EE.A.2

Next
Students will add linear expressions and express the sum in simplest form.
7.EE.A.1

Rigor

The Three Pillars of Rigor

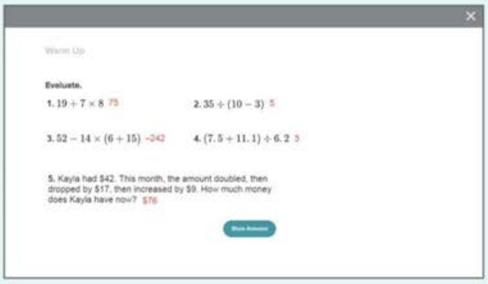
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of operations with algebraic expressions to *understand* identifying like terms and distributing integers across algebraic expressions. They will use this understanding to gain *fluency* in simplifying algebraic expressions through distributing and combining like terms.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- evaluating expressions using the order of operations (Exercises 1–4)
- writing and evaluating expressions using the order of operations (Exercise 5)

Answers

1. 75
2. 5
3. −242
4. 3
5. $76

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an expression to represent a youth organization's cookie sales.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does the prefix *co-* mean? Sample answer: Co- means together or joined together.
- What is the everyday meaning of the word *constant*? Sample answer: For something to be constant, it means that it is unchanging.
- What part of speech is the word *like* when used in *like terms*? How does it help you understand what *like terms* might be? Sample answer: Like is an adjective. I know that like terms must be terms that are similar, or alike in some manner.
- How can you use the meaning of the word *simple* to help you understand what the *simplest form* of an algebraic expression might be? Sample answer: Simple means easily done, or composed of one element. So, the simplest form of an algebraic expression might mean an expression that is composed in the most condensed form possible.
- Use the word *term* in a sentence outside of the study of mathematics. Sample answer: The U.S. President is elected to a four-year term.

Explore Simplify Algebraic Expressions

Objective

Students will use algebra tiles to explore how to simplify algebraic expressions.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the number of hours that three students worked in a week. The hours worked are represented by algebraic expressions. Throughout this activity, students will explore how to use algebra tiles to simplify the algebraic expressions. Students will use their observations to make a conjecture as to how algebra tiles can be used to simplify expressions, in general.

Inquiry Question

How can algebra tiles be used to simplify an expression? Sample answer: I can model an expression with algebra tiles, group the like tiles together, then remove any zero pairs. The tiles that are left represent the simplified expression.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

How do you think you could you use algebra tiles to simplify the expression? Sample answer: Combine the x-tiles together. There are four x-tiles in all. Then combine the 1-tiles and −1-tiles, removing any zero pairs as needed. There will be one 1-tile left.

(continued on next page)

Interactive Presentation

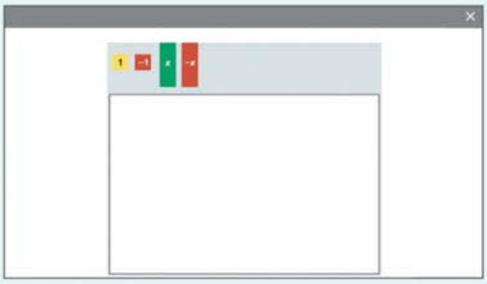
Explore, Slide 1 of 8

Explore, Slide 3 of 8

DRAG & DROP

On Slides 3 and 4, students drag algebra tiles to simplify an algebraic expression.

Interactive Presentation



Explore, Slide 6 of 8

DRAG & DROP

On Slides 5 and 6, students drag algebra tiles to model and simplify algebraic expressions.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Simplify Algebraic Expressions *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Explain to students the benefit of using algebra tiles as they can manipulate the tiles to represent and simplify expressions, visualize results, and make conjectures about how to use algebra tiles when simplifying expressions.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

What steps did you take to simplify the expression? Sample answer: First, I represented the original expression using algebra tiles. Next, I combined like tiles. I removed the zero pair of x-tiles and the two zero pairs of 1-tiles. There are two $-x$-tiles and three 1-tiles left on the mat.

What is the simplified expression? $-2x + 3$

Learn Like Terms

Objective

Students will understand what a term is and how to identify like terms.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to correctly and carefully attend to the definition of *like terms* when sorting each given term on Slide 2. Encourage them to explain their reasoning as to why and how they sorted each term.

As students discuss the *Talk About It!* question on Slide 3, encourage them to carefully attend to the meaning of *like terms* to explain why the given expressions are not like terms.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Why are the expressions $3x$ and $3x^2$ *not* like terms? Sample answer: $3x$ and $3x^2$ are not like terms because the variables have different exponents.

DIFFERENTIATE

Reteaching Activity AL

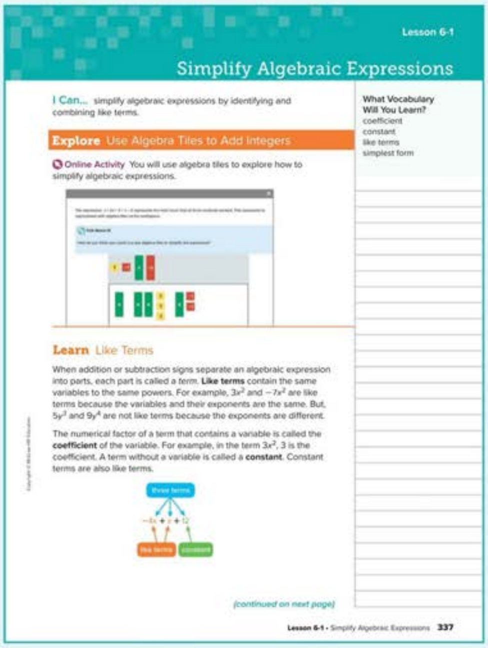
For students that may be struggling to identify like terms, explain that variable like terms only differ by coefficients. Have students work with a partner to generate sample like terms by using different coefficients for each of the following. Sample answers given.

x^2 $-5x^2$, $2x^2$

y^5 $3y^5$, $4y^5$

x^3 $-2x^3$, $-x^3$

y^{12} $-7y^{12}$, $7y^{12}$



Lesson 6-1

Simplify Algebraic Expressions

I Can... simplify algebraic expressions by identifying and combining like terms.

What Vocabulary Will You Learn?
coefficient
constant
like terms
simplest form

Explore Use Algebra Tiles to Add Integers

Online Activity You will use algebra tiles to explore how to simplify algebraic expressions.

Learn Like Terms

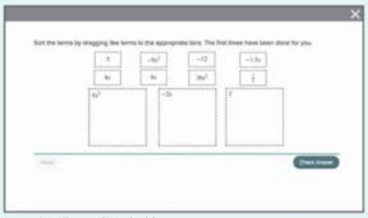
When addition or subtraction signs separate an algebraic expression into parts, each part is called a term. **Like terms** contain the same variables to the same powers. For example, $3x^2$ and $-7x^2$ are like terms because the variables and their exponents are the same. But, $5y^3$ and $9y^4$ are not like terms because the exponents are different.

The numerical factor of a term that contains a variable is called the **coefficient** of the variable. For example, in the term $3x^2$, 3 is the coefficient. A term without a variable is called a **constant**. Constant terms are also like terms.

(continued on next page)

Lesson 6-1 • Simplify Algebraic Expressions 337

Interactive Presentation



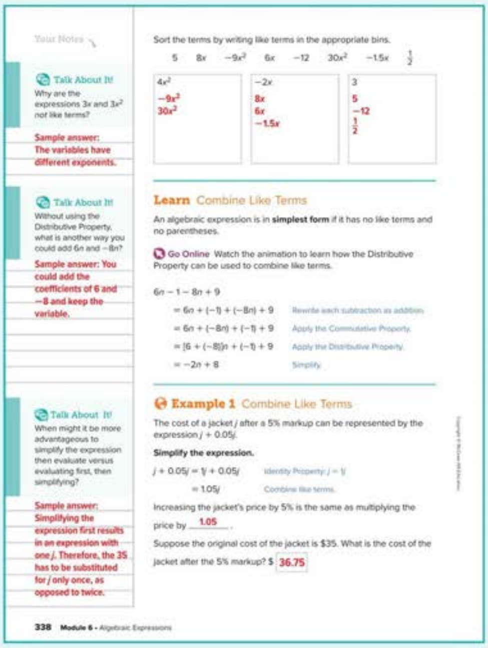
Learn, Like Terms, Slide 2 of 3

CLICK

On Slide 1, students select buttons to see examples of terms, like terms, and a constant for a given expression.

DRAG & DROP

On Slide 2, students drag to sort terms into groups of like terms.



Your Notes

Talk About It! Why are the expressions $3x$ and $3x^2$ not like terms?

Sample answer: The variables have different exponents.

Sort the terms by writing like terms in the appropriate bins.

$5 \quad 8x \quad -9x^2 \quad 6x \quad -12 \quad 30x^2 \quad -1.5x \quad \frac{1}{2}$

$4x^2$ $-9x^2$ $30x^2$	$-2x$ $8x$ $6x$ $-1.5x$	3 5 -12 $\frac{1}{2}$

Learn Combine Like Terms

An algebraic expression is in **simplest form** if it has no like terms and no parentheses.

Go Online Watch the animation to learn how the Distributive Property can be used to combine like terms.

Talk About It! Without using the Distributive Property, what is another way you could add $6n$ and $-8n$?

Sample answer: You could add the coefficients of 6 and −8 and keep the variable.

$6n - 1 - 8n + 9$

$= 6n + (-1) + (-8n) + 9$ Rewrite each subtraction as addition.

$= 6n + (-8n) + (-1) + 9$ Apply the Commutative Property.

$= [6 + (-8)]n + (-1) + 9$ Apply the Distributive Property.

$= -2n + 8$ Simplify.

Example 1 Combine Like Terms

The cost of a jacket j after a 5% markup can be represented by the expression $j + 0.05j$.

Simplify the expression.

$j + 0.05j = 1j + 0.05j$ Identity Property: $j = 1j$

$= 1.05j$ Combine like terms.

Increasing the jacket's price by 5% is the same as multiplying the price by 1.05.

Suppose the original cost of the jacket is \$35. What is the cost of the jacket after the 5% markup? \$ 36.75

Talk About It! When might it be more advantageous to simplify the expression then evaluate versus evaluating first, then simplifying?

Sample answer: Simplifying the expression first results in an expression with one j. Therefore, the 35 has to be substituted for j only once, as opposed to twice.

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Interactive Presentation

Learn, Combine Like Terms, Slide 1 of 2

WATCH

On Slide 1 of the Learn, students watch an animation to learn how the Distributive Property can be used to combine like terms.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Combine Like Terms

Objective

Students will learn how to combine like terms.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them As students discuss the *Talk About It!* question on Slide 2, encourage them to listen and understand the approaches of others and to identify the correspondences between different approaches.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

In Steps 3 and 4 of the animation, the Distributive Property was used to combine like terms. Without using the Distributive Property, what is another way you could add $6n$ and $-8n$? Sample answer: You could add the coefficients of 6 and −8 and keep the variable.

Example 1 Combine Like Terms

Objective

Students will combine like terms to simplify an expression representing a real-world scenario.

Questions for Mathematical Discourse

SLIDE 2

AL What is the coefficient of j? How do you know? The coefficient of j is 1 because j refers to 1 whole j.

AL What is the coefficient of $0.05j$? How does it relate to j? 0.05; Sample answer: $0.05j$ is smaller than j.

OL Why is the correct expression $j + 0.05j$ and not $j + 5j$? The 5% markup should be expressed as a decimal, 0.05.

OL What are the like terms? j and $0.05j$

BL What expression would represent the total cost of the jacket after a 15% markup of the original price? What would be the total cost if the original cost is \$35? $j + 0.15j$, or $115j$; \$40.25

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Combine Like Terms

Objective

Students will combine like terms with integer coefficients and constants.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the terms given in the expression, why any terms that are subtracted are rewritten as addition of the additive inverse, and how that step is needed when using the Commutative Property.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use precise and clear mathematical language when explaining why the Commutative Property was used, and why the subtraction needed to be written as addition prior to using the property.

Questions for Mathematical Discourse

SLIDE 2

AL What is the coefficient of the term y? Explain. The coefficient of the term y is 1, because it represents 1 whole y.

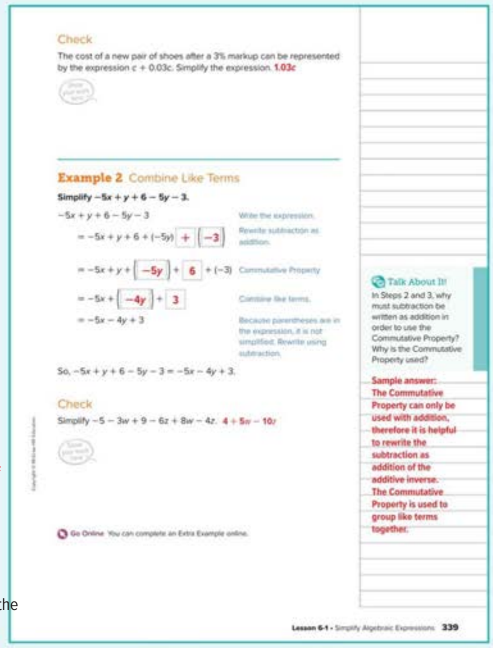
OL How many sets of like terms are there? How do you know? There are three sets of like terms, expressions with the variable x, expressions with the variable y, and constants.

OL When the like terms with the variable y are combined, why is the result not $6y$? The terms are y and $-5y$, not y and $5y$.

BL How could you alter the expression so that the simplest form only has two terms in it, not three? Sample answer: Rewrite the expression so that one of the like terms combines to be 0, as in $-5x + y + 6 - y - 3$, which simplifies to be $-5x + 3$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

The cost of a new pair of shoes after a 3% markup can be represented by the expression $c + 0.03c$. Simplify the expression. $1.03c$

Example 2 Combine Like Terms

Simplify $-5x + y + 6 - 5y - 3$.

$-5x + y + 6 - 5y - 3$ Write the expression.

$= -5x + y + 6 + (-5y) + (-3)$ Rewrite subtraction as addition.

$= -5x + y + (-5y) + 6 + (-3)$ Commutative Property

$= -5x + (-4y) + 3$ Combine like terms.

$= -5x - 4y + 3$ Because parentheses are in the expression, it is not simplified. Rewrite using subtraction.

So, $-5x + y + 6 - 5y - 3 = -5x - 4y + 3$.

Check

Simplify $-5 - 3w + 9 - 6z + 8w - 4z$. $4 + 5w - 10z$

Go Online You can complete an Extra Example online.

Talk About It! In Steps 2 and 3, why must subtraction be written as addition in order to use the Commutative Property? Why is the Commutative Property used?

Sample answer: The Commutative Property can only be used with addition, therefore it is helpful to rewrite the subtraction as addition of the additive inverse. The Commutative Property is used to group like terms together.

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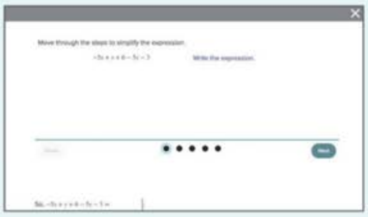
Interactive Presentation

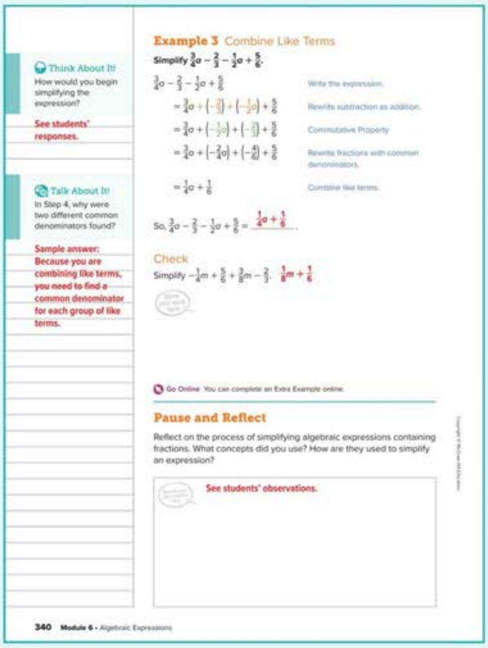


Example 2, Combine Like Terms, Slide 2 of 4

CLICK On Slide 2, students move through the steps to simplify the expression.

TYPE On Slide 2, students type to enter the simplified expression.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It! How would you begin simplifying the expression?

See students' responses.

Talk About It! In Step 4, why were two different common denominators found?

Sample answer: Because you are combining like terms, you need to find a common denominator for each group of like terms.

Example 3 Combine Like Terms

Simplify $\frac{3}{4}a - \frac{2}{3} - \frac{1}{2}a + \frac{5}{6}$.

$\frac{3}{4}a - \frac{2}{3} - \frac{1}{2}a + \frac{5}{6}$ — Write the expression.

$= \frac{3}{4}a + \left(-\frac{2}{3}\right) + \left(-\frac{1}{2}a\right) + \frac{5}{6}$ — Rewrite subtraction as addition.

$= \frac{3}{4}a + \left(-\frac{1}{2}a\right) + \left(-\frac{2}{3}\right) + \frac{5}{6}$ — Commutative Property

$= \frac{3}{4}a + \left(-\frac{2}{4}a\right) + \left(-\frac{4}{6}\right) + \frac{5}{6}$ — Rewrite fractions with common denominators.

$= \frac{1}{4}a + \frac{1}{6}$ — Combine like terms.

So, $\frac{3}{4}a - \frac{2}{3} - \frac{1}{2}a + \frac{5}{6} =$ $\frac{1}{4}a + \frac{1}{6}$.

Check

Simplify $-\frac{1}{4}m + \frac{5}{6} + \frac{3}{8}m - \frac{2}{3}$. $\frac{1}{8}m + \frac{1}{6}$

Go Online You can complete an Extra Example online.

Pause and Reflect

Reflect on the process of simplifying algebraic expressions containing fractions. What concepts did you use? How are they used to simplify an expression?

See students' observations.

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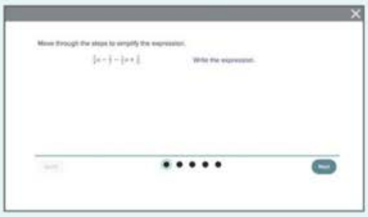
Interactive Presentation



Example 3, Combine Like Terms, Slide 2 of 4

CLICK On Slide 2, students move through the steps to simplify the expression.

TYPE On Slide 2, students type to write the simplified expression.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Combine Like Terms

Objective

Students will combine like terms with rational coefficients and constants.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the terms given in the expression, why any terms that are subtracted are rewritten as addition of the additive inverse, and how that step is needed when using the Commutative Property.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use precise and clear mathematical language when explaining why two different common denominators were found when combining like terms.

Questions for Mathematical Discourse

SLIDE 2

AL How many sets of like terms are there? Explain. There are two sets of like terms, the two terms with the variable *a* and the two constants.

AL Why do we rewrite subtraction as addition of the additive inverse? In order to use the Commutative Property to group the like terms together.

OL Why do we rewrite the fractions with common denominators? In order to add the fractions, we need to find common denominators.

BL Suppose a classmate simplified the expression as $\frac{1}{4}a + 1\frac{1}{2}$. Describe their error. Sample answer: They added $\frac{2}{3}$ and $\frac{5}{6}$, but the term is $-\frac{2}{3}$, not $\frac{2}{3}$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of combining like terms, have students identify whether or not each of the following pairs of terms can be combined, and if so, have them identify the resulting term.

$5xy^2 + 12xy^2$ $17xy^2$

$3x^3y + 8xy^3$ not like terms

$6xy - xy$ $5xy$

Learn Expand Linear Expressions

Objective

Students will understand how to expand linear expressions using the Distributive Property.

Teaching Notes

SLIDE 1

Students will learn how the Distributive Property can be used to expand linear expressions. You may wish to have a student volunteer select each card to show how the Distributive Property can be described using words, symbols, and examples.

Go Online to find additional teaching notes.

Example 4 Distribute Over Addition

Objective

Students will expand linear expressions by distributing over addition.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to accurately use the Distributive Property when expanding the expression, paying attention to which terms are multiplied by 4, and paying attention to the sign of each term.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is indicated by the parentheses? multiplication

AL Is $-3x$ the only term that is multiplied by 4? Explain. no; $-3x$ and 6 are each multiplied by 4

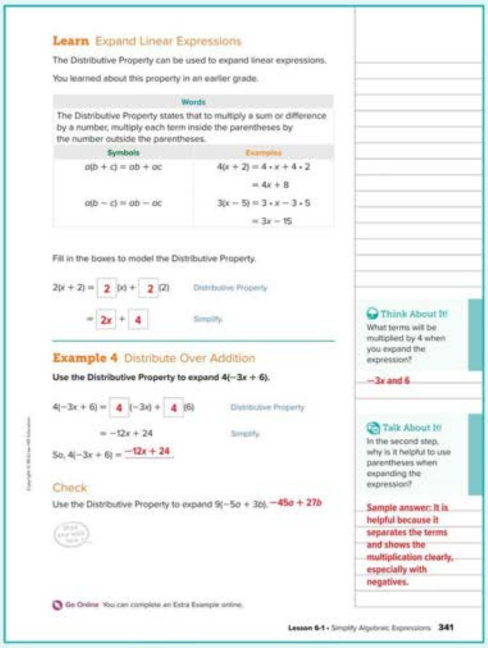
OL On the last step, are you able to combine $-12x$ and 24? Explain. No; they are not like terms.

OL Is the expression $(-3x + 6)4$ equivalent to the given expression in this example? Explain. yes; Sample answer: Both expressions simplify to $-12x + 24$. The Commutative Property allows us to multiply in any order. The two factors are $(-3x + 6)$ and 4. The factors can be in any order.

BL Generate another expression containing parentheses that, when expanded, equals $-12x + 24$. Sample answer: $3(-4x + 8)$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Expand Linear Expressions

The Distributive Property can be used to expand linear expressions.

You learned about this property in an earlier grade.

Words	
The Distributive Property states that to multiply a sum or difference by a number, multiply each term inside the parentheses by the number outside the parentheses.	
Symbols	**Examples**
$a(b + c) = ab + ac$	$4(x + 2) = 4 \cdot x + 4 \cdot 2$ $= 4x + 8$
$a(b - c) = ab - ac$	$3(x - 5) = 3 \cdot x - 3 \cdot 5$ $= 3x - 15$

Fill in the boxes to model the Distributive Property.

$2(x + 2) = \boxed{2}(x) + \boxed{2}(2)$ Distributive Property

$= \boxed{2x} + \boxed{4}$ Simplify.

Example 4 Distribute Over Addition

Use the Distributive Property to expand $4(-3x + 6)$.

$4(-3x + 6) = \boxed{4}(-3x) + \boxed{4}(6)$ Distributive Property

$= -12x + 24$ Simplify.

So, $4(-3x + 6) = -12x + 24$

Check

Use the Distributive Property to expand $9(-5a + 3b)$. $-45a + 27b$

Go Online You can complete an Extra Example online.

Think About It! What terms will be multiplied by 4 when you expand the expression? $-3x$ and 6

Talk About It! In the second step, why is it helpful to use parentheses when expanding the expression? Sample answer: It is helpful because it separates the terms and shows the multiplication clearly, especially with negatives.

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Interactive Presentation

Learn, Expand Linear Expressions, Slide 1 of 2

FLASHCARDS

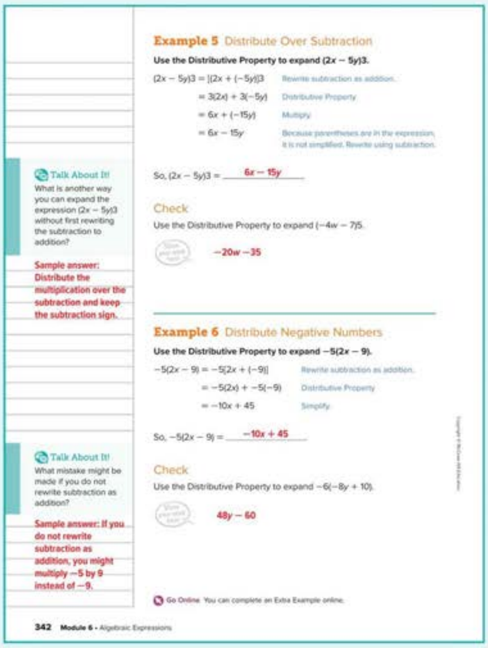
On Slide 1 of the Learn, students use Flashcards to learn about the Distributive Property through words, symbols, and examples.

CLICK

On Slide 2 of Example 4, students move through the steps to simplify an expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 5 Distribute Over Subtraction

Use the Distributive Property to expand $(2x - 5y)3$.

$(2x - 5y)3 = [2x + (-5y)]3$ Rewrite subtraction as addition.

$= 3(2x) + 3(-5y)$ Distributive Property

$= 6x + (-15y)$ Multiply

$= 6x - 15y$ Because parentheses are in the expression, it is not simplified. Rewrite using subtraction.

So, $(2x - 5y)3 =$ $6x - 15y$

Talk About It! What is another way you can expand the expression $(2x - 5y)3$ without first rewriting the subtraction to addition?

Sample answer: Distribute the multiplication over the subtraction and keep the subtraction sign.

Check

Use the Distributive Property to expand $(-4w - 7)5$.

$-20w - 35$

Example 6 Distribute Negative Numbers

Use the Distributive Property to expand $-5(2x - 9)$.

$-5(2x - 9) = -5[2x + (-9)]$ Rewrite subtraction as addition.

$= -5(2x) + -5(-9)$ Distributive Property

$= -10x + 45$ Simplify

So, $-5(2x - 9) =$ $-10x + 45$

Talk About It! What mistake might be made if you do not rewrite subtraction as addition?

Sample answer: If you do not rewrite subtraction as addition, you might multiply -5 by 9 instead of -9.

Check

Use the Distributive Property to expand $-6(-8y + 10)$.

$48y - 60$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 5, Distribute Over Subtraction, Slide 2 of 4

CLICK

On Slide 2 of Example 5, students move through the steps to simplify the expression.

TYPE

On Slide 2 of Example 6, students type to show a simplified expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Example 5 Distribute Over Subtraction

Objective

Students will expand linear expressions by distributing over subtraction.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is indicated by the parentheses? multiplication

AL What does it mean when the constant 3 is on the right-hand side of the parentheses? Sample answer: It means that each term, $2x$ and $-5y$, is multiplied by 3. It does not matter if it is on the left or the right side of the parentheses.

OL On the last step, why is $6x + (-15y)$ equivalent to $6x - 15y$? Addition of a negative number is the same as subtracting the additive inverse of that number.

BL Suppose a classmate simplified the expression as $2x - 15y$. Describe the error that they made. Sample answer: They did not distribute the 3 to each term, only the second term.

Example 6 Distribute Negative Numbers

Objective

Students will expand linear expressions by distributing a negative number.

Questions for Mathematical Discourse

SLIDE 2

AL What number is each term, $2x$ and -9, multiplied by? -5

OL Suppose a classmate simplified the expression as $-10x - 45$. Describe the error they made. Sample answer: They incorrectly multiplied -5 by 9. The product should be 45, not -45.

BL How would the simplified expression change if the original expression was $-5(2x + 9)$? The simplified expression would be $-10x - 45$, since $-5(9) = -45$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.

Apply Geometry

Objective

Students will come up with their own strategy to solve an application problem involving side lengths of triangles.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students time to make sure they understand the task, think of possible strategies, and work to solve the problem.

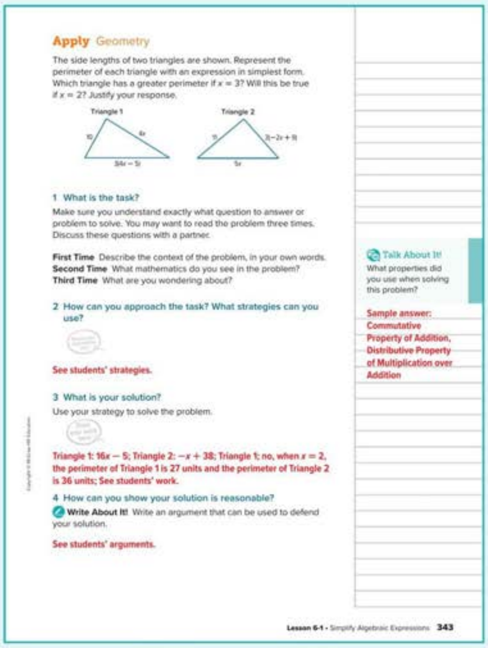
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you find the perimeter of a figure?
- How can you use the Distributive Property to help you?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they used to defend their solution.



Apply Geometry

The side lengths of two triangles are shown. Represent the perimeter of each triangle with an expression in simplest form. Which triangle has a greater perimeter if $x = 3$? Will this be true if $x = 2$? Justify your response.

Triangle 1: 10, $6x$, $3(4x - 5)$
Triangle 2: 11, $3(-2x + 9)$, $5x$

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

Triangle 1: $16x - 5$; Triangle 2: $-x + 38$; Triangle 1; no, when $x = 2$, the perimeter of Triangle 1 is 27 units and the perimeter of Triangle 2 is 36 units; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! What properties did you use when solving this problem?

Sample answer: Commutative Property of Addition, Distributive Property of Multiplication over Addition

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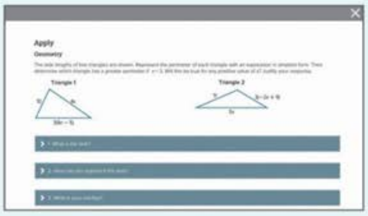
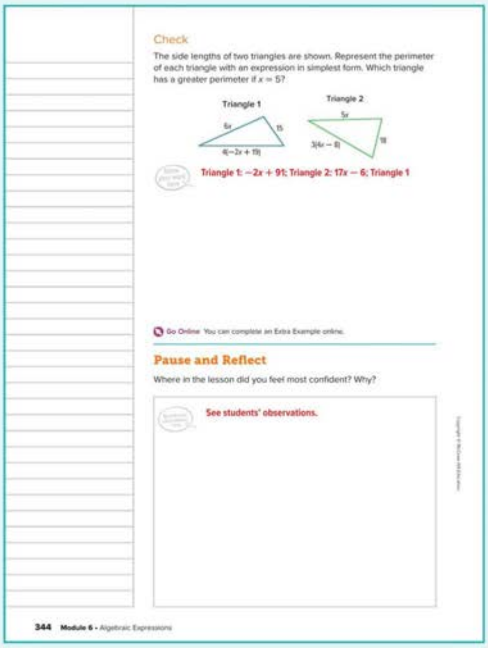
Interactive Presentation



Apply, Geometry

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The side lengths of two triangles are shown. Represent the perimeter of each triangle with an expression in simplest form. Which triangle has a greater perimeter if $x = 5$?

Triangle 1: $-2x + 91$; Triangle 2: $17x - 6$; Triangle 1

Go Online You can complete an Extra Example online.

Pause and Reflect

Where in the lesson did you feel most confident? Why?

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

Why is it beneficial to rewrite expressions in different forms?

In this lesson, students learned how to simplify algebraic expressions by combining like terms. Encourage them to discuss with a partner how combining like terms is beneficial. For example, they may state that combining like terms reduces the number of terms in an expression to a minimum.

Exit Ticket

Refer to the Exit Ticket slide. Let x represent the price of a box of coconut cookies and y represent the price of a box of shortbread cookies. Then the expression $2x + 7y$ represents Sarah selling 2 boxes of coconut cookies and 7 boxes of shortbread cookies. The expression $5x + 3y$ represents Natalie selling 5 boxes of coconut cookies and 3 boxes of shortbread cookies. Write an expression that represents the cookie sales for both girls. Write a mathematical argument that can be used to defend your solution. $7x + 10y$; Sample answer: Add the expressions $2x + 7y$ and $5x + 3y$ by combining like terms. $2x$ and $5x$ are like terms with a sum of $7x$. $7y$ and $3y$ are like terms with a sum of $10y$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 15, 17–20
- Extension: The FOIL Method
- ALEKS® Simplifying Algebraic Expressions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–14, 16, 18
- Extension: The FOIL Method
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–6
- ALEKS® The Distributive Property

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® The Distributive Property

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

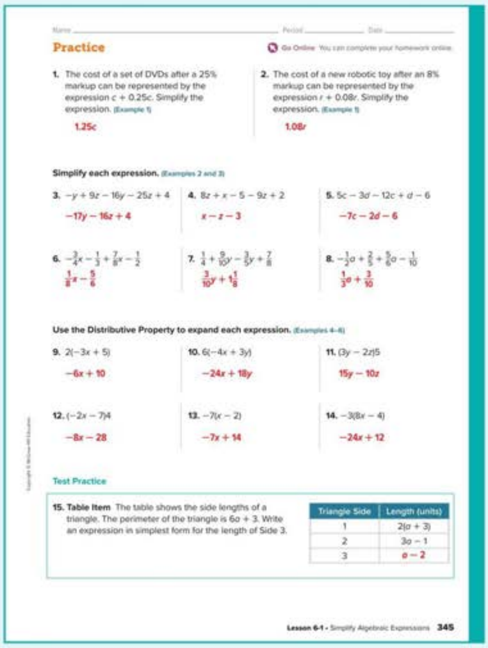
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	combine like terms in a real-world scenario	1, 2
1	combine like terms with integer and rational coefficients and constants	3–8
1	expand linear expressions by distributing	9–14
2	extend concepts learned in class to apply them in new contexts	15
3	solve application problems involving simplifying algebraic expressions	16, 17
3	higher-order and critical thinking skills	18–20

Common Misconception

Some students may not identify like terms correctly when one of the coefficients is 1 or −1 since the number 1 is not written explicitly. In Exercise 3, students may not combine the y terms correctly by either misidentifying like terms or by failing to recognize the coefficient of $-y$ as −1.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The cost of a set of DVDs after a 25% markup can be represented by the expression $c + 0.25c$. Simplify the expression. (Example 1)
   $1.25c$

2. The cost of a new robotic toy after an 8% markup can be represented by the expression $r + 0.08r$. Simplify the expression. (Example 1)
   $1.08r$

Simplify each expression. (Examples 2 and 3)

3. $-y + 9z - 16y - 25z + 4$
   $-17y - 16z + 4$
4. $8z + x - 5 - 9z + 2$
   $x - z - 3$
5. $5c - 3d - 12c + d - 6$
   $-7c - 2d - 6$
6. $-\frac{3}{4}x - \frac{1}{3} + \frac{7}{8}x - \frac{1}{2}$
   $\frac{1}{8}x - \frac{5}{6}$
7. $\frac{1}{4} + \frac{9}{10}y - \frac{3}{5}y + \frac{7}{8}$
   $\frac{3}{10}y + 1\frac{1}{8}$
8. $-\frac{1}{2}a + \frac{2}{5} + \frac{5}{6}a - \frac{1}{10}$
   $\frac{1}{3}a + \frac{3}{10}$

Use the Distributive Property to expand each expression. (Examples 4–6)

9. $2(-3x + 5)$
   $-6x + 10$
10. $6(-4x + 3y)$
    $-24x + 18y$
11. $(3y - 2z)5$
    $15y - 10z$
12. $(-2x - 7)4$
    $-8x - 28$
13. $-7(x - 2)$
    $-7x + 14$
14. $-3(8x - 4)$
    $-24x + 12$

Test Practice

15. **Table Item** The table shows the side lengths of a triangle. The perimeter of the triangle is $6a + 3$. Write an expression in simplest form for the length of Side 3.

Triangle Side	Length (units)
1	$2(a + 3)$
2	$3a - 1$
3	$a - 2$

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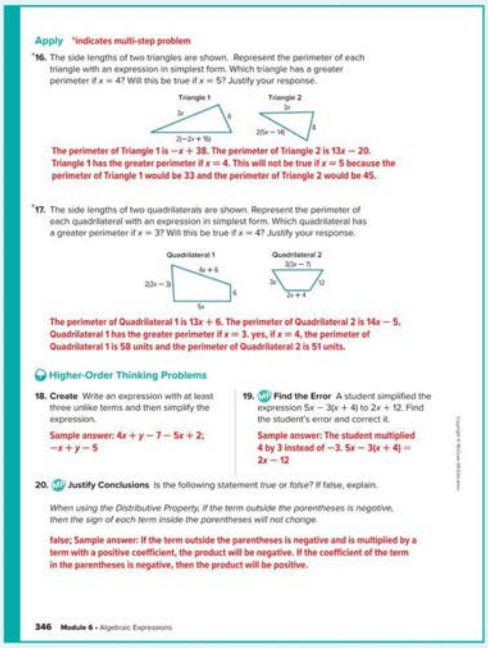

Apply *indicates multi-step problem

*16. The side lengths of two triangles are shown. Represent the perimeter of each triangle with an expression in simplest form. Which triangle has a greater perimeter if $x = 4$? Will this be true if $x = 5$? Justify your response.

The perimeter of Triangle 1 is $-x + 38$. The perimeter of Triangle 2 is $13x - 20$. Triangle 1 has the greater perimeter if $x = 4$. This will not be true if $x = 5$ because the perimeter of Triangle 1 would be 33 and the perimeter of Triangle 2 would be 45.

*17. The side lengths of two quadrilaterals are shown. Represent the perimeter of each quadrilateral with an expression in simplest form. Which quadrilateral has a greater perimeter if $x = 3$? Will this be true if $x = 4$? Justify your response.

The perimeter of Quadrilateral 1 is $13x + 6$. The perimeter of Quadrilateral 2 is $14x - 5$. Quadrilateral 1 has the greater perimeter if $x = 3$. yes, if $x = 4$, the perimeter of Quadrilateral 1 is 58 units and the perimeter of Quadrilateral 2 is 51 units.

Higher-Order Thinking Problems

18. **Create** Write an expression with at least three unlike terms and then simplify the expression.

Sample answer: $4x + y - 7 - 5x + 2$; $-x + y - 5$

19. **MP Find the Error** A student simplified the expression $5x - 3(x + 4)$ to $2x + 12$. Find the student's error and correct it.

Sample answer: The student multiplied 4 by 3 instead of −3. $5x - 3(x + 4) = 2x - 12$

20. **MP Justify Conclusions** Is the following statement *true* or *false*? If false, explain.

When using the Distributive Property, if the term outside the parentheses is negative, then the sign of each term inside the parentheses will not change.

false; Sample answer: If the term outside the parentheses is negative and is multiplied by a term with a positive coefficient, the product will be negative. If the coefficient of the term in the parentheses is negative, then the product will be positive.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 19, students find and correct a student's error in simplifying an expression.

In Exercise 20, students determine if a statement is true or false about signs when distributing a negative number.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 16 Have students work together to prepare a brief demonstration that illustrates why this is an application problem. For example, before they can determine the triangle with the greater perimeter if $x = 4$, they must first generate a simplified expression for each triangle. Have each pair or group of students present their response to the class.

Listen and ask clarifying questions.

Use with Exercise 19 Have students work in pairs. Have students individually read Exercise 19 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection.

Lesson 6-2

Add Linear Expressions

7.EE.A.1

LESSON GOAL

Students will add linear expressions and express the sum in simplest form.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Add Expressions

Learn: Add Linear Expressions
Example 1: Add Linear Expressions
Example 2: Add Linear Expressions
Apply: Theater

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 34 of the *Language Development Handbook* to help your students build mathematical language related to addition of linear expressions.

ELL You can use the tips and suggestions on page T34 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7.EE.A** by adding linear expressions and expressing the sum in simplest form.
Standards for Mathematical Content: **7 .EE.A.1**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students simplified algebraic expressions by combining like terms and using the Distributive Property.
7.EE.A.1, 7.EE.A.2

Now
Students add linear expressions and express the sum in simplest form.
7.EE.A.1

Next
Students will subtract linear expressions and express the difference in simplest form.
7.EE.A.1

Rigor

The Three Pillars of Rigor

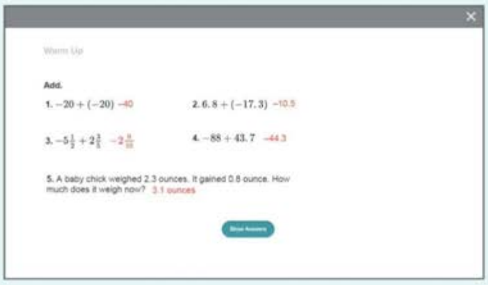
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of simplifying algebraic expressions to develop an *understanding* of adding linear expressions. They will gain *fluency* with adding algebraic expressions through continued practice.

Mathematical Background

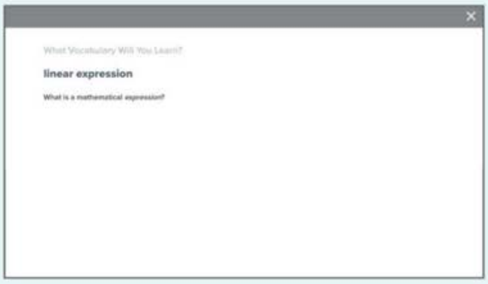
A *linear expression* is an algebraic expression in which the variable is raised to the first power and the variables are not multiplied or divided. To add linear expressions, add like terms. To add $(6x + 4) + (7x + 5)$, add the like terms $6x$ and $7x$ to obtain $13x$. Then add the constants 4 and 5 to obtain 9. The sum of the two linear expressions is $13x + 9$.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- adding rational numbers (Exercises 1–5)

Answers

1. −40
2. −10 .5
3. $-2\frac{9}{10}$
4. −44.3
5. 3.1 ounces

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using expressions to find total numbers of new species discovered in the rain forest.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What is a mathematical expression? Sample answer: A mathematical expression contains numbers, variables, and at least one operation.

Explore Add Expressions

Objective

Students will use Web Sketchpad to explore how to add linear expressions.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch called the Magic Square puzzle. A completed Magic Square puzzle displays the variable terms of two algebraic expressions and their sum in one column, the constant terms and their sum in another column, and the terms themselves and the total sum in the last column. Throughout this activity, students will complete Magic Square puzzles with varying inputs to explore adding algebraic expressions.

Inquiry Question

How can you use a Magic Square puzzle to add expressions?
Sample answer: A magic square groups the variable terms separately from the constant terms. Add each group of terms vertically to determine the sum.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

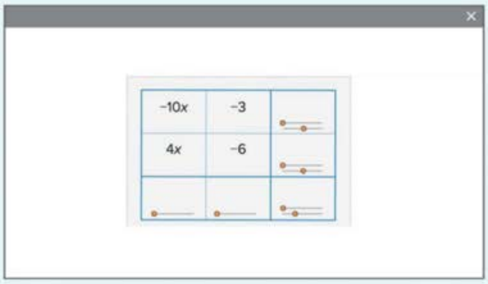
Mathematical Discourse

What is the sum of the two expressions? What strategy did you use to complete the puzzle? Sample answer: The sum of the two expressions is $-6x - 9$. I added the terms in the rows and the columns to complete each puzzle.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

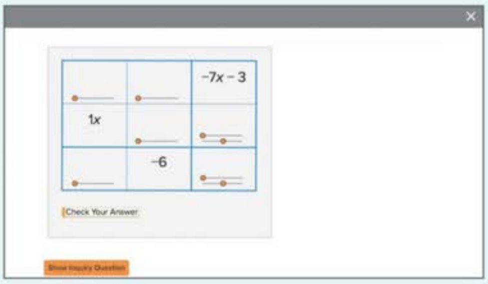


Explore, Slide 2 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad and Magic Square puzzles to explore adding algebraic expressions.

Interactive Presentation



Explore, Slide 5 of 7

WEB SKETCHPAD

On Slide 6, students use Web Sketchpad and Magic Square puzzles to practice adding algebraic expressions.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Add Expressions *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use the Magic Square puzzle as a tool to help them find the sum of two algebraic expressions, or missing elements when some parts of the expressions and/or the sum are given.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What strategy did you use to complete the puzzle? Sample answer: Because the bottom center box is −6, then I know the sum of the constant terms is −6. The other constant term is −3. So, I need to determine what constant plus −3 has a sum of −6. I know one variable term is $-7x$ and the other is $1x$, so I can add these terms together.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Add Linear Expressions

Objective

Students will understand what a linear expression is and how to add linear expressions.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to adhere to the definition of *linear expression* as they analyze each expression on Slide 1 to determine if it is linear or nonlinear.

7 Look for and Make Use of Structure Students should look closely at each expression to analyze its structure prior to sorting them. Have students explain the difference in the structures of the expressions $\frac{1}{2}x - 5$ and $\frac{6}{x}$.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 3. The animation illustrates how to add linear expressions.

Talk About It!

SLIDE 4

Mathematical Discourse

When you add the expressions $4x + 2$ and $5x - 7$, the answer is $9x + (-5)$ or $9x - 5$. Why can we rewrite $9x + (-5)$ as $9x - 5$? Sample answer: $9x + (-5)$ can be rewritten as $9x - 5$ because adding the additive inverse is the same as subtracting.

DIFFERENTIATE

Language Development Activity ELL

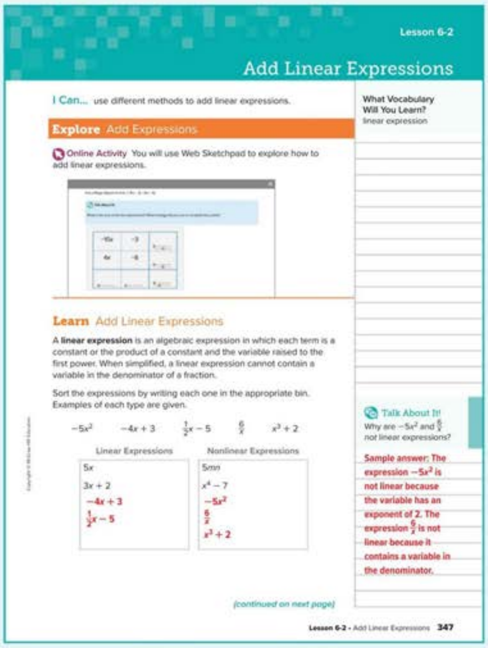
To support students in identifying linear and nonlinear expressions, explain to them that a linear expression can only have variable terms to the first power. Have students work with a partner to generate their own linear expressions using each of the following variables. Then have them generate a nonlinear expression for each.

x Sample answer: linear: $-4x + 3$; nonlinear: $3x^2$

z Sample answer: linear: $3z$; nonlinear: $-5 + z^3$

m Sample answer: linear: $9 - 2m$; nonlinear: $\frac{4}{m}$

t Sample answer: linear: $t - 12$; nonlinear: $t^2 + 1$


Lesson 6-2

Add Linear Expressions

I Can... use different methods to add linear expressions.

What Vocabulary Will You Learn?
linear expression

Explore Add Expressions

Online Activity You will use Web Sketchpad to explore how to add linear expressions.

Learn Add Linear Expressions

A **linear expression** is an algebraic expression in which each term is a constant or the product of a constant and the variable raised to the first power. When simplified, a linear expression cannot contain a variable in the denominator of a fraction.

Sort the expressions by writing each one in the appropriate bin. Examples of each type are given.

$-5x^2$ $-4x + 3$ $\frac{1}{2}x - 5$ $\frac{6}{x}$ $x^3 + 2$

Linear Expressions	Nonlinear Expressions
$5x$	$5mn$
$3x + 2$	$x^4 - 7$
$-4x + 3$	$-5x^2$
$\frac{1}{2}x - 5$	$\frac{6}{x}$
	$x^3 + 2$

Talk About It! Why are $-5x^2$ and $\frac{6}{x}$ not linear expressions?

Sample answer: The expression $-5x^2$ is not linear because the variable has an exponent of 2. The expression $\frac{6}{x}$ is not linear because it contains a variable in the denominator.

(continued on next page)

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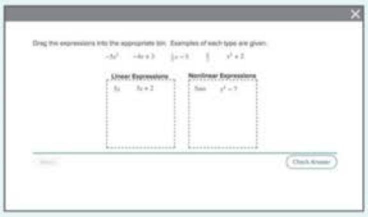
Interactive Presentation

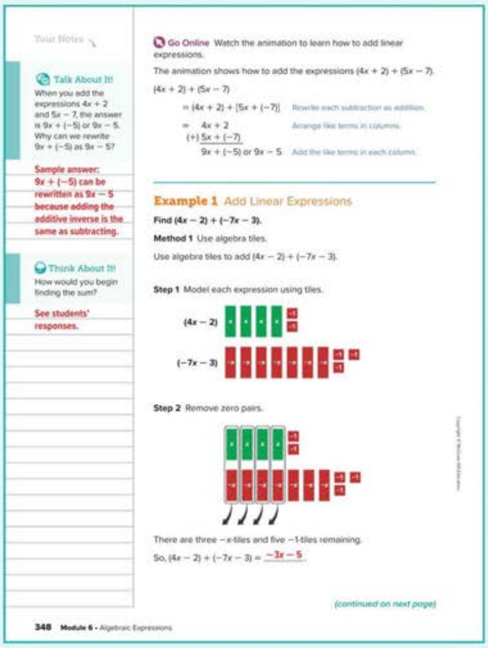

Learn, Add Linear Expressions, Slide 1 of 4

DRAG & DROP

On Slide 1, students drag to sort expressions as linear or nonlinear.

WATCH

On Slide 3, students watch an animation that explains how to add linear expressions.



Your Notes

Talk About It!
When you add the expressions $4x + 2$ and $5x - 7$, the answer is $9x + (-5)$ or $9x - 5$. Why can we rewrite $9x + (-5)$ as $9x - 5$?

Sample answer: $9x + (-5)$ can be rewritten as $9x - 5$ because adding the additive inverse is the same as subtracting.

Think About It!
How would you begin finding the sum?

See students' responses.

Go Online Watch the animation to learn how to add linear expressions.

The animation shows how to add the expressions $(4x + 2) + (5x - 7)$.

$(4x + 2) + (5x - 7)$

$= (4x + 2) + [5x + (-7)]$ Rewrite each subtraction as addition.

$= 4x + 2$ Arrange like terms in columns.
$(+)\ 5x + (-7)$

$9x + (-5)$ or $9x - 5$ Add the like terms in each column.

Example 1 Add Linear Expressions

Find $(4x - 2) + (-7x - 3)$.

Method 1 Use algebra tiles.

Use algebra tiles to add $(4x - 2) + (-7x - 3)$.

Step 1 Model each expression using tiles.

$(4x - 2)$

$(-7x - 3)$

Step 2 Remove zero pairs.

There are three $-x$-tiles and five -1-tiles remaining.

So, $(4x - 2) + (-7x - 3) = -3x - 5$

(continued on next page)

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Interactive Presentation

Example 1, Add Linear Expressions, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag algebra tiles to add linear expressions.

CLICK

On Slide 3, students move through the steps to find the sum of the expressions.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Add Linear Expressions

Objective

Students will add linear expressions with integer coefficients and constants.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to understand how they can use either method: algebra tiles or grouping and combining like terms, to add the expressions. Encourage them to explain how using algebra tiles helps to deepen their understanding of combining like terms.

Questions for Mathematical Discourse

SLIDE 2

AL To model $4x - 2$, how many x-tiles should you use? -1-tiles? four x-tiles; two -1-tiles

AL How can you use the algebra tiles to model $-7x - 3$? Model $-7x$ by using seven $-x$-tiles and model -3 by using three -1-tiles.

OL How can you use the algebra tiles to add the expressions once you've modeled each expression? Sample answer: Remove zero pairs of x- and $-x$-tiles, leaving three $-x$-tiles. Combining the constants, there are a total of five -1-tiles. So, the sum is $-3x - 5$, or $-3x + (-5)$.

BL A classmate used algebra tiles to model the sum of two different expressions. Suppose the sum was represented by two x-tiles and four -1-tiles. If one expression is $3x + 1$, what is the other expression? $-x - 5$ or $-x + (-5)$

(continued on next page)

Example 1 Add Linear Expressions *(continued)*

Questions for Mathematical Discourse

SLIDE 3

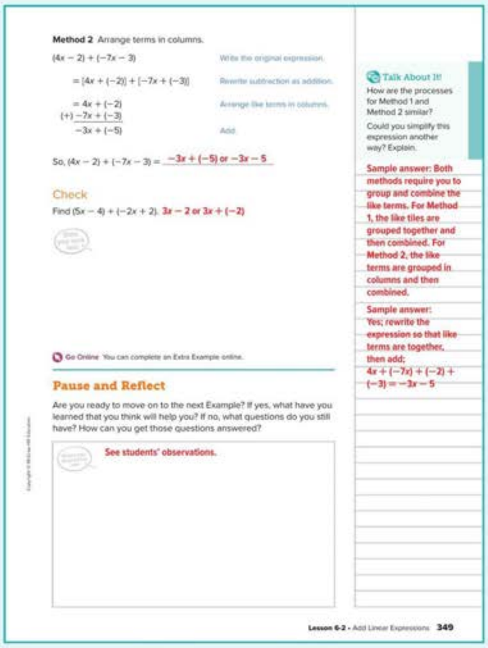
AL Why can you rewrite subtraction as addition? Subtracting a number is the same as adding the number's additive inverse.

OL Why is it helpful to arrange the like terms in columns? Sample answer: I can easily see the variable terms and the constants.

BL Which method of finding the sum: using algebra tiles or arranging like terms in columns, do you prefer? Explain. See students' preferences.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Method 2 Arrange terms in columns.

$(4x - 2) + (-7x - 3)$ Write the original expression.

$= [4x + (-2)] + [-7x + (-3)]$ Rewrite subtraction as addition.

$= 4x + (-2)$
$(+)\ -7x + (-3)$ Arrange like terms in columns.

$-3x + (-5)$ Add.

So, $(4x - 2) + (-7x - 3) =$ $-3x + (-5)$ or $-3x - 5$

Check

Find $(5x - 4) + (-2x + 2)$. $3x - 2$ or $3x + (-2)$

Go Online You can complete an Extra Example online.

Pause and Reflect

Are you ready to move on to the next Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

See students' observations.

Talk About It!

How are the processes for Method 1 and Method 2 similar?

Could you simplify this expression another way? Explain.

Sample answer: Both methods require you to group and combine the like terms. For Method 1, the like tiles are grouped together and then combined. For Method 2, the like terms are grouped in columns and then combined.

Sample answer: Yes; rewrite the expression so that like terms are together, then add; $4x + (-7x) + (-2) + (-3) = -3x - 5$

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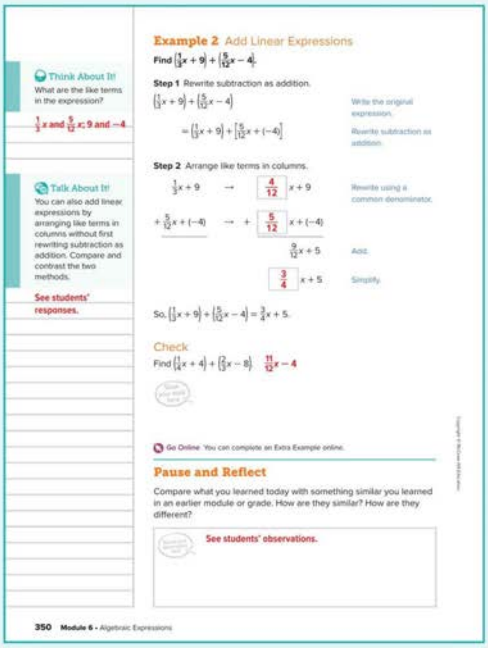
1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION



Interactive Presentation

Example 2, Add Linear Expressions, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the sum of the expressions.

TYPE

On Slide 2, students type to enter the sum of the expressions.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Add Linear Expressions

Objective

Students will add linear expressions with rational coefficients and constants.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to flexibly use their knowledge of additive inverses to rewrite subtraction as addition, the Commutative Property to group like terms together, and their knowledge of addition of fractions by finding a common denominator.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to use clear and precise mathematical vocabulary as they compare and contrast the two methods for adding linear expressions.

Questions for Mathematical Discourse

SLIDE 2

AL How many sets of like terms are there? Identify them. There are two sets of like terms, $\frac{1}{3}x$ and $\frac{5}{12}x$, and 9 and −4.

OL What are the two coefficients of *x*? $\frac{1}{3}$ and $\frac{5}{12}$

OL Why do we need to find a common denominator? We need to find a common denominator so that we can add the variable terms together.

BL Generate two expressions that have rational coefficients in which the sum of the two expressions has a negative rational coefficient. Then find the sum. Sample answer: $\left(-\frac{1}{2} + 3 +\right) - \left(x -\frac{1}{4} - \frac{1}{2} = -\right)x + 1\frac{3}{4}$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Theater

Objective

Students will come up with their own strategy to solve an application problem involving ticket sales and donations.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

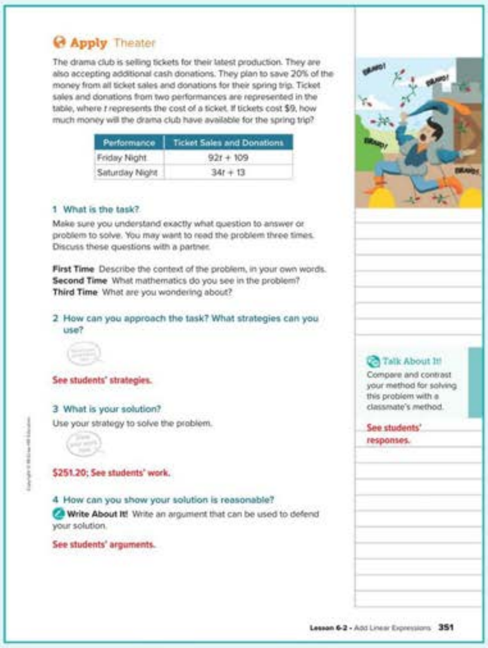
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Do the expressions contain any like terms?
- If t represents the cost of a ticket, what does $92t$ mean?
- What percent of the money are they saving?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they used to defend their solution.



Apply Theater

The drama club is selling tickets for their latest production. They are also accepting additional cash donations. They plan to save 20% of the money from all ticket sales and donations for their spring trip. Ticket sales and donations from two performances are represented in the table, where t represents the cost of a ticket. If tickets cost $9, how much money will the drama club have available for the spring trip?

Performance	Ticket Sales and Donations
Friday Night	$92t + 109$
Saturday Night	$34t + 13$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$251.20; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Compare and contrast your method for solving this problem with a classmate's method.

See students' responses.

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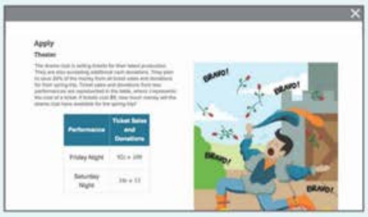
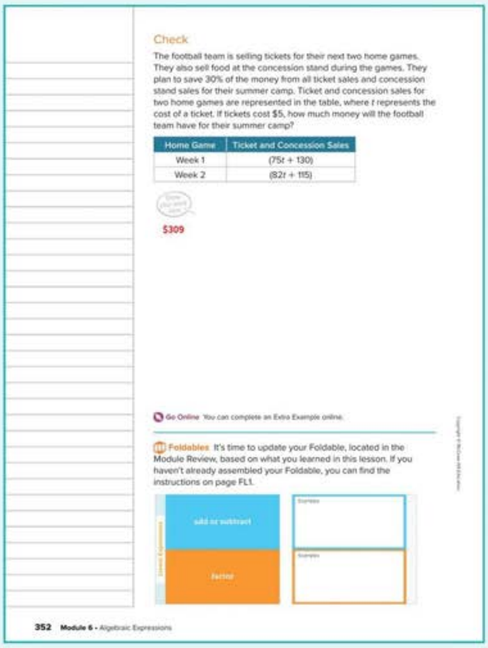
Interactive Presentation



Apply, Theater

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The football team is selling tickets for their next two home games. They also sell food at the concession stand during the games. They plan to save 30% of the money from all ticket sales and concession stand sales for their summer camp. Ticket and concession sales for two home games are represented in the table, where t represents the cost of a ticket. If tickets cost $5, how much money will the football team have for their summer camp?

Home Game	Ticket and Concession Sales
Week 1	$(75t + 130)$
Week 2	$(82t + 115)$

$309

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information about adding linear expressions. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

Why is it beneficial to rewrite expressions in different forms?

In this lesson, students learned how to add linear expressions. Encourage them to brainstorm with a partner some advantages of simplifying an expression such as $(2x + 7) + (-3x - 9)$. For example, they may state that adding the expressions involves combining like terms which removes the parentheses and reduces the number of terms in the expression to a minimum.

Exit Ticket

Refer to the Exit Ticket slide. Over a span of just a few years, scientists discovered x new bird species, $4x + 18$ new fish, $3x + 4$ new amphibians, and $2x - 14$ new reptiles. Write and simplify an expression that represents the total number of species that were discovered during this time period. Show the steps that you used. Sample answer:

$x + (4x + 18) + (3x + 4) + (2x - 14)$
$= x + 4x + 18 + 3x + 4 + 2x + (-14)$
$= 10x + 8$

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 13, 15–19
- ALEKS® Simplifying Algebraic Expressions

IF students score 66-89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–12, 14, 16, 17
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Simplifying Algebraic Expressions

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS® Simplifying Algebraic Expressions

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

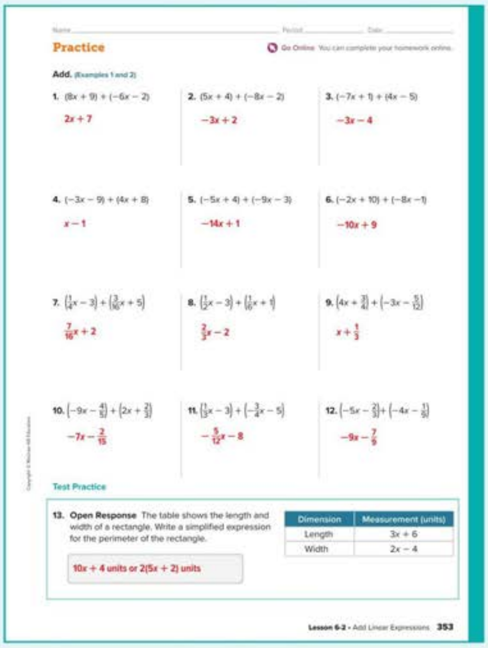
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	add linear expressions	1–12
2	extend concepts learned in class to apply them in new contexts	13
3	solve application problems involving adding linear expressions	14, 15
3	higher-order and critical thinking skills	16–19

Common Misconception

Some students may not immediately recognize that they need to find a common denominator in order to add the linear expressions. In Exercises 7–12, encourage students to identify a common denominator before combining like terms in order to add the expressions.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Add. (Examples 1 and 2)

1. $(8x + 9) + (-6x - 2)$ $2x + 7$
2. $(5x + 4) + (-8x - 2)$ $-3x + 2$
3. $(-7x + 1) + (4x - 5)$ $-3x - 4$
4. $(-3x - 9) + (4x + 8)$ $x - 1$
5. $(-5x + 4) + (-9x - 3)$ $-14x + 1$
6. $(-2x + 10) + (-8x - 1)$ $-10x + 9$
7. $\left(\frac{1}{4}x - 3\right) + \left(\frac{3}{16}x + 5\right)$ $\frac{7}{16}x + 2$
8. $\left(\frac{1}{2}x - 3\right) + \left(\frac{1}{6}x + 1\right)$ $\frac{2}{3}x - 2$
9. $\left(4x + \frac{3}{4}\right) + \left(-3x - \frac{5}{12}\right)$ $x + \frac{1}{3}$
10. $\left(-9x - \frac{4}{5}\right) + \left(2x + \frac{2}{3}\right)$ $-7x - \frac{2}{15}$
11. $\left(\frac{1}{3}x - 3\right) + \left(-\frac{3}{4}x - 5\right)$ $-\frac{5}{12}x - 8$
12. $\left(-5x - \frac{2}{3}\right) + \left(-4x - \frac{1}{9}\right)$ $-9x - \frac{7}{9}$

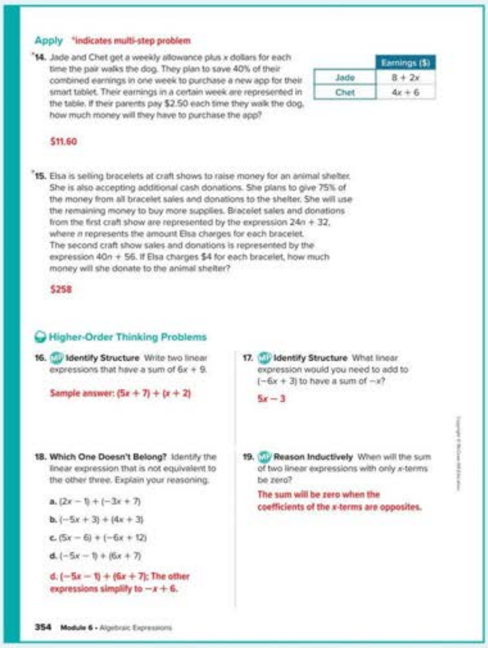
Test Practice

13. **Open Response** The table shows the length and width of a rectangle. Write a simplified expression for the perimeter of the rectangle.

Dimension	Measurement (units)
Length	$3x + 6$
Width	$2x - 4$

$10x + 4$ units or $2(5x + 2)$ units

Lesson 6-2 • Add Linear Expressions 353



Apply *indicates multi-step problem

*14. Jade and Chet get a weekly allowance plus x dollars for each time the pair walks the dog. They plan to save 40% of their combined earnings in one week to purchase a new app for their smart tablet. Their earnings in a certain week are represented in the table. If their parents pay $2.50 each time they walk the dog, how much money will they have to purchase the app?

	Earnings ($)
Jade	$8 + 2x$
Chet	$4x + 6$

$11.60

*15. Elsa is selling bracelets at craft shows to raise money for an animal shelter. She is also accepting additional cash donations. She plans to give 75% of the money from all bracelet sales and donations to the shelter. She will use the remaining money to buy more supplies. Bracelet sales and donations from the first craft show are represented by the expression $24n + 32$, where n represents the amount Elsa charges for each bracelet. The second craft show sales and donations is represented by the expression $40n + 56$. If Elsa charges $4 for each bracelet, how much money will she donate to the animal shelter?

$258

Higher-Order Thinking Problems

16. **Identify Structure** Write two linear expressions that have a sum of $6x + 9$.

Sample answer: $(5x + 7) + (x + 2)$

17. **Identify Structure** What linear expression would you need to add to $(-6x + 3)$ to have a sum of $-x$?

$5x - 3$

18. **Which One Doesn't Belong?** Identify the linear expression that is not equivalent to the other three. Explain your reasoning.

a. $(2x - 1) + (-3x + 7)$

b. $(-5x + 3) + (4x + 3)$

c. $(5x - 6) + (-6x + 12)$

d. $(-5x - 1) + (6x + 7)$

d. $(-5x - 1) + (6x + 7)$; The other expressions simplify to $-x + 6$.

19. **Reason Inductively** When will the sum of two linear expressions with only x-terms be zero?

The sum will be zero when the coefficients of the x-terms are opposites.

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Teaching the Mathematical Practices

7 Look For and Make Use of Structure In Exercise 16, students write two linear expressions that satisfy the given requirement.

In Exercise 17, students use multiple steps to find a linear expression representing a missing addend.

2 Reason Abstractly and Quantitatively In Exercise 19, students use reasoning to determine when the sum of two linear expressions with only x-terms will be zero.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercises 14–15 Have students work in groups of 3-4. After completing Exercise 14, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method produces a viable solution. If the methods were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is correct. Repeat this process for Exercise 15.

Be sure everyone understands.

Use with Exercises 16–17 Have students work in groups of 3-4 to solve the problem in Exercise 16. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 17.

Lesson 6-3

Subtract Linear Expressions

7.EE.A.1

LESSON GOAL

Students will subtract linear expressions and express the difference in simplest form.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Additive Inverses of Expressions
Example 1: Find the Additive Inverse of Expressions
Learn: Subtract Linear Expressions
Example 2: Subtract Linear Expressions
Example 3: Subtract Linear Expressions
Apply: Sales

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

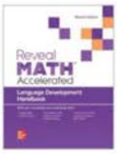
View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Add and Subtract Rational Expressions		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 35 of the *Language Development Handbook* to help your students build mathematical language related to subtraction of linear expressions.

ELL You can use the tips and suggestions on page T35 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7 .EE.A** by subtracting linear expressions and expressing the difference in simplest form.
Standards for Mathematical Content: **7 .EE.A.1**
Standards for Mathematical Practice: **MP1, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students added linear expressions and expressed the sum in simplest form.
7.EE.A.1

Now
Students subtract linear expressions and express the difference in simplest form.
7.EE.A.1

Next
Students will find the GCF of monomials and factor algebraic expressions.
7.EE.A.1

Rigor

The Three Pillars of Rigor

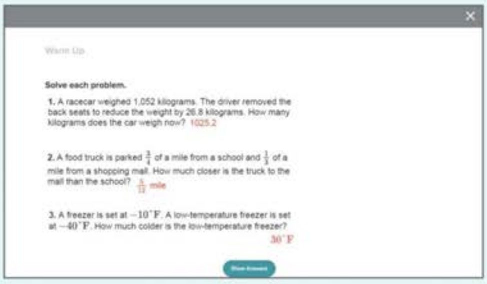
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of integers to develop an *understanding* of finding the additive inverse of linear expressions. They will gain *fluency* in subracting linear expressions through continued practice.

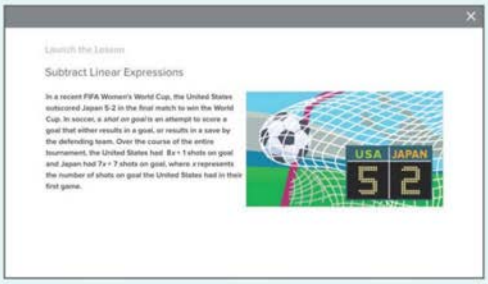
Mathematical Background

To subtract linear expressions, add the opposite, or the *additive inverse*. To subtract $(9x - 4) - (7x - 5)$, use the Distributive Property to rewrite the difference as $9x - 4 - 7x + 5$. Combine like terms by subtracting $7x$ from $9x$ to obtain $2x$. Then combine the constants by adding -4 and 5 to obtain 1. The difference of the two expressions $9x - 4$ and $7x - 5$ is $2x + 1$.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- subtracting rational numbers (Exercises 1–3)

Answers

1. 1052.2 kg
2. $\frac{5}{12}$ mile
3. 30°F

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using expressions to determine the difference between two countries' shots on goal in soccer.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Define *inverse* in your own words. How does it help you remember the meaning of the term *additive inverse*? Sample answer: *Inverse* means reversed in position, order, direction, or operation. This helps me to remember that the additive inverse of a number is the number that when added to the original number has a result of zero.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Additive Inverses of Expressions

Objective

Students will understand that when two expressions are additive inverses, their sum is zero.

Teaching the Mathematical Practices

8 Look for and Express Regularity in Repeated Reasoning
As students discuss the *Talk About It!* question on Slide 2, encourage them to realize that when they find the additive inverse of an expression, such as $4x + 2$, they are essentially finding the additive inverse of each term, $4x$ and 2. So, they should notice that these steps are repeated every time they find the additive inverse of an expression. You may wish to give them several examples with which to practice.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

How can you find the additive inverse of an expression mentally? Sample answer: Find the additive inverse of the coefficient and the constant.

DIFFERENTIATE

Reteaching Activity AL

For students that may be struggling to identify additive inverses of expressions, have students consider the additive inverses of the coefficients. For each of the following terms, have students identify the additive inverse of the coefficient and then write the additive inverse of the expression.

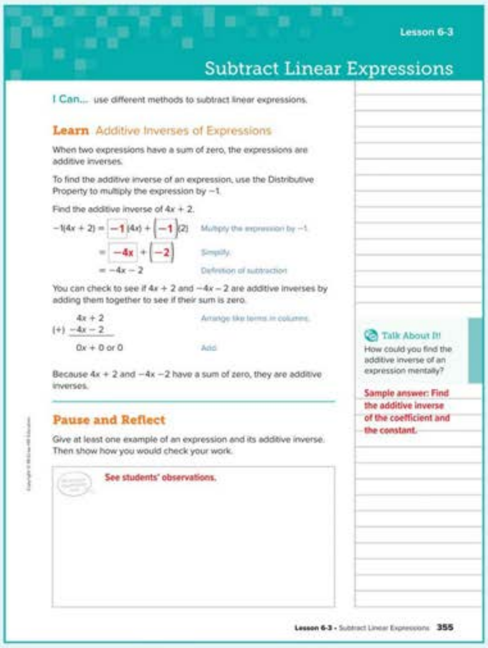
$7x$ -7; $-7x$

$-2t$ 2; $2t$

z -1; $-z$

$3m$ -3; $-3m$

$-n$ 1; n


Lesson 6-3

Subtract Linear Expressions

I Can... use different methods to subtract linear expressions.

Learn Additive Inverses of Expressions

When two expressions have a sum of zero, the expressions are additive inverses.

To find the additive inverse of an expression, use the Distributive Property to multiply the expression by -1.

Find the additive inverse of $4x + 2$.

$-1(4x + 2) = -1(4x) + (-1)(2)$ Multiply the expression by -1.

$= -4x + (-2)$ Simplify.

$= -4x - 2$ Definition of subtraction

You can check to see if $4x + 2$ and $-4x - 2$ are additive inverses by adding them together to see if their sum is zero.

$4x + 2$ Arrange like terms in columns.
$(+)\ -4x - 2$
$0x + 0$ or 0 Add.

Because $4x + 2$ and $-4x - 2$ have a sum of zero, they are additive inverses.

Talk About It! How could you find the additive inverse of an expression mentally? Sample answer: Find the additive inverse of the coefficient and the constant.

Pause and Reflect

Give at least one example of an expression and its additive inverse. Then show how you would check your work.

See students' observations.

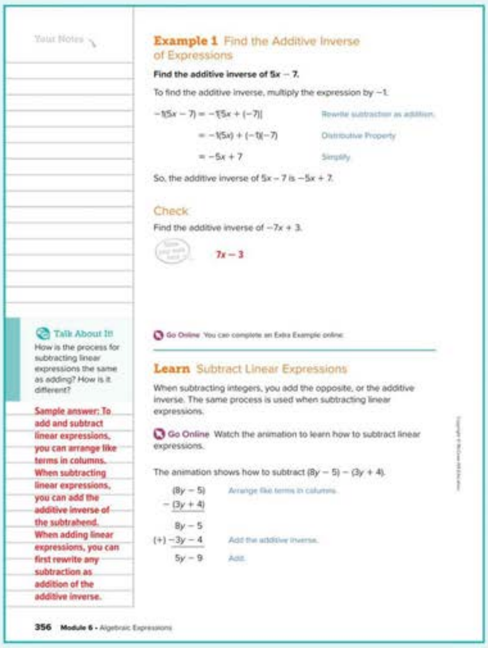
Lesson 6-3 • Subtract Linear Expressions 355

Interactive Presentation

Learn, Additive Inverses of Expressions, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to find the additive inverse of the linear expression.


Your Notes

Example 1 Find the Additive Inverse of Expressions

Find the additive inverse of $5x - 7$.

To find the additive inverse, multiply the expression by -1.

$-1(5x - 7) = -1[5x + (-7)]$ Rewrite subtraction as addition.

$= -1(5x) + (-1)(-7)$ Distributive Property

$= -5x + 7$ Simplify.

So, the additive inverse of $5x - 7$ is $-5x + 7$.

Check

Find the additive inverse of $-7x + 3$.

$7x - 3$

Go Online You can complete an Extra Example online.

Talk About It!

How is the process for subtracting linear expressions the same as adding? How is it different?

Sample answer: To add and subtract linear expressions, you can arrange like terms in columns. When subtracting linear expressions, you can add the additive inverse of the subtrahend. When adding linear expressions, you can first rewrite any subtraction as addition of the additive inverse.

Learn Subtract Linear Expressions

When subtracting integers, you add the opposite, or the additive inverse. The same process is used when subtracting linear expressions.

Go Online Watch the animation to learn how to subtract linear expressions.

The animation shows how to subtract $(8y - 5) - (3y + 4)$.

$(8y - 5)$ $-(3y + 4)$ Arrange like terms in columns.

$8y - 5$ $(+) -3y - 4$ Add the additive inverse.

$5y - 9$ Add.

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Interactive Presentation

Example 1, Find the Additive Inverse of Expressions, Slide 1 of 2

CLICK

On Slide 1 of Example 1, students move through the steps to find the additive inverse of an expression.

WATCH

On Slide 1 of the Learn, students watch an animation that explains how to subtract linear expressions.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find the Additive Inverse of Expressions

Objective

Students will find the additive inverse of linear expressions.

Questions for Mathematical Discourse

SLIDE 1

AL What is the additive inverse of 5? How can you use this knowledge to find the additive inverse of $5x$? The additive inverse of 5 is -5. The additive inverse of $5x$ is $-5x$ because $5x + (-5x) = 0$.

AL What is the additive inverse of -7? 7

OL How can you verify that the expression $-5x + 7$ is the additive inverse of $5x - 7$? Find the sum of the expressions. Since $(5x - 7) + (-5x + 7) = 0$, the expressions are additive inverses.

OL Explain how finding the additive inverse of an expression is related to the Distributive Property. Sample answer: To find the additive inverse, multiply the expression by -1. To do so, you need to use the Distributive Property.

BL Generate two different expressions that are additive inverses. Sample answer: $3x + 2$ and $-3x - 2$.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Subtract Linear Expressions

Objective

Students will learn how to subtract linear expressions using the additive inverse.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to subtract linear expressions using the additive inverse.

Talk About It!

SLIDE 2

Mathematical Discourse

How is the process for subtracting linear expressions the same as adding? How is it different? Sample answer: To add and subtract linear expressions, you need to arrange like terms in columns. When subtracting linear expressions, you can add the inverse. When adding linear expressions, you can first rewrite subtraction as addition.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Subtract Linear Expressions

Objective

Students will subtract linear expressions with integer coefficients and constants.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to understand how algebra tiles can be used to model the process for subtracting linear expressions.

8 Look for and Express Regularity in Repeated Reasoning Encourage students to look for repeated reasoning when arranging like terms in columns and how this method relates to the method of using algebra tiles.

Questions for Mathematical Discourse

SLIDE 2

AL Which expression should you model first? Why? $6x + 5$; This expression represents the minuend, the quantity from which the second expression is subtracted.

AL How can you use algebra tiles to model $6x + 5$? Use six x-tiles to model $6x$ and five 1-tiles to model 5.

OL Explain how to use algebra tiles to subtract $3x - 2$ from $6x + 5$. Sample answer: Remove three x-tiles from six x-tiles. There are $3x$ tiles left. To represent subtracting -2, add two zero pairs of 1- and -1-tiles to five 1-tiles. Then remove two -1-tiles. There are seven 1-tiles left. The difference is $3x + 7$.

BL Suppose algebra tiles were used to subtract $3x - 2$ from an unknown expression. The result was two x-tiles and four 1-tiles. What is the expression that represents the minuend? $5x + 2$

SLIDE 3

AL What is the additive inverse of $3x - 2$? $-3x + 2$

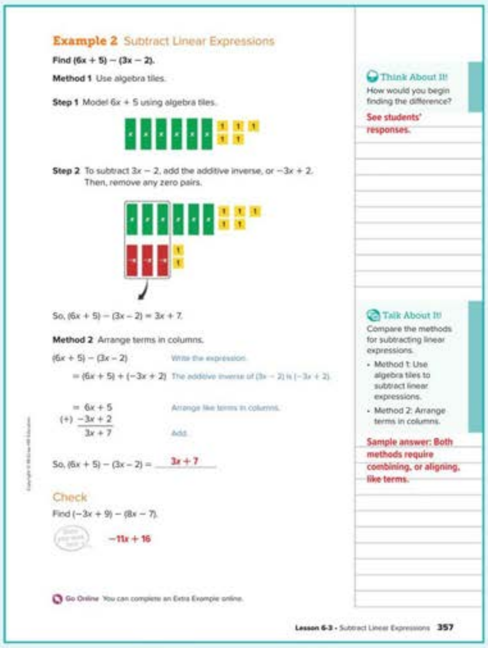
OL Why do we rewrite the expressions using the additive inverse? Sample answer: The second expression is being subtracted. Subtraction is the same as adding the additive inverse.

OL Why is it helpful to arrange the like terms in columns? Sample answer: It helps to visualize the like terms.

BL Describe two different ways that you could verify that you found the correct difference. Sample answer: Add $3x + 7$ and $3x - 2$ and verify that the sum is $6x + 5$. Another way is to substitute a value for x, such as $x = 3$, and verify that the difference of $6(3) + 5 - [3(3) - 2]$ equals $3(3) + 7$. Since $23 - 7 = 16$, the difference is correct.

Go Online

- Find additional teaching notes, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Subtract Linear Expressions

Find $(6x + 5) - (3x - 2)$.

Method 1 Use algebra tiles.

Step 1 Model $6x + 5$ using algebra tiles.

Step 2 To subtract $3x - 2$, add the additive inverse, or $-3x + 2$. Then, remove any zero pairs.

So, $(6x + 5) - (3x - 2) = 3x + 7$.

Method 2 Arrange terms in columns.

$(6x + 5) - (3x - 2)$ Write the expression.

$= (6x + 5) + (-3x + 2)$ The additive inverse of $(3x - 2)$ is $(-3x + 2)$.

$= 6x + 5$ Arrange like terms in columns.

$(+)\ -3x + 2$

$3x + 7$ Add.

So, $(6x + 5) - (3x - 2) =$ $3x + 7$

Check

Find $(-3x + 9) - (8x - 7)$.

$-11x + 16$

Go Online You can complete an Extra Example online.

Think About It! How would you begin finding the difference? See students' responses.

Talk About It! Compare the methods for subtracting linear expressions.
- Method 1: Use algebra tiles to subtract linear expressions.
- Method 2: Arrange terms in columns.

Sample answer: Both methods require combining, or aligning, like terms.

Lesson 6-3 • Subtract Linear Expressions 357

Interactive Presentation

Example 2, Subtract Linear Expressions, Slide 2 of 5

WATCH

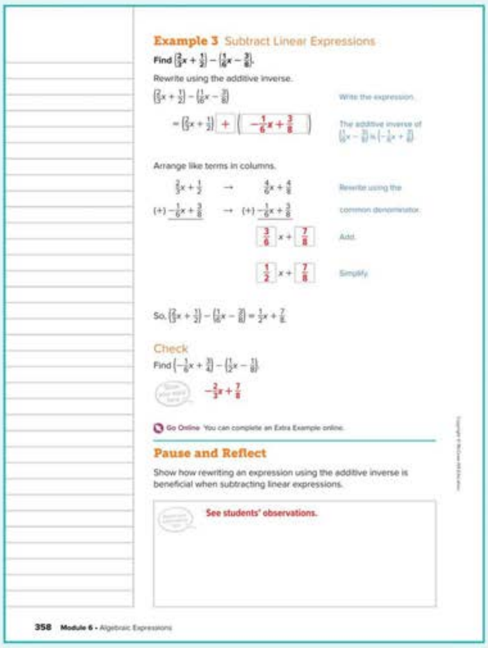
On Slide 2, students watch a video that explains how to use algebra tiles to subtract linear expressions.

DRAG & DROP

On Slide 2, students drag algebra tiles to subtract linear expressions.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Subtract Linear Expressions

Find $\left(\frac{2}{3}x + \frac{1}{2}\right) - \left(\frac{1}{6}x - \frac{3}{8}\right)$.

Rewrite using the additive inverse.

$\left(\frac{2}{3}x + \frac{1}{2}\right) - \left(\frac{1}{6}x - \frac{3}{8}\right)$ Write the expression.

$= \left(\frac{2}{3}x + \frac{1}{2}\right) + \left(-\frac{1}{6}x + \frac{3}{8}\right)$ The additive inverse of $\left(\frac{1}{6}x - \frac{3}{8}\right)$ is $\left(-\frac{1}{6}x + \frac{3}{8}\right)$.

Arrange like terms in columns.

$\frac{2}{3}x + \frac{1}{2} \rightarrow \frac{4}{6}x + \frac{4}{8}$ Rewrite using the

$(+) -\frac{1}{6}x + \frac{3}{8} \rightarrow (+) -\frac{1}{6}x + \frac{3}{8}$ common denominator.

$\frac{3}{6}x + \frac{7}{8}$ Add.

$\frac{1}{2}x + \frac{7}{8}$ Simplify.

So, $\left(\frac{2}{3}x + \frac{1}{2}\right) - \left(\frac{1}{6}x - \frac{3}{8}\right) = \frac{1}{2}x + \frac{7}{8}$.

Check

Find $\left(-\frac{1}{6}x + \frac{3}{4}\right) - \left(\frac{1}{2}x - \frac{1}{8}\right)$.

$-\frac{2}{3}x + \frac{7}{8}$

Go Online You can complete an Extra Example online.

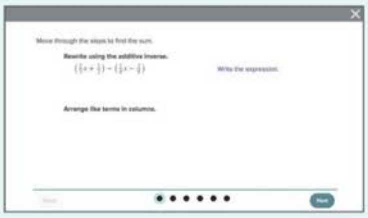
Pause and Reflect

Show how rewriting an expression using the additive inverse is beneficial when subtracting linear expressions.

See students' observations.

358 Module 6 • Algebraic Expressions

Interactive Presentation



Example 3, Subtract Linear Expressions, Slide 1 of 2

CLICK

On Slide 1, students move through the steps to find the difference.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Subtract Linear Expressions

Objective

Students will subtract linear expressions with rational coefficients and constants.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to see how this example using rational coefficients and rational constants compares to examples that use only integers. Students should see that the process for finding the difference is the same, except they will need to find common denominators to combine the rational terms. Students should perform the calculations accurately and explain each step using precise mathematical language, such as *rational*, *coefficient*, *constant*, and *common denominator*.

Questions for Mathematical Discourse

SLIDE 1

AL What is the first step in finding the difference? Rewrite the subtraction as addition of the additive inverse.

OL Why do we rewrite the expressions using common denominators? Common denominators are needed to combine like terms, since there are rational coefficients and constants.

OL Why are there two sets of common denominators? We need a common denominator for the coefficients and one for the constants.

BL Why would algebra tiles not be a good way to find this difference? Sample answer: Algebra tiles are used when there are integer coefficients and constants. It is not possible to represent a fraction of a tile.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Sales

Objective

Students will come up with their own strategy to solve an application problem involving sales of T-shirts.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- If w represents the number of weeks, what does $5w$ mean?
- What operation will you use to simplify the expressions?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Sales

Bonnie owns a T-shirt shop where she sells plain T-shirts and printed T-shirts. The table shows the cost per shirt and the number of each type of shirt sold over w weeks. After 25 weeks, how much more did she earn in sales of printed T-shirts than in sales of plain T-shirts?

Style	Cost ($)	Number Sold
Plain	15	$5w - 4$
Printed	15	$8w + 3$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$1,230 more; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How do you know just by looking at the expressions that Bonnie sold more printed T-shirts than plain T-shirts?

Sample answer: For all positive values of w, 8 times w plus 3 is always greater than 5 times w minus 4.

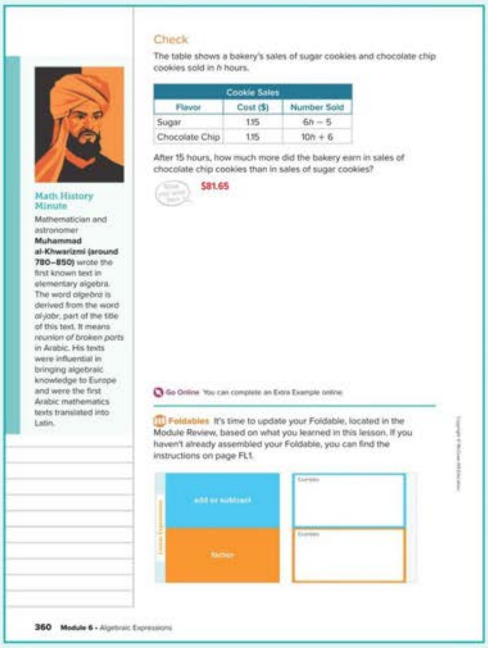
Lesson 6-3 • Subtract Linear Expressions **359**

Interactive Presentation

Apply, Sales

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows a bakery's sales of sugar cookies and chocolate chip cookies sold in h hours.

Cookie Sales		
Flavor	Cost ($)	Number Sold
Sugar	1.15	$6h - 5$
Chocolate Chip	1.15	$10h + 6$

After 15 hours, how much more did the bakery earn in sales of chocolate chip cookies than in sales of sugar cookies?

$81.65

Math History Minute

Mathematician and astronomer **Muhammad al-Khwarizmi (around 780–850)** wrote the first known text in elementary algebra. The word *algebra* is derived from the word *al-jabr*, part of the title of this text. It means *reunion of broken parts* in Arabic. His texts were influential in bringing algebraic knowledge to Europe and were the first Arabic mathematics texts translated into Latin.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

360 Module 6 • Algebraic Expressions

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information about subtracting linear expressions. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

Why is it beneficial to rewrite expressions in different forms?

In this lesson, students learned how to subtract linear expressions. Encourage them to discuss with a partner the advantage(s) of simplifying an expression such as $(-6y + 1) - (-2y - 8)$. For example, they may state that adding the expressions involves combining like terms which removes the parentheses and reduces the number of terms in the expression to a minimum.

Exit Ticket

Refer to the Exit Ticket slide. Write and simplify an expression that represents how many more shots on goal Japan had than the United States. Explain the steps that you used. $-x + 6$; Sample answer: Subtract the expression for the United States from the expression for Japan: $(7x + 7) - (8x + 1)$. Rewrite the subtraction by adding the additive inverse: $(7x + 7) + (-8x - 1)$. Simplify by adding the like terms and adding the constants: $7x + (-8x) + 7 + (-1) = -x + 6$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 13, 15–19
- Extension: Add and Subtract Rational Expressions
- ALEKS® Simplifying Algebraic Expressions

IF students score 66-89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–12, 14, 16, 18
- Extension: Add and Subtract Rational Expressions
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® The Distributive Property

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS® The Distributive Property

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

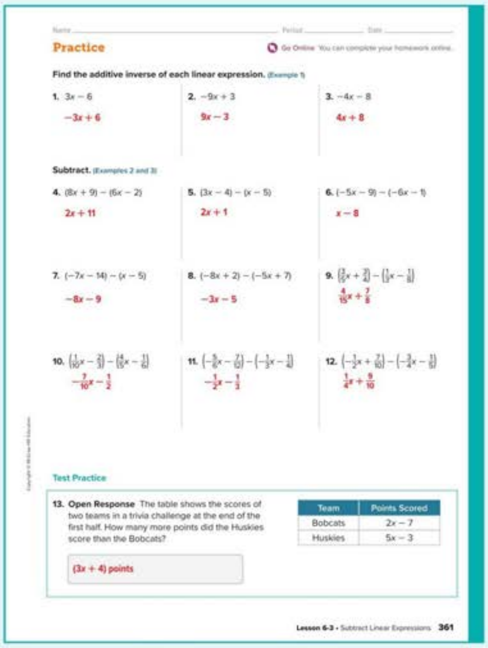
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the additive inverse of linear expressions	1–3
1	subtract linear expressions	4–12
2	extend concepts learned in class to apply them in new contexts	13
3	solve application problems involving subtracting linear expressions	14, 15
3	higher-order and critical thinking skills	16–19

Common Misconception

When students subtract linear expressions, they may do so by incorrectly writing the additive inverse of an expression. In Exercise 4, students may write the additive inverse of $6x - 2$ as $-6x - 2$ rather than $-6x + 2$, essentially only distributing the negative to the first term. Remind students that additive inverses are found by considering all of the terms in the expression.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find the additive inverse of each linear expression. (Example 1)

1. $3x - 6$ — $-3x + 6$
2. $-9x + 3$ — $9x - 3$
3. $-4x - 8$ — $4x + 8$

Subtract. (Examples 2 and 3)

4. $(8x + 9) - (6x - 2)$ — $2x + 11$
5. $(3x - 4) - (x - 5)$ — $2x + 1$
6. $(-5x - 9) - (-6x - 1)$ — $x - 8$
7. $(-7x - 14) - (x - 5)$ — $-8x - 9$
8. $(-8x + 2) - (-5x + 7)$ — $-3x - 5$
9. $\left(\frac{3}{5}x + \frac{3}{4}\right) - \left(\frac{1}{3}x - \frac{1}{8}\right)$ — $\frac{4}{15}x + \frac{7}{8}$
10. $\left(\frac{1}{10}x - \frac{2}{3}\right) - \left(\frac{4}{5}x - \frac{1}{6}\right)$ — $-\frac{7}{10}x - \frac{1}{2}$
11. $\left(-\frac{5}{6}x - \frac{7}{12}\right) - \left(-\frac{1}{3}x - \frac{1}{4}\right)$ — $-\frac{1}{2}x - \frac{1}{3}$
12. $\left(-\frac{1}{2}x + \frac{7}{10}\right) - \left(-\frac{3}{4}x - \frac{1}{5}\right)$ — $\frac{1}{4}x + \frac{9}{10}$

Test Practice

13. **Open Response** The table shows the scores of two teams in a trivia challenge at the end of the first half. How many more points did the Huskies score than the Bobcats?

Team	Points Scored
Bobcats	$2x - 7$
Huskies	$5x - 3$

$(3x + 4)$ points

Lesson 6-3 • Subtract Linear Expressions 361

Apply *indicates multi-step problem

*14. The table shows the sales of plain and Asiago cheese bagels at a bakery for h hours. After 6 hours, how much more will the bakery have made in sales of Asiago cheese bagels than the sales of plain bagels?

Bagel Sales		
Bagel	Cost ($)	Number Sold After h hours
Asiago Cheese	1.50	$12h + 7$
Plain	1.50	$7h - 4$

$61.50

*15. Derek owns a snack shop where he sells tins of buttered and caramel popcorn. The table shows the number of each type of popcorn sold over w weeks. After 12 weeks, how much more will he have made in sales of buttered popcorn than the sales of caramel popcorn?

Popcorn Sales		
Popcorn	Cost ($)	Number Sold Over w Weeks
Buttered	11	$8w + 9$
Caramel	11	$6w - 1$

$374

Higher-Order Thinking Problems

16. **Identify Structure** Write two linear expressions that have a difference of $x + 1$.

Sample answer: $(2x - 6) - (x - 7)$

17. **Identify Structure** What linear expression would you need to subtract from $(5x + 3)$ to have a difference of $-x$?

$6x + 3$

18. **Which One Doesn't Belong?** Identify the linear expression that does not belong with the other three. Explain your reasoning.

a. $(-5x + 3) - (-7x - 1)$

b. $(-3x + 3) - (-5x - 2)$

c. $(x - 6) - (-x - 10)$

d. $(-7x + 2) - (-9x - 2)$

b. $(-3x + 3) - (-5x - 2)$; The other expressions simplify to $2x + 4$ or $2(x + 2)$.

19. **Find the Error** A student simplified the expression $(6x - 2) - (-x + 5)$ to $7x + 3$. Find the student's error and correct it.

Sample answer: The student only added the additive inverse of $-x$ and not 5. $(6x - 2) - (-x + 5) = 7x - 7$

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Teaching the Mathematical Practices

7 Look For and Make Use of Structure In E xercise 16, students write two linear expressions that satisfy a given requirement.

In Exercise 17, students use multiple steps to find the missing linear expression from a subtraction problem.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 19, students find a student's error in subtracting linear expressions and correct it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 14 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain to their partner how they would solve the problem without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Explore the truth of statements created by others.

Use with Exercises 16–19 Have students work in pairs. After completing the exercises, have students write two true statements about subtracting linear expressions and one false statement. An example of a true statement might be, "When subtracting linear expressions, it is necessary to add the additive inverse." An example of a false statement might be, "The additive inverse of an expression is always negative." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. For each false statement, have them generate a counterexample. Have them discuss and resolve any differences.

Lesson 6-4

Factor Linear Expressions

7.EE.A.1

LESSON GOAL

Students will find the GCF of monomials and factor algebraic expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Greatest Common Factor of Monomials
Example 1: Find the GCF of Monomials
Example 2: Find the GCF of Monomials

Explore: Factor Linear Expressions

Learn: Factor Linear Expressions
Example 3: Factor Linear Expressions
Example 4: Expressions with No Common Factors
Example 5: Factor Linear Expressions

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 36 of the *Language Development Handbook* to help your students build mathematical language related to factoring linear expressions.

ELL You can use the tips and suggestions on page T36 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7.EE.A** by finding the GCF of monomials and factoring algebraic expressions.
Standards for Mathematical Content: **7 .EE.A.1**
Standards for Mathematical Practice: **MP3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students subtracted linear expressions and expressed the difference in simplest form.
7.EE.A.1

Now
Students find the GCF of monomials and factor algebraic expressions.
7.EE.A.1

Next
Students will combine operations to simplify linear expressions.
7.EE.A.1

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

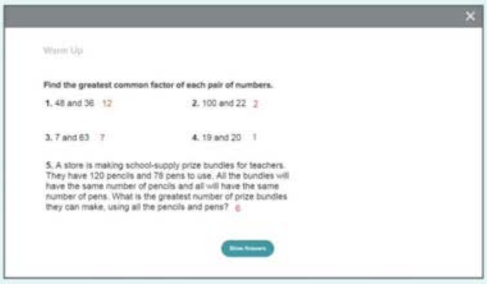
Conceptual Bridge In this lesson, students will draw on their knowledge of the greatest common factor and the distributive property to gain an *understanding* of using the greatest common factor and the distributive property to factor linear expressions. They will gain *fluency* in factoring linear expressions through continued practice.

Mathematical Background

A *monomial* is a number, a variable, or a product of a number and one or more variables. To *factor* a number means to write it as a product of its factors. A monomial can be factored using the same method you would use to factor a number. The greatest common factor (GCF) of two monomials is the greatest monomial that is a factor of both. The greatest common factor also includes any variables that the monomials have in common.

You can work backward and use the Distributive Property to express a linear expression as a product of its factors. A linear expression is in *factored form* when it is expressed as the product of its factors.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding the greatest common factor of two numbers (Exercises 1–5)

Answers

1. 12

2. 2

3. 7

4. 1

5. 6

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an expression to find the cost of a group of friends attending a concert.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Identify the factors of 12. 1, 2, 3, 4, 6, and 12
- What part of speech is *factored* in *factored form*? How will this help you understand what *factored form* might mean? adjective; Sample answer: Factored form might mean that a number or expression is written in the form of its factors.
- Use what you know about common factors to help you define *greatest common factor*. Sample answer: When factors of two or more numbers are the same, they are common factors. So, the greatest common factor is the greatest factor that is the same for two or more numbers.
- What does the prefix *mono-* mean? Sample answer: Mono- means alone, single, or one.

momcilog/iStock/Getty Images

Learn Greatest Common Factor of Monomials

Objective

Students will find the greatest common factor of two monomials.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

Based on what you know about finding the GCF of numbers, how do you think you can find the GCF of monomials? Sample answer: Finding the GCF of the coefficients of the monomials is the same as finding the GCF of numbers. To find the GCF of the variables, find variables that are common in each monomial.

Example 1 Find the GCF of Monomials

Objective

Students will find the greatest common factor of monomials by identifying the GCF of the coefficients and the variables.

Questions for Mathematical Discourse

SLIDE 2

AL What are all of the factors of 12? 30? The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

OL What factors do 12 and 30 have in common? 1, 2, 3, and 6

OL What is the greatest common factor of the coefficients? 6

BL What is another way to find the GCF of the coefficients? Sample answer: Find the prime factors of 12 and 30. Then multiply their common prime factors, 2 and 3. and 2 • 3 = 6.

SLIDE 3

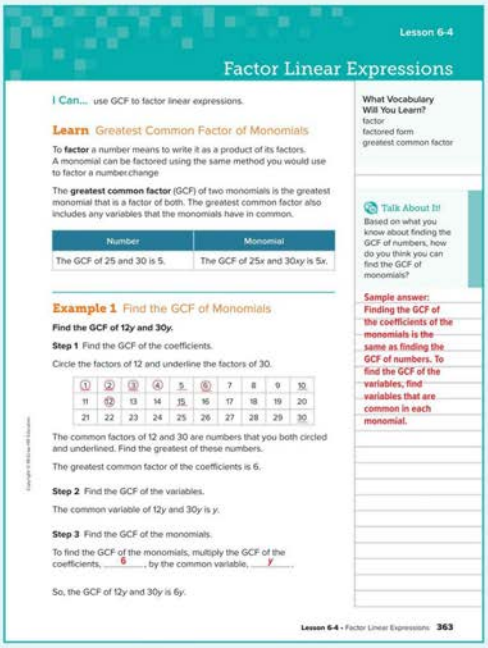
AL Identify the variable(s) in each expression. *y*

OL Identify any common variables and explain your reasoning. *y*; It is the only variable they each have, and thus the only variable they have in common.

BL Would the greatest common factor of the variables be different if the expressions were 12*y* and 30*xy*? Explain. no; Sample answer: Even though 30*xy* has factors of *x* and *y*, *y* is still the only variable factor that the two expressions have in common.

Go Online

- Find additional teaching notes and Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 6-4

Factor Linear Expressions

I Can... use GCF to factor linear expressions.

What Vocabulary Will You Learn?
factor
factored form
greatest common factor

Learn Greatest Common Factor of Monomials

To **factor** a number means to write it as a product of its factors. A monomial can be factored using the same method you would use to factor a number.change

The **greatest common factor** (GCF) of two monomials is the greatest monomial that is a factor of both. The greatest common factor also includes any variables that the monomials have in common.

Number	Monomial
The GCF of 25 and 30 is 5.	The GCF of 25*x* and 30*xy* is 5*x*.

Talk About It! Based on what you know about finding the GCF of numbers, how do you think you can find the GCF of monomials?

Sample answer: Finding the GCF of the coefficients of the monomials is the same as finding the GCF of numbers. To find the GCF of the variables, find variables that are common in each monomial.

Example 1 Find the GCF of Monomials

Find the GCF of 12*y* and 30*y*.

Step 1 Find the GCF of the coefficients.

Circle the factors of 12 and underline the factors of 30.

①	②	③	④	5	⑥	7	8	9	10
11	⑫	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

The common factors of 12 and 30 are numbers that you both circled and underlined. Find the greatest of these numbers.

The greatest common factor of the coefficients is 6.

Step 2 Find the GCF of the variables.

The common variable of 12*y* and 30*y* is *y*.

Step 3 Find the GCF of the monomials.

To find the GCF of the monomials, multiply the GCF of the coefficients, 6, by the common variable, *y*.

So, the GCF of 12*y* and 30*y* is 6*y*.

Lesson 6-4 • Factor Linear Expressions 363

Interactive Presentation

Example 1, Find the GCF of Monomials, Slide 2 of 5

FLASHCARDS

On Slide 1 of the Learn, students use Flashcards to see an example of the GCF of numbers and monomials.

CLICK

On Slide 2 of Example 1, students shade squares to show the factors of 12 and 30.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.EE.A.1

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION


Your Notes

Check

Find the GCF of $24a$ and $32a$. $8a$

Example 2 Find the GCF of Monomials

Use prime factorization to find the GCF of $18a$ and $20ab$.

Step 1 Identify common factors.

Complete the prime factorization for each term.

$18a = 2 \cdot 3 \cdot 3 \cdot a$

$20ab = 2 \cdot 2 \cdot 5 \cdot a \cdot b$

The common factors in each term are 2 and a.

Step 2 Multiply the common factors.

The GCF of $18a$ and $20ab$ is $2 \cdot a$ or $2a$.

Check

Use prime factorization to find the GCF of $42xy$ and $14y$. $14y$

Think About It! What variable(s) are common in each term? a

Talk About It! The factor 2 is common in each term. Why can only one 2 of the term $20ab$ be selected as a common factor?

Sample answer: Because there is only one factor of 2 in the term $18a$, then only one factor of 2 can be selected in the term $20ab$.

Go Online You can complete Extra Examples online.

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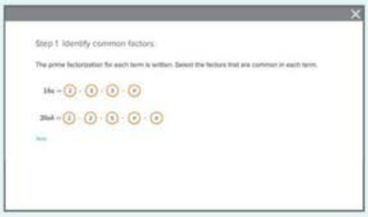
Interactive Presentation



Example 2, Find the GCF of Monomials, Slide 2 of 5

CLICK

On Slide 2, students select the factors that are common in each term.

TYPE

On Slide 3, students type to write the GCF of $18a$ and $20ab$.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find the GCF of Monomials

Objective

Students will find the greatest common factor of monomials using prime factorization.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to analyze the structure of the prime factorization of a monomial that contains variables in order to see the relationship between the coefficients and their prime factorization, and between the variables and their prime factorization. Students should be able to explain how the structure of the prime factorizations can be used to identify the GCF of the monomials.

Questions for Mathematical Discourse

SLIDE 2

AL What are prime factors? Prime factors are factors of a number that are also prime numbers.

AL Give an example of a prime factor. Sample answer: One prime factor of the number 6 is 2.

OL Why is 3 not a common factor? It is only a factor for $18a$, not for $20ab$.

OL In the list of factors for $20ab$, why is the second 2 not a common factor? Sample answer: Only one 2 is a common factor for both expressions. For the second 2 to be a common factor, it would mean that 2×2, or 4 would have to be a factor of both $18a$ and $20ab$, and 4 is not a factor of 18.

BL How could you alter the expression $18a$ so that its prime factors include both 2s? Sample answer: If the expression was $36a$, its prime factors would be 2, 2, 3, 3, and a.

SLIDE 3

AL What is 2 times a? $2a$

OL Why do we multiply the common factors? Sample answer: They are factors of the GCF. By definition, factors are multiplied to obtain a product.

BL Generate two different monomials whose GCF is $4ab$. Sample answer: $36abc$ and $20ab$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Factor Linear Expressions

Objective

Students will use algebra tiles to explore how to factor linear expressions.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a video showing how to use algebra tiles to factor linear expressions. Throughout this activity, students will use algebra tiles to model and factor three linear expressions.

Inquiry Question

How can algebra tiles help you factor linear expressions? Sample answer: Algebra tiles help to visualize the factors as if they were the length and width of a rectangle.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

Compare your arrangement with your partner's arrangement. Are they the same? How many different ways can the tiles be arranged?
See students' work; the arrangements should be the same. Sample answer: The tiles can be arranged in a rectangle one way. The rectangle has a width of 2 and a length of $x + 3$.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

Explore, Slide 3 of 7

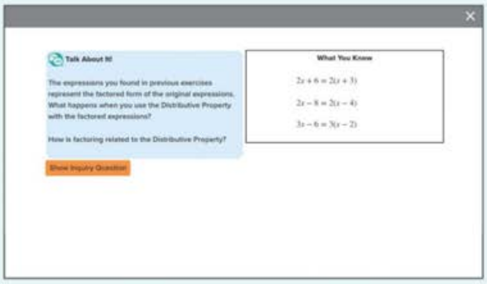
DRAG & DROP

Throughout the Explore, students drag algebra tiles to factor algebraic expressions.

WATCH

On Slide 2, students watch a video that demonstrates how to use algebra tiles to factor linear expressions.

Interactive Presentation



Explore, Slide 6 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Factor Linear Expressions *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically St udents should understand the structure of a linear expression and how it can be represented using algebra tiles.

7 Look For and Make Use of Structure Through exploration, students should come to realize the benefit of using algebra tiles as they can manipulate the tiles and visualize the results when factoring linear expressions.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

The expressions you found in previous exercises represent the factored form of the original expressions. What happens when you apply the Distributive Property to the factored expressions? Sample answer: When you use apply Distributive Property to the factored expressions, the result is the original expression.

How is factoring related to the Distributive Property? Sample answer: Factoring is the Distributive Property in reverse.

DIFFERENTIATE

Language Development Activity ELL

Students previously used the term *factor* as a noun. When multiplying, each number being multiplied is a *factor*. In this lesson, students will use the term *factor* as a verb, but the meanings are related. To *factor* an expression means to express the product (the expression) as a multiplication expression of its factors.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Factor Linear Expressions

Objective

Students will use the Distributive Property to factor a linear expression.

Go Online to have students watch the animation on Slide 1. The animation illustrates factoring linear expressions.

Teaching Notes

SLIDE 1

Play the animation about factoring linear expressions for the class. Remind students that the factored form of the expression is equivalent to the original expression. There should be the same number of terms inside the parentheses of the factored form as there were terms in the original expression, but is written as a multiplication expression.

Explore Factor Linear Expressions

Online Activity You will use algebra tiles to explore how to factor linear expressions.

Learn Factor Linear Expressions

You can use the Distributive Property and work backward to express a linear expression as a product of its factors. A linear expression is in **factored form** when it is expressed as the product of its factors.

Go Online Watch the animation to learn how to factor $12a + 6b$.

Step 1 Find the GCF of the terms.

$12a = 2 \cdot 2 \cdot 3 \cdot a$ Write the prime factorization of each term.

$6b = 2 \cdot 3 \cdot b$ Circle the common factors.

$2 \cdot 3$ or 6 Multiply the common factors to find the GCF.

Step 2 Write each term as a product with the GCF as a factor.

$12a + 6b = 6(2a) + 6(b)$

Step 3 Apply the Distributive Property.

$6(2a) + 6(b) = 6(2a + b)$

So, the factored form of $12a + 6b$ is $6(2a + b)$.

Lesson 6-4 • Factor Linear Expressions 365

Interactive Presentation

Learn, Factor Linear Expressions

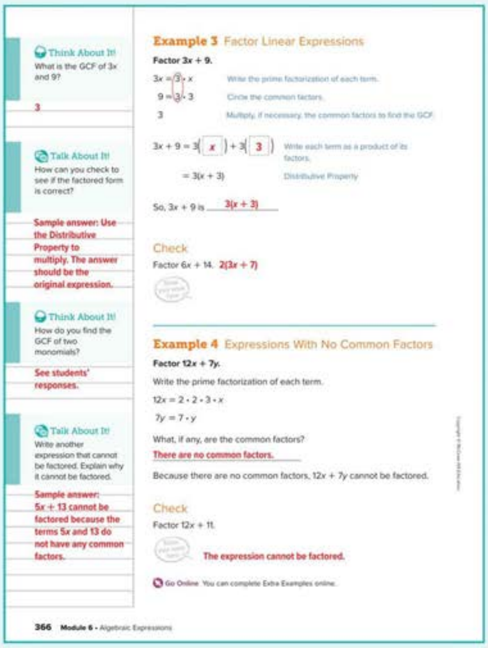
WATCH

On Slide 1 of the Learn, students watch an animation that demonstrates how to factor a linear expression.

DIFFERENTIATE

Enrichment Activity

Have students work in pairs to write three different linear expressions. Two of the expressions should be prime, while one of the expressions should be able to be factored. Have pairs exchange their sets of expressions with another pair. Each pair should determine which expressions that the other pair wrote cannot be factored. Have them explain why.



Think About It! What is the GCF of $3x$ and 9?

3

Talk About It! How can you check to see if the factored form is correct?

Sample answer: Use the Distributive Property to multiply. The answer should be the original expression.

Think About It! How do you find the GCF of two monomials?

See students' responses.

Talk About It! Write another expression that cannot be factored. Explain why it cannot be factored.

Sample answer: $5x + 13$ cannot be factored because the terms $5x$ and 13 do not have any common factors.

Example 3 Factor Linear Expressions

Factor $3x + 9$.

$3x = 3 \cdot x$ Write the prime factorization of each term.

$9 = 3 \cdot 3$ Circle the common factors.

3 Multiply, if necessary, the common factors to find the GCF.

$3x + 9 = 3(x) + 3(3)$ Write each term as a product of its factors.

$= 3(x + 3)$ Distributive Property

So, $3x + 9$ is $3(x + 3)$.

Check

Factor $6x + 14$. $2(3x + 7)$

Example 4 Expressions With No Common Factors

Factor $12x + 7y$.

Write the prime factorization of each term.

$12x = 2 \cdot 2 \cdot 3 \cdot x$

$7y = 7 \cdot y$

What, if any, are the common factors?

There are no common factors.

Because there are no common factors, $12x + 7y$ cannot be factored.

Check

Factor $12x + 11$.

The expression cannot be factored.

Go Online You can complete Extra Examples online.

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Interactive Presentation

Example 4, Expressions with No Common Factors, Slide 2 of 4

TYPE

On Slide 2 of Example 3, students type to factor $3x + 9$.

CLICK

On Slide 2 of Example 4 students select from a drop-down menu to indicate if the terms have any common factors.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Factor Linear Expressions

Objective

Students will factor linear expressions.

Questions for Mathematical Discourse

SLIDE 2

AL What are the prime factors of $3x$? 9? The prime factors of $3x$ are 3 and x. The prime factors of 9 are 3 and 3.

OL Why do we only place one 3 on the outside of the parentheses? Sample answer: Only one 3 is a factor of both expressions.

BL Describe how you think you could factor $3x^2 + 9x$. Sample answer: The prime factors of $3x^2$ are 3, x, and x. The prime factors of $9x$ are 3, 3, and x. So, the common factors are 3 and x. The factored form would be $3x(x + 3)$.

Example 4 Expressions with No Common Factors

Objective

Students will determine that linear expressions with no common factors cannot be factored.

Questions for Mathematical Discourse

SLIDE 2

AL What are the prime factors of $12x$? $7y$? The prime factors of $12x$ are 2, 2, 3, and x. The prime factors of $7y$ are 7 and y.

OL Are there any factors in common? no

OL What does it mean when there are no factors in common? The expression cannot be factored.

BL When an expression cannot be factored, it is considered *prime*. Is $12x + 7y$ prime? yes

BL Generate another expression that is considered *prime*. Sample answer: $8a + 9b$

Go Online

- Find additional teaching notes. Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 5 Factor Linear Expressions

Objective

Students will factor linear expressions with rational numbers written as fractions.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to perform the steps in factoring the expression accurately and efficiently, clearly articulating the mathematical reason(s) that support each step.

Questions for Mathematical Discourse

SLIDE 2

AL How is the term $\frac{1}{4}x$ expressed as a product of its factors? $\frac{1}{4}(x)$

AL How is the term $\frac{1}{4}$ expressed as a product of its factors? $\frac{1}{4}(1)$

OL Why is $\frac{1}{4}$ placed outside of the parentheses? It is a factor of both expressions.

OL Why is the number 1 left on the inside of the parentheses? When you factor $\frac{1}{4}$ out of $\frac{1}{4}$, the only number left is 1.

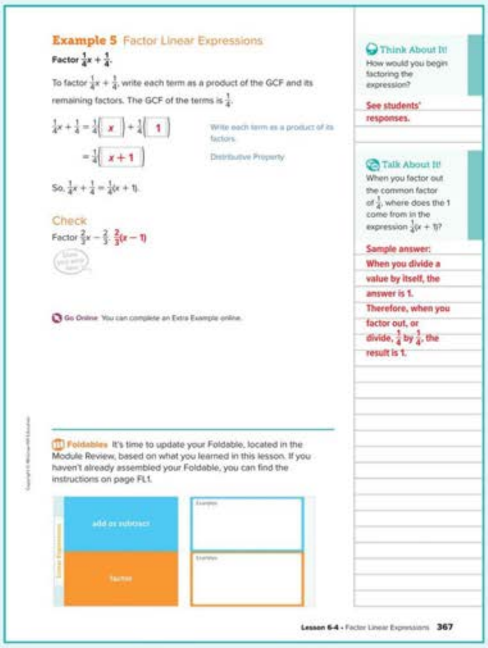
BL Create and factor your own expression that contains a rational coefficient and a rational constant. Sample answer: $\frac{1}{3}y + \frac{1}{3}$; The factored form is $\frac{1}{3}(y + 1)$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information about factoring linear expressions. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.



Example 5 Factor Linear Expressions

Factor $\frac{1}{4}x + \frac{1}{4}$.

To factor $\frac{1}{4}x + \frac{1}{4}$, write each term as a product of the GCF and its remaining factors. The GCF of the terms is $\frac{1}{4}$.

$\frac{1}{4}x + \frac{1}{4} = \frac{1}{4}(x) + \frac{1}{4}(1)$ Write each term as a product of its factors.

$= \frac{1}{4}(x + 1)$ Distributive Property

So, $\frac{1}{4}x + \frac{1}{4} = \frac{1}{4}(x + 1)$.

Check

Factor $\frac{2}{3}x - \frac{2}{3}$. $\frac{2}{3}(x - 1)$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

Think About It! How would you begin factoring the expression?

See students' responses.

Talk About It! When you factor out the common factor of $\frac{1}{4}$, where does the 1 come from in the expression $\frac{1}{4}(x + 1)$?

Sample answer: When you divide a value by itself, the answer is 1. Therefore, when you factor out, or divide, $\frac{1}{4}$ by $\frac{1}{4}$, the result is 1.

Lesson 6-4 • Factor Linear Expressions 367

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Pause and Reflect

Create a graphic organizer to record the steps for factoring linear expressions. Be sure to include examples of expressions that can be factored and expressions that cannot be factored.

See students' graphic organizers.

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Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. How could you rewrite the expression $4x + 8$ using a common factor that represents the total cost and the cost for each friend? Write a mathematical argument that can be used to defend your solution. Sample answer: Factor out the common factor of 4 and rewrite the expression using the Distributive Property. So, $4(x + 2)$ represents the total cost and cost for each friend.

Essential Question Follow-Up

Why is it beneficial to rewrite expressions in different forms?

In this lesson, students learned how to factor linear expressions. Encourage them to brainstorm with a partner a real-world problem in which facotring an expression such as $6a + 24ab$ can help solve the problem. One possible example could be finding the dimensions of a rectangle whose area is represented by the expression $(6a + 24ab)$ feet. If the width is $6a$ feet, then the length must be $(1 + 4b)$ feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 17, 19–22
- ALEKS® Simplifying Algebraic Expressions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–15, 18–20
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS® The Distributive Property

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® The Distributive Property

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

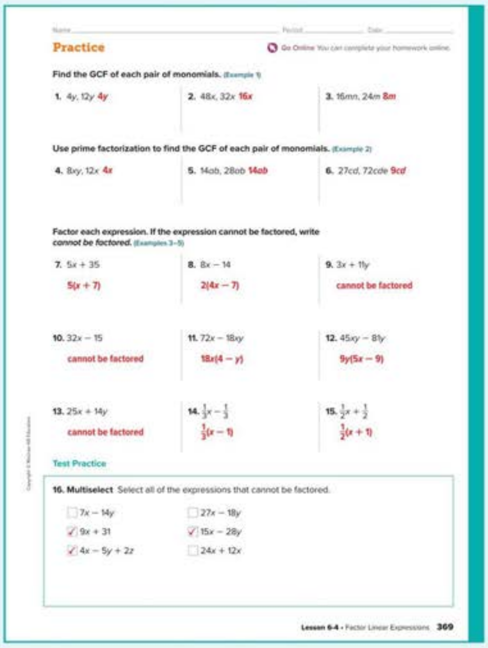
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the greatest common factor of monomials by identifying the GCF of the coefficients and the variables	1–6
2	factor linear expressions	7–15
2	extend concepts learned in class to apply them in new contexts	16
3	solve application problems involving factoring linear expressions	17, 18
3	higher-order and critical thinking skills	19–22

Common Misconception

When students factor expressions, they may use the wrong factor which would result in the incorrect answer. In Exercises 7–15 remind students to find the GCF and not just a common factor.



Practice

Find the GCF of each pair of monomials. (Example 1)

1. $4y, 12y$ **$4y$**
2. $48x, 32x$ **$16x$**
3. $16mn, 24m$ **$8m$**

Use prime factorization to find the GCF of each pair of monomials. (Example 2)

4. $8xy, 12x$ **$4x$**
5. $14ab, 28ab$ **$14ab$**
6. $27cd, 72cde$ **$9cd$**

Factor each expression. If the expression cannot be factored, write *cannot be factored.* (Examples 3–5)

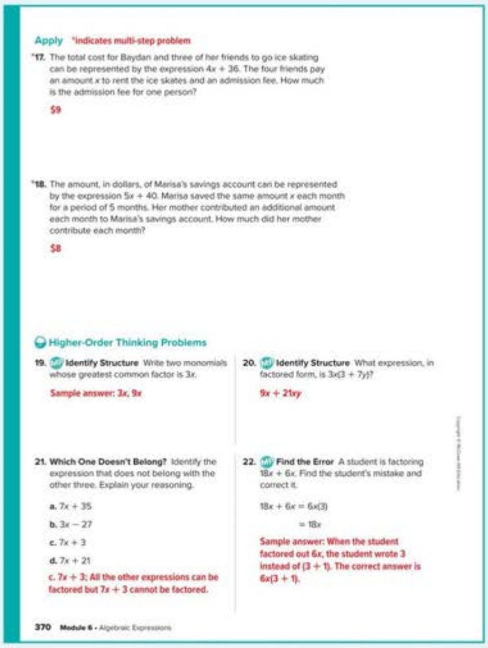
7. $5x + 35$ **$5(x + 7)$**
8. $8x - 14$ **$2(4x - 7)$**
9. $3x + 11y$ **cannot be factored**
10. $32x - 15$ **cannot be factored**
11. $72x - 18xy$ **$18x(4 - y)$**
12. $45xy - 81y$ **$9y(5x - 9)$**
13. $25x + 14y$ **cannot be factored**
14. $\frac{1}{3}x - \frac{1}{3}$ **$\frac{1}{3}(x - 1)$**
15. $\frac{1}{2}x + \frac{1}{2}$ **$\frac{1}{2}(x + 1)$**

Test Practice

16. **Multiselect** Select all of the expressions that cannot be factored.

- ☐ $7x - 14y$
- ☑ $9x + 31$
- ☑ $4x - 5y + 2z$
- ☐ $27x - 18y$
- ☑ $15x - 28y$
- ☐ $24x + 12x$

Lesson 6-4 • Factor Linear Expressions **369**



Apply *indicates multi-step problem

*17. The total cost for Baydan and three of her friends to go ice skating can be represented by the expression $4x + 36$. The four friends pay an amount x to rent the ice skates and an admission fee. How much is the admission fee for one person?

$9

*18. The amount, in dollars, of Marisa's savings account can be represented by the expression $5x + 40$. Marisa saved the same amount x each month for a period of 5 months. Her mother contributed an additional amount each month to Marisa's savings account. How much did her mother contribute each month?

$8

Higher-Order Thinking Problems

19. **Identify Structure** Write two monomials whose greatest common factor is $3x$.

Sample answer: $3x$, $9x$

20. **Identify Structure** What expression, in factored form, is $3x(3 + 7y)$?

$9x + 21xy$

21. **Which One Doesn't Belong?** Identify the expression that does not belong with the other three. Explain your reasoning.

a. $7x + 35$

b. $3x - 27$

c. $7x + 3$

d. $7x + 21$

c. $7x + 3$; All the other expressions can be factored but $7x + 3$ cannot be factored.

22. **Find the Error** A student is factoring $18x + 6x$. Find the student's mistake and correct it.

$18x + 6x = 6x(3)$

$= 18x$

Sample answer: When the student factored out $6x$, the student wrote 3 instead of $(3 + 1)$. The correct answer is $6x(3 + 1)$.

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Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 19, students write two monomials that satisfy the given requirement.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 22, students find the error in a student's reasoning and correct it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.
Use with Exercises 17–18 Have students work in groups of 3-4. After completing Exercise 17, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 18.

Create your own higher-order thinking problem.
Use with Exercises 19–22 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Lesson 6-5

Combine Operations with Linear Expressions

7.EE.A.1

LESSON GOAL

Students will combine operations to simplify linear expressions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Example 1: Combine Operations to Simplify Expressions
Example 2: Combine Operations to Simplify Expressions
Example 3: Combine Operations to Simplify Expressions
Apply: Gardening

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

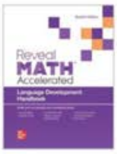
View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Simplify Rational Expressions		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 37 of the *Language Development Handbook* to help your students build mathematical language related to combining operations with linear expressions.

ELL You can use the tips and suggestions on page T37 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7.EE.A** by combining operations to simplify linear expressions.
Standards for Mathematical Content: **7.EE.A.1**
Standards for Mathematical Practice: **MP1, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the GCF of monomials and factored algebraic expressions.
7.EE.A.1

Now
Students combine operations to simplify linear expressions.
7.EE.A.1

Next
Students will apply the use of expressions to write and solve equations and formulas.
7.EE.B.4

Rigor

The Three Pillars of Rigor

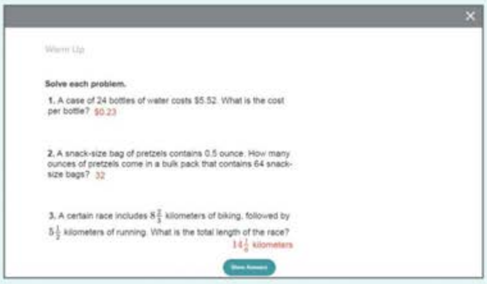
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of adding and subtracting linear expressions and factoring linear expressions to gain an *understanding* of combining linear expressions. Students will gain *fluency* in combining the procedures of combining like terms, adding and subtracting linear expressions, distributing integers over linear expressions and factoring linear expressions.

Mathematical Background

Use properties, the order of operations, and knowledge of adding, subtracting, and factoring linear expressions to combine operations to *simplify* linear expressions. To simplify the expression $-2(x + 3) + 8x$, first use the Distributive Property to rewrite the expression as $-2x - 6 + 8x$, then combine like terms. The expression in simplest form is $6x - 6$.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- adding, multiplying, and dividing rational numbers (Exercises 1–3)

Answers

1. $0 .23
2. 32
3. $14\frac{1}{6}$ kilometers

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an expression to find the number of leftover decorations from classroom parties.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- When is an expression *simplified*? Sample answer: An expression is simplified when no more operations can be performed without knowing the value(s) of any variables.

Example 1 Combine Operations to Simplify Expressions

Objective

Students will simplify linear expressions by using the Distributive Property, combining like terms, and writing the answer in factored form.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the parts that make up this expression. Sample answer: There is a product of –2 and (x + 3). Then 8x is added to that product.

OL After using the Distributive Property and combining like terms, what is the expression? 6x – 6

OL How do you know that you can factor 6x – 6? There is a common factor in each term, 6.

BL Generate your own expression in which you need to perform multiple operations to simplify it. Write your result in factored form. Sample answer: 4(x + 1) + (5x + 2); In factored form, this is 3(3x + 2).

Example 2 Combine Operations to Simplify Expressions

Objective

Students will simplify expressions with rational numbers by using the Distributive Property and combining like terms.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the parts that make up this expression. Sample answer: There is a product of $\frac{1}{2}$ and $\left(\frac{1}{4}x - 2\right)$. Then $\frac{3}{8}x$ is added to that product.

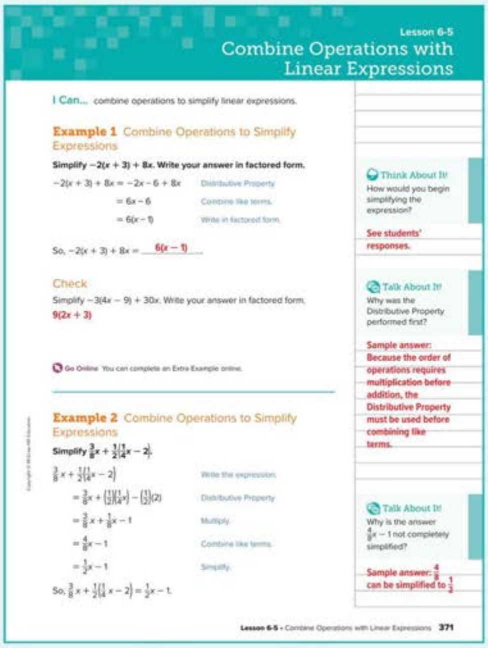
OL After using the Distributive Property and combining like terms, what is the expression? $\frac{1}{2}x - 1$

OL Is $\frac{1}{2}x - 1$ in simplest form? Explain. yes; Sample answer: There are no like terms.

BL Does it matter that $\frac{3}{8}x$ is added at the beginning of the expression instead of at the end? Explain. no; Sample answer: The Commutative Property states that you can add in any order.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.



Lesson 6-5

Combine Operations with Linear Expressions

I Can... combine operations to simplify linear expressions.

Example 1 Combine Operations to Simplify Expressions

Simplify −2(x + 3) + 8x. Write your answer in factored form.

$-2(x+3) + 8x = -2x - 6 + 8x$ Distributive Property

$= 6x - 6$ Combine like terms.

$= 6(x - 1)$ Write in factored form.

So, $-2(x+3) + 8x =$ $6(x - 1)$

Think About It! How would you begin simplifying the expression?

See students' responses.

Talk About It! Why was the Distributive Property performed first?

Sample answer: Because the order of operations requires multiplication before addition, the Distributive Property must be used before combining like terms.

Check

Simplify $-3(4x - 9) + 30x$. Write your answer in factored form.

$9(2x + 3)$

Go Online You can complete an Extra Example online.

Example 2 Combine Operations to Simplify Expressions

Simplify $\frac{3}{8}x + \frac{1}{2}\left(\frac{1}{4}x - 2\right)$.

$\frac{3}{8}x + \frac{1}{2}\left(\frac{1}{4}x - 2\right)$ Write the expression.

$= \frac{3}{8}x + \left(\frac{1}{2}\right)\left(\frac{1}{4}x\right) - \left(\frac{1}{2}\right)(2)$ Distributive Property

$= \frac{3}{8}x + \frac{1}{8}x - 1$ Multiply.

$= \frac{4}{8}x - 1$ Combine like terms.

$= \frac{1}{2}x - 1$ Simplify.

So, $\frac{3}{8}x + \frac{1}{2}\left(\frac{1}{4}x - 2\right) = \frac{1}{2}x - 1$.

Talk About It! Why is the answer $\frac{4}{8}x - 1$ not completely simplified?

Sample answer: $\frac{4}{8}$ can be simplified to $\frac{1}{2}$.

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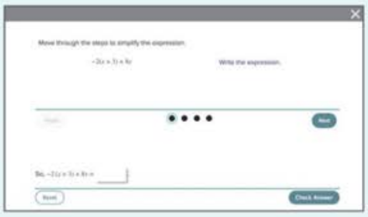
Interactive Presentation



Example 1, Combine Operations to Simplify Expressions, Slide 2 of 4

TYPE

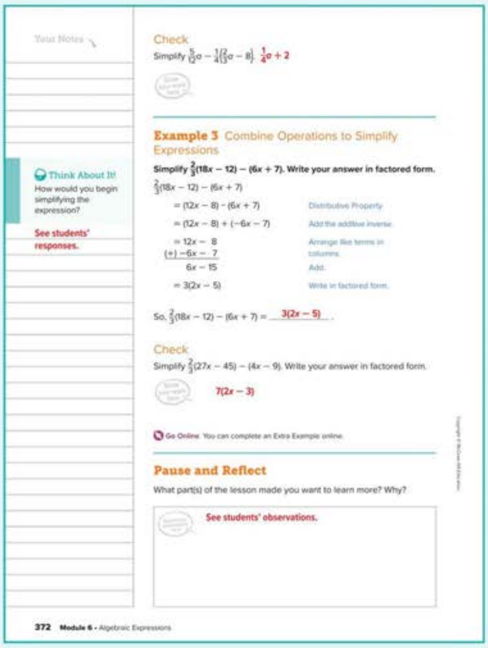
On Slide 2 of Example 1, students type to enter the simplified expression.

CLICK

On Slide 2 of Example 2, students move through the steps to simplify an expression.

CHECK

Students complete the Check exercises online to determine if they are ready to move on.



Your Notes

Check

Simplify $\frac{5}{12}a - \frac{1}{4}\left(\frac{2}{3}a - 8\right)$. $\frac{1}{4}a + 2$

Example 3 Combine Operations to Simplify Expressions

Simplify $\frac{2}{3}(18x - 12) - (6x + 7)$. Write your answer in factored form.

$\frac{2}{3}(18x - 12) - (6x + 7)$

$= (12x - 8) - (6x + 7)$ — Distributive Property

$= (12x - 8) + (-6x - 7)$ — Add the additive inverse.

$= 12x - 8$
$(+) -6x - 7$ — Arrange like terms in columns.

$6x - 15$ — Add.

$= 3(2x - 5)$ — Write in factored form.

So, $\frac{2}{3}(18x - 12) - (6x + 7) =$ $3(2x - 5)$.

Think About It! How would you begin simplifying the expression?

See students' responses.

Check

Simplify $\frac{2}{3}(27x - 45) - (4x - 9)$. Write your answer in factored form.

$7(2x - 3)$

Go Online You can complete an Extra Example online.

Pause and Reflect

What part(s) of the lesson made you want to learn more? Why?

See students' observations.

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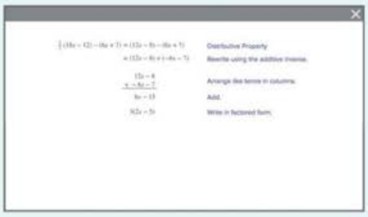
Interactive Presentation



Example 3, Combine Operations to Simplify Expressions, Slide 2 of 3

TYPE

On Slide 2, students type to enter the simplified expression.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Combine Operations to Simplify Expressions

Objective

Students will simplify linear expressions by using the Distributive Property, adding or subtracting, and writing the answer in factored form.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to carefully attend to each operation and perform the operations accurately and efficiently, paying special attention to the sign of each term.

7 Look For and Make Use of Structure Encourage students to analyze the structure of the given expression prior to simplifying it. They should understand how the expression is structured in order to know which operations must be performed first.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the parts that make up this expression. Sample answer: There is a product of $\frac{2}{3}$ and $(18x - 12)$. Then $(6x + 7)$ is subtracted from that product.

OL What operation should be performed first? Use the Distributive Property to simplify $\frac{2}{3}(18x - 12)$.

OL What operation should be performed second? Rewrite the subtraction of the second expression as addition of the additive inverse.

BL Generate an expression that involves multiple operations and one in which there is at least one rational number. Trade your expression with another student and have them explain to you how they would simplify it. See students' work.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of simplifying expressions, have them work with a partner to simplify the following expression without completely expanding it. Have them first identify any common factors between the two addends, then factor and simplify the resulting expression.

$(4a - 2) + 2\left(a - \frac{1}{2}\right)$ $3(a - 1)$

Apply Gardening

Objective

Students will come up with their own strategy to solve an application problem involving the area of a flower border of a garden.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

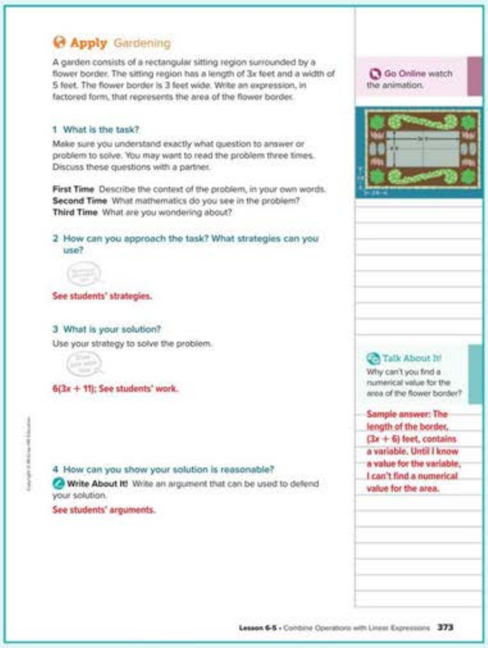
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What are the dimensions of the garden?
- What operations will you use to find the area of the garden without the sitting region?
- How can the Distributive Property help you?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Gardening

A garden consists of a rectangular sitting region surrounded by a flower border. The sitting region has a length of 3x feet and a width of 5 feet. The flower border is 3 feet wide. Write an expression, in factored form, that represents the area of the flower border.

Go Online watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

6(3x + 11); See students' work.

Talk About It! Why can't you find a numerical value for the area of the flower border?

Sample answer: The length of the border, (3x + 6) feet, contains a variable. Until I know a value for the variable, I can't find a numerical value for the area.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Lesson 6-5 • Combine Operations with Linear Expressions 373

Interactive Presentation

Apply, Gardening

WATCH Students watch an animation that illustrates the problem they are about to solve.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

The diagram shows a walkway that is 5 feet wide surrounding a rectangular swimming pool. Write an expression, in factored form, that represents the area of the walkway.

$10(4x + 25)$

Go Online You can complete an Extra Example online.

Pause and Reflect

Write a real-world problem that uses the concepts from today's lesson. Explain how you came up with that problem. Exchange problems with a classmate and solve each other's problem.

See students' observations.

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Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. The expression $6x + 17y - (3x + 5y)$ represents the number of decorations and streamers that would be left over. Find the simplified expression that represents the number of decorations and streamers that were given to each of the three neighboring classrooms. Write a mathematical argument that can be used to defend your solution. $x + 4y$; Sample answer: First, rewrite the expression $6x + 17y - (3x+5y)$ by adding the additive inverse: $6x + 17y + (-3x - 5y)$. Simplify the expression by combining like terms: $3x + 12y$. Then factor out 3 which results in a final simplified expression of $3(x + 4y)$. Each classroom will receive $x + 4y$ number of decorations and streamers.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 13, 15–19
- Extension: Simplify Rational Expressions
- ALEKS Simplifying Algebraic Expressions

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–12, 14, 16, 18
- Extension: Simplify Rational Expressions
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS The Distributive Property

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS The Distributive Property

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

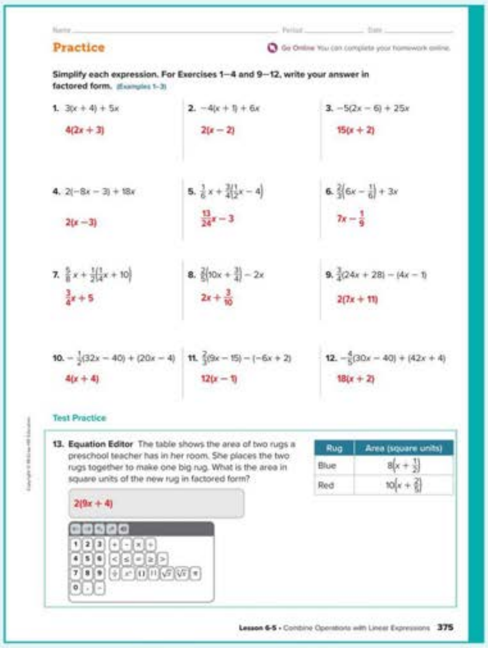
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	simplify linear expressions by using the Distributive Property, combining like terms, and adding or subtracting	1–12
2	extend concepts learned in class to apply them in new contexts	13
3	solve application problems involving combining operations with linear expressions	14, 15
3	higher-order and critical thinking skills	16–19

Common Misconception

Some students may attempt to factor the expression before simplifying the expression. Review the steps to simplifying an expression that includes factoring as the last step.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Simplify each expression. For Exercises 1–4 and 9–12, write your answer in factored form. (Examples 1–3)

1. $3(x + 4) + 5x$ — $4(2x + 3)$
2. $-4(x + 1) + 6x$ — $2(x - 2)$
3. $-5(2x - 6) + 25x$ — $15(x + 2)$
4. $2(-8x - 3) + 18x$ — $2(x - 3)$
5. $\frac{1}{6}x + \frac{3}{4}\left(\frac{1}{2}x - 4\right)$ — $\frac{13}{24}x - 3$
6. $\frac{2}{3}\left(6x - \frac{1}{6}\right) + 3x$ — $7x - \frac{1}{9}$
7. $\frac{5}{8}x + \frac{1}{2}\left(\frac{1}{4}x + 10\right)$ — $\frac{3}{4}x + 5$
8. $\frac{2}{5}\left(10x + \frac{3}{4}\right) - 2x$ — $2x + \frac{3}{10}$
9. $\frac{3}{4}(24x + 28) - (4x - 1)$ — $2(7x + 11)$
10. $-\frac{1}{4}(32x - 40) + (20x - 4)$ — $4(x + 4)$
11. $\frac{2}{3}(9x - 15) - (-6x + 2)$ — $12(x - 1)$
12. $-\frac{4}{5}(30x - 40) + (42x + 4)$ — $18(x + 2)$

Test Practice

13. **Equation Editor** The table shows the area of two rugs a preschool teacher has in her room. She places the two rugs together to make one big rug. What is the area in square units of the new rug in factored form?

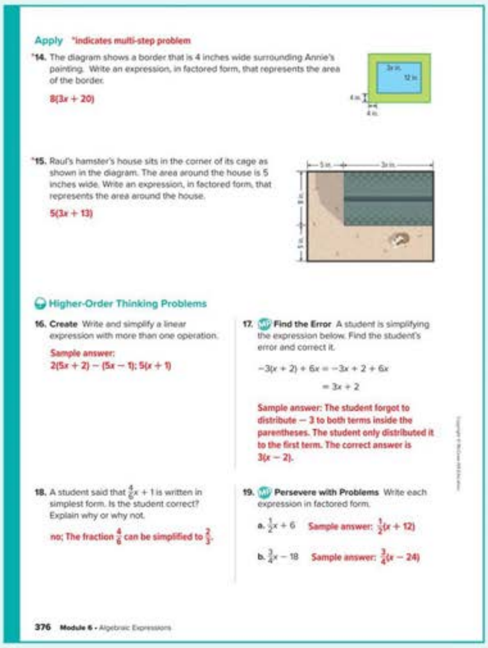
Rug	Area (square units)
Blue	$8\left(x + \frac{1}{2}\right)$
Red	$10\left(x + \frac{2}{5}\right)$

$2(9x + 4)$

Lesson 6-5 • Combine Operations with Linear Expressions 375

7.EE.A.1

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*14. The diagram shows a border that is 4 inches wide surrounding Annie's painting. Write an expression, in factored form, that represents the area of the border.

$8(3x + 20)$

*15. Raul's hamster's house sits in the corner of its cage as shown in the diagram. The area around the house is 5 inches wide. Write an expression, in factored form, that represents the area around the house.

$5(3x + 13)$

Higher-Order Thinking Problems

16. **Create** Write and simplify a linear expression with more than one operation.

Sample answer: $2(5x + 2) - (5x - 1)$; $5(x + 1)$

17. MP **Find the Error** A student is simplifying the expression below. Find the student's error and correct it.

$$-3(x + 2) + 6x = -3x + 2 + 6x$$
$$= 3x + 2$$

Sample answer: The student forgot to distribute -3 to both terms inside the parentheses. The student only distributed it to the first term. The correct answer is $3(x - 2)$.

18. A student said that $\frac{4}{6}x + 1$ is written in simplest form. Is the student correct? Explain why or why not.

no; The fraction $\frac{4}{6}$ can be simplified to $\frac{2}{3}$.

19. MP **Persevere with Problems** Write each expression in factored form.

a. $\frac{1}{2}x + 6$ Sample answer: $\frac{1}{2}(x + 12)$

b. $\frac{3}{4}x - 18$ Sample answer: $\frac{3}{4}(x - 24)$

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students find a student's error in simplifying an expression and correct it.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 19, students factor two expressions with rational coefficients.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 14–15 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 15 might be "How would you find the expressions for the length and width?"

Clearly and precisely explain.
Use with Exercise 18 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to think about the requirements for an expression to be simplified.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples and explanations of operations with linear expressions. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

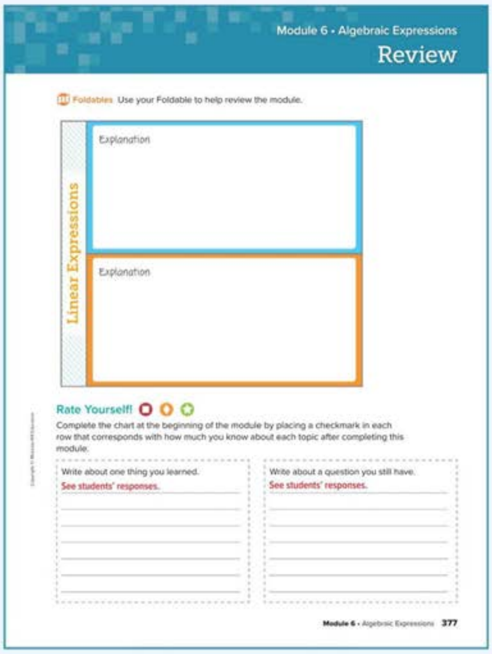
Assessment Resources

Put It All Together: Lessons 6-1 through 6-3
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with this topic for **Expressions and Equations**.

• Equivalent Algebraic Expressions


Module 6 • Algebraic Expressions

Review

Foldables Use your Foldable to help review the module.

Linear Expressions

Explanation

Explanation

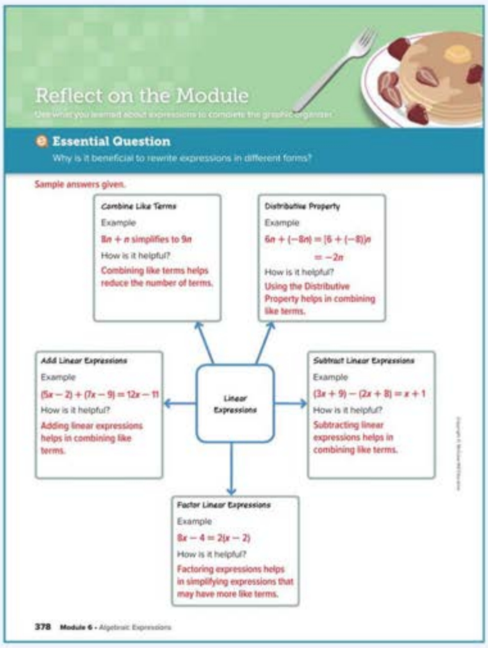
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 6 • Algebraic Expressions 377



Reflect on the Module

Use what you learned about expressions to complete the graphic organizer.

Essential Question

Why is it beneficial to rewrite expressions in different forms?

Sample answers given.

Combine Like Terms
Example
$8n + n$ simplifies to $9n$
How is it helpful?
Combining like terms helps reduce the number of terms.

Distributive Property
Example
$6n + (-8n) = [6 + (-8)]n$
$= -2n$
How is it helpful?
Using the Distributive Property helps in combining like terms.

Linear Expressions

Add Linear Expressions
Example
$(5x - 2) + (7x - 9) = 12x - 11$
How is it helpful?
Adding linear expressions helps in combining like terms.

Subtract Linear Expressions
Example
$(3x + 9) - (2x + 8) = x + 1$
How is it helpful?
Subtracting linear expressions helps in combining like terms.

Factor Linear Expressions
Example
$8x - 4 = 2(x - 2)$
How is it helpful?
Factoring expressions helps in simplifying expressions that may have more like terms.

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can we communicate algebraic relationships with mathematical symbols? See students' graphic organizers.

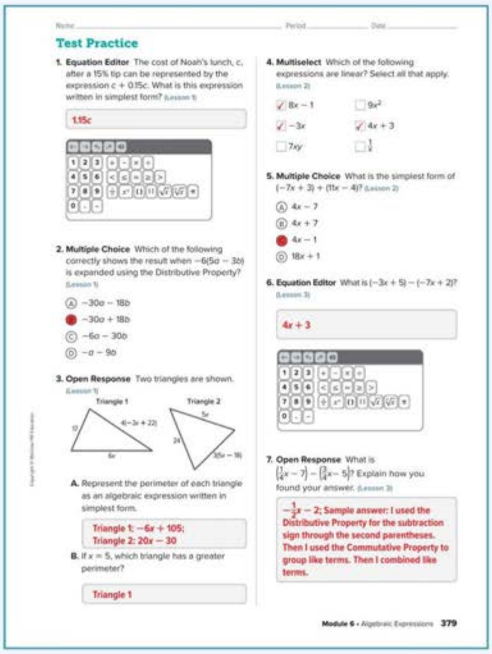
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–14 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	2, 5, 9, 11
Multiselect	Multiple answers may be correct. Students must select all correct answers.	4
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	1, 6, 10, 13
Open Response	Students construct their own response in the area provided.	3, 7, 8, 12, 14

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.EE.A.1	6-1, 6-2, 6-3, 6-4, 6-5	1–14
7.EE.A.2	6-1	1, 3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Test Practice

1. **Equation Editor** The cost of Noah's lunch, c, after a 15% tip can be represented by the expression $c + 0.15c$. What is this expression written in simplest form? (Lesson 1)

   1.15c

2. **Multiple Choice** Which of the following correctly shows the result when $-6(5a - 3b)$ is expanded using the Distributive Property? (Lesson 1)

   (A) $-30a - 18b$
   (B) $-30a + 18b$
   (C) $-6a - 30b$
   (D) $-a - 9b$

3. **Open Response** Two triangles are shown. (Lesson 1)

   Triangle 1 Triangle 2

   A. Represent the perimeter of each triangle as an algebraic expression written in simplest form.

   Triangle 1: $-6x + 105$;
   Triangle 2: $20x - 30$

   B. If $x = 5$, which triangle has a greater perimeter?

   Triangle 1

4. **Multiselect** Which of the following expressions are linear? Select all that apply. (Lesson 2)

   ☑ $8x - 1$ ☐ $9x^2$
   ☑ $-3x$ ☑ $4x + 3$
   ☐ $7xy$ ☐ $\frac{1}{x}$

5. **Multiple Choice** What is the simplest form of $(-7x + 3) + (11x - 4)$? (Lesson 2)

   (A) $4x - 7$
   (B) $4x + 7$
   (C) $4x - 1$
   (D) $18x + 1$

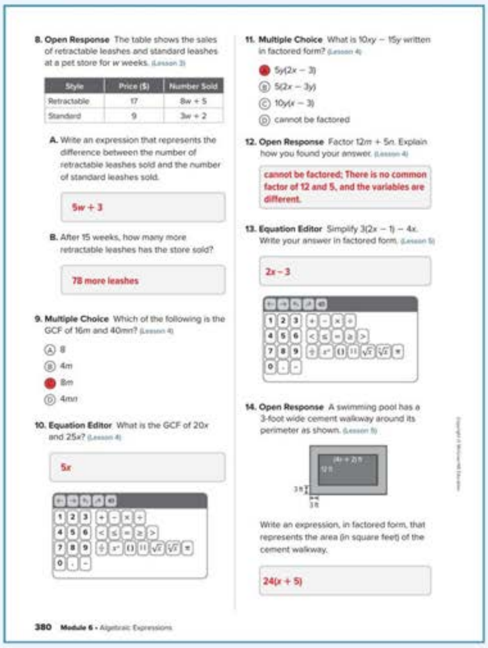
6. **Equation Editor** What is $(-3x + 5) - (-7x + 2)$? (Lesson 3)

   $4x + 3$

7. **Open Response** What is $\left(\frac{1}{4}x - 7\right) - \left(\frac{3}{4}x - 5\right)$? Explain how you found your answer. (Lesson 3)

   $-\frac{1}{2}x - 2$; Sample answer: I used the Distributive Property for the subtraction sign through the second parentheses. Then I used the Commutative Property to group like terms. Then I combined like terms.

Module 6 • Algebraic Expressions 379



8. Open Response The table shows the sales of retractable leashes and standard leashes at a pet store for w weeks. (Lesson 3)

Style	Price ($)	Number Sold
Retractable	17	$8w + 5$
Standard	9	$3w + 2$

A. Write an expression that represents the difference between the number of retractable leashes sold and the number of standard leashes sold.

$5w + 3$

B. After 15 weeks, how many more retractable leashes has the store sold?

78 more leashes

9. Multiple Choice Which of the following is the GCF of $16m$ and $40mn$? (Lesson 4)

(A) 8
(B) $4m$
(C) $8m$
(D) $4mn$

10. Equation Editor What is the GCF of $20x$ and $25x$? (Lesson 4)

$5x$

11. Multiple Choice What is $10xy - 15y$ written in factored form? (Lesson 4)

(A) $5y(2x - 3)$
(B) $5(2x - 3y)$
(C) $10y(x - 3)$
(D) cannot be factored

12. Open Response Factor $12m + 5n$. Explain how you found your answer. (Lesson 4)

cannot be factored; There is no common factor of 12 and 5, and the variables are different.

13. Equation Editor Simplify $3(2x - 1) - 4x$. Write your answer in factored form. (Lesson 5)

$2x - 3$

14. Open Response A swimming pool has a 3-foot wide cement walkway around its perimeter as shown. (Lesson 5)

Write an expression, in factored form, that represents the area (in square feet) of the cement walkway.

$24(x + 5)$

Module 7

Equations and Inequalities

Module Goal

Write and solve equations and inequalities.

Focus

Domain: Expressions and Equations

Major Cluster(s):

7.EE.B S olve real-life and mathematical problems using numerical and algebraic expressions and equations.

8.EE.C Analyze and solve linear equations and pairs of simultaneous linear equations.

Standards for Mathematical Content:

7.EE.B.4.A S olve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *Also addresses 7.NS.A.3, 7.EE.B.4, 7.EE.B.4.B, 8.EE.C.7, 8.EE.C.7.A, 8.EE.C.7.B.*

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- solve one-step equations
- graph an inequality on the number line

Use the Module Pretest to diagnose students' readiness for this module.

Coherence

Vertical Alignment

Previous
Students wrote and solved one-step equations. **6.EE.B.7**

Now
Students write and solve equations and inequalities.
7.EE.B.4, 7.EE.B.4.A, 7.EE.B.4.B, 8.EE.C.7, 8.EE.C.7.A,8.EE.C.7.B

Next
Students will solve linear equations in one variable, including equations with coefficients represented by letters. **HSA.REI.B.3**

Rigor

The Three Pillars of Rigor

In this module, students will draw on their knowledge of solving one-step equations (gained in Grade 6) to develop an *understanding* of solving equations and inequalities. They will use this understanding to gain *fluency* in writing and solving equations and inequalities. They will *apply* their understanding to solve real-world problems.

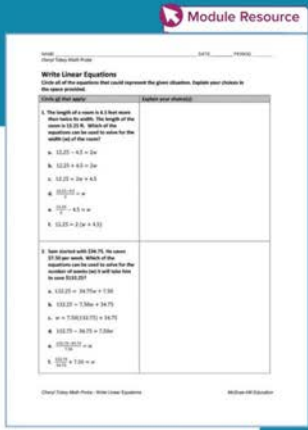
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
7-1	Write and Solve Two-Step Equations: $px + q = r$	7.EE.B.4, 7.EE.B.4.A	2	1
7-2	Write and Solve Two-Step Equations: $p(x + q) = r$	7.EE.B.4, 7.EE.B.4.A	2	1
7-3	Write and Solve Equations with Variables on Each Side	8.EE.C.7, 8.EE.C.7.B, *Also addresses 8.EE.C.7.A*	2	1
7-4	Write and Solve Multi-Step Equations	8.EE.C.7, 8.EE.C.7.B, *Also addresses 8.EE.C.7.A*	2	1
7-5	Determine the Number of Solutions	8.EE.C.7, 8.EE.C.7.A	2	1
Put It All Together 1: Lessons 7-1 through 7-5			0.5	0.25
7-6	Write and Solve One-Step Addition and Subtraction Inequalities	7.EE.B.4, 7.EE.B.4.B	1	0.5
7-7	Write and Solve One-Step Multiplication and Division Inequalities	7.EE.B.4, 7.EE.B.4.B	2	1
7-8	Write and Solve Two-Step Inequalities	7.EE.B.4, 7.EE.B.4.B	1	0.5
Put It All Together 2: Lessons 7-6 through 7-8			0.5	0.25
Module Review and Assessment			2	1
		Total Days	18	9

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe
Write Linear Equations



Correct Answers:
1. a, c, d
2. b, d, e

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which equations can be used to represent each situation, and explain their choices.

Targeted Concepts Understand the mathematical meaning of words used to describe relationships between quantities in real-life situations and recognize different ways to write algebraic equations that represent the same mathematical relationship.

Targeted Misconceptions
- Students may misrepresent words and/or relationships described with inaccurate mathematical operations.
- Students may inaccurately represent the described situation by a direct translation equation (i.e. equations that have numbers and variables in the order in which they appear in the description).
- Students may not recognize equivalent algebraic representations.

Assign the probe after Lesson 2.

Collect and Assess Student Answers

If the student selects...	Then the student likely...
1. b and/or f	looked for key words and associated operations, such as "more" means "add," and then selected equations without considering the relationship described.
Various incorrect choices	saw only one equation as being correct. Once they have found an equation that reflects their thinking, they do not consider whether other equations accurately represent the relationship.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Equations and Inequalities
- Lesson 1, Examples 1–4
- Lesson 2, Examples 1–5
- Lesson 3, Examples 1–4
- Lesson 4, Examples 1–5

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can equations be used to solve everyday problems?
See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

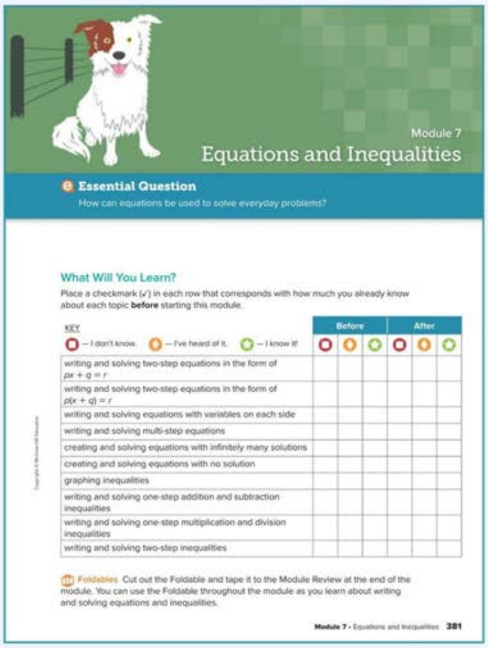
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of saving money, purchasing music online, and building a fence to introduce the idea of writing and solving equations. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 7

Equations and Inequalities

Essential Question

How can equations be used to solve everyday problems?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

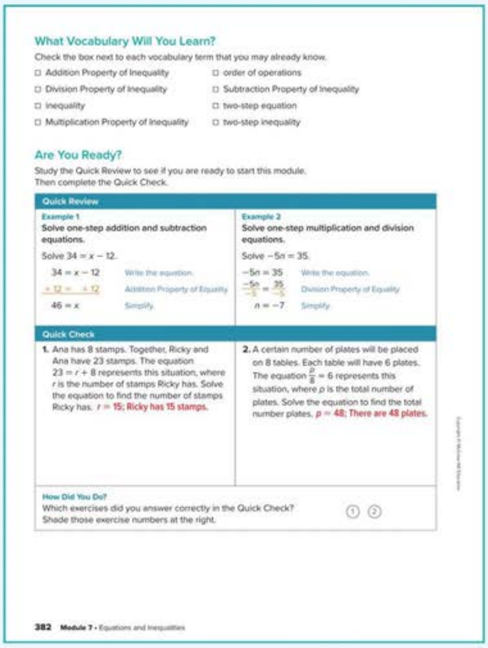
KEY: I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know.	I've heard of it.	I know it!	I don't know.	I've heard of it.	I know it!
writing and solving two-step equations in the form of $px + q = r$						
writing and solving two-step equations in the form of $p(x + q) = r$						
writing and solving equations with variables on each side						
writing and solving multi-step equations						
creating and solving equations with infinitely many solutions						
creating and solving equations with no solution						
graphing inequalities						
writing and solving one-step addition and subtraction inequalities						
writing and solving one-step multiplication and division inequalities						
writing and solving two-step inequalities						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about writing and solving equations and inequalities.

Module 7 • Equations and Inequalities 381

Interactive Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ Addition Property of Inequality
- ☐ order of operations
- ☐ Division Property of Inequality
- ☐ Subtraction Property of Inequality
- ☐ inequality
- ☐ two-step equation
- ☐ Multiplication Property of Inequality
- ☐ two-step inequality

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1	Example 2
Solve one-step addition and subtraction equations. Solve $34 = x - 12$. $34 = x - 12$ Write the equation. $+12 = +12$ Addition Property of Equality. $46 = x$ Simplify.	**Solve one-step multiplication and division equations.** Solve $-5n = 35$. $-5n = 35$ Write the equation. $\frac{-5n}{-5} = \frac{35}{-5}$ Division Property of Equality. $n = -7$ Simplify.

Quick Check

1. Ana has 8 stamps. Together, Ricky and Ana have 23 stamps. The equation $23 = r + 8$ represents this situation, where r is the number of stamps Ricky has. Solve the equation to find the number of stamps Ricky has. $r = 15$; Ricky has 15 stamps.	2. A certain number of plates will be placed on 8 tables. Each table will have 6 plates. The equation $\frac{p}{8} = 6$ represents this situation, where p is the total number of plates. Solve the equation to find the total number plates. $p = 48$; There are 48 plates.

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define A **two-step equation** has two different operations paired with the variable.

Example The equation $3x - 6 = 12$ is an example of a two-step equation. The two operations paired with the variable are multiplication and subtraction.

Ask Give another example of a two-step equation. Explain the operations that are paired with the variable. Sample answer: $-2x + 7 = 11$; The two operations paired with the variable are multiplication and addition.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- adding, subtracting, multiplying, and dividing rational numbers
- solving one-step equations
- writing one-step equations
- using the Distributive Property to evaluate numerical expressions containing rational numbers

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Equations and Inequalities** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Reward Effort, Not Talent

When adults praise students for their hard work toward a solution, rather than praising them for being smart or talented, it supports students' development of a growth mindset.

How Can I Apply It?

Have students complete the **Performance Task** for the module. Allow students a forum to discuss their process or strategy that they used and give them positive feedback on their diligence in completing the task.

Lesson 7-1 7.EE.B.4, 7.EE.B.4.A

Write and Solve Two-Step Equations: $px + q = r$

LESSON GOAL

Students will write and solve two-step equations of the form $px + q = r$.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Solve Two-Step Equations Using Algebra Tiles

Learn: Two-Step Equations
Learn: Properties of Equality
Example 1: Solve Two-Step Equations
Example 2: Solve Two-Step Equations
Learn: Two-Step Equations: Arithmetic Method and Algebraic Method

Explore: Write Two-Step Equations

Learn: Write Two-Step Equations
Example 3: Write and Solve Two-Step Equations
Example 4: Write and Solve Two-Step Equations
Apply: Budgets

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Consecutive Integers Equations		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 38 of the *Language Development Handbook* to help your students build mathematical language related to writing and solving two-step equations of the form $px + q = r$.

ELL You can use the tips and suggestions on page T38 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7.EE.B** by writing and solving two-step equations of the form $px + q = r$.
Standards for Mathematical Content: **7. EE.B.4, 7.EE.B.4.A,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students wrote and solved one-step equations.
6.EE.B.7

Now
Students write and solve two-step equations of the form $px + q = r$.
7.EE.B.4, 7.EE.B.4.A

Next
Students will write and solve two-step equations of the form $p(x + q) = r$.
7.EE.B.4, 7.EE.B.4.A

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of solving one-step equations to develop an *understanding* of solving two-step equations of the form $px + q = r$. They will *apply* this understanding to write and solve two-step equations in the form $px + q = r$ with real-world problems.

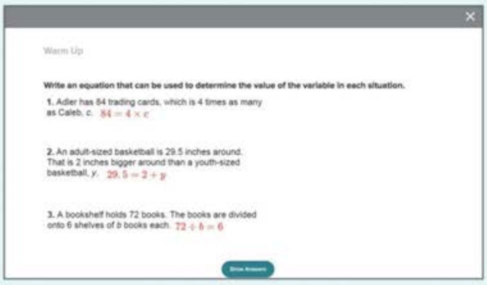
Mathematical Background

A *two-step equation*, such as $5x + 3 = 13$, has two different operations performed on the variable. In this case, the operations are multiplication and addition. To solve a two-step equation, undo the operations in reverse order of the order of operations.

First, undo the addition or subtraction: $5x + 3 - 3 = 13 - 3 \rightarrow 5x = 10$.

Then undo the multiplication or division: $\frac{5x}{5} = \frac{10}{5} \rightarrow x = 2$.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing one-step equations (Exercises 1–3)

Answers

1. $84 = 4 \cdot c$

2. $29.5 = 2 + y$

3. $72 \div b = 6$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an equation to determine the specifics of joining the ski club.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Why do you think the *order* in which operations are performed is important? Sample answer: If the order in which operations are performed in an expression changes, the value of the expression may also change.
- A *one-step equation* has one operation paired with the variable. What do you think will be true about a *two-step equation*? Sample answer: I think a two-step equation will have two operations paired with the variable.

Explore Solve Two-Step Equations with Parentheses Using Algebra Tiles

Objective

Students will use algebra tiles to explore how to model and solve two-step equations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with an algebra tiles tool that includes an equation mat and four types of algebra tiles: an x-tile, $-x$-tile, 1-tile, and -1-tile. Throughout this activity students will use the algebra tiles tool to model and solve two-step equations.

Inquiry Question

How can algebra tiles help you solve equations that involve two operations? Sample answer: Algebra tiles provide a visual aid when deciding the steps needed to solve the equation.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

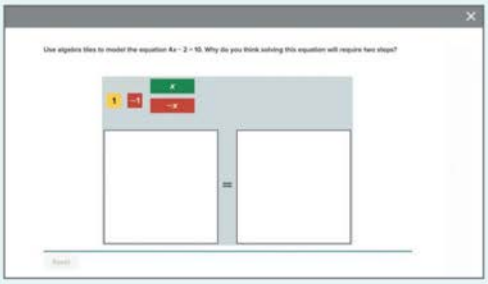
Mathematical Discourse

Make a plan to solve the equation using algebra tiles. Watch the video if you need help. Sample answer: Add two 1-tiles on each side of the mat to form zero pairs on the left side, and then remove the zero pairs. Then separate the tiles into four equal groups.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 5



Explore, Slide 2 of 5

DRAG & DROP

Throughout the Explore, students drag algebra tiles to solve equations that involve two operations.

Interactive Presentation

Explore, Slide 4 of 5

CLICK

On Slide 4, students move through the slides to practice solving equations using algebra tiles.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Solve Two-Step Equations Using Algebra Tiles *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically E xplain to students the benefit of using algebra tiles is that they can manipulate the tiles and visualize the results when solving equations.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Describe the steps you took to solve the equations and explain why that method works. Sample answer: Model the equations, then form zero pairs on the left side of the mat to isolate the *x*-tiles. Then separate the tiles into equal groups. You can add or subtract zero pairs from either side of an equation without changing its value, and you can use the properties of equality to separate tiles into groups on each side of the mat.

Did you use any of the Properties of Equality in your solution? Explain. Yes; Sample answer: the Subtraction Property of Equality to remove the same number of 1-tiles from each side of the mat; the Division Property of Equality to separate the tiles into equal groups

Learn Two-Step Equations

Objective

Students will learn how to solve two-step equations.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, encourage them to use mathematical reasoning to formulate their comparisons.

Teaching Notes

SLIDE 1

Students will learn how to solve a two-step equation. Play the animation for the class. Ask students to list the general steps for solving a two-step equation. Students should note they must first undo the addition or subtraction, then undo the multiplication or division, and finally check their solution. Point out that these steps are valid for two-step equations without parentheses. As a general rule, tell students to undo the operations in an expression in the reverse order that they would use to evaluate the expression.

Talk About It!

SLIDE 2

Mathematical Discourse

Compare and contrast the equations $5x + 3 = 13$ and $5x = 10$.
Sample answer: They are equivalent equations because both have the same solution, $x = 2$. The first equation is a two-step equation, and the second equation is a one-step equation.

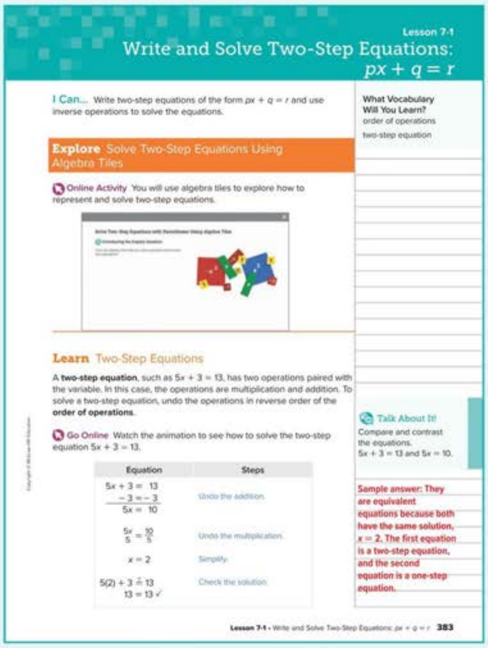
Go Online to have students watch the animation on Slide 1. The animation illustrates how to solve a two-step equation.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of how to use models to solve two-step equations of the form $px + q = r$, have them work with a partner to complete the following activity.

Write a real-world problem in which two operations are needed to find the solution. One of the operations should be addition or subtraction. The other operation should be multiplication or division. Trade problems with another pair of students. Have each pair use their own strategy to solve the problem and be prepared to defend how their strategy works. Have pairs determine if either a bar diagram or algebra tiles, or both, can be used to model and solve the problem. Have them compare and contrast all of the stregies and determine if there are any correspondences between them.


Lesson 7-1

Write and Solve Two-Step Equations: $px + q = r$

I Can... Write two-step equations of the form $px + q = r$ and use inverse operations to solve the equations.

What Vocabulary Will You Learn?
order of operations
two-step equation

Explore Solve Two-Step Equations Using Algebra Tiles

Online Activity You will use algebra tiles to explore how to represent and solve two-step equations.

Learn Two-Step Equations

A **two-step equation**, such as $5x + 3 = 13$, has two operations paired with the variable. In this case, the operations are multiplication and addition. To solve a two-step equation, undo the operations in reverse order of the **order of operations**.

Go Online Watch the animation to see how to solve the two-step equation $5x + 3 = 13$.

Equation	Steps
$5x + 3 = 13$ $-3 = -3$ $5x = 10$	Undo the addition.
$\frac{5x}{5} = \frac{10}{5}$	Undo the multiplication.
$x = 2$	Simplify.
$5(2) + 3 \stackrel{?}{=} 13$ $13 = 13$ ✓	Check the solution.

Talk About It! Compare and contrast the equations. $5x + 3 = 13$ and $5x = 10$.

Sample answer: They are equivalent equations because both have the same solution, $x = 2$. The first equation is a two-step equation, and the second equation is a one-step equation.

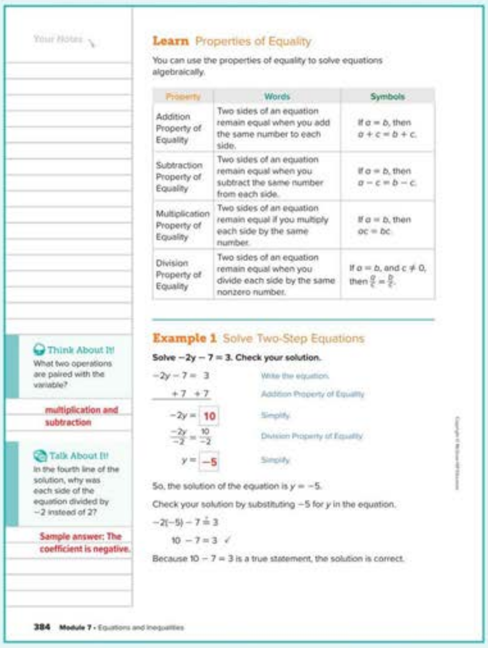
Lesson 7-1 • Write and Solve Two-Step Equations: $px + q = r$ 383

Interactive Presentation

Learn, Two-Step Equations, Slide 1 of 2

WATCH

On Slide 1 of the Learn, students watch an animation to learn how to solve a two-step equation.



Your Notes

Learn Properties of Equality

You can use the properties of equality to solve equations algebraically.

Property	Words	Symbols
Addition Property of Equality	Two sides of an equation remain equal when you add the same number to each side.	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	Two sides of an equation remain equal when you subtract the same number from each side.	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	Two sides of an equation remain equal if you multiply each side by the same number.	If $a = b$, then $ac = bc$.
Division Property of Equality	Two sides of an equation remain equal when you divide each side by the same nonzero number.	If $a = b$, and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Example 1 Solve Two-Step Equations

Solve $-2y - 7 = 3$. Check your solution.

$-2y - 7 = 3$ Write the equation.

$+7 \quad +7$ Addition Property of Equality

$-2y = 10$ Simplify.

$\frac{-2y}{-2} = \frac{10}{-2}$ Division Property of Equality

$y = -5$ Simplify.

So, the solution of the equation is $y = -5$.

Check your solution by substituting -5 for y in the equation.

$-2(-5) - 7 \stackrel{?}{=} 3$

$10 - 7 = 3$ ✓

Because $10 - 7 = 3$ is a true statement, the solution is correct.

Think About It! What two operations are paired with the variable? multiplication and subtraction

Talk About It! In the fourth line of the solution, why was each side of the equation divided by -2 instead of 2? Sample answer: The coefficient is negative.

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Interactive Presentation

Example 1, Solve Two-Step Equations, Slide 2 of 4

TYPE

On Slide 2 of Example 1, students enter the missing value to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Properties of Equality

Objective

Students will learn about the properties of equality.

Teaching Notes

SLIDES 1-4

Students will learn about the Addition, Subtraction, Multiplication, and Division Properties of Equality. Have them select each flashcard to view how each property can be represented in words and symbolically. Prior to selecting each Symbols flashcard, have students make a conjecture as to what equation will be shown for each property. Then have them select the flashcard to verify their thinking.

Example 1 Solve Two-Step Equations

Objective

Students will solve two-step equations of the form $px - q = r$ with integers.

Questions for Mathematical Discourse

SLIDE 2

AL What operations are paired with the variable? multiplication and subtraction

AL According to the order of operations, which operation would be performed first if you were evaluating the expression $-2y - 7$? Explain. multiplication; Multiplication is performed before subtraction in the order of operations.

OL Explain how to isolate the variable. Sample answer: Undo the operations in the reverse order of the order of operations. So, undo the subtraction of 7 by adding 7 to each side. Then undo the multiplication by dividing each side of the equation by -2.

OL How can you check your answer? Substitute $y = -5$ into the original equation to verify the statement $-2(-5) - 7 = 3$ is true, which it is.

BL If $-2y - 7 = 3$, what does $-2y - 10$ equal? Explain without calculating the value of y. 0; Sample answer: Because 3 is subtracted from one side of the equation ($-2y - 7 - 3 = -2y - 10$), subtract 3 from the other side of the equation. Because $3 - 3 = 0$, then $-2y - 10 = 0$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Solve Two-Step Equations

Objective

Students will solve two-step equations with fractional coefficients.

MP Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to pay careful attention to the coefficient in a two-step equation when the coefficient is a fraction. Ask students to explain each step as they find the solution, encouraging them to use mathematical language, such as *reciprocal* and *inverse operation*.

Questions for Mathematical Discourse

SLIDE 2

AL What operations are paired with the variable? multiplication and addition

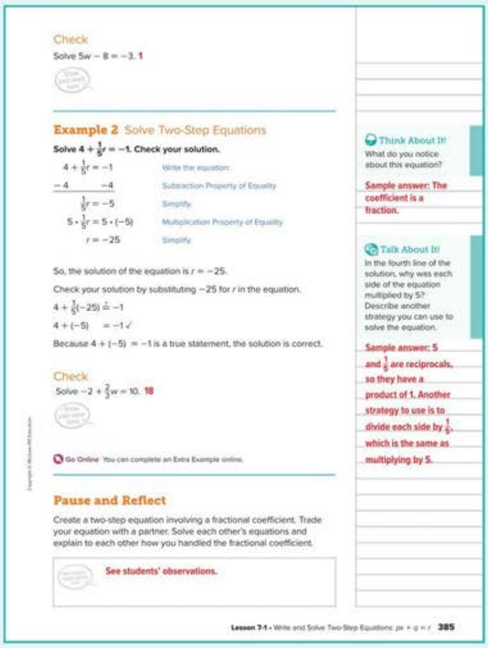
AL According to the order of operations, which operation would be performed first if you were evaluating the expression $4 + \frac{1}{5}r$? Explain. multiplication; Multiplication is performed before addition in the order of operations.

OL How can you write this equation so that the two operations paired with the variable are division and addition? Explain. How does rewriting the equation affect the solution process? Sample answer: I can write the equation as $4 + \frac{r}{5} = -1$, because multiplying r by $\frac{1}{5}$ is the same as dividing r by 5. To solve this equation, subtract 4 from each side. Then multiply each side by 5.

BL A classmate stated that you can multiply each side of the equation by 5 to eliminate the fraction. Is this method correct? Explain. yes; Sample answer: Because the denominator is 5, multiplying both sides by 5 yields the resulting equation $20 + r = -5$. The fraction is eliminated, and I can solve the equation by subtracting 20 from each side. So, $r = -25$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

Solve $5w - 8 = -3$. 1

Example 2 Solve Two-Step Equations

Solve $4 + \frac{1}{5}r = -1$. Check your solution.

$4 + \frac{1}{5}r = -1$ Write the equation.

$-4 \quad -4$ Subtraction Property of Equality

$\frac{1}{5}r = -5$ Simplify.

$5 \cdot \frac{1}{5}r = 5 \cdot (-5)$ Multiplication Property of Equality

$r = -25$ Simplify.

So, the solution of the equation is $r = -25$.

Check your solution by substituting -25 for r in the equation.

$4 + \frac{1}{5}(-25) \stackrel{?}{=} -1$

$4 + (-5) = -1$ ✓

Because $4 + (-5) = -1$ is a true statement, the solution is correct.

Check

Solve $-2 + \frac{2}{3}w = 10$. 18

Go Online You can complete an Extra Example online.

Think About It! What do you notice about this equation? Sample answer: The coefficient is a fraction.

Talk About It! In the fourth line of the solution, why was each side of the equation multiplied by 5? Describe another strategy you can use to solve the equation. Sample answer: 5 and $\frac{1}{5}$ are reciprocals, so they have a product of 1. Another strategy to use is to divide each side by $\frac{1}{5}$, which is the same as multiplying by 5.

Pause and Reflect

Create a two-step equation involving a fractional coefficient. Trade your equation with a partner. Solve each other's equations and explain to each other how you handled the fractional coefficient.

See students' observations.

Lesson 7-1 • Write and Solve Two-Step Equations: $px + q = r$ 385

Interactive Presentation

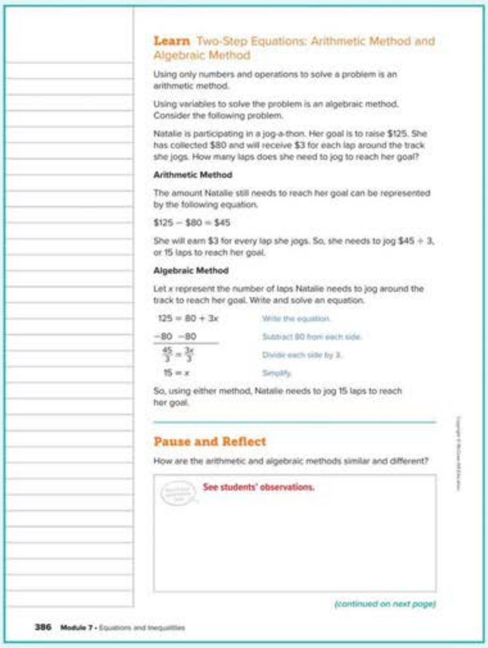
Example 3, Solve Two-Step Equations, Slide 2 of 4

TYPE

On Slide 2, students enter the missing values to solve the eqation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Two-Step Equations: Arithmetic Method and Algebraic Method

Using only numbers and operations to solve a problem is an arithmetic method.

Using variables to solve the problem is an algebraic method. Consider the following problem.

Natalie is participating in a jog-a-thon. Her goal is to raise $125. She has collected $80 and will receive $3 for each lap around the track she jogs. How many laps does she need to jog to reach her goal?

Arithmetic Method

The amount Natalie still needs to reach her goal can be represented by the following equation.

$125 – $80 = $45

She will earn $3 for every lap she jogs. So, she needs to jog $45 ÷ 3, or 15 laps to reach her goal.

Algebraic Method

Let x represent the number of laps Natalie needs to jog around the track to reach her goal. Write and solve an equation.

$125 = 80 + 3x$ Write the equation.

$-80 \quad -80$ Subtract 80 from each side.

$\frac{45}{3} = \frac{3x}{3}$ Divide each side by 3.

$15 = x$ Simplify.

So, using either method, Natalie needs to jog 15 laps to reach her goal.

Pause and Reflect

How are the arithmetic and algebraic methods similar and different?

See students' observations.

(continued on next page)

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Interactive Presentation

Learn, Two-Step Equations: Arithmetic Method and Algebraic Method, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to compare the Arithmetic Method and the Algebraic Method.

Learn Two-Step Equations: Arithmetic Method and Algebraic Method

Objective

Students will understand how the arithmetic method and algebraic method of solving a two-step equation compare.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to understand the two different methods for solving the problem, and why each approach leads to the correct solution. They should be able to explain how the information can be decontextualized in order to represent it with an equation.

Teaching Notes

SLIDE 1

Students will learn about the arithmetic method and the algebraic method for solving problems. Select the flashcards to show how the given problem can be solved using each method.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

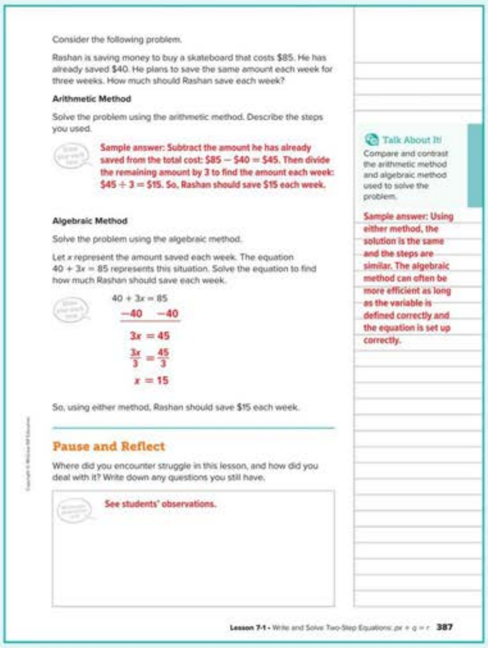
Learn Two-Step Equations: Arithmetic Method and Algebraic Method *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

Compare and contrast the arithmetic method and algebraic method used to solve the problem. Sample answer: Using either method, the solution is the same and the steps are similar. The algebraic method can often be more efficient as long as the variable is defined correctly and the equation is set up correctly.



Consider the following problem.

Rashan is saving money to buy a skateboard that costs $85. He has already saved $40. He plans to save the same amount each week for three weeks. How much should Rashan save each week?

Arithmetic Method

Solve the problem using the arithmetic method. Describe the steps you used.

Sample answer: Subtract the amount he has already saved from the total cost: $85 − $40 = $45. Then divide the remaining amount by 3 to find the amount each week: $45 ÷ 3 = $15. So, Rashan should save $15 each week.

Algebraic Method

Solve the problem using the algebraic method.

Let x represent the amount saved each week. The equation $40 + 3x = 85$ represents this situation. Solve the equation to find how much Rashan should save each week.

$$40 + 3x = 85$$
$$-40 \quad -40$$
$$3x = 45$$
$$\frac{3x}{3} = \frac{45}{3}$$
$$x = 15$$

So, using either method, Rashan should save $15 each week.

Talk About It! Compare and contrast the arithmetic method and algebraic method used to solve the problem.

Sample answer: Using either method, the solution is the same and the steps are similar. The algebraic method can often be more efficient as long as the variable is defined correctly and the equation is set up correctly.

Pause and Reflect

Where did you encounter struggle in this lesson, and how did you deal with it? Write down any questions you still have.

See students' observations.

Lesson 7-1 • Write and Solve Two-Step Equations: $px + q = r$ 387

DIFFERENTIATE

Language Development Activity ELL

Some students may struggle identifying the operations that are paired with the variable, and in which order to undo them. They may also struggle to identify which property of equality allows them to undo those operations. Have students work with a partner to study the following two equations. Have them cover up the variable and any multiplication or division that is paired with that variable using a slip of paper. Then ask them which operation remains. That is the operation that should be undone first using the appropriate property of equality. Have them determine the properties of equality that can be used to solve each equation, and in which order to perform which operations.

$\frac{x}{5} + 2 = 16$

1. Subtraction Property of Equality; Subtract 2 from each side.

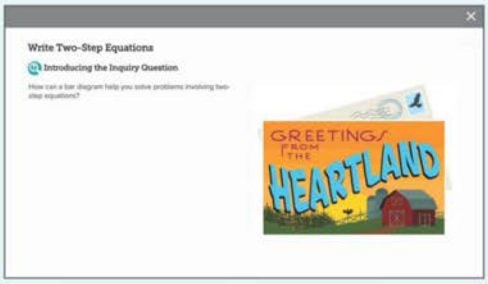
2. Multiplication Property of Equality; Multiply each side by 5.

$8t - 4 = 10$

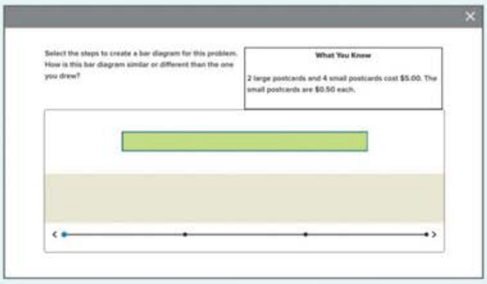
1. Addition Property of Equality; Add 4 to each side.

2. Division Property of Equality; Divide each side by 8.

Interactive Presentation



Explore, Slide 1 of 7



Explore, Slide 4 of 7

CLICK

On Slide 4, students move through the steps to create a bar diagram for the problem.

Explore Write Two-Step Equations

Objective

Students will use bar diagrams to explore how to write two-step equations.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a real-world problem. Throughout this activity, students will investigate how a bar diagram and an equation can be used to represent and solve the problem.

Inquiry Question

How can a bar diagram help you solve problems involving two-step equations? Sample answer: I can visualize all parts of the problem using a bar diagram and then use the bar diagram to write an equation. I can then solve the equation arithmetically or algebraically.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

How do you know what size to make each section in the bar diagram? Sample answer: The cost of one small postcard is $0.50 so four of them cost $2.00. Since $2.00 is less than half of the total cost of $5.00, the sections for the cost of the small postcards needs to be smaller than the sections for the cost of the large postcards.

How can you use the bar diagram to find the cost of one large postcard? Sample answer: Subtract the cost of the 4 small postcards from the total cost to find the cost of the 2 large postcards. Then divide that cost by 2 to find the cost of one large postcard. So, each postcard costs $1.50.

Where is the solution on the bar diagram? The solution is shown on each section of the large postcards.

(continued on next page)

Explore Write Two-Step Equations *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to think about the meaning of the different sections of the bar diagram and how the diagram can help when writing two-step equations.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

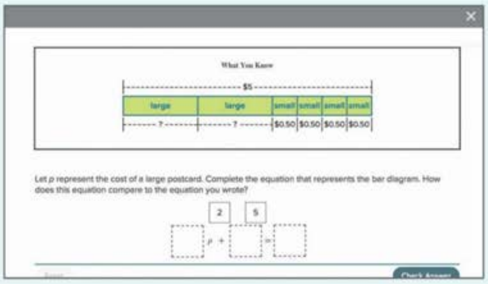
SLIDE 5

Mathematical Discourse

Share your equation with a partner. How is each part of the equation illustrated on the bar diagram? Sample answer: $2x + 2 = 5$; x represents the cost of one large postcard; \$2 represents the cost of the 4 small postcards; \$5 represents the total cost.

Are there different equations you could write? If so, how are they similar and different? Sample answer: Other equations could be non-simplified forms of $2x + 2 = 5$, such as $x + x + 2 = 5$ or $x + x + 0.5 + 0.5 + 0.5 + 0.5 = 5$. They all represent the same situation and have the same solution, but are written in different forms.

Interactive Presentation



Explore, Slide 6 of 7

TYPE

On Slide 5, students type to explain how the bar diagram helped them write an equation.

DRAG & DROP

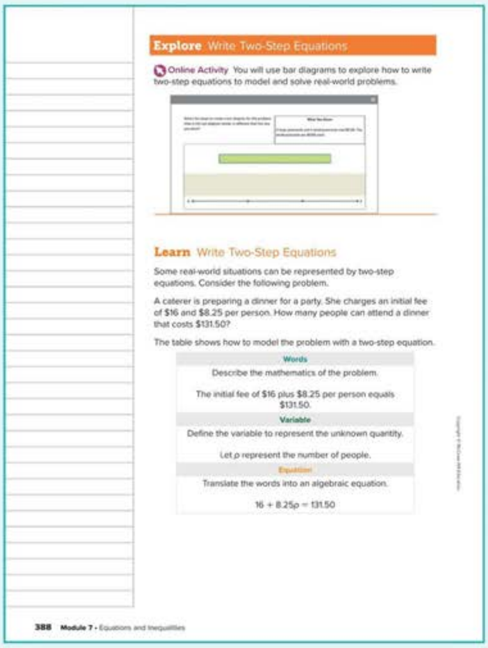
On Slide 6, students drag to complete an equation represented by the bar diagram.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

7.EE.B.4, 7.EE.B.4.A

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Explore Write Two-Step Equations

Online Activity You will use bar diagrams to explore how to write two-step equations to model and solve real-world problems.

Learn Write Two-Step Equations

Some real-world situations can be represented by two-step equations. Consider the following problem.

A caterer is preparing a dinner for a party. She charges an initial fee of \$16 and \$8.25 per person. How many people can attend a dinner that costs \$131.50?

The table shows how to model the problem with a two-step equation.

Words
Describe the mathematics of the problem.
The initial fee of \$16 plus \$8.25 per person equals \$131.50.
Variable
Define the variable to represent the unknown quantity.
Let p represent the number of people.
Equation
Translate the words into an algebraic equation.
$16 + 8.25p = 131.50$

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Interactive Presentation

Learn, Write Two-Step Equations

FLASHCARDS

On Slide 1, students will select the Flashcards to see the steps for modeling a real-world problem with a two-step equation.

Learn Write Two-Step Equations

Objective

Students will learn how to model a real-world problem with a two-step equation of the form $px + q = r$.

Teaching Notes

SLIDE 1

Have students select each flashcard to view the steps for modeling a real-world problem with a two-step equation of the form $px + q = r$. An important step in writing the equation is to define the variable. Remind students that the variable can be any letter. In this case, p is used because the variable represents the number of people. To avoid confusion, point out that if p is used to represent the variable, it will replace x in the general form of a two-step equation, $px + q = r$. The letter p in the equation $px + q = r$ is the coefficient of the variable x.

DIFFERENTIATE

Reteaching Activity AL

To help students that may be struggling to model real-world problems with two-step equations, have them first review how to model a real-world problem with a one-step equation. Then adjust the real-world scenario in order to add in the additional operation that will make it a two-step problem. Have them work with a partner to complete the following activity.

1. Present them with the following one-step problem: The cost of renting a jet ski at a local marina is \$18 per hour. If Jackson spent a total of \$54, for how many hours did he rent the jet ski?
2. Have them model the problem with a one-step equation. Be sure they first define the variable. Sample answer: Let h represent the number of hours Jackson rented the jet ski; $18h = 54$
3. Have them adjust part of the problem so that the local marina also charges an application fee on top of the hourly rental fee. Sample answer: The marina charges an application fee of \$5.
4. Have them discuss what this would mean for Jackson's total cost. It would increase from \$54 to \$59.
5. Have them adjust their one-step equation so that it now models the new scenario, and becomes a two-step equation. Sample answer: $18h + 5 = 59$

Example 3 Write and Solve Two-Step Equations

Objective

Students will write and solve two-step equations of the form $px + q = r$.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to use a bar diagram to help make sense of the important quantities in the real-world problem. Students will decontextualize the information by representing it symbolically using a two-step equation.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown quantity you need to find? How will you represent this in the equation? the number of friends at the party; with a variable

OL Why is the expression that represents the ticket cost $8.50n$? Sample answer: The ticket cost for one friend is \$8.50 and there are n friends. To find the total cost, multiply \$8.50 by n.

OL A classmate wrote the equation $8.50n + 27 = 78$. Is this correct? Explain. yes; Sample answer: Addition is commutative, so the terms $8.50n$ and 27 can be added in any order.

BL A classmate wrote the equation $8.50n = 78 - 27$. Is this correct? Explain. yes; Sample answer: The cost of the pizza will eventually be subtracted from the total cost, so it is correct to set it up as a subtraction expression from the start.

SLIDE 3

AL What operations are paired with the variable? addition and multiplication

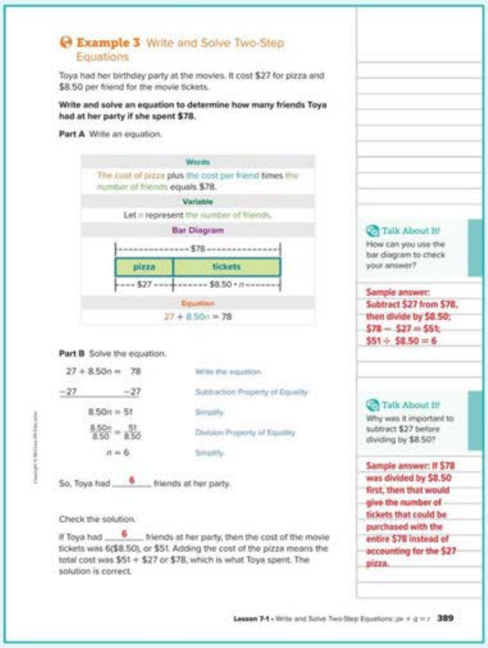
OL Explain how to isolate the variable. Sample answer: Subtract 27 from each side. Then divide each side by 8.50.

OL How can you check your answer? Sample answer: Replace n with 6 into the original equation and verify that the statement $27 + 8.50(6) = 78$ is true, which it is.

BL Suppose next year, Toya wants to invite the same number of friends to her party, but the price of each ticket increases by \$2.50. If the cost of the pizza remains the same, how much will Toya need to spend? $\$27 + \$11(6) = \$93$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Write and Solve Two-Step Equations

Toya had her birthday party at the movies. It cost \$27 for pizza and \$8.50 per friend for the movie tickets.

Write and solve an equation to determine how many friends Toya had at her party if she spent \$78.

Part A Write an equation.

Words

The cost of pizza plus the cost per friend times the number of friends equals \$78.

Variable

Let n represent the number of friends.

Bar Diagram

Equation

$27 + 8.50n = 78$

Part B Solve the equation.

$27 + 8.50n = 78$	Write the equation.
$-27 \quad -27$	Subtraction Property of Equality
$8.50n = 51$	Simplify.
$\frac{8.50n}{8.50} = \frac{51}{8.50}$	Division Property of Equality
$n = 6$	Simplify.

So, Toya had 6 friends at her party.

Check the solution.

If Toya had 6 friends at her party, then the cost of the movie tickets was 6(\$8.50), or \$51. Adding the cost of the pizza means the total cost was \$51 + \$27 or \$78, which is what Toya spent. The solution is correct.

Talk About It! How can you use the bar diagram to check your answer?

Sample answer: Subtract \$27 from \$78, then divide by \$8.50; \$78 − \$27 = \$51; \$51 ÷ \$8.50 = 6

Talk About It! Why was it important to subtract \$27 before dividing by \$8.50?

Sample answer: If \$78 was divided by \$8.50 first, then that would give the number of tickets that could be purchased with the entire \$78 instead of accounting for the \$27 pizza.

Lesson 7-1 • Write and Solve Two-Step Equations: $px + q = r$ 389

Interactive Presentation

Example 3, Write and Solve Two-Step Equations, Slide 2 of 5

FLASHCARDS

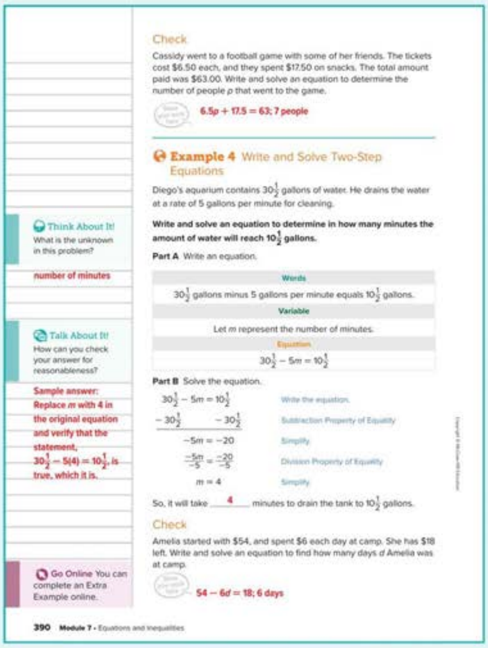
On Slide 2, students use Flashcards to see the steps for writing the equation.

CLICK

On Slide 3, students move through the steps to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Cassidy went to a football game with some of her friends. The tickets cost \$6.50 each, and they spent \$17.50 on snacks. The total amount paid was \$63.00. Write and solve an equation to determine the number of people p that went to the game.

$6.5p + 17.5 = 63$; 7 people

Example 4 Write and Solve Two-Step Equations

Diego's aquarium contains $30\frac{1}{2}$ gallons of water. He drains the water at a rate of 5 gallons per minute for cleaning.

Write and solve an equation to determine in how many minutes the amount of water will reach $10\frac{1}{2}$ gallons.

Think About It! What is the unknown in this problem? number of minutes

Part A Write an equation.

Words
$30\frac{1}{2}$ gallons minus 5 gallons per minute equals $10\frac{1}{2}$ gallons.
Variable
Let m represent the number of minutes.
Equation
$30\frac{1}{2} - 5m = 10\frac{1}{2}$

Part B Solve the equation.

$30\frac{1}{2} - 5m = 10\frac{1}{2}$ Write the equation.

$-30\frac{1}{2} \quad -30\frac{1}{2}$ Subtraction Property of Equality

$-5m = -20$ Simplify.

$\frac{-5m}{-5} = \frac{-20}{-5}$ Division Property of Equality

$m = 4$ Simplify.

So, it will take 4 minutes to drain the tank to $10\frac{1}{2}$ gallons.

Talk About It! How can you check your answer for reasonableness? Sample answer: Replace m with 4 in the original equation and verify that the statement, $30\frac{1}{2} - 5(4) = 10\frac{1}{2}$, is true, which it is.

Check

Amelia started with \$54, and spent \$6 each day at camp. She has \$18 left. Write and solve an equation to find how many days d Amelia was at camp.

$54 - 6d = 18$; 6 days

Go Online You can complete an Extra Example online.

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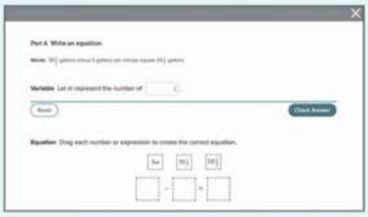
Interactive Presentation



Example 4, Write and Solve Two-Step Equations, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag to create the correct equation.

CLICK

On Slide 3, students enter the missing values to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Example 4 Write and Solve Two-Step Equations

Objective

Students will write and solve two-step equations of the form $px + q = r$ with negative coefficients.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the important quantities in the real-world problem in order to abstract them and represent them symbolically.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown quantity you need to find? How will you represent this in the equation? in how many minutes will the amount of water reach $10\frac{1}{2}$ gallons; with a variable

OL Why is the expression that represents the starting amount minus the amount that is draining equal to $30\frac{1}{2} - 5m$? Sample answer: The aquarium originally has $30\frac{1}{2}$ gallons of water and is draining at the rate of 5 gallons per minute m.

OL A classmate wrote the equation $5m - 30\frac{1}{2} = 10\frac{1}{2}$. Is this correct? Explain. no; Sample answer: Subtraction is not commutative, so the terms $30\frac{1}{2}$ and $5m$ cannot be subtracted in any order.

BL A classmate wrote the equation $10\frac{1}{2} + 5m = 30\frac{1}{2}$. Is this correct? Explain. yes; Sample answer: If you start with the final amount of water and add the rate at which the water had been drained times the number of minutes, you will end up with the original amount of water.

SLIDE 3

AL What operations are paired with the variable? Explain. addition and multiplication; The variable m is multiplied by -5 and added to $30\frac{1}{2}$.

OL Explain how to isolate the variable. Sample answer: Subtract $30\frac{1}{2}$ from each side. Then divide both sides by -5.

OL Why do you divide each side by -5 instead of 5? Sample answer: Because $5m$ is being subtracted, the coefficient of m is -5, not 5.

BL How many gallons were drained in 4 minutes? 20 gallons

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Budgets

Objective

Students will come up with their own strategy to solve an application problem involving the cost to rent a moon bounce.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

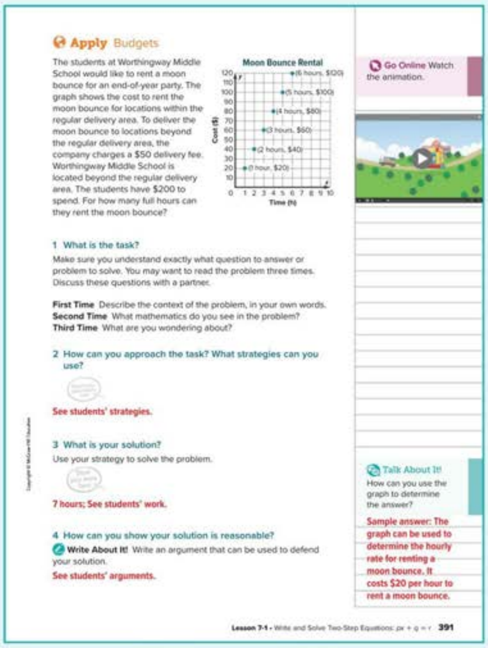
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you determine the hourly charge?
- What other costs, besides the hourly charge do you need to include?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Budgets

The students at Worthingway Middle School would like to rent a moon bounce for an end-of-year party. The graph shows the cost to rent the moon bounce for locations within the regular delivery area. To deliver the moon bounce to locations beyond the regular delivery area, the company charges a $50 delivery fee. Worthingway Middle School is located beyond the regular delivery area. The students have $200 to spend. For how many full hours can they rent the moon bounce?

Go Online Watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

7 hours; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How can you use the graph to determine the answer?

Sample answer: The graph can be used to determine the hourly rate for renting a moon bounce. It costs $20 per hour to rent a moon bounce.

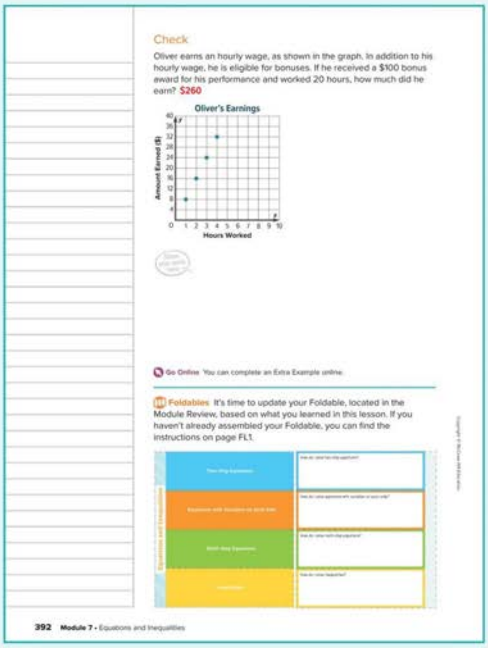
Lesson 7-1 • Write and Solve Two-Step Equations: $px + q = r$ 391

Interactive Presentation

Apply, Budgets

WATCH Students watch an animation that illustrates the problem they are about to solve.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. The cost to join the Ski Club is $270. You paid a deposit of $95.50 and will save an additional $20 per week to pay for the cost. The equation $95.5 + 20w = 270$ can be used to find the number of weeks w you will need to save. Solve the equation and interpret the solution within the context of the problem. Describe the steps you used. $w = 8.725$; I will need to save for 9 weeks; Sample answer: First, use the Subtraction Property of Equality to subtract 95.5 from each side; $20w = 174.5$. Then use the Division Property of Equality to divide each side of the equation by 20; $w = 8.725$.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information on solving two-step equations of the form $px + q = r$. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can equations be used to solve everyday problems?

In this lesson, students learned how to write and solve a two-step equation, of the form $px + q = r$ that represents a real-world problem involving rational numbers. Encourage them to brainstorm an example of a problem arising in everyday life in which they can use a two-step equation of this form to model and solve the problem. For example, the equation $32.50p + 5 = 167.50$ can model the cost of attending a waterpark in which the cost per person p is $32.50 and the parking fee is $5.00.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve two-step equations of the form $px + q = r$	1–6
1	write and solve two-step equations of the form $px + q = r$	7–10
2	write and solve two-step equations of the form $px + q = r$	7–10
2	extend concepts learned in class to apply them in new contexts	11
3	solve application problems involving writing and solving two-step equations of the form $px + q = r$	12, 13
3	higher-order and critical thinking skills	14–17

Common Misconception

Students may struggle to identify the correct value of p and the correct value of q when writing the equation for each exercise. For example, in Exercise 7, students may incorrectly write the equation as $15p + 29.50 = 133$. Remind them to look for key words in the problem. The phrase "a total of $15 spent on parking" means that there was a one-time parking fee of $15, which is added on after finding the total cost of the tickets. The phrase "$29.50 each" means that for each person who attends the concert, there is a ticket price of $29.50, so 29.50 will need to be multiplied by the number of people, p.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each equation. Check your solution. (Examples 1–3)

1. $5x + 2 = 17$ — $x = 3$
2. $-6x - 7 = 17$ — $x = -4$
3. $-5 = 3x - 14$ — $x = 3$
4. $3.8 = 2x - 11.2$ — $x = 7.5$
5. $2 + \frac{1}{6}x = -4$ — $x = -36$
6. $-9 = \frac{2}{7}x + 5$ — $x = -49$

Write and solve an equation for each exercise. Check your solution. (Examples 4 and 5)

7. Easton went to a concert with some of his friends. The tickets cost $29.50 each, and they spent a total of $15 on parking. The total amount spent was $133. Determine how many people went to the concert.
   $29.50p + 15 = 133$; 4 people
8. Ishi bought a $6.95 canvas and 8 tubes of paint. She spent a total of $24.95 on the canvas and paints. Determine the cost of each tube of paint.
   $6.95 + 8c = 24.95$; $2.25
9. A hot air balloon is at an altitude of $100\frac{1}{5}$ yards. The balloon's altitude decreases by $10\frac{4}{5}$ yards every minute. Determine the number of minutes it will take the balloon to reach an altitude of 57 yards.
   $100\frac{1}{5} - 10\frac{4}{5}m = 57$; 4 min
10. The current temperature is 48°F. It is expected to drop 1.5°F each hour. Determine in how many hours the temperature will be 36°F.
   $48 - 1.5h = 36$; 8 h

Test Practice

11. **Open Response** The table shows the costs of a membership and fruit baskets at a discount warehouse club. Mrs. Williams paid a total of $105 for her annual membership fee and several fruit baskets as gifts for her coworkers. Solve the equation $15x + 30 = 105$ to find the number of fruit baskets Mrs. Williams purchased.

Item	Cost ($)
Membership Fee	30
Fruit Basket	15

5 fruit baskets

Lesson 7-1 • Write and Solve Two-Step Equations: $px + q = r$ 393

Apply *indicates multi-step problem

12. A face painting artist's fee for parties is shown in the table. Customers are also charged a $25 reservation fee. A school has $175 to spend on a face painting artist for their carnival. For how many full hours can they hire the artist?

Hours	Fee ($)
2	49.00
3	73.50
4	98.00
5	122.50

6 hours

13. The table shows the cost of boarding a dog at a dog kennel. Owners are also charged a $10 registration fee. The Kittle family boarded their dog at the kennel for 7 days. What was the total, with registration fee, of boarding their dog?

Days	Cost ($)
3	97.50
4	130.00
5	162.50
6	195.00

$237.50

Higher-Order Thinking Problems

14. Write a real-world problem that could be represented by the equation $2x + 5 = 35$. Then solve the equation.

Sample answer: You and your friend spent a total of $35 on dinner. Your dinners were the same cost and you ordered a $5 dessert. What was the cost of your dinner?; $15

15. MP **Persevere with Problems** Mia discovered that if she takes half her age and adds 3, it produces the same results as taking one-fourth of her age and adding 7. How old is Mia?

16 years old

16. MP **Multiple Representations** At the start of a school trip, a minibus contains 28 gallons of gas. Each hour, the minibus uses 6 gallons of gas. The bus will stop when there are 4 gallons of gas left.

a. Make a table to show how many gallons of gas are remaining after 1, 2, and 3 hours.

Number of Hours	1	2	3
Gallons Left	22	16	10

b. Write and solve an equation to find how many hours will pass before the minibus will have to stop for gas.

$28 - 6x = 4$; 4 h

17. MP **Find the Error** A student is solving $-5 + 2x = 15$. Find the student's mistake and correct it.

$$-5 + 2x = 15$$
$$-5 + (-5) + 2x = 15 + (-5)$$
$$2x = 10$$
$$x = 5$$

The student added −5 to both sides instead of adding +5 to both sides. The solution is $x = 10$.

394 Module 7 • Equations and Inequalities

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 15, students use multiple steps to write and solve an equation for a real-world problem.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students solve a multi-step word problem involving writing and solving a two-step equation.
In Exercise 17, students find and correct a student's mistake.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Solve the problem another way.
Use with Exercise 15 Have students work in groups of 3–4. After completing Exercise 15, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is viable. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 12, 14–17
- Extension: Consecutive Integers Equations
- ALEKS Multi-Step Equations, Applications of Equations

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–10, 13, 16, 17
- Extension: Consecutive Integers Equations
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS One-Step Equations

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS One-Step Equations

Lesson 7-2 7.EE.B.4, 7.EE.B.4.A

Write and Solve Two-Step Equations: $p(x + q) = r$

LESSON GOAL

Students will write and solve two-step equations of the form $p(x + q) = r$.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Solve Two-Step Equations Using Algebra Tiles

Learn: Two-Step Equations
Examples 1-3: Solve Two-Step Equations
Learn: Two-Step Equations: Arithmetic Method and Algebraic Method

Explore: Write Two-Step Equations

Learn: Write Two-Step Equations
Examples 4-5: Write and Solve Two-Step Equations
Apply: Perimeter

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Systems of Equations		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 39 of the *Language Development Handbook* to help your students build mathematical language related to equations of the form $p(x + q) = r$.

ELL You can use the tips and suggestions on page T39 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7. EE.B** by writing and solving two-step equations of the form $p(x + q) = r$.
Standards for Mathematical Content: **7.EE.B.4, 7.EE.B.4.A,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5**

Coherence

Vertical Alignment

Previous
Students wrote and solved two-step equations of the form $px + q = r$.
7.EE.B.4, 7.EE.B.4.A

Now
Students write and solve two-step equations of the form $p(x + q) = r$.
7.EE.B.4, 7.EE.B.4.A

Next
Students will write and solve equations with variables on each side.
8.EE.C.7, 8.EE.C.7.B

Rigor

The Three Pillars of Rigor

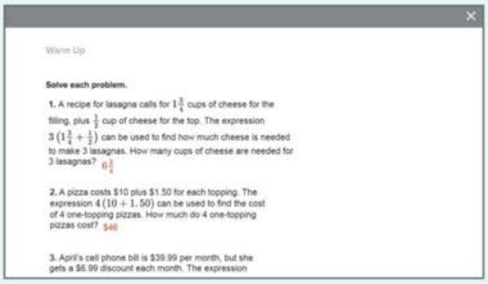
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of solving two-step equations of the form $px + q = r$ and the Distributive Property to develop an *understanding* of solving two-step equations of the form $p(x + q) = r$. They will use this understanding to gain *fluency* in solving two-step equations of the form $p(x + q) = r$ with rational numbers. They will *apply* this understanding to write and solve two-step equations in the form $p(x + q) = r$ with real-world problems.

Mathematical Background

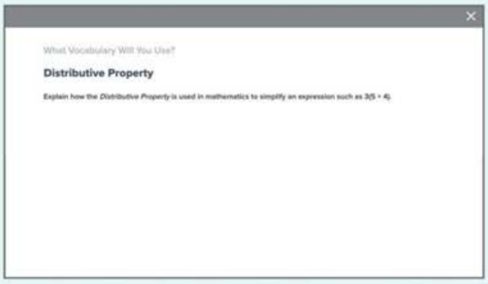
An equation like $2(x + 6) = 14$ is in the form $p(x + q) = r$. It contains two factors, p and $(x + q)$, and is considered a two-step equation. You can solve equations like this using the reverse order of operations and the properties of equality or by using the Distributive Property.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- using the Distributive Property to evaluate numerical expressions containing rational numbers (Exercises 1–3)

Answers

1. $6\frac{3}{4}$
2. $46
3. $396

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an equation to find the cost per game of bowling.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Explain how the *Distributive Property* is used in mathematics to simplify an expression such as $3(5 + 4)$. Sample answer: The term outside the parentheses is multiplied by each term inside the parentheses. So, $3(5 + 4) = 3(5) + 3(4)$, or $15 + 12$, which is 27.

Explore Solve Two-Step Equations Using Algebra Tiles

Objective

Students will use algebra tiles to explore how to model and solve two-step equations with parentheses.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use an algebra tiles tool that includes an equation mat and algebra tiles that represent x, $-x$, 1, and -1. Throughout this activity, students will use the algebra tiles tool to model and solve two-step equations with parentheses.

Inquiry Question

How can algebra tiles help you solve two-step equations containing parentheses? Sample answer: Algebra tiles provide a visual aid when deciding the steps needed to solve the equation.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

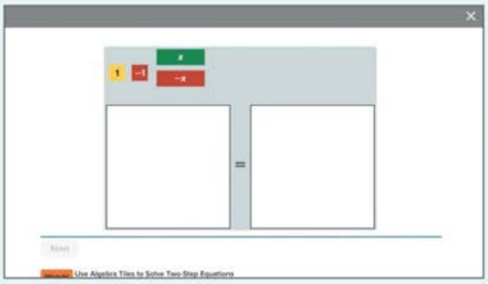
How can you model the equation $2(x + 6) = 16$ using algebra tiles? Sample answer: On the left mat, make 2 groups that each have one x-tile and six 1-tiles. On the right mat, place 16 1-tiles.

Make a plan to solve the equation using algebra tiles. Watch the video if you need help. Sample answer: Divide the tiles into 2 equal groups on each side of the mat. Each group is equal to $x + 6 = 8$. Then solve for x in each group by removing six 1-tiles from each side.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 5

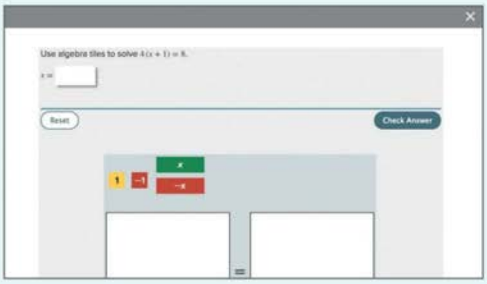


Explore, Slide 2 of 5

DRAG & DROP

On Slide 2 and 3, students drag algebra tiles to solve two-step equations.

Interactive Presentation



Explore, Slide 4 of 5

DRAG & DROP

On Slide 4, students drag algebra tiles to practice solving two-step equations.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Solve Two-Step Equations Using Algebra Tiles *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to think about the meaning of the different colors and sizes of algebra tiles and how the manipulation of them can help when solving a two-step equation containing parentheses.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Did you use the same method to solve each equation? See students' responses.

Describe the method(s) you used to solve the equations and explain why they work. Sample answer: There are two methods to solve the equations using algebra tiles. One method is to first divide the tiles into equal groups, and then solve for x in each group. Another method is to first isolate the x-tiles on one side of the mat by removing tiles and/or creating zero pairs, and then divide the remaining tiles into equal groups. Both methods use the properties of equality to isolate the x-tiles, and I can add or subtract zero pairs from either side of an equation without changing its value.

Will the process you used to solve the previous equations work for solving $4(x + 2) = 15$? Why or why not? Sample answer: No; I cannot separate 15 into 4 equal groups.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Two-Step Equations

Objective

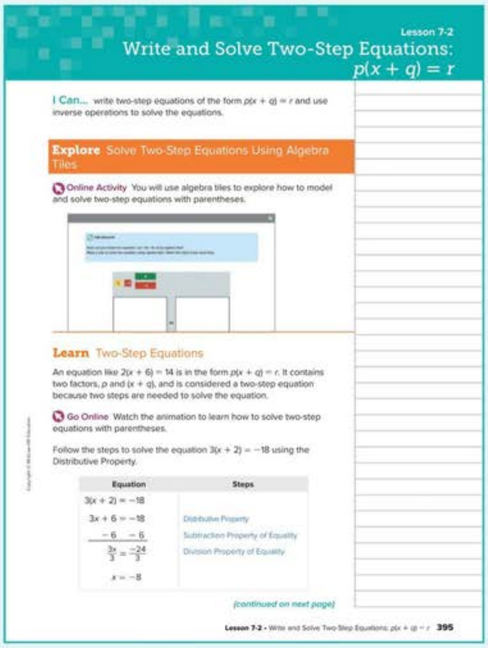
Students will learn how to solve two-step equations of the form $p(x + q) = r$.

Go Online to have students watch the animation on Slide 1. The animation illustrates how to solve a two-step equation of the form $p(x + q) = r$.

Teaching Notes

SLIDE 1

Play the animation for the class. Students will learn how to solve a two-step equation of the form $p(x + q) = r$. Have students compare and contrast the two ways the equation was solved. Students should note that both methods obtained the same correct solution. You may wish to have students discuss which method they prefer.



Lesson 7-2

Write and Solve Two-Step Equations: $p(x + q) = r$

I Can... write two-step equations of the form $p(x + q) = r$ and use inverse operations to solve the equations.

Explore Solve Two-Step Equations Using Algebra Tiles

Online Activity You will use algebra tiles to explore how to model and solve two-step equations with parentheses.

Learn Two-Step Equations

An equation like $2(x + 6) = 14$ is in the form $p(x + q) = r$. It contains two factors, p and $(x + q)$, and is considered a two-step equation because two steps are needed to solve the equation.

Go Online Watch the animation to learn how to solve two-step equations with parentheses.

Follow the steps to solve the equation $3(x + 2) = -18$ using the Distributive Property.

Equation	Steps
$3(x + 2) = -18$	
$3x + 6 = -18$	Distributive Property
$-6 \quad -6$	Subtraction Property of Equality
$\frac{3x}{3} = \frac{-24}{3}$	Division Property of Equality
$x = -8$	

(continued on next page)

Lesson 7-2 • Write and Solve Two-Step Equations: $p(x + q) = r$ 395

Interactive Presentation

Learn, Two-Step Equations

WATCH

On Slide 1, students watch an animation that explains how to solve equations of the form $p(x + q) = r$.

DIFFERENTIATE

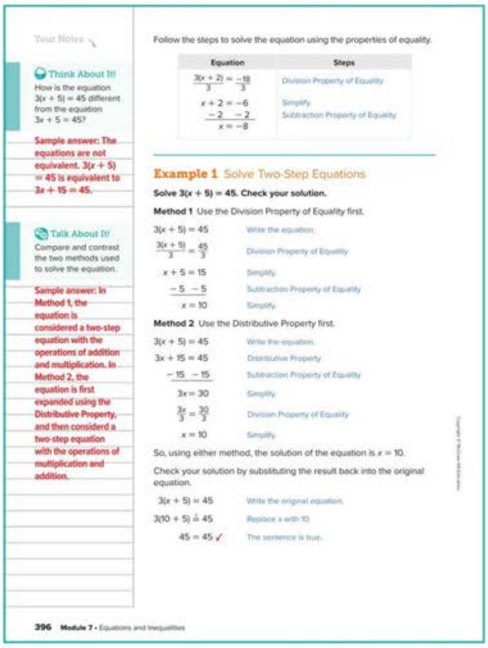
Reteaching Activity AL

If students are struggling with recognizing the difference between equations of the form $px + q = r$ and equations of the form $p(x + q) = r$, have them work with a partner to complete the following activity.

Have students choose a value for each variable p, q, and r and substitute each of those values into both forms of two-step equations. Have students solve each equation to recognize the different values of x.

For example, let $p = 3$, $q = 9$, and $r = 15$.

$px + q = r$	$p(x + q) = r$
$3x + 9 = 15$	$3(x + 9) = 15$
$3x = 6$	$x + 9 = 5$
$x = 2$	$x = -4$



Your Notes

Think About It!
How is the equation $3(x + 5) = 45$ different from the equation $3x + 5 = 45$?

Sample answer: The equations are not equivalent. $3(x + 5) = 45$ is equivalent to $3x + 15 = 45$.

Talk About It!
Compare and contrast the two methods used to solve the equation.

Sample answer: In Method 1, the equation is considered a two-step equation with the operations of addition and multiplication. In Method 2, the equation is first expanded using the Distributive Property, and then considerd a two-step equation with the operations of multiplication and addition.

Follow the steps to solve the equation using the properties of equality.

Equation	Steps
$\frac{3(x+2)}{3} = \frac{-18}{3}$	Division Property of Equality
$x + 2 = -6$	Simplify.
$-2 \quad -2$	Subtraction Property of Equality
$x = -8$	

Example 1 Solve Two-Step Equations

Solve $3(x + 5) = 45$. Check your solution.

Method 1 Use the Division Property of Equality first.

$3(x + 5) = 45$	Write the equation.
$\frac{3(x+5)}{3} = \frac{45}{3}$	Division Property of Equality
$x + 5 = 15$	Simplify.
$-5 \quad -5$	Subtraction Property of Equality
$x = 10$	Simplify.

Method 2 Use the Distributive Property first.

$3(x + 5) = 45$	Write the equation.
$3x + 15 = 45$	Distributive Property
$-15 \quad -15$	Subtraction Property of Equality
$3x = 30$	Simplify.
$\frac{3x}{3} = \frac{30}{3}$	Division Property of Equality
$x = 10$	Simplify.

So, using either method, the solution of the equation is $x = 10$.

Check your solution by substituting the result back into the original equation.

$3(x + 5) = 45$	Write the original equation.
$3(10 + 5) \stackrel{?}{=} 45$	Replace x with 10.
$45 = 45$ ✓	The sentence is true.

396 **Module 7** • Equations and Inequalities

Interactive Presentation

Example 1, Solve Two-Step Equations, Slide 2 of 5

CLICK

On Slide 2, students move through the steps to solve the equation.

CLICK

On Slide 3, students move through the steps to use the Distributive Property to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Solve Two-Step Equations

Objective

Students will solve two-step equations of the form $p(x + q) = r$ with integers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to understand and flexibly use the properties of equality and the Distributive Property. Students should be able to use reasoning to explain how either method can be used to solve the equation $3(x + 5) = 45$.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is indicated by the number outside the parentheses? multiplication

OL Explain how the Division Property of Equality helps you begin to solve this equation. Sample answer: I can divide each side of the equation by 3 to undo the multiplication of 3 on the left side of the equation.

OL Even though there are parentheses, can you think of this method as solving a two-step equation? Explain. yes; Sample answer: By considering the two operations of adding 5 and multiplying by 3, I can think of this equation as a two-step equation.

BL Is there another way you can solve this equation for x? Explain. yes; Sample answer: Distribute 3 first to obtain the equation $3x + 15 = 45$. Subtract 15 from each side and then divide each side by 3.

SLIDE 3

AL Describe how the Distributive Property is used in the first step. Sample answer: The Distributive Property is used to expand the expression $3(x + 5)$ to obtain $3x + 15$.

OL How does this method compare to the previous method? Sample answer: In this method, I use the Distributive Property first to expand the expression.

OL How can you check your answer? Sample answer: Replace x with 10 into the original equation to verify that $3(10 + 5) = 45$ is a true statement, which it is.

BL How do you think you might use a similar method to solve an equation such as $\frac{x+1}{2} = 5$? Sample answer: Multiply both sides by 2 to eliminate the fraction. Then subtract 1 from each side; $x = 9$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Solve Two-Step Equations

Objective

Students will solve equations of the form $p(x - q) = r$ with integers.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is indicated by the number outside the parentheses? multiplication

OL Explain how the Division Property of Equality helps you begin to solve this equation. Sample answer: I can divide each side of the equation by 5 to undo the multiplication of 5 on the left side.

OL Explain how to consider this equation a two-step equation. Sample answer: By considering subtracting 2 and multiplying by 5, I can think of this equation as a two-step equation.

BL Suppose a classmate states that when solving this equation, before multiplication, you should undo the subtraction first because the order of operations should be undone in reverse order. Explain why this is incorrect. Sample answer: The order of operations states that parentheses should be done first. So, the operation inside the parentheses must be undone last.

SLIDE 3

AL Describe how the Distributive Property is used in the first step. Sample answer: The Distributive Property is used to expand the expression $5(n - 2)$ to obtain $5n - 10$.

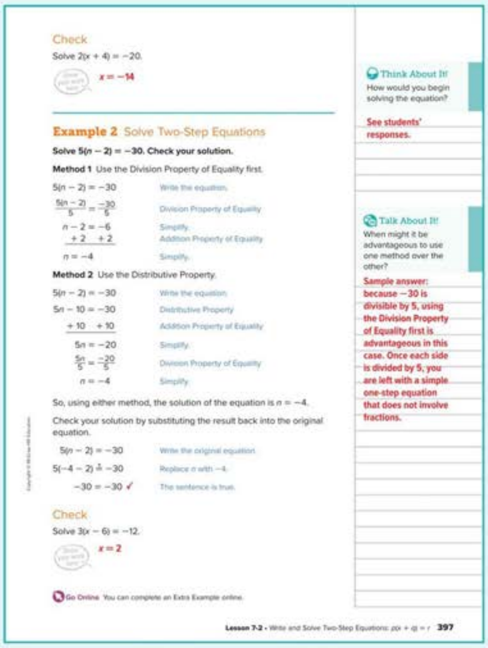
OL How does this method compare to the previous method? Sample answer: In this method, I use the Distributive Property first to expand the expression.

OL How can you check your answer? Sample answer: Replace n with -4 into the original equation to verify that $5(-4 - 2) = -30$ is a true statement, which it is.

BL How do you think you might use a similar method to solve an equation such as $\frac{n-3}{-6} = 1$? Sample answer: Multiply both sides by -6 to eliminate the fraction. Then add 3 to each side; $n = -3$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

Solve $2(x + 4) = -20$.

$x = -14$

Example 2 Solve Two-Step Equations

Solve $5(n - 2) = -30$. Check your solution.

Method 1 Use the Division Property of Equality first.

$5(n - 2) = -30$ Write the equation.

$\frac{5(n-2)}{5} = \frac{-30}{5}$ Division Property of Equality

$n - 2 = -6$ Simplify.

$+2 \quad +2$ Addition Property of Equality

$n = -4$ Simplify.

Method 2 Use the Distributive Property.

$5(n - 2) = -30$ Write the equation.

$5n - 10 = -30$ Distributive Property

$+10 \quad +10$ Addition Property of Equality

$5n = -20$ Simplify.

$\frac{5n}{5} = \frac{-20}{5}$ Division Property of Equality

$n = -4$ Simplify.

So, using either method, the solution of the equation is $n = -4$.

Check your solution by substituting the result back into the original equation.

$5(n - 2) = -30$ Write the original equation.

$5(-4 - 2) \stackrel{?}{=} -30$ Replace n with -4.

$-30 = -30$ ✓ The sentence is true.

Check

Solve $3(x - 6) = -12$.

$x = 2$

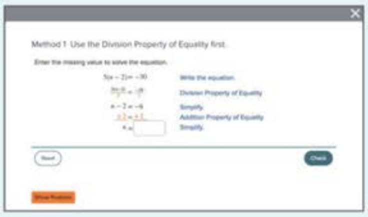
Go Online You can complete an Extra Example online.

Think About It! How would you begin solving the equation? See students' responses.

Talk About It! When might it be advantageous to use one method over the other? Sample answer: because −30 is divisible by 5, using the Division Property of Equality first is advantageous in this case. Once each side is divided by 5, you are left with a simple one-step equation that does not involve fractions.

Lesson 7-2 • Write and Solve Two-Step Equations: $p(x + q) = r$ 397

Interactive Presentation



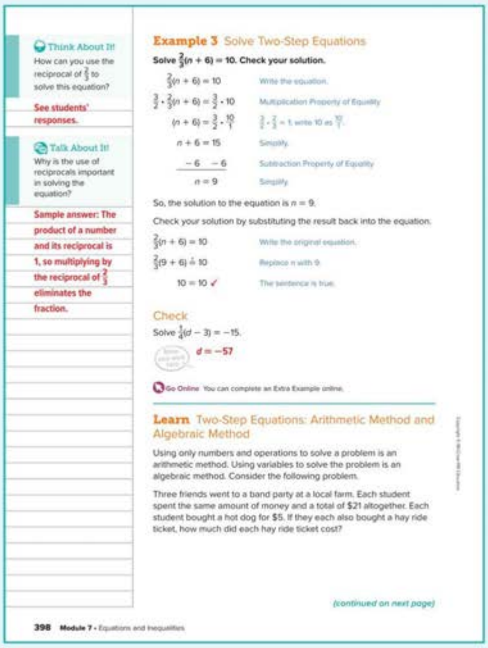
Example 2, Solve Two-Step Equations, Slide 2 of 5

TYPE

On Slides 2 and 3, students enter the missing values to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Think About It!
How can you use the reciprocal of $\frac{2}{3}$ to solve this equation?
See students' responses.

Talk About It!
Why is the use of reciprocals important in solving the equation?
Sample answer: The product of a number and its reciprocal is 1, so multiplying by the reciprocal of $\frac{2}{3}$ eliminates the fraction.

Example 3 Solve Two-Step Equations

Solve $\frac{2}{3}(n + 6) = 10$. Check your solution.

$\frac{2}{3}(n + 6) = 10$ Write the equation.

$\frac{3}{2} \cdot \frac{2}{3}(n + 6) = \frac{3}{2} \cdot 10$ Multiplication Property of Equality

$(n + 6) = \frac{3}{2} \cdot \frac{10}{1}$ $\frac{3}{2} \cdot \frac{2}{3} = 1$; write 10 as $\frac{10}{1}$.

$n + 6 = 15$ Simplify.

$-6 \quad -6$ Subtraction Property of Equality

$n = 9$ Simplify.

So, the solution to the equation is $n = 9$.

Check your solution by substituting the result back into the equation.

$\frac{2}{3}(n + 6) = 10$ Write the original equation.

$\frac{2}{3}(9 + 6) \stackrel{?}{=} 10$ Replace n with 9.

$10 = 10$ ✓ The sentence is true.

Check

Solve $\frac{1}{4}(d - 3) = -15$.

$d = -57$

Go Online You can complete an Extra Example online.

Learn Two-Step Equations: Arithmetic Method and Algebraic Method

Using only numbers and operations to solve a problem is an arithmetic method. Using variables to solve the problem is an algebraic method. Consider the following problem.

Three friends went to a band party at a local farm. Each student spent the same amount of money and a total of $21 altogether. Each student bought a hot dog for $5. If they each also bought a hay ride ticket, how much did each hay ride ticket cost?

(continued on next page)

398 Module 7 • Equations and Inequalities

Interactive Presentation

Example 3, Solve Two-Step Equations, Slide 2 of 4

CLICK
On Slide 2, students move through the steps to solve the equation.

TYPE
On Slide 2, students enter the missing values to solve the equation.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Solve Two-Step Equations

Objective

Students will solve two-step equations, of the form $p(x + q) = r$, with rational numbers written as fractions.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the number being multiplied outside of the parentheses? It is a fraction.

AL What are the two methods that can be used to solve the equation? Sample answer: First divide each side by $\frac{2}{3}$, and then subtract 6. Or I can use the Distributive Property to expand the expression on the left side of the equation, and then solve using inverse operations.

OL Which method do you prefer to use in this case? Sample answer: I prefer to divide each side of the equation by $\frac{2}{3}$ first so that I can eliminate the fraction from the left side of the equation.

OL How can you check your solution? Sample answer: Replace n with 9 in the original equation, simplify, and determine if the final statement is true.

BL Describe another way to solve the equation. Sample answer: Multiply each side of the equation by 3 to eliminate the denominator of 3. The equation becomes $2(n + 6) = 30$. Then divide each side by 2, and then subtract 6 from each side.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Two-Step Equations: Arithmetic Method and Algebraic Method

Objective

Students will understand how the arithmetic method and algebraic method of solving a two-step equation with parentheses compare.

MP Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them.
6 Attend to Precision As students discuss the *Talk About It!* question, they should be able to clearly communicate the similarities and differences of each method in order to compare and contrast them.

(continued on next page)

Learn Two-Step Equations: Arithmetic Method and Algebraic Method *(continued)*

Talk About It!

SLIDE 1

Mathematical Discourse

Compare and contrast the arithmetic method and algebraic method used to solve the problem. Sample answer: Using either method, the solution is the same and the steps are similar. In both cases, you can divide by 3 first and then subtract 5. The algebraic method is more abstract than the arithmetic method because it uses an equation to represent the problem.

Learn Write Two-Step Equations

Objective

Students will learn how to model a real-world problem with a two-step equation of the form $p(x + q) = r$.

Teaching Notes

SLIDE 1

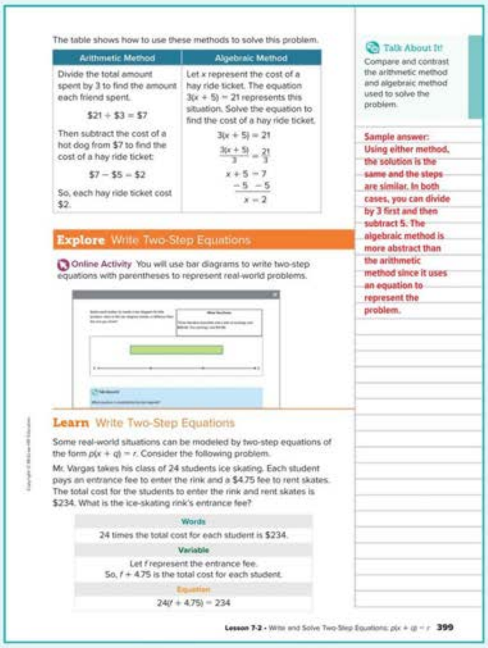
Have students select each flashcard to view the steps for modeling a real-world problem with a two-step equation of the form $p(x + q) = r$. An important step in writing the equation is to define the variable. Remind students that the variables can be any letter. In this case, f is used because the variable represents the entrance fee.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of how to model a real-world problem with a two-step equation of the form $p(x + q) = r$, have them compare and contrast it with the form $px + q = r$. Have them work with a partner to complete the following activity.

1. Present them with the same real-world problem that is presented in the Learn. Mr. Vargas takes his class of 24 students ice skating. Each student pays an entrance fee to enter the rink and a $4.75 fee to rent skates. The total cost for the students to enter the rink and rent skates is $234. What is the ice-skating rink's entrance fee?
2. Have them strategize as to how they could model the problem with an equation in the form $px + q = r$. Be sure they first define the variable. Have them explain how they determined the equation. Sample answer: Let f represent the entrance fee; $24f + 114 = 234$; A total of 24 students each paid an entrance fee which can be represented by $24f$. Each of the 24 students also paid $4.75 to rent skates, which is 24($4.75), or $114. The total cost is $234.
3. Have them compare and contrast the equation they just wrote and the one presented in the Learn. Sample answer: The equation from the Learn, $24(f + 4.75) = 234$, is a factored form of the equation I just wrote, $24f + 114 = 234$. They are equivalent.



The table shows how to use these methods to solve this problem.

Arithmetic Method	Algebraic Method
Divide the total amount spent by 3 to find the amount each friend spent. $\$21 \div \$3 = \$7$ Then subtract the cost of a hot dog from $7 to find the cost of a hay ride ticket: $\$7 - \$5 = \$2$ So, each hay ride ticket cost $2.	Let x represent the cost of a hay ride ticket. The equation $3(x + 5) = 21$ represents this situation. Solve the equation to find the cost of a hay ride ticket. $3(x + 5) = 21$; $\frac{3(x+5)}{3} = \frac{21}{3}$; $x + 5 = 7$; $-5 \quad -5$; $x = 2$

Talk About It! Compare and contrast the arithmetic method and algebraic method used to solve the problem.

Sample answer: Using either method, the solution is the same and the steps are similar. In both cases, you can divide by 3 first and then subtract 5. The algebraic method is more abstract than the arithmetic method since it uses an equation to represent the problem.

Explore Write Two-Step Equations

Online Activity You will use bar diagrams to write two-step equations with parentheses to represent real-world problems.

Learn Write Two-Step Equations

Some real-world situations can be modeled by two-step equations of the form $p(x + q) = r$. Consider the following problem.

Mr. Vargas takes his class of 24 students ice skating. Each student pays an entrance fee to enter the rink and a $4.75 fee to rent skates. The total cost for the students to enter the rink and rent skates is $234. What is the ice-skating rink's entrance fee?

Words
24 times the total cost for each student is $234.
Variable
Let f represent the entrance fee. So, $f + 4.75$ is the total cost for each student.
Equation
$24(f + 4.75) = 234$

Lesson 7-2 • Write and Solve Two-Step Equations: $p(x + q) = r$ 399

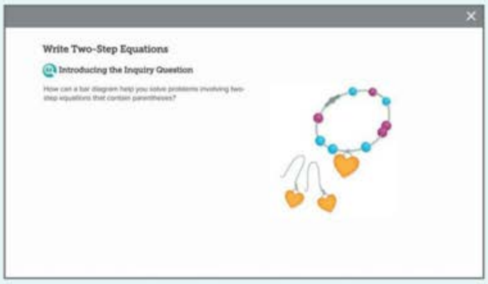
Interactive Presentation

Learn, Write Two-Step Equations

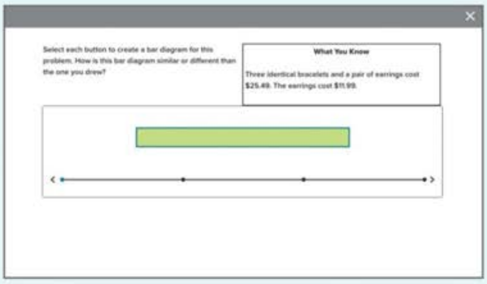
FLASHCARDS

On Slide 1 of Learn, Write Two-Step Equations, students select the Flashcards to see the steps for modeling a real-world problem with a two-step equation.

Interactive Presentation



Explore, Slide 1 of 7



Explore, Slide 3 of 7

CLICK

On Slide 3, students move through the steps to create a bar diagram for the problem.

Explore Write Two-Step Equations

Objective

Students will explore how to write two-step equations with parentheses.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a real-world problem. Throughout this activity, students will investigate how a bar diagram can be used to help write a two-step equation with parentheses to representthe problem.

Inquiry Question

How can a bar diagram help you solve problems involving two-step equations that contain parentheses? Sample answer: I can visualize all parts of the problem using a bar diagram and then use the bar diagram to write an equation. I can then solve the equation to solve the problem.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

What equation is modeled by the bar diagram? $3x + 11.99 = 25.49$

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Explore Write Two-Step Equations *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to discover and be able to explain the benefit of using bar diagrams as they can visualize how each part of the problem is represented in the bar diagram.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

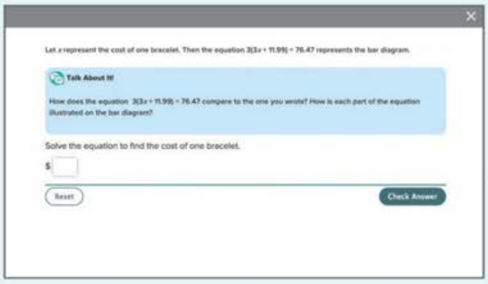
SLIDE 6

Mathematical Discourse

How does the equation $3(3x + 11.99) = 76.47$ compare to the one you wrote? See students' responses.

How is each part of the equation illustrated on the bar diagram? Sample answer: The three bar diagrams are represented by the number outside of the parentheses, 3. The expression $3x + 11.99$ represents the cost of three bracelets and one pair of earrings, labeled on each bar diagram. The decimal 76.47 represents the cost of all three sets, labeled on the side of the bar diagrams.

Interactive Presentation



Explore, Slide 6 of 7

TYPE

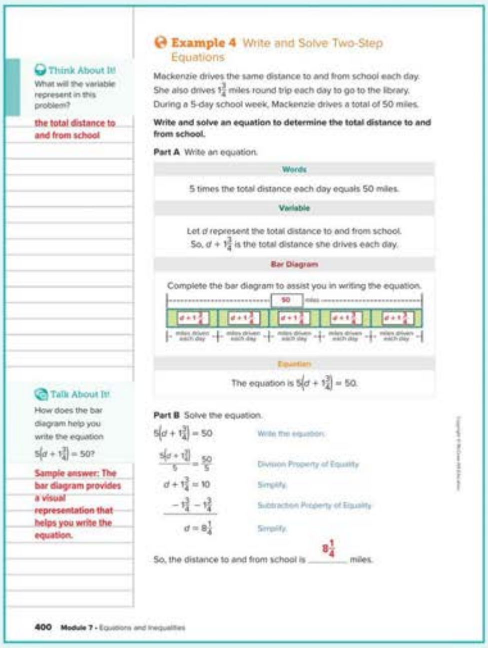
On Slide 5, students explain how the bar diagram helped them write the equation.

TYPE

On Slide 6, students type to solve an equation.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.


Example 4 Write and Solve Two-Step Equations

Mackenzie drives the same distance to and from school each day. She also drives $1\frac{3}{4}$ miles round trip each day to go to the library. During a 5-day school week, Mackenzie drives a total of 50 miles.

Write and solve an equation to determine the total distance to and from school.

Part A Write an equation.

Words

5 times the total distance each day equals 50 miles.

Variable

Let d represent the total distance to and from school. So, $d + 1\frac{3}{4}$ is the total distance she drives each day.

Bar Diagram

Complete the bar diagram to assist you in writing the equation.

Equation

The equation is $5\left(d + 1\frac{3}{4}\right) = 50$.

Part B Solve the equation.

$5\left(d + 1\frac{3}{4}\right) = 50$ Write the equation.

$\frac{5\left(d + 1\frac{3}{4}\right)}{5} = \frac{50}{5}$ Division Property of Equality

$d + 1\frac{3}{4} = 10$ Simplify.

$-1\frac{3}{4} \quad -1\frac{3}{4}$ Subtraction Property of Equality

$d = 8\frac{1}{4}$ Simplify.

So, the distance to and from school is $8\frac{1}{4}$ miles.

Think About It! What will the variable represent in this problem? the total distance to and from school

Talk About It! How does the bar diagram help you write the equation $5\left(d + 1\frac{3}{4}\right) = 50$? Sample answer: The bar diagram provides a visual representation that helps you write the equation.

400 Module 7 • Equations and Inequalities

Interactive Presentation

Example 4, Write and Solve Two-Step Equations, Slide 2 of 5

FLASHCARDS

On Slide 2, students use Flashcards to see the steps for writing the equation.

TYPE

On Slide 3, students enter the missing values to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Write and Solve Two-Step Equations

Objective

Students will write and solve two-step equations of the form $p(x + q) = r$.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown quantity you need to find? How will you represent this in the equation? the distance to and from school; with a variable

OL Why is the expression that represents the distance Mackenzie drives each day $d + 1\frac{3}{4}$? Sample answer: She drives $1\frac{3}{4}$ miles round trip each day to go to the library, plus a certain distance d to and from school each day.

OL Explain why the entire quantity $d + 1\frac{3}{4}$ is multiplied by 5, and not just d. Sample answer: She drives both distances, d and $1\frac{3}{4}$, five days a week, not just d.

BL A classmate wrote the equation $d + 1\frac{3}{4} = 10$ because this represents the distance each day. Is this correct? Explain. yes; Sample answer: The total distance, 50 miles, can be divided by 5 days per week, from the start. So, this equation is correct.

SLIDE 3

AL What are the two different methods you can use to solve this equation? Sample answer: First divide each side of the equation by 5. Then subtract $1\frac{3}{4}$. Or I can use the Distributive Property to expand the expression on the left side of the equation. Then use inverse operations to continue to solve.

OL Choose a method that can be used to solve the equation. Describe the steps you need to take. Sample answer: First divide each side of the equation by 5. Then subtract $1\frac{3}{4}$.

OL How can you check your answer? Sample answer: Replace d with $8\frac{1}{4}$ into the original equation. Verify the statement $5\left(8\frac{1}{4} + 1\frac{3}{4}\right) = 50$ is true, which it is.

BL What percentage of the total amount that Mackenzie drives each week is to and from school? Explain. 82.5%; Sample answer: 41.25 out of 50 is 82.5%.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

Mrs. Byers is making 5 costumes that each require $1\frac{3}{8}$ yards of blue fabric and a certain amount of red fabric. She will use $8\frac{3}{4}$ yards in all. Write and solve an equation to determine the number of yards of red fabric r she will need for each costume.

$5\left(1\frac{3}{8} + r\right) = 8\frac{3}{4}$; $\frac{3}{8}$ yard

Go Online You can complete an Extra Example online.

Pause and Reflect

Explain how you wrote and solved the equation in the Check problem. Did you use a bar diagram? Why or why not? Can you solve the problem in a different way from what was shown in the Example? If so, explain.

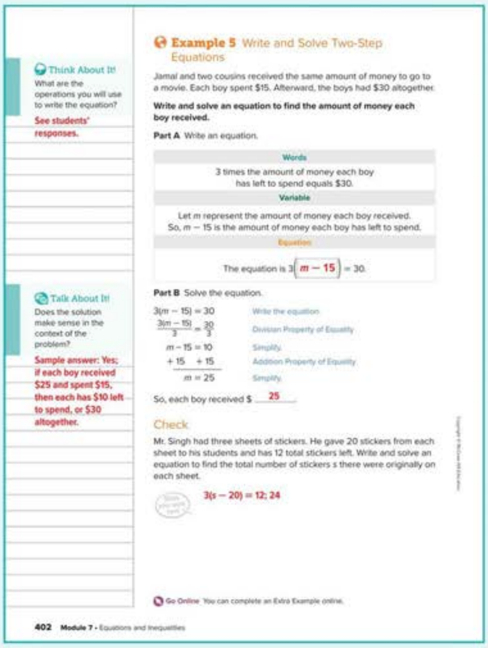
See students' observations.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of solving two-step equations, have them work with a partner to strategize and solve the following three-step equations.

$2(x - 1) + 1 = 7$ $x = 4$ $\quad$ $-3(4 - x) - 1 = 5$ $x = 6$



Example 5 Write and Solve Two-Step Equations

Jamal and two cousins received the same amount of money to go to a movie. Each boy spent \$15. Afterward, the boys had \$30 altogether.

Write and solve an equation to find the amount of money each boy received.

Part A Write an equation.

Words
3 times the amount of money each boy has left to spend equals \$30.
Variable
Let m represent the amount of money each boy received. So, $m - 15$ is the amount of money each boy has left to spend.
Equation
The equation is $3(m - 15) = 30$.

Part B Solve the equation.

$3(m - 15) = 30$ Write the equation.

$\frac{3(m-15)}{3} = \frac{30}{3}$ Division Property of Equality

$m - 15 = 10$ Simplify.

$+15 \quad +15$ Addition Property of Equality

$m = 25$ Simplify.

So, each boy received \$ 25

Check

Mr. Singh had three sheets of stickers. He gave 20 stickers from each sheet to his students and has 12 total stickers left. Write and solve an equation to find the total number of stickers s there were originally on each sheet.

$3(s - 20) = 12$; 24

Think About It! What are the operations you will use to write the equation? See students' responses.

Talk About It! Does the solution make sense in the context of the problem? Sample answer: Yes; if each boy received \$25 and spent \$15, then each has \$10 left to spend, or \$30 altogether.

Go Online You can complete an Extra Example online.

402 Module 7 • Equations and Inequalities

Interactive Presentation

Example 5, Write and Solve Two-Step Equations, Slide 2 of 5

FLASHCARDS

On Slide 2, students use Flashcards to see the steps for writing the equation.

CLICK

On Slide 3, students move through the steps to solve the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Write and Solve Two-Step Equations

Objective

Students will write and solve two-step equations of the form $p(x - q) = r$.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students will decontextualize the information by representing it symbolically with a correct two-step equation. Encourage students to make sure their solution makes sense within the context of the real-world scenario.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown quantity? How will you represent it in the equation? the amount of money each boy received; with a variable

OL Explain why the expression $m - 15$ is multiplied by 3. Sample answer: There are three boys and each boy has $m - 15$ left.

BL Is there another way you can write this equation? Explain. yes; Sample answer: $3m - 45 = 30$ represents this problem because there are three boys, and if each boy has $m - 15$ left, then the total amount left is $3m - 45$.

SLIDE 3

AL What are the two different methods you can use to solve this equation? Sample answer: First divide each side of the equation by 3. Then add 15 to each side. Or I can use the Distributive Property to expand the expression on the left side of the equation. Then use inverse operations to continue to solve.

OL Choose a method that can be used to solve the equation. Describe the steps you need to take. Sample answer: First divide each side of the equation by 3. Then add 10 to each side.

OL How can you check your answer? Sample answer: Replace m with 25 into the original equation. Verify the statement $3(25 - 15) = 30$ is true, which it is.

BL What percentage of the money each boy received did he spend? Explain. 60%; Sample answer: Each boy spent \$15 and received \$25; \$15 out of \$25 is 60%.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Perimeter

Objective

Students will come up with their own strategy to solve an application problem involving perimeter.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

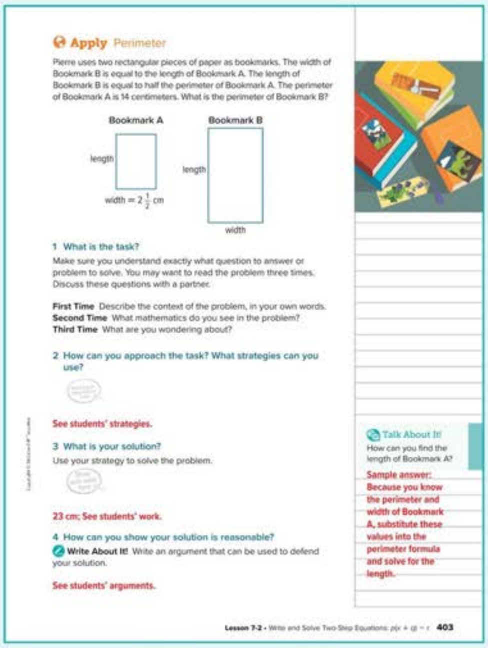
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What do you know about Bookmark A?
- How can you find the length of Bookmark A?
- How can you find the width of Bookmark B?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Perimeter

Pierre uses two rectangular pieces of paper as bookmarks. The width of Bookmark B is equal to the length of Bookmark A. The length of Bookmark B is equal to half the perimeter of Bookmark A. The perimeter of Bookmark A is 14 centimeters. What is the perimeter of Bookmark B?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

23 cm; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

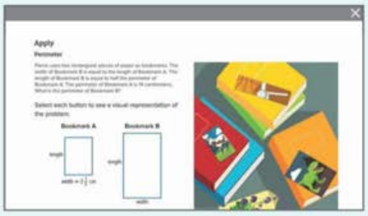
See students' arguments.

Talk About It!
How can you find the length of Bookmark A?

Sample answer: Because you know the perimeter and width of Bookmark A, substitute these values into the perimeter formula and solve for the length.

Lesson 7-2 • Write and Solve Two-Step Equations: $p(x + q) = r$ 403

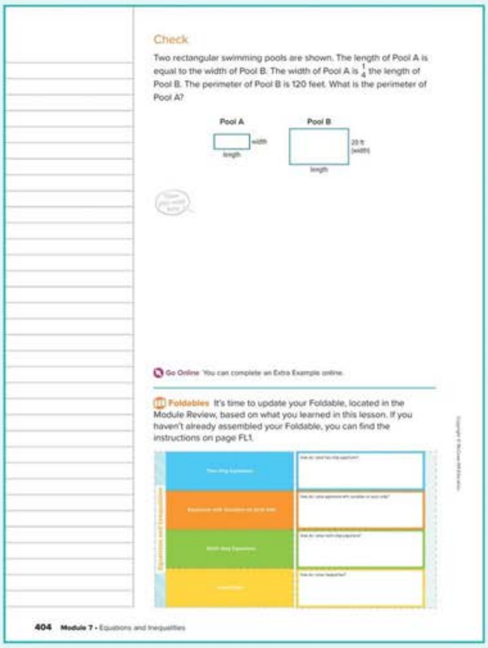
Interactive Presentation



Apply, Perimeter

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Exit Ticket

Refer to the Exit Ticket slide. Suppose a bowling alley charges $5 per person to rent shoes, and a certain amount per game per person. Six friends each pay for one game and rent a pair of shoes. The total cost is $49.50. Explain how to write an equation that can be used to find the cost of a game. Then find the cost of a game. Sample answer: Let x represent the cost of a game. Then $x + 5$ represents the amount each friend pays for a game and shoes. Therefore, $6(x + 5)$ represents the amount six friends pay for a game and shoes. Set this expression equal to the total cost; $6(x + 5) = 49.5$; $x = 3.25$, so it costs $3.25 per game.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record information on solving equations of the form $p(x + q) = r$. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can equations be used to solve everyday problems?

In this lesson, students learned how to write and solve a two-step equation of the form $p(x + q) = r$ that represents a real-world problem involving rational numbers. Encourage them to brainstorm an example of a problem arising in everyday life in which they can use a two-step equation of this form to model and solve the problem. For example, the equation $28(s + 6.50) = 490$ can model the cost of 28 students to visit a museum in which s represents the cost of admission to the museum per student, and $6.50 is the cost per student for lunch.

Interactive Presentation

Exit Ticket

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

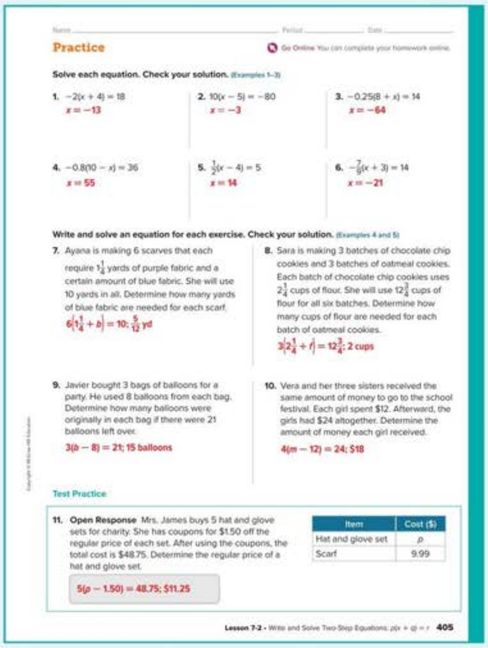
Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve two-step equations of the form $p(x + q) = r$	1–6
2	write and solve two-step equations of the form $p(x + q) = r$	7–10
2	extend concepts learned in class to apply them in new contexts	11
3	solve application problems involving writing and solving two-step equations of the form $p(x + q) = r$	12–13
3	higher-order and critical thinking skills	14–17

Common Misconception

Some students may struggle to handle parentheses correctly when solving an equation. In Exercise 2, some students may try to add 5 to both sides of the equation before dividing by 10 or expanding the left side. They may also incorrectly eliminate the parentheses by only multiplying x by 10. Remind them to adhere to the Distributive Property when expanding the expression to remove the parentheses. Alternatively, students could begin by dividing each side of the equation by 10. This will also eliminate the parentheses.

Students may struggle to write each equation in the correct form for each exercise. For example, in Exercise 7, students may incorrectly write the equation as $6b + 1\frac{1}{4} = 10$. Remind them to look for key words in the problem. Because there are 6 scarves, both the number of yards of purple fabric and the number of yards of blue fabric need to be multiplied by 6 to equal a total of 10 yards of fabric. Students should recognize that they can use parentheses to represent the number of yards of fabric for one scarf, $\left(1\frac{1}{4} + b\right)$ Then the equation for the total number of yards of fabric can be written as $6\left(1\frac{1}{4} + b\right) = 10$.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each equation. Check your solution. (Examples 1–3)

1. $-2(x + 4) = 18$
   $x = -13$
2. $10(x - 5) = -80$
   $x = -3$
3. $-0.25(8 + x) = 14$
   $x = -64$
4. $-0.8(10 - x) = 36$
   $x = 55$
5. $\frac{1}{2}(x - 4) = 5$
   $x = 14$
6. $-\frac{7}{9}(x + 3) = 14$
   $x = -21$

Write and solve an equation for each exercise. Check your solution. (Examples 4 and 5)

7. Ayana is making 6 scarves that each require $1\frac{1}{4}$ yards of purple fabric and a certain amount of blue fabric. She will use 10 yards in all. Determine how many yards of blue fabric are needed for each scarf.
   $6\left(1\frac{1}{4} + b\right) = 10$; $\frac{5}{12}$ yd
8. Sara is making 3 batches of chocolate chip cookies and 3 batches of oatmeal cookies. Each batch of chocolate chip cookies uses $2\frac{1}{4}$ cups of flour. She will use $12\frac{3}{4}$ cups of flour for all six batches. Determine how many cups of flour are needed for each batch of oatmeal cookies.
   $3\left(2\frac{1}{4} + f\right) = 12\frac{3}{4}$; 2 cups
9. Javier bought 3 bags of balloons for a party. He used 8 balloons from each bag. Determine how many balloons were originally in each bag if there were 21 balloons left over.
   $3(b - 8) = 21$; 15 balloons
10. Vera and her three sisters received the same amount of money to go to the school festival. Each girl spent $12. Afterward, the girls had $24 altogether. Determine the amount of money each girl received.
    $4(m - 12) = 24$; $18

Test Practice

11. **Open Response** Mrs. James buys 5 hat and glove sets for charity. She has coupons for $1.50 off the regular price of each set. After using the coupons, the total cost is $48.75. Determine the regular price of a hat and glove set.

Item	Cost ($)
Hat and glove set	p
Scarf	9.99

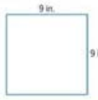
$5(p - 1.50) = 48.75$; $11.25

Lesson 7-2 • Write and Solve Two-Step Equations: $p(x + q) = r$ **405**

Apply

12. Olive and Ryan have picture frames with the same perimeter. Olive's picture frame has a width 1.25 times the width of Ryan's picture frame. What is the length ℓ of Olive's picture frame?

Ryan's Picture Frame



6.75 in.

13. A landscape architect designed a flower garden in the shape of a trapezoid. The area of the garden is 13.92 square meters. A fence is planned around the perimeter of the garden. How many meters of fencing are needed?

15.88 m

Higher-Order Thinking Problems

14. MP **Justify Conclusions** Suppose for some value of x the solution to the equation $2.5(y - x) = 0$ is $y = 6$. What must be true about x? Justify your conclusion.

$x = 6$; Sample answer: The value of the expression inside the parentheses must be equal to 0. So, if $6 - x = 0$, then $x = 6$.

15. MP **Persevere with Problems** Keith is 5 years older than Trina. Two times the sum of their ages is 62. Write and solve an equation to find Keith's age.

Let Keith's age $= x$. Trina's age is $x - 5$. The sum of their ages is $(x) + (x - 5)$. $2(2x - 5) = 62$; Keith is 18 years old.

16. MP **Find the Error** A student is solving $-2(x - 5) = 12$. Find the student's mistake and correct it.

$$-2(x - 5) = 12$$
$$-2x - 5 = 12$$
$$-2x - 5 + 5 = 12 + 5$$
$$-2x = 17$$
$$x = -8.5$$

Sample answer: The student only distributed the −2 to the first term inside the parentheses instead of both terms. The correct solution is $x = -1$.

17. **Create** Write a real-world problem that could be represented by the equation $12(x + 2.50) = 78$. Then solve the equation.

Sample answer: You and 11 friends go bowling. Shoe rental costs \$2.50. The total cost of one game and one shoe rental for everyone is \$78. What is the cost of one game of bowling for one person?; \$4

406 Module 7 • Equations and Inequalities

MP Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 15, students use multiple steps to write and solve an equation for a real-world problem.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students find and correct a student's mistake. In Exercise 18, students determine the value of one variable and justify their conclusion given the value of the other variable and an equation relating them.

Collaborative Practice

Have students work in pairs or small groups to complete the following.

Solve the problem another way.

Use with Exercises 12–13 Have students work in groups of 3–4. After completing Exercise 12, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is viable. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 13.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 7–13 odd, 14–17
- Extension: Systems of Equations
- ALEKS Multi-Step Equations, Applications of Equations

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–12, 16, 17
- Extension: Systems of Equations
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS One-Step Equations

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS One-Step Equations

Lesson 7-3

8.EE.C.7

Write and Solve Equations with Variables on Each Side

LESSON GOAL

Students will write and solve equations with variables on each side.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Equations with Variables on Each Side

Learn: Equations with Variables on Each Side
Example 1: Equations with Variables on Each Side
Example 2: Equations with Variables on Each Side

Explore: Write and Solve Equations with Variables on Each Side

Learn: Write and Solve Equations with Variables on Each Side
Example 3: Write and Solve Equations with Variables on Each Side
Example 4: Write and Solve Equations with Variables on Each Side
Apply: Home Improvement

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 40 of the *Language Development Handbook* to help your students build mathematical language related to writing and solving equations with variables on each side.

ELL You can use the tips and suggestions on page T40 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.C** by writing and solving equations with variables on each side.
Standards for Mathematical Content: **8.E E.C.7, 8.EE.C.7.B,** *Also addresses 8.EE.C.7.A*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6**

Coherence

Vertical Alignment

Previous
Students wrote and solved two-step equations.
7.EE.B.4, 7.EE.B.4.A

Now
Students write and solve equations with variables on each side.
8.EE.C.7, 8.EE.C.7.B

Next
Students will write and solve multi-step equations.
8.EE.C.7, 8.EE.C.7.B

Rigor

The Three Pillars of Rigor

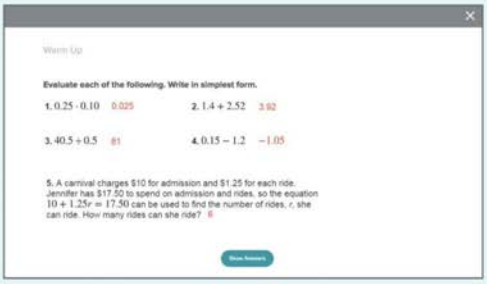
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of two-step equations to build *fluency* with solving equations that have variables on each side of the equals sign, using the Properties of Equality. They *apply* their fluency by writing and solving equations with variables on each side to solve real-world problems.

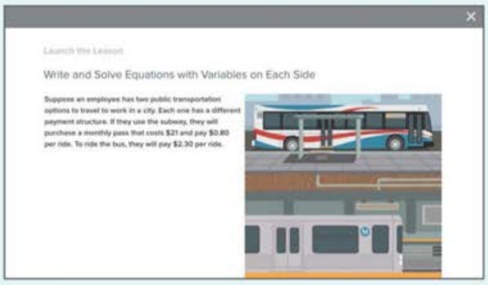
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- using operations with rational numbers (Exercises 1–4)
- solving two-step equations with rational coefficients (Exercise 5)

Answers

1. 0.025
2. 3.92
3. 81
4. −1.05
5. 6

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an equation to find which form of public transportation is the best option.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The verb *express* means to convey a thought or feeling through words or gestures. How can you use this term to describe how it might relate to a mathematical *expression*? Sample answer: Mathematical expressions are used to express, or describe, quantities and relationships, using numbers, variables, and operations.

Explore Equations with Variables on Each Side

Objective

Students will use Web Sketchpad to explore how to use a balance to solve equations with variables on each side.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations

Summary of Activity

Students will be presented with weights and balloons representing expressions or numbers that can be placed on a balance representing an equation. Throughout this activity, students will add and remove weights and balloons to balance two expressions or to find the value of the variable in an equation.

Inquiry Question

How can you solve an equation with variables on each side of the equals sign? Sample answer: Add or subtract equal objects from both sides of the equation until there is only an x on one side of the equals sign and a number on the other side.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

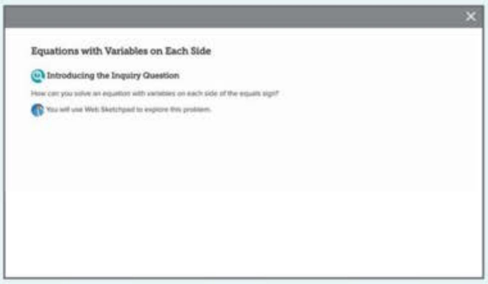
SLIDE 2

Mathematical Discourse

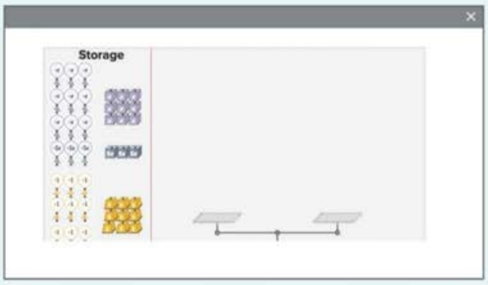
Which objects weigh a pan down, and which ones pull it up?
Sample answer: The weight blocks weigh it down, the balloons pull it up.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8

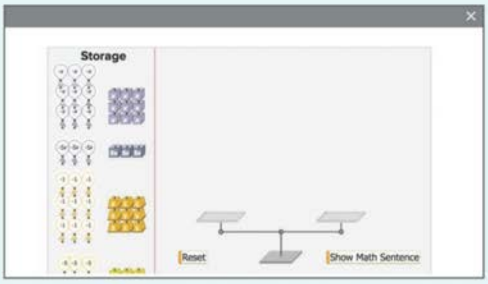


Explore, Slide 2 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to use a balance to solve equations with variables on each side.

Interactive Presentation



Explore, Slide 6 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Equations with Variables on Each Side *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use a Web Sketchpad balance to explore and examine solving equations with variables on each side. Encourage students to think about why weights are used to represent expressions such as 1 and *x* and balloons are used to represent expressions such as -1 and $-x$.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

Talk About It!

SLIDE

Mathematical Discourse

How could you add or subtract balloons or weights to keep weights balanced? Sample answer: I could add a value and its opposite to one of the pans or add the same value to each side of the pan.

Learn Equations with Variables on Each Side

Objective

Students will learn how to solve equations with variables on each side.

Teaching Notes

SLIDE 1

Students will use the *Subtraction Property of Equality* and the *Division Property of Equality*, which can be used to solve an equation algebraically. Move through the slides to learn how the properties are used when there are variables on each side of the equation.

After presenting the equation, you may wish to ask students to think about different strategies they can use to solve the equation. Encourage students to come up with various ways they can solve the problem. Encourage students to share their strategies and solutions with the class.

Have the students move through the steps. Ask students which properties were used to find the solution of the given equation. Students should note the *Subtraction Property of Equality* and the *Division Property of Equality*. Then have students replace the solution of 3 into the original equation, simplify, and determine if the resulting statement is true. Both sides should equal 17. Students may forget whether to add or subtract to solve the equation. Remind them to use the inverse of the operation in the equation.

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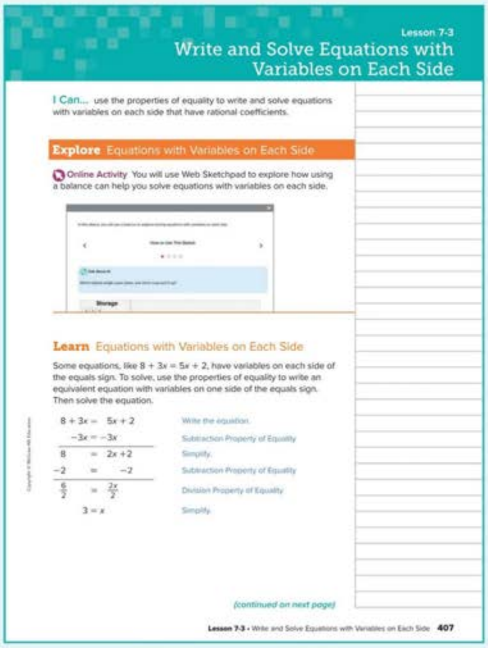


Lesson 7-3

Write and Solve Equations with Variables on Each Side

I Can... use the properties of equality to write and solve equations with variables on each side that have rational coefficients.

Explore Equations with Variables on Each Side

Online Activity You will use Web Sketchpad to explore how using a balance can help you solve equations with variables on each side.

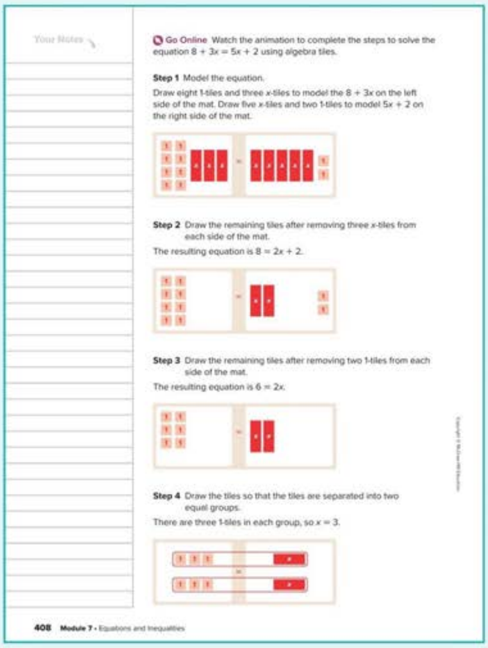
Learn Equations with Variables on Each Side

Some equations, like $8 + 3x = 5x + 2$, have variables on each side of the equals sign. To solve, use the properties of equality to write an equivalent equation with variables on one side of the equals sign. Then solve the equation.

$8 + 3x = 5x + 2$	Write the equation.
$-3x = -3x$	Subtraction Property of Equality
$8 = 2x + 2$	Simplify.
$-2 = -2$	Subtraction Property of Equality
$\frac{6}{2} = \frac{2x}{2}$	Division Property of Equality
$3 = x$	Simplify.

(continued on next page)

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION



Your Notes

Go Online Watch the animation to complete the steps to solve the equation $8 + 3x = 5x + 2$ using algebra tiles.

Step 1 Model the equation.

Draw eight 1-tiles and three x-tiles to model the $8 + 3x$ on the left side of the mat. Draw five x-tiles and two 1-tiles to model $5x + 2$ on the right side of the mat.

Step 2 Draw the remaining tiles after removing three x-tiles from each side of the mat.

The resulting equation is $8 = 2x + 2$.

Step 3 Draw the remaining tiles after removing two 1-tiles from each side of the mat.

The resulting equation is $6 = 2x$.

Step 4 Draw the tiles so that the tiles are separated into two equal groups.

There are three 1-tiles in each group, so $x = 3$.

408 Module 7 • Equations and Inequalities

Interactive Presentation

Learn, Equations with Variables on Each Side, Slide 2 of 2

WATCH

On Slide 2, students watch an animation that illustrates the steps used to solve an equation with variables on each side.

Learn Equations with Variables on Each Side *(continued)*

Go Online to have students watch the animation on Slide 2. The animation illustrates solving an equation with variables on each side.

Teaching Notes

SLIDE 2

The animation illustrates how to solve an equation with variables on each side of the equals sign, by visually demonstrating the properties of equality using algebra tiles. You may wish to pause the animation after the equation $8 + 3x = 5x + 2$ is shown, and ask students to strategize with a partner how they can model the equation using algebra tiles. Then ask them how they can manipulate the algebra tiles in order to solve for the variable x. They may use any strategy they wish, but should be prepared to explain their strategy to the class and defend why it works. Have student volunteers explain their strategy to the class. Then have students watch the animation to see if their strategy was used. If not, have them compare their strategy to the one used in the animation.

DIFFERENTIATE

Language Development Activity ELL

If any of your students need more of a challenge, provide the following equations. Encourage students to work with a partner to strategize how they might be able to solve these equations. They may use any strategy they wish, but must explain their strategy and defend why it works, using clear and precise mathematical language. Have students present their strategies and explanations with another pair of students, or the entire class.

$8 + x + 3x = 5x + x + 2$ $x = 3$

$x + x + 2 = -14 - x + x$ $x = -8$

$-6x - 20 + x = -2x + 4(1 - 3x) + x$ $x = 3$

Example 1 Solve Equations with Variables on Each Side

Objective

Students will solve equations with variables on each side that have integer coefficients.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to understand and flexibly use the properties of equality, such as the Subtraction Property of Equality, the Addition Property of Equality, and the Division Property of Equality.

Questions for Mathematical Discourse

SLIDE 1

AL Explain the location of the expressions $6n$ and $4n$ with respect to the equals sign. They are on opposite sides of the equals sign.

AL Why do you need to move one of the variable terms to the other side? Sample answer: We need to isolate the variable so that we can solve the equation.

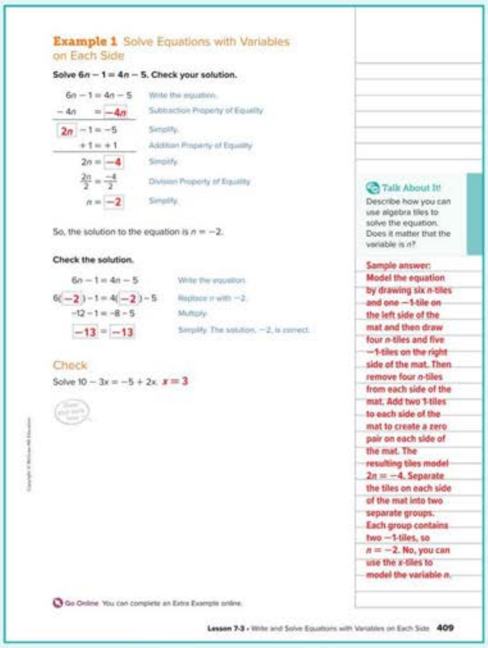
OL How can you check your answer? Sample answer: Substitute the value of the variable into the original equation to make sure it is a true equation.

OL Are there other ways to begin to solve the equation? yes; Sample answer: I can subtract $6n$ from each side first, or I can add 1 to each side, or add 5 to each side.

BL What happens if you subtract $6n$ from each side as the first step? Do you prefer to do this? Explain. Sample answer: By subtracting $6n$ from each side, $-2n$ will remain on the right side. I prefer to subtract $4n$ from each side so that only positive coefficients of n remain.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Solve Equations with Variables on Each Side

Solve $6n - 1 = 4n - 5$. Check your solution.

$6n - 1 = 4n - 5$	Write the equation.
$-4n \quad = -4n$	Subtraction Property of Equality
$2n - 1 = -5$	Simplify.
$+1 = +1$	Addition Property of Equality
$2n = -4$	Simplify.
$\frac{2n}{2} = \frac{-4}{2}$	Division Property of Equality
$n = -2$	Simplify.

So, the solution to the equation is $n = -2$.

Check the solution.

$6n - 1 = 4n - 5$	Write the equation.
$6(-2) - 1 = 4(-2) - 5$	Replace n with -2.
$-12 - 1 = -8 - 5$	Multiply.
$-13 = -13$	Simplify. The solution, -2, is correct.

Check

Solve $10 - 3x = -5 + 2x$. $x = 3$

Talk About It! Describe how you can use algebra tiles to solve the equation. Does it matter that the variable is n?

Sample answer: Model the equation by drawing six n-tiles and one -1-tile on the left side of the mat and then draw four n-tiles and five -1-tiles on the right side of the mat. Then remove four n-tiles from each side of the mat. Add two 1-tiles to each side of the mat to create a zero pair on each side of the mat. The resulting tiles model $2n = -4$. Separate the tiles on each side of the mat into two separate groups. Each group contains two -1-tiles, so $n = -2$. No, you can use the x-tiles to model the variable n.

Go Online You can complete an Extra Example online.

Lesson 7-3 • Write and Solve Equations with Variables on Each Side 409

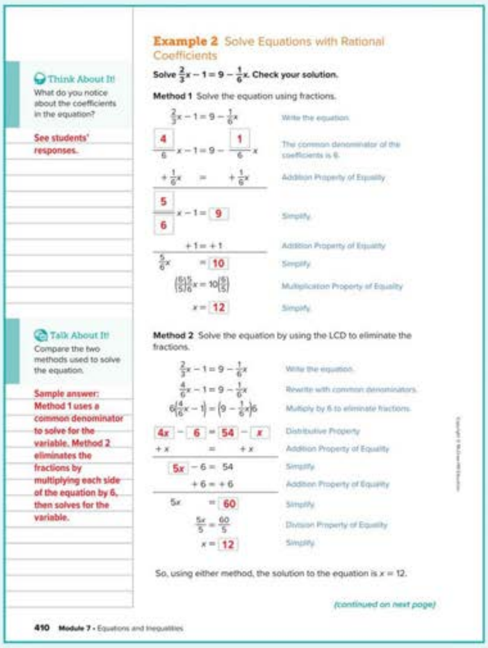
Interactive Presentation

Example 1, Solve Equations with Variables on Each Side, Slide 1 of 3

CLICK On Slide 1, students move through the steps to solve the equation.

TYPE On Slide 1, students determine the solution of the equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Example 2 Solve Equations with Rational Coefficients

Solve $\frac{2}{3}x - 1 = 9 - \frac{1}{6}x$. Check your solution.

Think About It! What do you notice about the coefficients in the equation?

See students' responses.

Method 1 Solve the equation using fractions.

$\frac{2}{3}x - 1 = 9 - \frac{1}{6}x$ Write the equation.

$\frac{4}{6}x - 1 = 9 - \frac{1}{6}x$ The common denominator of the coefficients is 6.

$+\frac{1}{6}x = +\frac{1}{6}x$ Addition Property of Equality

$\frac{5}{6}x - 1 = 9$ Simplify.

$+1 = +1$ Addition Property of Equality

$\frac{5}{6}x = 10$ Simplify.

$\left(\frac{6}{5}\right)\frac{5}{6}x = 10\left(\frac{6}{5}\right)$ Multiplication Property of Equality

$x = 12$ Simplify.

Talk About It! Compare the two methods used to solve the equation.

Sample answer: Method 1 uses a common denominator to solve for the variable. Method 2 eliminates the fractions by multiplying each side of the equation by 6, then solves for the variable.

Method 2 Solve the equation by using the LCD to eliminate the fractions.

$\frac{2}{3}x - 1 = 9 - \frac{1}{6}x$ Write the equation.

$\frac{4}{6}x - 1 = 9 - \frac{1}{6}x$ Rewrite with common denominators.

$6\left(\frac{4}{6}x - 1\right) = \left(9 - \frac{1}{6}x\right)6$ Multiply by 6 to eliminate fractions.

$4x - 6 = 54 - x$ Distributive Property

$+x = +x$ Addition Property of Equality

$5x - 6 = 54$ Simplify.

$+6 = +6$ Addition Property of Equality

$5x = 60$ Simplify.

$\frac{5x}{5} = \frac{60}{5}$ Division Property of Equality

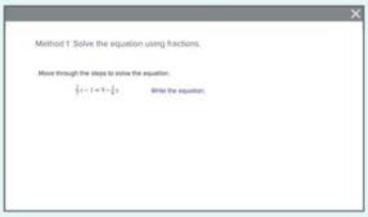
$x = 12$ Simplify.

So, using either method, the solution to the equation is $x = 12$.

(continued on next page)

410 Module 7 • Equations and Inequalities

Interactive Presentation



Example 2, Solve Equations with Rational Coefficients, Slide 2 of 5

CLICK On Slides 2 and 3, students move through the steps to solve the equation.

TYPE On Slides 2 and 3, students determine the solution of the equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Solve Equations with Rational Coefficients

Objective

Students will solve equations with variables on each side that have rational coefficients written as fractions.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them Encourage students to understand that different approaches may be used to solve the equation, and understand why those approaches are valid. As students discuss the *Talk About It!* question on Slide 4, encourage them to analyze each method, note any correspondences, and make a conjecture as to when one method might be more advantageous to use than the other.

7 Look For and Make Use of Structure Students should analyze the structure of the equation in order to determine what method they will use, and which operation they choose to undo first.

Questions for Mathematical Discourse

SLIDE 2

AL What must be true of fractions before they can be added or subtracted? They must have the same denominator.

OL In the last step, why do you multiply both sides of the equation by $\frac{6}{5}$? Sample answer: To eliminate the fractional coefficient of x, multiply by its reciprocal since $\frac{5}{6}$ multiplied by $\frac{6}{5}$ is 1.

BL A classmate added $\frac{1}{6}x$ to each side, and then subtracted 9 from each side. The result was $\frac{5}{6}x - 10 = 0$. Describe how what they did was not helpful in solving the equation. Sample answer: The variable is still not isolated. They should have added 1 to each side as the second step.

SLIDE 3

AL Does this method still require finding common denominators? Explain why. yes; Sample answer: In order to eventually combine the variable terms, I need to find common denominators since the coefficients are fractions.

OL Why is it helpful to multiply both sides by 6 in the third step? Sample answer: The common denominator of the fractional coefficients is 6. Multiplying each term by 6 eliminates the denominator.

BL Could you have multiplied each term in the original equation by 6 prior to finding a common denominator? Explain. yes; Sample answer: The result will be the same, as long as the LCD is used.

(continued on next page)

Example 2 Solve Equations with Rational Coefficients *(continued)*

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check the solution.

$\frac{2}{3}x - 1 = 9 - \frac{1}{6}x$	Write the equation.
$\frac{2}{3} \cdot 12 - 1 = 9 - \frac{1}{6} \cdot 12$	Replace x with 12.
$8 - 1 = 9 - 2$	Multiply.
$7 = 7$	Simplify. The solution, 12, is correct.

Check

Solve $\frac{2}{3}x + 5 = \frac{2}{5}x - 3$. $x = -30$

Go Online You can complete an Extra Example online.

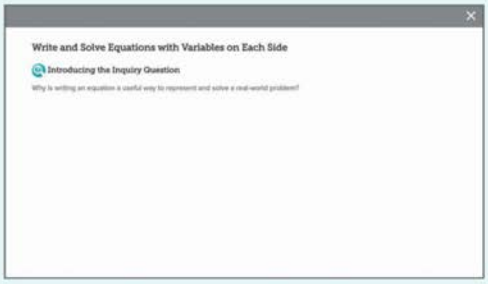
Explore Write and Solve Equations with Variables on Each Side

Online Activity You will explore how to write an equation with variables on each side to solve a real-world problem.

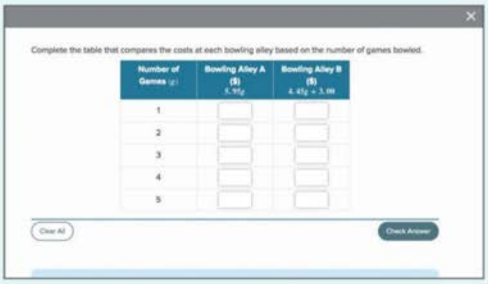
Math History Minute

Alicia Dickenstein (1955-) is an Argentine mathematician and professor at the University of Buenos Aires. In 2015, she received the TWAS (The World Academy of Sciences) Prize for mathematics, which is awarded to individuals from developing countries who make outstanding contributions to science or mathematics. In 2018, Dickenstein was invited to speak at the World Meeting for Women in Mathematics in Rio de Janeiro, Brazil.

Interactive Presentation



Explore, Slide 1 of 6



Explore, Slide 3 of 6

DRAG & DROP

On Slide 2, students drag the algebraic expression to the appropriate bin to represent each situation.

TYPE

On Slide 3, students enter the missing values in the table.

Explore Write and Solve Equations with Variables on Each Side

Objective

Students will explore how to write an equation with variables on each side to solve a real-world problem.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the cost to bowl at two bowling alleys, each with a charge per game and one with a fixed charge. Throughout this activity, students will evaluate the costs based on the number of games bowled, and they will write and solve an equation representing the point at which the two costs are the same.

Inquiry Question

Why is writing an equation a useful way to solve a real-world problem? Sample answer: It can take a long time to solve a real-world problem with a table. An equation that models the real-world problem can be a more efficient way to find a solution.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

Mathematical Discourse

When would you rather bowl at Bowling Alley A? Bowling Alley B? Justify your answer. See students' responses.

(continued on next page)

Explore Write and Solve Equations with Variables on Each Side *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to pay careful attention to the quantities in order to determine the expression that models the cost at each bowling alley.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

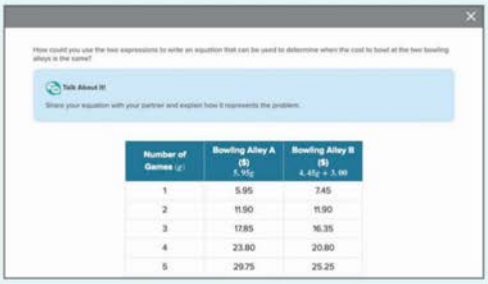
Talk About It!

SLIDE 5

Mathematical Discourse

Solve the equation $5.95g = 4.45g + 3.00$. What does the solution mean in the context of the problem? What are some advantages to using an equation to solve the problem, rather than a table? Sample answer: It means that the cost at each bowling alley is the same if you bowl 2 games. There is less work involved and less guessing and checking. Setting up and solving an equation is a more direct way of finding a solution.

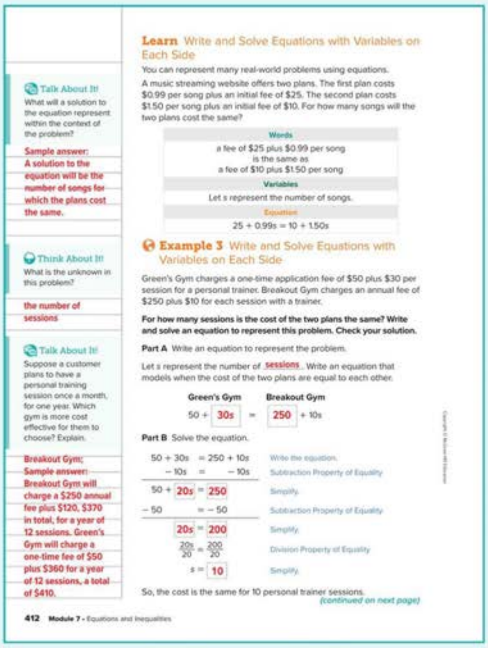
Interactive Presentation



Explore, Slide 4 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.



Learn Write and Solve Equations with Variables on Each Side

You can represent many real-world problems using equations.

A music streaming website offers two plans. The first plan costs $0.99 per song plus an initial fee of $25. The second plan costs $1.50 per song plus an initial fee of $10. For how many songs will the two plans cost the same?

Words
a fee of $25 plus $0.99 per song is the same as a fee of $10 plus $1.50 per song
Variables
Let s represent the number of songs.
Equation
$25 + 0.99s = 10 + 1.50s$

Talk About It! What will a solution to the equation represent within the context of the problem?

Sample answer: A solution to the equation will be the number of songs for which the plans cost the same.

Example 3 Write and Solve Equations with Variables on Each Side

Green's Gym charges a one-time application fee of $50 plus $30 per session for a personal trainer. Breakout Gym charges an annual fee of $250 plus $10 for each session with a trainer.

For how many sessions is the cost of the two plans the same? Write and solve an equation to represent this problem. Check your solution.

Think About It! What is the unknown in this problem?

the number of sessions

Part A Write an equation to represent the problem.

Let s represent the number of sessions. Write an equation that models when the cost of the two plans are equal to each other.

Green's Gym Breakout Gym

$50 + 30s = 250 + 10s$

Part B Solve the equation.

$50 + 30s = 250 + 10s$	Write the equation.
$-10s = -10s$	Subtraction Property of Equality
$50 + 20s = 250$	Simplify.
$-50 = -50$	Subtraction Property of Equality
$20s = 200$	Simplify.
$\frac{20s}{20} = \frac{200}{20}$	Division Property of Equality
$s = 10$	Simplify.

So, the cost is the same for 10 personal trainer sessions.

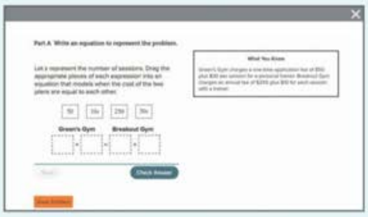
(continued on next page)

Talk About It! Suppose a customer plans to have a personal training session once a month, for one year. Which gym is more cost effective for them to choose? Explain.

Breakout Gym; Sample answer: Breakout Gym will charge a $250 annual fee plus $120, $370 in total, for a year of 12 sessions. Green's Gym will charge a one-time fee of $50 plus $360 for a year of 12 sessions, a total of $410.

412 Module 7 • Equations and Inequalities

Interactive Presentation



Example 3, Write and Solve Equations with Variables on Each Side, Slide 2 of 5

FLASHCARDS

On Slide 1 of the Learn, students use Flashcards to learn how to model a real-world problem with an equation with variables on each side.

DRAG & DROP

In Slide 2 of Example 3, students drag the terms to create the correct equation for the problem.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Write and Solve Equations with Variables on Each Side

Objective

Students will understand that they can model a real-world problem with an equation that has variables on each side.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

What will a solution to the equation represent within the context of the problem? Sample answer: A solution to the equation will be the number of songs for which the plans cost the same.

Example 3 Write and Solve Equations with Variables on Each Side

Objective

Students will write and solve equations with variables on each side that have integer coefficients.

Questions for Mathematical Discourse

SLIDE 2

AL What is the cost per session for each gym? Green's Gym charges $30 per session and Breakout Gym charges $10 per session.

AL Does either gym have an additional charge, other than the charge per session? Explain. yes; Green's Gym charges a one-time application fee of $50. Breakout Gym charges an annual fee of $250.

OL Can you write the equation as $50 + 30s = 250 + 10s$, or as $30s + 50 = 10s + 250$? Explain. yes; Since addition is commutative, I can write the sum of the terms on either side of the equals sign in any order.

BL Is the equation that represents the problem valid for any amount of time, such as 2 years? Explain. no; Sample answer: Breakout Gym charges the $250 fee annually, so the equation represents the problem when the time is one year or less.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Write and Solve Equations with Variables on Each Side

Objective

Students will write and solve equations with variables on each side that have rational coefficients.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to generate the correct equation to model the real-world problem, and interpret the solution to the equation within the real-world context.

While discussing the *Talk About It!* question, encourage students to make sense of the expressions that represent the cost for each car rental company, in order to determine which company is less expensive to rent for each number of miles traveled in one day.

6 Attend to Precision Students should be able to solve the equation efficiently and accurately, adhering to the properties of operations.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that 0.25 and 0.45 are coefficients, not constants? Sample answer: They both represent the costs *per mile*, and the number of miles is the variable.

AL Why aren't the quantities 40 and 25 coefficients, since they represent the costs per day? Sample answer: The variable quantity is the number of miles, not the number of days.

OL A classmate wrote the equation as $40 + 0.45m = 25 + 0.25m$. Describe the error that was made. Sample answer: The classmate wrote the wrong rate per mile with each daily rate. The rate per mile that goes with the daily rate of 40 is $0.25m$. The rate per mile that goes with the daily rate of 25 is $0.45m$.

BL Is the equation that represents the problem valid for any amount of time, such as 3 days? Explain. no; Sample answer: The fees \$40 and \$25 are fees per day, so the equation represents the problem when the time is one day or less.

BL Suppose that during very busy periods, each car company charges twice their normal daily rate, but the same rate per mile. What equation models the cost of the two rentals being equal during a busy period? $80 + 0.25m = 50 + 0.45m$

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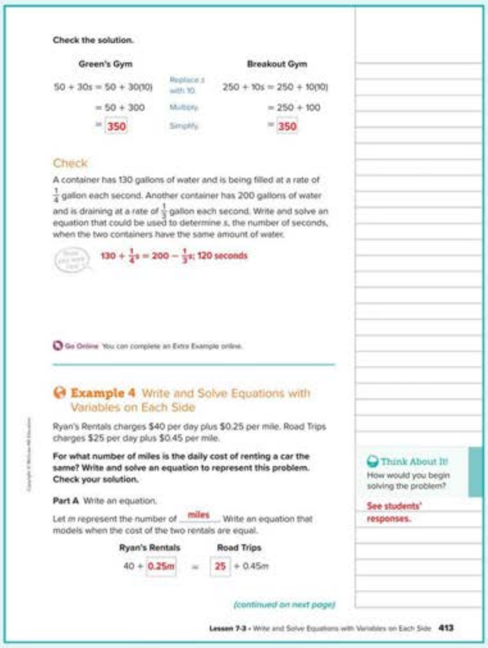

Check the solution.

Green's Gym		Breakout Gym
$50 + 30s = 50 + 30(10)$	Replace s with 10.	$250 + 10s = 250 + 10(10)$
$= 50 + 300$	Multiply.	$= 250 + 100$
$= 350$	Simplify.	$= 350$

Check

A container has 130 gallons of water and is being filled at a rate of $\frac{1}{4}$ gallon each second. Another container has 200 gallons of water and is draining at a rate of $\frac{1}{3}$ gallon each second. Write and solve an equation that could be used to determine s, the number of seconds, when the two containers have the same amount of water.

$130 + \frac{1}{4}s = 200 - \frac{1}{3}s$; 120 seconds

Go Online You can complete an Extra Example online.

Example 4 Write and Solve Equations with Variables on Each Side

Ryan's Rentals charges \$40 per day plus \$0.25 per mile. Road Trips charges \$25 per day plus \$0.45 per mile.

For what number of miles is the daily cost of renting a car the same? Write and solve an equation to represent this problem. Check your solution.

Part A Write an equation.

Let m represent the number of miles. Write an equation that models when the cost of the two rentals are equal.

Ryan's Rentals = Road Trips

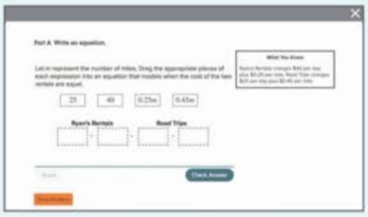
$40 + 0.25m = 25 + 0.45m$

Think About It! How would you begin solving the problem? See students' responses.

(continued on next page)

Lesson 7-3 • Write and Solve Equations with Variables on Each Side 413

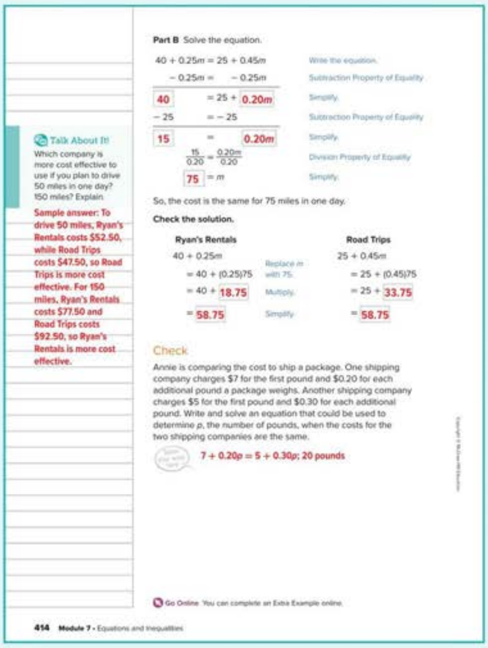
Interactive Presentation



Example 4, Write and Solve Equations with Variables on Each Side, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag the terms to create the correct equation for the problem.



Part B Solve the equation.

$40 + 0.25m = 25 + 0.45m$	Write the equation.
$-0.25m = \quad -0.25m$	Subtraction Property of Equality
$40 = 25 + 0.20m$	Simplify.
$-25 = -25$	Subtraction Property of Equality
$15 = 0.20m$	Simplify.
$\frac{15}{0.20} = \frac{0.20m}{0.20}$	Division Property of Equality
$75 = m$	Simplify.

So, the cost is the same for 75 miles in one day.

Check the solution.

Ryan's Rentals		Road Trips
$40 + 0.25m$		$25 + 0.45m$
$= 40 + (0.25)75$	Replace m with 75.	$= 25 + (0.45)75$
$= 40 + 18.75$	Multiply.	$= 25 + 33.75$
$= 58.75$	Simplify.	$= 58.75$

Talk About It! Which company is more cost effective to use if you plan to drive 50 miles in one day? 150 miles? Explain.

Sample answer: To drive 50 miles, Ryan's Rentals costs $52.50, while Road Trips costs $47.50, so Road Trips is more cost effective. For 150 miles, Ryan's Rentals costs $77.50 and Road Trips costs $92.50, so Ryan's Rentals is more cost effective.

Check

Annie is comparing the cost to ship a package. One shipping company charges $7 for the first pound and $0.20 for each additional pound a package weighs. Another shipping company charges $5 for the first pound and $0.30 for each additional pound. Write and solve an equation that could be used to determine p, the number of pounds, when the costs for the two shipping companies are the same.

$7 + 0.20p = 5 + 0.30p$; 20 pounds

Go Online You can complete an Extra Example online.

414 Module 7 • Equations and Inequalities

Interactive Presentation

Example 2, Write and Solve Equations with Variables on Each Side, Slide 3 of 5

TYPE

On Slide 3, students determine the solution of the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Write and Solve Equations with Variables on Each Side *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL How do you know when you have finished solving the equation? Sample answer: When the variable m is isolated on one side of the equals sign, and a number is on the other side, then I know I have finished solving the equation.

OL Can you solve this equation differently? Explain. yes; Sample answer: I could have subtracted $0.45m$ from each side first. Or I could have subtracted 25 from each side first, or 40 from each side first.

OL How can you check your answer? Sample answer: Substitute the solution into each side of the equation to verify it is a true statement.

BL Can there be any other number of miles for which the cost of a one-day rental from each company will be the same? Explain. No; Sample answer: The equation has only one solution, so the cost for using each company is only the same for 75 miles.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Home Improvement

Objective

Students will come up with their own strategy to solve an application problem that involves calculating the total cost of carpeting a living room.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

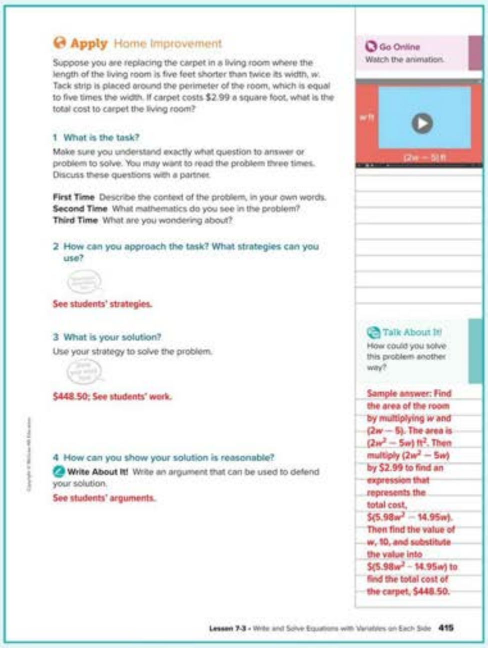
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the equation for the perimeter of the living room?
- What are the measurements of the living room?
- What dimensions are needed for calculating the area of the living room?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Home Improvement

Suppose you are replacing the carpet in a living room where the length of the living room is five feet shorter than twice its width, w. Tack strip is placed around the perimeter of the room, which is equal to five times the width. If carpet costs \$2.99 a square foot, what is the total cost to carpet the living room?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

\$448.50; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It! How could you solve this problem another way?

Sample answer: Find the area of the room by multiplying w and $(2w - 5)$. The area is $(2w^2 - 5w)$ ft^2. Then multiply $(2w^2 - 5w)$ by \$2.99 to find an expression that represents the total cost, $\$(5.98w^2 - 14.95w)$. Then find the value of w, 10, and substitute the value into $\$(5.98w^2 - 14.95w)$ to find the total cost of the carpet, \$448.50.

Lesson 7-3 • Write and Solve Equations with Variables on Each Side 415

Interactive Presentation

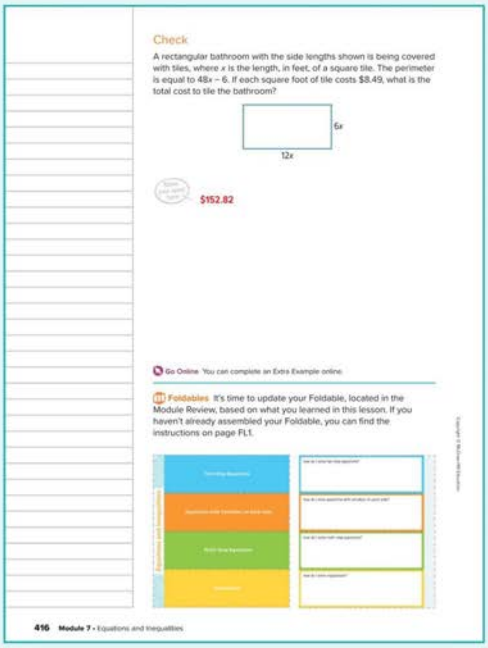
Apply, Home Improvement

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

A rectangular bathroom with the side lengths shown is being covered with tiles, where x is the length, in feet, of a square tile. The perimeter is equal to $48x - 6$. If each square foot of tile costs $8.49, what is the total cost to tile the bathroom?

6x
12x

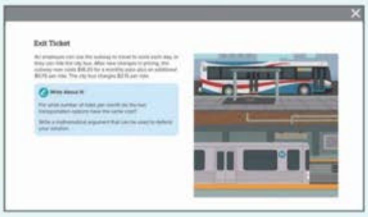
$152.82

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

416 Module 7 • Equations and Inequalities

Interactive Presentation



Exit Ticket

Essential Question Follow-Up

How can equations be used to solve everyday problems?

In this lesson, students learned how to write equations with variables on each side. Some of the examples compared two different scenarios to determine when the two would be equal. Encourage them to discuss with a partner when this is beneficial in real life. For example, you know in Example 1, the two costs are the same when you pay for 10 training sessions. They could use the solution as a starting point to find which one would cost less if they wanted more than 10 training sessions.

Exit Ticket

Refer to the Exit Ticket slide. For what number of rides per month do the two transportation options have the same cost? Write a mathematical argument that can be used to defend your solution. 13; Sample answer: The expression $18.20 + 0.75x$ gives the cost to ride the subway x times in one month. The expression $2.15x$ gives the cost to ride the city bus x times in one month. Use the equation $18.20 + 0.75x = 2.15x$ to find the number of rides when the costs are the same. By solving the equation, $x = 13$. So, for 13 rides, it costs the same to ride the subway or the bus.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–11 odd, 12
- Extension : Extension Resources
- ALEKS Equations with Variables on Both Sides, Applications of Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 9, 12
- Extension : Extension Resources
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Equations with Variables on Both Sides

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Equations with Variables on Both Sides

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

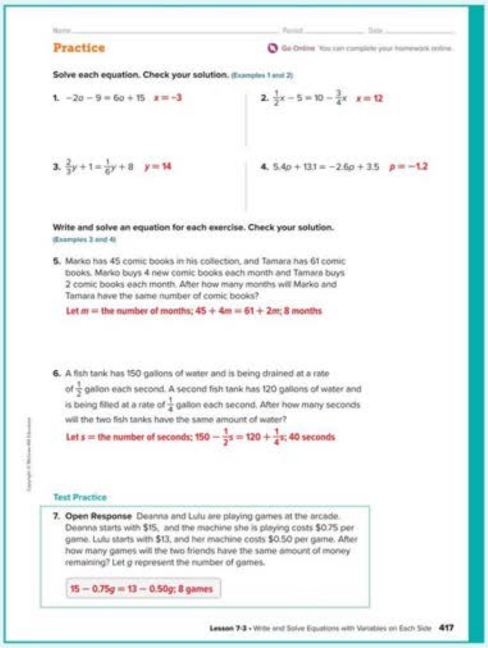
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve equations with variables on each side	1–4
2	write and solve equations with variables on each side that have rational coefficients	5, 6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems that involve solving equations with variables on each side	8, 9
3	higher-order and critical thinking skills	10–12

Common Misconception

Some students may incorrectly use the Subtraction Property of Equality by only subtracting a value from one side of the equation. Remind students that to use the Subtraction Property of Equality correctly, they must subtract the same value from each side of the equation. Encourage them to understand and be able to explain that if they perform one operation to one side of an equation, they must perform the same operation to the other side, in order for the equation to remain true.



Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Solve each equation. Check your solution. (Examples 1 and 2)

1. $-2x - 9 = 6x + 15$ $x = -3$
2. $\frac{1}{2}x - 5 = 10 - \frac{3}{4}x$ $x = 12$
3. $\frac{2}{3}y + 1 = \frac{1}{6}y + 8$ $y = 14$
4. $5.4p + 13.1 = -2.6p + 3.5$ $p = -1.2$

Write and solve an equation for each exercise. Check your solution. (Examples 3 and 4)

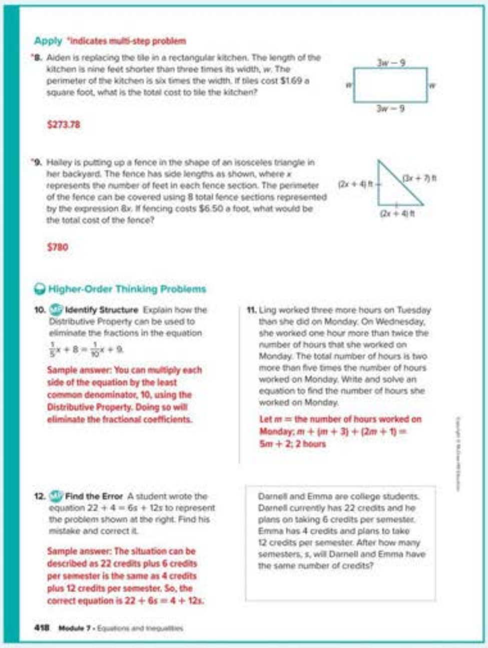
5. Marko has 45 comic books in his collection, and Tamara has 61 comic books. Marko buys 4 new comic books each month and Tamara buys 2 comic books each month. After how many months will Marko and Tamara have the same number of comic books?
Let m = the number of months; $45 + 4m = 61 + 2m$; 8 months

6. A fish tank has 150 gallons of water and is being drained at a rate of $\frac{1}{2}$ gallon each second. A second fish tank has 120 gallons of water and is being filled at a rate of $\frac{1}{4}$ gallon each second. After how many seconds will the two fish tanks have the same amount of water?
Let s = the number of seconds; $150 - \frac{1}{2}s = 120 + \frac{1}{4}s$; 40 seconds

Test Practice

7. **Open Response** Deanna and Lulu are playing games at the arcade. Deanna starts with \$15, and the machine she is playing costs \$0.75 per game. Lulu starts with \$13, and her machine costs \$0.50 per game. After how many games will the two friends have the same amount of money remaining? Let g represent the number of games.
$15 - 0.75g = 13 - 0.50g$; 8 games

Lesson 7-3 • Write and Solve Equations with Variables on Each Side **417**



Apply *indicates multi-step problem

*8. Aiden is replacing the tile in a rectangular kitchen. The length of the kitchen is nine feet shorter than three times its width, w. The perimeter of the kitchen is six times the width. If tiles cost $1.69 a square foot, what is the total cost to tile the kitchen?

$273.78

*9. Hailey is putting up a fence in the shape of an isosceles triangle in her backyard. The fence has side lengths as shown, where x represents the number of feet in each fence section. The perimeter of the fence can be covered using 8 total fence sections represented by the expression $8x$. If fencing costs $6.50 a foot, what would be the total cost of the fence?

$780

Higher-Order Thinking Problems

10. MP Identify Structure Explain how the Distributive Property can be used to eliminate the fractions in the equation $\frac{1}{5}x + 8 = \frac{1}{10}x + 9$.

Sample answer: You can multiply each side of the equation by the least common denominator, 10, using the Distributive Property. Doing so will eliminate the fractional coefficients.

11. Ling worked three more hours on Tuesday than she did on Monday. On Wednesday, she worked one hour more than twice the number of hours that she worked on Monday. The total number of hours is two more than five times the number of hours worked on Monday. Write and solve an equation to find the number of hours she worked on Monday.

Let m = the number of hours worked on Monday; $m + (m + 3) + (2m + 1) = 5m + 2$; 2 hours

12. MP Find the Error A student wrote the equation $22 + 4 = 6s + 12s$ to represent the problem shown at the right. Find his mistake and correct it.

Darnell and Emma are college students. Darnell currently has 22 credits and he plans on taking 6 credits per semester. Emma has 4 credits and plans to take 12 credits per semester. After how many semesters, s, will Darnell and Emma have the same number of credits?

Sample answer: The situation can be described as 22 credits plus 6 credits per semester is the same as 4 credits plus 12 credits per semester. So, the correct equation is $22 + 6s = 4 + 12s$.

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MP Teaching the Mathematical Practices

7 Look for and Make Use of Structure In E xercise 10, students will explain how the Distributive Property can be used to eliminate the fractions in the equation. Encourage students to use the similar structure in the fractions to eliminate them.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students will find the mistake in the problem and correct it. Encourage students to determine the error by analyzing the real-world problem and explain how they could fix it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 8-9 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 12 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that the terms in the equation can be interchanged. Have each pair or group of students present their explanations to the class.

Lesson 7-4

Write and Solve Multi-Step Equations

8.EE.C.7

LESSON GOAL

Students will write and solve multi-step equations with variables on each side.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Solve Multi-Step Equations
Example 1: Solve Multi-Step Equations
Example 2: Solve Multi-Step Equations
Example 3: Solve Multi-Step Equations

Explore: Translate Problems into Equations

Learn: Write and Solve Multi-Step Equations
Example 4: Write and Solve Multi-Step Equations
Example 5: Write and Solve Multi-Step Equations
Apply: Business Finance

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 41 of the *Language Development Handbook* to help your students build mathematical language related to writing and solving multi-step equations.

ELL You can use the tips and suggestions on page T41 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.C** by writing and solving multi-step equations with variables on each side.
Standards for Mathematical Content: **8.EE.C.7, 8.EE.C.7.B,** *Also addresses 8.EE.C.7.A*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP7**

Coherence

Vertical Alignment

Previous
Students wrote and solved equations with variables on each side.
8.EE.C.7, 8.EE.C.7.B

Now
Students write and solve multi-step equations.
8.EE.C.7, 8.EE.C.7.B

Next
Students will determine the number of solutions to an equation.
8.EE.C.7, 8.EE.C.7.A

Rigor

The Three Pillars of Rigor

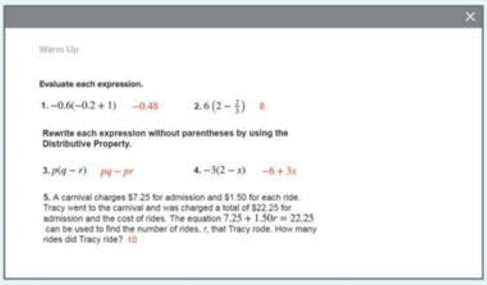
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their *fluency* of solving equations with variables on each side. Students use their understanding of the Distributive Property and combining like terms to simplify each side of a multi-step equation before solving it. They *apply* their fluency with multi-step equations to write and solve equations that model real-world problems.

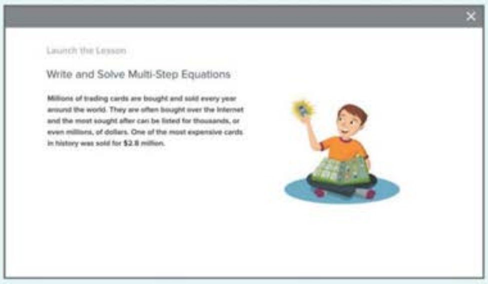
Mathematical Background

In an equation or expression, *like terms* are monomials of the same power. To solve multi-step equations, grouping symbols such as parentheses can often be eliminated using the Distributive Property: $a(b + c) = ab + ac$. The properties of equality should be used as well as combining like terms, which is adding the like terms on one side of the equation, in order to isolate the variable on one side.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- using operations with rational numbers (Exercises 1–2)
- using the Distributive Property (Exercises 3–4)
- solving two-step equations with rational coefficients (Exercise 5)

Answers

1. -0.48
2. 8
3. $pq - pr$
4. $-6 + 3x$
5. 10

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the cost of buying and shipping trading cards, using a multi-step equation.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Give an example of two like terms. Then give an example of two terms that are not alike. Explain. Sample answer: The terms $3x$ and $5x$ are alike because they have the same variables, raised to the same power (the power of 1, in this case). The terms $3x$ and $5xy$ are not alike because they do not have the same variables.

Learn Solve Multi-Step Equations

Objective

Students will learn how to solve multi-step equations with variables on each side, and grouping symbols on one or both sides.

Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* question on Slide 2, encourage students to use clear and precise mathematical language, such as *substitute* or *replace*, when describing how they can check to verify they solved the equation correctly.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates solving a multi-step equation.

Talk About It!

SLIDE 2

Mathematical Discourse

How can you make sure that you solved the equation correctly?
Sample answer: Substitute the value of the variable, $x = -4$, into the original equation. Simplify the expressions on each side of the equals sign to verify that the statement is true.

DIFFERENTIATE

Enrichment Activity BL

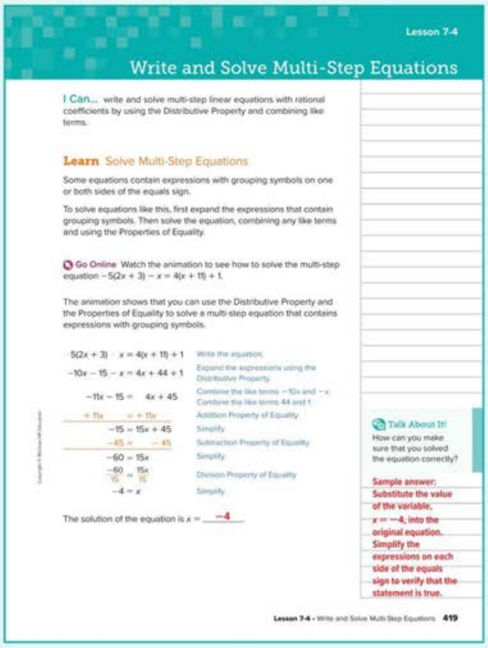
If any of your students need more of a challenge, have students determine which of the following equations, if any, have the same solution. They should be able to explain and defend their reasoning.

A. $-3(4x + 3) + 4(6x + 1) = 43$

B. $5x + 34 - 2x = -2(1 - 7x) - 2x$

C. $-5(1 - 5x) + 5(-8x - 2) = -4x - 8x$

A and B have the same solution; See students' explanations.



Lesson 7-4

Write and Solve Multi-Step Equations

I Can... write and solve multi-step linear equations with rational coefficients by using the Distributive Property and combining like terms.

Learn Solve Multi-Step Equations

Some equations contain expressions with grouping symbols on one or both sides of the equals sign.

To solve equations like this, first expand the expressions that contain grouping symbols. Then solve the equation, combining any like terms and using the Properties of Equality.

Go Online Watch the animation to see how to solve the multi-step equation $-5(2x + 3) - x = 4(x + 11) + 1$.

The animation shows that you can use the Distributive Property and the Properties of Equality to solve a multi-step equation that contains expressions with grouping symbols.

$-5(2x + 3) - x = 4(x + 11) + 1$	Write the equation.
$-10x - 15 - x = 4x + 44 + 1$	Expand the expressions using the Distributive Property.
$-11x - 15 = 4x + 45$	Combine the like terms $-10x$ and $-x$. Combine the like terms 44 and 1.
$+11x \quad = +11x$	Addition Property of Equality
$-15 = 15x + 45$	Simplify.
$-45 = \quad -45$	Subtraction Property of Equality
$-60 = 15x$	Simplify.
$\frac{-60}{15} = \frac{15x}{15}$	Division Property of Equality
$-4 = x$	Simplify.

The solution of the equation is $x =$ -4.

Talk About It! How can you make sure that you solved the equation correctly?

Sample answer: Substitute the value of the variable, $x = -4$, into the original equation. Simplify the expressions on each side of the equals sign to verify that the statement is true.

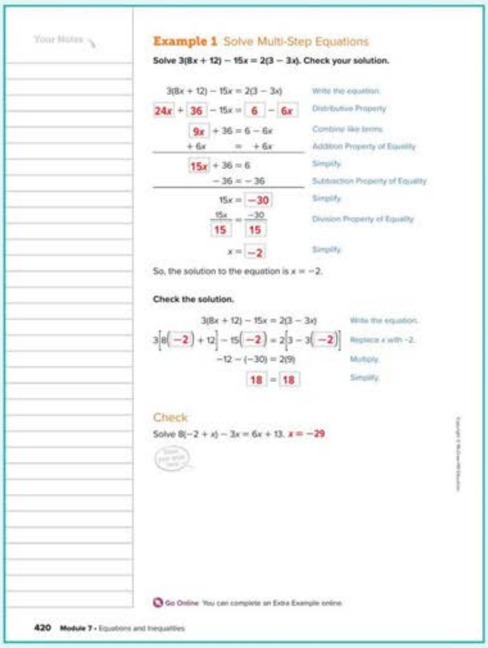
Lesson 7-4 • Write and Solve Multi-Step Equations 419

Interactive Presentation

Learn, Solve Multi-Step Equations, Slide 1 of 2

WATCH

On Slide 1, students watch the animation to see how to solve a multi-step equation.


Your Notes

Example 1 Solve Multi-Step Equations

Solve $3(8x + 12) - 15x = 2(3 - 3x)$. Check your solution.

$3(8x + 12) - 15x = 2(3 - 3x)$	Write the equation.
$24x + 36 - 15x = 6 - 6x$	Distributive Property
$9x + 36 = 6 - 6x$	Combine like terms.
$+ 6x \quad = \quad + 6x$	Addition Property of Equality
$15x + 36 = 6$	Simplify.
$- 36 = - 36$	Subtraction Property of Equality
$15x = -30$	Simplify.
$\frac{15x}{15} = \frac{-30}{15}$	Division Property of Equality
$x = -2$	Simplify.

So, the solution to the equation is $x = -2$.

Check the solution.

$3(8x + 12) - 15x = 2(3 - 3x)$	Write the equation.
$3[8(-2) + 12] - 15(-2) = 2[3 - 3(-2)]$	Replace x with -2.
$-12 - (-30) = 2(9)$	Multiply.
$18 = 18$	Simplify.

Check

Solve $8(-2 + x) - 3x = 6x + 13$. $x = -29$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 1, Solve Multi-Step Equations, Slide 1 of 2

CLICK On Slide 1, students move through the steps to solve the equation.

TYPE On Slide 1, students determine the solution of the equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Solve Multi-Step Equations

Objective

Students will solve multi-step equations with variables on each side that have integer coefficients.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to reason whether they can use the Distributive Property to expand each expression.

6 Attend to Precision Students should be able to fluently simplify each side of the equation and accurately use the properties of operations to isolate the variable.

7 Look for and Make Use of Structure Encourage students to study the structure of the equation in order to determine that they can use the Distributive Property to expand each expression.

Questions for Mathematical Discourse

SLIDE 1

AL Study the structure of the equation. Why is using the Distributive Property helpful? There are two sets of parentheses, one on either side of the equals sign. Use the Distributive Property to expand each expression.

OL After expanding $3(8x + 12)$, what remains on the left side of the equation? $24x + 36 - 15x$

OL In the equation $24x + 36 - 15x = 6 - 6x$, what is another step you can take if you do not combine like terms first? Sample answer: Add $6x$ to each side. Then combine like terms.

BL Generate an equation, with variables on each side of the equals sign, in which you need to use the Distributive Property at least twice in order to solve the equation. Sample answer: $4(5x - 10) + 2x = 6(x - 12) - 16$

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Solve Multi-Step Equations

Objective

Students will solve multi-step equations with variables on each side that have rational coefficients written as decimals.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively St udents should determine that they can find the additive inverse of the expression on the right side of the equation.

6 Attend to Precision Students should be able to fluently simplify each side of the equation and accurately use the properties of operations to isolate the variable. While discussing the *Talk About It!* question on Slide 3, encourage students to use clear and precise mathematical language to explain how additive inverses are found and why the concept applies in this situation.

7 Look for and Make Use of Structure Encourage students to study the structure of the equation in order to determine that they can use the Distributive Property to expand the expression on the left side of the equation.

Questions for Mathematical Discourse

SLIDE 2

AL On the left side of the equation, what number is distributed when the expression $0.3(10 - 5x)$ is expanded? The number 0.3 is distributed to each term inside the parentheses.

OL On the right side of the equation, why can you write $-(8x + 9)$ as $(-8x - 9)$? Sample answer: The expression $8x + 9$ is being subtracted. To subtract an expression, add its additive inverse. The additive inverse of $8x + 9$ is $-8x - 9$.

BL A classmate claimed that you can use the Distributive Property to expand $-(8x + 9)$ as $-8x - 9$ by distributing the number -1 to each term inside the parentheses. Is the classmate correct? Explain. yes; Sample answer: $-(8x + 9) = -1(8x + 9)$ which means that you can distribute the -1 to both $8x$ and 9.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Solve Multi-Step Equations

Solve $0.3(10 - 5x) = 31.5 - (8x + 9)$. Check your solution.

$0.3(10 - 5x) = 31.5 - (8x + 9)$	Write the equation.
$3 - 1.5x = 31.5 - (8x + 9)$	Distributive Property
$3 - 1.5x = 31.5 + (-8x - 9)$	Rewrite using the additive inverse.
$3 - 1.5x = 22.5 - 8x$	Combine like terms.
$-3 \quad = -3$	Subtraction Property of Equality
$-1.5x = 19.5 - 8x$	Simplify.
$+8x = \quad +8x$	Addition Property of Equality
$6.5x = 19.5$	Simplify.
$\frac{6.5x}{6.5} = \frac{19.5}{6.5}$	Division Property of Equality
$x = 3$	Simplify.

So, the solution to the equation is $x = 3$.

Check the solution.

To check your solution, replace x with 3 in the original equation.

$0.3(10 - 5x) = 31.5 - (8x + 9)$	Write the equation.
$0.3(10 - 5 \cdot 3) = 31.5 - (8 \cdot 3 + 9)$	Substitute.
$0.3(10 - 15) = 31.5 - (24 + 9)$	Multiply.
$0.3(-5) = 31.5 - 33$	Simplify.
$-1.5 = -1.5$	Simplify.

Check

Solve $15 - (7x + 20) = 0.5(6x - 14)$. $x = 0.2$

Go Online You can complete an Extra Example online.

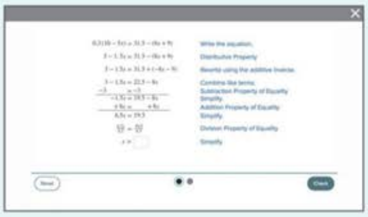
Think About It!
How would you begin solving the equation?

See students' responses.

Talk About It!
How does the Additive Inverse Property allow the parentheses to be removed in order to simplify the right side of the equation?

Sample answer: The negative sign outside of the parentheses indicates to add the additive inverse of the expression.

Interactive Presentation



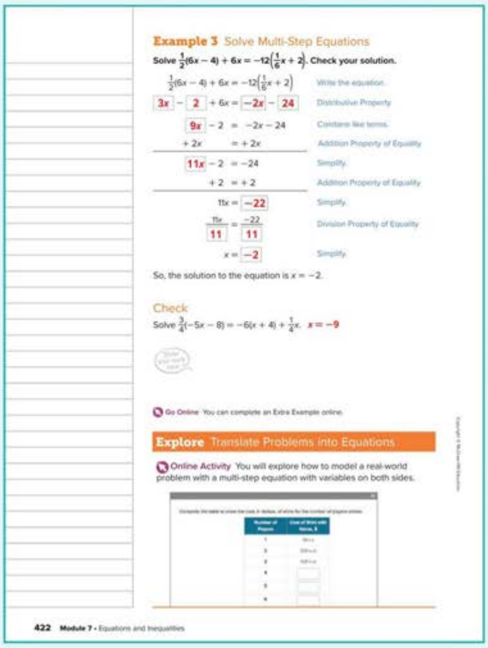
Example 2, Solve Multi-Step Equations, Slide 2 of 4

TYPE

On Slide 2, students determine the solution of the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Solve Multi-Step Equations

Solve $\frac{1}{2}(6x - 4) + 6x = -12\left(\frac{1}{6}x + 2\right)$. Check your solution.

$\frac{1}{2}(6x - 4) + 6x = -12\left(\frac{1}{6}x + 2\right)$ Write the equation.

$3x - 2 + 6x = -2x - 24$ Distributive Property

$9x - 2 = -2x - 24$ Combine like terms.

$+2x \quad = +2x$ Addition Property of Equality

$11x - 2 = -24$ Simplify.

$+2 = +2$ Addition Property of Equality

$11x = -22$ Simplify.

$\frac{11x}{11} = \frac{-22}{11}$ Division Property of Equality

$x = -2$ Simplify.

So, the solution to the equation is $x = -2$.

Check

Solve $\frac{3}{4}(-5x - 8) = -6(x + 4) + \frac{1}{4}x$. $x = -9$

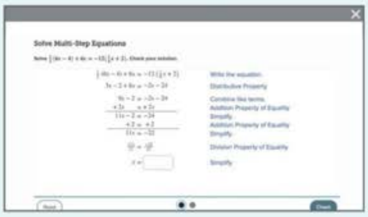
Go Online You can complete an Extra Example online.

Explore Translate Problems into Equations

Online Activity You will explore how to model a real-world problem with a multi-step equation with variables on both sides.

422 Module 7 • Equations and Inequalities

Interactive Presentation



Example 3, Solve Multi-Step Equations, Slide 1 of 2

TYPE

On Slide 1, students determine the solution of the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Solve Multi-Step Equations

Objective

Students will solve multi-step equations with variables on each side that have rational coefficients written as fractions.

Questions for Mathematical Discourse

SLIDE 1

AL On the left side of the equation, what number is distributed when the expression $\frac{1}{2}(6x - 4)$ is expanded? The number $\frac{1}{2}$ is distributed to each term inside the parentheses.

AL On the right side of the equation, what number is distributed when the expression $-12\left(\frac{1}{6}x + 2\right)$ is expanded? The number -12 is distributed to each term inside the parentheses.

OL How can you mentally distribute $\frac{1}{2}$ to each term inside the parentheses of the expression $\frac{1}{2}(6x - 4)$? Sample answer: One half of $6x$ is $3x$, and one half of 4 is 2. So, one half of the expression $6x - 4$ is $3x - 2$.

OL How can you mentally distribute -12 to each term inside the parentheses of the expression $-12\left(\frac{1}{6}x + 2\right)$? Sample answer: One sixth of -12 is -2, and -12 times 2 is -24. So, $-12\left(\frac{1}{6}x + 2\right) = -2x - 24$.

BL A classmate found the LCD of $\frac{1}{2}$ and $\frac{1}{6}$ and multiplied both sides of the equation by 6 to eliminate the fractions. If they performed all of the calculations correctly, what would be the resulting equation after the multiplication and use of the Distributive Property was complete? $18x - 12 + 36x = -12x - 144$

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Translate Problems into Equations

Objective

Students will explore how to model a real-world problem with a multi-step equation with variables on both sides.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with information about the cost of shirts for a lacrosse team. Throughout this activity, students will choose variables and write expressions for the cost of shirts for different numbers of players, and write equations to include the total cost.

Inquiry Question

How can you translate a real-world problem into a multi-step equation? Sample answer: Identify the relevant given and unknown information, define any variables for the unknown information, and then use the constants and the variables to write an equation that relates the quantities.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

Refer to the original problem. Do you know how many players there are on the lacrosse team? How could you alter the expression $20 + n$ to represent the total cost for any number of players? What do you need to do first, before writing the expression? Sample answer: The number of players on the team is unknown. The expression $20 + n$ could be multiplied by p players to represent the total cost for any number of players. To write the expression that represents the total cost for any number of players I can use parentheses around $20 + n$ to indicate that p is multiplied by the entire value $20 + n$.

(continued on next page)

Interactive Presentation

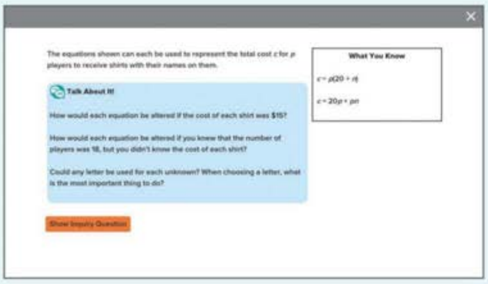
Explore, Slide 1 of 6

Explore, Slide 3 of 6

TYPE

On Slide 3, student enter missing values into the table.

Interactive Presentation



Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Translate Problems into Equations *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to practice writing expressions with known values and then generalizing those expressions by using variables.

Encourage students to think about the correspondences between the variables, expressions, and equations that would help to translate the real-world scenario into a multi-step equation.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

How would each equation be altered if the cost of each shirt was $15? Sample answer: The equation would become $c = p(15 + n)$ if the cost of each shirt were $15.

How would each equation be altered if you knew that the number of players was 18, but you didn't know the cost of each shirt? Sample answer: If the number of players was 18, then 18 could replace p in the equation and x could replace the cost of each shirt in the equation. The equation would be written as $c = 18(x + n)$.

Could any letter be used for each unknown? When choosing a letter, what is the most important thing to do? Sample answer: When writing an expression, any letter can be used as a variable. It is important to use a different variable for each value and to assign letters in a way that makes it clear what they represent.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Write and Solve Multi-Step Equations

Objective

Students will understand that they can model and solve a real-world problem by using a multi-step equation that has variables on each side.

Go Online to find additional teaching notes and Teaching the Mathematical practices.

Talk About It!

SLIDE 2

Mathematical Discourse

Describe another real-world situation in which you might need to solve a multi-step equation in order to solve a problem. See students' responses.

Example 4 Write and Solve Multi-Step Equations

Objective

Students will model and solve a real-world problem by using a multi-step equation with integer coefficients, variables on both sides, and grouping symbols.

Teaching the Mathematical Practices

4 Model with Mathematics Encourage students to model the real-world situation with a correct multi-step equation.

7 Look For and Make Use of Structure Students should study the structure of the equation in order to determine that they can use the Distributive Property to expand the expression on the right side of the equation. As students discuss the *Talk About It!* question on Slide 4, encourage them to think about the form of the equation and how it might be alternatively factored or expanded.

Questions for Mathematical Discourse

SLIDE 2

AL What is the formula for the perimeter of a rectangle? Sample answer: $P = 2\ell + 2w$

AL Why is it useful to write w in terms of ℓ? Sample answer: The perimeter formula has two variables, but I need to write an equation with one variable so that I can solve it.

OL What is the width w equal to in terms of ℓ? Explain how you determined this. $2\ell - 11\frac{1}{2}$; The width of the garden is twice the length minus $11\frac{1}{2}$ feet.

BL Can you write an equation in terms of the width, instead of the length? Explain. yes; Sample answer: If $w = 2\ell - 11\frac{1}{2}$, then $\ell = \frac{1}{2}\left(w + 11\frac{1}{2}\right)$

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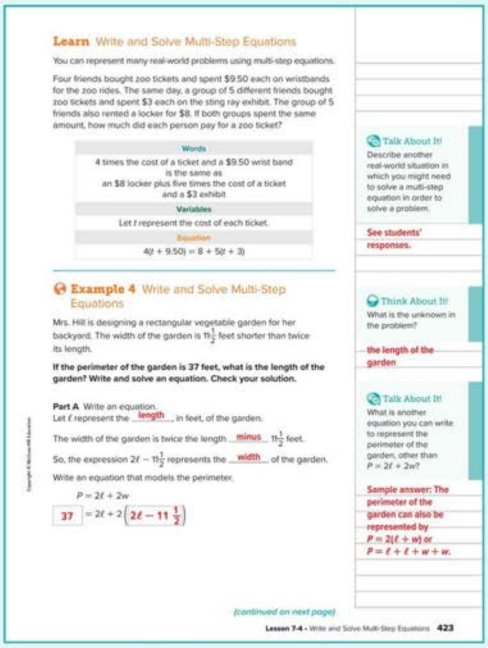


Learn Write and Solve Multi-Step Equations

You can represent many real-world problems using multi-step equations.

Four friends bought zoo tickets and spent \$9.50 each on wristbands for the zoo rides. The same day, a group of 5 different friends bought zoo tickets and spent \$3 each on the sting ray exhibit. The group of 5 friends also rented a locker for \$8. If both groups spent the same amount, how much did each person pay for a zoo ticket?

Words
4 times the cost of a ticket and a \$9.50 wrist band is the same as an \$8 locker plus five times the cost of a ticket and a \$3 exhibit
Variables
Let t represent the cost of each ticket.
Equation
$4(t + 9.50) = 8 + 5(t + 3)$

Talk About It! Describe another real-world situation in which you might need to solve a multi-step equation in order to solve a problem. See students' responses.

Example 4 Write and Solve Multi-Step Equations

Mrs. Hill is designing a rectangular vegetable garden for her backyard. The width of the garden is $11\frac{1}{2}$ feet shorter than twice its length.

If the perimeter of the garden is 37 feet, what is the length of the garden? Write and solve an equation. Check your solution.

Think About It! What is the unknown in the problem? the length of the garden

Part A Write an equation.

Let ℓ represent the length, in feet, of the garden.

The width of the garden is twice the length minus $11\frac{1}{2}$ feet.

So, the expression $2\ell - 11\frac{1}{2}$ represents the width of the garden.

Write an equation that models the perimeter.

$P = 2\ell + 2w$

$37 = 2\ell + 2\left(2\ell - 11\frac{1}{2}\right)$

Talk About It! What is another equation you can write to represent the perimeter of the garden, other than $P = 2\ell + 2w$? Sample answer: The perimeter of the garden can also be represented by $P = 2(\ell + w)$ or $P = \ell + \ell + w + w$.

(continued on next page)

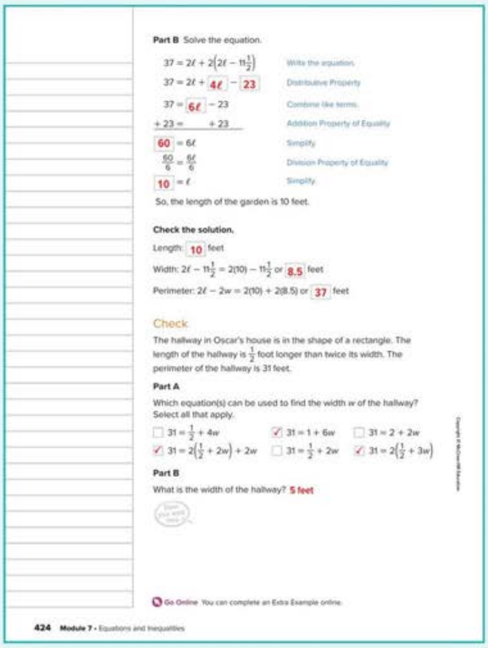
Lesson 7-4 • Write and Solve Multi-Step Equations 423

Interactive Presentation

Learn, Write and Solve Multi-Step Equations, Slide 1 of 2

FLASHCARDS

On Slide 1 of the Learn, students use Flashcards to view the steps used to write an equation that models a real-world problem.



Part B Solve the equation.

$37 = 2\ell + 2\left(2\ell - 11\frac{1}{2}\right)$	Write the equation.
$37 = 2\ell + \boxed{4\ell} - \boxed{23}$	Distributive Property
$37 = \boxed{6\ell} - 23$	Combine like terms.
$+23 = \quad +23$	Addition Property of Equality
$\boxed{60} = 6\ell$	Simplify.
$\frac{60}{6} = \frac{6\ell}{6}$	Division Property of Equality
$\boxed{10} = \ell$	Simplify.

So, the length of the garden is 10 feet.

Check the solution.

Length: $\boxed{10}$ feet

Width: $2\ell - 11\frac{1}{2} = 2(10) - 11\frac{1}{2}$ or $\boxed{8.5}$ feet

Perimeter: $2\ell - 2w = 2(10) + 2(8.5)$ or $\boxed{37}$ feet

Check

The hallway in Oscar's house is in the shape of a rectangle. The length of the hallway is $\frac{1}{2}$ foot longer than twice its width. The perimeter of the hallway is 31 feet.

Part A

Which equation(s) can be used to find the width w of the hallway? Select all that apply.

- [ ] $31 = \frac{1}{2} + 4w$
- [x] $31 = 1 + 6w$
- [ ] $31 = 2 + 2w$
- [x] $31 = 2\left(\frac{1}{2} + 2w\right) + 2w$
- [ ] $31 = \frac{1}{2} + 2w$
- [x] $31 = 2\left(\frac{1}{2} + 3w\right)$

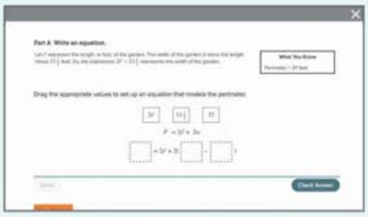
Part B

What is the width of the hallway? **5 feet**

Go Online You can complete an Extra Example online.

424 Module 7 • Equations and Inequalities

Interactive Presentation



Example 4, Write and Solve Multi-Step Equations, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag the appropriate values to set up an equation that models the situation.

TYPE

On Slide 3, students determine the length of the garden.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Write and Solve Multi-Step Equations *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Why is the Distributive Property needed to simplify the right side of the equation? Sample answer: The parentheses indicate that the entire expression $2\ell - 11\frac{1}{2}$ is multiplied by 2, so the Distributive Property is needed to expand $2\left(2\ell - 11\frac{1}{2}\right)$

OL If the solution represents the length, what does the expression $2\ell + 2\left(2\ell - 11\frac{1}{2}\right)$ represent? the perimeter

BL If the length is 10 feet, what is the width? 8.5 feet

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 5 Write and Solve Multi-Step Equations

Objective

Students will model and solve a real-world problem by using a multi-step equation with rational coefficients, variables on both sides, and grouping symbols.

Questions for Mathematical Discourse

SLIDE 2

AL How many students are going to the science center? How many are going to the art museum? 20 students are going to the science center, 15 students are going to the art museum

OL Why does the expression $x + 2.50$ represent the total cost of one student's trip to the science center? The cost for each student is the entry fee, x, plus the cost of the 3-D movie, \$2.50.

OL Why is the total cost for 20 students to go to the science center represented by $20(x + 2.50)$, instead of $20x + 2.50$? Each student must pay $x + 2.50$, so 20 students will pay a total of $20(x + 2.50)$. The expression $20x + 2.50$ represents 20 entry fees plus only one 3-D movie.

BL Suppose only 19 students are going to the science center and 23 students are going to the art museum. What equation would model this situation? $19(x + 2.50) = 23(2.5x + 1)$

SLIDE 3

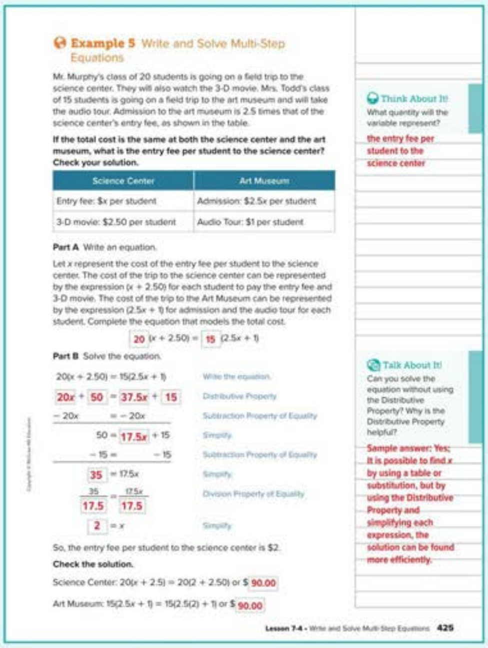
AL How many times will you use the Distributive Property to expand an expression before solving this equation? Explain. twice; Sample answer: There are two sets of parentheses, one on each side of the equation.

OL After expanding each expression, is there another way to continue to solve the equation? Explain. yes; Sample answer: I can subtract $37.5x$ from each side of the equation, or I can subtract 50 from each side, or I can subtract 15 from each side.

BL What is the admission cost of the art museum? How did you determine this? \$5; The admission cost of the art museum is 2.5 times that of the entry fee to the science center, and $2.5(\$2) = \$5$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 5 Write and Solve Multi-Step Equations

Mr. Murphy's class of 20 students is going on a field trip to the science center. They will also watch the 3-D movie. Mrs. Todd's class of 15 students is going on a field trip to the art museum and will take the audio tour. Admission to the art museum is 2.5 times that of the science center's entry fee, as shown in the table.

If the total cost is the same at both the science center and the art museum, what is the entry fee per student to the science center? Check your solution.

Science Center	Art Museum
Entry fee: \$x per student	Admission: \$2.5x per student
3-D movie: \$2.50 per student	Audio Tour: \$1 per student

Part A Write an equation.

Let x represent the cost of the entry fee per student to the science center. The cost of the trip to the science center can be represented by the expression $(x + 2.50)$ for each student to pay the entry fee and 3-D movie. The cost of the trip to the Art Museum can be represented by the expression $(2.5x + 1)$ for admission and the audio tour for each student. Complete the equation that models the total cost.

$$20(x + 2.50) = 15(2.5x + 1)$$

Part B Solve the equation.

$20(x + 2.50) = 15(2.5x + 1)$ Write the equation.

$20x + 50 = 37.5x + 15$ Distributive Property

$-20x \quad = -20x$ Subtraction Property of Equality

$50 = 17.5x + 15$ Simplify.

$-15 = \quad -15$ Subtraction Property of Equality

$35 = 17.5x$ Simplify.

$\frac{35}{17.5} = \frac{17.5x}{17.5}$ Division Property of Equality

$2 = x$ Simplify.

So, the entry fee per student to the science center is \$2.

Check the solution.

Science Center: $20(x + 2.5) = 20(2 + 2.50)$ or \$ 90.00

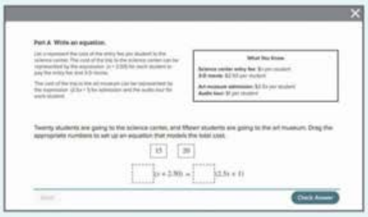
Art Museum: $15(2.5x + 1) = 15(2.5(2) + 1)$ or \$ 90.00

Think About It! What quantity will the variable represent? the entry fee per student to the science center

Talk About It! Can you solve the equation without using the Distributive Property? Why is the Distributive Property helpful? Sample answer: Yes; It is possible to find x by using a table or substitution, but by using the Distributive Property and simplifying each expression, the solution can be found more efficiently.

Lesson 7-4 • Write and Solve Multi-Step Equations 425

Interactive Presentation



Example 5, Write and Solve Multi-Step Equations, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag the appropriate numbers to set up an equation that models the total cost.

TYPE

On Slide 3, students determine the entry fee per person.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A group of 6 friends went to a basketball tournament game. They each bought a ticket for x dollars and spent \$8.50 each on snacks. Another group of 4 friends went to the championship game and paid twice as much for each of their tickets as the first group. The group of 4 friends also spent \$6.50 each on snacks. Write and solve an equation that can be used to find the cost of each ticket for the group of 6 friends.

$6(x + 8.50) = 4(2x + 6.50)$; \$12.50

Go Online You can complete an Extra Example online.

Pause and Reflect

Write one sentence about today's lesson for each of the categories: Who, What, Where, How, and Why.

See students' observations.

426 Module 7 • Equations and Inequalities

Apply Business Finance

Objective

Students will come up with their own strategy to solve an application problem that involves finding the total payroll of a coffee shop.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

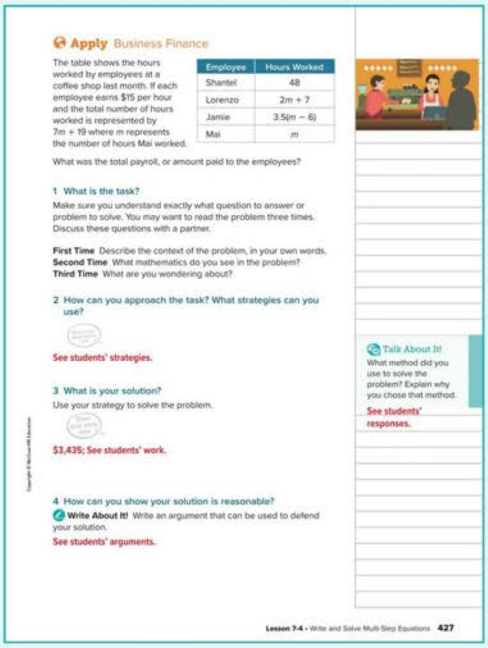
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What expression can be written to represent the sum of the hours worked by all employees?
- What equation represents the total number of hours worked?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Business Finance

The table shows the hours worked by employees at a coffee shop last month. If each employee earns $15 per hour and the total number of hours worked is represented by $7m + 19$ where m represents the number of hours Mai worked.

Employee	Hours Worked
Shantel	48
Lorenzo	$2m + 7$
Jamie	$3.5(m - 6)$
Mai	m

What was the total payroll, or amount paid to the employees?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$3,435; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! What method did you use to solve the problem? Explain why you chose that method.

See students' responses.

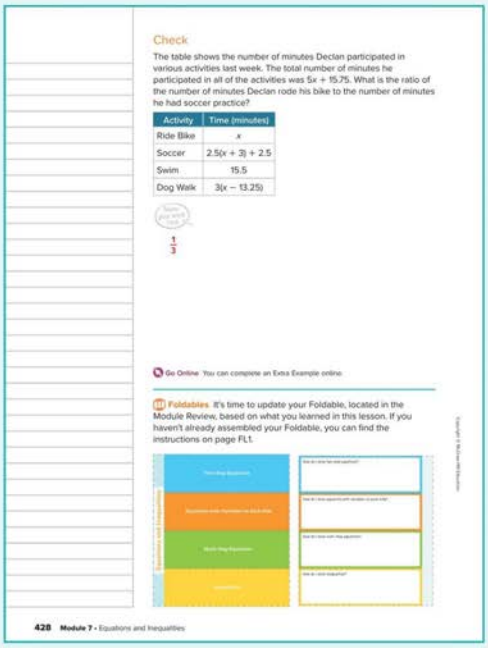
Lesson 7-4 • Write and Solve Multi-Step Equations 427

Interactive Presentation

Apply, Business Finance

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The table shows the number of minutes Declan participated in various activities last week. The total number of minutes he participated in all of the activities was $5x + 15.75$. What is the ratio of the number of minutes Declan rode his bike to the number of minutes he had soccer practice?

Activity	Time (minutes)
Ride Bike	x
Soccer	$2.5(x + 3) + 2.5$
Swim	15.5
Dog Walk	$3(x - 13.25)$

$\frac{1}{3}$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can equations be used to solve everyday problems?

In this lesson, students learned how to write more complex equations with variables on each side. Encourage them to brainstorm an example of a problem arising in everyday life in which they could use the equation $15(n - 3) = 10n + 15$. For example, they may say, "At one online store, I can buy games for $15 each with no shipping charge. I have a coupon for three free games. At a different online store, I pay $10 per game plus a $15 shipping charge."

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students should be able to solve the equation. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. If you spent $22.50 on shipping, how much is the cost of domestic shipping? Define a variable, write an equation, and solve the problem. $3.50; Sample answer: Let c represent the cost of domestic shipping, then the equation $3c + 2(c + 2.50) = 22.50$ can be used to model the problem. Use the Distributive Property and the properties of operations to solve the equation. Since $c = 3.5$, the cost of domestic shipping is $3.50.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 10–12
- Extension: Extension Resources
- ALEKS Multi-Step Equations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8, 10, 12
- Extension: Mixture Problems
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS The Distributive Property, Simplifying Algebraic Expressions

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS The Distributive Property, Simplifying Algebraic Expressions

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

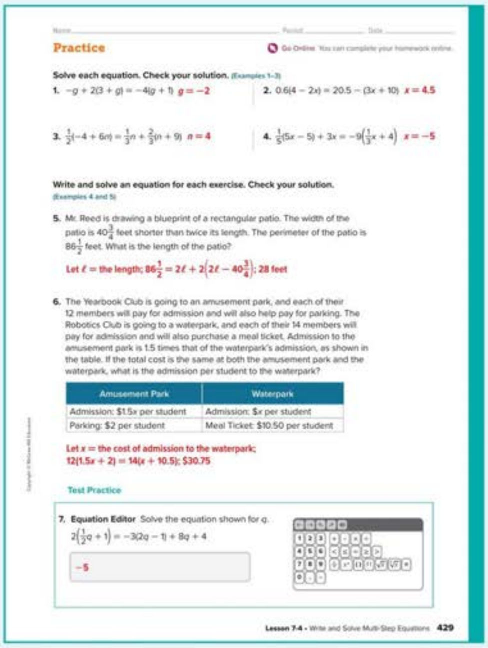
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve multi-step equations	1–4
2	model and solve real-world problems by using a multi-step equation with rational coefficients, variables on both sides, and grouping symbols	5, 6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems that involve writing and solving multi-step equations	8, 9
3	higher-order and critical thinking skills	10–12

Common Misconception

Some students may incorrectly write a multi-step equation that illustrates a given situation. Encourage students to underline the important information in the word problem and to check that the information is displayed correctly in the equation they write.


Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

Solve each equation. Check your solution. (Examples 1–3)

1. $-g + 2(3 + g) = -4(g + 1)$ $g = -2$
2. $0.6(4 - 2x) = 20.5 - (3x + 10)$ $x = 4.5$
3. $\frac{1}{2}(-4 + 6n) = \frac{1}{3}n + \frac{2}{3}(n + 9)$ $n = 4$
4. $\frac{1}{5}(5x - 5) + 3x = -9\left(\frac{1}{3}x + 4\right)$ $x = -5$

Write and solve an equation for each exercise. Check your solution. (Examples 4 and 5)

5. Mr. Reed is drawing a blueprint of a rectangular patio. The width of the patio is $40\frac{3}{4}$ feet shorter than twice its length. The perimeter of the patio is $86\frac{1}{2}$ feet. What is the length of the patio?

   Let ℓ = the length; $86\frac{1}{2} = 2\ell + 2\left(2\ell - 40\frac{3}{4}\right)$; 28 feet

6. The Yearbook Club is going to an amusement park, and each of their 12 members will pay for admission and will also help pay for parking. The Robotics Club is going to a waterpark, and each of their 14 members will pay for admission and will also purchase a meal ticket. Admission to the amusement park is 1.5 times that of the waterpark's admission, as shown in the table. If the total cost is the same at both the amusement park and the waterpark, what is the admission per student to the waterpark?

Amusement Park	Waterpark
Admission: \$1.5x per student	Admission: \$x per student
Parking: \$2 per student	Meal Ticket: \$10.50 per student

   Let x = the cost of admission to the waterpark; $12(1.5x + 2) = 14(x + 10.5)$; \$30.75

Test Practice

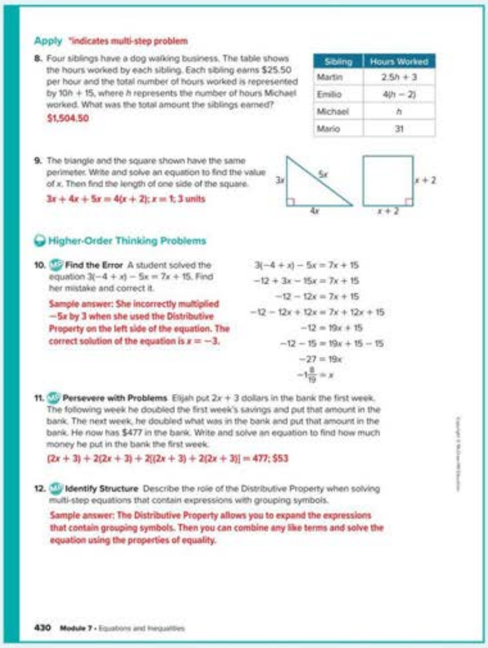
7. **Equation Editor** Solve the equation shown for q.

   $2\left(\frac{1}{2}q + 1\right) = -3(2q - 1) + 8q + 4$

   -5

Lesson 7-4 • Write and Solve Multi-Step Equations 429

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**


Apply *indicates multi-step problem

8. Four siblings have a dog walking business. The table shows the hours worked by each sibling. Each sibling earns $25.50 per hour and the total number of hours worked is represented by $10h + 15$, where h represents the number of hours Michael worked. What was the total amount the siblings earned?
$1,504.50

Sibling	Hours Worked
Martin	$2.5h + 3$
Emilio	$4(h - 2)$
Michael	h
Mario	31

9. The triangle and the square shown have the same perimeter. Write and solve an equation to find the value of x. Then find the length of one side of the square.
$3x + 4x + 5x = 4(x + 2)$; $x = 1$; 3 units

Higher-Order Thinking Problems

10. **Find the Error** A student solved the equation $3(-4 + x) - 5x = 7x + 15$. Find her mistake and correct it.

$$3(-4 + x) - 5x = 7x + 15$$
$$-12 + 3x - 15x = 7x + 15$$
$$-12 - 12x = 7x + 15$$
$$-12 - 12x + 12x = 7x + 12x + 15$$
$$-12 = 19x + 15$$
$$-12 - 15 = 19x + 15 - 15$$
$$-27 = 19x$$
$$-1\tfrac{8}{19} = x$$

Sample answer: She incorrectly multiplied $-5x$ by 3 when she used the Distributive Property on the left side of the equation. The correct solution of the equation is $x = -3$.

11. **Persevere with Problems** Elijah put $2x + 3$ dollars in the bank the first week. The following week he doubled the first week's savings and put that amount in the bank. The next week, he doubled what was in the bank and put that amount in the bank. He now has $477 in the bank. Write and solve an equation to find how much money he put in the bank the first week.
$(2x + 3) + 2(2x + 3) + 2[(2x + 3) + 2(2x + 3)] = 477$; $53

12. **Identify Structure** Describe the role of the Distributive Property when solving multi-step equations that contain expressions with grouping symbols.
Sample answer: The Distributive Property allows you to expand the expressions that contain grouping symbols. Then you can combine any like terms and solve the equation using the properties of equality.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students will find the mistake in the problem and correct it. Encourage students to determine the error by analyzing the worked-out solution and explain how they could fix it.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 11, students will write and solve an equation to solve the real-world problem. Encourage students to plan a solution pathway they can implement to solve the problem.

7 Look for and Make Use of Structure In Exercise 12, students will describe the role of the Distributive Property when solving multi-step equations that contain expressions with grouping symbols. Encourage students to explain why they use the Distributive Property when solving equations.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 8 Have students work together to prepare a brief demonstration that illustrates why this problem might require multiple steps to solve. For example, before they can find the total amount, they must first write and solve an equation to find the value of h, the number of hours Michael worked. Have each pair or group of students present their response to the class.

Solve the problem another way.
Use with Exercise 11 Have students work in groups of 3–4. After completing Exercise 11, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Lesson 7-5

Determine the Number of Solutions

8.EE.C.7

LESSON GOAL

Students will determine the number of solutions to an equation.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Number of Solutions

Learn: Number of Solutions
Example 1: Equations with Infinitely Many Solutions
Example 2: Equations with No Solution
Learn: Analyze Equations to Determine the Number of Solutions
Example 3: Create Equations with Infinitely Many Solutions
Example 4: Create Equations with No Solution
Apply: School

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Determine the Number of Solutions by Graphing		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 42 of the *Language Development Handbook* to help your students build mathematical language related to determining the number of solutions to an equation.

ELL You can use the tips and suggestions on page T42 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.C** by determining the number of solutions to an equation.
Standards for Mathematical Content: **8.E E.C.7, 8.EE.C.7.A**
Standards for Mathematical Practice: **MP1, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students wrote and solved multi-step equations.
8.EE.C.7, 8.EE.C.7.B

Now
Students determine the number of solutions to an equation.
8.EE.C.7, 8.EE.C.7.A

Next
Students will write and solve one-step inequalities.
7.EE.B.4, 7.EE.B.4.B

Rigor

The Three Pillars of Rigor

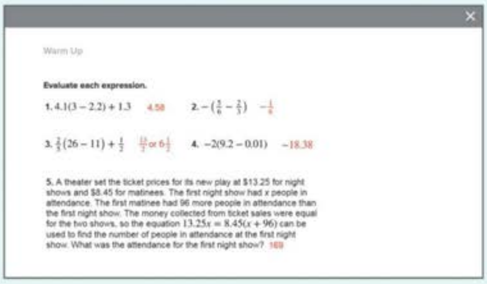
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand their *understanding* of equations by examining the number of possible solutions that an equation can have. They come to understand that an equation with one variable may have one solution, no solution, or infinitely many solutions. Students develop *fluency* by creating multi-step equations that have infinite or no solutions.

Mathematical Background

Linear equations of one variable may have no solution, one solution, or infinitely many solutions. To determine the number of solutions, simplify the expressions on each side of the equation so that the form of the equation is $ax + b = cx + d$.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- performing operations with rational numbers (Exercises 1–4)
- solving multi-step equations with rational coefficients (Exercise 5)

Answers

1. 4.58

2. $-\frac{1}{6}$

3. $\frac{13}{2}$ or $6\frac{1}{2}$

4. −18.38

5. 169

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the cost of a birthday party, using an equation.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What are some synonyms for the word *constant?* How does this help you remember the meaning of the term *constant* in mathematics? Sample answer: unchanging, consistent, unvarying; It helps me remember that a constant (unlike a variable) always has the same, unchanging value - such as a given number.
- The Latin root *solut* is a variant of the root *solv* which means to loosen. What is another math term related to the term *solution*? Sample answer: solve

Explore Number of Solutions

Objective
Students will use Web Sketchpad to explore equations with one solution, no solution, and infinitely many solutions.

Ideas for Use
Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity
Students will be presented with three equations with varying numbers of solutions. Throughout this sketch, students will make observations about the equations and how the number of solutions can be determined.

Inquiry Question
How many solutions can an equation have? Sample answer: In some cases, an equation will have one solution. In other cases, an equation might have infinitely many solutions, or no solution.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!
SLIDE 3

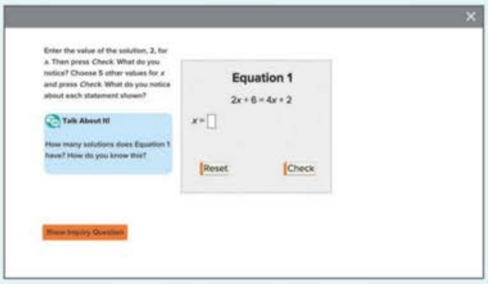
Mathematical Discourse

How many solutions does Equation 1 have? How do you know this?
Sample answer: Equation 1 has one solution, because only the value of 2 makes the equation true.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

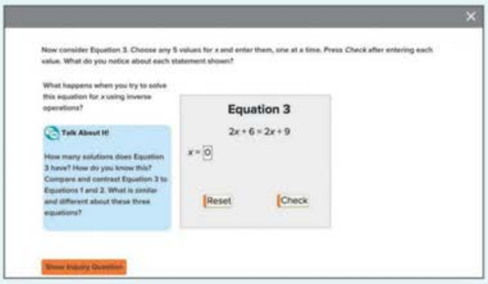


Explore, Slide 3 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore equations with one solution, no solution, and infinitely many solutions.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Number of Solutions *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore and examine three different equations with varying numbers of solutions.

7 Look for and Make Use of Structure Encourage students to examine the similarities and differences in the forms of the equations and the number of solutions the equations have.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

How many solutions does Equation 3 have? How do you know this? Compare and contrast Equation 3 to Equations 1 and 2. What is similar and different about these three equations? Sample answer: Equation 3 does not have any solutions. There are no numbers that can be substituted for x to make the equation true. All three equations have variables on each side of the equation. Equation 3, like Equation 2, has coefficients that are the same, while Equation 3 and Equation 1 have constants that are different.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Number of Solutions

Objective

Students will understand that an equation can have one solution, no solution, or infinitely many solutions.

Teaching the Mathematical Practices

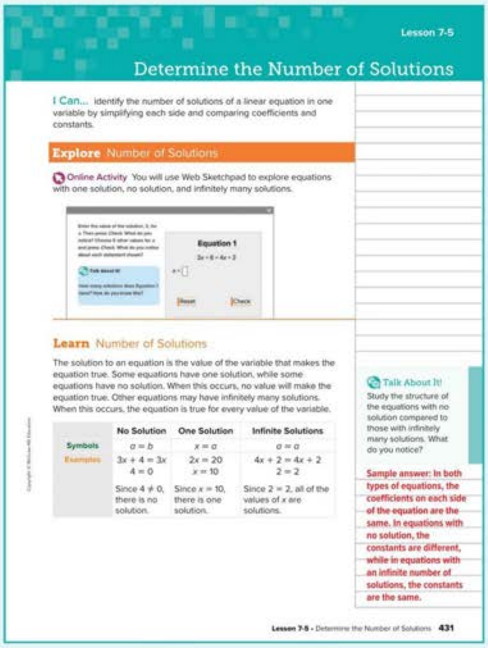
7 Look for and Make Use of Structure While discussing the *Talk About It!* question, encourage students to study the structure of each type of equation and how the structure can be used to identify the number of solutions without actually solving the equation.

Talk About It!

SLIDE 1

Mathematical Discourse

Study the structure of the equations with no solution compared to those with infinitely many solutions. What do you notice? Sample answer: In both types of equations, the coefficients on each side of the equation are the same. In equations with no solution, the constants are different, while in equations with an infinite number of solutions, the constants are the same.



Lesson 7-5

Determine the Number of Solutions

I Can... identify the number of solutions of a linear equation in one variable by simplifying each side and comparing coefficients and constants.

Explore Number of Solutions

Online Activity You will use Web Sketchpad to explore equations with one solution, no solution, and infinitely many solutions.

Learn Number of Solutions

The solution to an equation is the value of the variable that makes the equation true. Some equations have one solution, while some equations have no solution. When this occurs, no value will make the equation true. Other equations may have infinitely many solutions. When this occurs, the equation is true for every value of the variable.

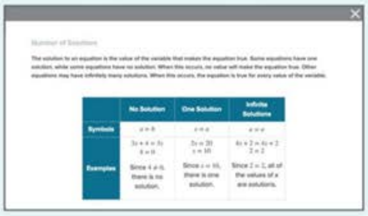
	No Solution	One Solution	Infinite Solutions
Symbols	$a = b$	$x = a$	$a = a$
Examples	$3x + 4 = 3x$ $4 = 0$	$2x = 20$ $x = 10$	$4x + 2 = 4x + 2$ $2 = 2$
	Since $4 \neq 0$, there is no solution.	Since $x = 10$, there is one solution.	Since $2 = 2$, all of the values of x are solutions.

Talk About It! Study the structure of the equations with no solution compared to those with infinitely many solutions. What do you notice?

Sample answer: In both types of equations, the coefficients on each side of the equation are the same. In equations with no solution, the constants are different, while in equations with an infinite number of solutions, the constants are the same.

Lesson 7-5 • Determine the Number of Solutions 431

Interactive Presentation

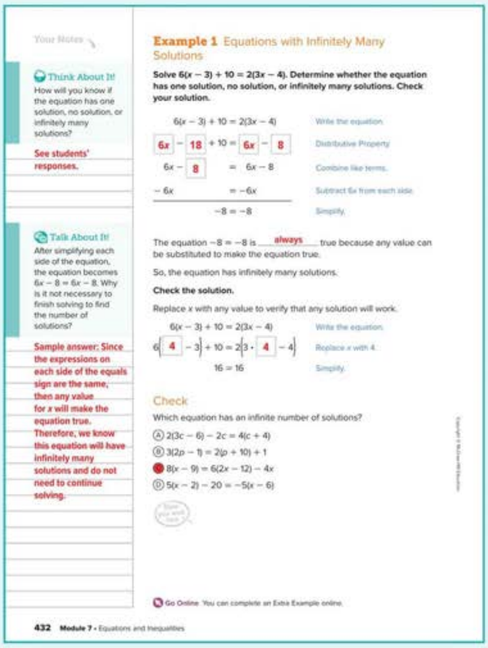


Learn, Number of Solutions

DIFFERENTIATE

Reteaching Activity AL

To support students' understanding of what it means for an equation to have either no solution, or infinitely many solutions, have them work with a partner to study the equations presented in the Learn. For each equation, have them substitute varying possible values of x to see if the equation is true. They should try at least 5 possible values for each equation. In the first equation, they should note that none of their possibilities result in a true equation. In the third equation, they should note that every one of their possibilities result in a true equation.



Your Notes

Think About It! How will you know if the equation has one solution, no solution, or infinitely many solutions?
See students' responses.

Talk About It! After simplifying each side of the equation, the equation becomes $6x - 8 = 6x - 8$. Why is it not necessary to finish solving to find the number of solutions?
Sample answer: Since the expressions on each side of the equals sign are the same, then any value for x will make the equation true. Therefore, we know this equation will have infinitely many solutions and do not need to continue solving.

Example 1 Equations with Infinitely Many Solutions

Solve $6(x - 3) + 10 = 2(3x - 4)$. Determine whether the equation has one solution, no solution, or infinitely many solutions. Check your solution.

$6(x - 3) + 10 = 2(3x - 4)$ Write the equation.
$6x - 18 + 10 = 6x - 8$ Distributive Property
$6x - 8 = 6x - 8$ Combine like terms.
$-6x \quad = -6x$ Subtract 6x from each side.
$-8 = -8$ Simplify.

The equation $-8 = -8$ is always true because any value can be substituted to make the equation true.
So, the equation has infinitely many solutions.

Check the solution.
Replace x with any value to verify that any solution will work.
$6(x - 3) + 10 = 2(3x - 4)$ Write the equation.
$6(4 - 3) + 10 = 2(3 \cdot 4 - 4)$ Replace x with 4.
$16 = 16$ Simplify.

Check
Which equation has an infinite number of solutions?
Ⓐ $2(3c - 6) - 2c = 4(c + 4)$
Ⓑ $3(2p - 1) = 2(p + 10) + 1$
● $8(x - 9) = 6(2x - 12) - 4x$
Ⓓ $5(x - 2) - 20 = -5(x - 6)$

Go Online You can complete an Extra Example online.

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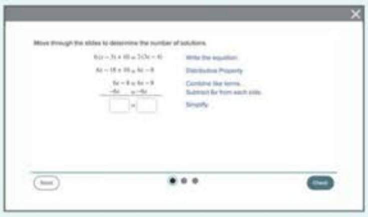
Interactive Presentation



Example 1, Equations with Infinitely Many Solutions, Slide 2 of 4

CLICK
On Slide 2, students determine the equation has an infinite number of solutions.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** 3 APPLICATION

Example 1 Equations with Infinitely Many Solutions

Objective

Students will determine algebraically that an equation has an infinite number of solutions.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to simplify the expressions on each side of the equation efficiently and accurately, adhering to the properties of operations.

7 Look for and Make Use of Structure Students should notice the structure of the expressions as they near the end of their solution process in order to determine that the equation has infinitely many solutions.

While discussing the *Talk About It!* question on Slide 3, encourage students to recognize the structure of equations with infinitely many solutions. Such equations will have equivalent expressions on either side of the equals sign.

Questions for Mathematical Discourse

SLIDE 2

AL What property can you use to eliminate the parentheses on each side of the equation? the Distributive Property

OL What does it mean that the last step says $-8 = -8$? Sample answer: The statement $-8 = -8$ is always true. No matter what value you substitute for x in the original equation, the expressions on each side of the equals sign are equivalent. This means that any value substituted for x in this equation will make a true statement.

BL A classmate substitutes $x = 1$ into the equation and concludes that the equation has only one solution since it is true for $x = 1$. What is wrong with the classmate's reasoning? Sample answer: The classmate should not base their conclusion on that one value. In this case, $x = 1$ is only one of the infinitely many solutions.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Equations with No Solution

Objective

Students will determine algebraically that an equation has no solution.

Teaching the Mathematical Practices

6 Attend to Precision Enco urage students to simplify the expressions on each side of the equation efficiently and accurately, adhering to the properties of operations.

7 Look for and Make Use of Structure Students should notice the structure of the expressions as they near the end of their solution process in order to determine that the equation has no solution.

While discussing the *Talk About It!* question on Slide 3, encourage students to learn to recognize equations with no solutions by considering their coefficients and constants.

Questions for Mathematical Discourse

SLIDE 2

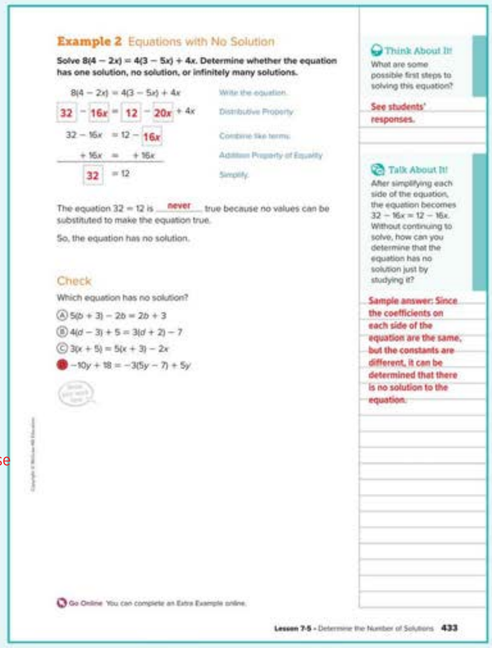
AL What is the first step to solve this equation? Sample answer: Use the Distributive Property to eliminate the parentheses.

OL What does it mean that the last step says $32 = 12$? Sample answer: The statement $32 = 12$ is not true. No matter what value you substitute for x in the original equation, the expressions on each side of the equals sign are not equivalent. This means that any value substituted for x in this equation will make a false statement. The equation has no solution.

BL Generate an equation that has no solution. Sample answer: $3(x + 9) = 3x - 12$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Equations with No Solution

Solve $8(4 - 2x) = 4(3 - 5x) + 4x$. Determine whether the equation has one solution, no solution, or infinitely many solutions.

$8(4 - 2x) = 4(3 - 5x) + 4x$	Write the equation.
$32 - 16x = 12 - 20x + 4x$	Distributive Property
$32 - 16x = 12 - 16x$	Combine like terms.
$+ 16x = + 16x$	Addition Property of Equality
$32 = 12$	Simplify.

The equation $32 = 12$ is never true because no values can be substituted to make the equation true.

So, the equation has no solution.

Think About It! What are some possible first steps to solving this equation?

See students' responses.

Talk About It! After simplifying each side of the equation, the equation becomes $32 - 16x = 12 - 16x$. Without continuing to solve, how can you determine that the equation has no solution just by studying it?

Sample answer: Since the coefficients on each side of the equation are the same, but the constants are different, it can be determined that there is no solution to the equation.

Check

Which equation has no solution?

A. $5(b + 3) - 2b = 2b + 3$

B. $4(d - 3) + 5 = 3(d + 2) - 7$

C. $3(x + 5) = 5(x + 3) - 2x$

D. $-10y + 18 = -3(5y - 7) + 5y$

Go Online You can complete an Extra Example online.

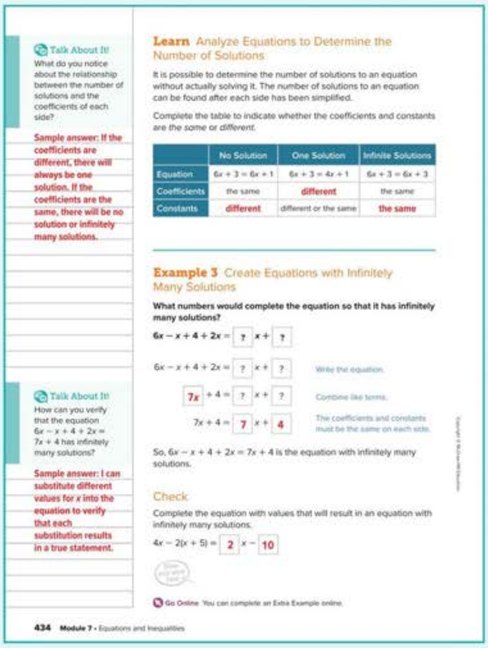
Lesson 7-5 • Determine the Number of Solutions 433

Interactive Presentation

Example 2, Equations with No Solution, Slide 2 of 4

CLICK On Slide 2, students determine the equation has no solution.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Talk About It! What do you notice about the relationship between the number of solutions and the coefficients of each side?

Sample answer: If the coefficients are different, there will always be one solution. If the coefficients are the same, there will be no solution or infinitely many solutions.

Learn Analyze Equations to Determine the Number of Solutions

It is possible to determine the number of solutions to an equation without actually solving it. The number of solutions to an equation can be found after each side has been simplified.

Complete the table to indicate whether the coefficients and constants are the *same* or *different*.

	No Solution	One Solution	Infinite Solutions
Equation	$6x + 3 = 6x + 1$	$6x + 3 = 4x + 1$	$6x + 3 = 6x + 3$
Coefficients	the same	different	the same
Constants	different	different or the same	the same

Example 3 Create Equations with Infinitely Many Solutions

What numbers would complete the equation so that it has infinitely many solutions?

$6x - x + 4 + 2x = ?\,x + ?$

$6x - x + 4 + 2x = ?\,x + ?$ Write the equation.

$7x + 4 = ?\,x + ?$ Combine like terms.

$7x + 4 = 7\,x + 4$ The coefficients and constants must be the same on each side.

So, $6x - x + 4 + 2x = 7x + 4$ is the equation with infinitely many solutions.

Talk About It! How can you verify that the equation $6x - x + 4 + 2x = 7x + 4$ has infinitely many solutions?

Sample answer: I can substitute different values for x into the equation to verify that each substitution results in a true statement.

Check

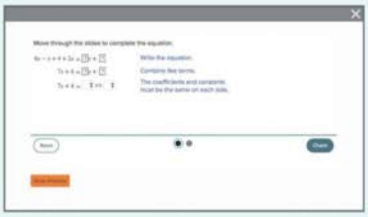
Complete the equation with values that will result in an equation with infinitely many solutions.

$4x - 2(x + 5) = 2\,x - 10$

Go Online You can complete an Extra Example online.

434 Module 7 • Equations and Inequalities

Interactive Presentation



Example 3, Create Equations with Infinitely Many Solutions, Slide 2 of 4

CLICK

On Slide 2 of Example 3, students construct the correct equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Analyze Equations to Determine the Number of Solutions

Objective

Students will understand how the structure of an equation indicates whether it has one solution, no solution, or infinitely many solutions.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Find a sample answer for the *Talk About It!* question.

Example 3 Create Equations with Infinitely Many Solutions

Objective

Students will construct an equation that has infinitely many solutions.

MP Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language, such as *substitution*, in order to explain how they can verify the equation has infinitely many solutions.

7 Look for and Make Use of Structure Students should use the structure of the simplified expression on the left side to determine the expression that should be on the right side of the equation, in order for the equation to have infinitely many solutions.

Questions for Mathematical Discourse

SLIDE 2

AL How can you simplify the left side of the equation? Combine like terms to obtain $7x + 4$.

OL What must be true of the coefficients and constants on each side of the equation, in order for the equation to have infinitely many solutions? After the equation is simplified, the coefficients on each side must be the same and the constants on each side must be the same.

BL Is there only one expression of the form $ax + b$ for the right side of the equation in order for the equation to have infinitely many solutions? Explain. yes; Sample answer: The left side of the equation simplifies to $7x + 4$, so if the right side is of the form $ax + b$, it must also be $7x + 4$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Create Equations with No Solution

Objective

Students will construct an equation that has no solution.

Teaching the Mathematical Practices

6 Attend to Precision Enco urage students to simplify the expression on the left side of the equation efficiently and accurately.

While discussing the *Talk About It!* questions on Slide 3, encourage students to use clear and precise mathematical language to explain how they can verify the equation has no solution.

7 Look for and Make Use of Structure Students should use the structure of the simplified expression on the left side to determine the expression that should be on the right side of the equation, in order for the equation to have no solution.

While discussing the *Talk About It!* questions on Slide 3, students should use their understanding of the structure of an equation with no solution to generate other possibilities for the expression on the right side of the equation.

Questions for Mathematical Discourse

SLIDE 2

AL How can you simplify the left side of the equation? Use the Distributive Property and combine like terms.

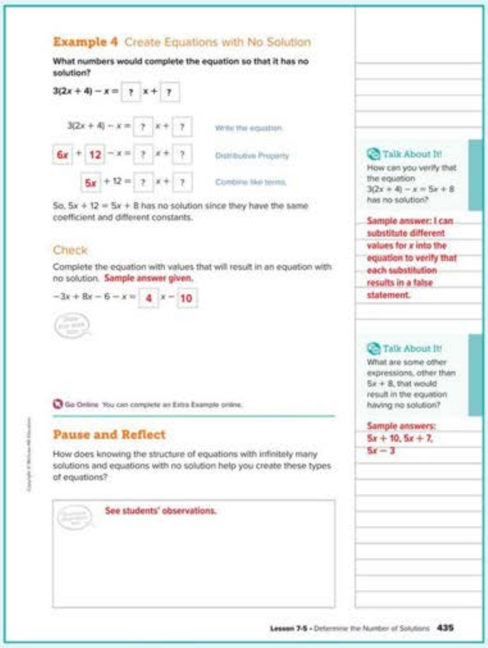
OL How do the coefficient and constant of the right side relate to the coefficient and constant of the left side, in order for the equation to have no solution? The coefficients must be equal, but the constants cannot be equal.

OL Is 8 the only possible value for the constant? Explain. no; Sample answer: As long as the coefficient is 5, and the constant is any number except 12, then the constant can be any other number.

BL What are some possible coefficients of x for the right side of the equation that would ensure that the equation has exactly one solution? Explain. Sample answers: 1, 2, 8, etc. As long as the coefficient is not 5, then the equation will have exactly one solution.

Go Online

- Find additional teaching notes and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Create Equations with No Solution

What numbers would complete the equation so that it has no solution?

$3(2x + 4) - x = \boxed{?}\,x + \boxed{?}$

$3(2x + 4) - x = \boxed{?}\,x + \boxed{?}$ Write the equation.

$\boxed{6x} + \boxed{12} - x = \boxed{?}\,x + \boxed{?}$ Distributive Property

$\boxed{5x} + 12 = \boxed{?}\,x + \boxed{?}$ Combine like terms.

So, $5x + 12 = 5x + 8$ has no solution since they have the same coefficient and different constants.

Talk About It! How can you verify that the equation $3(2x + 4) - x = 5x + 8$ has no solution?

Sample answer: I can substitute different values for x into the equation to verify that each substitution results in a false statement.

Check

Complete the equation with values that will result in an equation with no solution. Sample answer given.

$-3x + 8x - 6 - x = \boxed{4}\,x - \boxed{10}$

Talk About It! What are some other expressions, other than $5x + 8$, that would result in the equation having no solution?

Sample answers: $5x + 10$, $5x + 7$, $5x - 3$

Go Online You can complete an Extra Example online.

Pause and Reflect

How does knowing the structure of equations with infinitely many solutions and equations with no solution help you create these types of equations?

See students' observations.

Lesson 7-5 • Determine the Number of Solutions 435

Interactive Presentation

Example 4, Create Equations with No Solution, Slide 2 of 4

CLICK On Slide 2, students construct a correct equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Pause and Reflect

Create a graphic organizer that will help you study the concepts you learned today in class.

See students' observations.

Apply School

Objective

Students will come up with their own strategy to solve an application problem that involves analyzing expressions.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

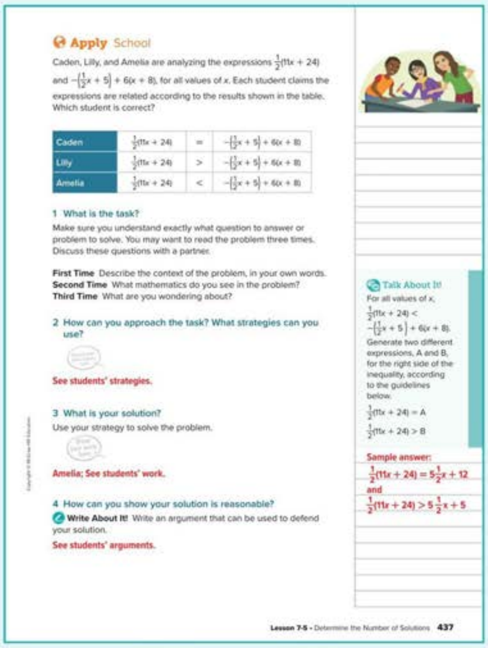
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you simplify each expression?
- What properties are needed to simplify each expression?
- Is it possible for more than one student to be correct?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply School

Caden, Lilly, and Amelia are analyzing the expressions $\frac{1}{2}(11x + 24)$ and $-\left(\frac{1}{2}x + 5\right) + 6(x + 8)$, for all values of x. Each student claims the expressions are related according to the results shown in the table. Which student is correct?

Caden	$\frac{1}{2}(11x + 24)$	$=$	$-\left(\frac{1}{2}x + 5\right) + 6(x + 8)$
Lilly	$\frac{1}{2}(11x + 24)$	$>$	$-\left(\frac{1}{2}x + 5\right) + 6(x + 8)$
Amelia	$\frac{1}{2}(11x + 24)$	$<$	$-\left(\frac{1}{2}x + 5\right) + 6(x + 8)$

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Amelia; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

For all values of x, $\frac{1}{2}(11x + 24) < -\left(\frac{1}{2}x + 5\right) + 6(x + 8)$. Generate two different expressions, A and B, for the right side of the inequality, according to the guidelines below.

$\frac{1}{2}(11x + 24) = A$

$\frac{1}{2}(11x + 24) > B$

Sample answer:
$\frac{1}{2}(11x + 24) = 5\frac{1}{2}x + 12$
and
$\frac{1}{2}(11x + 24) > 5\frac{1}{2}x + 5$

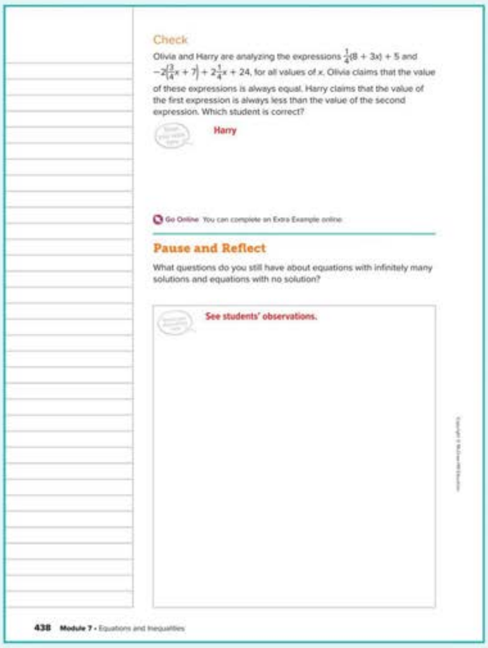
Lesson 7-5 • Determine the Number of Solutions 437

Interactive Presentation

Apply, School

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Olivia and Harry are analyzing the expressions $\frac{1}{4}(8 + 3x) + 5$ and $-2\left(\frac{3}{4}x + 7\right) + 2\frac{1}{4}x + 24$, for all values of x. Olivia claims that the value of these expressions is always equal. Harry claims that the value of the first expression is always less than the value of the second expression. Which student is correct?

Harry

Go Online You can complete an Extra Example online.

Pause and Reflect

What questions do you still have about equations with infinitely many solutions and equations with no solution?

See students' observations.

438 Module 7 • Equations and Inequalities

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can equations be used to solve everyday problems?

In this lesson, students learned how to determine if an equation in one variable would have no solutions, one solution, or an infinite number of solutions. Encourage them to discuss with a partner why it is important in real life to know the number of solutions to an equation. Some students may observe that when comparing two costs, if there are infinite solutions, then the costs are always the same.

Exit Ticket

Refer to the Exit Ticket slide. If each party has 16 guests and the total amount spent for each party is the same, what did they spend on food per person? Write an equation to represent this situation. How can you interpret the result?

$20 + 16(c + 3) = 68 + 16c$; Sample answer: The amount spent on food for each person cannot be determined. The equation to find the value of c is $20 + 16(c + 3) = 68 + 16c$, which simplifies to $68 + 16c = 68 + 16c$. The equation has infinitely many solutions, meaning that they paid the same amount for any cost of food per person.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 9, 11, 12–15
- Extension: Determine the Number of Solutions by Graphing
- ALEKS® Equations with Variables on Both Sides, Applications of Equations

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–8, 10, 12, 15
- Extension: Determine the Number of Solutions by Graphing
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Equations with Variables on Both Sides

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Equations with Variables on Both Sides

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

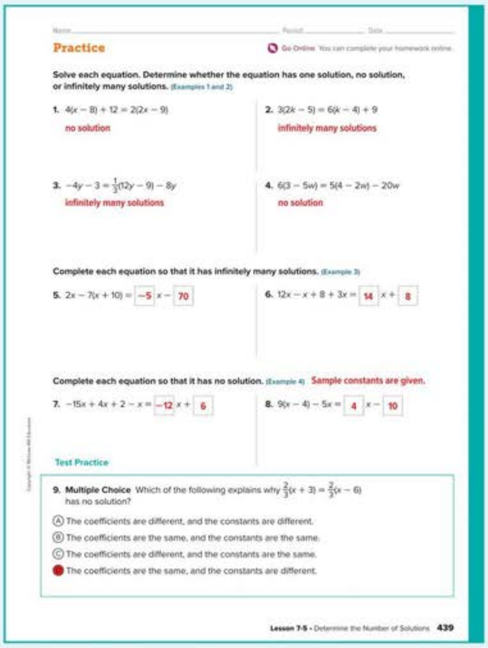
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	determine algebraically that an equation has an infinite number of solutions	2, 3
1	determine algebraically that an equation has no solution	1, 4
1	construct equations that have infinitely many solutions	5, 6
1	construct equations that have no solution	7, 8
2	extend concepts learned in class to apply them in new contexts	9
3	solve application problems that involve equations with one solution, no solution, and infinitely many solutions	10, 11
3	higher-order and critical thinking skills	12–15

Common Misconception

Students may incorrectly identify an equation that has a solution of 0 as an equation that does not have a solution. Remind students that when no value makes the equation true, the equation has no solution. An equation that has a solution of 0 means the equation either has one solution or infinitely many solutions.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each equation. Determine whether the equation has one solution, no solution, or infinitely many solutions. (Examples 1 and 2)

1. $4(x - 8) + 12 = 2(2x - 9)$
no solution

2. $3(2k - 5) = 6(k - 4) + 9$
infinitely many solutions

3. $-4y - 3 = \frac{1}{3}(12y - 9) - 8y$
infinitely many solutions

4. $6(3 - 5w) = 5(4 - 2w) - 20w$
no solution

Complete each equation so that it has infinitely many solutions. (Example 3)

5. $2x - 7(x + 10) = \boxed{-5}\,x - \boxed{70}$

6. $12x - x + 8 + 3x = \boxed{14}\,x + \boxed{8}$

Complete each equation so that it has no solution. (Example 4) Sample constants are given.

7. $-15x + 4x + 2 - x = \boxed{-12}\,x + \boxed{6}$

8. $9(x - 4) - 5x = \boxed{4}\,x - \boxed{10}$

Test Practice

9. **Multiple Choice** Which of the following explains why $\frac{2}{3}(x + 3) = \frac{2}{3}(x - 6)$ has no solution?

(A) The coefficients are different, and the constants are different.
(B) The coefficients are the same, and the constants are the same.
(C) The coefficients are different, and the constants are the same.
(D) The coefficients are the same, and the constants are different.

Lesson 7-5 • Determine the Number of Solutions 439

Apply *indicates multi-step problem

*10. Three students in the Math Club are analyzing the expressions $\frac{1}{4}(10x + 8)$ and $-(\frac{1}{2}x + 13) + 3(x + 5)$, for all values of x. Each student claims the expressions are related according to the results shown in the table. Which student is correct? **Student 1**

Student 1	$\frac{1}{4}(10x + 8)$	$=$	$-(\frac{1}{2}x + 13) + 3(x + 5)$
Student 2	$\frac{1}{4}(10x + 8)$	$>$	$-(\frac{1}{2}x + 13) + 3(x + 5)$
Student 3	$\frac{1}{4}(10x + 8)$	$<$	$-(\frac{1}{2}x + 13) + 3(x + 5)$

*11. Daniel and Fatima are analyzing the expressions below for all values of x. Daniel claims that the value of these expressions is always equal. Fatima claims that the value of the expression on the left is always greater than the value of the expression on the right. Which student is correct?

$0.6(-7x + 9)$ and $4(x - 3) - (-8 + 8.2x)$ **Fatima**

Higher-Order Thinking Problems

12. MP **Find the Error** A student solved the equation and determined that the solution was −2. Find her error and correct it.

$$1.5x - 2 = -2 + 1.5x$$
$$-2 = -2$$

Sample answer: The equation $-2 = -2$ is always true, so every value of x will make the equation true. The correct solution of the equation is infinitely many solutions.

13. MP **Make an Argument** Suppose the solution to an equation is $x = 0$. Explain why it is incorrect to conclude that the equation has no solution.

Sample answer: The solution $x = 0$ means that 0 is the solution to the equation and the equation has one solution, 0.

14. Determine if the statement is *true* or *false*. Justify your response.

An equation will always have at least one solution.

false; Sample answer: An equation can have no solution, one solution, or infinitely many solutions.

15. MP **Identify Structure** What values of a, b, c, and d will make the equation have one solution? Then alter your equation so that it has no solution. Finally, alter the equation again so that it has infinitely many solutions.

$$ax + b = cx + d$$

Sample answer:
one solution: $-4x + 7 = -5x + 9$;
no solution: $-4x + 7 = -4x + 8$;
infinite solutions: $-4x + 7 = -4x + 7$

440 Module 7 • Equations and Inequalities

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students will find the error in the problem and correct it. Encourage students to determine the error by analyzing the solution of the equation and explain how they could correct the error. In Exercise 13, students will explain why a proposed solution is correct or incorrect. Encourage students to support their answer with an explanation that uses information they learned in the lesson.

7 Look for and Make Use of Structure In Exercise 15, students will identify values of *a, b, c,* and *d* that will make the equation have one solution, no solution, and then infinitely many solutions. Encourage students to identify structures in equations that make them have infinitely many solutions.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.
Use with Exercises 10–11 Have students work in pairs. Have students individually read Exercise 10 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 11.

Be sure everyone understands.
Use with Exercises 14–15 Have students work in groups of 3–4 to solve the problem in Exercise 14. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 15.

Lesson 7-6

7.EE.B.4, 7.EE.B.4.B

Write and Solve One-Step Addition and Subtraction Inequalit

LESSON GOAL

Students will write and solve one-step addition and subtraction inequalities.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Addition and Subtraction Properties of Inequality

Learn: Inequalities
Learn: Graph Inequalities
Learn: Subtraction and Addition Properties of Inequality
Example 1: Solve and Graph Addition Inequalities
Example 2: Solve and Graph Subtraction Inequalities
Learn: Write Inequalities
Example 3: Write and Solve Addition Inequalities
Example 4: Write and Solve One-Step Subtraction Inequalities
Apply: Elevators

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Solve Compound Inequalities		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 43 of the *Language Development Handbook* to help your students build mathematical language related to writing and solving one-step addition and subtraction inequalities.

ELL You can use the tips and suggestions on page T43 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7.EE.B** by writing and solving one-step addition and subtraction inequalities.
Standards for Mathematical Content: **7 .EE.B.4, 7.EE.B.4.B,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students wrote inequalities to represent real-world and mathematical problems. **6.EE.B.8**

Now
Students write and solve one-step addition and subtraction inequalities. **7.EE.B.4, 7.EE.B.4.B**

Next
Students will write and solve one-step multiplication and division inequalities. **7.EE.B.4.B**

Rigor

The Three Pillars of Rigor

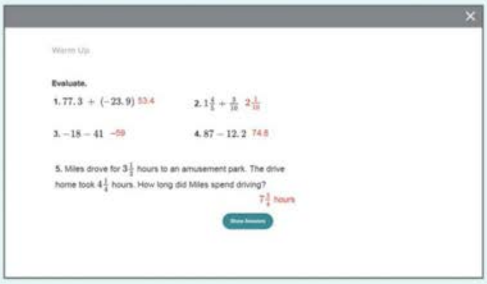
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of inequalities and solving one-step addition and subtraction equations to build *understanding* of solving and graphing one-step addition and subtraction inequalities. They will then use this understanding to build *fluency* to write and solve one-step addition and subtraction inequalities.

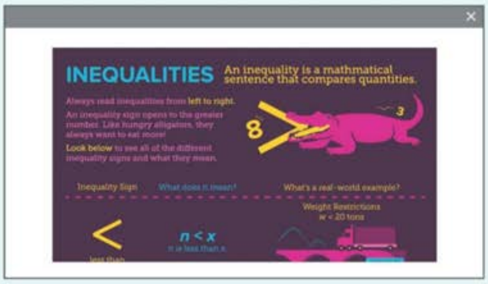
Mathematical Background

Inequalities involving only addition or subtraction can be solved using the *Addition Property of Inequality* and *Subtraction Property of Inequality*. The inequality remains true when you add or subtract the same number from each side. Solutions of inequalities can be graphed on a number line. When graphing the solution, use an open dot to show $>$ or $<$. Use a closed dot to indicate the solution is $\geq$ or $\leq$.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 1

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- adding and subtracting rational numbers (Exercises 1–5)

Answers

1. 53.4

2. $2\frac{1}{10}$

3. −59

4. 74.8

5. $7\frac{3}{4}$ hours

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about inequalities, using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Using what you know about the *Addition Property of Equality*, how do you think the *Addition Property of Inequality* might be similar or different? Sample answer: I think the Addition Property of Inequality will still relate to adding a number to each side, but it will be related to an inequality, not an equation.
- The prefix *in-* means *not*. Use what you know about the term *equality* to predict what an *inequality* might be. Sample answer: Equality means the state of being equal. So, inequality might mean the state of *not* being equal.
- Using what you know about the *Subtraction Property of Equality*, how do you think the *Subtraction Property of Inequality* might be similar or different? Sample answer: I think the Subtraction Property of Inequality will still relate to subtracting a number from each side, but it will be related to an inequality, not an equation.

Explore Addition and Subtraction Properties of Inequality

Objective

Students will use Web Sketchpad to explore how inequalities behave when adding or subtracting the same number from each side.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a Web Sketchpad containing a bar graph that represents values on both sides of an equation or inequality. Throughout this activity, students will use a sketch to investigate the effect of adding a number to each side of an inequality or subtracting a number from each side of an inequality.

Inquiry Question

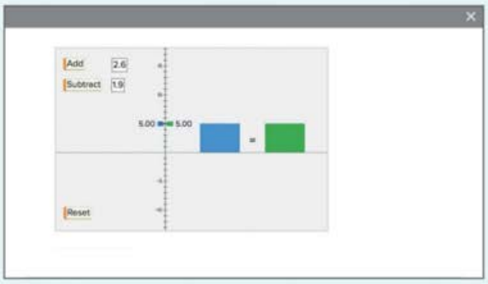
How does adding or subtracting the same number from each side of an inequality affect the inequality? Sample answer: Adding or subtracting the same number from each side of an inequality keeps the inequality true.

Go Online to find additional teaching notes.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 5



Explore, Slide 3 of 5

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how adding or subtracting the same number from each side of an inequality affects the inequality.

TYPE

On Slide 3, students make a conjecture about which operations and numbers can be used without changing the inequality.

Interactive Presentation

Explore, Slide 4 of 5

TYPE

On Slide 4, students type to explain if their experiments support their conjecture.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Addition and Subtraction Properties of Inequality *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore and deepen their understanding of inequalities. Students should notice how an inequality changes or remains true when they subtract or add a certain value to each side of the inequality.

Go Online to find additional teaching notes.

Learn Inequalities

Objective

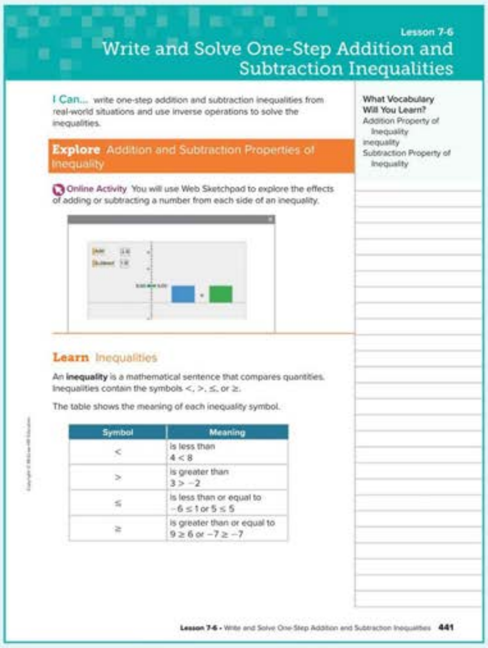
Students will understand the definition of inequality and the different meanings of the inequality symbols.

Teaching Notes

SLIDE 1

Encourage students to brainstorm different words or phrases that could be represented by each inequality symbol. This will help them recognize key words and phrases when they are asked to write inequalities later in the module.

Discuss with students the differences between the related symbols ($>$ and $\geq$, $<$ and $\leq$). Point out that the symbol with the bar under the inequality symbol also means *or equal to* and mimics part of the equals sign. They will need to differentiate between the two related symbols when they write and graph inequalities. It is interesting to note that if you keyboard the $>$ symbol and then the $=$ sign, some word processing programs translate that to $\geq$.



Lesson 7-6

Write and Solve One-Step Addition and Subtraction Inequalities

I Can... write one-step addition and subtraction inequalities from real-world situations and use inverse operations to solve the inequalities.

What Vocabulary Will You Learn?
Addition Property of Inequality
inequality
Subtraction Property of Inequality

Explore Addition and Subtraction Properties of Inequality

Online Activity You will use Web Sketchpad to explore the effects of adding or subtracting a number from each side of an inequality.

Learn Inequalities

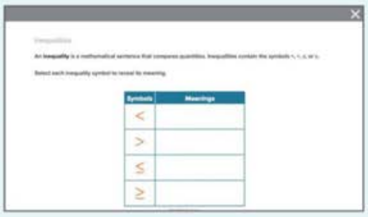
An **inequality** is a mathematical sentence that compares quantities. Inequalities contain the symbols <, >, ≤, or ≥.

The table shows the meaning of each inequality symbol.

Symbol	Meaning
$<$	is less than $4 < 8$
$>$	is greater than $3 > -2$
$\leq$	is less than or equal to $-6 \leq 1$ or $5 \leq 5$
$\geq$	is greater than or equal to $9 \geq 6$ or $-7 \geq -7$

Lesson 7-6 • Write and Solve One-Step Addition and Subtraction Inequalities 441

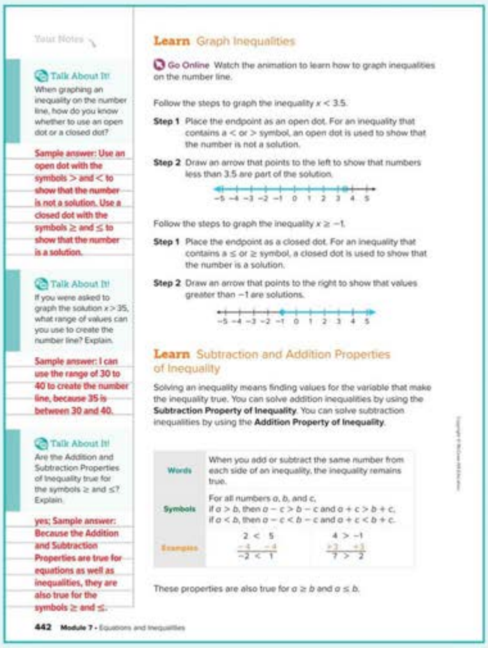
Interactive Presentation



Learn, Inequalities

CLICK

Students select inequality symbols to reveal their meanings.



Your Notes

Talk About It!
When graphing an inequality on the number line, how do you know whether to use an open dot or a closed dot?

Sample answer: Use an open dot with the symbols > and < to show that the number is not a solution. Use a closed dot with the symbols ≥ and ≤ to show that the number is a solution.

Talk About It!
If you were asked to graph the solution $x > 35$, what range of values can you use to create the number line? Explain.

Sample answer: I can use the range of 30 to 40 to create the number line, because 35 is between 30 and 40.

Talk About It!
Are the Addition and Subtraction Properties of Inequality true for the symbols ≥ and ≤? Explain.

yes; Sample answer: Because the Addition and Subtraction Properties are true for equations as well as inequalities, they are also true for the symbols ≥ and ≤.

Learn Graph Inequalities

Go Online Watch the animation to learn how to graph inequalities on the number line.

Follow the steps to graph the inequality $x < 3.5$.

Step 1 Place the endpoint as an open dot. For an inequality that contains a < or > symbol, an open dot is used to show that the number is not a solution.

Step 2 Draw an arrow that points to the left to show that numbers less than 3.5 are part of the solution.

Follow the steps to graph the inequality $x \geq -1$.

Step 1 Place the endpoint as a closed dot. For an inequality that contains a ≤ or ≥ symbol, a closed dot is used to show that the number is a solution.

Step 2 Draw an arrow that points to the right to show that values greater than −1 are solutions.

Learn Subtraction and Addition Properties of Inequality

Solving an inequality means finding values for the variable that make the inequality true. You can solve addition inequalities by using the **Subtraction Property of Inequality**. You can solve subtraction inequalities by using the **Addition Property of Inequality**.

Words	When you add or subtract the same number from each side of an inequality, the inequality remains true.
Symbols	For all numbers a, b, and c, if $a > b$, then $a - c > b - c$ and $a + c > b + c$. if $a < b$, then $a - c < b - c$ and $a + c < b + c$.
Examples	$2 < 5$, -4 -4, $-2 < 1$; $4 > -1$, $+3$ $+3$, $7 > 2$

These properties are also true for $a \geq b$ and $a \leq b$.

442 Module 7 • Equations and Inequalities

Interactive Presentation

Learn, Subtraction and Addition Properties of Inequality, Slide 1 of 2

WATCH

On Slide 1 of Learn, Graph Inequalities, students watch an animation that explains how to graph inequalities.

FLASHCARDS

On Slide 1 of Learn, Subtraction and Addition Properties of Inequality, students use Flashcards to view multiple representations of properties.

Learn Graph Inequalities

Objective

Students will understand how to graph an inequality on a number line.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Have students watch the animation on Slide 1. The animation illustrates how to graph inequalities on a number line.

Talk About It!

SLIDE 2

Mathematical Discourse

When graphing an inequality on the number line, how do you know whether to use an open dot or a closed dot? Sample answer: Use an open dot with the symbols > and < to show that the number is not a solution. Use a closed dot with the symbols ≥ and ≤ to show that the number is a solution.

If you were asked to graph the solution $x > 35$, what range of values can you use to create the number line? Explain. Sample answer: I can use the range of 30 to 40 to create the number line, because 35 is between 30 and 40.

Learn Subtraction and Addition Properties of Inequality

Objective

Students will understand the Subtraction and Addition Properties of Inequality.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

Are the Addition and Subtraction Properties of Inequality true for the symbols ≥ and ≤? Explain. yes; Sample answer: Because the Addition and Subtraction Properties are true for equations as well as inequalities, they are also true for the symbols ≥ and ≤.

Example 1 Solve and Graph Addition Inequalities

Objective

Students will solve and graph one-step addition inequalities with rational numbers.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the inequality symbol and the value –10 when explaining an appropriate range for the number line.

5 Use Appropriate Tools Strategically Students will use the number line eTool to graph the solution set on the number line.

6 Attend to Precision Encourage students to accurately represent the solution set by using a closed dot because the solution set does contain the number –10.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is paired with the variable? addition

AL Read the inequality in words. Sample answer: *four tenths plus y is less than or equal to negative nine and six tenths.*

OL How will you undo the addition of 0.4? Why? Subtract 0.4 from each side of the inequality. Sample answer: Addition and subtraction are inverse operations.

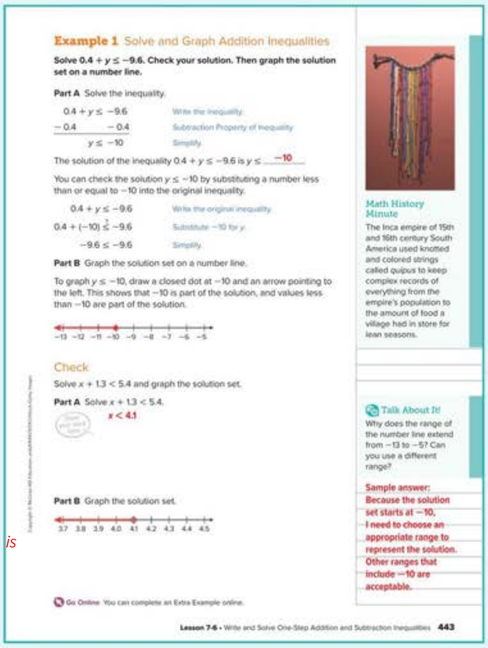
OL What property of inequality is used to subtract 0.4 from each side? Subtraction Property of Inequality

OL How can you check your answer? Sample answer: Replace *y* with –10 in the original inequality to verify the inequality is a true statement, which it is.

BL If $0.4 + y \leq -9.6$, what must be true about $0.1 + y$? Explain without solving the inequality $0.4 + y \leq -9.6$. Sample answer: If $0.4 + y \leq -9.6$, then $0.1 + y$ must be less than or equal to –9.9. The expression $0.1 + y$ is 03 less than the expression $0.4 + y$. Subtracting 0.3 from the left side of the inequality means that I must subtract 0.3 from the right side of the inequality. Because $-9.6 - 0.3 = -9.9$, then $0.1 + y \leq -9.9$.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Solve and Graph Addition Inequalities

Solve $0.4 + y \leq -9.6$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$0.4 + y \leq -9.6$ — Write the inequality.
$-0.4 \quad -0.4$ — Subtraction Property of Inequality
$y \leq -10$ — Simplify.

The solution of the inequality $0.4 + y \leq -9.6$ is $y \leq$ –10.

You can check the solution $y \leq -10$ by substituting a number less than or equal to –10 into the original inequality.

$0.4 + y \leq -9.6$ — Write the original inequality.
$0.4 + (-10) \stackrel{?}{\leq} -9.6$ — Substitute –10 for y.
$-9.6 \leq -9.6$ — Simplify.

Part B Graph the solution set on a number line.

To graph $y \leq -10$, draw a closed dot at –10 and an arrow pointing to the left. This shows that –10 is part of the solution, and values less than –10 are part of the solution.

–13 –12 –11 –10 –9 –8 –7 –6 –5

Check

Solve $x + 1.3 < 5.4$ and graph the solution set.

Part A Solve $x + 1.3 < 5.4$.

$x < 4.1$

Part B Graph the solution set.

3.7 3.8 3.9 4.0 4.1 4.2 4.3 4.4 4.5

Go Online You can complete an Extra Example online.

Math History Minute

The Inca empire of 15th and 16th century South America used knotted and colored strings called quipus to keep complex records of everything from the empire's population to the amount of food a village had in store for lean seasons.

Talk About It!

Why does the range of the number line extend from –13 to –5? Can you use a different range?

Sample answer: Because the solution set starts at –10, I need to choose an appropriate range to represent the solution. Other ranges that include –10 are acceptable.

Lesson 7-6 • Write and Solve One-Step Addition and Subtraction Inequalities **443**

Example 1, Solve and Graph Addition Inequalities, Slide 3 of 5

TYPE

On Slide 2, students determine the missing value to solve an inequality.

eTOOLS

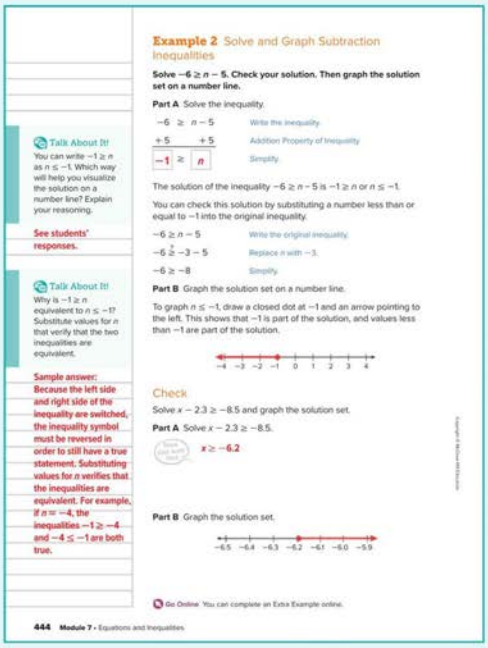
On Slide 3, students use the Number Line eTool to graph the inequality on the number line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.EE.B.4, 7.EE.B.4.B

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION



Talk About It!

You can write $-1 \geq n$ as $n \leq -1$. Which way will help you visualize the solution on a number line? Explain your reasoning.

See students' responses.

Talk About It!

Why is $-1 \geq n$ equivalent to $n \leq -1$? Substitute values for n that verify that the two inequalities are equivalent.

Sample answer: Because the left side and right side of the inequality are switched, the inequality symbol must be reversed in order to still have a true statement. Substituting values for n verifies that the inequalities are equivalent. For example, if $n = -4$, the inequalities $-1 \geq -4$ and $-4 \leq -1$ are both true.

Example 2 Solve and Graph Subtraction Inequalities

Solve $-6 \geq n - 5$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$-6 \geq n - 5$ Write the inequality.

$+5 \quad +5$ Addition Property of Inequality

$-1 \geq n$ Simplify.

The solution of the inequality $-6 \geq n - 5$ is $-1 \geq n$ or $n \leq -1$.

You can check this solution by substituting a number less than or equal to -1 into the original inequality.

$-6 \geq n - 5$ Write the original inequality.

$-6 \overset{?}{\geq} -3 - 5$ Replace n with -3.

$-6 \geq -8$ Simplify.

Part B Graph the solution set on a number line.

To graph $n \leq -1$, draw a closed dot at -1 and an arrow pointing to the left. This shows that -1 is part of the solution, and values less than -1 are part of the solution.

Check

Solve $x - 2.3 \geq -8.5$ and graph the solution set.

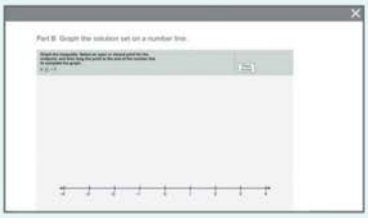
Part A Solve $x - 2.3 \geq -8.5$.

$x \geq -6.2$

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

444 Module 7 • Equations and Inequalities



Example 2, Solve and Graph Subtraction Inequalities, Slide 3 of 5

TYPE

On Slide 2, students determine the missing values to solve an inequality.

eTOOLS

On Slide 3, students use the Number Line eTool to graph the inequality on the number line.

CLICK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Solve and Graph Subtraction Inequalities

Objective

Students will solve and graph one-step subtraction inequalities with rational numbers.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Number Line eTool to graph the solution set on the number line.

6 Attend to Precision Encourage students to accurately represent the solution set by using a closed dot because the solution set does contain the number –1.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is paired with the variable? subtraction

AL Read the inequality in words. Sample answer: *negative six is greater than or equal to n minus five.*

OL How will you undo the subtraction of 5? Why? Add 5 to each side of the inequality. Sample answer: Addition and subtraction are inverse operations.

OL What property of inequality is used to add 5 to each side? Addition Property of Inequality

OL How can you check your answer? Sample answer: Replace *n* with –1 in the original inequality to verify the inequality is a true statement, which it is.

BL Is zero a solution for the inequality? Explain. no; Sample answer: $0 - 5 = -5$ and -6 is not greater than or equal to -5.

SLIDE 3

AL To graph $n \leq -1$, will the endpoint be open or closed? Explain. closed; Any number that is less than or equal to –1 is a solution.

OL Is –1 part of the solution? Explain. yes; Sample answer: –1 is less than or equal to –1.

BL Explain why the graph of the solution set never ends in the direction to the left of the closed dot. Sample answer: The graph never ends in this direction because any number that is less than or equal to –1 is a solution.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Write Inequalities

Objective

Students will understand how to write inequalities from a real-world problem.

Teaching Notes

SLIDE 1

You may wish to have students practice identifying the appropriate symbol when given a phrase. For example, ask students which inequality symbol should be used to represent the phrase *at least*. Students should note that the greater than or equal to symbol (≥) should be used.

SLIDE 2

Ask students to compare and contrast the steps for writing an equation from a real-world situation with the steps for writing an inequality from a real-world situation. Point out the steps are the same, but the symbols used are different. Students should pay close attention to the phrases used in an inequality situation to determine the correct symbol to use.

Talk About It!

SLIDE 3

Mathematical Discourse

How do you know which inequality symbol to use when representing a real-world situation? Sample answer: Identify key phrases in the problem like *more than, less than, at least,* or *at most*. Determine which symbol best represents the situation in the context of the problem.

Example 3 Write and Solve One-Step Addition Inequalities

Objective

Students will write one-step addition inequalities from real-world problems and interpret the solution.

Questions for Mathematical Discourse

SLIDE 2

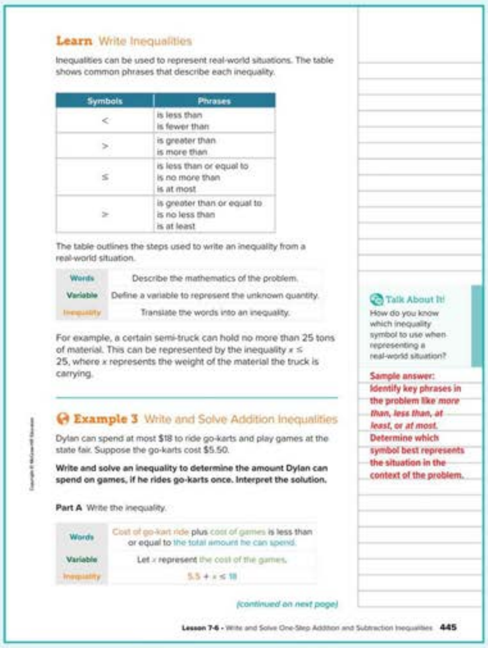
AL What is the unknown quantity? How will you represent it in the inequality? the amount Dylan can spend on games; with a variable

OL Why is the inequality symbol ≤ used? Sample answer: The most Dylan can spend is $18, so he can spend any amount that is less than or equal to $18.

OL A classmate wrote the inequality $x + 5.5 \leq 18$. Is this inequality correct? Explain. yes; Addition is commutative, so the terms 5.5 and x can be added in any order.

BL Suppose Dylan wants to ride the go-karts twice. What inequality can be used to find how much he can spend on games now? $11 + x \leq 18$

(continued on next page)



Learn Write Inequalities

Inequalities can be used to represent real-world situations. The table shows common phrases that describe each inequality.

Symbols	Phrases
<	is less than is fewer than
>	is greater than is more than
≤	is less than or equal to is no more than is at most
≥	is greater than or equal to is no less than is at least

The table outlines the steps used to write an inequality from a real-world situation.

Words	Describe the mathematics of the problem.
Variable	Define a variable to represent the unknown quantity.
Inequality	Translate the words into an inequality.

For example, a certain semi-truck can hold no more than 25 tons of material. This can be represented by the inequality $x \leq 25$, where x represents the weight of the material the truck is carrying.

Talk About It! How do you know which inequality symbol to use when representing a real-world situation?

Sample answer: Identify key phrases in the problem like *more than, less than, at least, or at most.* Determine which symbol best represents the situation in the context of the problem.

Example 3 Write and Solve Addition Inequalities

Dylan can spend at most $18 to ride go-karts and play games at the state fair. Suppose the go-karts cost $5.50.

Write and solve an inequality to determine the amount Dylan can spend on games, if he rides go-karts once. Interpret the solution.

Part A Write the inequality.

Words	Cost of go-kart ride plus cost of games is less than or equal to the total amount he can spend.
Variable	Let x represent the cost of the games.
Inequality	$5.5 + x \leq 18$

(continued on next page)

Lesson 7-6 • Write and Solve One-Step Addition and Subtraction Inequalities 445

Interactive Presentation

Learn, Write Inequalities, Slide 2 of 3

CLICK

On Slide 1 of the Learn, students select inequality symbols to see common phrases that describe each inequality.

FLASHCARDS

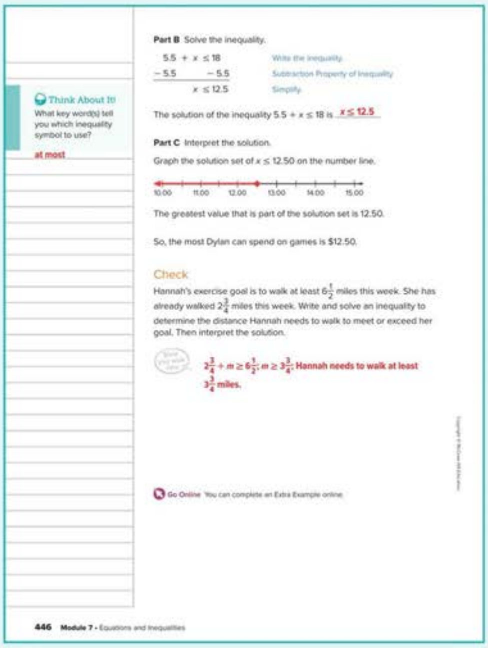
On Slide 2 of the Learn, students use Flashcards to learn how to write any inequality from a real-world situation.

FLASHCARDS

On Slide 2 of the Example, students use Flashcards to view thesteps used to write the inequality.

7.EE.B.4, 7.EE.B.4.B

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION


Part B Solve the inequality.

$5.5 + x \leq 18$ — Write the inequality.
$-5.5 \quad -5.5$ — Subtraction Property of Inequality
$x \leq 12.5$ — Simplify.

The solution of the inequality $5.5 + x \leq 18$ is $x \leq 12.5$.

Part C Interpret the solution.

Graph the solution set of $x \leq 12.50$ on the number line.

The greatest value that is part of the solution set is 12.50.

So, the most Dylan can spend on games is $12.50.

Think About It! What key word(s) tell you which inequality symbol to use? at most

Check

Hannah's exercise goal is to walk at least $6\frac{1}{2}$ miles this week. She has already walked $2\frac{3}{4}$ miles this week. Write and solve an inequality to determine the distance Hannah needs to walk to meet or exceed her goal. Then interpret the solution.

$2\frac{3}{4} + m \geq 6\frac{1}{2}$; $m \geq 3\frac{3}{4}$; Hannah needs to walk at least $3\frac{3}{4}$ miles.

Go Online You can complete an Extra Example online.

446 Module 7 • Equations and Inequalities

Interactive Presentation

Example 3, Write and Solve One-Step Addition Inequalities, Slide 2 of 5

eTOOLS

On Slide 4, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Write and Solve Addition Inequalities *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL What operation is paired with the variable? addition

AL Read the inequality in words. Sample answer: *five and five tenths plus x is less than or equal to eighteen.*

OL Describe how to solve the inequality. Sample answer: Use the Subtraction Property of Inequality to subtract 5.5 from each side. So, $x \leq 12.5$.

OL How can you check your answer? Sample answer: Replace *x* with 12.5 in the original inequality to verify the statement is true.

BL A classmate interpreted the solution to mean that Dylan can play 12.5 games, or 12 games since he cannot play half of a game. Is this reasoning correct? Explain. no; Sample answer: The variable is not the number of games. The variable represents how much money Dylan can spend on the games. So, he can spend $12.50.

SLIDE 4

AL What does the solution $x \leq 12.50$ mean, within the context of the problem? It means that Dylan can spend an amount less than or equal to $12.50 on games.

OL Explain how to graph the inequality. Sample answer: Place a closed dot at 12.5 because the solution contains 12.5. Then extend the arrow forever to the left because any number that is less than or equal to 12.5 is a solution.

OL Does it make sense, within the context of the problem, that the graph will extend forever to the left? Explain. no; Sample answer: Dylan cannot spend an amount less than $0, so any point left of zero does not make sense within the context of the problem.

BL If each game costs $0.75, what is the maximum number of games Dylan can play? 16 games

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 4 Write and Solve One-Step Subtraction Inequalities

Objective

Students will write one-step subtraction inequalities from real-world problems and interpret the solution.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively S students will decontextualize the information, represent it symbolically with a one-step inequality, and interpret the solution.

5 Use Appropriate Tools Strategically Students will use the Number Line eTool to graph the solution set of the inequality.

6 Attend to Precision Encourage students to solve the inequality by adhering to the properties of operations.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown quantity? How will you represent it in the inequality? the height of the Great Dane; with a variable

AL What word or phrase indicates which inequality symbol to use? What inequality symbol will you use? at least; $\geq$

OL A classmate wrote the inequality $6\frac{1}{4} - h \geq 25$. Is this inequality correct? Explain. no; Subtraction is not commutative, so the terms $6\frac{1}{4}$ and h cannot be subtracted in any order.

OL How can you write the inequality using the less than or equal to symbol? $25 \leq h - 6\frac{1}{4}$

BL Write and solve the inequality using decimals. $h - 6.25 \geq 25$; $h \geq 31.25$

SLIDE 3

AL What operation is paired with the variable? subtraction

AL Read the inequality in words. Sample answer: *h minus six and one fourth is greater than or equal to twenty-five.*

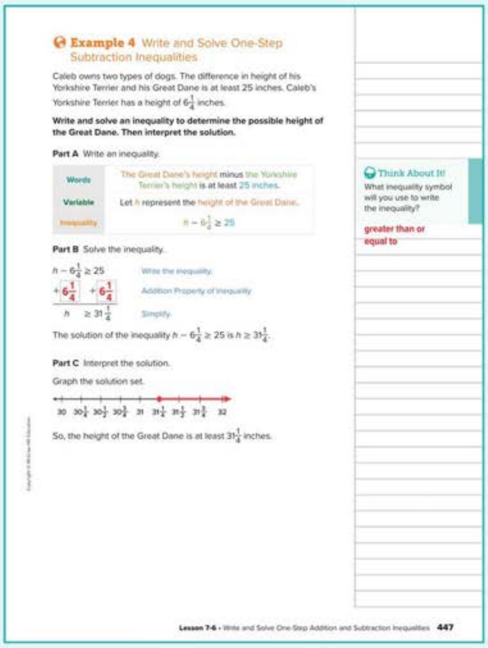
OL Describe how to solve the inequality. Sample answer: Use the Addition Property of Inequality to add $6\frac{1}{4}$ to each side. So, $h \geq 31\frac{1}{4}$.

OL How can you check your answer? Sample answer: Replace h with $31\frac{1}{4}$ in the original inequality to verify the statement is true.

BL A classmate interpreted the solution to mean that the height of the Great Dane is exactly $31\frac{1}{4}$ inches. Is this reasoning correct? Explain. no; Sample answer: The height of the dog is at least $31\frac{1}{4}$ inches. We don't know if the dog is exactly $31\frac{1}{4}$ inches tall.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Write and Solve One-Step Subtraction Inequalities

Caleb owns two types of dogs. The difference in height of his Yorkshire Terrier and his Great Dane is at least 25 inches. Caleb's Yorkshire Terrier has a height of $6\frac{1}{4}$ inches.

Write and solve an inequality to determine the possible height of the Great Dane. Then interpret the solution.

Part A Write an inequality.

Words	The Great Dane's height minus the Yorkshire Terrier's height is at least 25 inches.
Variable	Let h represent the height of the Great Dane.
Inequality	$h - 6\frac{1}{4} \geq 25$

Part B Solve the inequality.

$h - 6\frac{1}{4} \geq 25$ Write the inequality.

$+6\frac{1}{4} \quad +6\frac{1}{4}$ Addition Property of Inequality

$h \geq 31\frac{1}{4}$ Simplify.

The solution of the inequality $h - 6\frac{1}{4} \geq 25$ is $h \geq 31\frac{1}{4}$.

Part C Interpret the solution.

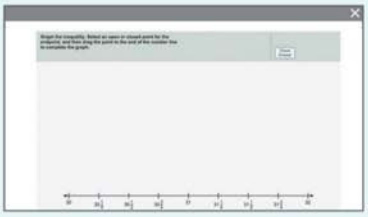
Graph the solution set.

So, the height of the Great Dane is at least $31\frac{1}{4}$ inches.

Think About It! What inequality symbol will you use to write the inequality? greater than or equal to

Lesson 7-6 • Write and Solve One-Step Addition and Subtraction Inequalities 447

Interactive Presentation



Example 4, Write and Solve One-Step Subtraction Inequalities, Slide 4 of 5

FLASHCARDS

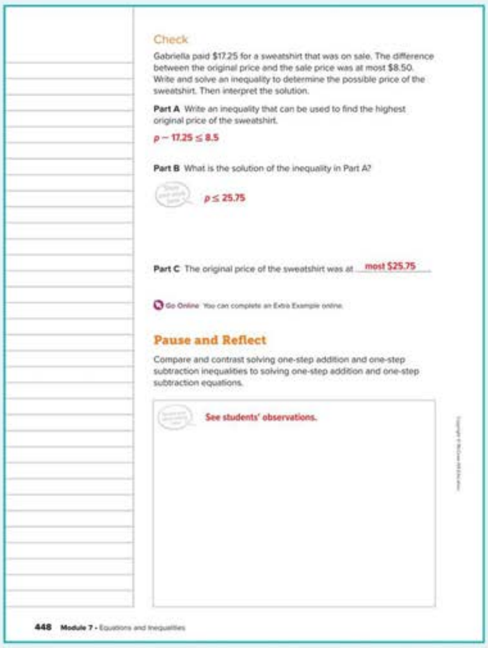
On Slide 2, students use Flashcards to view the steps for writing the inequality.

eTOOLS

On Slide 4, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Gabriella paid $17.25 for a sweatshirt that was on sale. The difference between the original price and the sale price was at most $8.50. Write and solve an inequality to determine the possible price of the sweatshirt. Then interpret the solution.

Part A Write an inequality that can be used to find the highest original price of the sweatshirt.

$p - 17.25 \leq 8.5$

Part B What is the solution of the inequality in Part A?

$p \leq 25.75$

Part C The original price of the sweatshirt was at most $25.75

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare and contrast solving one-step addition and one-step subtraction inequalities to solving one-step addition and one-step subtraction equations.

See students' observations.

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DIFFERENTIATE

Reteaching Activity AL

To further students' understanding of graphing inequalities, have them create a flow chart or an outline that walks through the steps needed to graph and check an inequality. A sample outline using the value 3 is shown.

1. Place the endpoint at 3.
   a. Open dot
      i. $>$
      ii. $<$
   b. Closed dot
      i. $\geq$
      ii. $\leq$

2. Draw an arrow.
   a. Arrow points to the left
      i. $x < 3$
      ii. $x \leq 3$
      iii. $3 > x$
      iv. $3 \geq x$
   b. Arrow points to the right
      i. $x > 3$
      ii. $x \geq 3$
      iii. $3 < x$
      iv. $3 \leq x$

3. Check the graph.
   a. Choose a point that is on the arrow and test the value in the inequality. It should produce a true statement.
   b. Choose a point that is not on the arrow and test the value in the inequality. It should produce a false statement.

Some students may need help understanding parts iii and iv in sections 2a and 2b. Encourage them to examine what happens when you rewrite an inequality like $3 < 7$ as $7 > 3$. Then have them discuss with a partner why the inequality symbol changes when the sides of an inequality like $x \geq 9$ are switched.

Apply Elevators

Objective

Students will come up with their own strategy to solve an application problem involving the weight capacity of an elevator.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

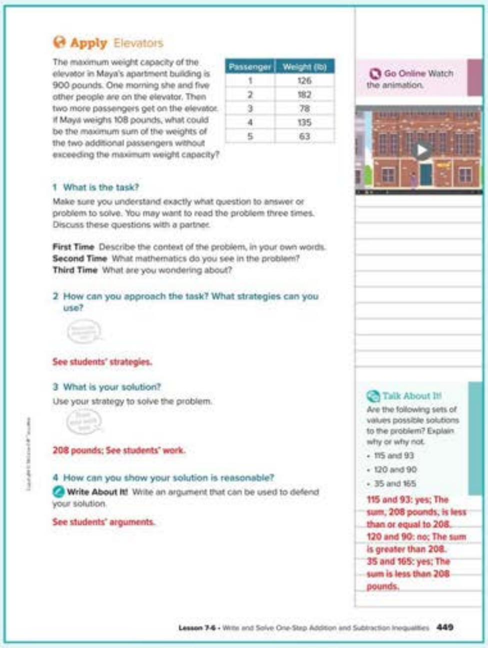
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can finding the sum of the six passengers help find the weight of the remaining two?
- What symbol should be used in the inequality expressing maximum weight of the two passengers?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Elevators

The maximum weight capacity of the elevator in Maya's apartment building is 900 pounds. One morning she and five other people are on the elevator. Then two more passengers get on the elevator. If Maya weighs 108 pounds, what could be the maximum sum of the weights of the two additional passengers without exceeding the maximum weight capacity?

Passenger	Weight (lb)
1	126
2	182
3	78
4	135
5	63

Go Online Watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

208 pounds; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

Are the following sets of values possible solutions to the problem? Explain why or why not.

- 115 and 93
- 120 and 90
- 35 and 165

115 and 93: yes; The sum, 208 pounds, is less than or equal to 208.
120 and 90: no; The sum is greater than 208.
35 and 165: yes; The sum is less than 208 pounds.

Lesson 7-6 • Write and Solve One-Step Addition and Subtraction Inequalities 449

Interactive Presentation

Apply, Elevators

WATCH

Students watch an animation that illustrates the problem they are about to solve.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Hayley's reading log is shown. She is required to read at least 2 hours each week. Which could be the amount of time she needs to read on Saturday and Sunday to meet her reading goal?

Day	Number of Minutes
Monday	20
Tuesday	25
Wednesday	15
Thursday	10
Friday	20
Saturday	x
Sunday	y

Ⓐ Saturday: 5 minutes; Sunday: 10 minutes

Ⓑ Saturday: 10 minutes; Sunday: 10 minutes

Ⓒ Saturday: 10 minutes; Sunday: 15 minutes

● Saturday: 15 minutes; Sunday: 20 minutes

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

450 Module 7 • Equations and Inequalities

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about solving one-step addition and subtraction inequalities. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. If Zoe is $32\frac{1}{2}$ inches tall, what inequality can be used to find the number of inches she needs to grow so that she is at least 36 inches tall? Write a mathematical argument that can be used to defend your solution. $32\frac{1}{2} + x \geq 36$; Sample answer: The phrase "at least" means to use the ≥ symbol. Because Zoe needs to grow, use addition as the operation.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 11, 13–16
- Extension: Solve Compound Inequalities
- ALEKS® One-Step Inequalities, Applications of Inequalities

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–10, 12, 14
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Writing and Graphing Inequalities

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Writing and Graphing Inequalities

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

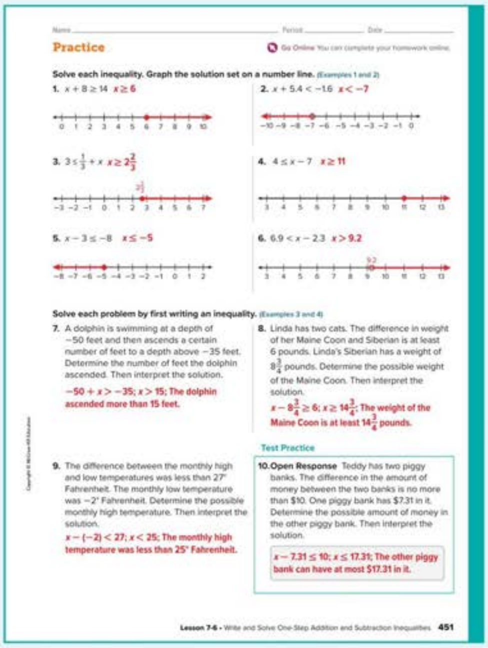
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve and graph one-step addition and subtraction inequalities	1–6
2	write and solve one-step addition inequalities from real-world problems and interpret the solution	7
2	write and solve one-step subtraction inequalities from real-world problems and interpret the solution	8, 9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving writing and solving one-step addition and subtraction inequalities	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may incorrectly solve inequalities in which the variable is on the right side of the inequality. In Exercise 2, students might get the inequality $-5 > x$, and incorrectly rewrite it as $x > -5$. Encourage students to read the inequality aloud and accurately identify how the two sides of the inequality are related.



Name ________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Solve each inequality. Graph the solution set on a number line. (Examples 1 and 2)

1. $x + 8 \geq 14$ $x \geq 6$
2. $x + 5.4 < -1.6$ $x < -7$
3. $3 \leq \frac{1}{3} + x$ $x \geq 2\frac{2}{3}$
4. $4 \leq x - 7$ $x \geq 11$
5. $x - 3 \leq -8$ $x \leq -5$
6. $6.9 < x - 2.3$ $x > 9.2$

Solve each problem by first writing an inequality. (Examples 3 and 4)

7. A dolphin is swimming at a depth of −50 feet and then ascends a certain number of feet to a depth above −35 feet. Determine the number of feet the dolphin ascended. Then interpret the solution.
   $-50 + x > -35$; $x > 15$; The dolphin ascended more than 15 feet.

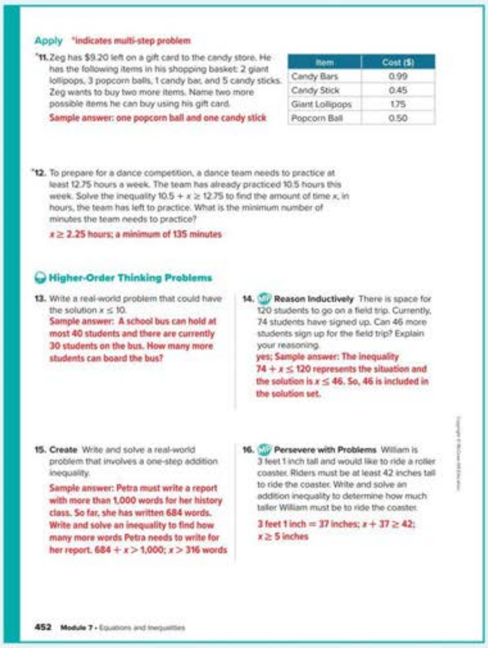
8. Linda has two cats. The difference in weight of her Maine Coon and Siberian is at least 6 pounds. Linda's Siberian has a weight of $8\frac{3}{4}$ pounds. Determine the possible weight of the Maine Coon. Then interpret the solution.
   $x - 8\frac{3}{4} \geq 6$; $x \geq 14\frac{3}{4}$; The weight of the Maine Coon is at least $14\frac{3}{4}$ pounds.

9. The difference between the monthly high and low temperatures was less than 27° Fahrenheit. The monthly low temperature was −2° Fahrenheit. Determine the possible monthly high temperature. Then interpret the solution.
   $x - (-2) < 27$; $x < 25$; The monthly high temperature was less than 25° Fahrenheit.

Test Practice

10. **Open Response** Teddy has two piggy banks. The difference in the amount of money between the two banks is no more than $10. One piggy bank has $7.31 in it. Determine the possible amount of money in the other piggy bank. Then interpret the solution.
   $x - 7.31 \leq 10$; $x \leq 17.31$; The other piggy bank can have at most $17.31 in it.

Lesson 7-6 • Write and Solve One-Step Addition and Subtraction Inequalities 451



Apply *indicates multi-step problem

*11. Zeg has $9.20 left on a gift card to the candy store. He has the following items in his shopping basket: 2 giant lollipops, 3 popcorn balls, 1 candy bar, and 5 candy sticks. Zeg wants to buy two more items. Name two more possible items he can buy using his gift card.

Sample answer: one popcorn ball and one candy stick

Item	Cost ($)
Candy Bars	0.99
Candy Stick	0.45
Giant Lollipops	1.75
Popcorn Ball	0.50

*12. To prepare for a dance competition, a dance team needs to practice at least 12.75 hours a week. The team has already practiced 10.5 hours this week. Solve the inequality $10.5 + x \geq 12.75$ to find the amount of time x, in hours, the team has left to practice. What is the minimum number of minutes the team needs to practice?

$x \geq 2.25$ hours; a minimum of 135 minutes

Higher-Order Thinking Problems

13. Write a real-world problem that could have the solution $x \leq 10$.

Sample answer: A school bus can hold at most 40 students and there are currently 30 students on the bus. How many more students can board the bus?

14. **Reason Inductively** There is space for 120 students to go on a field trip. Currently, 74 students have signed up. Can 46 more students sign up for the field trip? Explain your reasoning.

yes; Sample answer: The inequality $74 + x \leq 120$ represents the situation and the solution is $x \leq 46$. So, 46 is included in the solution set.

15. **Create** Write and solve a real-world problem that involves a one-step addition inequality.

Sample answer: Petra must write a report with more than 1,000 words for her history class. So far, she has written 684 words. Write and solve an inequality to find how many more words Petra needs to write for her report. $684 + x > 1{,}000$; $x > 316$ words

16. **Persevere with Problems** William is 3 feet 1 inch tall and would like to ride a roller coaster. Riders must be at least 42 inches tall to ride the coaster. Write and solve an addition inequality to determine how much taller William must be to ride the coaster.

3 feet 1 inch = 37 inches; $x + 37 \geq 42$; $x \geq 5$ inches

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7.EE.B.4, 7.EE.B.4.B

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 14, students use reasoning with inequalities to determine if 46 additional students can sign up for a field trip.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 16, students use multiple steps to write and solve an inequality for a real-world problem.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercises 11–12 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 9 might be, “Two giant lollipops cost $3.50.” An example of a false statement might be, “Five candy sticks cost less than $2.00.” Have students trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Solve the problem another way.
Use with Exercise 14 Have students work in groups of 3–4. After completing Exercise 14, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is viable. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable methods to the class, and explain why each method is a correct method.

Lesson 7-7 7.EE.B.4, 7.EE.B.4.B

Write and Solve One-Step Multiplication and Division Inequalities

LESSON GOAL

Students will write and solve one-step multiplication and division inequalities.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Multiplication and Division Properties of Inequality

Learn: Division and Multiplication Properties of Inequality
Example 1: Solve and Graph Multiplication Inequalities
Example 2: Solve and Graph Division Inequalities

Explore: Multiply and Divide Inequalities by Negative Numbers

Learn: Division and Multiplication Properties of Inequality
Example 3: Multiplication Inequalities with Negative Coefficients
Example 4: Division Inequalities with Negative Coefficients
Example 5: Write and Solve One-Step Multiplication Inequalities
Example 6: Write and Solve One-Step Division Inequalities
Apply: Fundraising

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Interval Notation and Set Notation		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 44 of the *Language Development Handbook* to help your students build mathematical language related to writing and solving one-step multiplication and division inequalities.

ELL You can use the tips and suggestions on page T44 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7.EE.B** by writing and solving one-step multiplication and division inequalities.
Standards for Mathematical Content: **7 .EE.B.4, 7.EE.B.4.B,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP7**

Coherence

Vertical Alignment

Previous
Students wrote and solved one-step addition and subtraction inequalities.
7.EE.B.4, 7.EE.B.4.B

Now
Students write and solve one-step multiplication and division inequalities.
7.EE.B.4, 7.EE.B.4.B

Next
Students will write and solve two-step inequalities.
7.EE.B.4.B

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

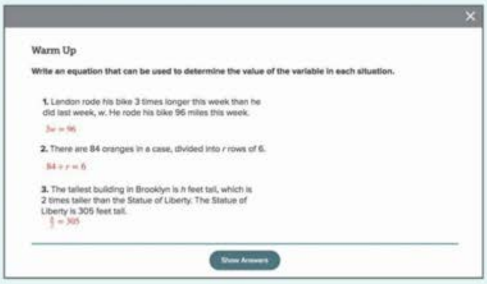
Conceptual Bridge In this lesson, students draw on their knowledge of solving one-step addition and subtraction inequalities to build *understanding* of solving and graphing one-step multiplication and division inequalities. They will then use this understanding to build *fluency* in solving one-step multiplication and division inequalities. They *apply* their understanding to write, solve, and graph multiplication and division inequalities that represent real-world situations.

Mathematical Background

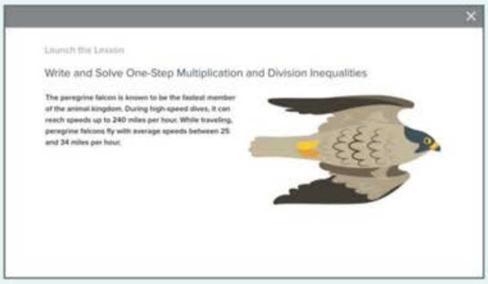
Inequalities involving only multiplication or division can be solved using the Properties of Inequality, Multiplication Property of Inequality and Division Property of Inequality.

When you multiply or divide each side of an inequality by a positive number, the inequality remains true. When you multiply or divide each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

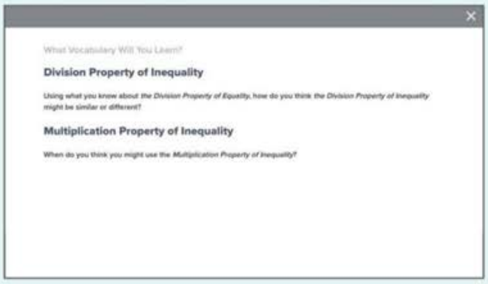
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing one-step multiplication and division equations (Exercises 1–3)

Answers

1. $3w = 96$

2. $84 \div r = 6$

3. $\frac{h}{2} = 305$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the flying speeds of peregrine falcons.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Using what you know about the *Division Property of Equality*, how do you think the *Division Property of Inequality* might be similar or different? Sample answer: I can use either property to divide each side by the same number. I will use the Division Property of Equality if I'm solving an equation, and the Division Property of Inequality if I'm solving an inequality.
- When do you think you might use the *Multiplication Property of Inequality*? Sample answer: I might use the Multiplication Property of Inequality when I'm solving an inequality that requires me to multiply each side of the inequality by the same number.

Explore Multiplication and Division Properties of Inequality

Objective

Students will explore how multiplying and dividing each side of an inequality by the same positive number affects the inequality.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the amounts of money of three friends. Throughout this activity, students will use the money values from the table and Web Sketchpad bar graphs to investigate how dividing or multiplying each side of an inequality by the same positive number affects the inequality's truth value.

Inquiry Question

How does multiplying or dividing each side of an inequality by the same positive number affect the inequality? Sample answer: Multiplying or dividing each side of an inequality by the same positive number keeps the inequality true.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

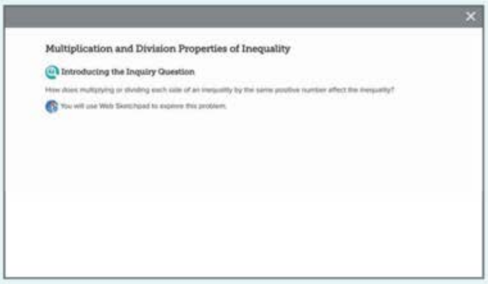
SLIDE 4

Mathematical Discourse

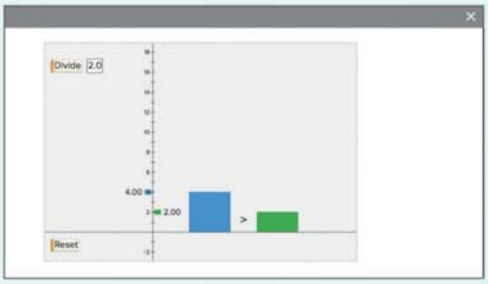
Does an inequality remain true when you divide each side by the same positive number? Explain. Sample answer: Yes. Dividing each side of an inequality by the same positive number results in a true inequality.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 8



Explore, Slide 4 of 8

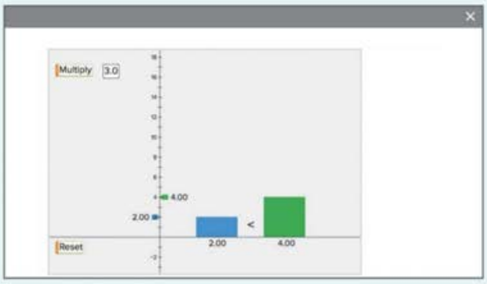
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how multiplying or dividing each side of an inequality by the same positive number affects the inequality.

TYPE

On Slide 2, students write an inequality to represent a comparison between two different amounts of money.

Interactive Presentation



Explore, Slide 7 of 8

TYPE

On Slide 5, students write an inequality to compare the amount of money.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Multiplication and Division Properties of Inequality *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore and deepen their understanding of inequalities that involve multiplication and division.

6 Attend to Precision Students should notice how an inequality changes or remains true when they multiply or divide a certain value from each side of the inequality.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Does an inequality remain true when you multiply each side by the same positive number? Explain. Sample answer: Yes. Multiplying each side of an inequality by the same positive number results in a true inequality.

Learn Division and Multiplication Properties of Inequality

Objective

Students will understand the Division and Multiplication Properties of Inequality when the coefficients are positive.

Teaching Notes

SLIDE 1

Before you move through the flashcards that describe the Multiplication and Division Properties of Inequality, you may wish to have students make a conjecture about how multiplying or dividing each side of an inequality will affect the inequality. Suggest they first practice by using an example, $3 < 4$. Ask students to multiply each side of the inequality by 2 and then note any changes to the inequality. Then ask them for their conjectures. Some students may say that the inequality remains true.

Talk About It!

SLIDE 2

Mathematical Discourse

The Division Property of Inequality states: For all numbers a, b, and c, where $c > 0$,

1. if $a > b$, then $\frac{a}{c} > \frac{b}{c}$.
2. if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

What does the inequality $c > 0$ mean? Sample answer: It means that c is a positive number.

Go Online to find Teaching the Mathematical Practices.

DIFFERENTIATE

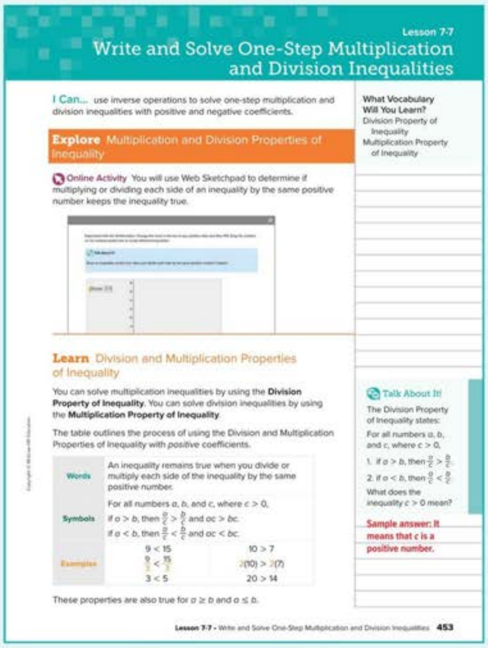
Reteaching Activity AL

To help students better understand the *Symbols* description of the Division and Multiplication Properties of Inequality using positive coefficients, ask students the following questions about the statement *If* $a > b$, *then* $\frac{a}{c} > \frac{b}{c}$, *where* $c > 0$.

- What does $c > 0$ mean? c is a positive number
- What does $a > b$ mean? a is greater than b
- What do $\frac{a}{c}$ and $\frac{b}{c}$ mean? a is divided by c; b is divided by c
- What do you notice about the inequality $a > b$, when each side is divided by c? The inequality is true.

Have students work with a partner to repeat the activity with the remaining inequalities when $c > 0$:

- If $a > b$, then $ac > bc$.
- If $a < b$, then $\frac{a}{c} < \frac{b}{c}$.
- If $a < b$, then $ac < bc$.


Lesson 7-7

Write and Solve One-Step Multiplication and Division Inequalities

I Can... use inverse operations to solve one-step multiplication and division inequalities with positive and negative coefficients.

What Vocabulary Will You Learn?
Division Property of Inequality
Multiplication Property of Inequality

Explore Multiplication and Division Properties of Inequality

Online Activity You will use Web Sketchpad to determine if multiplying or dividing each side of an inequality by the same positive number keeps the inequality true.

Learn Division and Multiplication Properties of Inequality

You can solve multiplication inequalities by using the **Division Property of Inequality**. You can solve division inequalities by using the **Multiplication Property of Inequality**.

The table outlines the process of using the Division and Multiplication Properties of Inequality with *positive* coefficients.

Words	An inequality remains true when you divide or multiply each side of the inequality by the same positive number.	
Symbols	For all numbers a, b, and c, where $c > 0$, If $a > b$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$. If $a < b$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$.	
Examples	$9 < 15$ $\frac{9}{3} < \frac{15}{3}$ $3 < 5$	$10 > 7$ $2(10) > 2(7)$ $20 > 14$

These properties are also true for $a \geq b$ and $a \leq b$.

Talk About It!
The Division Property of Inequality states:
For all numbers a, b, and c, where $c > 0$,
1. If $a > b$, then $\frac{a}{c} > \frac{b}{c}$
2. If $a < b$, then $\frac{a}{c} < \frac{b}{c}$
What does the inequality $c > 0$ mean?
Sample answer: It means that c is a positive number.

Lesson 7-7 • Write and Solve One-Step Multiplication and Division Inequalities 453

Interactive Presentation

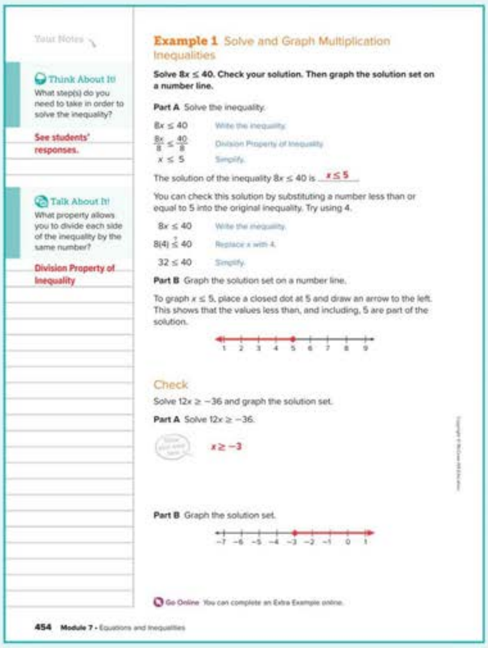
Learn, Division and Multiplication Properties of Inequality, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of the Division and Multiplication Properties of Inequality with positive coefficients.

7.EE.B.4, 7.EE.B.4.B

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION


Your Notes

Think About It! What step(s) do you need to take in order to solve the inequality? See students' responses.

Talk About It! What property allows you to divide each side of the inequality by the same number? Division Property of Inequality

Example 1 Solve and Graph Multiplication Inequalities

Solve $8x \leq 40$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$8x \leq 40$ Write the inequality.

$\frac{8x}{8} \leq \frac{40}{8}$ Division Property of Inequality

$x \leq 5$ Simplify.

The solution of the inequality $8x \leq 40$ is $x \leq 5$.

You can check this solution by substituting a number less than or equal to 5 into the original inequality. Try using 4.

$8x \leq 40$ Write the inequality.

$8(4) \stackrel{?}{\leq} 40$ Replace x with 4.

$32 \leq 40$ Simplify.

Part B Graph the solution set on a number line.

To graph $x \leq 5$, place a closed dot at 5 and draw an arrow to the left. This shows that the values less than, and including, 5 are part of the solution.

Check

Solve $12x \geq -36$ and graph the solution set.

Part A Solve $12x \geq -36$.

$x \geq -3$

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

454 Module 7 • Equations and Inequalities

Interactive Presentation

Example 1, Solve and Graph Multiplication Inequalities, Slide 2 of 5

CLICK On Slide 2, students move through the steps to solve the inequality.

eTOOLS On Slide 3, students use the Number Line eTool to graph the inequality.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Solve and Graph Multiplication Inequalities

Objective

Students will solve and graph one-step multiplication inequalities with positive coefficients.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Number Line eTool to graph the solution set of the inequality on the number line.

6 Attend to Precision Students should solve the inequality efficiently and accurately, using the correct property of inequality and paying careful attention to the inequality symbol.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is paired with the variable? multiplication

AL If you were solving the equation $8x = 40$, what would you do? Divide each side by 8.

OL What operation will undo multiplication? Why? division; Multiplication and division are inverse operations.

OL What property allows you to divide each side by 8 in this inequality? Division Property of Inequality

BL If $8x \leq 40$, what must be true about $8x + 5$? Explain without calculating the value of x. Sample answer: $8x + 5 \leq 45$; Because 5 is added to the left side of the inequality, $8x$, 5 must also be added to the right side of the inequality, 40.

SLIDE 3

AL Explain whether 5 is or is not a solution to this inequality. 5 is a solution to this inequality because the inequality symbol means less than or equal to.

OL Explain how to graph the inequality. Sample answer: Place a closed dot at 5, because the solution set includes the number 5. Then extend the arrow to the left because any number that is less than or equal to 5 is a solution of this inequality.

BL Write a real-world problem that can be represented by this inequality. See students' responses.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Solve and Graph Division Inequalities

Objective

Students will solve and graph one-step division inequalities with positive coefficients.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Number Line eTool to graph the solution set of the inequality on the number line.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is paired with the variable? division

AL What operation will undo division? Why? multiplication; Multiplication and division are inverse operations.

OL Describe how to solve the inequality. Sample answer: Multiply each side by 2. The solution is $d > 14$.

OL How can you check your answer? Sample answer: Replace d with a number greater than 14 in the original inequality to verify that it is a true statement, which it is.

BL Write a real world problem that can be represented by this inequality. Sample answer: Two friends will split evenly the money that they earn. How much do they need to earn altogether in order to have more than $7 each after they split the amount?

SLIDE 3

AL List three numbers that are solutions to this inequality. Then list three numbers that are not solutions to this inequality. Sample answer: 14.1, 16, 25 are solutions; 9, 11, and 12.7 are not solutions.

OL Describe how to graph the solution set for this inequality. Sample answer: Place an open dot at 14 because the solution set does not include 14. Then extend the arrow to the right because any number greater than 14 is a solution.

OL Are the statements $d > 14$ and $14 < d$ equivalent? Explain. yes; Sample answer: They both state that the solution is greater than 14.

BL If the inequality was $\frac{d}{2} \geq 7$, how will the graph change? The endpoint will be closed.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Solve and Graph Division Inequalities

Solve $\frac{d}{2} > 7$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$\frac{d}{2} > 7$ — Write the inequality.

$2\left(\frac{d}{2}\right) > 2(7)$ — Multiplication Property of Inequality

$d > 14$ — Simplify.

The solution of the inequality $\frac{d}{2} > 7$ is $d > 14$.

You can check this solution by substituting a number greater than 14 into the original inequality. Try using 15.

$\frac{d}{2} > 7$ — Write the inequality.

$\frac{15}{2} \overset{?}{>} 7$ — Replace d with 15.

$7.5 > 7$ — Simplify.

Part B Graph the solution set on a number line.

To graph $d > 14$, place an open dot on 14 and draw an arrow pointing to the right to show that all values greater than, but not including, 14 are part of the solution.

Think About It! What step(s) do you need to take in order to solve the inequality?

See students' responses.

Talk About It! Would the inequality be true if 14 was substituted for d? Why or why not?

Sample answer: No, because $14 > 14$ is not a true statement.

Check

Solve $-6 \geq \frac{x}{7}$ and graph the solution set.

Part A Solve $-6 \geq \frac{x}{7}$.

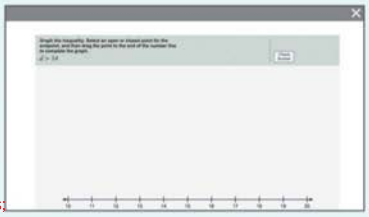
$x \leq -42$

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

Lesson 7-7 • Write and Solve One-Step Multiplication and Division Inequalities 455

Interactive Presentation



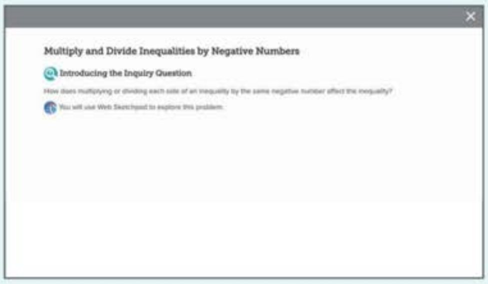
Example 2, Solve and Graph Division Inequalities, Slide 3 of 5

CLICK On Slide 2, students move through the steps to solve the inequality.

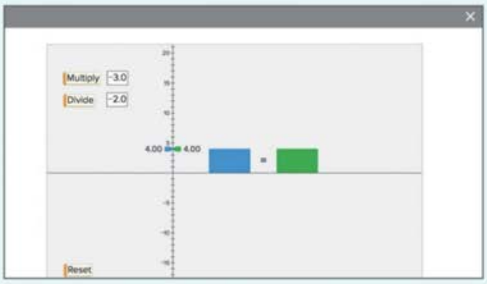
eTOOLS On Slide 3, students use the Number Line eTool to graph the inequality.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Interactive Presentation



Explore, Slide 1 of 5



Explore, Slide 3 of 5

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how multiplying or dividing each side of an inequality by the same negative number affects the inequality.

TYPE

On Slide 3, students make a conjecture about what happens when each side of an inequality is multiplied or divided by the same negative number.

Explore Multiply and Divide Inequalities by Negative Numbers

Objective

Students will use Web Sketchpad to explore how multiplying and dividing each side of an inequality by the same negative number affects the inequality.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations

Summary of Activity

Students will be presented with a Web Sketchpad bar graph that represents values on each side of an equation or inequality. Throughout this activity, students will use sketches to investigate the effect of multiplying or dividing each side of an inequality by a negative number.

Inquiry Question

How does multiplying or dividing each side of an inequality by the same negative number affect the inequality? Sample answer: When I multiply or divide each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

Go Online to find additional teaching notes.

(continued on next page)

Explore Multiply and Divide Inequalities by Negative Numbers *(continued)*

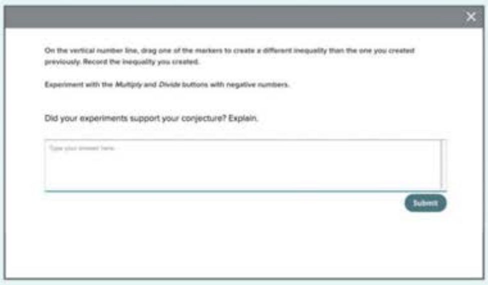
Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore and deepen their understanding of multiplication and division inequalities with negative coefficients.

6 Attend to Precision Students should notice how an inequality changes when they multiply or divide each side of the inequality by a certain value. Students should calculate and compare accurately and efficiently, paying careful attention to the symbols of inequalities.

Go Online to find additional teaching notes.

Interactive Presentation



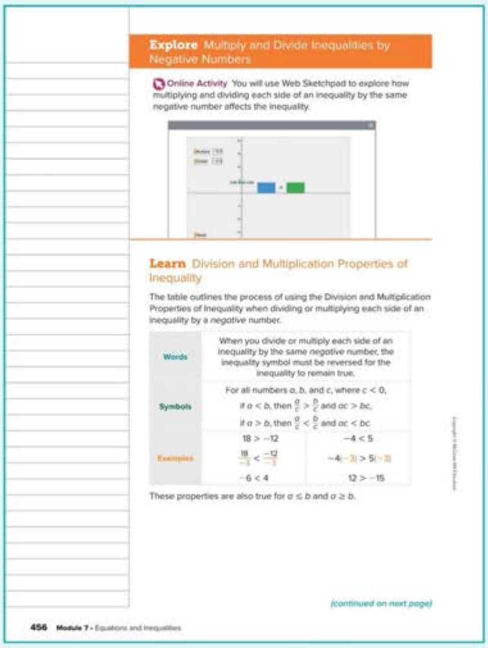
Explore, Slide 4 of 5

TYPE

On Slide 4, students explain if their experiments support their conjecture.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.



Explore Multiply and Divide Inequalities by Negative Numbers

Online Activity You will use Web Sketchpad to explore how multiplying and dividing each side of an inequality by the same negative number affects the inequality.

Learn Division and Multiplication Properties of Inequality

The table outlines the process of using the Division and Multiplication Properties of Inequality when dividing or multiplying each side of an inequality by a negative number.

Words	When you divide or multiply each side of an inequality by the same *negative* number, the inequality symbol must be reversed for the inequality to remain true.	
Symbols	For all numbers a, b, and c, where $c < 0$, if $a < b$, then $\frac{a}{c} > \frac{b}{c}$ and $ac > bc$, if $a > b$, then $\frac{a}{c} < \frac{b}{c}$ and $ac < bc$	
Examples	$18 > -12$ $\frac{18}{-3} < \frac{-12}{-3}$ $-6 < 4$	$-4 < 5$ $-4(-3) > 5(-3)$ $12 > -15$

These properties are also true for $a \leq b$ and $a \geq b$.

(continued on next page)

456 **Module 7** • Equations and Inequalities

Interactive Presentation

Learn, Division and Multiplication Properties of Inequality, Slide 1 of 5

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of the Division and Multiplication Properties of Inequality with negative coefficients.

Learn Division and Multiplication Properties of Inequality

Objective

Students will understand the Division and Multiplication Properties of Inequality when the coefficients are negative.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question, it may be helpful for them to use an example in their reasoning to justify their conclusions. Students should construct an argument based on the information they gathered from previous slides.

Teaching Notes

SLIDE 1

Students will learn how to use the *Division Property of Inequality* and the *Multiplication Property of Inequality* when dividing or multiplying each side of an inequality by a negative number. Select each flashcard to show how these properties can be represented using words, symbols, and examples. You may wish to have students practice using an example. Give students the inequality $5 < 6$ and ask them to multiply each side by -2. Ask them if the new statement is true. Students should note no. Then ask students how they could make it a true statement without changing the numbers or the signs of the numbers. Students should note that reversing the inequality symbol will make the statement true.

(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

If students need more of a challenge, have them solve the following inequalities and ask them to write the solution so the variable is on the left side of the inequality:

- $-35 < -\frac{5}{6}x$
- $75 \geq -5x$

Have students explain to a partner why the inequality sign did not change for either inequality.

Learn Division and Multiplication Properties of Inequality *(continued)*

Go Online to have your students watch the animation on Slide 2. The animation illustrates why the inequality symbol must be reversed when dividing by a negative number.

Teaching Notes

SLIDE 2

You may wish to pause the animation when the notation $\frac{-6}{-2} \overset{?}{<} \frac{4}{-2}$ first appears. Ask students why the question mark is above the inequality sign. Some students may say that you don't know what the inequality will be after you have divided by a negative number. Then, before continuing the animation, ask them to simplify each side of the inequality, and to make a conjecture about how dividing by a negative number affects an inequality.

Talk About It!

SLIDE 3

Mathematical Discourse

Will the inequality symbol need to be reversed when solving the inequality $3x > -9$? Why or why not? no; Sample answer: To solve the inequality, divide each side by 3, which is a positive number. You only reverse the inequality symbol when dividing by a negative number.

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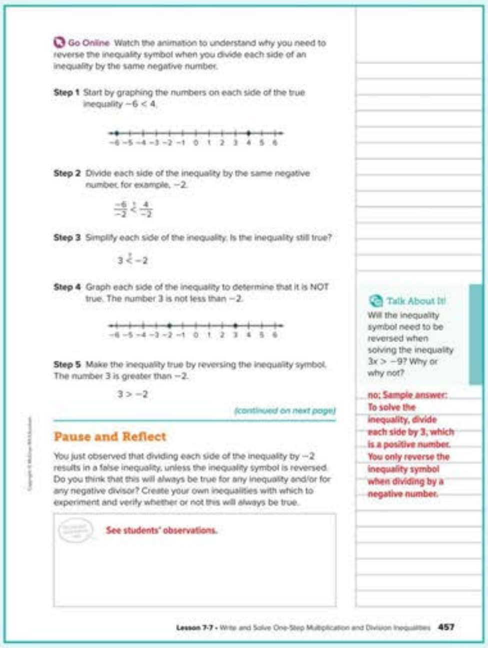


Go Online Watch the animation to understand why you need to reverse the inequality symbol when you divide each side of an inequality by the same negative number.

Step 1 Start by graphing the numbers on each side of the true inequality $-6 < 4$.

Step 2 Divide each side of the inequality by the same negative number, for example, −2.

$\frac{-6}{-2} \overset{?}{<} \frac{4}{-2}$

Step 3 Simplify each side of the inequality. Is the inequality still true?

$3 \overset{?}{<} -2$

Step 4 Graph each side of the inequality to determine that it is NOT true. The number 3 is not less than −2.

Step 5 Make the inequality true by reversing the inequality symbol. The number 3 is greater than −2.

$3 > -2$

(continued on next page)

Pause and Reflect

You just observed that dividing each side of the inequality by −2 results in a false inequality, unless the inequality symbol is reversed. Do you think that this will always be true for any inequality and/or for any negative divisor? Create your own inequalities with which to experiment and verify whether or not this will always be true.

See students' observations.

Talk About It! Will the inequality symbol need to be reversed when solving the inequality $3x > -9$? Why or why not?

no; Sample answer: To solve the inequality, divide each side by 3, which is a positive number. You only reverse the inequality symbol when dividing by a negative number.

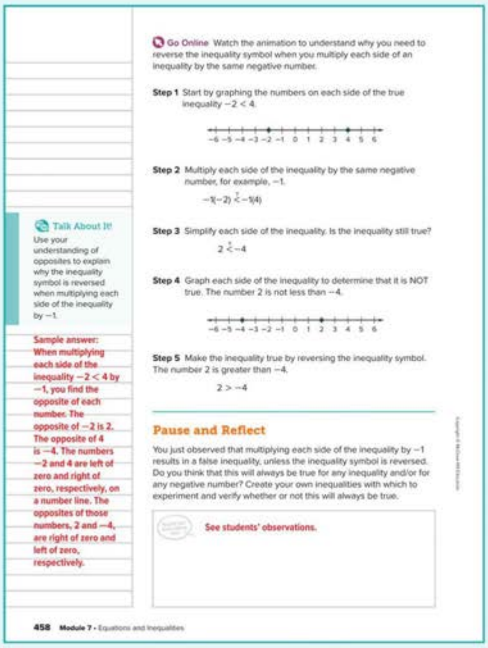
Lesson 7-7 • Write and Solve One-Step Multiplication and Division Inequalities 457

Interactive Presentation

Learn, Division and Multiplication Properties of Inequality, Slide 2 of 5

WATCH

On Slide 2, students watch an animation that illustrates the Division Property of Inequality.



Go Online Watch the animation to understand why you need to reverse the inequality symbol when you multiply each side of an inequality by the same negative number.

Step 1 Start by graphing the numbers on each side of the true inequality $-2 < 4$.

Step 2 Multiply each side of the inequality by the same negative number, for example, -1.

$-1(-2) \overset{?}{<} -1(4)$

Step 3 Simplify each side of the inequality. Is the inequality still true?

$2 \overset{?}{<} -4$

Step 4 Graph each side of the inequality to determine that it is NOT true. The number 2 is not less than -4.

Step 5 Make the inequality true by reversing the inequality symbol. The number 2 is greater than -4.

$2 > -4$

Talk About It! Use your understanding of opposites to explain why the inequality symbol is reversed when multiplying each side of the inequality by -1.

Sample answer: When multiplying each side of the inequality $-2 < 4$ by -1, you find the opposite of each number. The opposite of -2 is 2. The opposite of 4 is -4. The numbers -2 and 4 are left of zero and right of zero, respectively, on a number line. The opposites of those numbers, 2 and -4, are right of zero and left of zero, respectively.

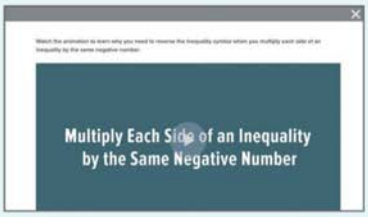
Pause and Reflect

You just observed that multiplying each side of the inequality by -1 results in a false inequality, unless the inequality symbol is reversed. Do you think that this will always be true for any inequality and/or for any negative number? Create your own inequalities with which to experiment and verify whether or not this will always be true.

See students' observations.

458 Module 7 • Equations and Inequalities

Interactive Presentation



Learn, Division and Multiplication Properties of Inequality, Slide 4 of 5

WATCH

On Slide 4, students watch an animation that illustrates the Multiplication Property of Inequality.

Learn Division and Multiplication Properties of Inequality *(continued)*

Go Online to have your students watch the animation on Slide 4. The animation illustrates why the inequality symbol must be reversed when multiplying by a negative number.

Teaching Notes

SLIDE 4

Before you play the animation for the class, you may wish to ask students if what they learned about the Division Property of Inequality and negative coefficients can help them make a conjecture about how the Multiplication Property of Inequality applies to inequalities with negative coefficients. Some students may say that when you multiply each side of an inequality by a negative number, the inequality symbol reverses. Ask them to explain why they think this happens. Some students may say that multiplication and division are related/inverse operations so they have similar effects on the inequalities.

Talk About It!

SLIDE 5

Mathematical Discourse

Use your understanding of opposites to explain why the inequality symbol is reversed when multiplying each side of the inequality by -1. Sample answer: When multiplying each side of the inequality $-2 < 4$ by -1, you find the opposite of each number. The opposite of -2 is 2. The opposite of 4 is -4. The numbers -2 and 4 are left of zero and right of zero, respectively, on a number line. The opposites of those numbers, 2 and -4, are right of zero and left of zero, respectively.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Multiplication Inequalities with Negative Coefficients

Objective

Students will solve and graph one-step multiplication inequalities with negative coefficients.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Number Line eTool to graph the solution to the inequality.

6 Attend to Precision Students should be able to solve the inequality efficiently, paying careful attention to what the negative sign of the coefficient means.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is paired with the variable? multiplication

AL What do you notice about the coefficient of the variable? It is negative.

OL Explain how to solve the inequality. Sample answer: Divide each side of the inequality by –2 and reverse the symbol. So, $x > -5$.

OL Why do you need to reverse the inequality symbol? Sample answer: Because I am dividing each side by a negative number, reversing the inequality symbol keeps the inequality true.

BL If $-2x < 10$, what must be true about $2x$? Explain without calculating the value of x. $2x > -10$; Sample answer: Divide each side by –1. When doing so, reverse the inequality symbol.

SLIDE 3

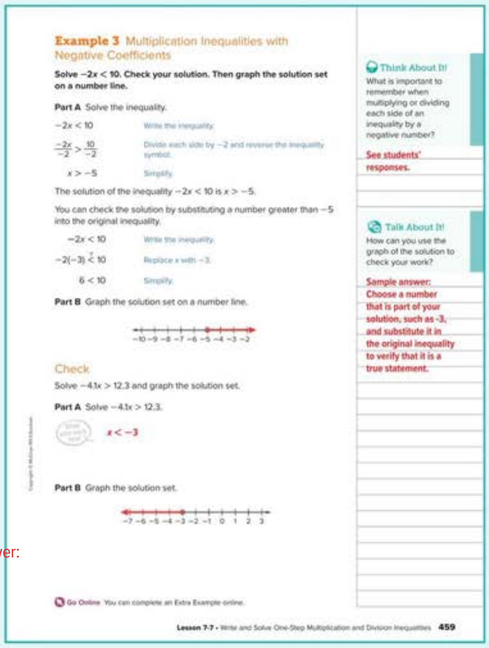
AL List three possible solutions to the inequality. Then list three numbers that are not solutions. Sample answer: –4, –3, and 0 are solutions; –5, –6, and –10 are not solutions.

OL Describe how to graph the inequality. Sample answer: Place an open dot at –5 because –5 is not a solution to the inequality. Then extend the arrow to the right because any number greater than –5 is a solution.

BL Think about the inequality $|x| < -5$. Describe what you think the solution set of this inequality might be. Sample answer: There is no solution to this inequality because the absolute value of any number can never be negative. Therefore, x can never be less than a negative number.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Multiplication Inequalities with Negative Coefficients

Solve $-2x < 10$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$-2x < 10$ Write the inequality.

$\frac{-2x}{-2} > \frac{10}{-2}$ Divide each side by −2 and reverse the inequality symbol.

$x > -5$ Simplify.

The solution of the inequality $-2x < 10$ is $x > -5$.

You can check the solution by substituting a number greater than −5 into the original inequality.

$-2x < 10$ Write the inequality.

$-2(-3) \stackrel{?}{<} 10$ Replace x with −3.

$6 < 10$ Simplify.

Part B Graph the solution set on a number line.

Check

Solve $-4.1x > 12.3$ and graph the solution set.

Part A Solve $-4.1x > 12.3$.

$x < -3$

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

Think About It! What is important to remember when multiplying or dividing each side of an inequality by a negative number?

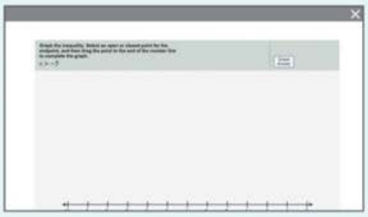
See students' responses.

Talk About It! How can you use the graph of the solution to check your work?

Sample answer: Choose a number that is part of your solution, such as -3, and substitute it in the original inequality to verify that it is a true statement.

Lesson 7-7 • Write and Solve One-Step Multiplication and Division Inequalities 459

Interactive Presentation



Example 3, Multiplication Inequalities with Negative Coefficients, Slide 3 of 5

TYPE

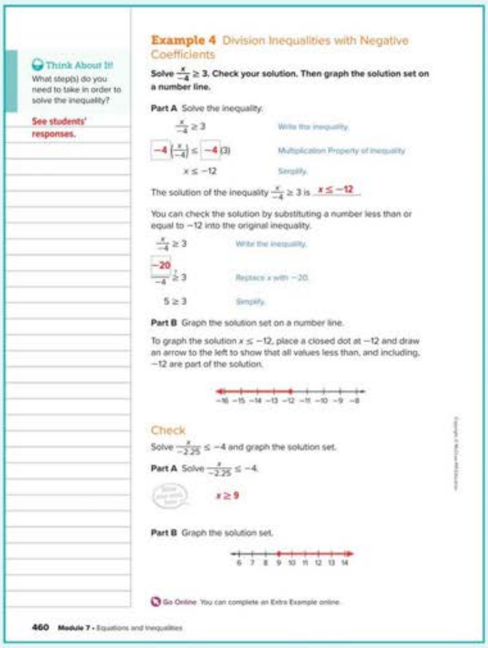
On Slide 2, students determine the missing value to solve the inequality.

eTOOLS

On Slide 3, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It!
What step(s) do you need to take in order to solve the inequality?
See students' responses.

Example 4 Division Inequalities with Negative Coefficients

Solve $\frac{x}{-4} \geq 3$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$\frac{x}{-4} \geq 3$ Write the inequality.

$-4\left(\frac{x}{-4}\right) \leq -4(3)$ Multiplication Property of Inequality

$x \leq -12$ Simplify.

The solution of the inequality $\frac{x}{-4} \geq 3$ is $x \leq -12$.

You can check the solution by substituting a number less than or equal to −12 into the original inequality.

$\frac{x}{-4} \geq 3$ Write the inequality.

$\frac{-20}{-4} \overset{?}{\geq} 3$ Replace x with −20.

$5 \geq 3$ Simplify.

Part B Graph the solution set on a number line.

To graph the solution $x \leq -12$, place a closed dot at −12 and draw an arrow to the left to show that all values less than, and including, −12 are part of the solution.

Check

Solve $\frac{x}{-2.25} \leq -4$ and graph the solution set.

Part A Solve $\frac{x}{-2.25} \leq -4$.

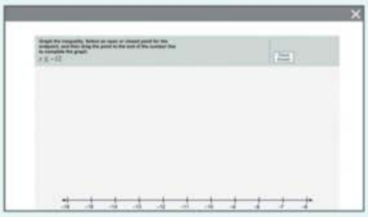
$x \geq 9$

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

460 Module 7 • Equations and Inequalities

Interactive Presentation



Example 4, Division Inequalities with Negative Coefficients, Slide 3 of 5

CLICK

On Slide 2, students move through the steps to solve the inequality.

eTOOLS

On Slide 3, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Division Inequalities with Negative Coefficients

Objective

Students will solve and graph one-step division inequalities with negative coefficients.

Teaching the Mathematical Practices

6 Attend to Precision Students should be able to solve the inequality efficiently, paying careful attention to what it means for the variable to be divided by a negative number.

Questions for Mathematical Discourse

SLIDE 2

AL What operation is paired with the variable? division

AL What do you notice about this inequality? The number by which the variable is divided is negative.

OL Explain how to solve the inequality. Sample answer: Multiply each side of the inequality by −4 and reverse the symbol. So, $x \leq -12$.

OL Why do you need to reverse the inequality symbol? Sample answer: Because I am multiplying each side by a negative number, reversing the inequality symbol keeps the inequality true.

BL A classmate says that because the number on the right side of the inequality is positive, the inequality symbol does not need to be reversed. How can you explain to your classmate their reasoning is incorrect? Sample answer: It doesn't matter whether the number on the right side of the inequality is positive or negative. Because we multiplied by a negative number, we need to reverse the inequality symbol.

SLIDE 3

AL List three possible solutions to the inequality. Then list three numbers that are not solutions. Sample answer: −12, −15, and −25 are solutions; −5, 0, and 2 are not solutions.

OL Describe how to graph the inequality. Sample answer: Place a closed dot at −12 because −12 is a solution to the inequality. Then extend the arrow to the left because any number less than or equal to −12 is a solution.

BL Write the inequality $\frac{x}{-4} \geq 3$ as a multiplication inequality. How can you solve it? Sample answer: $-\frac{1}{4}x \geq 3$; Divide each side by $-\frac{1}{4}$.

Go Online

- Find additional teaching notes and the Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 5 Write and Solve One-Step Multiplication Inequalities

Objective

Students will write one-step multiplication inequalities from real-world problems and interpret the solution.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the information in the real-world problem by representing it symbolically with a correct inequality.

5 Use Appropriate Tools Strategically Students will graph the solution set using the Number Line eTool.

7 Look For and Make Use of Structure As students discuss the *Talk About It!* question on Slide 5, encourage them to use the structure of the inequality to explain why they do not need to reverse the inequality symbol when solving the inequality.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown quantity you need to find? How will you represent it in the inequality? the number of hours Ling needs to work to earn at least $225; with a variable

AL What key words from the problem indicate which inequality symbol to use? What inequality symbol is indicated by these key words? at least; the greater than or equal to symbol

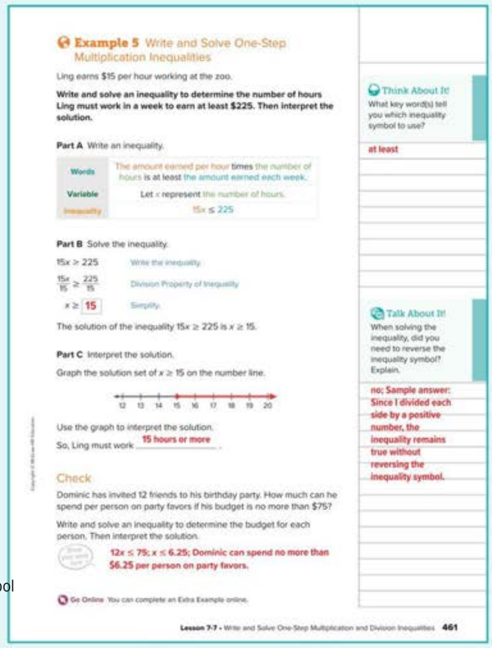
OL Explain why the expression $15x$ represents the quantity that needs to be compared with 225. Sample answer: Ling earns $15 per hour and x represents the number of hours. So, $15x$ represents how much money she will earn after x hours. This quantity needs to be greater than or equal to 225.

OL Suppose a classmate wrote the inequality $225 \leq 15x$. Is this inequality correct? Explain. yes; Sample answer: If $15x$ is greater than or equal to 225, this means that 225 is less than or equal to $15x$.

BL What would the inequality $15x \leq 225$ represent within the context of this problem? the number of hours Ling must work to make no more than $225

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 5 Write and Solve One-Step Multiplication Inequalities

Ling earns $15 per hour working at the zoo.

Write and solve an inequality to determine the number of hours Ling must work in a week to earn at least $225. Then interpret the solution.

Part A Write an inequality.

Words	The amount earned per hour times the number of hours is at least the amount earned each week.
Variable	Let x represent the number of hours.
Inequality	$15x \leq 225$

Part B Solve the inequality.

$15x \geq 225$ Write the inequality.

$\frac{15x}{15} \geq \frac{225}{15}$ Division Property of Inequality

$x \geq 15$ Simplify.

The solution of the inequality $15x \geq 225$ is $x \geq 15$.

Part C Interpret the solution.

Graph the solution set of $x \geq 15$ on the number line.

12 13 14 15 16 17 18 19 20

Use the graph to interpret the solution.

So, Ling must work 15 hours or more.

Check

Dominic has invited 12 friends to his birthday party. How much can he spend per person on party favors if his budget is no more than $75?

Write and solve an inequality to determine the budget for each person. Then interpret the solution.

$12x \leq 75$; $x \leq 6.25$; Dominic can spend no more than $6.25 per person on party favors.

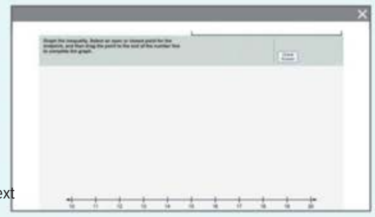
Go Online You can complete an Extra Example online.

Think About It! What key word(s) tell you which inequality symbol to use? at least

Talk About It! When solving the inequality, did you need to reverse the inequality symbol? Explain. no; Sample answer: Since I divided each side by a positive number, the inequality remains true without reversing the inequality symbol.

Lesson 7-7 • Write and Solve One-Step Multiplication and Division Inequalities 461

Interactive Presentation



Example 5, Write and Solve One-Step Multiplication Inequalities, Slide 4 of 6

FLASHCARDS

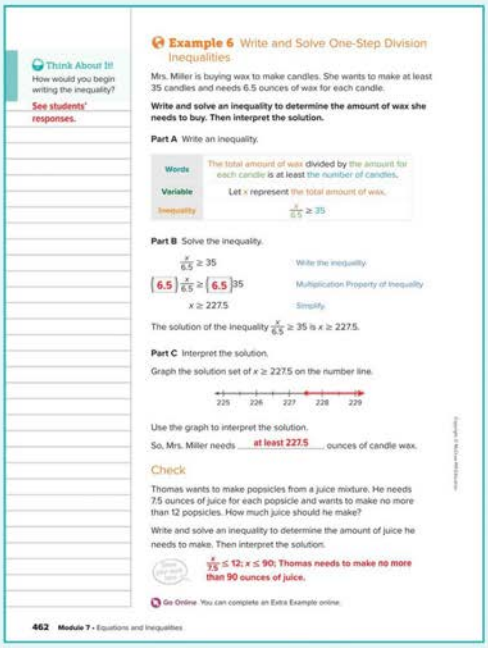
On Slide 2, students use Flashcards to view the steps for writing the inequality.

eTOOLS

On Slide 4, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! How would you begin writing the inequality? See students' responses.

Example 6 Write and Solve One-Step Division Inequalities

Mrs. Miller is buying wax to make candles. She wants to make at least 35 candles and needs 6.5 ounces of wax for each candle.

Write and solve an inequality to determine the amount of wax she needs to buy. Then interpret the solution.

Part A Write an inequality.

Words	The total amount of wax divided by the amount for each candle is at least the number of candles.
Variable	Let x represent the total amount of wax.
Inequality	$\frac{x}{6.5} \geq 35$

Part B Solve the inequality.

$\frac{x}{6.5} \geq 35$ Write the inequality.

$(6.5)\frac{x}{6.5} \geq (6.5)35$ Multiplication Property of Inequality

$x \geq 227.5$ Simplify.

The solution of the inequality $\frac{x}{6.5} \geq 35$ is $x \geq 227.5$.

Part C Interpret the solution.

Graph the solution set of $x \geq 227.5$ on the number line.

225 226 227 228 229

Use the graph to interpret the solution.

So, Mrs. Miller needs at least 227.5 ounces of candle wax.

Check

Thomas wants to make popsicles from a juice mixture. He needs 7.5 ounces of juice for each popsicle and wants to make no more than 12 popsicles. How much juice should he make?

Write and solve an inequality to determine the amount of juice he needs to make. Then interpret the solution.

$\frac{x}{7.5} \leq 12$; $x \leq 90$; Thomas needs to make no more than 90 ounces of juice.

Go Online You can complete an Extra Example online.

462 Module 7 • Equations and Inequalities

Interactive Presentation

Example 6, Write and Solve One-Step Division Inequalities, Slide 2 of 5

FLASHCARDS

On Slide 2, students use Flashcards to view the steps for writing the inequality.

eTOOLS

On Slide 4, students use an eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 6 Write and Solve One-Step Division Inequalities

Objective

Students will write one-step division inequalities from real-world problems and interpret the solution.

Questions for Mathematical Discourse

SLIDE 2

AL What is the unknown quantity you need to find? How will you represent it in the inequality? the amount of wax Mrs. Miller needs to buy; with a variable

AL What key words from the problem indicate which inequality symbol to use? What inequality symbol is indicated by these key words? at least; the greater than or equal to symbol

OL Explain why the variable is divided by 6.5 in the inequality. Sample answer: The total amount of wax is divided by 6.5 ounces of wax per candle.

OL A classmate wrote the inequality $35 \leq \frac{x}{6.5}$. Is this inequality correct? Explain. yes; Sample answer: If $\frac{x}{6.5}$ is greater than or equal to 35, this means that 35 is less than or equal to $\frac{x}{6.5}$.

BL What would the inequality $\frac{x}{6.5} \leq 35$ represent within the context of this problem? the amount of wax x Mrs. Miller needs to buy in order to make no more than 35 candles

Go Online

- Find additional teaching notes, discussion questions, and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of one-step multiplication and division inequalities, have them solve the following problem.

Mrs. Hawthorne is organizing a food drive and will need to have volunteers in groups of 5. To keep things manageable, Mrs. Hawthorne wants to have fewer than 15 groups of volunteers. Each volunteer will be responsible for packing 10 boxes of food, and Mrs. Hawthorne wants to have more than 650 boxes of food packed during the drive. If there are no volunteers left over when split into groups of 5, how many volunteers will Mrs. Hawthorne need? 70

Apply Fundraising

Objective

Students will come up with their own strategy to solve an application problem involving fundraising for buses.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

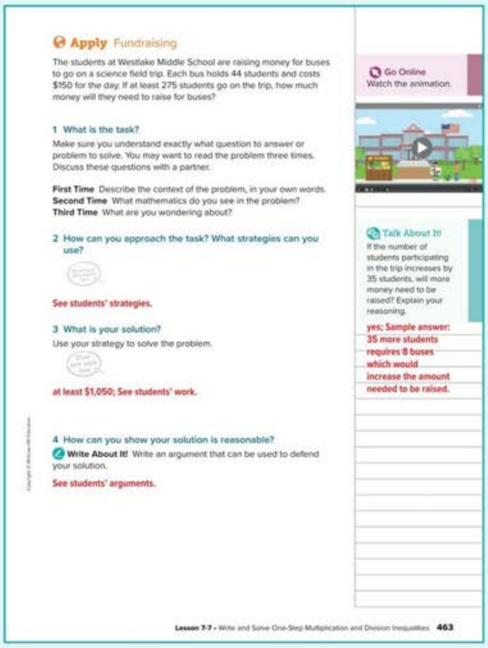
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What operation(s) should be used to find the amount of money needed for the buses?
- What inequality symbol should be used to best represent the situation?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Fundraising

The students at Westlake Middle School are raising money for buses to go on a science field trip. Each bus holds 44 students and costs $150 for the day. If at least 275 students go on the trip, how much money will they need to raise for buses?

Go Online Watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

at least $1,050; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

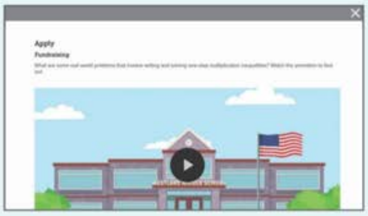
See students' arguments.

Talk About It!
If the number of students participating in the trip increases by 35 students, will more money need to be raised? Explain your reasoning.

yes; Sample answer: 35 more students requires 8 buses which would increase the amount needed to be raised.

Lesson 7-7 • Write and Solve One-Step Multiplication and Division Inequalities 463

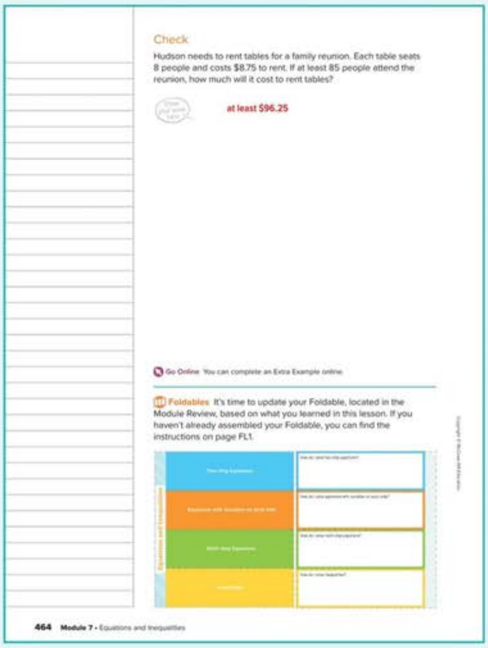
Interactive Presentation



Apply, Fundraising

WATCH Students watch an animation that illustrates the problem they are about to solve.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Check

Hudson needs to rent tables for a family reunion. Each table seats 8 people and costs $8.75 to rent. If at least 85 people attend the reunion, how much will it cost to rent tables?

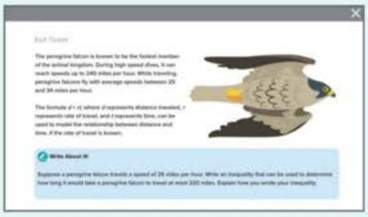
at least $96.25

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

464 Module 7 • Equations and Inequalities

Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about solving one-step multiplication and division inequalities. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Suppose a peregrine falcon travels at a speed of 25 miles per hour. Write an inequality that can be used to determine how long it would take a peregrine falcon to travel at most 220 miles. Write a mathematical argument that can be used to defend your solution. $25t \leq 220$; Sample answer: The phrase *at most* indicates that the $\leq$ symbol should be used if 220 is on the right side of the inequality.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 11, 13–16
- Extension: Interval Notation and Set Notation
- ALEKS One-Step Inequalities, Applications of Inequalities

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–11, 14
- Remediation: Review Resources
- Extension: Interval Notation and Set Notation
- Personal Tutor
- Extra Examples 1–6
- ALEKS Writing and Graphing Inequalities

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Writing and Graphing Inequalities

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

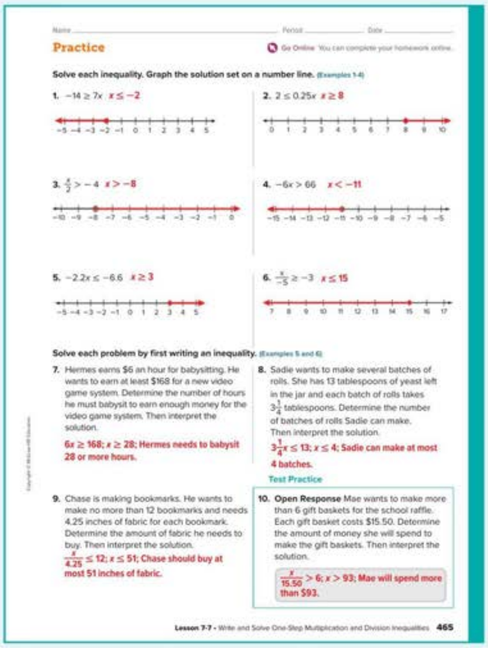
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve one-step multiplication and division inequalities with positive coefficients	1–3
1	solve one-step multiplication and division inequalities with negative coefficients	4–6
2	write and solve one-step multiplication inequalities from real-world problems and interpret the solution	7, 8
2	write and solve one-step division inequalities from real-world problems and interpret the solution	9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving writing and solving one-step multiplication and division inequalities	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may incorrectly reverse the direction of the inequality symbol when the inequality contains a negative number. In Exercise 1, students may reverse the inequality symbol when dividing by 7 because the negative number –14 is being divided. Explain to students that the inequality symbol is not reversed when multiplying or dividing by a positive number.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each inequality. Graph the solution set on a number line. (Examples 1-4)

1. $-14 \geq 7x$ $x \leq -2$

2. $2 \leq 0.25x$ $x \geq 8$

3. $\frac{x}{2} > -4$ $x > -8$

4. $-6x > 66$ $x < -11$

5. $-2.2x \leq -6.6$ $x \geq 3$

6. $\frac{x}{-5} \geq -3$ $x \leq 15$

Solve each problem by first writing an inequality. (Examples 5 and 6)

7. Hermes earns $6 an hour for babysitting. He wants to earn at least $168 for a new video game system. Determine the number of hours he must babysit to earn enough money for the video game system. Then interpret the solution.
$6x \geq 168$; $x \geq 28$; Hermes needs to babysit 28 or more hours.

8. Sadie wants to make several batches of rolls. She has 13 tablespoons of yeast left in the jar and each batch of rolls takes $3\frac{1}{4}$ tablespoons. Determine the number of batches of rolls Sadie can make. Then interpret the solution.
$3\frac{1}{4}x \leq 13$; $x \leq 4$; Sadie can make at most 4 batches.

9. Chase is making bookmarks. He wants to make no more than 12 bookmarks and needs 4.25 inches of fabric for each bookmark. Determine the amount of fabric he needs to buy. Then interpret the solution.
$\frac{x}{4.25} \leq 12$; $x \leq 51$; Chase should buy at most 51 inches of fabric.

Test Practice

10. **Open Response** Mae wants to make more than 6 gift baskets for the school raffle. Each gift basket costs $15.50. Determine the amount of money she will spend to make the gift baskets. Then interpret the solution.
$\frac{x}{15.50} > 6$; $x > 93$; Mae will spend more than $93.

Lesson 7-7 • Write and Solve One-Step Multiplication and Division Inequalities 465

Apply *indicates multi-step problems

*11. Greg's grandmother is knitting scarves for charity. The table shows the number of yards needed for different types of scarves. She plans to make bulky scarves and has no more than 2,250 yards of yarn. If the charity plans to sell the scarves for $24.50, how much money will the charity make?

Yarn Type	Number of Yards
Bulky	150
Light	250
Medium	200

at most $367.50

*12. At a school outing, a group decides to go rafting. Each raft holds 8 people and costs $25 for the day. If at least 70 people go rafting, how much money will they need for the rafts?

at least $225

Higher-Order Thinking Problems

13. Write a real-world problem involving multiplication properties of an inequality that would have a solution $x \geq 25$.

Sample answer: Kail earns $4 for every dog he walks. He needs to earn at least $100 for new ski boots. Write and solve an inequality to determine the number of dogs he must walk to earn enough for the ski boots.

14. **Reason Abstractly** You are asked to draw a rectangle with a width of 5 inches and an area less than 55 square inches. Can the length of the rectangle be 11 inches? Explain your reasoning.

no; Sample answer: The inequality $5x < 55$ represents the situation and the solution is $x < 11$. So, 11 is not included in the solution set.

15. **Which One Doesn't Belong?** Identify the inequality that does not belong with the other three. Explain your reasoning.

A. $-6x \leq -36$

B. $\frac{x}{2} \geq 3$

C. $\frac{x}{3} \geq 2$

D. $\frac{x}{-3} \geq -2$

$\frac{x}{-3} \geq -2$; This inequality's solution is $x \leq 6$. The other three inequalities' solutions are $x \geq 6$.

16. **Find the Error** A student solved the inequality shown below. Find the student's mistake and correct it.

$$-25 \leq -5x$$
$$\frac{-25}{-5} \leq \frac{-5x}{-5}$$
$$5 \leq x$$

The student forgot to reverse the inequality symbol when dividing by a negative coefficient. The correct solution is $x \leq 5$.

466 Module 7 • Equations and Inequalities

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 14, students use reasoning with inequalities to make conclusions about the dimensions of a rectangle.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students find and correct another student's mistake.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 12 Have students work together to prepare a brief demonstration that illustrates how they solved this problem. For example, before they can determine how much money the group will need for the rafts, they must first determine the minimum number of rafts they will need. Have each pair or group of students present their response to the class.

Listen and ask clarifying questions.
Use with Exercises 13–14 Have students work in pairs. Have students individually read Exercise 13 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 14.

Lesson 7-8

Write and Solve Two-Step Inequalities

7.EE.B.4, 7.EE.B.4.B

LESSON GOAL

Students will write and solve two-step inequalities.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Solve Two-Step Inequalities
Example 1: Solve Two-Step Inequalities
Example 2: Solve Two-Step Inequalities
Example 3: Solve Two-Step Inequalities
Example 4: Write and Solve Two-Step Inequalities
Example 5: Write and Solve Two-Step Inequalities
Apply: School

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Solve Two-step Absolute Value Inequalities		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 45 of the *Language Development Handbook* to help your students build mathematical language related to writing and solving two-step inequalities.

ELL You can use the tips and suggestions on page T45 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address major cluster **7. EE.B** by writing and solving two-step inequalities.
Standards for Mathematical Content: **7 .EE.B.4, 7.EE.B.4.B,** *Also addresses 7.EE.B.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students wrote and solved one-step multiplication and division inequalities.
7.EE.B.4, 7.EE.B.4.B

Now
Students write and solve two-step inequalities.
7.EE.B.4, 7.EE.B.4.B

Next
Students will graph the solution to a linear inequality in two variables as a half-plane.
HSA.REI.D.12

Rigor

The Three Pillars of Rigor

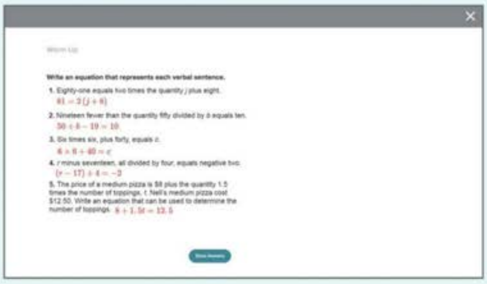
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of solving one-step inequalities and solving two-step equations to build *understanding* of solving two-step inequalities. They will use this understanding to build *fluency* in solving two-step inequalities. They will *apply* this fluency to write, solve, and graph two-step inequalities that represent real-world situations.

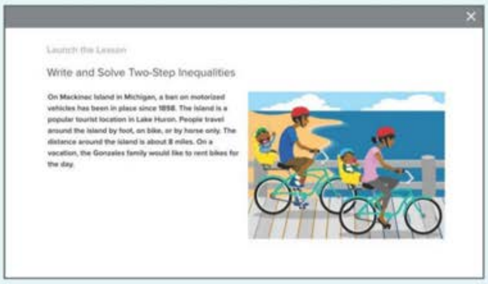
Mathematical Background

A *two-step inequality* is an inequality that contains two operations. To solve a two-step inequality, use inverse operations to undo each operation in reverse order of the order of operations.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing two-step equations (Exercises 1–5)

Answers

1. $81 = 2(f + 8)$
2. $50 \div b - 19 = 10$
3. $6 \times 6 + 40 = c$
4. $(r - 17) \div 4 = -2$
5. $8 + 1.5t = 12.5$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about renting bicycles to travel around Mackinac Island in Michigan.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- How do you think a *two-step inequality* might differ from a *one-step inequality*? Sample answer: A two-step inequality has two operations, whereas a one-step inequality has one operation.

Learn Solve Two-Step Inequalities

Objective

Students will understand how to solve two-step inequalities.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, encourage them to construct an argument that justifies whether or not -9 is a solution to the inequality.

Go Online

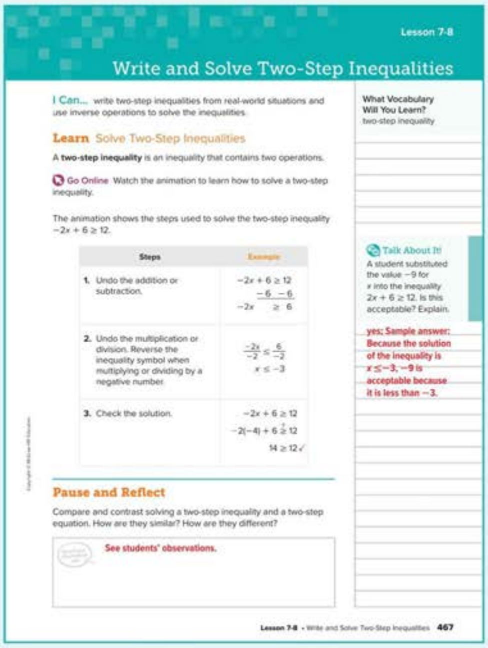
- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to solve a two-step inequality.

Talk About It!

SLIDE 2

Mathematical Discourse

A student substituted the value -9 for x into the inequality $2x + 6 \geq 12$. Is this acceptable? Explain. yes; Sample answer: Since the solution of the inequality is $x \leq -3$, -9 is acceptable because it is less than -3.



Lesson 7-8

Write and Solve Two-Step Inequalities

I Can... write two-step inequalities from real-world situations and use inverse operations to solve the inequalities.

What Vocabulary Will You Learn?
two-step inequality

Learn Solve Two-Step Inequalities

A **two-step inequality** is an inequality that contains two operations.

Go Online Watch the animation to learn how to solve a two-step inequality.

The animation shows the steps used to solve the two-step inequality $-2x + 6 \geq 12$.

Steps	Example
1. Undo the addition or subtraction.	$-2x + 6 \geq 12$ $-6 \quad -6$ $-2x \geq 6$
2. Undo the multiplication or division. Reverse the inequality symbol when multiplying or dividing by a negative number.	$\frac{-2x}{-2} \leq \frac{6}{-2}$ $x \leq -3$
3. Check the solution.	$-2x + 6 \geq 12$ $-2(-4) + 6 \overset{?}{\geq} 12$ $14 \geq 12$ ✓

Talk About It!
A student substituted the value -9 for x into the inequality $2x + 6 \geq 12$. Is this acceptable? Explain.

yes; Sample answer: Because the solution of the inequality is $x \leq -3$, -9 is acceptable because it is less than -3.

Pause and Reflect

Compare and contrast solving a two-step inequality and a two-step equation. How are they similar? How are they different?

See students' observations.

Lesson 7-8 • Write and Solve Two-Step Inequalities 467

Interactive Presentation

Learn, Solve Two-Step Inequalities, Slide 1 of 2

WATCH

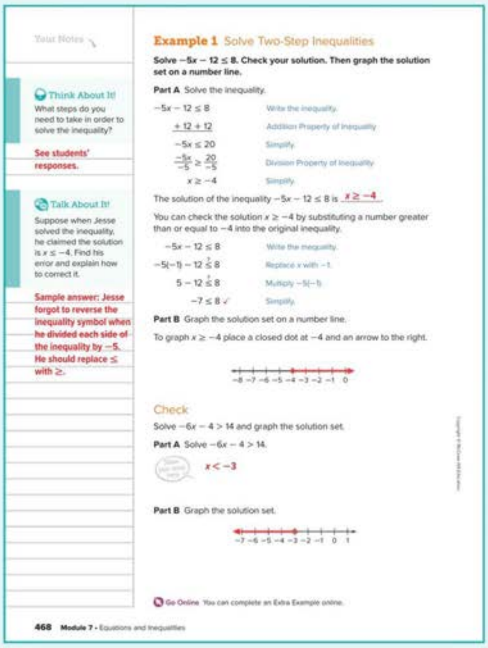
On Slide 1, students watch an animation that explains how to solve a two-step inequality.

DIFFERENTIATE

Language Development Activity ELL

To further students' understanding of how to solve two-step inequalities, have them work with a partner to compare and contrast solving two-step inequalities and two-step equations that involve addition or subtraction and multiplication or division. They should create a poster or graphic organizer that illustrates the similarities and differences between the steps involved in solving each of these mathematical statements. They should include examples of each that contain negative and positive coefficients and the Properties they use to solve each statement. They can present their work to the class, or you can hang the posters or graphic organizers around the classroom. Some sample similarities and differences are shown.

- Both require "undoing" addition or subtraction first.
- Both require "undoing" multiplication or division next.
- Y ou can multiply each side of an equation by a negative number without changing the equals sign, but you must reverse the inequality symbol when you do the same to solve an inequality.



Your Notes

Think About It! What steps do you need to take in order to solve the inequality?

See students' responses.

Talk About It! Suppose when Jesse solved the inequality, he claimed the solution is $x \le -4$. Find his error and explain how to correct it.

Sample answer: Jesse forgot to reverse the inequality symbol when he divided each side of the inequality by −5. He should replace ≤ with ≥.

Example 1 Solve Two-Step Inequalities

Solve $-5x - 12 \le 8$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$-5x - 12 \le 8$	Write the inequality.
$+12 \quad +12$	Addition Property of Inequality
$-5x \le 20$	Simplify.
$\frac{-5x}{-5} \ge \frac{20}{-5}$	Division Property of Inequality
$x \ge -4$	Simplify.

The solution of the inequality $-5x - 12 \le 8$ is $x \ge -4$.

You can check the solution $x \ge -4$ by substituting a number greater than or equal to −4 into the original inequality.

$-5x - 12 \le 8$	Write the inequality.
$-5(-1) - 12 \overset{?}{\le} 8$	Replace x with −1.
$5 - 12 \overset{?}{\le} 8$	Multiply −5(−1).
$-7 \le 8$ ✓	Simplify.

Part B Graph the solution set on a number line.

To graph $x \ge -4$ place a closed dot at −4 and an arrow to the right.

−8 −7 −6 −5 −4 −3 −2 −1 0

Check

Solve $-6x - 4 > 14$ and graph the solution set.

Part A Solve $-6x - 4 > 14$.

$x < -3$

Part B Graph the solution set.

−7 −6 −5 −4 −3 −2 −1 0 1

Go Online You can complete an Extra Example online.

468 **Module 7** • Equations and Inequalities

Interactive Presentation

Example 1, Solve Two-Step Inequalities, Slide 2 of 5

CLICK On Slide 2, students move through the steps to solve the inequality.

eTOOLS On Slide 3, students use the Number Line eTool to graph the inequality.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

7.EE.B.4, 7.EE.B.4.B

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 1 Solve Two-Step Inequalities

Objective

Students will solve and graph two-step inequalities involving integers.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to use correct mathematical vocabulary when explaining the flaw in Jesse's work.

5 Use Appropriate Tools Strategically Students will use the Number Line eTool to graph the solution set to this inequality.

6 Attend to Precision Students should use the properties of operations when solving this inequality and pay special attention to whether or not they need to reverse the inequality symbol.

Questions for Mathematical Discourse

SLIDE 2

AL What operations are paired with the variable? multiplication and subtraction

AL Which operation will you undo first? Why? Add 12 to each side to undo the subtraction; Undo the operations in the reverse order of the order of operations.

OL Explain why you need to reverse the inequality symbol when solving this inequality. Sample answer: In the second step, I divided each side of the inequality by −5. When multiplying or dividing by a negative number, the inequality symbol must be reversed for the inequality to remain true.

OL How can you check your answer? Sample answer: Replace *x* with any number greater than or equal to −4 into the original inequality to verify the inequality is true.

BL If $-5x - 12 \le 8$, what must be true about $-5x + 12$? Explain without calculating the value of *x*. $-5x + 12 \le 32$; Sample answer: Because 24 was added to the left side of the inequality, add 24 to the right side of the inequality.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Solve Two-Step Inequalities

Objective

Students will solve and graph two-step inequalities involving decimals.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Number Line eTool to graph the solution set to this inequality.

6 Attend to Precision Students should use to the properties of operations when solving this inequality and pay special attention to the whether or not they need to reverse the inequality symbol.

Questions for Mathematical Discourse

SLIDE 2

AL What operations are paired with the variable? multiplication and subtraction

AL Which operation will you undo first? Why? Add 3.25 to each side to undo the subtraction; undo the operations in the reverse order of the order of operations.

OL Explain why you do not reverse the inequality symbol when solving this inequality. Sample answer: In the second step, I divided each side of the inequality by 4.7. Because 4.7 is a positive number, I do not reverse the inequality symbol.

OL How can you check your answer? Sample answer: Replace x with any number less than or equal to 3 in the original inequality to verify the inequality is true.

BL If $4.7x - 3.25 \leq 10.85$, what must be true about $4.7x - 3$? Explain without calculating the value of x. $4.7x - 3 \leq 11.1$; Sample answer: Because 0.25 was added to the left side of the inequality, add 0.25 to the right side of the inequality.

SLIDE 3

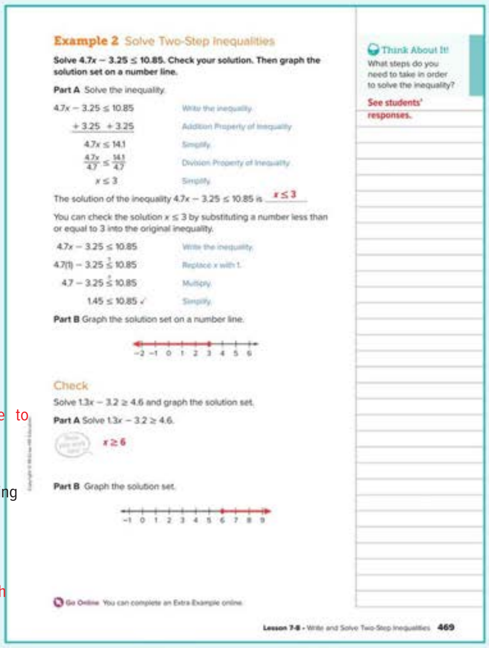
AL Name three numbers that are solutions to the inequality. Then name three numbers that are not solutions. Sample answer: 3, 0, and −2 are solutions; 3.5, 4, and 7 are not solutions.

OL Describe how to graph the inequality. Sample answer: Place a closed dot at 3, because 3 is part of the solution. Then extend the arrow to the left because any number less than or equal to 3 is part of the solution.

BL Write a two-step inequality, different from $4.7x - 3.25 \leq 10.85$, that has the same solution. Sample answer: $2.3x - 1.8 \leq 5.1$

Go Online

- Find additional teaching notes and discussion questions.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Example 2 Solve Two-Step Inequalities

Solve $4.7x - 3.25 \leq 10.85$. Check your solution. Then graph the solution set on a number line.

Part A Solve the inequality.

$4.7x - 3.25 \leq 10.85$	Write the inequality.
$+3.25 \quad +3.25$	Addition Property of Inequality
$4.7x \leq 14.1$	Simplify.
$\frac{4.7x}{4.7} \leq \frac{14.1}{4.7}$	Division Property of Inequality
$x \leq 3$	Simplify.

The solution of the inequality $4.7x - 3.25 \leq 10.85$ is $x \leq 3$.

You can check the solution $x \leq 3$ by substituting a number less than or equal to 3 into the original inequality.

$4.7x - 3.25 \leq 10.85$	Write the inequality.
$4.7(1) - 3.25 \stackrel{?}{\leq} 10.85$	Replace x with 1.
$4.7 - 3.25 \stackrel{?}{\leq} 10.85$	Multiply.
$1.45 \leq 10.85$ ✓	Simplify.

Part B Graph the solution set on a number line.

−2 −1 0 1 2 3 4 5 6

Check

Solve $1.3x - 3.2 \geq 4.6$ and graph the solution set.

Part A Solve $1.3x - 3.2 \geq 4.6$.

$x \geq 6$

Part B Graph the solution set.

−1 0 1 2 3 4 5 6 7 8 9

Go Online You can complete an Extra Example online.

Think About It! What steps do you need to take in order to solve the inequality?

See students' responses.

Lesson 7-8 • Write and Solve Two-Step Inequalities 469

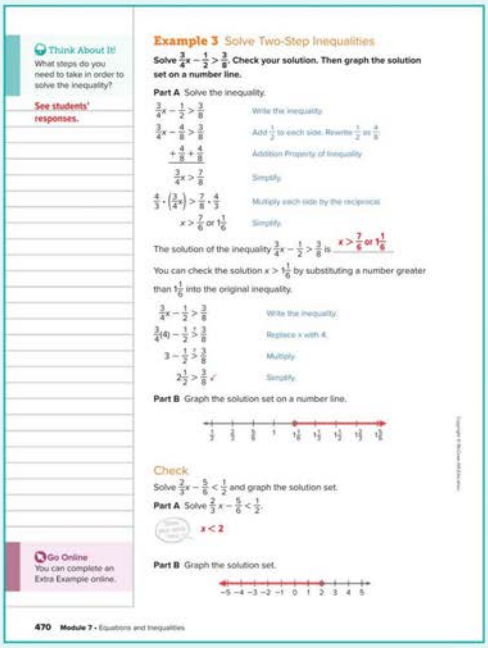
Interactive Presentation

Example 2, Solve Two-Step Inequalities, Slide 2 of 4

CLICK On Slide 2, students move through the steps to solve the inequality.

eTOOLS On Slide 3, students use the Number Line eTool to graph the inequality.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Solve Two-Step Inequalities

Solve $\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$. Check your solution. Then graph the solution set on a number line.

Think About It! What steps do you need to take in order to solve the inequality? See students' responses.

Part A Solve the inequality.

$\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$ Write the inequality.

$\frac{3}{4}x - \frac{4}{8} > \frac{3}{8}$ Add $\frac{1}{2}$ to each side. Rewrite $\frac{1}{2}$ as $\frac{4}{8}$.

$+\frac{4}{8} \quad +\frac{4}{8}$ Addition Property of Inequality

$\frac{3}{4}x > \frac{7}{8}$ Simplify.

$\frac{4}{3}\cdot\left(\frac{3}{4}x\right) > \frac{7}{8}\cdot\frac{4}{3}$ Multiply each side by the reciprocal.

$x > \frac{7}{6}$ or $1\frac{1}{6}$ Simplify.

The solution of the inequality $\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$ is $x > \frac{7}{6}$ or $1\frac{1}{6}$.

You can check the solution $x > 1\frac{1}{6}$ by substituting a number greater than $1\frac{1}{6}$ into the original inequality.

$\frac{3}{4}x - \frac{1}{2} > \frac{3}{8}$ Write the inequality.

$\frac{3}{4}(4) - \frac{1}{2} \overset{?}{>} \frac{3}{8}$ Replace x with 4.

$3 - \frac{1}{2} \overset{?}{>} \frac{3}{8}$ Multiply.

$2\frac{1}{2} > \frac{3}{8}$ ✓ Simplify.

Part B Graph the solution set on a number line.

Check

Solve $\frac{2}{3}x - \frac{5}{6} < \frac{1}{2}$ and graph the solution set.

Part A Solve $\frac{2}{3}x - \frac{5}{6} < \frac{1}{2}$.

$x < 2$

Part B Graph the solution set.

Go Online You can complete an Extra Example online.

470 Module 7 • Equations and Inequalities

Interactive Presentation

Example 3, Solve Two-Step Inequalities, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to solve the inequality.

eTOOLS

On Slide 3, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.EE.B.4, 7.EE.B.4.B

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Solve Two-Step Inequalities

Objective

Students will solve and graph two-step inequalities involving fractions.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Number Line eTool to graph the solution set to this inequality.

6 Attend to Precision Students should use to the properties of operations when solving this inequality and pay special attention to whether or not they need to reverse the inequality symbol.

Questions for Mathematical Discourse

SLIDE 2

AL What operations are paired with the variable? multiplication and subtraction

AL Which operation will you undo first? Why? Add $\frac{1}{2}$ to each side to undo the subtraction; Undo the operations in the reverse order of the order of operations.

OL Explain why you do not reverse the inequality symbol when solving this inequality. Sample answer: In the second step, I multiplied each side of the inequality by $\frac{4}{3}$. Because $\frac{4}{3}$ is a positive number, I do not reverse the inequality symbol.

OL How can you check your answer? Sample answer: Replace x with any number greater than $1\frac{1}{6}$ in the original inequality to verify the inequality is true.

BL Explain how to solve this inequality by eliminating the fractions. Sample answer: The greatest denominator is 8, and the other denominators are factors of 8. So, multiply each term by 8. The inequality becomes $6x - 4 > 3$. Then solve by adding 4 to each side, and dividing by 6.

Go Online

- Find additional teaching notes and discussion questions.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of solving inequalities, have them solve the following three-step inequality. They should be able to cite the properties of operations as they explain the steps they used.

$-3(x + 5) + 7 \leq -44$ $x \geq 12$

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 4 Write and Solve Two-Step Inequalities

Objective

Students will write two-step inequalities from real-world problems and interpret the solution.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to decontextualize the information in the real-world problem by representing it symbolically with a correct two-step inequality. As students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of the quantities given in the problem in order to understand why Stewart cannot bowl part of a game.

5 Use Appropriate Tools Strategically Students will use the number line eTool to graph the solution set to this inequality.

6 Attend to Precision Students should use the properties of operations to solve the inequality and precisely interpret its solution.

Questions for Mathematical Discourse

SLIDE 2

AL Explain this problem in your own words. Sample answer: Stewart has 34 strikes. At the average rate of 2 strikes per game, how many more games will Stewart need to play in order to have a total of at least 61 strikes?

AL What phrase indicates this is an inequality? What inequality symbol will you use? at least; greater than or equal to symbol

OL Explain why this is a two-step inequality. Sample answer: Stewart already has 34 strikes. He will add *2 strikes per game* to this number; the phrase *2 strikes per game* indicates multiplication. So, the two operations (steps) are multiplication and addition.

OL Write this inequality another way. Sample answer: $2g + 34 \geq 61$

BL Does Stewart have to bowl exactly 2 strikes per game in order to beat the record? Explain. no; Sample answer: His average rate is 2 strikes per game. He can have more or fewer strikes as long as he has a total of at least 61 strikes by the end.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Write and Solve Two-Step Inequalities

Halfway through the bowling league season, Stewart has 34 strikes. He averages 2 strikes per game. He needs at least 61 strikes to beat the league record.

Write and solve an inequality to determine the possible number of additional games Stewart should bowl to have at least 61 strikes. Then interpret the solution.

Part A Write an inequality.

Words	The number of strikes Stewart has plus two strikes per game is at least the total number of strikes needed.
Variable	Let g represent the number of games.
Inequality	$34 + 2g \geq 61$

Part B Solve the inequality.

$34 + 2g \geq 61$ Write the inequality.

$-34 \quad -34$ Subtraction Property of Inequality

$2g \geq 27$ Simplify.

$\frac{2g}{2} \geq \frac{27}{2}$ Division Property of Inequality

$g \geq 13.5$ Simplify.

The solution of the inequality $34 + 2g \geq 61$ is $g \geq$ 13.5

Part C Interpret the solution.

Graph the solution set of $g \geq 13.5$ on the number line.

Use the graph to interpret the solution.

So, in order to beat the record, Stewart will have to bowl at least 14 games.

Think About It! What is the unknown in the problem?

number of additional games

Talk About It! When interpreting the solution of the inequality, why is the least number of games Stewart needs to bowl equal to 14, and not 13 or 13.5?

Sample answer: The number of possible games must be greater than or equal to 13.5, so 13 cannot be a possible solution. Stewart cannot bowl part of a game, so 13.5 cannot be a solution. Only whole numbers greater than or equal to 13.5 can be solutions, so the least number of games he needs to bowl is 14.

Lesson 7-8 • Write and Solve Two-Step Inequalities 471

Interactive Presentation

Example 4, Write and Solve Two-Step Inequalities, Slide 2 of 6

FLASHCARDS On Slide 2, students use Flashcards to view the steps for writing the inequality.

eTOOLS On Slide 4, students use the Number Line eTool to graph the inequality.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

Peter can spend no more than $100 on new clothes for school. He spends $35 on a new pair of shoes. Shirts cost $15.

Write and solve an inequality to determine how many shirts Peter can purchase. Then interpret the solution.

$35 + 15x \le 100$; $x \le 4.\overline{3}$; Peter can buy no more than 4 shirts.

Go Online You can complete an Extra Example online.

Example 5 Write and Solve Two-Step Inequalities

Meredith is given a $50 monthly allowance to buy lunch at school. Any remaining money can be spent on entertainment. Meredith would like to have at least $12 left at the end of the month to go to the movies with her friends. It costs Meredith $2.50 per lunch that she buys at school.

Write and solve an inequality to determine the number of lunches Meredith can buy and have at least $12 left. Then interpret the solution.

Part A Write an inequality to determine the number of lunches Meredith can buy.

Words	Monthly allowance minus the cost per lunch is at least the amount remaining.
Variable	Let x represent the number of lunches.
Inequality	$50 - 2.50x \ge 12$

(continued on next page)

Example 5 Write and Solve Two-Step Inequalities

Objective

Students will write two-step inequalities with negative coefficients from real-world problems and interpret the solution.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to decontextualize the information in the real-world problem by representing it symbolically with a correct two-step inequality.

5 Use Appropriate Tools Strategically Students will use the Number Line eTool to graph the solution set to this inequality.

Questions for Mathematical Discourse

SLIDE 2

AL Explain this problem in your own words. Sample answer: Meredith has $50. At the rate of spending $2.50 per lunch, how many lunches can she buy in order to have at least $12 left?

AL What phrase indicates this is an inequality? What inequality symbol will you use? at least; greater than or equal to symbol

OL Explain why this is a two-step inequality. Sample answer: Meredith has $50. After paying for x lunches, she will subtract $2.50x$ from this amount. So, the two operations (steps) are multiplication and subtraction.

OL A classmate wrote the inequality $2.50x + 50 \ge 12$. Explain why this is incorrect. Sample answer: The cost of x lunches, $2.50x$, must be subtracted from 50, not added to it. Because she has $50, after paying for the lunches, she will have an amount less than $50.

BL A classmate claims that because Meredith wants to have $12 left, she can spend at most $38 on lunches. They then wrote the inequality $2.50x \ge 38$. Is this approach and inequality correct? Explain. Sample answer: The approach is correct, but the inequality is not correct. Because she can spend *at most* $38, the one-step inequality should use the *less than or equal to symbol* instead. This is because after 50 is subtracted from each side of the two-step inequality $50 - 2.50x \ge 12$, the inequality becomes $-2.50x \ge -38$. To divide each side by -1 means the inequality symbol must be reversed. So, the correct one-step inequality is $2.50x \le 38$, not $2.50x \ge 38$.

(continued on next page)

Example 5 Write and Solve Two-Step Inequalities *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL What operations are paired with the variable? multiplication and addition

AL Which operation will you undo first? Why? Undo addition by subtracting 50 from each side; Addition and subtraction are inverse operations.

OL How can you check your answer? Replace *x* with 15.2 in the original inequality and verify that the inequality is true.

OL Even though the inequality symbol is $\leq$, does an answer of 15.2 make sense within the context of the problem? Explain. no; Sample answer: Meredith cannot purchase part of a lunch. Since she can only purchase whole-number lunches less than or equal to 15.2, the maximum number of lunches she can purchase is 15.

BL If Meredith purchases 15 lunches, how much money will she have left? $12.50

SLIDE 4

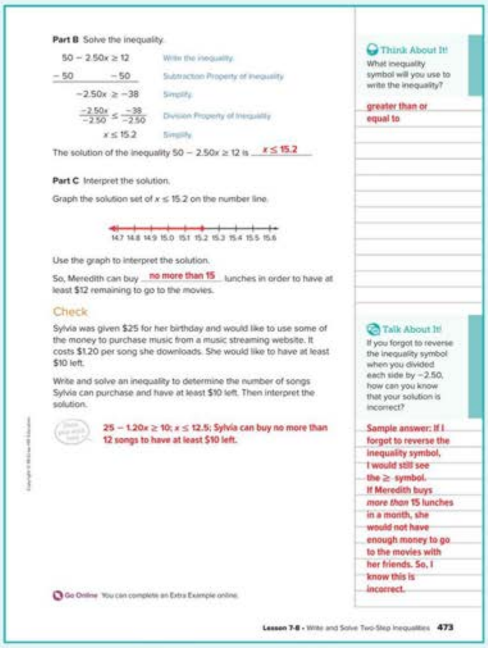
AL Describe in your own words what $x \leq 15.2$ means. Sample answer: The set of all numbers that are less than or equal to 15.2.

OL The solution set $x \leq 15.2$ includes *all numbers* that are less than or equal to 15.2. Explain why not every number in the solution set can be a solution to the real-world problem. Sample answer: Meredith can only purchase whole-number lunches. So, the solution set within the context of the problem can only be whole numbers that are less than or equal to 15.2, such as 15, 14, and so on. Additionally, negative numbers will not make sense within the context of this problem beause she cannot purchase a negative number of lunches.

BL Suppose the cost per lunch increases to $3.25. How would this affect the number of lunches Meredith can buy and still have at least $12 left? Sample answer: The number of lunches Meredith can buy will decrease. The maximum number of lunches she can buy at this price is 11 lunches.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Part B Solve the inequality.

$50 - 2.50x \geq 12$ — Write the inequality.

$-50 \quad -50$ — Subtraction Property of Inequality

$-2.50x \geq -38$ — Simplify.

$\frac{-2.50x}{-2.50} \leq \frac{-38}{-2.50}$ — Division Property of Inequality

$x \leq 15.2$ — Simplify.

The solution of the inequality $50 - 2.50x \geq 12$ is $x \leq 15.2$.

Part C Interpret the solution.

Graph the solution set of $x \leq 15.2$ on the number line.

14.7 14.8 14.9 15.0 15.1 15.2 15.3 15.4 15.5 15.6

Use the graph to interpret the solution.

So, Meredith can buy no more than 15 lunches in order to have at least $12 remaining to go to the movies.

Check

Sylvia was given $25 for her birthday and would like to use some of the money to purchase music from a music streaming website. It costs $1.20 per song she downloads. She would like to have at least $10 left.

Write and solve an inequality to determine the number of songs Sylvia can purchase and have at least $10 left. Then interpret the solution.

$25 - 1.20x \geq 10$; $x \leq 12.5$; Sylvia can buy no more than 12 songs to have at least $10 left.

Go Online You can complete an Extra Example online.

Think About It! What inequality symbol will you use to write the inequality? greater than or equal to

Talk About It! If you forgot to reverse the inequality symbol when you divided each side by −2.50, how can you know that your solution is incorrect? Sample answer: If I forgot to reverse the inequality symbol, I would still see the ≥ symbol. If Meredith buys *more than* 15 lunches in a month, she would not have enough money to go to the movies with her friends. So, I know this is incorrect.

Lesson 7-8 • Write and Solve Two-Step Inequalities 473

Interactive Presentation

Example 5, Write and Solve Two-Step Inequalities, Slide 2 of 6

FLASHCARDS

On Slide 2, students use Flashcards to view the steps for writing the inequality.

eTOOLS

On Slide 4, students use the Number Line eTool to graph the inequality.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.EE.B.4, 7.EE.B.4.B

Pause and Reflect

Create a graphic organizer that describes how to solve inequalities. Be sure to include examples of solving addition, subtraction, multiplication, division, and two-step inequalities.

See students' graphic organizers.

474 Module 7 • Equations and Inequalities

Apply School

Objective

Students will come up with their own strategy to solve an application problem involving the average score needed in a class.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

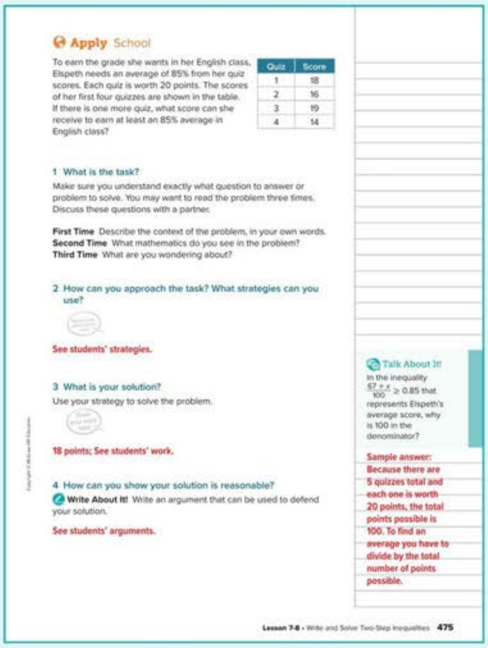
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How do you find the average score of the four existing quizzes?
- What is the maximum number of points available for five quizzes?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply School

To earn the grade she wants in her English class, Elspeth needs an average of 85% from her quiz scores. Each quiz is worth 20 points. The scores of her first four quizzes are shown in the table. If there is one more quiz, what score can she receive to earn at least an 85% average in English class?

Quiz	Score
1	18
2	16
3	19
4	14

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

18 points; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! In the inequality $\frac{67 + x}{100} \geq 0.85$ that represents Elspeth's average score, why is 100 in the denominator?

Sample answer: Because there are 5 quizzes total and each one is worth 20 points, the total points possible is 100. To find an average you have to divide by the total number of points possible.

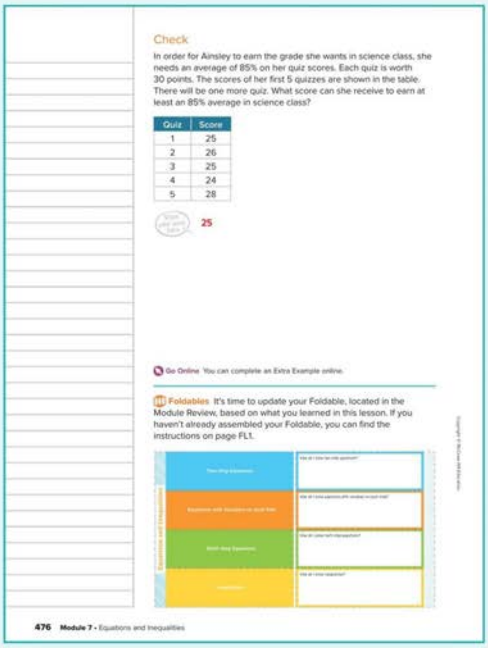
Lesson 7-8 • Write and Solve Two-Step Inequalities 475

Interactive Presentation

Apply, School

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

In order for Ainsley to earn the grade she wants in science class, she needs an average of 85% on her quiz scores. Each quiz is worth 30 points. The scores of her first 5 quizzes are shown in the table. There will be one more quiz. What score can she receive to earn at least an 85% average in science class?

Quiz	Score
1	25
2	26
3	25
4	24
5	28

25

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

476 Module 7 • Equations and Inequalities

Interactive Presentation


Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information on solving two-step inequalities. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Write and solve an inequality that can be used to determine the number of hours for which they can rent the bikes. Interpret the solution within the context of the problem. $25 + 8x \leq 75$; $x \leq 6.25$; The Gonzales family can spend no more than 6.25 hours riding the bikes.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 9, 11, 13–16
- Extension: Solve Two-Step Absolute Value Inequalities
- ALEKS Multi-Step Inequalities, Applications of Inequalities

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–9, 12, 15
- Extension: Solve Two-Step Absolute Value Inequalities
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS One-Step Inequalities

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS One-Step Inequalities

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

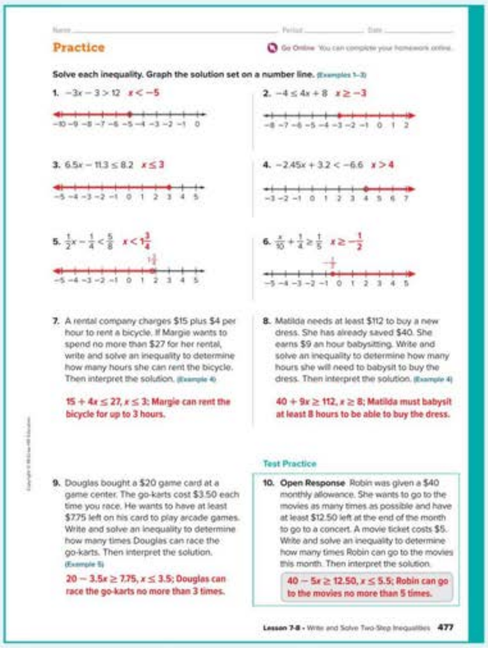
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve and graph two-step inequalities with rational numbers	1–6
2	write and solve two-step inequalities from real-world problems and interpret the solution	7, 8
2	write and solve two-step inequalities with negative coefficients from real-world problems and interpret the solution	9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving writing and solving two-step inequalities	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may incorrectly graph the solution to an inequality. In Exercise 1, students may use a solid circle rather than an open circle on the number line. Explain that solid circles are used to indicate the inclusion of a value.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Solve each inequality. Graph the solution set on a number line. (Examples 1–3)

1. $-3x - 3 > 12$ $x < -5$
2. $-4 \leq 4x + 8$ $x \geq -3$
3. $6.5x - 11.3 \leq 8.2$ $x \leq 3$
4. $-2.45x + 3.2 < -6.6$ $x > 4$
5. $\frac{1}{2}x - \frac{1}{4} < \frac{5}{8}$ $x < 1\frac{3}{4}$
6. $\frac{x}{10} + \frac{1}{4} \geq \frac{1}{5}$ $x \geq -\frac{1}{2}$

7. A rental company charges $15 plus $4 per hour to rent a bicycle. If Margie wants to spend no more than $27 for her rental, write and solve an inequality to determine how many hours she can rent the bicycle. Then interpret the solution. (Example 4)

   $15 + 4x \leq 27$, $x \leq 3$; Margie can rent the bicycle for up to 3 hours.

8. Matilda needs at least $112 to buy a new dress. She has already saved $40. She earns $9 an hour babysitting. Write and solve an inequality to determine how many hours she will need to babysit to buy the dress. Then interpret the solution. (Example 4)

   $40 + 9x \geq 112$, $x \geq 8$; Matilda must babysit at least 8 hours to be able to buy the dress.

9. Douglas bought a $20 game card at a game center. The go-karts cost $3.50 each time you race. He wants to have at least $7.75 left on his card to play arcade games. Write and solve an inequality to determine how many times Douglas can race the go-karts. Then interpret the solution. (Example 5)

   $20 - 3.5x \geq 7.75$, $x \leq 3.5$; Douglas can race the go-karts no more than 3 times.

Test Practice

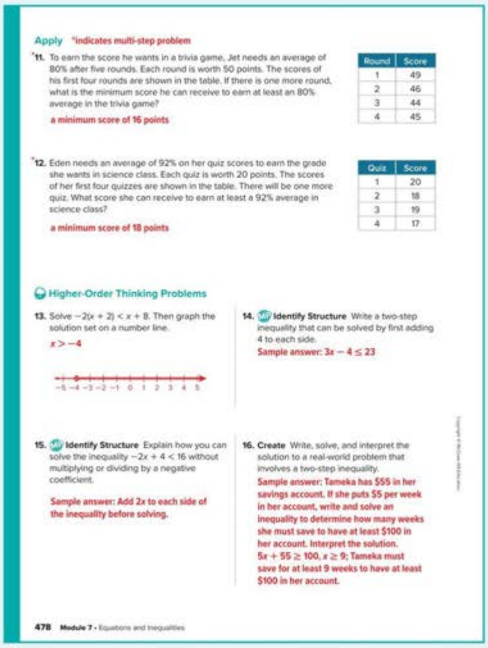
10. **Open Response** Robin was given a $40 monthly allowance. She wants to go to the movies as many times as possible and have at least $12.50 left at the end of the month to go to a concert. A movie ticket costs $5. Write and solve an inequality to determine how many times Robin can go to the movies this month. Then interpret the solution.

   $40 - 5x \geq 12.50$, $x \leq 5.5$; Robin can go to the movies no more than 5 times.

Lesson 7-8 • Write and Solve Two-Step Inequalities 477

7.EE.B.4, 7.EE.B.4.B

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*11. To earn the score he wants in a trivia game, Jet needs an average of 80% after five rounds. Each round is worth 50 points. The scores of his first four rounds are shown in the table. If there is one more round, what is the minimum score he can receive to earn at least an 80% average in the trivia game?

Round	Score
1	49
2	46
3	44
4	45

a minimum score of 16 points

*12. Eden needs an average of 92% on her quiz scores to earn the grade she wants in science class. Each quiz is worth 20 points. The scores of her first four quizzes are shown in the table. There will be one more quiz. What score she can receive to earn at least a 92% average in science class?

Quiz	Score
1	20
2	18
3	19
4	17

a minimum score of 18 points

Higher-Order Thinking Problems

13. Solve $-2(x + 2) < x + 8$. Then graph the solution set on a number line.

$x > -4$

14. **Identify Structure** Write a two-step inequality that can be solved by first adding 4 to each side.

Sample answer: $3x - 4 \le 23$

15. **Identify Structure** Explain how you can solve the inequality $-2x + 4 < 16$ without multiplying or dividing by a negative coefficient.

Sample answer: Add $2x$ to each side of the inequality before solving.

16. **Create** Write, solve, and interpret the solution to a real-world problem that involves a two-step inequality.

Sample answer: Tameka has $55 in her savings account. If she puts $5 per week in her account, write and solve an inequality to determine how many weeks she must save to have at least $100 in her account. Interpret the solution. $5x + 55 \ge 100$, $x \ge 9$; Tameka must save for at least 9 weeks to have at least $100 in her account.

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Teaching the Mathematical Practices

7 Look For and Make Use of Structure In E xercise 14, students write a two-step inequality given the first step of solving the inequality.

In Exercise 15, students explain how they can solve an inequality without multiplying or dividing by a negative coefficient.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.

Use with Exercises 11–12 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 11 might be, "A score of 80% on a 50-point game would be a score of 40." An example of a false statement might be, "Jet has already scored a total of 180 points." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Create your own higher-order thinking problem

Use with Exercises 13–16 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of how to write and solve equations and inequalities. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

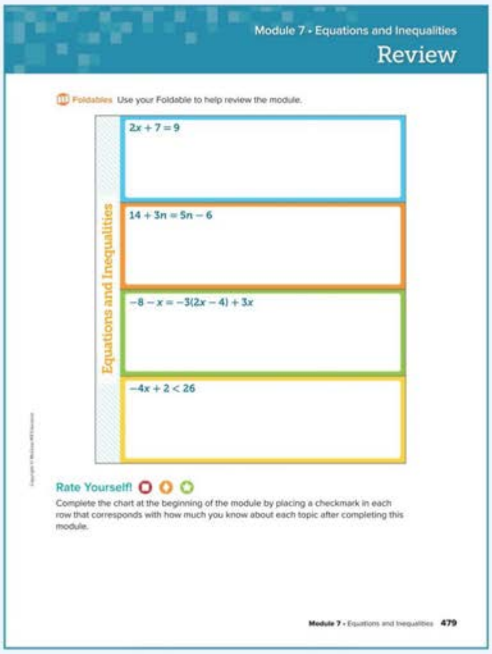
Assessment Resources

Put It All Together 1: Lessons 7-1 through 7-5
Put It All Together 2: Lessons 7-6 through 7-8
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with this topic for **Expressions and Equations**.

- Solve Simple Equations
- Inequalities
- Linear Equations in One Variable


Module 7 • Equations and Inequalities

Review

Foldables Use your Foldable to help review the module.

Equations and Inequalities

$2x + 7 = 9$

$14 + 3n = 5n - 6$

$-8 - x = -3(2x - 4) + 3x$

$-4x + 2 < 26$

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

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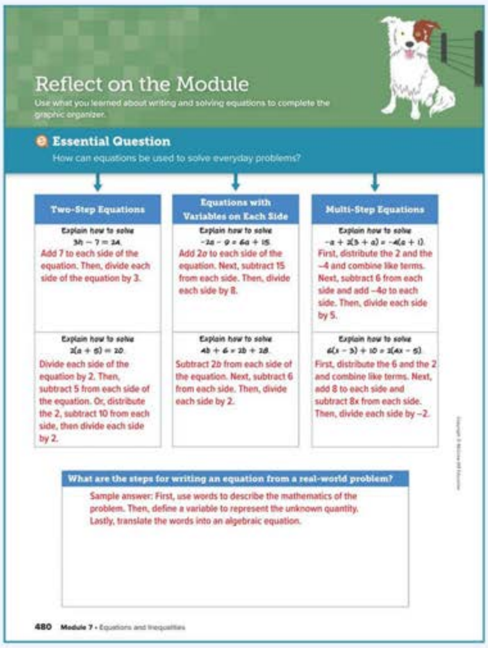

Reflect on the Module

Use what you learned about writing and solving equations to complete the graphic organizer.

Essential Question

How can equations be used to solve everyday problems?

Two-Step Equations	Equations with Variables on Each Side	Multi-Step Equations
Explain how to solve $3h - 7 = 34$. Add 7 to each side of the equation. Then, divide each side of the equation by 3.	Explain how to solve $-2a - 9 = 6a + 15$. Add $2a$ to each side of the equation. Next, subtract 15 from each side. Then, divide each side by 8.	Explain how to solve $-a + 2(3 + a) = -4(a + 1)$. First, distribute the 2 and the -4 and combine like terms. Next, subtract 6 from each side and add $-4a$ to each side. Then, divide each side by 5.
Explain how to solve $2(a + 5) = 20$. Divide each side of the equation by 2. Then, subtract 5 from each side of the equation. Or, distribute the 2, subtract 10 from each side, then divide each side by 2.	Explain how to solve $4b + 6 = 2b + 28$. Subtract $2b$ from each side of the equation. Next, subtract 6 from each side. Then, divide each side by 2.	Explain how to solve $6(x - 3) + 10 = 2(4x - 5)$. First, distribute the 6 and the 2 and combine like terms. Next, add 8 to each side and subtract $8x$ from each side. Then, divide each side by -2.

What are the steps for writing an equation from a real-world problem?

Sample answer: First, use words to describe the mathematics of the problem. Then, define a variable to represent the unknown quantity. Lastly, translate the words into an algebraic equation.

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole-class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How are the solutions to inequalities different from the solutions to equations? See students' graphic organizers.

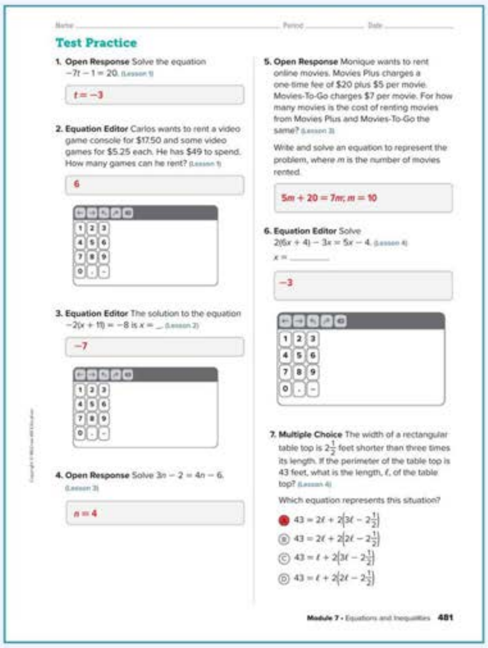
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–13 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	7
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	2, 3, 6, 11
Grid	Students graph an inequality on an online number line.	10, 12
Table item	Students complete a table to indicate the number of solutions.	8
Open Response	Students construct their own response in the area provided.	1, 4, 5, 9, 13

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.EE.B.4	7-1, 7-2, 7-6, 7-7, 7-8	1-3, 9-13
7.EE.B.4.A	7-1, 7-2	2
7.EE.B.4.B	7-6, 7-7, 7-8	9-13
8.EE.C.7	7-3, 7-4, 7-5	4-8
8.EE.C.7.A	7-5	8
8.EE.C.7.B	7-3, 7-4	4-7


Name ______ Period ______ Date ______

Test Practice

1. **Open Response** Solve the equation $-7t - 1 = 20$. (Lesson 1)

$t = -3$

2. **Equation Editor** Carlos wants to rent a video game console for $17.50 and some video games for $5.25 each. He has $49 to spend. How many games can he rent? (Lesson 1)

6

3. **Equation Editor** The solution to the equation $-2(x + 11) = -8$ is $x =$ __. (Lesson 2)

-7

4. **Open Response** Solve $3n - 2 = 4n - 6$. (Lesson 3)

$n = 4$

5. **Open Response** Monique wants to rent online movies. Movies Plus charges a one-time fee of $20 plus $5 per movie. Movies-To-Go charges $7 per movie. For how many movies is the cost of renting movies from Movies Plus and Movies-To-Go the same? (Lesson 3)

Write and solve an equation to represent the problem, where m is the number of movies rented.

$5m + 20 = 7m$; $m = 10$

6. **Equation Editor** Solve $2(6x + 4) - 3x = 5x - 4$. (Lesson 4)
$x =$ ______

-3

7. **Multiple Choice** The width of a rectangular table top is $2\frac{1}{2}$ feet shorter than three times its length. If the perimeter of the table top is 43 feet, what is the length, ℓ, of the table top? (Lesson 4)

Which equation represents this situation?

- (A) $43 = 2\ell + 2\left(3\ell - 2\frac{1}{2}\right)$
- (B) $43 = 2\ell + 2\left(2\ell - 2\frac{1}{2}\right)$
- (C) $43 = \ell + 2\left(3\ell - 2\frac{1}{2}\right)$
- (D) $43 = \ell + 2\left(2\ell - 2\frac{1}{2}\right)$

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8. Table Item Determine whether each equation has one solution, no solution, or infinitely many solutions. (Lesson 5)

	One Solution	No Solution	Infinitely Many Solutions
$4(x + 8) = 2(4 + 2x)$		X	
$3(2x + 1) = 3 + 6x$			X
$2(x + 5) = 5x + 1$	X		

9. Open Response Jessica has a department store gift card worth $75. The table shows the items that she has picked out so far. (Lesson 6)

Item	Cost ($)
bracelet	8
sunglasses	15
beach towel	12
hat	10
sandals	14

Jessica also wants to buy two T-shirts. Write and solve an inequality to represent how much she can spend on the T-shirts. Interpret the solution.

Let x = cost of T-shirt; $59 + 2x \leq 75$; $x \leq 8$; Sample answer: Based upon the solution, each T-shirt can cost $8 or less.

10. Grid The difference between the high and low temperatures on Sunday was at least 35°F. The low temperature on Sunday was −8°F. (Lesson 6)

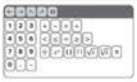
A. Write and solve an inequality to find t, the high temperature on Sunday.

$t - (-8) \geq 35$; $t \geq 27$

B. Graph the solution of the inequality.

11. Equation Editor Write the correct symbol and value that represents the solution to the inequality $9x < 27$. (Lesson 7)

< 3

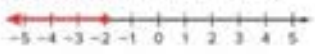


12. Grid Consider the inequality $-11x \geq 22$. (Lesson 7)

A. Solve the inequality. Explain why or why not the direction of the inequality symbol is reversed.

$x \leq -2$; Sample answer: The direction of the inequality symbol is reversed because I divided each side of the inequality by a negative number in order to solve for x.

B. Graph the solution of the inequality.



13. Open Response Solve the inequality $6.6z - 2.25 \leq -10.5$. Explain why or why not the direction of the inequality symbol is reversed. (Lesson 8)

$z \leq -1.25$; Sample answer: The direction of the inequality symbol is not reversed because I did not have to multiply or divide each side of the inequality by a negative number in order to solve for z.

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Module 8

Linear Relationships and Slope

Module Goal

Graph and write equations to represent linear relationships.

Focus

Domain: Expressions and Equations
Major Cluster(s): **8.EE.B** Under stand the connections between proportional relationships, lines, and linear equations.
Standards for Mathematical Content: **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
8.EE.B.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- identify relationships as proportional or nonproportional
- find unit rates
- write fractions in simplest form
- express relationships using multiple representations (tables, graphs, and equations)

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students recognized and represented proportional relationshipsbetween quantities.
7.RP.A.2

Now
Students graph and write equations to represent linear relationships.
8.EE.B.5, 8.EE.B.6

Next
Students will interpret expressions that represent a quantity in terms of its context.
HSA.SSE.A.1

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of proportional relationships to develop *understanding* of the concept of slope. They use this understanding to build *fluency* with finding the slope of a line, and writing and graphing linear equations. They *apply* their fluency to solve multi-step real-world problems.

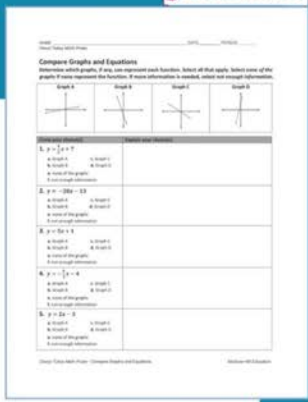
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			**1**	**0.5**
8-1	Proportional Relationships and Slope	8.EE.B.5	2	1
8-2	Slope of a Line	Foundational for 8.EE.B.6	2	1
8-3	Similar Triangles and Slope	8.EE.B.6	1	0.5
8-4	Direct Variation	8.EE.B.6, *Also addresses 8.EE.B.5*	2	1
Put It All Together 1: Lessons 8-1 through 8-4			**0.5**	**0.25**
8-5	Slope-Intercept Form	8.EE.B.6	2	1
8-6	Graph Linear Equations	8.EE.B.6	2	1
Put It All Together 2: Lessons 8-5 and 8-6			**0.5**	**0.25**
Module Review			**1**	**0.5**
Module Assessment			**1**	**0.5**
		Total Days	15	7.5

Formative Assessment Math Probe
Compare Graphs and Equations

Module Resource


Compare Graphs and Equations

Correct Answers: **1.** Graphs A, D; **2.** Graphs B, C; **3.** Graphs A, D; **4.** Graphs B, C; **5.** e

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which graphs can represent each equation, and explain their choices.

Targeted Concept Understand how the slope and *y*-intercept in an equation can give you information about the general shape of the graph.

Targeted Misconceptions

- Students may incorrectly think that without numbers on the graph, they cannot determine a possible slope and *y*-intercept.
- Students may incorrectly assume that any graph without numbers shows a small range, such as –5 to 5 or –10 to 10.

Assign the probe after Lesson 6.

Collect and Assess Student Work

If the student selects...	**Then** the student likely...
Chooses *not enough information* for each item.	believes that without numbers on the graph, they cannot determine a possible slope or *y*-intercept.
1. Only chooses either Graph A or Graph D, but not both **2.** Only chooses e **3.** Only chooses either Graph A or Graph D, but not both **4.** Only chooses either Graph B or Graph C, but not both	assumes the range on the graphs must be smaller than a certain amount. **Example:** For item 2, –20 and –13 are too large to appear on any of the graphs.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Graphing, Functions, and Sequences
- Lesson 6, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How are linear relationships related to proportional relationships? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

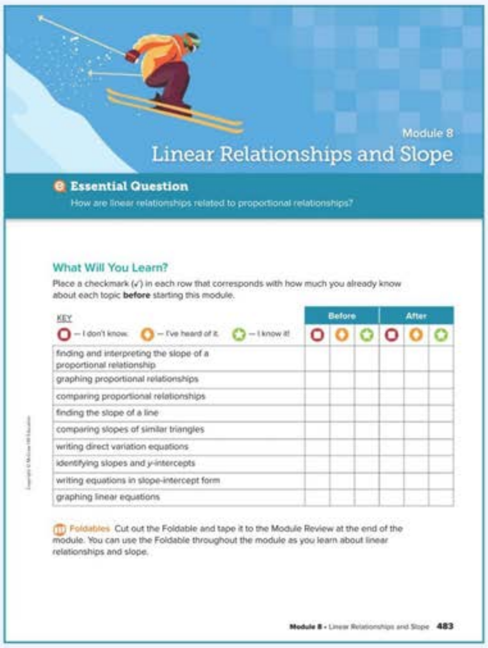
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of skiing and airplanes to introduce the idea of linear relationships and slope. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 8

Linear Relationships and Slope

Essential Question

How are linear relationships related to proportional relationships?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know.	I've heard of it.	I know it!	I don't know.	I've heard of it.	I know it!
finding and interpreting the slope of a proportional relationship						
graphing proportional relationships						
comparing proportional relationships						
finding the slope of a line						
comparing slopes of similar triangles						
writing direct variation equations						
identifying slopes and y-intercepts						
writing equations in slope-intercept form						
graphing linear equations						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about linear relationships and slope.

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Interactive Student Presentation

What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ constant of variation	☐ linear relationships	☐ slope
☐ constant rate of change	☐ rate of change	☐ slope-intercept form
☐ corresponding parts	☐ rise	☐ slope triangles
☐ direct variation	☐ run	☐ solution
☐ initial value	☐ similar figures	☐ y-intercept
☐ linear equation		

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1	Example 2
Subtract integers. Find $-24 - 9$. $-24 - 9 = -24 + (-9)$ To subtract 9, add -9. $= -33$ Simplify.	**Evaluate expressions.** Evaluate $\frac{15+5}{9-5}$. $\frac{15+5}{9-5} = \frac{20}{4}$ Simplify the numerator and denominator. $= 5$ Simplify.

Quick Check

1. A fish was 7 feet below sea level and descended 12 feet. The expression $-7 - 12$ represents this situation. Find $-7 - 12$ to determine the location of the fish compared to sea level. -19; 19 ft below sea level	2. The expression $\frac{40-13}{22-19}$ represents the number of points Cia earned on an assignment. How many points did she earn? 9 points

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine.

Define Slope is the rate of change between any two points on a line. The ratio of the rise, or vertical change, to the run, or horizontal change.

Example The slope of the line that passes through the points (2, −3) and (−4, 5) is $\frac{5-(-3)}{-4-2}$, or $\frac{8}{-6}$, which simplifies to $-\frac{4}{3}$.

Ask Find the slope of the line that passes through the points (−1, 5) and (−7, −5). $\frac{-5-5}{-7-(-1)}$ or $\frac{-10}{-6}$, which simplifies to $\frac{5}{3}$.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- identifying proportional relationships
- finding unit rates
- using constant ratios
- subtracting integers
- graphing on the coordinate plane
- using the slope formula
- identifying nonproportional linear relationships

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Graphing, Functions, and Sequences** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Regular Reflection

When students are asked to regularly explain their thinking about a strategy they used, they are engaging in thought organization, concise consolidation of knowledge, and deductive and inductive reasoning.

How Can I Apply It?

Use the **Think About It!** and the **Talk About It!** questions throughout each lesson to encourage students to reflect about what they just learned, or what they might do next.

Throughout the lesson, **Pause and Reflect** questions are included at point-of-use in the *Interactive Student Edition*. Encourage students to not skip over these questions, but to actually *pause* and *reflect* on the concept(s) they just learned and what questions they still might have.

Have students complete the **Exit Tickets** at the end of each lesson to reflect on their learning about the topics covered in each lesson. Have students share their reflections with a partner or in small groups.

Lesson 8-1

Proportional Relationships and Slope

8.EE.B.5

LESSON GOAL

Students will graph and compare proportional relationships, interpreting the unit rate as the slope of the line.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Rate of Change

Learn: Proportional Relationships
Learn: Unit Rate and Slope
Example 1: Proportional Relationships and Slope
Examples 2-3: Graph Proportional Relationships
Learn: Compare Proportional Relationships
Examples 4-5: Compare Proportional Relationships
Apply: Utilities

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 46 of the *Language Development Handbook* to help your students build mathematical language related to proportional relationships and slope .

ELL You can use the tips and suggestions on page T46 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.B** by graphing and comparing proportional relationships, interpreting the unit rate as the slope of the line.
Standards for Mathematical Content: **8.E E.B.5**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students recognized and represented proportional relationships between quantities.
7.RP.A.2

Now
Students graph and compare proportional relationships, interpreting the unit rate as the slope of the line.
8.EE.B.5

Next
Students will find the slope of a line from a graph, table, and using the formula.
Foundational for 8.EE.B.6

Rigor

The Three Pillars of Rigor

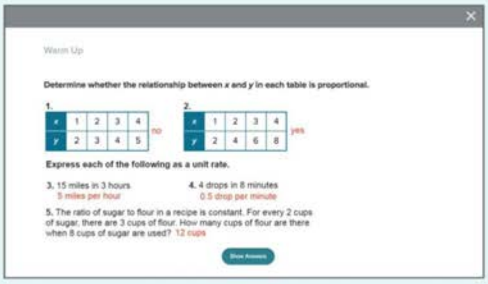
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of unit rates and proportional relationships to develop *understanding* of slope. They come to understand that the constant rate of change, or unit rate, in a proportional relationship is the same as the slope of the line representing the proportional relationship. They build *fluency* with slope by comparing proportional relationships written in different forms, and *apply* it to real-world problems.

Mathematical Background

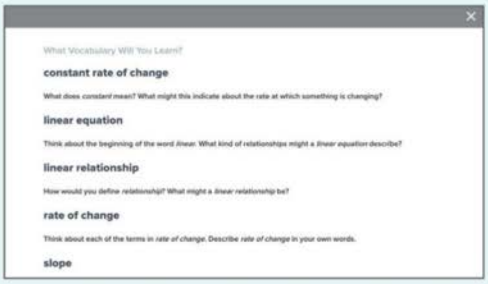
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- identifying proportional relationships (Exercises 1–2)
- finding unit rates (Exercises 3–4)
- using constant ratios (Exercise 5)

Answers

1. no
2. yes
3. 5 miles per hour
4. 0.5 drop per minute
5. 12 cups

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about using an equation to find the cost of entrance fees to two different national parks.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- What does *constant* mean? What might this indicate about the rate at which something is changing? Sample answer: Constant means steady or unchanging. A constant rate might be a rate that doesn't change.
- Think about the beginning of the word *linear*. What kind of relationship might a *linear equation* describe? Sample answer: A linear equation might describe a line.
- How would you define *relationship?* What might a *linear relationship* be? Sample answer: A relationship describes how two quantities are related. A linear relationship might be a relationship between two quantities that looks like a line when graphed.
- Think about each of the terms in *rate of change*. Describe *rate of change* in your own words. Sample answer: A rate of change is a measurement of how something is changing.

Explore Rate of Change

Objective

Students will explore how one quantity changes in relation to another quantity.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a rate at which Marcus can download songs from the Internet. Throughout this activity, students will use a table, graph, and ratio to compare the number of minutes and the number of songs downloaded.

Inquiry Question

How can you describe how one quantity changes in relation to another quantity? Sample answer: To describe how one quantity changes in relation to another, you can use a rate.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

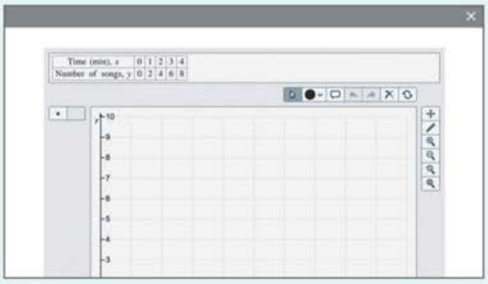
Describe the pattern shown in the graph. Sample answer: The points appear to make a line.

Use the graph to examine any two consecutive points. By how much does y change? By how much does x change? y by 2 units and x by 1 unit

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 5



Explore, Slide 3 of 5

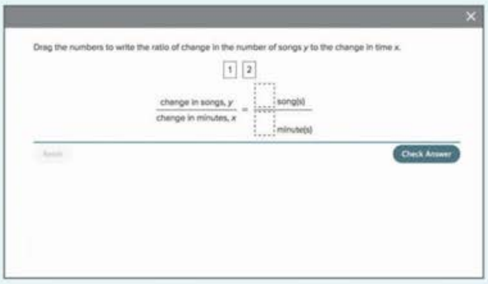
TYPE

On Slide 2, students complete the table to find the number of songs for 0, 1, 2, 3, and 4 minutes.

eTOOL

On Slide 3, students use the Coordinate Graphing eTool to graph ordered pairs from the table.

Interactive Presentation



Explore, Slide 4 of 5

DRAG & DROP

On Slide 4, students drag the numbers to write the ratio of change.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Rate of Change *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively St udents should understand how tables, graphs, and ratios can each be used to represent real-world proportional relationships.

5 Use Appropriate Tools Strategically Students will use the Coordinate Graphing eTool to explore the patterns in the graph and examine how one quantity changes in relation to another quantity.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Where is the ratio found in the table and the graph? In the table, the ratio is the number of songs, 2, downloaded in 1 minute. In the graph, it is the ordered pair (1, 2).

What are some other names for this ratio? Sample answer: unit rate, constant of proportionality, slope

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Proportional Relationships

Objective

Students will understand how proportional relationships are related to linear relationships.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, encourage them to clearly explain how they can determine whether the ordered pair is a solution to the equation.

Teaching Notes

SLIDE 1

Remind students they have previously learned that proportional relationships have a constant ratio, or unit rate. Point out that proportional relationships are a special kind of linear relationship, in which the graph of the relationship passes through the origin. All linear relationships, including proportional relationships, have graphs that are straight lines. Have students select the *Words, Table, Symbols, Graph*, and *Example* flashcards to view how proportional relationships can be represented using these multiple representations.

(continued on next page)

DIFFERENTIATE

Language Development Activity ELL

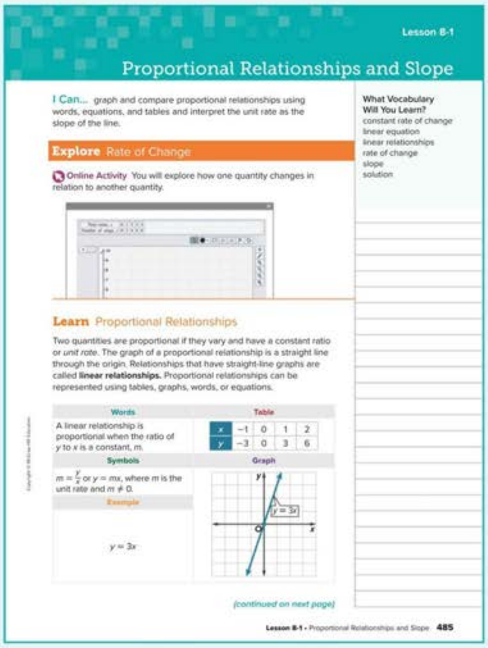
To further students' understanding of how proportional relationships are related to linear relationships, have them create a flowchart, Venn diagram, or other type of graphic organizer that conveys the following information.

A linear relationship has a graph that is a straight line.

A proportional relationship has a graph that is a straight line that passes through the origin.

For example, if students choose to create a flowchart, sample questions along the flowchart could be...

1. Is the graph of the relationship a straight line? If yes, the relationship is linear. Proceed to question 2. If no, the relationship is neither linear nor proportional.
2. Does the graph of the relationship pass through the origin? If yes, the relationship is proportional. If no, the relationship is linear, but not proportional.


Lesson 8-1

Proportional Relationships and Slope

I Can... graph and compare proportional relationships using words, equations, and tables and interpret the unit rate as the slope of the line.

What Vocabulary Will You Learn?
constant rate of change
linear equation
linear relationships
rate of change
slope
solution

Explore Rate of Change

Online Activity You will explore how one quantity changes in relation to another quantity.

Learn Proportional Relationships

Two quantities are proportional if they vary and have a constant ratio or *unit rate*. The graph of a proportional relationship is a straight line through the origin. Relationships that have straight-line graphs are called **linear relationships.** Proportional relationships can be represented using tables, graphs, words, or equations.

Words	Table
A linear relationship is proportional when the ratio of y to x is a constant, m.	x: −1, 0, 1, 2; y: −3, 0, 3, 6
Symbols	**Graph**
$m = \frac{y}{x}$ or $y = mx$, where m is the unit rate and $m \neq 0$.	(graph of $y = 3x$)
Example	
$y = 3x$	

(continued on next page)

Lesson 8-1 • Proportional Relationships and Slope 485

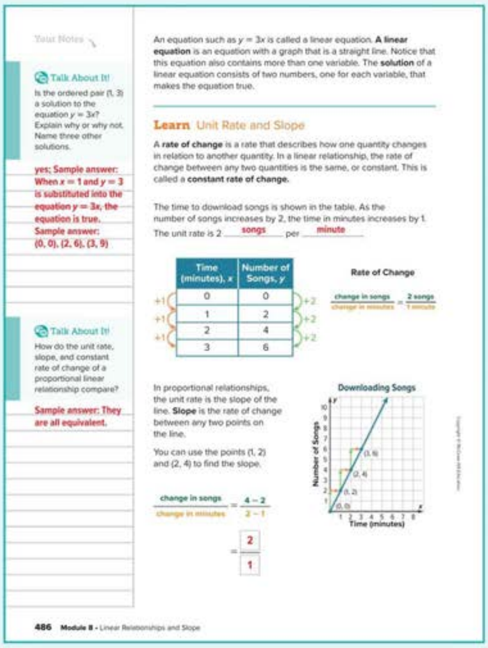
Interactive Presentation

Learn, Proportional Relationships, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of proportional relationships.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Your Notes

Talk About It!

Is the ordered pair (1, 3) a solution to the equation $y = 3x$? Explain why or why not. Name three other solutions.

yes; Sample answer: When $x = 1$ and $y = 3$ is substituted into the equation $y = 3x$, the equation is true. Sample answer: (0, 0), (2, 6), (3, 9)

Talk About It!

How do the unit rate, slope, and constant rate of change of a proportional linear relationship compare?

Sample answer: They are all equivalent.

An equation such as $y = 3x$ is called a linear equation. A **linear equation** is an equation with a graph that is a straight line. Notice that this equation also contains more than one variable. The **solution** of a linear equation consists of two numbers, one for each variable, that makes the equation true.

Learn Unit Rate and Slope

A **rate of change** is a rate that describes how one quantity changes in relation to another quantity. In a linear relationship, the rate of change between any two quantities is the same, or constant. This is called a **constant rate of change.**

The time to download songs is shown in the table. As the number of songs increases by 2, the time in minutes increases by 1. The unit rate is 2 songs per minute.

Time (minutes), x	Number of Songs, y
0	0
1	2
2	4
3	6

Rate of Change

$\frac{\text{change in songs}}{\text{change in minutes}} = \frac{2 \text{ songs}}{1 \text{ minute}}$

In proportional relationships, the unit rate is the slope of the line. **Slope** is the rate of change between any two points on the line.

You can use the points (1, 2) and (2, 4) to find the slope.

$\frac{\text{change in songs}}{\text{change in minutes}} = \frac{4-2}{2-1} = \frac{2}{1}$

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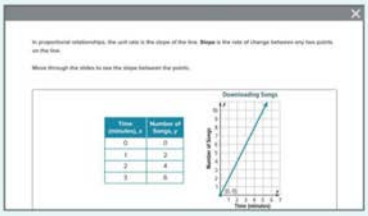
Interactive Presentation



Learn, Unit Rate and Slope, Slide 2 of 3

CLICK

On Slide 2 of the Learn, *Unit Rate and Slope*, students click to move through the slides to see the slope between the points.

Learn Proportional Relationships *(continued)*

Talk About It!

SLIDE 2

Mathematical Discourse

Is the ordered pair (1, 3) a solution to the equation $y = 3x$? Explain why or why not. Name three other solutions. yes; Sample answer: When $x = 1$ and $y = 3$ are substituted into the equation $y = 3x$, the equation is true; (0, 0), (2, 6), (3, 9)

Learn Unit Rate and Slope

Objective

Students will understand the relationship between the unit rate of a proportional relationship and the slope of the line.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to think about what each term means (unit rate, slope, and constant rate of change) in order to determine and clearly explain that they are equivalent for proportional linear relationships.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

How do the unit rate, slope, and constant rate of change of a proportional linear relationship compare? They are all equivalent.

Example 1 Proportional Relationships and Slope

Objective

Students will find and interpret the slope of a graph of a proportional relationship and compare it to the unit rate.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to use precision when finding slope, paying careful attention to the order in which they subtract the *x*- and *y*-coordinates.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the graph of Ava's savings? Sample answer: The graph is a straight line and it passes through the origin.

OL In this example, we chose the points (1, 15) and (2, 30). Can you choose any other two points on the line to find the slope? Explain. yes; Sample answer: The slope of a line is the same no matter which two points you choose.

BL Based on the constant rate of change, what do you expect the next point with whole number coordinates on the graph to be? Explain. (4, 60); Sample answer: The last point is (3, 45), representing \$45 after 3 weeks. After one more week, Ava will have \$15 more in savings, or a total of \$45 + \$15 = \$60. The next point will be (4, 60) representing 4 weeks and \$60.

SLIDE 3

AL What is a unit rate? Sample answer: a simplified rate comparing one quantity to one unit of another quantity.

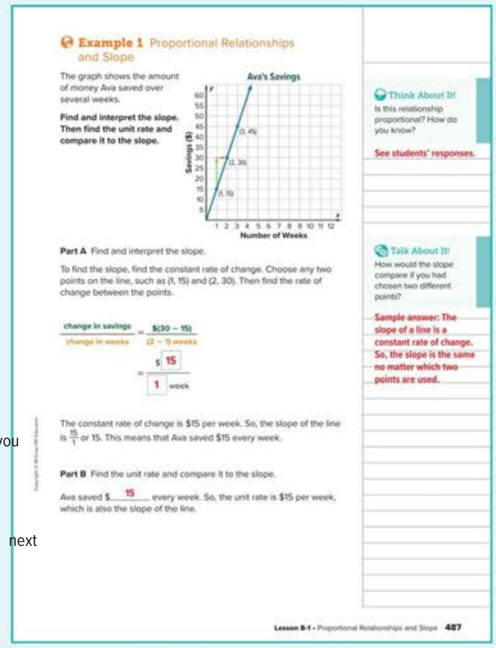
OL How do you know that the unit rate in this problem is \$15 per week? Sample answer: It compares the amount saved, \$15, to one week.

OL A classmate stated that since the point (2, 30) falls on the line, the unit rate can also be described as \$30 for 2 weeks. Describe the error in the classmate's reasoning. Sample answer: \$30 for 2 weeks is a rate, but it is not the unit rate. The unit rate compares the amount saved for one week.

BL What point on the line corresponds to the unit rate? (1, 15)

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Proportional Relationships and Slope

The graph shows the amount of money Ava saved over several weeks.

Find and interpret the slope. Then find the unit rate and compare it to the slope.

Part A Find and interpret the slope.

To find the slope, find the constant rate of change. Choose any two points on the line, such as (1, 15) and (2, 30). Then find the rate of change between the points.

$$\frac{\text{change in savings}}{\text{change in weeks}} = \frac{\$(30-15)}{(2-1)\text{ weeks}} = \frac{\$\,15}{1\text{ week}}$$

The constant rate of change is \$15 per week. So, the slope of the line is $\frac{15}{1}$ or 15. This means that Ava saved \$15 every week.

Part B Find the unit rate and compare it to the slope.

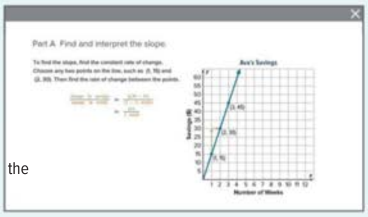
Ava saved \$ 15 every week. So, the unit rate is \$15 per week, which is also the slope of the line.

Think About It! Is this relationship proportional? How do you know? See students' responses.

Talk About It! How would the slope compare if you had chosen two different points? Sample answer: The slope of a line is a constant rate of change. So, the slope is the same no matter which two points are used.

Lesson 8-1 • Proportional Relationships and Slope 487

Interactive Presentation



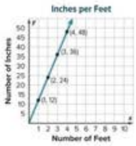
Example 1, Proportional Relationships and Slope, Slide 2 of 5

TYPE On Slide 2, students determine the constant rate of change.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

The graph shows the relationship between feet and inches.



Part A

Find and interpret the slope.

The slope of the line is $\frac{12}{1}$. This means that there are 12 inches in 1 foot.

Part B

Find the unit rate and compare it to the slope.

The unit rate is 12 inches per foot, which is also the slope of the line.

Go Online You can complete an Extra Example online.

Pause and Reflect

Reflect on what you have learned about ratios and rates in previous grades. How do the terms *rate of change* and *slope* relate to what you already know?

See students' observations.

Example 2 Graph Proportional Relationships

Objective

Students will graph the equation of a proportional relationship and interpret the slope.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 5, encourage them to be able to understand and explain how the table and graph each represent and model the unit rate, 12 miles per hour.

5 Use Appropriate Tools Strategically Students will use the Coordinate Graphing eTool to graph the ordered pairs.

Questions for Mathematical Discourse

SLIDE 2

AL What do x and y represent? x represents the number of hours and y represents the number of miles

OL How can you find the value of y for a given value of x? How do you know this? multiply the value of x by 12; Sample answer: The equation is $y = 12x$, which means y is equal to 12 multiplied by the value of x.

BL What would be the number of miles for 3 hours? 4 hours? 36 miles; 48 miles

BL By studying the table, can you use a different pattern to find the number of miles for 3 and 4 hours instead of multiplying by 12? Explain. yes; Sample answer: The rate of change is 12 miles per hour, so I can add 12 miles to each successive increase of 1 hour in the table. For 3 hours, add 12 to 24, which is 36. For 4 hours, add 12 to 36, which is 48.

SLIDE 3

AL To what part of the coordinate plane does the point (0, 0) correspond? the origin

OL Do you need all three points to graph the line? Explain. no; Sample answer: The line is straight, so it can be graphed using at least two points.

BL How can you interpret the point (2, 24) in context? The cyclist rides 24 miles in 2 hours.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Graph Proportional Relationships

The distance y in miles that a certain cyclist can ride and the time x in hours are in a proportional relationship. This can be represented by the equation $y = 12x$.

Graph the equation. Then find and interpret the slope.

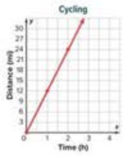
Part A Graph the equation.

Step 1 Make a table of values to find the distance for 0, 1, and 2 hours.

Hours, x	$y = 12x$	Miles, y
0	$y = 12(0)$	0
1	$y = 12(1)$	12
2	$y = 12(2)$	24

Step 2 Graph the ordered pairs.

Graph the ordered pairs (0, 0), (1, 12), and (2, 24) from the table. Then draw a line through the points.



Part B Find and interpret the slope.

In the equation of a proportional linear relationship $y = mx$, m represents the unit rate or slope. The equation $y = 12x$ represents the distance y in miles that the cyclist can ride in x hours.

So, the slope of the line is $\frac{12}{1}$ or 12. This means that the cyclist can ride 12 miles per hour.

Think About It! How does the slope and unit rate compare, for proportional relationships?

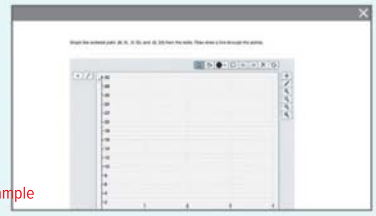
They are the same.

Talk About It! How is the unit rate represented in the table? How is the unit rate represented in the graph?

Sample answer: In the table, the unit rate is the constant rate of change. In the graph, the unit rate, 12 miles per hour, is the slope of the line.

Lesson 8-1 • Proportional Relationships and Slope 489

Interactive Presentation



Example 2, Graph Proportional Relationships, Slide 3 of 6

eTOOLS On Slide 3, students use the Coordinate Graphing eTool to graph the relationship.

TYPE On Slide 4, students determine the number of miles per hour the cyclist can ride.

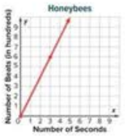
CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

The number of times y a honeybee can beat its wings and the time in seconds x are in a proportional linear relationship. This situation can be represented by $y = 200x$.

Part A

Graph the equation on the coordinate plane.



Part B

Find and interpret the slope.

The slope of the line is $\frac{200}{1}$. This means the honeybee beats its wings 200 times per second.

Go Online You can complete an Extra Example online.

Example 3 Graph Proportional Relationships

Objective

Students will graph a proportional relationship from a verbal description, and interpret the slope.

Teaching Notes

Data that can be graphed as any real number are continuous. Data that can only be graphed as whole numbers are discrete. In this Example, a dashed line is used to indicate the graph of discrete data.

Questions for Mathematical Discourse

SLIDE 2

AL What is the rate? How can you find the unit rate? The rate is $3.75 for 3 songs. Divide $3.75 by 3 to find the unit rate (cost) per song.

OL How can you mentally find the unit rate? Sample answer: $3.75 can be thought of as $3 + $0.75. Dividing $3 by 3 yields $1. Dividing $0.75 (75 cents) by 3 yields $0.25 (25 cents). So, the unit rate is $1.25 per song.

BL What does it mean to assume that the cost is proportional to the number of songs? Sample answer: It means that the cost of each song is always the same.

SLIDE 3

AL What is the unit rate? $1.25 per song

OL Explain how to find the cost of 2 songs, given the unit rate. Sample answer: Multiply 2 by the cost of one song, $1.25; 2($1.25) = $2.50

BL What expression can you write that represents the cost of *n* songs? $1.25n$

SLIDE 4

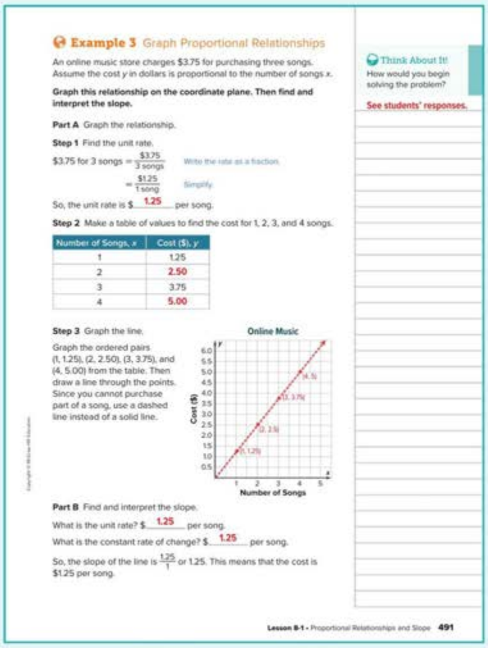
AL What does the point (1, 1.25) mean in context of the problem? 1 song costs $1.25.

OL Explain how to graph the point (1, 1.25). Sample answer: Start at the origin. Moving to the right, locate 1 on the *x*-axis. From there, move up and locate 1.25 on the *y*-axis. Plot the point.

BL Explain why it makes sense, within the context of the problem, that the graph is a straight line. Sample answer: Each song costs the same amount. So, there is a constant rate of change.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and discussion questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Graph Proportional Relationships

An online music store charges $3.75 for purchasing three songs. Assume the cost *y* in dollars is proportional to the number of songs *x*.

Graph this relationship on the coordinate plane. Then find and interpret the slope.

Part A Graph the relationship.

Step 1 Find the unit rate.

$$\$3.75 \text{ for 3 songs} = \frac{\$3.75}{3 \text{ songs}} \quad \text{Write the rate as a fraction.}$$

$$= \frac{\$1.25}{1 \text{ song}} \quad \text{Simplify.}$$

So, the unit rate is $ 1.25 per song.

Step 2 Make a table of values to find the cost for 1, 2, 3, and 4 songs.

Number of Songs, x	Cost ($), y
1	1.25
2	2.50
3	3.75
4	5.00

Step 3 Graph the line.

Graph the ordered pairs (1, 1.25), (2, 2.50), (3, 3.75), and (4, 5.00) from the table. Then draw a line through the points. Since you cannot purchase part of a song, use a dashed line instead of a solid line.

Part B Find and interpret the slope.

What is the unit rate? $ 1.25 per song.

What is the constant rate of change? $ 1.25 per song.

So, the slope of the line is $\frac{1.25}{1}$ or 1.25. This means that the cost is $1.25 per song.

Think About It! How would you begin solving the problem?

See students' responses.

Lesson 8-1 • Proportional Relationships and Slope 491

Interactive Presentation

Example 3, Graph Proportional Relationships, Slide 4 of 6

TYPE

On Slide 3, students enter values in the table. On Slide 5, students enter the unit rate and the constant rate of change.

eTOOLS

On Slide 4, students use the Coordinate Graphing eTool to graph the relationship.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A manatee can swim an average of 10 miles every 2 hours. Assume the distance y in miles is proportional to the number of hours x the manatee swam.

Part A Graph this relationship on the coordinate plane.

Part B
Find and interpret the slope.

The slope of the line is 5. This means that a manatee can swim 5 miles per hour.

Go Online You can complete an Extra Example online.

Learn Compare Proportional Relationships

You can use tables, graphs, words, or equations to represent and compare proportional relationships.

- In the equation of a proportional linear relationship, $y = mx$, m represents the slope or unit rate.
- In the table, the slope is the constant rate of change or unit rate.
- In the graph, the unit rate or slope is the constant rate of change or the constant ratio $\frac{y}{x}$. You can also find the unit rate from the point $(1, r)$, where r is the unit rate.

(continued on next page)

492 Module 8 • Linear Relationships and Slope

Interactive Presentation

Learn, Compare Proportional Relationships

FLASHCARDS Students use Flashcards to view multiple representations of proportional relationships.

Learn Compare Proportional Relationships

Objective

Students will understand that they can compare two proportional relationships that are represented in different ways.

Teaching Notes

SLIDE 1

Students will learn that proportional relationships can be represented using tables, graphs, words, or equations. The equation of a proportional relationship has the form $y = mx$. In a table, the slope is the constant rate of change or unit rate. In a graph, the unit rate or slope is the constant rate of change $\frac{y}{x}$. Have students select the *Words, Equation, Table,* and *Graph* flashcards to help them understand how to compare the different representations of proportional relationships.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling to compare proportional relationships that are expressed in different forms, have them work with a partner to generate their own example of a real-world proportional relationship. Then have them create a graphic organizer or poster that illustrates how the unit rate, or slope, of their proportional relationship can be expressed in these multiple representations: verbal description, table, equation, and graph. Have pairs share their poster or graphic organizer with the class, explaining how each representation illustrates the unit rate. Some students may be uncomfortable speaking in front of other students. Encourage them to make appropriate eye contact, and to articulate loudly enough for others to hear.

Example 4 Compare Proportional Relationships

Objective

Students will compare two different proportional relationships represented in different ways.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
Encourage students to make sense of the problem in order to plan a pathway, rather than just jumping into a solution attempt. They should be able to reason that both relationships are proportional. If they can find each relationship's unit rate, they can compare them.

As students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of the different methods they can use to find the speed of the grizzly bear. They should be able to understand that each method is valid, even if they prefer one over the others.

Questions for Mathematical Discourse

SLIDE 2

AL What do x and y represent? x represents the number of hours and y represents the number of miles.

AL What equation models the rabbit's travel? $y = 35x$

OL What does the form of the equation $y = 35x$ indicate about the relationship between x and y? The relationship is proportional.

OL Identify the rabbit's unit rate and describe what it represents in the context of this problem. The unit rate is 35 miles per hour; it means that the rabbit can travel at a rate of 35 miles every hour.

BL Do you think that it is realistic for a rabbit to travel at a constant speed for any number of hours? Why do you think we use an equation that represents x as any number of hours? Sample answer: It is not realistic to assume that a rabbit will be traveling at a constant speed for hours. However, we can use the equation to model its rate of travel for small quantities of time, such as 0.1 hour or 0.2 hour.

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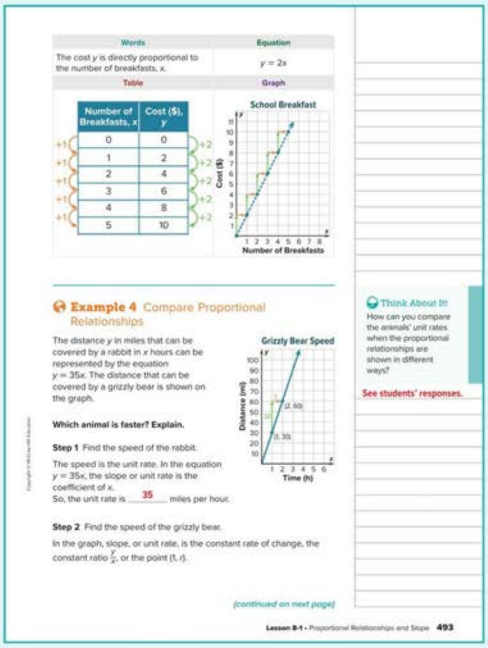


Words	Equation
The cost y is directly proportional to the number of breakfasts, x.	$y = 2x$

Number of Breakfasts, x	Cost ($), y
0	0
1	2
2	4
3	6
4	8
5	10

Example 4 Compare Proportional Relationships

The distance y in miles that can be covered by a rabbit in x hours can be represented by the equation $y = 35x$. The distance that can be covered by a grizzly bear is shown on the graph.

Which animal is faster? Explain.

Step 1 Find the speed of the rabbit.

The speed is the unit rate. In the equation $y = 35x$, the slope or unit rate is the coefficient of x.
So, the unit rate is 35 miles per hour.

Step 2 Find the speed of the grizzly bear.

In the graph, slope, or unit rate, is the constant rate of change, the constant ratio $\frac{y}{x}$, or the point (1, r).

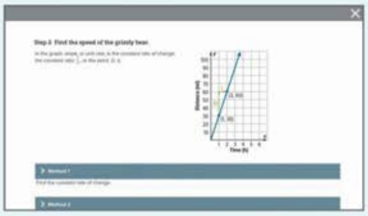
Think About It! How can you compare the animals' unit rates when the proportional relationships are shown in different ways?

See students' responses.

(continued on next page)

Lesson 8-1 • Proportional Relationships and Slope 493

Interactive Presentation



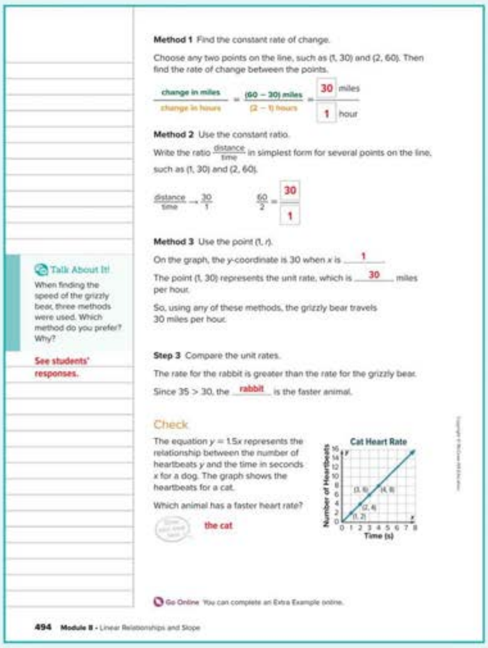
Example 4, Compare Proportional Relationships, Slide 3 of 6

EXPAND

On Slide 3, students expand to learn three different methods for determining the faster animal.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Method 1 Find the constant rate of change.

Choose any two points on the line, such as (1, 30) and (2, 60). Then find the rate of change between the points.

$\frac{\text{change in miles}}{\text{change in hours}} = \frac{(60 - 30)\text{ miles}}{(2 - 1)\text{ hours}} = \frac{30 \text{ miles}}{1 \text{ hour}}$

Method 2 Use the constant ratio.

Write the ratio $\frac{\text{distance}}{\text{time}}$ in simplest form for several points on the line, such as (1, 30) and (2, 60).

$\frac{\text{distance}}{\text{time}} \rightarrow \frac{30}{1}$ $\frac{60}{2} = \frac{30}{1}$

Method 3 Use the point (1, *r*).

On the graph, the *y*-coordinate is 30 when *x* is 1.

The point (1, 30) represents the unit rate, which is 30 miles per hour.

So, using any of these methods, the grizzly bear travels 30 miles per hour.

Talk About It!

When finding the speed of the grizzly bear, three methods were used. Which method do you prefer? Why?

See students' responses.

Step 3 Compare the unit rates.

The rate for the rabbit is greater than the rate for the grizzly bear.

Since 35 > 30, the rabbit is the faster animal.

Check

The equation $y = 1.5x$ represents the relationship between the number of heartbeats *y* and the time in seconds *x* for a dog. The graph shows the heartbeats for a cat.

Which animal has a faster heart rate? the cat

Go Online You can complete an Extra Example online.

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Example 4 Compare Proportional Relationships *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL What ordered pairs are graphed? Interpret them within the context of the problem. (1, 30) means the bear travels 30 miles in 1 hour. (2, 60) means the bear travels 60 miles in 2 hours.

OL What are three methods you can use to find the bear's speed in miles per hour? Sample answer: I can find the constant rate of change between the two ordered pairs. I can find the constant ratio $\frac{y}{x}$. Or I can use the point (1, 30) since it represents the unit rate.

BL Do you think that it is realistic for a bear to travel at a constant speed for any number of hours? Why do you think we use these equations in this example? Sample answer: It is not realistic to assume that a bear will be traveling at a constant speed for hours. However, we can use these equations to compare the speeds of each animal. Since the unit rate for a rabbit is greater than that of a bear, a rabbit can travel faster.

SLIDE 4

AL What is the unit rate for each animal? 35 miles per hour for the rabbit and 30 miles per hour for the grizzly bear

OL Explain how you know which animal is faster? Sample answer: The unit rates are measured in miles per hour, which tells the speed. The animal with the greater unit rate is faster.

BL If the distance d in miles that a wolf can travel in h hours is modeled by the equation $d = 25h$, list the animals in order from slowest to fastest. Explain. wolf, grizzly bear, rabbit; Sample answer: The rate for the wolf is 25 miles per hour, so it is slower than the rabbit and the grizzly bear.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 5 Compare Proportional Relationships

Objective

Students will compare two different proportional relationships represented in different ways.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
Encourage students to make sense of the problem in order to plan a pathway, rather than just jumping into a solution attempt. They should be able to reason that both relationships are proportional. If they can find each relationship's unit cost, they can compare them.

Questions for Mathematical Discourse

SLIDE 2

AL By finding the constant rate of change, what else are you also finding? Why is this true? The slope and the unit rate; Since the relationship is proportional, the constant rate of change is equal to the slope and the unit rate.

OL By what amount does the cost increase for each additional hour? What does this mean? $25; This is the unit rate.

BL What equation could be used to relate *x* and *y* for Computer Access? $y = 25x$

SLIDE 3

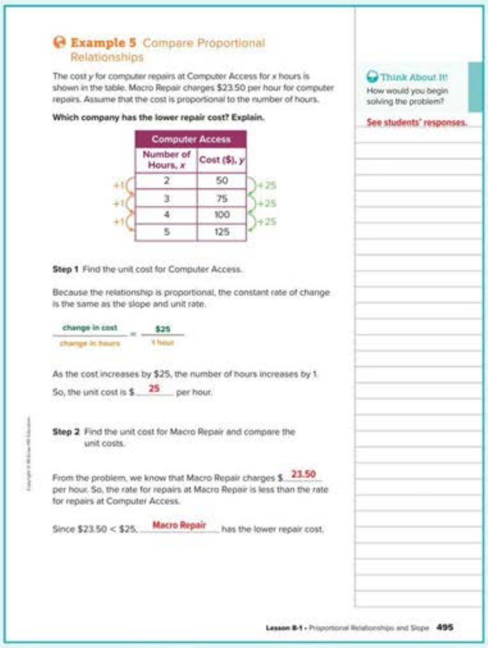
AL How many units does the term *per hour* represent? one

OL What is the unit cost for Macro Repair? $23.50 per hour

BL What would each company charge for 8 hours of repairs? Macro Repair would charge $188 and Computer Access would charge $200.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 5 Compare Proportional Relationships

The cost *y* for computer repairs at Computer Access for *x* hours is shown in the table. Macro Repair charges $23.50 per hour for computer repairs. Assume that the cost is proportional to the number of hours.

Which company has the lower repair cost? Explain.

Computer Access	
Number of Hours, *x*	Cost ($), *y*
2	50
3	75
4	100
5	125

(+1 between consecutive hours; +25 between consecutive costs)

Step 1 Find the unit cost for Computer Access.

Because the relationship is proportional, the constant rate of change is the same as the slope and unit rate.

$\frac{\text{change in cost}}{\text{change in hours}} = \frac{\$25}{1 \text{ hour}}$

As the cost increases by $25, the number of hours increases by 1.

So, the unit cost is $ 25 per hour.

Step 2 Find the unit cost for Macro Repair and compare the unit costs.

From the problem, we know that Macro Repair charges $ 23.50 per hour. So, the rate for repairs at Macro Repair is less than the rate for repairs at Computer Access.

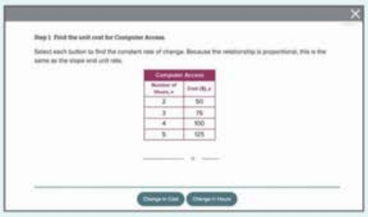
Since $23.50 < $25, Macro Repair has the lower repair cost.

Think About It! How would you begin solving the problem?

See students' responses.

Lesson 8-1 • Proportional Relationships and Slope **495**

Interactive Presentation



Example 5, Compare Proportional Relationships, Slide 2 of 4

CLICK
On Slide 2, students select each button to find the constant rate of change.

TYPE
On Slide 2, students determine the cost per hour.

CHECK
Students complete the Check exercise online to determine if they are ready to move on.

Check

Annie's current earnings are shown in the table. She was offered a new job that will pay $7.95 per hour. Assume that her earnings are proportional to the number of hours worked.

Current Earnings	
Hours, x	Money Earned ($), y
2	14.50
3	21.75
4	29.00
5	36.25

Which job pays more each hour?

new job offer

Go Online You can complete an Extra Example online.

Pause and Reflect

Write a real-world problem that uses the concepts from today's lesson. Explain how you came up with that problem. Exchange problems with a classmate and solve each other's problem.

See students' observations.

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Apply Utilities

Objective

Students will come up with their own strategy to solve an application problem that involves comparing utility prices.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

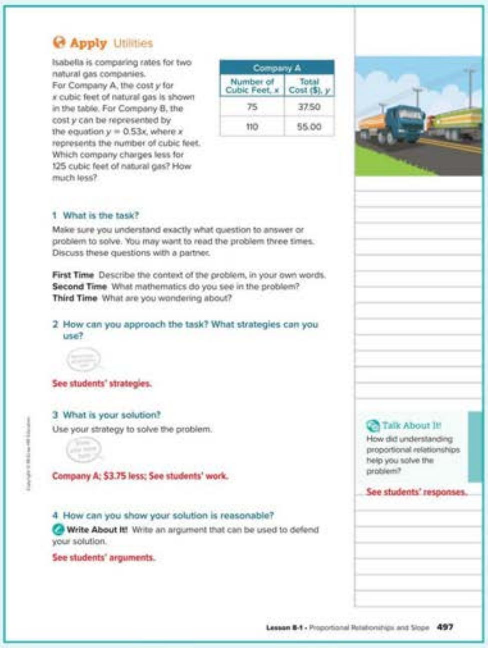
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does *rate* mean?
- How can you use the values in the table to find the unit rate for Company A?
- How can you use the equation to find the cost for Company B?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Utilities

Isabella is comparing rates for two natural gas companies. For Company A, the cost y for x cubic feet of natural gas is shown in the table. For Company B, the cost y can be represented by the equation $y = 0.53x$, where x represents the number of cubic feet. Which company charges less for 125 cubic feet of natural gas? How much less?

Company A	
Number of Cubic Feet, x	Total Cost ($), y
75	37.50
110	55.00

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Company A; $3.75 less; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How did understanding proportional relationships help you solve the problem?

See students' responses.

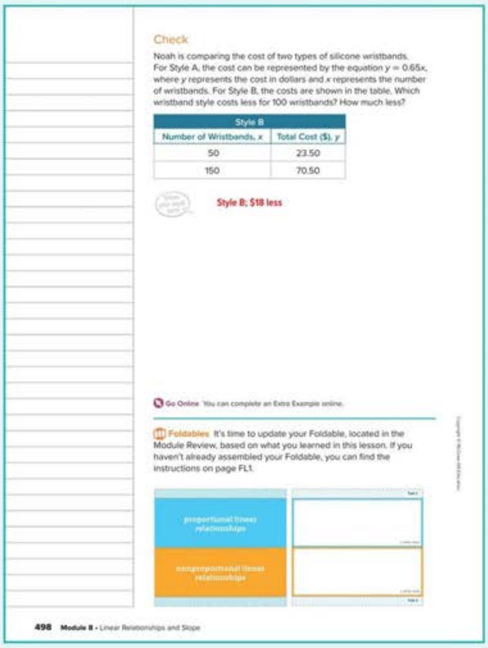
Lesson 8-1 • Proportional Relationships and Slope 497

Interactive Presentation

Apply, Utilities

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

Noah is comparing the cost of two types of silicone wristbands. For Style A, the cost can be represented by the equation $y = 0.65x$, where y represents the cost in dollars and x represents the number of wristbands. For Style B, the costs are shown in the table. Which wristband style costs less for 100 wristbands? How much less?

Style B	
Number of Wristbands, x	Total Cost ($), y
50	23.50
150	70.50

Style B; $18 less

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record an example of a proportional relationship. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are linear relationships related to proportional relationships?

In this lesson, students learned that in a linear equation in the form $y = mx$, the slope is the same as the unit rate in a proportional relationship. Encourage them to discuss with a partner if they prefer to compare the unit rates of two proportional relationships using words, equations, graphs, or tables. Some students might say equations easily show the unit rate as the slope m.

Exit Ticket

Refer to the Exit Ticket slide. Which park has the greater cost per vehicle? Write a mathematical argument that can be used to defend your solution.
Bryce Canyon National Park. The unit cost for Arches National Park is $25 per vehicle and the unit cost for Bryce Canyon National Park is $30 per vehicle.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–10
- ALEKS® Proportional Relationships, Slope

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–3, 4–10 even
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Proportions, Ratios, and Unit Rates

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Proportions, Ratios, and Unit Rates

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

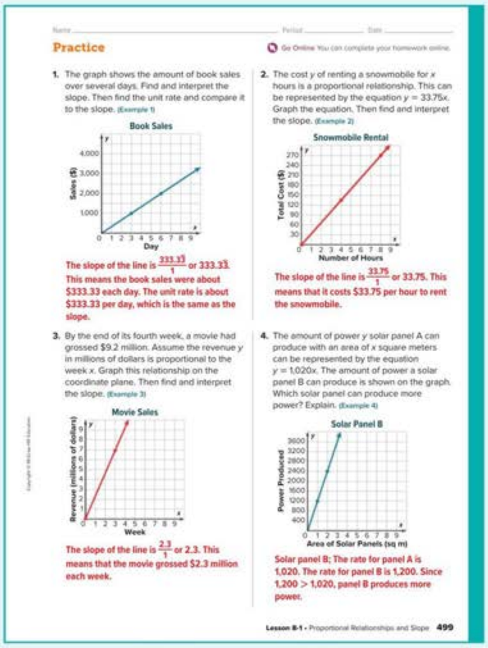
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find and interpret the slope of a graph of a proportional relationship and compare it to the unit rate	1
1	graph the equation of a proportional relationship and interpret the slope	2
1	graph verbal descriptions of proportional relationships and interpret the slope	3
2	compare two different proportional relationships represented in different ways	4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving proportional relationships and slope	6
3	higher-order and critical thinking skills	7–10

Common Misconception

Some students may incorrectly find and misinterpret the slope of a proportional relationship. Remind students that the slope of a proportional relationship is the unit rate.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The graph shows the amount of book sales over several days. Find and interpret the slope. Then find the unit rate and compare it to the slope. (Example 1)

The slope of the line is $\frac{333.3\bar{3}}{1}$ or $333.3\bar{3}$. This means the book sales were about $333.33 each day. The unit rate is about $333.33 per day, which is the same as the slope.

2. The cost y of renting a snowmobile for x hours is a proportional relationship. This can be represented by the equation $y = 33.75x$. Graph the equation. Then find and interpret the slope. (Example 2)

The slope of the line is $\frac{33.75}{1}$ or 33.75. This means that it costs $33.75 per hour to rent the snowmobile.

3. By the end of its fourth week, a movie had grossed $9.2 million. Assume the revenue y in millions of dollars is proportional to the week x. Graph this relationship on the coordinate plane. Then find and interpret the slope. (Example 3)

The slope of the line is $\frac{2.3}{1}$ or 2.3. This means that the movie grossed $2.3 million each week.

4. The amount of power y solar panel A can produce with an area of x square meters can be represented by the equation $y = 1{,}020x$. The amount of power a solar panel B can produce is shown on the graph. Which solar panel can produce more power? Explain. (Example 4)

Solar panel B; The rate for panel A is 1,020. The rate for panel B is 1,200. Since 1,200 > 1,020, panel B produces more power.

Lesson 8-1 • Proportional Relationships and Slope 499

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Test Practice

5. **Open Response** The distance y that Craig biked on x day trips is shown in the table. Rei biked 23.6 miles per day. Assume that the number of miles is proportional to the number of days. Who biked the lower number of miles each day? Explain. (Example 5)

Number of Day Trips	Distance (mi)
2	43.6
3	65.4
4	87.2

Craig; Sample answer: The unit rate of Craig's day trips is 21.8 miles per day. Since Rei biked 23.6 miles per day, and 23.6 > 21.8, Craig biked the lower number of miles each day.

Apply *indicates multi-step problem

*6. Nadia is comparing costs for two brands of garden compost. For Brand A, the cost y for x bags is shown in the table. For Brand B, the cost y can be represented by the equation $y = 1.99x$, where x represents the number of bags. Which brand costs less for 6 bags of compost? How much less?

Number of Bags, x	Total Cost ($), y
3	8.01
5	13.35

Brand B; $4.08 less

Higher-Order Thinking Problems

7. Explain how the unit rate and slope of a proportional relationship are related to each other.

Sample answer: The unit rate of a proportional relationship is the comparison of one quantity to one unit of another quantity. The slope of that relationship is that same comparison.

8. **Reason Abstractly** Determine if the statement is *true* or *false*. Justify your response.

The point (1, r) on the graph of a proportional relationship shows the unit rate r.

true; Sample answer: The unit rate of a proportional relationship is the comparison of one quantity to one unit of another quantity. Because the ordered pair is (1, r), the unit rate is $\frac{r}{1}$.

9. **Find the Error** A student graphs the relationship $y = x$ on a coordinate plane. She says the slope is 0 because there is no coefficient. Find her mistake and correct it.

Sample answer: The equation $y = x$ can be written as $y = 1x$ because $1 \cdot x = x$. So, the slope is 1, not 0.

10. **Identify Structure** The relationship $y = ax$ is graphed on a coordinate plane. If $a > b$, compare the graph of $y = bx$ to the graph of $y = ax$.

If $a > b$, then the slope of $y = ax$ is greater than the slope of $y = bx$. This means the line is steeper.

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Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 8, students determine if the statement is true or false. Encourage students to use what they know to reason whether the statement is true or false and provide an explanation for their answer.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students will find the mistake in the problem and correct it. Encourage students to determine the error and explain how they could fix it.

7 Look For and Make Use of Structure In Exercise 10, students compare the graphs of both equations. Encourage students to find similar structures between the graphs and use this to compare the graphs.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercise 6 After completing the application problem, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 9 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise incorrectly thinks that no coefficient on the variable x means that the slope is 0. Have each pair or group of students present their explanations to the class.

Lesson 8-2

Slope of a Line

Foundational for 8.EE.B.6

LESSON GOAL

Students will find the slope of a line from a graph, table, and using the formula.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Develop Concepts of Slope

Learn: Slope of a Line
Learn: Find Slope from a Graph
Examples 1-2: Find Slope from a Graph
Learn: Find Slope from a Table
Example 3: Find Slope from a Table
Learn: Find Slope Using the Slope Formula
Example 4: Find Slope Using the Slope Formula

Explore: Slope of Horizontal and Vertical Lines

Learn: Zero and Undefined Slope
Examples 5-6: Zero and Undefined Slope
Apply: Income

3 REFLECT AND PRACTICE

Exit Ticket

Practice

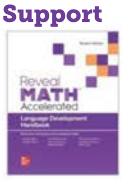
DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 47 of the *Language Development Handbook* to help your students build mathematical language related to the slope of a line.



ELL You can use the tips and suggestions on page T47 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.B** by finding the slope of a line from graphs, tables, and by using the formula.
Standards for Mathematical Content: **Founda tional for 8.EE.B.6**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students graphed and compared proportional relationships, interpreting the unit rate as the slope of the line. **8.EE.B.5**

Now
Students find the slope of a line from a graph, table, and using the formula. **Foundational for 8.EE.B.6**

Next
Students will relate the slope of a line to similar triangles. **8.EE.B.6**

Rigor

The Three Pillars of Rigor

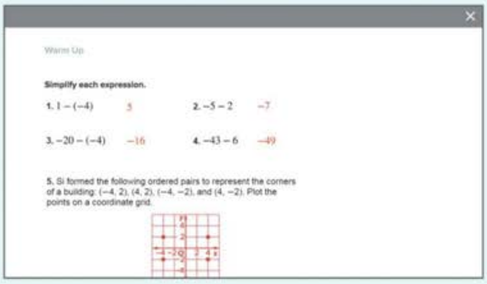
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to expand their *understanding* of slope. They come to understand that some lines have a slope equal to zero, some lines have an undefined slope, and that you can find the slope of a line from a graph, a table, and a formula.

Mathematical Background

Slope describes the steepness of a line by the ratio of rise (the change in *y*-values) to the run (the change in *x*-values). Slope can be positive or negative. Slope can be calculated from a graph or table by identifying two ordered pairs of the form (x, y) and writing the ratio of the change in *y*-values to the change in *x*-values as a fraction. The slope formula can also be used to calculate the slope. Based on this formula, it is easy to see that horizontal lines have a slope of 0 and vertical lines have an undefined slope.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- subtracting integers (Exercises 1–4)
- graphing on the coordinate plane (Exercise 5)

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the steepness of a roller coaster as related to rise and run.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- When you *rise* from a chair, does your body move primarily vertically or horizontally? vertically
- When you *run* on a track, does your body move primarily vertically or horizontally? horizontally

Explore Develop Concepts of Slope

Objective

Students will use Web Sketchpad to explore how horizontal and vertical steps are used to travel between points on a coordinate plane.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a starting point and a target and be asked to adjust vertical and horizontal step sizes in order to reach the target. Throughout this activity, students will use various starting locations and targets on the coordinate plane while adjusting horizontal and vertical step sizes to connect the idea of slope to the changes in vertical and horizontal components.

Inquiry Question

How can you demonstrate the concept of slope as you travel from one point to another on a coordinate plane? Sample answer: As you travel from one point to another on a coordinate plane, the ratio of the vertical change and horizontal change is the slope. The path between two points with the greatest number of vertical and horizontal steps is the slope in simplest form.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

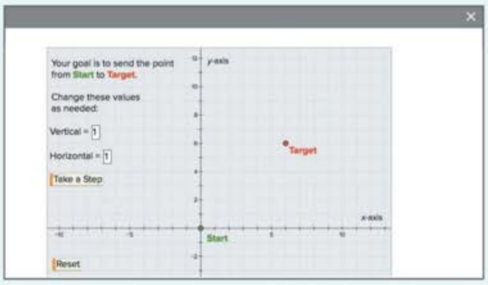
Mathematical Discourse

Which set or sets of values sent the point from Start to Target? Why do these values work? For each path, how many steps were taken? Sample answer: The Start is (0, 0) and the Target is (6, 6), so Vertical and Horizontal integer values of 2, 3, or 6 will work. For the path with vertical and horizontal integer values of 2, three steps were taken. For the path with vertical and horizontal integer values of 3, two steps were taken. For the path with vertical and horizontal integer values of 6, one step was taken.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8

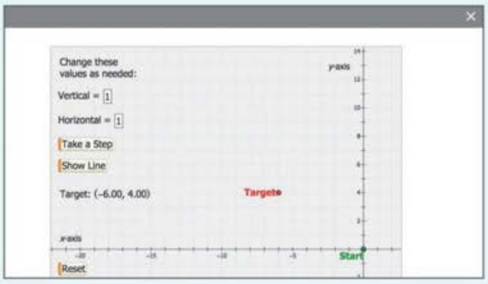


Explore, Slide 3 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how horizontal and vertical steps are used to travel between points on a coordinate plane.

Interactive Presentation



Explore, Slide 6 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Develop Concepts of Slope *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to consider how the vertical and horizontal step sizes each change the path from the starting point and explain their reasoning.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad as a tool to explore and examine how horizontal and vertical steps are used to travel between points on a coordinate plane.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

How did the Target being in Quadrant II affect your Vertical and Horizontal values? Sample answer: One of the values will be negative.

With the Start at (0, 0) and the Target at (−15, 5), how many paths did you find? For each path, how many steps were taken? Which path corresponds to the rate of change, in simplest form, between the points? Sample answer: two paths; For the path with a vertical value of 1 and a horizontal value of −3, five steps were taken. For the path with a vertical value of 5 and a horizontal value of −15, one step was taken; the path with the greatest number of steps.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Slope of a Line

Objective

Students will define slope as the ratio of the vertical change (rise) to the horizontal change (run) of a line.

Teaching Notes

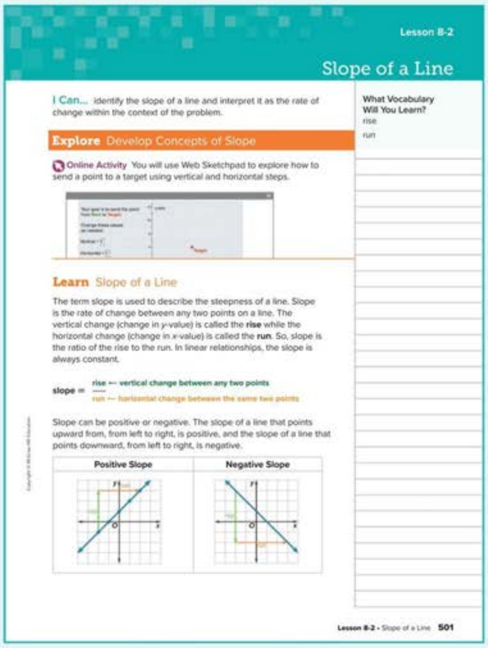
SLIDE 1

Point out that the slope of a line quantitatively describes the steepness of a line. In linear relationships, the slope is always constant. You may wish to ask students how they can use reasoning to explain why the slope of a line is always the same between any two points. Have students select the *Positive Slope* and *Negative Slope* flashcards to view an example of a line with a positive slope and one with a negative slope.

DIFFERENTIATE

Enrichment Activity

To further students' understanding of why the slope of a line is always the same between any two points, have them work with a partner to draw a line (either increasing or decreasing from left to right) using graph paper. Then have them pick three different pairs of points that fall on the line. Have them calculate the ratio of the rise to the run between each pair of points. They should see that the ratios are always the same. Have them explain why they can choose any two pairs of points that fall on a line in order to determine the slope of that line.



Lesson 8-2

Slope of a Line

I Can... identify the slope of a line and interpret it as the rate of change within the context of the problem.

What Vocabulary Will You Learn?
rise
run

Explore Develop Concepts of Slope

Online Activity You will use Web Sketchpad to explore how to send a point to a target using vertical and horizontal steps.

Learn Slope of a Line

The term slope is used to describe the steepness of a line. Slope is the rate of change between any two points on a line. The vertical change (change in *y*-value) is called the **rise** while the horizontal change (change in *x*-value) is called the **run**. So, slope is the ratio of the rise to the run. In linear relationships, the slope is always constant.

slope = rise ← vertical change between any two points / run ← horizontal change between the same two points

Slope can be positive or negative. The slope of a line that points upward from, from left to right, is positive, and the slope of a line that points downward, from left to right, is negative.

Positive Slope | Negative Slope

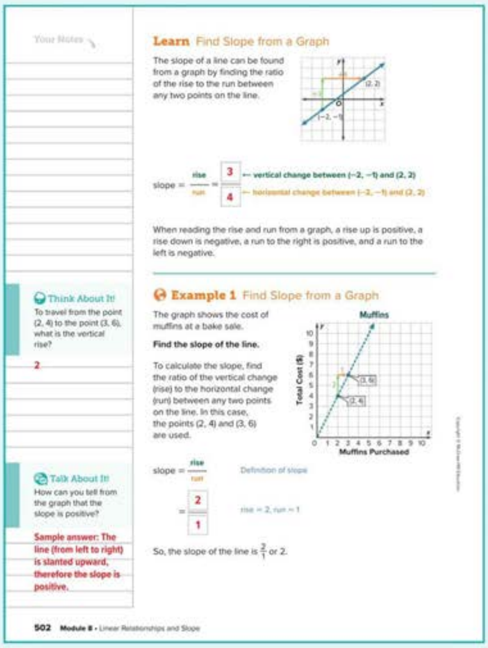
Lesson 8-2 • Slope of a Line 501

Interactive Presentation

Learn, Slope of a Line

FLASHCARDS

Students use Flashcards to see examples of positive and negative slope.



Your Notes

Learn Find Slope from a Graph

The slope of a line can be found from a graph by finding the ratio of the rise to the run between any two points on the line.

slope = $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ ← vertical change between (−2, −1) and (2, 2) ← horizontal change between (−2, −1) and (2, 2)

When reading the rise and run from a graph, a rise up is positive, a rise down is negative, a run to the right is positive, and a run to the left is negative.

Think About It! To travel from the point (2, 4) to the point (3, 6), what is the vertical rise?

2

Example 1 Find Slope from a Graph

The graph shows the cost of muffins at a bake sale.

Find the slope of the line.

To calculate the slope, find the ratio of the vertical change (rise) to the horizontal change (run) between any two points on the line. In this case, the points (2, 4) and (3, 6) are used.

slope = $\frac{\text{rise}}{\text{run}}$ Definition of slope

$= \frac{2}{1}$ rise = 2, run = 1

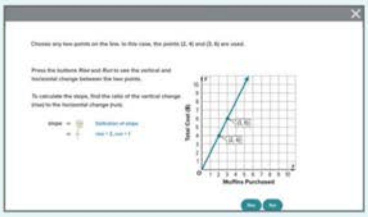
So, the slope of the line is $\frac{2}{1}$ or 2.

Talk About It! How can you tell from the graph that the slope is positive?

Sample answer: The line (from left to right) is slanted upward, therefore the slope is positive.

502 Module 8 • Linear Relationships and Slope

Interactive Presentation



Example 1, Find Slope from a Graph, Slide 2 of 4

CLICK

On Slide 2 of Example 1, students select the rise and run buttons to view the slope.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Find Slope from a Graph

Objective

Students will learn how to find the slope of a line from a graph.

Go Online to find additional teaching notes.

Example 1 Find Slope from a Graph

Objective

Students will find the positive slope of a line from a graph.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of what it means for the slope of a line to be positive or negative, and how positive and negative slopes are displayed visually on a graph.

6 Attend to Precision Encourage students to adhere to the definition of slope as the ratio of vertical to horizontal changes between two pairs of points.

Questions for Mathematical Discourse

SLIDE 2

AL What does the graph describe? the total cost based on the number of muffins purchased

OL What is the change in y-values between the two points? the change in x-values? The y-value increases by 2 and the x-value increases by 1.

OL Study the graph. Is this relationship proportional? Explain. If so, which point on the graph illustrates the slope? Yes, the relationship is proportional because the graph is a straight line that passes through the origin. The slope is illustrated by the point (1, 2), which is also the unit rate.

BL Find two other points that lie on this line. Sample answer: (4, 8) and (5, 10)

BL Describe the relationship between each x-value and its corresponding y-value using words and an equation. Sample answer: Each y-value is equal to twice its corresponding x-value; $y = 2x$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find Slope from a Graph

Objective

Students will find the negative slope of a line from a graph.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to pay attention to the fact that the line is decreasing from left to right, and that this indicates that the slope is negative. Students should adhere to the definition of slope as the ratio of the rise to the run between two points as they calculate the slope.

Questions for Mathematical Discourse

SLIDE 2

AL As you move from left to right on the graph, what is the vertical rise between the two indicated points? the horizontal run? The vertical rise is −4 units. The horizontal run is 6 units.

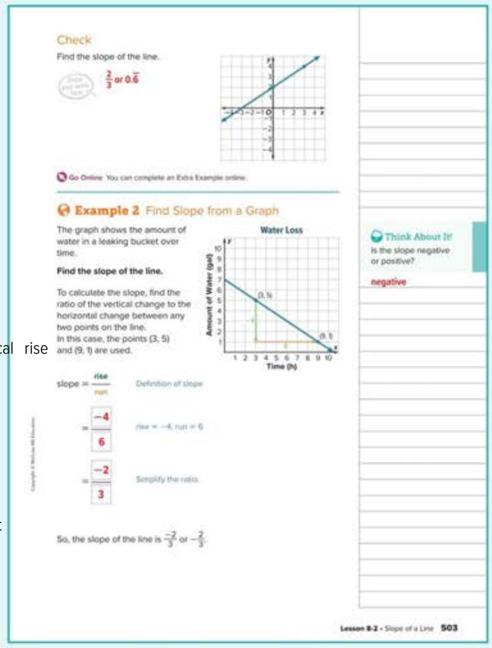
OL Why is the vertical rise negative? What does this mean? Sample answer: The line slopes downward from left to right. This means from the point (3, 5) to the point (9, 1), the line falls. So, the rise is negative.

OL Why does it make sense, given the context of the problem, that the slope is negative? Sample answer: The graph represents the amount of water remaining in the bucket. Since the bucket is leaking, the amount of water is decreasing.

BL Why isn't the slope $\frac{4}{6}$ or $\frac{2}{3}$? Sample answer: The slope is decreasing, so it must be negative, not positive.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Check

Find the slope of the line.

$\frac{2}{3}$ or $0.\overline{6}$

Go Online You can complete an Extra Example online.

Example 2 Find Slope from a Graph

The graph shows the amount of water in a leaking bucket over time.

Find the slope of the line.

To calculate the slope, find the ratio of the vertical change to the horizontal change between any two points on the line. In this case, the points (3, 5) and (9, 1) are used.

$\text{slope} = \frac{\text{rise}}{\text{run}}$ Definition of slope

$= \frac{-4}{6}$ rise = −4, run = 6

$= \frac{-2}{3}$ Simplify the ratio.

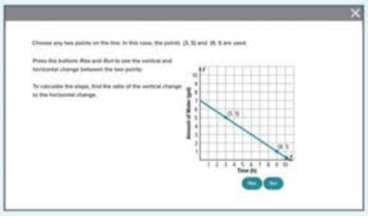
So, the slope of the line is $\frac{-2}{3}$ or $-\frac{2}{3}$.

Think About It! Is the slope negative or positive?

negative

Lesson 8-2 • Slope of a Line 503

Interactive Presentation



Example 2, Find Slope from a Graph, Slide 2 of 3

TYPE

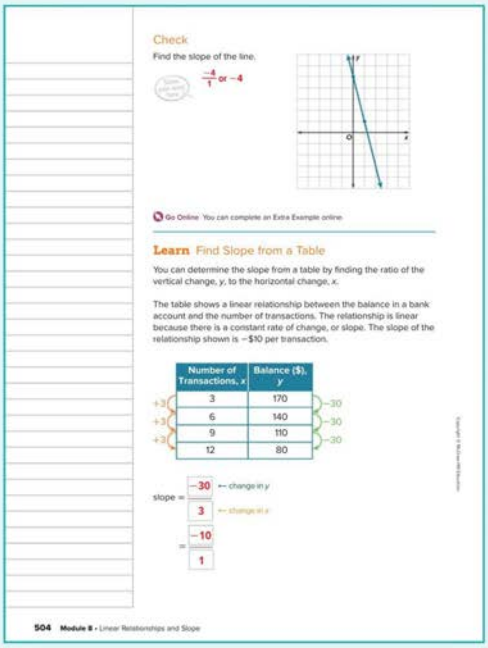
On Slide 2, students determine the simplified ratio.

CLICK

On Slide 2, students select the rise and run buttons to see the vertical and horizontal change between the two points.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Find the slope of the line.

$\frac{-4}{1}$ or -4

Go Online You can complete an Extra Example online.

Learn Find Slope from a Table

You can determine the slope from a table by finding the ratio of the vertical change, y, to the horizontal change, x.

The table shows a linear relationship between the balance in a bank account and the number of transactions. The relationship is linear because there is a constant rate of change, or slope. The slope of the relationship shown is –$10 per transaction.

Number of Transactions, x	Balance ($), y
3	170
6	140
9	110
12	80

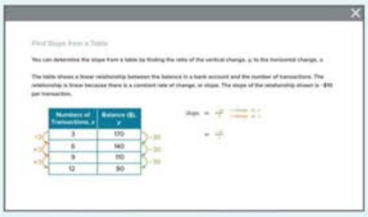
+3, +3, +3; −30, −30, −30

slope = $\frac{-30}{3}$ ← change in y / ← change in x

= $\frac{-10}{1}$

504 Module 8 • Linear Relationships and Slope

Interactive Presentation



Learn, Find Slope from a Table

Foundational for 8.EE.B.6

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Find Slope from a T able

Objective

Students will learn how to find the slope of a line from a table.

Teaching Notes

SLIDE 1

Encourage students to make sense of the equivalent relationship between the constant rate of change, as shown in the table, and the slope. Point out that since the table shows a constant rate of change, the relationship is linear. Ask students what the graph of this relationship will look like. They should note that the graph of any linear relationship is a straight line.

You may wish to ask students if they think the relationship is also proportional. Students should note that the relationship is not proportional because the slope, –10, is not the same as the unit rate. When the number of transactions is 1, the balance is not –$10 dollars. Instead, the balance is $150.

Example 3 Find Slope from a T able

Objective

Students will find the slope of a line from a table.

Teaching the Mathematical Practices

6 Attend to Precision Enco urage students to adhere to the process for finding the slope from a table of values which involves finding the simplified ratio of the vertical change, *y*, to the horizontal change, *x*. Students should carefully attend to the subtraction of any negative coordinates as they calculate the slope.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the pattern of *y*- values in the table? What might this tell you about the slope of the line that passes through these points? Sample answer: They *y*-values are increasing. This tells me that the slope of the line is positive.

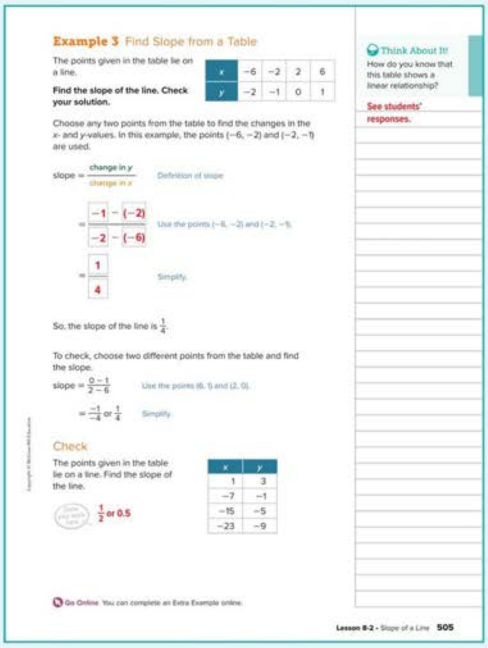
OL Can you use any pair of points from the table to calculate the slope? Why or why not? Y es; any pair of points can be used to calculate the slope, because the slope of a line is always the same through any two points on that line.

OL Suppose the table of values was graphed in the coordinate plane. Use your own words to describe the steepness of the slope of the line. Sample answer: The slope $\frac{1}{4}$ is less than 1 and close to 0, so it is not very steep.

BL Think about the graph that represents this relationship. Will the graph pass through all four quadrants? Explain. no; Sample answer: The graph passes through Quadrants III, IV, and I, but it will not pass through Quadrant II because when *x*-values are negative, the *y*-values are negative.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Find Slope from a Table

The points given in the table lie on a line.

Find the slope of the line. Check your solution.

x	−6	−2	2	6
y	−2	−1	0	1

Choose any two points from the table to find the changes in the *x*- and *y*-values. In this example, the points (−6, −2) and (−2, −1) are used.

$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$ Definition of slope

$= \frac{-1 - (-2)}{-2 - (-6)}$ Use the points (−6, −2) and (−2, −1).

$= \frac{1}{4}$ Simplify.

So, the slope of the line is $\frac{1}{4}$.

To check, choose two different points from the table and find the slope.

$\text{slope} = \frac{0 - 1}{2 - 6}$ Use the points (6, 1) and (2, 0).

$= \frac{-1}{-4}$ or $\frac{1}{4}$ Simplify.

Check

The points given in the table lie on a line. Find the slope of the line.

x	y
1	3
−7	−1
−15	−5
−23	−9

$\frac{1}{2}$ or 0.5

Go Online You can complete an Extra Example online.

Think About It! How do you know that this table shows a linear relationship?

See students' responses.

Lesson 8-2 • Slope of a Line 505

Interactive Presentation

Example 3, Find Slope from a Table, Slide 2 of 3

CLICK

On Slide 2, students move through the steps to find the slope.

TYPE

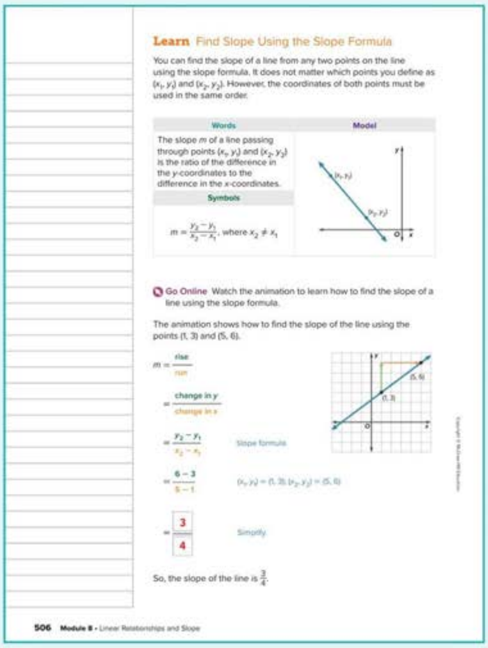
On Slide 2, students determine the slope of the line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Foundational for 8.EE.B.6

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Learn Find Slope Using the Slope Formula

You can find the slope of a line from any two points on the line using the slope formula. It does not matter which points you define as (x_1, y_1) and (x_2, y_2). However, the coordinates of both points must be used in the same order.

Words

The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the difference in the x-coordinates.

Symbols

$m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$

Model

Go Online Watch the animation to learn how to find the slope of a line using the slope formula.

The animation shows how to find the slope of the line using the points (1, 3) and (5, 6).

$m = \frac{\text{rise}}{\text{run}}$

$= \frac{\text{change in } y}{\text{change in } x}$

$= \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{6 - 3}{5 - 1}$ $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (5, 6)$

$= \frac{3}{4}$ Simplify

So, the slope of the line is $\frac{3}{4}$.

506 Module 8 • Linear Relationships and Slope

Learn Find Slope Using the Slope Formula

Objective

Students will learn how to find slope of a line from two points on that line, by using the slope formula.

Teaching Notes

SLIDE 1

Point out that the slope formula is a symbolic representation of writing the ratio of the rise to the run. Some students may struggle with remembering which coordinates to use in the numerator of the ratio, and which coordinates to use in the denominator. If they can make sense of the slope formula as this symbolic representation of the ratio of the rise to the run, they can remember that the rise represents the change in y-coordinates. So, they should use the y-coordinates in the numerator of the ratio.

Have students select the *Words*, *Symbols*, and *Model* flashcards to view how the slope formula can be expressed in these multiple representations.

Go Online Have students watch the animation on Slide 2. The animation illustrates finding the slope of a line using the slope formula.

SLIDE 2

Play the animation for the class. You may wish to pause the animation when the two points on the line (1, 3) and (5, 6) are shown, and have students work with a partner to find the slope using the slope formula. Some students may find the ratio $\frac{6-3}{5-1}$, w hile other students find the ratio $\frac{3-6}{1-5}$. Point out that, as long as they subtract the x- and y-coordinates in the same order, the slope will be the same.

Interactive Presentation

Learn, Find Slope Using the Slope Formula, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of the slope formula.

WATCH

On Slide 2, students watch an animation that illustrates how to find the slope of a line using the slope formula.

DIFFERENTIATE

Reteaching Activity AL

If students are struggling with remembering or correctly using the slope formula to find the slope of a line, have them create a graphic organizer that illustrates these other ways they have learned to find the slope in this lesson. Encourage them to provide an example for each.

- Find the ratio of $\frac{\text{rise}}{\text{run}}$ from the graph of a line.
- Find the constant rate of change $\frac{\text{change in } y}{\text{change in } x}$ from the table of a linear relationship.

Then have them discuss with a partner how the slope formula represents the same ratio, using the x- and y-coordinates of two points that fall on the line. Understanding how the slope formula represents the same $\frac{\text{rise}}{\text{run}}$ and $\frac{\text{change in } y}{\text{change in } x}$ ratios will help them remember it and use it correctly in the future.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 4 Find Slope Using the Slope Formula

Objective

Students will find the slope of a line from two points on the line, by using the slope formula.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to construct a viable argument that demonstrates a possible error in calculating the slope.

6 Attend to Precision Encourage students to adhere to the slope formula and pay attention to the order in which they subtract the x- and y-values as they calculate the slope.

Questions for Mathematical Discourse

SLIDE 2

AL Will the slope be negative or positive? How do you know? negative; The line slopes downward from left to right.

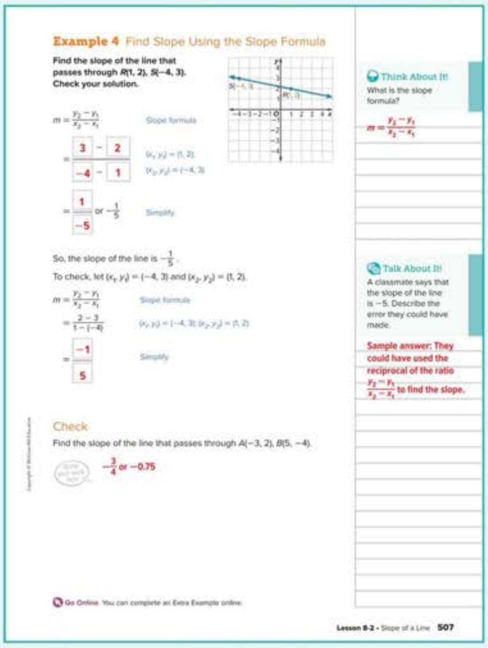
OL Why are the y-values in the numerator of the slope formula? Sample answer: Slope represents rise over run. The change in the y-values represents the rise.

OL Describe in your own words the steepness of the line. Sample answer: The line is not very steep.

BL Describe how the steepness of a line with a slope of $-\frac{1}{5}$ compares to a line with a slope of $\frac{1}{5}$. What is the significance of the negative sign? Sample answer: This line has the same steepness as a line with a slope of $\frac{1}{5}$, just in the other direction. The negative sign indicates that the slope falls from left to right, as opposed to rises.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Find Slope Using the Slope Formula

Find the slope of the line that passes through $R(1, 2)$, $S(-4, 3)$. Check your solution.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{3 - 2}{-4 - 1}$ $(x_1, y_1) = (1, 2)$; $(x_2, y_2) = (-4, 3)$

$= \frac{1}{-5}$ or $-\frac{1}{5}$ Simplify.

So, the slope of the line is $-\frac{1}{5}$.

To check, let $(x_1, y_1) = (-4, 3)$ and $(x_2, y_2) = (1, 2)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{2 - 3}{1 - (-4)}$ $(x_1, y_1) = (-4, 3)$; $(x_2, y_2) = (1, 2)$

$= \frac{-1}{5}$ Simplify.

Think About It! What is the slope formula? $m = \frac{y_2 - y_1}{x_2 - x_1}$

Talk About It! A classmate says that the slope of the line is −5. Describe the error they could have made. **Sample answer: They could have used the reciprocal of the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope.**

Check

Find the slope of the line that passes through $A(-3, 2)$, $B(5, -4)$.

$-\frac{3}{4}$ or -0.75

Go Online You can complete an Extra Example online.

Lesson 8-2 • Slope of a Line 507

Interactive Presentation

Example 4, Find Slope Using the Slope Formula, Slide 2 of 4

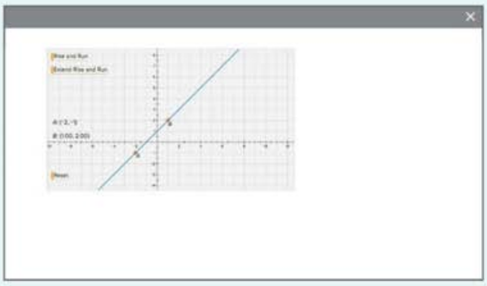
CLICK On Slide 2, students move through the steps to find the slope between the two points.

TYPE On Slide 2, students determine the slope of the line.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 2 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the slopes of horizontal and vertical lines.

Explore Slope of Horizontal and Vertical Lines

Objective

Students will use Web Sketchpad to explore the slopes of horizontal and vertical lines.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a series of lines whose slopes approach zero or infinity. Throughout this activity, students will view the patterns of slopes as they approach zero or infinity and make conjectures as to the slopes of all vertical and horizontal lines.

Inquiry Question

How can you determine the slope of a horizontal or a vertical line?
Sample answer: I can graph the points and determine the $\frac{\text{rise}}{\text{run}}$. For horizontal lines, there is no rise, so the rise is zero. This means the slope of a horizontal line is $\frac{0}{\text{run}}$, or 0. For vertical lines, there is no run, so the run is zero. This means the slope of a vertical line is $\frac{\text{rise}}{0}$, which is undefined.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

What is the slope of the horizontal line? zero

Why do you think this is the case? Sample answer: The slope is zero because the rise equals zero. If I find the ratio of rise to run when the rise is 0, then the ratio simplifies to 0.

(continued on next page)

Explore Slope of Horizontal and Vertical Lines *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore the patterns of slopes for horizontal and vertical lines.

8 Look For and Express Regularity in Repeated Reasoning Encourage students to notice the pattern in the slopes as each line is changed.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

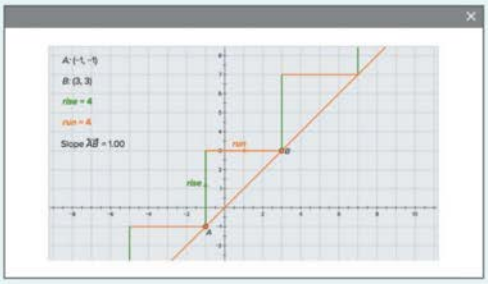
SLIDE 5

Mathematical Discourse

What is the slope of the vertical line? undefined

Why do you think this is the case? Sample answer: The slope is undefined because the run is 0. If I find the ratio of rise to run when the run is 0, then the ratio is undefined. I cannot divide by zero.

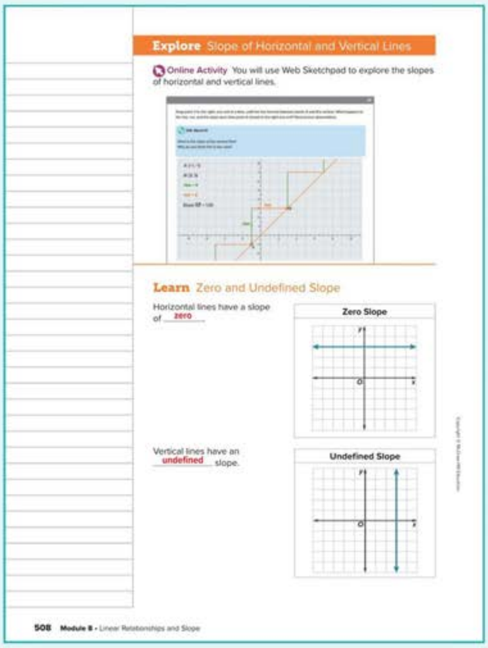
Interactive Presentation



Explore, Slide 5 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.



Interactive Presentation

Learn, Zero and Undefined Slope

FLASHCARDS

Students use Flashcards to view examples of graphs of lines with zero and undefined slopes.

Learn Zero and Undefined Slope

Objective

Students will understand that the slope of a horizontal line is zero and the slope of a vertical line is undefined.

Teaching Notes

SLIDE 1

Have students select the *Zero Slope* and *Undefined Slope* flashcards to illustrate that horizontal lines have slopes of zero, and vertical lines have undefined slopes. Ask students to discuss with a partner why it makes sense that a horizontal line has a slope of zero. Some students may say that since slope describes the steepness of a line, a horizontal line does not have any steepness. Other students may say that there is no change in *y*, even though the *x*-coordinates change between two points on the line. Other students may say there is no rise in the ratio of rise to run.

Ask students to discuss with a partner why it makes sense that a vertical line has an undefined slope. Some students may say that it is difficult or impossible to describe the steepness of a vertical line, or that it is infinitely steep. Other students may say that there is no change in *x*, even though the *y*-coordinates change between two points on the line. Other students may say there is no run in the ratio of rise to run.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 5 Zero Slope

Objective

Students will find the slope of a horizontal line by using the slope formula.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively
Encourage students to make sense of what it means for a horizontal line to have a slope of zero. They should pause and attend to the meaning of what slope represents and not just merely perform the calculations.

As students discuss the *Talk About It!* questions on Slide 3, encourage them to use reasoning to explain why the slope of this line is zero, and how they could use the graph to determine this without using the slope formula.

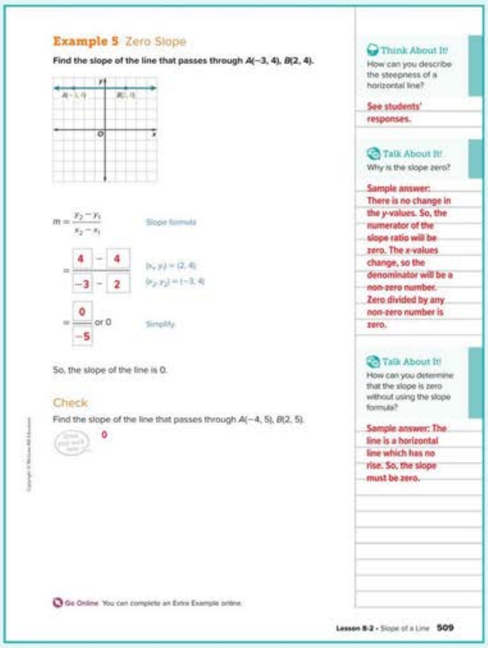
Questions for Mathematical Discourse

SLIDE 2

- **AL** What do you notice about the coordinates of the two points? Sample answer: The y-values are the same. They are both equal to 4.
- **OL** What is the change in y-values? What does this mean about the ratio of rise to run? 0; This means that there is no rise.
- **OL** What is the numerator when calculating the slope? How does this compare to the rise? The numerator is 0, which corresponds to the fact that the line does not rise.
- **BL** Does it matter what the denominator is, if the numerator is 0? Explain. Sample answer: If the numerator is 0, the denominator can be any value (except 0) and the slope will still be 0.
- **BL** Generate another point that lies on this horizontal line. Sample answer: (3, 4)

Go Online

- Find additional teaching notes and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Example 5 Zero Slope

Find the slope of the line that passes through $A(-3, 4)$, $B(2, 4)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{4 - 4}{-3 - 2}$ $(x_1, y_1) = (2, 4)$; $(x_2, y_2) = (-3, 4)$

$= \frac{0}{-5}$ or 0 Simplify.

So, the slope of the line is 0.

Check

Find the slope of the line that passes through $A(-4, 5)$, $B(2, 5)$. 0

Go Online You can complete an Extra Example online.

Think About It! How can you describe the steepness of a horizontal line? See students' responses.

Talk About It! Why is the slope zero? Sample answer: There is no change in the y-values. So, the numerator of the slope ratio will be zero. The x-values change, so the denominator will be a non-zero number. Zero divided by any non-zero number is zero.

Talk About It! How can you determine that the slope is zero without using the slope formula? Sample answer: The line is a horizontal line which has no rise. So, the slope must be zero.

Lesson 8-2 • Slope of a Line 509

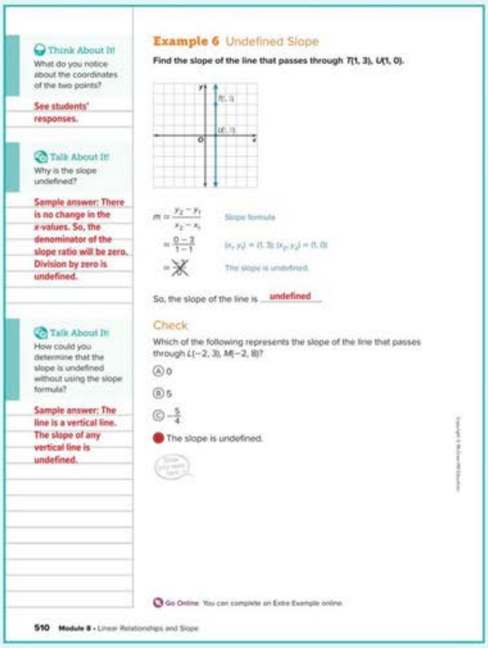
Interactive Presentation

Example 5, Zero Slope, Slide 2 of 4

CLICK On Slide 2, students move through the steps used to find the slope between the two points.

TYPE On Slide 2, students determine the slope.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It! What do you notice about the coordinates of the two points?

See students' responses.

Talk About It! Why is the slope undefined?

Sample answer: There is no change in the x-values. So, the denominator of the slope ratio will be zero. Division by zero is undefined.

Talk About It! How could you determine that the slope is undefined without using the slope formula?

Sample answer: The line is a vertical line. The slope of any vertical line is undefined.

Example 6 Undefined Slope

Find the slope of the line that passes through $T(1, 3)$, $U(1, 0)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{0 - 3}{1 - 1}$ $(x_1, y_1) = (1, 3); (x_2, y_2) = (1, 0)$

The slope is undefined.

So, the slope of the line is undefined.

Check

Which of the following represents the slope of the line that passes through $L(-2, 3)$, $M(-2, 8)$?

Ⓐ 0

Ⓑ 5

Ⓒ $-\frac{5}{4}$

● The slope is undefined.

Go Online You can complete an Extra Example online.

510 Module 8 • Linear Relationships and Slope

Interactive Presentation

Example 6, Undefined Slope, Slide 2 of 4

CLICK

On Slide 2, students move through the steps used to find the slope between the two points.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 6 Undefined Slope

Objective

Students will show that the slope of a vertical line is undefined by using the slope formula.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively

Encourage students to make sense of what it means for a vertical line to have an undefined slope. They should pause and attend to the meaning of what slope represents and not just merely perform the calculations.

As students discuss the *Talk About It!* questions on Slide 3, encourage them to use reasoning to explain why the slope of this line is undefined, and how they could use the graph to determine this without using the slope formula.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the coordinates of the two points? Sample answer: The *x*-values are the same. They are both equal to 1.

OL What is the change in *x*-values? What does this mean about the ratio of rise to run? 0; This means that there is no run.

OL What is the denominator when calculating the slope? What does it mean for the denominator of a fraction to be zero? The denominator is 0. Division by zero is undefined. So, the slope is undefined.

BL Generate another point that lies on this vertical line. Sample answer: (1, 6)

BL What must be true about every point that lies on this vertical line? Sample answer: The *x*-coordinate must be 1.

Go Online

- Find additional teaching notes and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Income

Objective

Students will come up with their own strategy to solve an application problem that involves comparing income.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

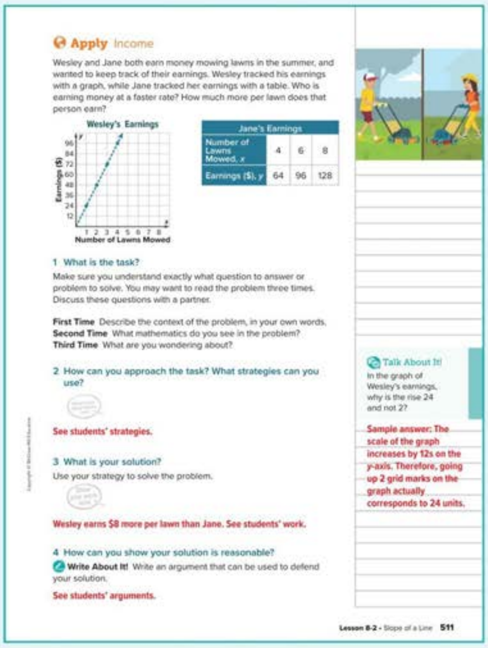
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you determine the rate shown in the graph?
- How can you use the values in the table to find the rate for Jane?
- What representation might make it easiest for you to compare the rates?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Income

Wesley and Jane both earn money mowing lawns in the summer, and wanted to keep track of their earnings. Wesley tracked his earnings with a graph, while Jane tracked her earnings with a table. Who is earning money at a faster rate? How much more per lawn does that person earn?

Jane's Earnings			
Number of Lawns Mowed, x	4	6	8
Earnings ($), y	64	96	128

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Wesley earns $8 more per lawn than Jane. See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
In the graph of Wesley's earnings, why is the rise 24 and not 2?

Sample answer: The scale of the graph increases by 12s on the y-axis. Therefore, going up 2 grid marks on the graph actually corresponds to 24 units.

Lesson 8-2 • Slope of a Line 511

Interactive Presentation

Apply, Income

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Lisa and Greg were comparing the gas mileage on their cars. Lisa recorded her gas mileage in a table, while Greg recorded his gas mileage on a graph. Assume the points lie on a line. Whose car gets better gas mileage?

Gas (gal), x	4	8	12	16
Distance (mi), y	108	216	324	432

Lisa's car

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare and contrast each of the methods used to find slope: from a graph, from a table, using the slope formula.

See students' observations.

512 Module 8 • Linear Relationships and Slope

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. A ride at an amusement park rises 8 feet for every horizontal change of 2 feet. What is the slope of the ride? Describe how you found it. 4; Sample answer: Slope is the ratio of rise to run. The rise is 8 feet and the run is 2 feet. Simplify the ratio 8 to 2, which is 4.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7–12
- Extension: Slope of Perpendicular Lines
- ALEKS Slope

IF students score 66–89% on the Checks. **THEN** assign: OL

- Practice, Exercises 1–6, 8, 9, 11
- Extension: Slope of Perpendicular Lines
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS Ratios and Unit Rates

IF students score 65% or below on the Checks. **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Ratios and Unit Rates

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- AL Practice Form B
- OL Practice Form A
- BL Practice Form C

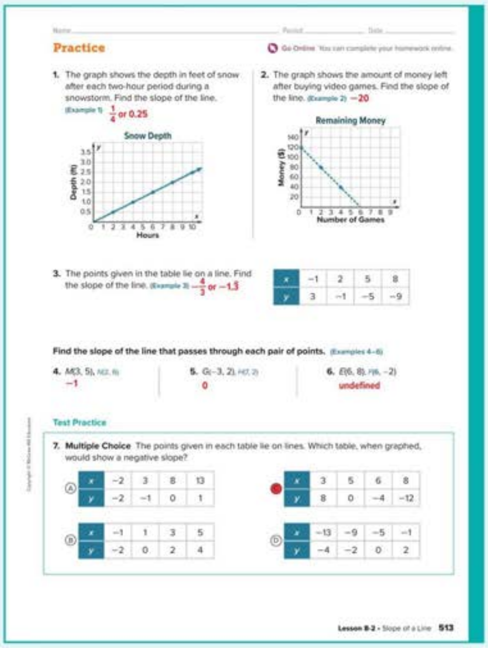
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the positive slope of a line from a graph	1
1	find the negative slope of a line from a graph	2
1	find the slope of a line from a table	3
1	find the slope of a line from two points on the line, by using the slope formula	4
1	find the slope of a horizontal line by using the slope formulas	5
1	show the slope of a vertical line is undefined by using the slope formula	6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems involving slope of a line	8
3	higher-order and critical thinking skills	9–12

Common Misconception

Some students may incorrectly calculate the slope because they either find the ratio of the change in x-values to the change in y-values, or because they do not consistently subtract the coordinates in the same order. In Exercise 4, students may incorrectly calculate the slope as 1 by finding 6 – 5 first and then finding 3 – 2 before writing the ratio. Remind them that if they subtract the *first* y-coordinate from the *second* y-coordinate to find the rise, then they should subtract the *first* x-coordinate from the *second* x-coordinate (not the *second* from the *first*) in order to find the run.



Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

1. The graph shows the depth in feet of snow after each two-hour period during a snowstorm. Find the slope of the line. (Example 1) $\frac{1}{4}$ or 0.25

2. The graph shows the amount of money left after buying video games. Find the slope of the line. (Example 2) −20

3. The points given in the table lie on a line. Find the slope of the line. (Example 3) $-\frac{4}{3}$ or $-1.\overline{3}$

x	−1	2	5	8
y	3	−1	−5	−9

Find the slope of the line that passes through each pair of points. (Examples 4–6)

4. $M(3, 5), N(2, 6)$ −1
5. $G(-3, 2), H(7, 2)$ 0
6. $E(6, 8), F(6, -2)$ undefined

Test Practice

7. **Multiple Choice** The points given in each table lie on lines. Which table, when graphed, would show a negative slope?

Ⓐ

x	−2	3	8	13
y	−2	−1	0	1

Ⓑ

x	−1	1	3	5
y	−2	0	2	4

● (C)

x	3	5	6	8
y	8	0	−4	−12

Ⓓ

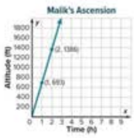
x	−13	−9	−5	−1
y	−4	−2	0	2

Lesson 8-2 • Slope of a Line 513

Apply *indicates multi-step problem

*8. Malik and Mila are both mountain climbing. The graph shows Malik's altitude at various points during the climb. The table shows Mila's altitude. Who is climbing at a faster rate? How much faster does that person climb per hour?

Mila's Ascension	
Time (h), x	Altitude (ft), y
0	0
1	721
2	1,442



Mila is climbing faster. She climbs 28 ft/h faster than Malik.

Higher-Order Thinking Problems

9. Give three points that lie on a line with a slope of $-\frac{2}{5}$.

Sample answers: (0, 0), (5, −2), (10, −4)

10. Explain why, when using the slope formula, it does not matter which point is (x_1, y_1) or (x_2, y_2).

Sample answer: When subtracting the y-coordinates and the x-coordinates, it does not matter which is (x_1, y_1) or (x_2, y_2) because both fractions will simplify to the same slope.

11. **Find the Error** A student finds the slope of the line that passes through the points (−3, 8) and (2, −4). Find the mistake and correct it.

$$m = \frac{-3-8}{2-(-4)}$$
$$m = \frac{-11}{6}$$

Sample answer: The student did not subtract the y-coordinates and x-coordinates. The numerator should be $-4-8$ and the denominator should be $2-(-3)$. The slope is $\frac{-12}{5}$.

12. **Identify Structure** Without computing, how can you tell by looking at the ordered pairs that a line will be horizontal or vertical?

Sample answer: If the x-coordinates are the same, the line is vertical. If the y-coordinates are the same, the line is horizontal.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students will find the mistake in the problem and correct it. Encourage students to determine the error and explain how they could fix it.

7 Look For and Make Use of Structure In Exercise 12, students will explain how they can tell a line will be horizontal or vertical by looking at ordered pairs. Encourage students to use the similar structure in ordered pairs and their corresponding slope to support their answer.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercise 8 Have pairs of students interview each other as they complete this application problem. Students take turns being the interviewer and interviewee. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 8 might be, “How is the rate of change represented for each person?”

Listen and ask clarifying questions.
Use with Exercises 9–10 Have students work in pairs. Have students individually read Exercise 9 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 10.

Lesson 8-3

Similar Triangles and Slope

8.EE.B.6

LESSON GOAL

Students will relate the slope of a line to similar triangles.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Right Triangles and Slope

Learn: Similar Triangles
Learn: Similar Triangles and Slope
Example 1: Compare Slopes of Similar Triangles
Example 2: Verify Slopes Using Slope Triangles

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 48 of the *Language Development Handbook* to help your students build mathematical language related to similar triangles and slope.

ELL You can use the tips and suggestions on page T48 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.B** by relating the slope of a line to similar triangles.
Standards for Mathematical Content: **8.E E.B.6**
Standards for Mathematical Practice: **MP 2, MP3, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students found the slope of a line from a graph, table, and using the formula.
Foundational for 8.EE.B.6

Now
Students relate the slope of a line to similar triangles.
8.EE.B.6

Next
Students will derive the equation $y = mx$ from graphs, tables, and verbal descriptions of proportional relationships.
8.EE.B.6

Rigor

The Three Pillars of Rigor

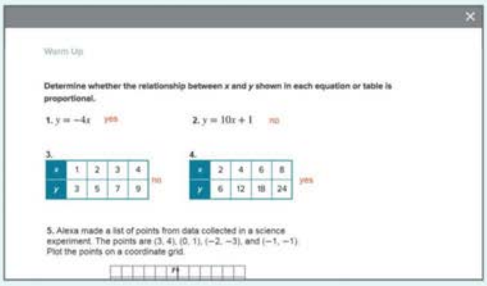
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop their *understanding* of slope. They learn that right triangles with hypotenuses on the same nonvertical line, or slope triangles, are similar. Since the ratio of the rise to the run for each triangle is the same, the slope is the same between any two points on the line.

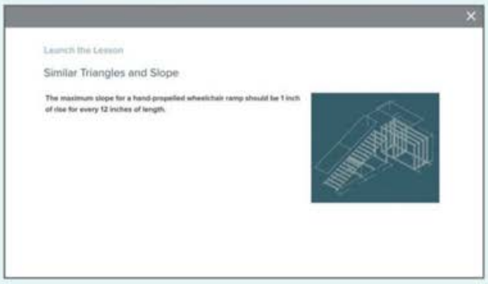
Mathematical Background

Figures with the same shape but not necessarily the same size are called *similar figures*. The angles and sides in the same relative positions of similar figures are called *corresponding parts*. One specific type of similar triangles is *slope triangles*. Slope triangles have hypotenuses that fall on the same line. Since these triangles are similar, each of them can be used to calculate the slope on which they lie. To calculate the slope of any line using slope triangles, find the ratio of the vertical side of one of the triangles to the horizontal side of that triangle, resulting in the ratio of rise to run.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding proportional relationships (Exercises 1–4)
- graphing on the coordinate plane (Exercise 5)

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the slope of a wheelchair ramp.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- In everyday life, what does it mean if you *correspond* with someone? What does it mean if two items or objects *correspond* to each other? Sample answer: If you correspond with someone, it means to communicate with them somehow. Items or objects that correspond to each other mean that they are related in some way, or have something in common.
- In everyday life, what does it mean for two objects to be *similar*? Sample answer: If two objects are similar, it means they are alike or resemble one another in some way.
- Picture a right triangle with the right angle at the bottom left. What part of a right triangle has a *slope* that is neither zero nor undefined? The longest side has a slope that is neither zero nor undefined.

Explore Right T riangles and Slope

Objective

Students will use Web Sketchpad to explore why the slope of a line is the same between any two points on the line.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a right triangle representing a pattern for a skateboard ramp. Throughout this Explore, students will extend the right triangle pattern to ramps of larger sizes with the same slope. They will compare the side lengths of the triangles to verify that the slopes of the extended ramps are equal to the original slope.

Inquiry Question

How does the slope compare between any two pairs of points on a line? Sample answer: The slope is the same between any two pairs of points on a line. Even though the values for the rise and the run change, the ratio between them remains the same.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

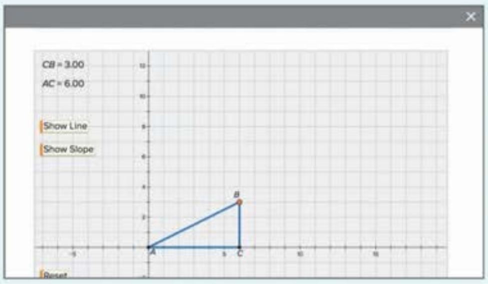
Press *Show Slope* to see the slope of line *AB*. How does the slope compare to the slope you found? Sample answer: Since 0.5 and $\frac{1}{2}$ are equivalent, the slope is the same as the one found.

As you drag point *B*, what happens to the slope? How does this compare to your earlier prediction? The slope is always 0.5 or $\frac{1}{2}$. See students' responses.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

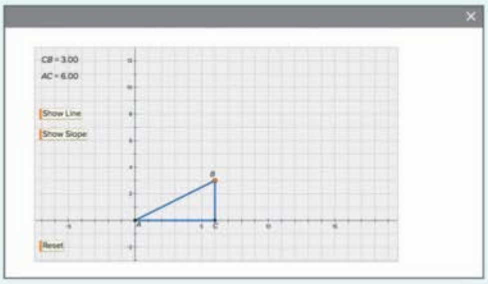


Explore, Slide 4 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore why the slope of a line is the same between any two points on the line.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Right Triangles and Slope *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore and examine how the slope is affected when the side lengths of the ramp change.

8 Look For and Express Regularity in Repeated Reasoning Encourage students to use the sketch to discover a pattern among the slopes as a constant ratio for any pair of side lengths.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Without using the sketch, how can you verify that the slopes of the ramps you found are all the same, 0.5? Sample answer: I can find the ratio of $\frac{\text{rise}}{\text{run}}$ for each of the three right triangles (ramps). Since the ratios all simplify to $\frac{1}{2}$, the slopes are all the same.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Similar Triangles

Objective

Students will understand the relationship between corresponding angles and sides of similar figures.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of the relationship between the side lengths of the similar triangles.

Teaching Notes

SLIDE 1

Students will learn that *similar figures* have the same shape but not necessarily the same size. When two figures are similar, their corresponding angles are congruent and their corresponding sides are proportional. Have students select the *Words*, *Symbols*, and *Model* flashcards to learn how similar triangles can be represented in multiple ways. Have students study the diagram on the back of the *Model* flashcard. The tic marks indicate the pairs of congruent angles. Ask students how they can use the side lengths to verify that the corresponding sides are proportional.

Talk About It!

SLIDE 2

Mathematical Discourse

The two triangles shown are similar. How do the side lengths of the smaller triangle compare to the side lengths of the larger triangle? Sample answer: The sides of the smaller triangle are half the length of the corresponding sides of the larger triangle, which means that the ratios of the corresponding sides are the same, $\frac{1}{2}$.

DIFFERENTIATE

Reteaching Activity AL

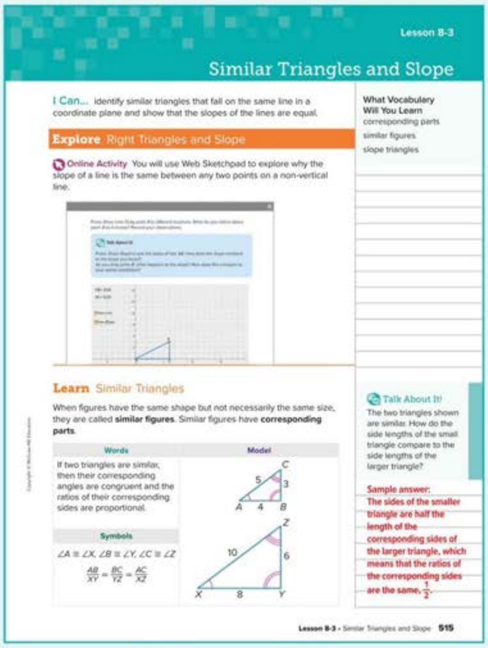
If any of your students are having difficulty understanding that the side lengths of similar figures are proportional, have them work with a partner to recreate the two triangles presented in the Learn by drawing them on graph paper. Then have students find the following ratios. Have them describe what they notice, and what this means.

The ratios of corresponding side lengths are equivalent. This means the side lengths are proportional.

$\frac{AB}{XY}$ $\frac{4}{8} = \frac{1}{2}$

$\frac{BC}{YZ}$ $\frac{3}{6} = \frac{1}{2}$

$\frac{AC}{XZ}$ $\frac{5}{10} = \frac{1}{2}$


Lesson 8-3

Similar Triangles and Slope

I Can... identify similar triangles that fall on the same line in a coordinate plane and show that the slopes of the lines are equal.

What Vocabulary Will You Learn
corresponding parts
similar figures
slope triangles

Explore Right Triangles and Slope

Online Activity You will use Web Sketchpad to explore why the slope of a line is the same between any two points on a non-vertical line.

Learn Similar Triangles

When figures have the same shape but not necessarily the same size, they are called **similar figures**. Similar figures have **corresponding parts**.

Words	Model
If two triangles are similar, then their corresponding angles are congruent and the ratios of their corresponding sides are proportional.	
Symbols	
$\angle A \cong \angle X, \angle B \cong \angle Y, \angle C \cong \angle Z$ $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$	

Talk About It! The two triangles shown are similar. How do the side lengths of the small triangle compare to the side lengths of the larger triangle?

Sample answer: The sides of the smaller triangle are half the length of the corresponding sides of the larger triangle, which means that the ratios of the corresponding sides are the same, $\frac{1}{2}$.

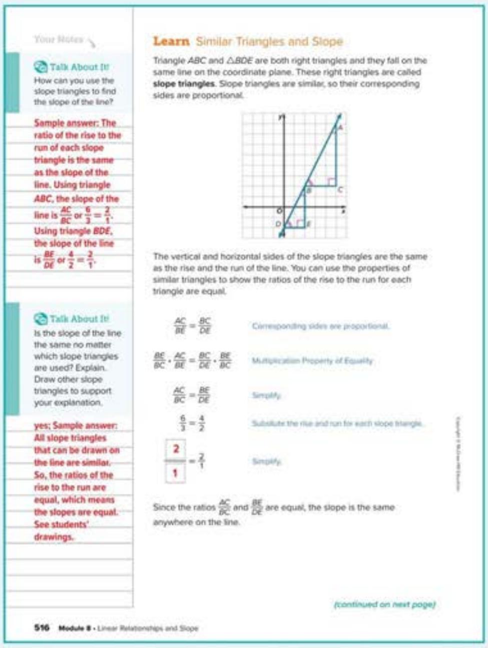
Lesson 8-3 • Similar Triangles and Slope 515

Interactive Presentation

Learn, Similar Triangles, Slide 1 of 2

FLASHCARDS


On Slide 1, students use Flashcards to view multiple representations of similar triangles.



Your Notes

Talk About It!
How can you use the slope triangles to find the slope of the line?

Sample answer: The ratio of the rise to the run of each slope triangle is the same as the slope of the line. Using triangle *ABC*, the slope of the line is $\frac{AC}{BC}$ or $\frac{6}{3} = \frac{2}{1}$. Using triangle *BDE*, the slope of the line is $\frac{BE}{DE}$ or $\frac{4}{2} = \frac{2}{1}$.

Talk About It!
Is the slope of the line the same no matter which slope triangles are used? Explain. Draw other slope triangles to support your explanation.

yes; Sample answer: All slope triangles that can be drawn on the line are similar. So, the ratios of the rise to the run are equal, which means the slopes are equal. See students' drawings.

Learn Similar Triangles and Slope

Triangle *ABC* and $\triangle BDE$ are both right triangles and they fall on the same line on the coordinate plane. These right triangles are called **slope triangles**. Slope triangles are similar, so their corresponding sides are proportional.

The vertical and horizontal sides of the slope triangles are the same as the rise and the run of the line. You can use the properties of similar triangles to show the ratios of the rise to the run for each triangle are equal.

$\frac{AC}{BE} = \frac{BC}{DE}$ Corresponding sides are proportional.

$\frac{BE}{BC} \cdot \frac{AC}{BE} = \frac{BC}{DE} \cdot \frac{BE}{BC}$ Multiplication Property of Equality

$\frac{AC}{BC} = \frac{BE}{DE}$ Simplify.

$\frac{6}{3} = \frac{4}{2}$ Substitute the rise and run for each slope triangle.

$\frac{2}{1} = \frac{2}{1}$ Simplify.

Since the ratios $\frac{AC}{BC}$ and $\frac{BE}{DE}$ are equal, the slope is the same anywhere on the line.

(continued on next page)

516 Module 8 • Linear Relationships and Slope

8.EE.B.6

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Similar Triangles and Slope

Objective

Students will understand the relationship between the slopes of similar slope triangles and the slope of the line.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* questions on Slide 2, students should be able to construct a plausible argument, with a supportive drawing, that illustrates why the slope of a line is the same no matter which slope triangle is used.

6 Attend to Precision As students discuss the *Talk About It!* questions on Slide 2, encourage students to use clear and precise mathematical language, such as *ratio, rise, run, slope*, and *similar*, to demonstrate their understanding of how slope triangles can be used to find the slope of a line.

Teaching Notes

SLIDE 1

Students will learn that *slope triangles* are right triangles that fall on the same line when a line is graphed on the coordinate plane. The vertical and horizontal sides of slope triangles are equal to the rise and run of the line. Slope triangles are similar, so their corresponding sides are proportional.

Talk About It!

SLIDE 2

Mathematical Discourse

How you can use the slope triangles to find the slope of the line? Sample answer: The ratio of the rise to the run of each slope triangle is the same as the slope of the line. Using triangle *ABC*, the slope of the line is $\frac{AC}{BC}$ or $\frac{6}{3} = \frac{2}{1}$. Using triangle *BDE*, the slope of the line is $\frac{BE}{DE}$ or $\frac{4}{2} = \frac{2}{1}$

Is the slope of the line the same no matter which slope triangles are used? Explain. Draw other slope triangles to support your explanation. yes; Sample answer: All slope triangles that can be drawn on the line are similar. So, the ratios of the rise to the run are equal, which means the slopes are equal. See students' drawings.

(continued on next page)

Learn Similar T riangles and Slope *(continued)*

Teaching Notes

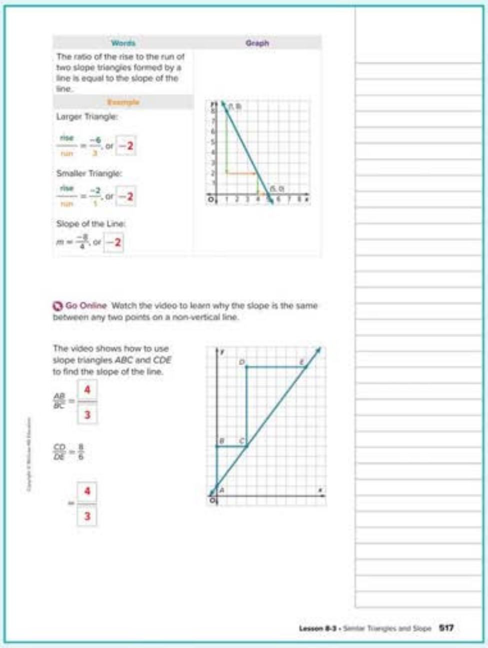
SLIDE 3

Have students select the *Words*, *Graph*, and *Example* flashcards to see the relationship between the rise and run of slope triangles and the slope of a line expressed in these multiple representations. You may wish to have students choose another pair of different slope triangles for this line, and have them determine the ratio of the rise to the run for each slope triangle. They should notice that the ratio is always equivalent to the slope of the line, no matter which slope triangles they use.

Go Online Have students watch the video on Slide 4. The video illustrates similar triangles and slope.

SLIDE 4

After watching the video, you may wish to have students create their own graph of two slope triangles that correspond to a line. Have them create their own argument for why their graph supports the fact that the slope is the same between any two points on a nonvertical line.



Words	Graph
The ratio of the rise to the run of two slope triangles formed by a line is equal to the slope of the line.	
Example	
Larger Triangle: $\frac{\text{rise}}{\text{run}} = \frac{-6}{3}$, or -2 Smaller Triangle: $\frac{\text{rise}}{\text{run}} = \frac{-2}{1}$, or -2 Slope of the Line: $m = \frac{-8}{4}$, or -2	

Go Online Watch the video to learn why the slope is the same between any two points on a non-vertical line.

The video shows how to use slope triangles *ABC* and *CDE* to find the slope of the line.

$\frac{AB}{BC} = \frac{4}{3}$

$\frac{CD}{DE} = \frac{8}{6}$

$= \frac{4}{3}$

Lesson 8-3 • Similar Triangles and Slope **517**

Interactive Presentation

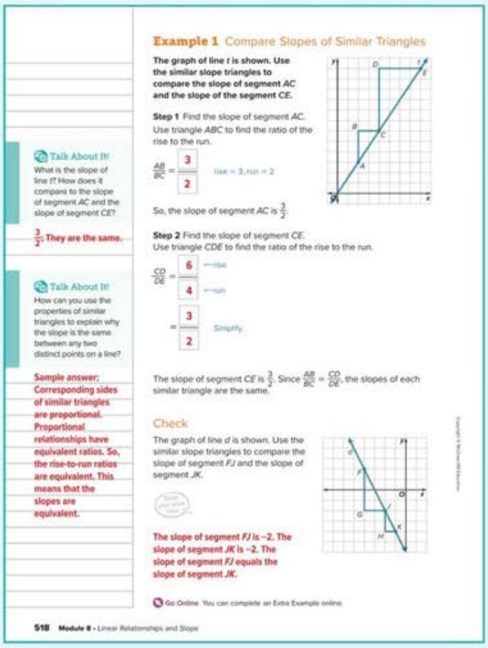
Learn, Similar Triangles and Slope, Slide 3 of 4

FLASHCARDS

On Slide 3, students use Flashcards to view multiple representations of slope triangles.

WATCH

On Slide 4, students watch a video to learn why the slope is the same between any two points on a non-vertical line.


Example 1 Compare Slopes of Similar Triangles

The graph of line *t* is shown. Use the similar slope triangles to compare the slope of segment *AC* and the slope of the segment *CE*.

Step 1 Find the slope of segment *AC*. Use triangle *ABC* to find the ratio of the rise to the run.

$\frac{AB}{BC} = \frac{3}{2}$ rise = 3, run = 2

So, the slope of segment *AC* is $\frac{3}{2}$.

Step 2 Find the slope of segment *CE*. Use triangle *CDE* to find the ratio of the rise to the run.

$\frac{CD}{DE} = \frac{6}{4}$ ←rise ←run

$= \frac{3}{2}$ Simplify.

The slope of segment *CE* is $\frac{3}{2}$. Since $\frac{AB}{BC} = \frac{CD}{DE}$, the slopes of each similar triangle are the same.

Talk About It! What is the slope of line *t*? How does it compare to the slope of segment *AC* and the slope of segment *CE*?

$\frac{3}{2}$; They are the same.

Talk About It! How can you use the properties of similar triangles to explain why the slope is the same between any two distinct points on a line?

Sample answer: Corresponding sides of similar triangles are proportional. Proportional relationships have equivalent ratios. So, the rise-to-run ratios are equivalent. This means that the slopes are equivalent.

Check

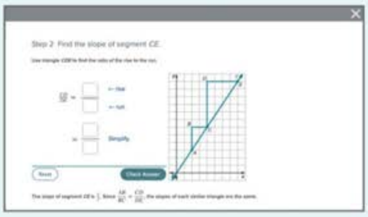
The graph of line *d* is shown. Use the similar slope triangles to compare the slope of segment *FJ* and the slope of segment *JK*.

The slope of segment *FJ* is −2. The slope of segment *JK* is −2. The slope of segment *FJ* equals the slope of segment *JK*.

Go Online You can complete an Extra Example online.

518 Module 8 • Linear Relationships and Slope

Interactive Presentation



Example 1, Compare Slopes of Similar Triangles, Slide 3 of 5

TYPE

On Slide 2, students determine the slope of segment *AC*.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Compare Slopes of Similar Triangles

Objective

Students will use similar slope triangles that correspond to the same line, to compare their slopes.

MP Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* questions on Slide 4, encourage them to use clear and precise mathematical language, such as *corresponding sides, proportional*, and *equivalent ratios*, to explain how the properties of similar triangles can be used to demonstrate that the slope between any two pairs of points on a line is the same.

Questions for Mathematical Discourse

SLIDE 2

AL What does $\frac{AB}{BC}$ represent? the ratio of the rise to the run for triangle *ABC*

AL What is the rise of slope triangle *ABC*? What is the run? 3; 2

OL A classmate wrote the slope of segment *AC* as 1.5. Explain why it may be more helpful to write the slope as a fraction. Sample answer: It is easier to see the rise and the run as separate quantities when the slope is written as a fraction.

BL Triangle *ABC* sits on top of the line. Can you have a slope triangle that sits below the line? What would be the third coordinate of a slope triangle that sits below the line, and has the same size as triangle *ABC*? (4, 3)

SLIDE 3

AL How can you identify the rise? Sample answer: Find the change in *y*-values from point *C* to point *D*.

OL How do you expect the slopes of the segments *AC* and *CE* to be related? Explain. Sample answer: The slopes should be equal, because the triangles are similar and they fall on the same line.

BL Suppose that to move from point *A* to point *C* is defined as *1 jump*. How many of these jumps are needed to move from point *C* to point *E*? How is this represented in the rise to run ratios for each slope triangle? 2 jumps; Sample answer: Before the ratios are simplified, the ratio $\frac{6}{4}$ has a rise twice that of the ratio $\frac{3}{2}$ and a run twice that of the ratio $\frac{3}{2}$.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Verify Slopes Using Slope Triangles

Objective

Students will graph slope triangles on the coordinate plane to show that the slope of a line is the same between any two points on a line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should make sense of the relationship between the ratio of the rise to the run for each slope triangle, and the slope of the line segment, which corresponds to the pitch of the roof.

6 Attend to Precision As students discuss the *Talk About It!* question, encourage them to use clear and precise mathematical language to explain why the slopes of the segments are the same.

Questions for Mathematical Discourse

SLIDE 2

AL How can you calculate the run from *T* to *U*? Sample answer: Count the number of horizontal units from *T* to *U*.

OL In construction terms, a roof with a rise of 3 inches and a run of 12 inches is called a $\frac{3}{12}$ *roof*. The pitches of most roofs are left with the units in inches, and not always simplified. What would the pitch of this roof be called, using this same terminology? a $\frac{6}{12}$ roof

BL Most roofs have a pitch between $\frac{4}{12}$ and $\frac{9}{12}$. Roofs with a pitch exceeding $\frac{9}{12}$ are considered steep slope roofs. Is this roof a steep slope roof? Explain. no; Sample answer: A $\frac{9}{12}$ roof would have a slope of $\frac{3}{4}$, and $\frac{1}{2}$ is less than $\frac{3}{4}$.

SLIDE 3

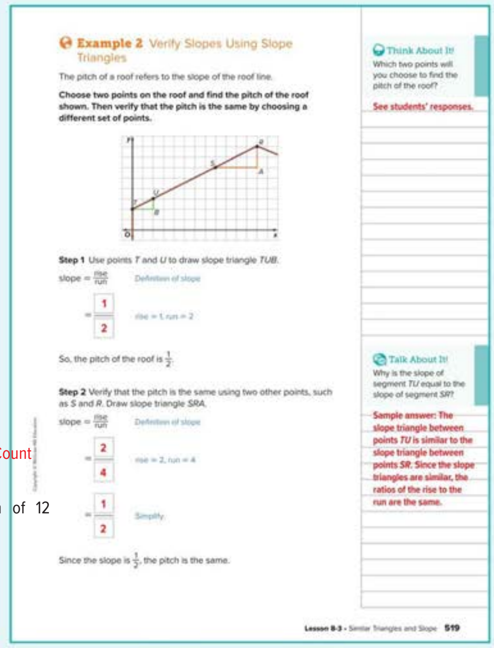
AL How can you calculate the rise from *S* to *R*? Sample answer: Count the number of vertical units from *S* to *R*.

OL Describe how to draw another slope triangle you could use to verify the slope. Sample answer: Draw a triangle that represents the rise and run to go from point *U* to point *S*.

BL In construction, the minimum pitch a roof can have is a $\frac{2}{12}$ pitch roof. Describe the slope of a roof that has a pitch of $\frac{2}{12}$. Sample answer: The slope of the roof is $\frac{1}{6}$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Verify Slopes Using Slope Triangles

The pitch of a roof refers to the slope of the roof line.

Choose two points on the roof and find the pitch of the roof shown. Then verify that the pitch is the same by choosing a different set of points.

Step 1 Use points *T* and *U* to draw slope triangle *TUB*.

slope $= \frac{\text{rise}}{\text{run}}$ Definition of slope

$= \frac{1}{2}$ rise = 1, run = 2

So, the pitch of the roof is $\frac{1}{2}$.

Step 2 Verify that the pitch is the same using two other points, such as *S* and *R*. Draw slope triangle *SRA*.

slope $= \frac{\text{rise}}{\text{run}}$ Definition of slope

$= \frac{2}{4}$ rise = 2, run = 4

$= \frac{1}{2}$ Simplify

Since the slope is $\frac{1}{2}$, the pitch is the same.

Think About It! Which two points will you choose to find the pitch of the roof?

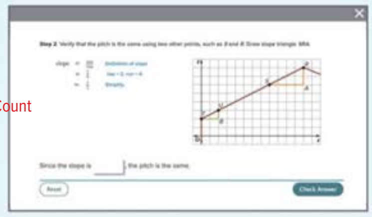
See students' responses.

Talk About It! Why is the slope of segment *TU* equal to the slope of segment *SR*?

Sample answer: The slope triangle between points *TU* is similar to the slope triangle between points *SR*. Since the slope triangles are similar, the ratios of the rise to the run are the same.

Lesson 8-3 • Similar Triangles and Slope 519

Interactive Presentation



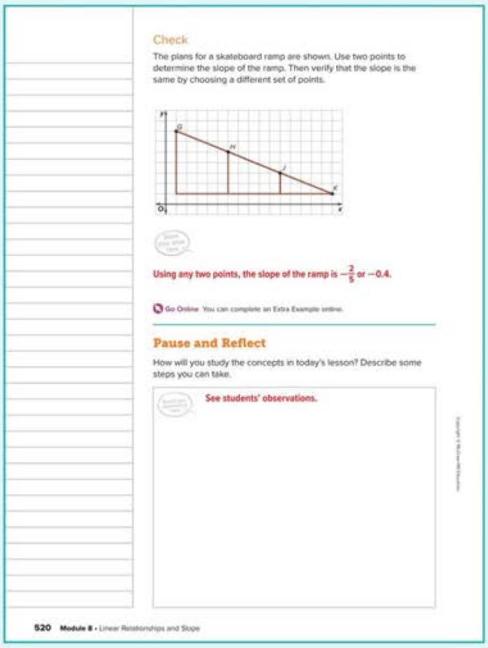
Example 2, Verify Slopes Using Triangles, Slide 3 of 5

TYPE

On Slide 2, students determine the pitch of the roof. On Slide 3, students determine the slope.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The plans for a skateboard ramp are shown. Use two points to determine the slope of the ramp. Then verify that the slope is the same by choosing a different set of points.

Using any two points, the slope of the ramp is $-\frac{2}{5}$ or -0.4.

Go Online You can complete an Extra Example online.

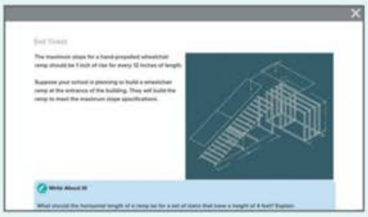
Pause and Reflect

How will you study the concepts in today's lesson? Describe some steps you can take.

See students' observations.

520 Module 8 • Linear Relationships and Slope

Interactive Presentation



Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. What should the horizontal length of a ramp be for a set of stairs that have a height of 4 feet? Write a mathematical argument that can be used to defend your solution. 48 feet; Sample answer: Write equivalent fractions using the given information that the height will be 4 feet, $\frac{1 \text{ in.}}{12 \text{ in.}} = \frac{48 \text{ in}}{x \text{ in.}}$. Solve for the unknown. Because 1 multiplied by 48 yields 48, multiply 12 by 48 to obtain 576. The length of the ramp should be 576 inches, or 48 feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 3–7 odd, 8–11
- ALEKS® Slope, Similar Figures

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 7–11 odd
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Proportions

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Proportions

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

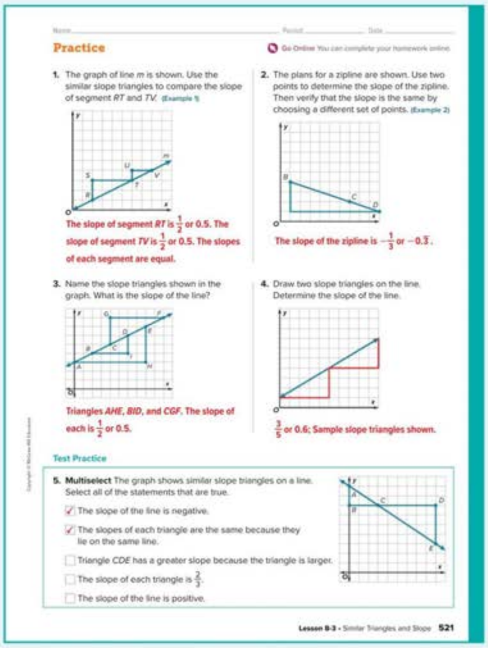
The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use similar slope triangles that correspond to the same line, to compare their slopes	1
1	graph slope triangles on the coordinate plane to show that the slope of a line is the same between any two points on a line	2
2	extend concepts learned in class to apply them in new contexts	3–7
3	higher-order and critical thinking skills	8–11


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The graph of line *m* is shown. Use the similar slope triangles to compare the slope of segment *RT* and *TV*. (Example 1)

The slope of segment *RT* is $\frac{1}{2}$ or 0.5. The slope of segment *TV* is $\frac{1}{2}$ or 0.5. The slopes of each segment are equal.

2. The plans for a zipline are shown. Use two points to determine the slope of the zipline. Then verify that the slope is the same by choosing a different set of points. (Example 2)

The slope of the zipline is $-\frac{1}{3}$ or $-0.\overline{3}$.

3. Name the slope triangles shown in the graph. What is the slope of the line?

Triangles *AHE*, *BID*, and *CGF*. The slope of each is $\frac{1}{2}$ or 0.5.

4. Draw two slope triangles on the line. Determine the slope of the line.

$\frac{3}{5}$ or 0.6; Sample slope triangles shown.

Test Practice

5. **Multiselect** The graph shows similar slope triangles on a line. Select all of the statements that are true.

- [x] The slope of the line is negative.
- [x] The slopes of each triangle are the same because they lie on the same line.
- [ ] Triangle *CDE* has a greater slope because the triangle is larger.
- [ ] The slope of each triangle is $\frac{2}{3}$.
- [ ] The slope of the line is positive.

Lesson 8-3 • Similar Triangles and Slope 521

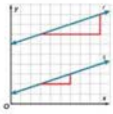
1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

8.EE.B.6

Apply

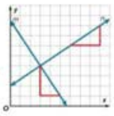
6. Lines *r* and *s* are parallel, meaning they will never intersect. Draw similar slope triangles on each line and find the slope of each line. What conclusion can you draw about the slopes of parallel lines?

Sample slope triangles shown. The slope of line *r* is $\frac{1}{3}$ or $0.\overline{3}$. The slope of line *s* is also $\frac{1}{3}$ or $0.\overline{3}$. The slopes of parallel lines are equal.



7. Lines *m* and *n* are perpendicular, meaning they form a right angle. Draw similar slope triangles on each line and find the slope of each line. What conclusion can you draw about the slopes of perpendicular lines?

Sample slope triangles shown. The slope of line *m* is $-\frac{3}{2}$ or -1.5. The slope of line *n* is $\frac{2}{3}$ or $0.\overline{6}$. The slopes of perpendicular lines are opposite reciprocals.



Higher-Order Thinking Problems

8. MP **Be Precise** How are slope triangles, corresponding sides, ratios, and $\frac{\text{rise}}{\text{run}}$ related?

Sample answer: The ratios of the corresponding sides $\left(\frac{\text{vertical}}{\text{horizontal}}\right)$ of similar slope triangles are the same as the ratio $\frac{\text{rise}}{\text{run}}$ for slope.

9. MP **Find the Error** A student found the slope of one segment on a line to be 4 and the slope of another segment on the same line to be $\frac{8}{2}$. He concludes that the slope is different at different points on the line. Correct his thinking.

Sample answer: Because $\frac{8}{2}$ simplifies to 4, the triangles are similar slope triangles on the same line. Therefore, the slope of the line is always 4.

10. Determine if the statement is *true* or *false*. Justify your response.

If one vertex of a slope triangle is at (0, 0), the other vertex on the line will be the point that represents the simplified unit rate or slope.

false; Sample answer: The other vertex will represent the slope if the *x*-coordinate is 1. If the *x*-coordinate is not 1, then the ratio of vertical to horizontal lengths may or may not need simplified.

11. MP **Identify Structure** Does the placement of the slope triangles matter when finding slope of a line? Explain your reasoning.

No; Sample answer: The placement of the slope triangles does not matter because even if the triangle is placed above or below the line, the ratio of the vertical side to the horizontal side will always be the same as the slope of the line.

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Teaching the Mathematical Practices

6 Attend to Precision In Exercise 8, students will explain how slope triangles, corresponding sides, ratios and $\frac{\text{rise}}{\text{run}}$ are related. Encourage students to be precise in their explanations.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students will find the mistake in the problem and correct it. Encourage students to determine the error and explain how they could correct the student's thinking.

7 Look For and Make Use of Structure In Exercise 11, students will determine if the placement of the slope triangles matter when finding the slope of a line. Encourage students to use the structure of the triangles to support their explanation.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.
Use with Exercises 6–7 Have students work in groups of 3–4. After completing Exercise 6, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 7.

Create your own higher-order thinking problem.
Use with Exercises 8–11 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Lesson 8-4

Direct Variation

8.EE.B.6

LESSON GOAL

Students will derive the equation $y = mx$ from graphs, tables, and verbal descriptions of proportional relationships.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Derive the Equation $y = mx$

Learn: Direct Variation
Example 1: Write Direct Variation Equations from Graphs
Example 2: Write Direct Variation Equations from Words
Example 3: Write Direct Variation Equations from Tables
Apply: Animal Care

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Solve Direct and Inverse Variation Problems		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 49 of the *Language Development Handbook* to help your students build mathematical language related to direct variation.

ELL You can use the tips and suggestions on page T49 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.B** by deriving equations of the form $y = mx$ from graphs, tables, and verbal descriptions of proportional relationships.
Standards for Mathematical Content: **8 .EE.B.6,** *Also addresses 8.EE.B.5*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students related the slope of a line to similar triangles.
8.EE.B.6

Now
Students derive the equation $y = mx$ from graphs, tables, and verbal descriptions of proportional relationships.
8.EE.B.6

Next
Students will write nonproportional linear relationships in the form $y = mx + b$.
8.EE.B.6

Rigor

The Three Pillars of Rigor

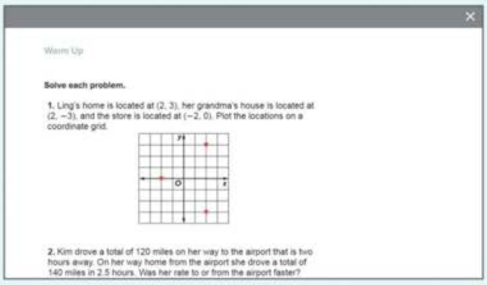
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of proportional relationships and slope to develop *understanding* of direct variations. They learn that an equation representing proportion relationships can be written in the form $y = mx$, where m represents the slope, or the constant of variation (proportionality). They build *fluency* with writing equations in the form $y = mx$ from graphs, tables, and verbal descriptions, and *apply* it to real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- graphing on the coordinate plane (Exercise 1)
- finding unit rates (Exercise 2)
- using the slope formula (Exercise 3)

1–3. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the number of hours a koala sleeps, as a unit rate.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- In mathematics, what is a *constant*? What does it mean for two quantities to be *proportional*? Sample answer: A constant is a number. When two quantities are proportional, they have a constant ratio.
- In mathematics, what is another term that is similar to *variation*? What does it mean for something to *vary*? Sample answer: variable; To vary means to change.
- If you communicate *directly* with someone, what does that mean? If you travel *directly* from one location to another, what does that mean? Sample answer: Communicating directly with someone means to talk to them yourself, not through another person. Traveling directly from one location to another means to travel the shortest distance between those two locations.

Explore Derive the Equation $y = mx$

Objective

Students will explore how to use the slope formula to derive the equation $y = mx$.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a table of values representing the amount of money raised for a Bike-a-Thon. Students will find the unit rate and the slope for the situation and then write an equation that represents the situation.

Inquiry Question

How can you use the slope formula to derive the equation of a proportional linear relationship? Sample answer: Use the coordinates of the points (0, 0) and (x, y) in the slope formula and simplify to get the equation $y = mx$, where m represents the slope.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

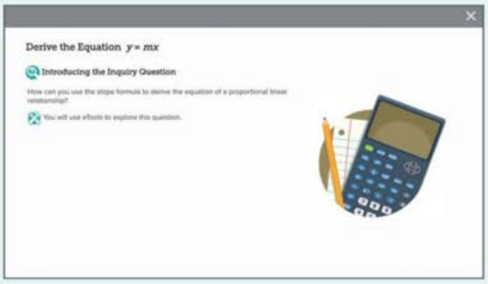
How do you know the relationship is proportional? Sample answer: The graph is a straight line through the origin.

What is the unit rate? $10 per hour

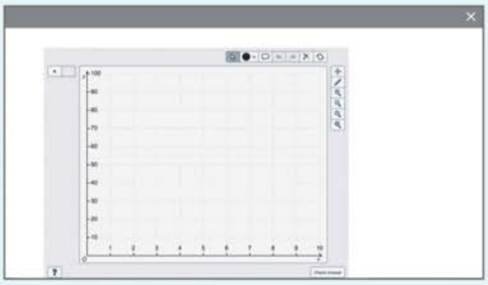
What is the slope of the line? $\frac{10}{1}$ or 10

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6

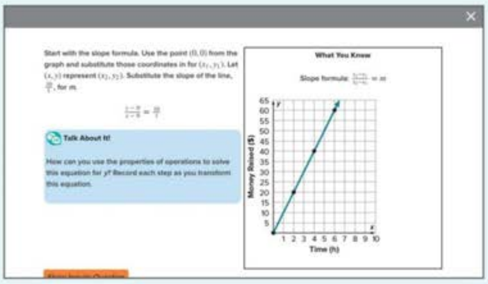


Explore, Slide 2 of 6

eTOOL

On Slide 2, students use the Coordinate Graphing eTool to graph the relationship on the coordinate plane.

Interactive Presentation



Explore, Slide 4 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Derive the Equation $y = mx$ *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively S tudents should make sense of the coordinates graphed in order to help derive the slope formula by reasoning about the points (0, 0) and any point on the graph (x, y).

5 Use Appropriate Tools Strategically Students will use the coordinate graphing eTool to graph the relationship given in the activity on the coordinate plane.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

How can you use the properties of operations to solve this equation for y? Record each step as you transform this equation. See students' work.

Learn Direct Variation

Objective

Students will understand that a direct variation is a proportional relationship, and how to derive the direct variation equation, $y = mx$.

Teaching Notes

SLIDE 1

When the ratio of two variable quantities is constant, a proportional linear relationship exists. Students have previously learned that the unit rate of a proportional relationship is also known as its slope. Point out that it is also known as the constant of proportionality. Be sure that students understand that a proportional relationship is also known as a direct variation, and the equation of a direct variation is in the general form $y = mx$, where m is the slope. Have students select the *Words*, *Symbols*, *Example*, and *Graph* flashcards to learn about the multiple ways in which a direct variation can be represented.

(continued on next page)

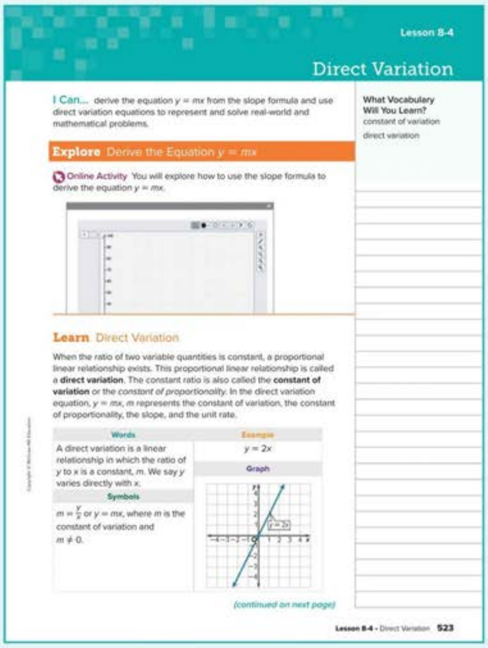
DIFFERENTIATE

Enrichment Activity

If students need more of a challenge, use the following activity.

Give students a constant of variation, such as 7, and have students create a table of values. Make sure to remind students that the graph of a proportional linear relationship passes through the origin. Encourage students to add as many values to their table as possible. Sample table shown.

x	y
0	0
1	7
2	14
3	21
4	28


Lesson 8-4

Direct Variation

I Can... derive the equation $y = mx$ from the slope formula and use direct variation equations to represent and solve real-world and mathematical problems.

What Vocabulary Will You Learn?
constant of variation
direct variation

Explore Derive the Equation $y = mx$

Online Activity You will explore how to use the slope formula to derive the equation $y = mx$.

Learn Direct Variation

When the ratio of two variable quantities is constant, a proportional linear relationship exists. This proportional linear relationship is called a **direct variation**. The constant ratio is also called the **constant of variation** or the *constant of proportionality*. In the direct variation equation, $y = mx$, m represents the constant of variation, the constant of proportionality, the slope, and the unit rate.

Words	Example
A direct variation is a linear relationship in which the ratio of y to x is a constant, m. We say y varies directly with x.	$y = 2x$
Symbols	**Graph**
$m = \frac{y}{x}$ or $y = mx$, where m is the constant of variation and $m \neq 0$.	

(continued on next page)

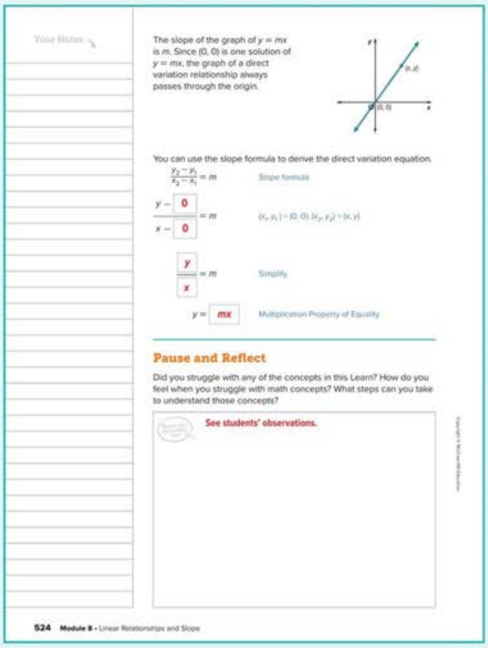
Lesson 8-4 • Direct Variation 523

Interactive Presentation

Learn, Direct Variation, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of direct variation.



Your Notes

The slope of the graph of $y = mx$ is m. Since (0, 0) is one solution of $y = mx$, the graph of a direct variation relationship always passes through the origin.

You can use the slope formula to derive the direct variation equation.

$\frac{y_2 - y_1}{x_2 - x_1} = m$ Slope formula

$\frac{y - 0}{x - 0} = m$ $(x_1, y_1) = (0, 0), (x_2, y_2) = (x, y)$

$\frac{y}{x} = m$ Simplify.

$y = mx$ Multiplication Property of Equality

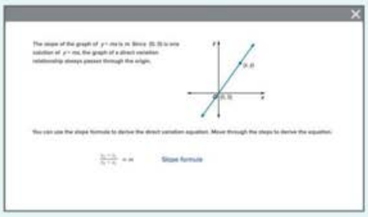
Pause and Reflect

Did you struggle with any of the concepts in this Learn? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

See students' observations.

524 Module 8 • Linear Relationships and Slope

Interactive Presentation



Learn, Direct Variation, Slide 2 of 2

TYPE

On Slide 2, students determine the direct variation equation.

CLICK

On Slide 2, students move through the steps to derive the equation.

Learn Direct Variation *(continued)*

Teaching Notes

SLIDE 2

Have students walk through the steps to learn how the slope formula and two points on a line can be used to derive the direct variation equation, $y = mx$.

Example 1 Write Direct Variation Equations from Graphs

Objective

Students will write direct variation equations from graphs and interpret the constant of variation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encour age students to identify the important information given in the graph in order to decontextualize the relationship by representing it symbolically with a direct variation equation.

As students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of the meanings of each of these terms and to be able to explain why they can use them interchangeably in the context of this problem.

6 Attend to Precision Students should be able to use the precise mathematical terminology, such as *direct variation*, *constant of variation*, *slope*, and *unit rate*, as they navigate through the steps to write the direct variation equation.

Questions for Mathematical Discourse

SLIDE 2

AL Is this relationship linear? Is it proportional? Explain. Y es, the relationship is linear because the graph is a straight line. The relationship is proportional, because it is a straight line that passes through the origin.

OL What point on the graph illustrates the slope? Explain. (1, 10); Sample answer: Since the relationship is proportional, the point (1, r) on a proportional graph represents the unit rate r, which is also the slope. So, the slope is 10.

BL Describe another way you can find the slope. Sample answer: I can use the slope formula to find the slope between the two points (1, 10) and (2, 20).

SLIDE 3

AL What is the form of a direct variation equation? What does m stand for? $y = mx$, where m represents the slope

OL Why is it useful to write the direct variation equation? Sample answer: I can use the equation to find the value of y for any value of x.

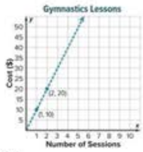
BL What is another way you can write the equation $y = 10x$? Sample answer: $x = \frac{y}{10}$

(continued on next page)

Example 1 Write Direct Variation Equations From Graphs

The cost y of gymnastics lessons varies directly with the number of sessions x as shown in the graph.

Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning.



Think About It! How do you know that this relationship is a direct variation?

See students' responses.

Part A Write a direct variation equation.

Step 1 Find the slope m using the graph.

Choose any two points on the line and find the ratio of the rise to the run between the two points.

slope $= \frac{\text{rise}}{\text{run}}$ Definition of slope

slope $= \frac{10}{1}$

So, the slope of the line is $\frac{10}{1}$ or 10.

Talk About It! In a direct variation relationship, how are the slope, unit rate, and constant of variation related?

They are all equivalent.

Step 2 Use the value of m to write the equation.

$y = mx$ Direct variation equation

$y = 10x$ Replace m with 10.

So, the direct variation equation is $y = 10x$.

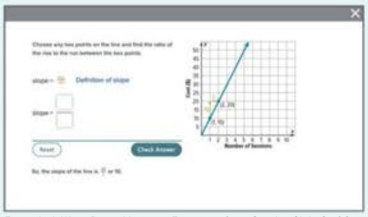
Part B Find the constant of variation and interpret its meaning.

The constant of variation is equal to the slope of the graph. So, the constant of variation is $\frac{10}{1}$ or 10.

This means that the unit rate, or cost per session, is $10.

Lesson 8-4 • Direct Variation 525

Interactive Presentation



Example 1, Write Direct Variation Equations from Graphs, Slide 2 of 6

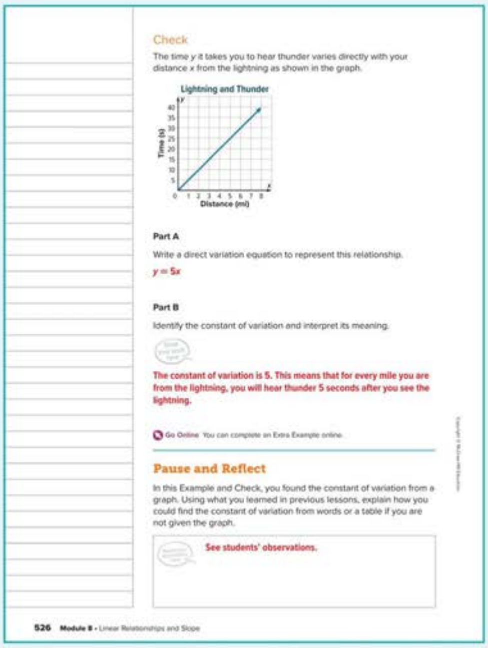
TYPE On Slide 2, students determine the slope.

CLICK On Slide 3, students select the slider to view the direct variation equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

8.EE.B.6

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Check

The time y it takes you to hear thunder varies directly with your distance x from the lightning as shown in the graph.

Part A

Write a direct variation equation to represent this relationship.

$y = 5x$

Part B

Identify the constant of variation and interpret its meaning.

The constant of variation is 5. This means that for every mile you are from the lightning, you will hear thunder 5 seconds after you see the lightning.

Go Online You can complete an Extra Example online.

Pause and Reflect

In this Example and Check, you found the constant of variation from a graph. Using what you learned in previous lessons, explain how you could find the constant of variation from words or a table if you are not given the graph.

See students' observations.

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Example 1 Write Direct Variation Equations from Graphs *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL In a direct variation, what is the slope equivalent to? Sample answer: the unit rate, the constant of variation, and the constant of proportionality

OL Is it easier for you to think of the cost per session as the unit rate, the constant of variation, the slope, or the constant of proportionality? Explain. Sample answer: I prefer to think of the cost per session as the unit rate, although in a direct variation, these terms are all equivalent.

BL How much will it cost to purchase 19 sessions? $190

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Write Direct Variation Equations from Words

Objective

Students will write a direct variation equation from a verbal description and interpret the constant of variation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to identify the important information given in the verbal description in order to decontextualize the relationship by representing it symbolically with a direct variation equation.

6 Attend to Precision Students should be able to use the precise mathematical terminology, such as *direct variation*, *constant of variation, slope*, and *unit rate*, as they navigate through the steps to write the direct variation equation.

7 Look For and Make Use of Structure As students discuss the *Talk About It!* question on slide 5, encourage them to use the structure of the equation in order to know which variable to substitute with the given quantity to find the cost of 7 pounds of peanuts.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? I need to find the unit rate, or how much one pound of peanuts costs.

OL Why is finding the unit rate helpful? Sample answer: In order to write the direct variation equation, I need to find the value of m. In a direct variation relationship, m is the slope, which is also the unit rate.

BL How would the unit rate change if 3 pounds of peanuts cost $6.00? The unit rate would be $2.00 per pound instead of $2.90.

SLIDE 3

AL What is the form of a direct variation equation? What does m stand for? $y = mx$, where m represents the slope

OL Why is the direct variation equation $y = 2.9x$, not $y = \$2.90x$? Sample answer: Units are not included in the equation. $2.90 = 2.9$, so the extra zero is not needed.

BL How would the equation change if 3 pounds of peanuts cost $6.00? The equation would change from $y = 2.9x$ to $y = 2x$.

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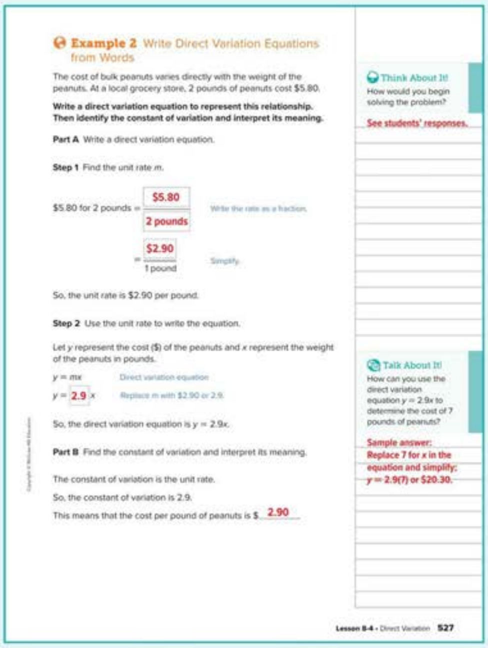

Example 2 Write Direct Variation Equations from Words

The cost of bulk peanuts varies directly with the weight of the peanuts. At a local grocery store, 2 pounds of peanuts cost $5.80.

Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning.

Part A Write a direct variation equation.

Step 1 Find the unit rate m.

$5.80 for 2 pounds $= \frac{\$5.80}{2 \text{ pounds}}$ Write the rate as a fraction.

$= \frac{\$2.90}{1 \text{ pound}}$ Simplify.

So, the unit rate is $2.90 per pound.

Step 2 Use the unit rate to write the equation.

Let y represent the cost ($) of the peanuts and x represent the weight of the peanuts in pounds.

$y = mx$ Direct variation equation

$y = 2.9x$ Replace m with $2.90 or 2.9.

So, the direct variation equation is $y = 2.9x$.

Part B Find the constant of variation and interpret its meaning.

The constant of variation is the unit rate.

So, the constant of variation is 2.9.

This means that the cost per pound of peanuts is $ 2.90

Think About It! How would you begin solving the problem?

See students' responses.

Talk About It! How can you use the direct variation equation $y = 2.9x$ to determine the cost of 7 pounds of peanuts?

Sample answer: Replace 7 for x in the equation and simplify: $y = 2.9(7)$ or $20.30.

Lesson 8-4 • Direct Variation 527

Interactive Presentation

Example 2, Write Direct Variation Equations from Words, Slide 2 of 6

TYPE On Slide 2, students determine the unit rate.

TYPE On Slide 3, students write the direct variation equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Math History Minute

In 1925, **Elbert Frank Cox (1895–1969)** became the first African American to earn a Ph.D. in mathematics. He was a mathematics professor at West Virginia State College and Howard University. After he retired, Howard University established a scholarship fund in his name to encourage young African Americans to pursue graduate studies in mathematics.

Check

The amount of money Olivia earns varies directly with the number of weeks she works. After 4 weeks, Olivia earned $3,000.

Part A

Write a direct variation equation to represent this relationship.

$y = 750x$

Part B

Identify the constant of variation and interpret its meaning.

The constant of variation is 750. This means Olivia earns $750 per week.

Go Online You can complete an Extra Example online.

Pause and Reflect

Are you ready to move on to the next Example? If yes, what have you learned that you think will help you? If no, what questions do you still have? How can you get those questions answered?

See students' observations.

528 **Module 8** • Linear Relationships and Slope

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Write Direct Variation Equations from Words *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL What other term, besides unit rate, can be used instead of constant of variation? slope

OL Would the constant of variation change if the price for two pounds of peanuts changed? Explain. yes; Sample answer: the constant of variation would change if the price of two pounds of peanuts changed. If the price of two pounds of peanuts changed, I would need to recalculate the unit rate before writing the equation.

BL With the constant of variation being 2.9, how many pounds of peanuts can you buy with $34.80? 12 pounds

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 3 Write Direct Variation Equations from Tables

Objective

Students will write a direct variation equation from a table and interpret the constant of variation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to identify the important information given in the table in order to decontextualize the relationship by representing it symbolically with a direct variation equation.

6 Attend to Precision Students should be able to use the precise mathematical terminology, such as *direct variation, constant of variation, slope,* and *unit rate*, as they navigate through the steps to write the direct variation equation.

Questions for Mathematical Discourse

SLIDE 2

AL What two points were used to find the slope? (2, 4) and (4, 8)

OL Can you use any two points from the table? Explain. Sample answer: Since the relationship is a direct variation, it will have a constant slope, no matter which two points are used.

BL What would be the next two rows in the table, written as ordered pairs? (10, 20) and (12, 24)

SLIDE 3

AL What other terms can be used instead of *slope*? Sample answers: *constant of variation* or *unit rate*

OL Describe this equation using words within the context of the problem. Sample answer: This equation can be used to find the number of cups of flour *y* that are needed for a certain number of cups of oats *x*.

BL How can you use this equation to find the number of cups of flour that are needed if you plan to use 16 cups of oats? Sample answer: Find 2(16), which is 32. So, 32 cups of flour are needed.

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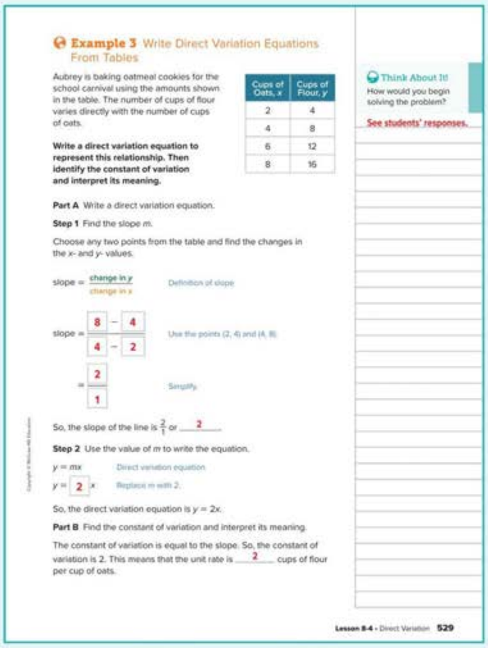


Example 3 Write Direct Variation Equations From Tables

Aubrey is baking oatmeal cookies for the school carnival using the amounts shown in the table. The number of cups of flour varies directly with the number of cups of oats.

Cups of Oats, x	Cups of Flour, y
2	4
4	8
6	12
8	16

Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning.

Part A Write a direct variation equation.

Step 1 Find the slope *m*.

Choose any two points from the table and find the changes in the *x*- and *y*- values.

slope = $\frac{\text{change in } y}{\text{change in } x}$ Definition of slope

slope = $\frac{8 - 4}{4 - 2}$ Use the points (2, 4) and (4, 8).

= $\frac{2}{1}$ Simplify.

So, the slope of the line is $\frac{2}{1}$ or 2.

Step 2 Use the value of *m* to write the equation.

$y = mx$ Direct variation equation

$y = 2x$ Replace *m* with 2.

So, the direct variation equation is $y = 2x$.

Part B Find the constant of variation and interpret its meaning.

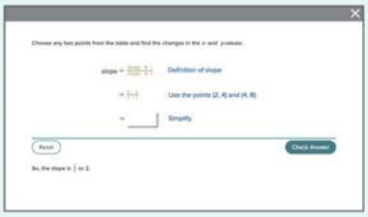
The constant of variation is equal to the slope. So, the constant of variation is 2. This means that the unit rate is 2 cups of flour per cup of oats.

Think About It! How would you begin solving the problem?

See students' responses.

Lesson 8-4 • Direct Variation 529

Interactive Presentation



Example 3, Write Direct Variation Equations from Tables, Slide 2 of 5

TYPE

On Slide 2, students determine the simplified slope.

TYPE

On Slide 3, students write the direct variation equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The time it takes Madeline to swim laps is shown in the table. The time spent swimming varies directly with the number of laps she swims.

Number of Laps, x	Time (seconds), y
1	45
2	90
3	135
4	180

Part A

Write a direct variation equation to represent this relationship.

$y = 45x$

Part B

Identify the constant of variation and interpret its meaning.

The constant of variation is 45. This means that it takes Madeline 45 seconds to swim each lap.

Go Online You can complete an Extra Example online.

Pause and Reflect

When you first saw this Check, what was your reaction? Did you think you could solve the problem? Did what you already know help you solve the problem?

See students' observations.

Example 3 Write Direct Variation Equations from Tables *(continued)*

Questions for Mathematical Discourse

SLIDE 4

AL What is the constant of variation? 2

OL What other terms can you use for this example, besides the constant of variation? Sample answer: unit rate, slope, or constant of proportionality

BL How many cups of oats will you need if you plan to use 22 cups of flour? 11 cups

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Animal Care

Objective

Students will come up with their own strategy to solve an application problem involving heart rates.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

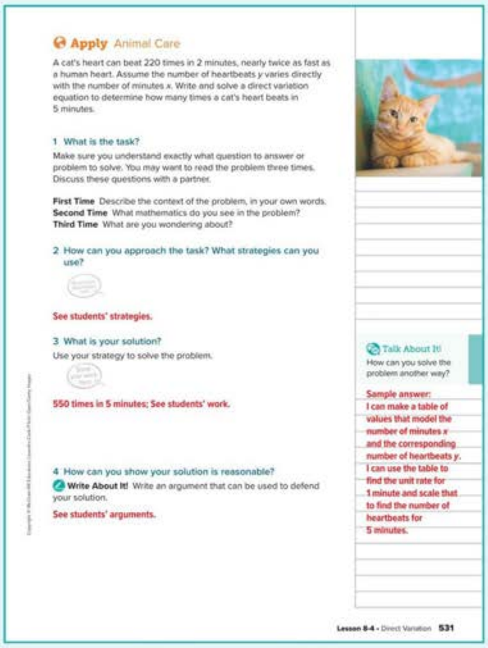
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Why might it help to determine how many times a cat's heart beats in 1 minute?
- What representation(s) could you use to help write the equation?
- How can you use the equation to determine the number of heartbeats in 5 minutes?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Animal Care

A cat's heart can beat 220 times in 2 minutes, nearly twice as fast as a human heart. Assume the number of heartbeats y varies directly with the number of minutes x. Write and solve a direct variation equation to determine how many times a cat's heart beats in 5 minutes.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

550 times in 5 minutes; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How can you solve the problem another way?

Sample answer: I can make a table of values that model the number of minutes x and the corresponding number of heartbeats y. I can use the table to find the unit rate for 1 minute and scale that to find the number of heartbeats for 5 minutes.

Lesson 8-4 • Direct Variation 531

Interactive Presentation

Apply, Animal Care

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A charter bus travels 210 miles in $3\frac{1}{2}$ hours. Assume the distance traveled varies directly with the time traveled. Write and solve a direct variation equation to find how far the bus will travel in 6 hours.

$y = 60x$; The bus will travel 360 miles in 6 hours.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

proportional linear relationships

nonproportional linear relationships

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can write about direct variation. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are linear relationships related to proportional relationships?

In this lesson, students learned how to write direct variation equations, identify the constant of variation, and interpret it in the context of a problem. Encourage them to discuss with a partner how direct variation equations represent proportional relationships and unit rates. For example, they may state that equations that represent direct variations and proportional relationships are the same, and the constant of variation is the same as the unit rate.

Exit Ticket

Refer to the Exit Ticket slide. Suppose the number of hours a koala sleeps is the unit rate. Write an equation that models the relationship between the total number of hours y a koala spends asleep for any number of days, x. Use 18 as the unit rate. How many hours can you expect a koala to sleep over an 8-day period? $y = 18x$; 144 hours

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–7 odd, 8–11
- Extension: Solve Direct and Inverse Variation Problems
- ALEKS® Direct and Inverse Variation

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 6, 10, 11
- Extension: Solve Direct and Inverse Variation Problems
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Proportional Relationships, Slope

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Proportional Relationships, Slope

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

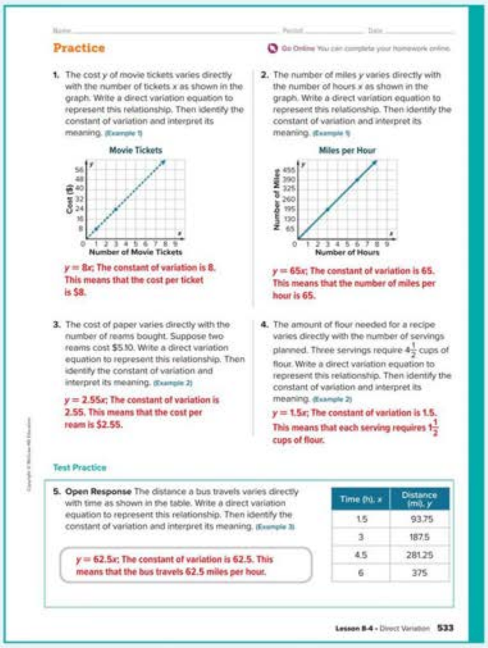
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	write a direct variation equation from a graph and interpret the constant of variation	1, 2
2	write a direct variation equation from a verbal description and interpret the constant of variation	3, 4
2	write a direct variation equation from a table and interpret the constant of variation	5
3	solve application problems involving direct variation	6, 7
3	higher-order and critical thinking skills	8–11

Common Misconception

Some students may incorrectly write a direct variation equation. Remind students that direct variation equations have the form $y = mx$. Encourage students to check that the equations they write are of this form.


Name Period Date

Practice

Go Online You can complete your homework online.

1. The cost y of movie tickets varies directly with the number of tickets x as shown in the graph. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 1)

Movie Tickets

$y = 8x$; The constant of variation is 8. This means that the cost per ticket is $8.

2. The number of miles y varies directly with the number of hours x as shown in the graph. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 1)

Miles per Hour

$y = 65x$; The constant of variation is 65. This means that the number of miles per hour is 65.

3. The cost of paper varies directly with the number of reams bought. Suppose two reams cost $5.10. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 2)

$y = 2.55x$; The constant of variation is 2.55. This means that the cost per ream is $2.55.

4. The amount of flour needed for a recipe varies directly with the number of servings planned. Three servings require $4\frac{1}{2}$ cups of flour. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 2)

$y = 1.5x$; The constant of variation is 1.5. This means that each serving requires $1\frac{1}{2}$ cups of flour.

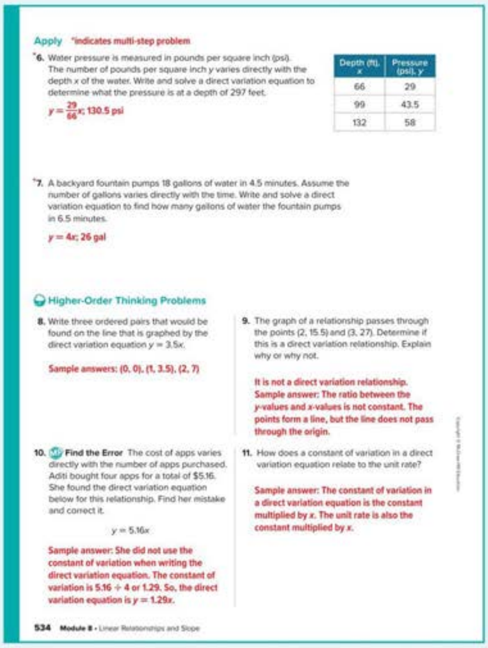
Test Practice

5. **Open Response** The distance a bus travels varies directly with time as shown in the table. Write a direct variation equation to represent this relationship. Then identify the constant of variation and interpret its meaning. (Example 3)

Time (h), x	Distance (mi), y
1.5	93.75
3	187.5
4.5	281.25
6	375

$y = 62.5x$; The constant of variation is 62.5. This means that the bus travels 62.5 miles per hour.

Lesson 8-4 • Direct Variation 533



Apply *indicates multi-step problem

*6. Water pressure is measured in pounds per square inch (psi). The number of pounds per square inch y varies directly with the depth x of the water. Write and solve a direct variation equation to determine what the pressure is at a depth of 297 feet.

$y = \frac{29}{66}x$; 130.5 psi

Depth (ft), x	Pressure (psi), y
66	29
99	43.5
132	58

*7. A backyard fountain pumps 18 gallons of water in 4.5 minutes. Assume the number of gallons varies directly with the time. Write and solve a direct variation equation to find how many gallons of water the fountain pumps in 6.5 minutes.

$y = 4x$; 26 gal

Higher-Order Thinking Problems

8. Write three ordered pairs that would be found on the line that is graphed by the direct variation equation $y = 3.5x$.

Sample answers: (0, 0), (1, 3.5), (2, 7)

9. The graph of a relationship passes through the points (2, 15.5) and (3, 27). Determine if this is a direct variation relationship. Explain why or why not.

It is not a direct variation relationship. Sample answer: The ratio between the y-values and x-values is not constant. The points form a line, but the line does not pass through the origin.

10. **Find the Error** The cost of apps varies directly with the number of apps purchased. Aditi bought four apps for a total of $5.16. She found the direct variation equation below for this relationship. Find her mistake and correct it.

$$y = 5.16x$$

Sample answer: She did not use the constant of variation when writing the direct variation equation. The constant of variation is 5.16 ÷ 4 or 1.29. So, the direct variation equation is $y = 1.29x$.

11. How does a constant of variation in a direct variation equation relate to the unit rate?

Sample answer: The constant of variation in a direct variation equation is the constant multiplied by x. The unit rate is also the constant multiplied by x.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students will find the mistake in the problem and correct it. Encourage students to determine the error and explain how they could fix it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercises 6–7 Have students work in groups of 3–4 to solve the problem in Exercise 6. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 7.

Clearly explain your strategy.

Use with Exercise 8 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would find an ordered pair that is found on the line, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Lesson 8-5

Slope-Intercept Form

8.EE.B.6

LESSON GOAL

Students will write equations to represent linear relationships in the form $y = mx + b$.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Derive the Equation $y = mx + b$

Learn: Slope-Intercept Form of a Line
Example 1: Identify Slope and *y*-Intercepts
Example 2: Write Equations in Slope-Intercept Form
Learn: Write Equations in Slope-Intercept Form From Graphs
Example 3: Write Equations in Slope-Intercept Form
Learn: Write Equations in Slope-Intercept Form From Verbal Descriptions
Example 4: Write Equations in Slope-Intercept Form
Learn: Write Equations in Slope-Intercept Form From Tables
Example 5: Write Equations in Slope-Intercept Form
Apply: Consumer Science

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Write Linear Equations in Point-Slope Form		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 50 of the *Language Development Handbook* to help your students build mathematical language related to slope-intercept form.

ELL You can use the tips and suggestions on page T50 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.B** by writing nonproportional linear relationships in the form $y = mx + b$.
Standards for Mathematical Content: **8.E E.B.6**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP7**

Coherence

Vertical Alignment

Previous
Students wrote the equation $y = mx$ from graphs, tables, and verbal descriptions of proportional relationships.
8.EE.B.6

Now
Students write the equation $y = mx + b$ from graphs, tables, and verbal descriptions of nonproportional relationships.
8.EE.B.6

Next
Students will graph lines in slope-intercept form, vertical lines, and horizontal lines.
8.EE.B.6

Rigor

The Three Pillars of Rigor

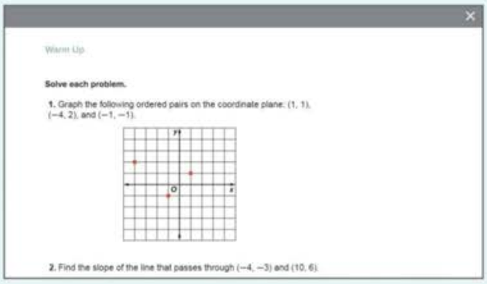
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their knowledge of slope and proportional relationships to develop *understanding* of how to represent nonproportional linear relationships with an equation. They come to understand that an equation representing a nonproportional linear relationship can be written in the form $y = mx + b$, where m represents the slope and b represents the *y*-intercept. They *apply* their understanding to solve real-world problems.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- graphing on the coordinate plane (Exercise 1)
- using the slope formula (Exercise 2)
- identifying nonproportional linear relationships (Exercise 3)

1–3. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about sightseeing in Washington, D.C., and the cost of a tour in relation to time.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *initial* means existing or occurring at the beginning. What do you think an *initial value* might be? Sample answer: An initial value might be the value (in a list or set of values) that occurs at the beginning.
- What do you know about the *slope* of a line? Sample answer: The slope of a line measures the steepness of a line.
- What does it mean to *intercept* an object? Sample answer: To intercept an object means to catch it, or interfere with the object continuing to its original destination.

Explore Derive the Equation $y = mx + b$

Objective

Students will explore how to use the slope formula to derive the equation $y = mx + b$.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will compare the graphs of two tables of values and then explore using the slope formula to write an equation that represents the nonproportional graph. Encourage students to observe how they can use the skills that they have previously learned to complete the activity.

Inquiry Question

How can you use the slope formula to derive the equation of a nonproportional linear relationship? Sample answer: Use the coordinates of any point on the line and (x, y) in the slope formula and simplify to get the equation $y = mx + b$, where m represents the slope and b represents the y-intercept.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

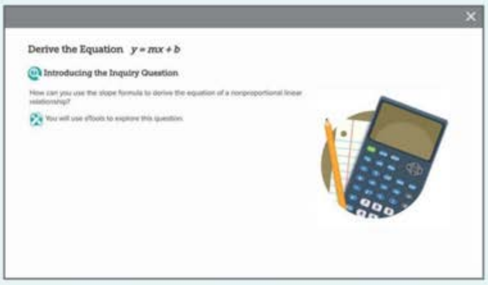
SLIDE 3

Mathematical Discourse

How does the graph for Company B compare to the graph for Company A? Sample answer: The relationship for Company A is proportional. The relationship for Company B is nonproportional. The slope of the line for each company is $\frac{2}{1}$ or 2. The graph for company A crosses the y-axis at (0, 0), whereas the graph for Company B crosses the y-axis at (0, 4).

(continued on next page)

Interactive Presentation



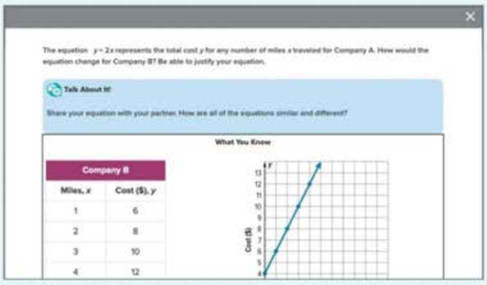
Explore, Slide 1 of 7

Explore, Slide 2 of 7

eTOOL

On Slides 2 and 3, students use the Coordinate Graphing eToolto graph the different relationships.

Interactive Presentation



Explore, Slide 4 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Derive the Equation $y = mx + b$ *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to compare and contrast the lines graphed in order to help derive the equation for a nonproportional linear relationship.

5 Use Appropriate Tools Strategically Students will use the Coordinate Graphing eTool to graph lines of proportional and nonproportional linear relationships.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

How does the equation $y = 2x + 4$ compare to the equation you wrote? Sample answer: See students' responses.

Can you choose any point on the graph and end up with the same equation? Explain. yes; Sample answer: It does not matter which point you choose since the equation will always have the same slope and the point (0, 4) will always make the equation a true sentence.

Where do you see the slope in the equation? Sample answer: The value for slope, 2, is found in the equation as the coefficient of x.

At what point does the line cross the y-axis? How does the y-coordinate of this point relate to the equation $y = 2x + 4$? (0, 4); Sample answer: the y-coordinate of the point, 4, is found in the equation.

Learn Slope-Intercept Form of a Line

Objective

Students will understand how to derive the slope-intercept form of a linear equation, $y = mx + b$.

Teaching Notes

SLIDE 1

Point out to students that not all linear relationships are proportional. Proportional relationships (or direct variations) can be written in the form $y = mx$, where m is the slope, unit rate, and constant of proportionality. Nonproportional linear relationships can be written in *slope-intercept form*, $y = mx + b$, where m is the slope and b is the y-intercept. You may wish to ask students why a nonproportional linear relationship has a y-intercept that is not equal to 0. Students should note that proportional relationships pass through the origin, and thus have a y-intercept of 0. Nonproportional linear relationships will not pass through the origin, and thus have a y-intercept that is not equal to 0. Have students select the *Equation* and *Graph* flashcards to view an example of a nonproportional linear relationship expressed in these multiple representations.

SLIDE 2

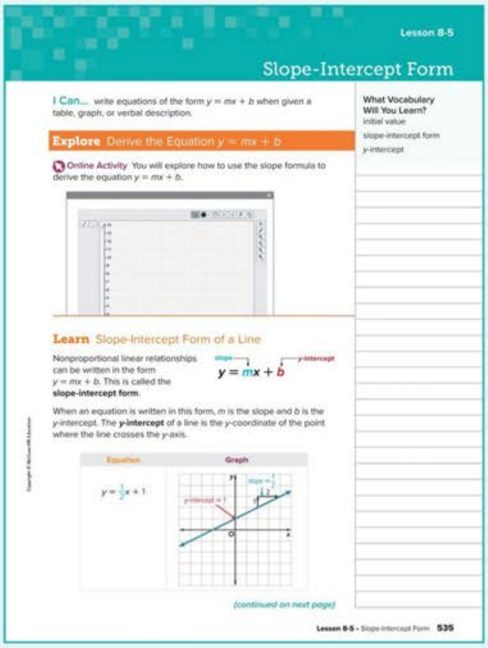
In nonproportional linear relationships, the graph passes through (0, b), not through the origin. Have students move through the steps to see how the equation for a nonproportional linear relationship, $y = mx + b$, is derived.

DIFFERENTIATE

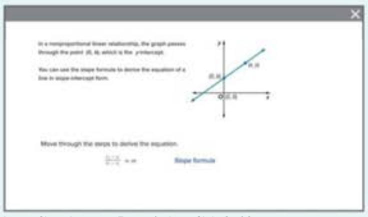
Language Developement Activity ELL

To further students' understanding of slope-intercept form, have them work with a partner to compare and contrast proportional relationships with nonproportional linear relationships. They should create a poster or graphic organizer that illustrates the similarities and differences between these two types of linear relationships. Have them include examples of each type of relationship, including the use of multiple representations (tables, graphs, and equations). Have them present their poster or graphic organizer to the class. Some sample similarities and differences are shown.

- Both have graphs that are straight lines. A proportional relationship passes through the origin, while a nonproportional linear relationship does not.
- Both have a slope, which is the constant rate of change. In a proportional relationship, the slope is also the unit rate, constant of proportionality, and constant of variation. In a nonproportional linear relationship, there is no constant ratio or unit rate.
- Both relationships can be written in slope-intercept form, $y = mx + b$, where m is the slope and b is the y-intercept. A proportional relationship has a y-intercept of 0, and thus can be written in the form $y = mx$. A nonproportional linear relationship cannot be written in the form $y = mx$, because the y-intercept is not 0.



Interactive Presentation



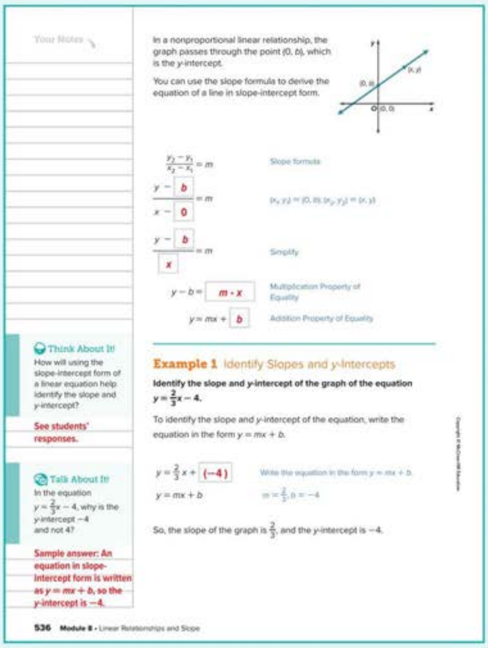
Learn, Slope Intercept Form of a Line, Slide 2 of 2

FLASHCARDS

On Slide 1, students use Flashcards to see an example of an equation in slope-intercept form and its graph.

TYPE

On Slide 2, students derive the slope-intercept form of a linear equation.



Your Notes

In a nonproportional linear relationship, the graph passes through the point (0, b), which is the y-intercept.

You can use the slope formula to derive the equation of a line in slope-intercept form.

$\frac{y_2 - y_1}{x_2 - x_1} = m$ Slope formula

$\frac{y - b}{x - 0} = m$ $(x_1, y_1) = (0, b); (x_2, y_2) = (x, y)$

$\frac{y - b}{x} = m$ Simplify

$y - b = m \cdot x$ Multiplication Property of Equality

$y = mx + b$ Addition Property of Equality

Think About It! How will using the slope-intercept form of a linear equation help identify the slope and y-intercept? See students' responses.

Talk About It! In the equation $y = \frac{2}{3}x - 4$, why is the y-intercept −4 and not 4? Sample answer: An equation in slope-intercept form is written as $y = mx + b$, so the y-intercept is −4.

Example 1 Identify Slopes and y-Intercepts

Identify the slope and y-intercept of the graph of the equation $y = \frac{2}{3}x - 4$.

To identify the slope and y-intercept of the equation, write the equation in the form $y = mx + b$.

$y = \frac{2}{3}x + (-4)$ Write the equation in the form $y = mx + b$.

$y = mx + b$ $m = \frac{2}{3}, b = -4$

So, the slope of the graph is $\frac{2}{3}$, and the y-intercept is −4.

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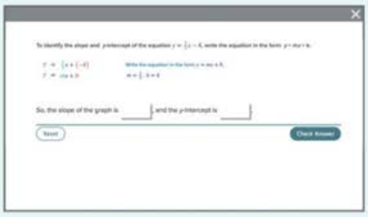
Interactive Presentation



Example 1, Identify Slopes and y-Intercepts, Slide 2 of 4

TYPE

On Slide 2, students determine the slope and y-intercept of the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Identify Slopes and y-Intercepts

Objective

Students will identify the slope and y-intercept of a line from the equation in slope-intercept form.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Encourage students to use the structure of the slope-intercept form of a linear equation to identify the slope and y-intercept of the given relationship. As students discuss the *Talk About It!* question on Slide 3, encourage them to understand the structure of the slope-intercept form of a linear equation. Students should notice that the y-intercept, b, is added to mx. If the y-intercept is negative, this means that b is negative, but still added to mx.

Questions for Mathematical Discourse

SLIDE 2

AL In slope-intercept form, which variable represents the slope? the y-intercept? *m* represents the slope; *b* represents the y-intercept

OL Why do we write $y = \frac{2}{3}x - 4$ as $y = \frac{2}{3}x + (-4)$? Sample answer: The equation needs to be written in slope-intercept form, $y = mx + b$.

OL Is this equation proportional? Explain. no; Sample answer: All proportional relationships pass through the origin, which means the y-intercept is 0. In this equation, the y-intercept is −4.

BL Imagine the graph of this line. What are some terms you can use to describe the graph? Sample answer: The graph crosses the y-axis at (0, −4). The line slopes upward from left to right.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Write Equations in Slope-Intercept Form

Objective

Students will write an equation in slope-intercept form given the slope and *y*-intercept.

MP Teaching the Mathematical Practices

7 Look For and Make Use of Structure Encourage students to use the structure of the slope-intercept form of a linear equation to accurately write the equation given the slope and *y*-intercept.

Questions for Mathematical Discourse

SLIDE 1

AL What do you notice about both the slope and *y*-intercept? They are both negative.

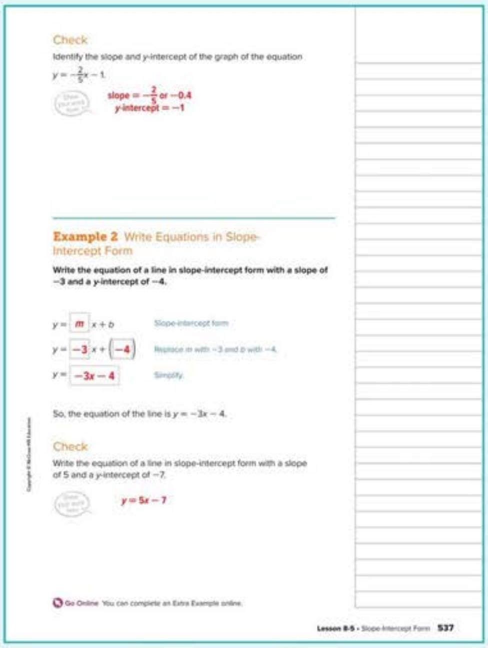
OL Explain what the slope of −3 means, in your own words. Sample answer: A slope of −3 means a rise of −3 units over a run of 1 unit. A rise that is negative means that the line slopes downward from left to right.

OL Why can you rewrite the equation $y = -3x + (-4)$ as $y = -3x - 4$? Sample answer: Adding a negative number is the same as subtracting its opposite, so I can rewrite the equation in simplest form.

BL Use the equation to name two points that would fall on this line. Sample answer: (0, −4) and (1, −7)

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

Identify the slope and *y*-intercept of the graph of the equation $y = -\frac{2}{5}x - 1$.

slope $= -\frac{2}{5}$ or -0.4
y-intercept $= -1$

Example 2 Write Equations in Slope-Intercept Form

Write the equation of a line in slope-intercept form with a slope of −3 and a *y*-intercept of −4.

$y = m x + b$ — Slope-intercept form

$y = -3 x + (-4)$ — Replace *m* with −3 and *b* with −4.

$y = -3x - 4$ — Simplify.

So, the equation of the line is $y = -3x - 4$.

Check

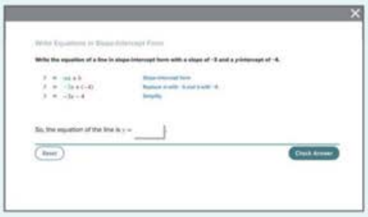
Write the equation of a line in slope-intercept form with a slope of 5 and a *y*-intercept of −7.

$y = 5x - 7$

Go Online You can complete an Extra Example online.

Lesson 8-5 • Slope-Intercept Form 537

Interactive Presentation



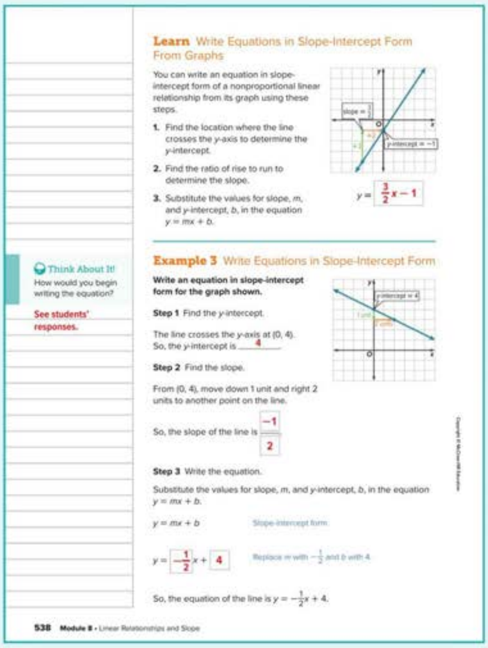
Example 2, Write Equations in Slope-Intercept Form, Slide 1 of 2

TYPE

On Slide 1, students determine the equation of the line.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Write Equations in Slope-Intercept Form From Graphs

You can write an equation in slope-intercept form of a nonproportional linear relationship from its graph using these steps.

1. Find the location where the line crosses the y-axis to determine the y-intercept.
2. Find the ratio of rise to run to determine the slope.
3. Substitute the values for slope, m, and y-intercept, b, in the equation $y = mx + b$.

$y = \frac{3}{2}x - 1$

Example 3 Write Equations in Slope-Intercept Form

Write an equation in slope-intercept form for the graph shown.

Step 1 Find the y-intercept.

The line crosses the y-axis at (0, 4). So, the y-intercept is 4.

Step 2 Find the slope.

From (0, 4), move down 1 unit and right 2 units to another point on the line.

So, the slope of the line is $\frac{-1}{2}$.

Step 3 Write the equation.

Substitute the values for slope, m, and y-intercept, b, in the equation $y = mx + b$.

$y = mx + b$ Slope-intercept form

$y = -\frac{1}{2}x + 4$ Replace m with $-\frac{1}{2}$ and b with 4.

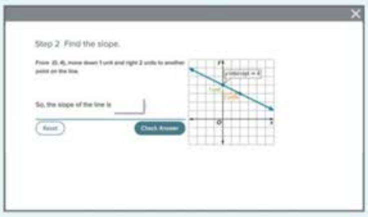
So, the equation of the line is $y = -\frac{1}{2}x + 4$.

Think About It! How would you begin writing the equation?

See students' responses.

538 Module 8 • Linear Relationships and Slope

Interactive Presentation



Example 3, Write Equations in Slope-Intercept Form, Slide 3 of 5

TYPE

On Slide 2 of Example 3, students determine the y-intercept. On Slide 3 of Example 3, students determine the slope.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Write Equations in Slope-Intercept Form From Graphs

Objective

Students will learn how to write an equation in slope-intercept form given the graph of a nonproportional linear relationship.

Go Online to find additional teaching notes.

Example 3 Write Equations in Slope-Intercept Form

Objective

Students will write an equation in slope-intercept form given the graph of a nonproportional linear relationship.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to use the structure of the graph to accurately identify the slope and y-intercept and form the correct equation.

Questions for Mathematical Discourse

SLIDE 2

AL Where does the line cross the y-axis? At the point (0, 4), which means the y-intercept is 4.

OL Why is the x-coordinate of the y-intercept 0? The x-coordinate of the y-intercept is 0 because the point is located on the y-axis.

BL If the slope of this line were positive instead of negative, would this change the y-intercept? No, if the sign of the slope only changed, the y-intercept would not change.

SLIDE 3

AL Is the slope positive or negative? Explain. negative; The line slopes downward from left to right.

OL How can you find the slope? Start at the y-intercept. Another point on the line is located 1 unit down and 2 units to the right. This means the slope is $-\frac{1}{2}$.

BL Using the slope, what are the coordinates of the next point (with whole-number coordinates) on the line to the right of (2, 3)? (4, 2)

Go Online

- Find additional teaching notes and discussion questions.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Write Equations in Slope-Intercept Form From Verbal Descriptions

Objective

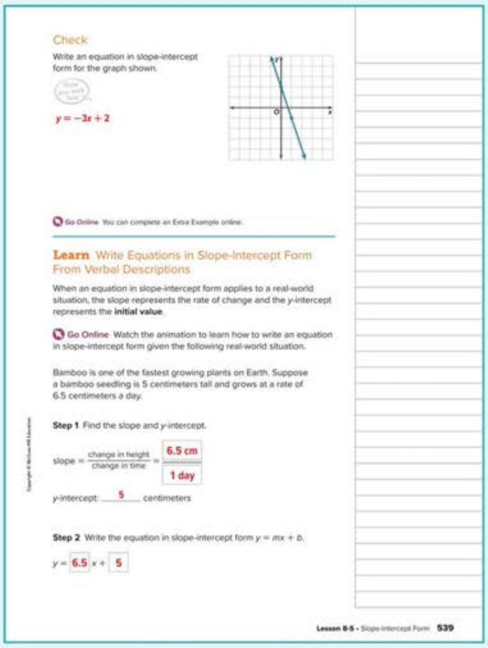
Students will learn how to write equations in slope-intercept form given a verbal description.

Go Online Have students watch the animation on Slide 1. The animation illustrates writing an equation in slope-intercept form.

Teaching Notes

SLIDE 1

When an equation in slope-intercept form applies to a real-world situation, the slope represents the *rate of change* and the *y*-intercept represents the *initial value*. Have students watch the animation to learn how to write an equation in slope-intercept form given a real-world situation.



Check

Write an equation in slope-intercept form for the graph shown.

$y = -3x + 2$

Go Online You can complete an Extra Example online.

Learn Write Equations in Slope-Intercept Form From Verbal Descriptions

When an equation in slope-intercept form applies to a real-world situation, the slope represents the rate of change and the *y*-intercept represents the **initial value**.

Go Online Watch the animation to learn how to write an equation in slope-intercept form given the following real-world situation.

Bamboo is one of the fastest growing plants on Earth. Suppose a bamboo seedling is 5 centimeters tall and grows at a rate of 6.5 centimeters a day.

Step 1 Find the slope and *y*-intercept.

$\text{slope} = \frac{\text{change in height}}{\text{change in time}} = \frac{6.5 \text{ cm}}{1 \text{ day}}$

y-intercept: 5 centimeters

Step 2 Write the equation in slope-intercept form $y = mx + b$.

$y = 6.5x + 5$

Lesson 8-5 • Slope-Intercept Form **539**

Interactive Presentation

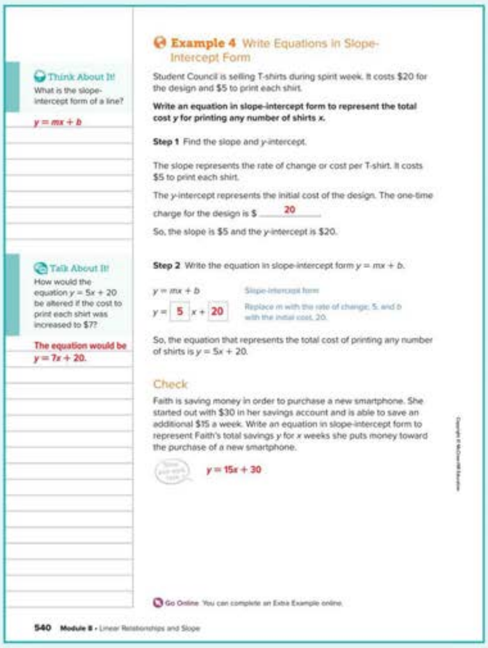
Learn, Write Equations in Slope-Intercept Form from Verbal Descriptions

WATCH

Students watch an animation to learn how to write an equation in slope-intercept form given a real-world situation.

8.EE.B.6

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**


Example 4 Write Equations in Slope-Intercept Form

Think About It! What is the slope-intercept form of a line? $y = mx + b$

Student Council is selling T-shirts during spirit week. It costs $20 for the design and $5 to print each shirt.

Write an equation in slope-intercept form to represent the total cost y for printing any number of shirts x.

Step 1 Find the slope and y-intercept.

The slope represents the rate of change or cost per T-shirt. It costs $5 to print each shirt.

The y-intercept represents the initial cost of the design. The one-time charge for the design is $ 20.

So, the slope is $5 and the y-intercept is $20.

Talk About It! How would the equation $y = 5x + 20$ be altered if the cost to print each shirt was increased to $7? The equation would be $y = 7x + 20$.

Step 2 Write the equation in slope-intercept form $y = mx + b$.

$y = mx + b$ — Slope-intercept form

$y = 5x + 20$ — Replace m with the rate of change, 5, and b with the initial cost, 20.

So, the equation that represents the total cost of printing any number of shirts is $y = 5x + 20$.

Check

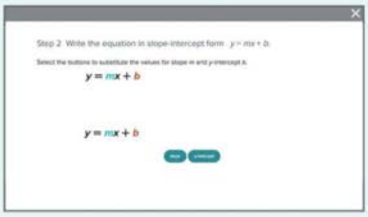
Faith is saving money in order to purchase a new smartphone. She started out with $30 in her savings account and is able to save an additional $15 a week. Write an equation in slope-intercept form to represent Faith's total savings y for x weeks she puts money toward the purchase of a new smartphone.

$y = 15x + 30$

Go Online You can complete an Extra Example online.

540 Module 8 • Linear Relationships and Slope

Interactive Presentation



Example 4, Write Equations in Slope-Intercept Form, Slide 3 of 5

CLICK

On Slide 3, students select the buttons to substitute the values for the slope and y-intercept.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Write Equations in Slope-Intercept Form

Objective

Students will write an equation in slope-intercept form given a verbal description that represents a linear relationship.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of how altering the cost per T-shirt would affect the equation.

7 Look For and Make Use of Structure Encourage students to use the structure of the slope-intercept form of a linear equation to accurately write the equation given the real-world context.

Questions for Mathematical Discourse

SLIDE 2

AL How much does it cost to print each T-shirt? $5

AL What is the initial cost of the design? $20

OL How do you know that the slope is 5? Sample answer: The slope is the rate of change, the cost per T-shirt. This value is $5.

OL How do you know that the initial value, or y-intercept is 20? Sample answer: The initial value is the cost when the number of shirts, represented by x, is 0. This value is $20. So, the y-intercept is 20.

BL If there was no initial cost to the design, and only the cost per T-shirt, what kind of relationship would this be? If there was no initial cost, and the only cost was $5 per T-shirt, this would be a proportional linear relationship, a direct variation.

SLIDE 3

AL What are the slope and y-intercept? The slope is 5 and the y-intercept is 20.

OL How would you explain what the equation represents in your own words? Sample answer: The equation gives the total cost of buying printed T-shirts. For any number of T-shirts that you buy, the cost is $20, plus $5 per T-shirt.

BL How much will it cost to buy 6 shirts? $50

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Write Equations in Slope-Intercept Form From Tables

Objective

Students will learn how to write an equation in slope-intercept form given a table of values that represents a linear relationship.

Go Online to find additional teaching notes.

Example 5 Write Equations in Slope-Intercept Form

Objective

Students will write an equation in slope-intercept form to represent a linear relationship expressed in a table.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* questions on Slide 5, encourage them to clearly interpret and explain what the slope and *y*-intercept represent in this situation.

7 Look For and Make Use of Structure Encourage students to use the structure of the table to accurately identify the slope and *y*-intercept and form the correct equation.

Questions for Mathematical Discourse

SLIDE 2

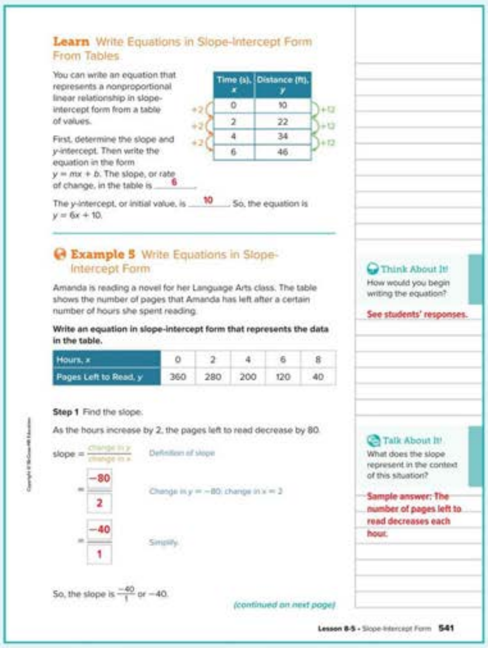
AL Why does it make sense that the slope will be negative? Sample answer: As Amanda spends more time reading, she will have fewer pages to read.

OL Why is the numerator –80 and not 80? Sample answer: The numerator represents the change in *y*. Since the *y*-values are decreasing, that change is negative.

OL What does the slope mean within the context of the problem? Sample answer: Amanda has 40 fewer pages left to read every hour.

BL How many more hours does Amanda have to read in order to finish the book? Explain. 1 more hour; Sample answer: The table shows that after 8 hours, she has 40 pages left. Since the slope is –40, she will read those 40 pages in one more hour.

(continued on next page)



Learn Write Equations in Slope-Intercept Form From Tables

You can write an equation that represents a nonproportional linear relationship in slope-intercept form from a table of values.

Time (s), x	Distance (ft), y
0	10
2	22
4	34
6	46

First, determine the slope and y-intercept. Then write the equation in the form $y = mx + b$. The slope, or rate of change, in the table is 6.

The y-intercept, or initial value, is 10. So, the equation is $y = 6x + 10$.

Example 5 Write Equations in Slope-Intercept Form

Amanda is reading a novel for her Language Arts class. The table shows the number of pages that Amanda has left after a certain number of hours she spent reading.

Write an equation in slope-intercept form that represents the data in the table.

Hours, x	0	2	4	6	8
Pages Left to Read, y	360	280	200	120	40

Step 1 Find the slope.

As the hours increase by 2, the pages left to read decrease by 80.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} \quad \text{Definition of slope}$$

$$= \frac{-80}{2} \quad \text{Change in } y = -80;\ \text{change in } x = 2$$

$$= \frac{-40}{1} \quad \text{Simplify.}$$

So, the slope is $\frac{-40}{1}$ or -40.

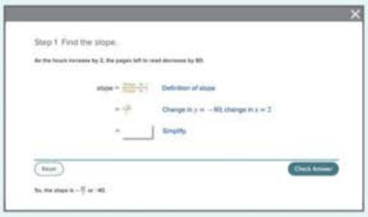
(continued on next page)

Think About It! How would you begin writing the equation? See students' responses.

Talk About It! What does the slope represent in the context of this situation? Sample answer: The number of pages left to read decreases each hour.

Lesson 8-5 • Slope-Intercept Form 541

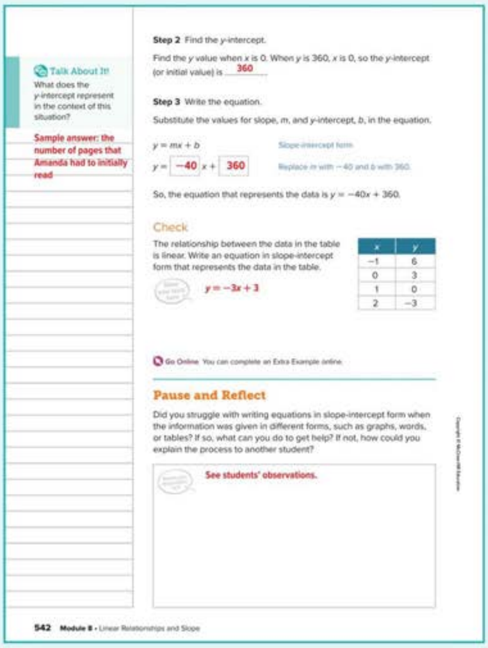
Interactive Presentation



Example 5, Write Equations in Slope-Intercept Form, Slide 2 of 6

TYPE

On Slide 2 of Example 5, students determine the slope.



Talk About It! What does the y-intercept represent in the context of this situation?

Sample answer: the number of pages that Amanda had to initially read

Step 2 Find the y-intercept.

Find the y value when x is 0. When y is 360, x is 0, so the y-intercept (or initial value) is 360.

Step 3 Write the equation.

Substitute the values for slope, m, and y-intercept, b, in the equation.

$y = mx + b$ Slope-intercept form

$y = -40x + 360$ Replace m with -40 and b with 360.

So, the equation that represents the data is $y = -40x + 360$.

Check

The relationship between the data in the table is linear. Write an equation in slope-intercept form that represents the data in the table.

x	y
−1	6
0	3
1	0
2	−3

$y = -3x + 3$

Go Online You can complete an Extra Example online.

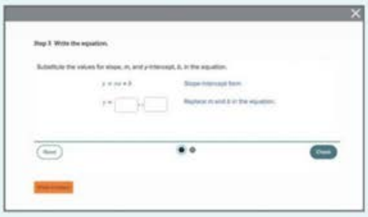
Pause and Reflect

Did you struggle with writing equations in slope-intercept form when the information was given in different forms, such as graphs, words, or tables? If so, what can you do to get help? If not, how could you explain the process to another student?

See students' observations.

542 Module 8 • Linear Relationships and Slope

Interactive Presentation



Example 5, Write Equations in Slope-Intercept Form, Slide 4 of 6

CLICK On Slide 3, students determine the y-intercept.

TYPE On Slide 4, students write the equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Write Equations in Slope-Intercept Form *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Where in the table can you find the y-intercept? Look for the y-value when the x-value is 0. When $x = 0$, $y = 360$. So, the y-intercept is 360.

OL What does the y-intercept mean within the context of the problem? Sample answer: It means the number of pages Amanda has to read before she even begins reading.

BL What else might the y-intercept tell you about this problem? Sample answer: Since she hasn't started reading, the number 360 might indicate the number of pages in the novel.

SLIDE 4

AL What is the slope and what is the y-intercept? The slope is –40. The y-intercept is 360.

OL Describe the equation in your own words, in terms of the context of the problem. Sample answer: Amanda has 360 pages to read. She reads at a rate of 40 pages per hour.

BL Why is the rate 40, but the slope is −40? Sample answer: The rate is 40, but the slope is negative because the number of pages left to read is decreasing.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Consumer Science

Objective

Students will come up with their own strategy to solve an application problem that involves comparing shipping companies.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

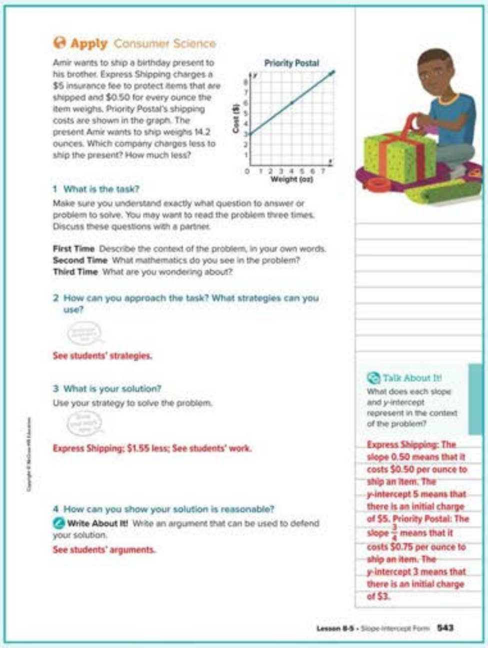
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you use the graph to determine the shipping cost for 1 ounce?
- What representation(s) could you use to help compare the charges?
- How can you use equations to determine the cost for 14.2 ounces?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Consumer Science

Amir wants to ship a birthday present to his brother. Express Shipping charges a $5 insurance fee to protect items that are shipped and $0.50 for every ounce the item weighs. Priority Postal's shipping costs are shown in the graph. The present Amir wants to ship weighs 14.2 ounces. Which company charges less to ship the present? How much less?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Express Shipping; $1.55 less; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

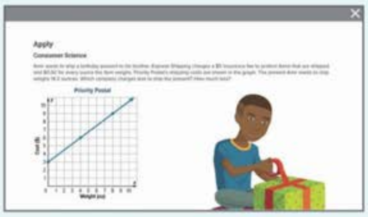
See students' arguments.

Talk About It! What does each slope and *y*-intercept represent in the context of the problem?

Express Shipping: The slope 0.50 means that it costs $0.50 per ounce to ship an item. The *y*-intercept 5 means that there is an initial charge of $5. Priority Postal: The slope $\frac{3}{4}$ means that it costs $0.75 per ounce to ship an item. The *y*-intercept 3 means that there is an initial charge of $3.

Lesson 8-5 • Slope-Intercept Form 543

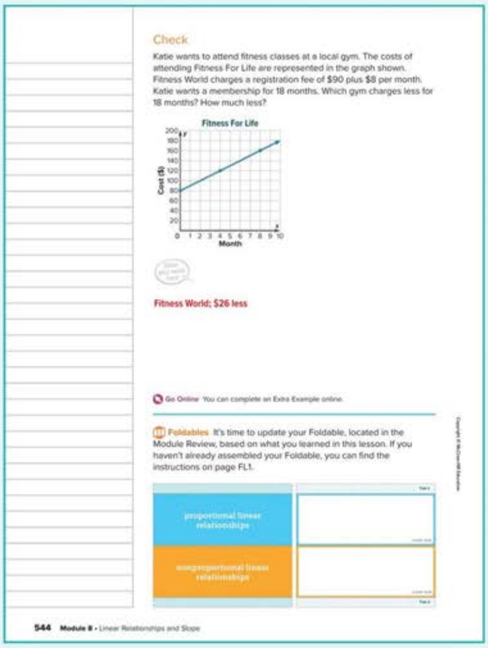
Interactive Presentation



Apply, Consumer Science

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Katie wants to attend fitness classes at a local gym. The costs of attending Fitness For Life are represented in the graph shown. Fitness World charges a registration fee of $90 plus $8 per month. Katie wants a membership for 18 months. Which gym charges less for 18 months? How much less?

Fitness World; $26 less

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

proportional linear relationships

nonproportional linear relationships

544 Module 8 • Linear Relationships and Slope

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record an example of a nonproportional relationship. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are linear relationships related to proportional relationships?

In this lesson, students learned how to write linear relationships using slope-intercept form. Encourage them to work with a partner to explain why $b = 0$ when the relationship is proportional. For example, they may state when $b = 0$, the y-intercept is 0, so the line passes through the origin.

Exit Ticket

Refer to the Exit Ticket slide. Write an equation that represents the total cost, y, of renting the scooter in relation to the number of hours, x.

$y = 7.50x + 45$

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 7—11 odd, 13—16
- Extension: Write Linear Equations in Point-Slope Form
- ALEKS® Equations of Lines

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–9, 11, 16
- Extension: Write Linear Equations in Point-Slope Form
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Slope

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Slope

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

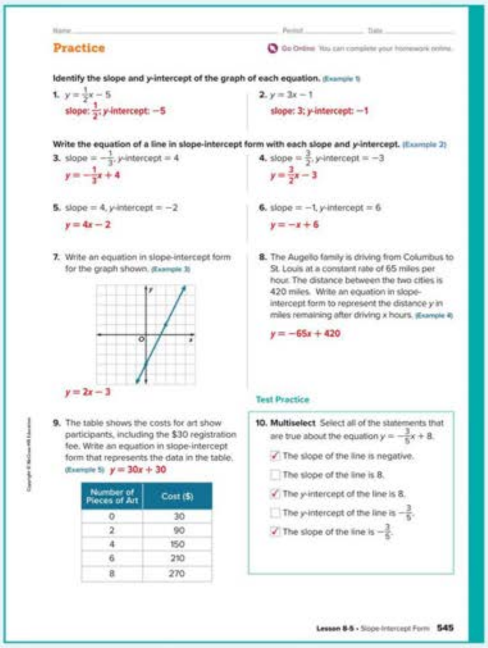
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify the slope and y-intercept of a line given an equation in slope-intercept form	1, 2
1	write an equation in slope-intercept form given the slope and y-intercept	3–6
2	write an equation in slope-intercept form given the graph of a nonproportional linear relationship	7
2	write an equation in slope-intercept form given a verbal description that represents a linear relationship	8
2	write an equation in slope-intercept form given a table of values that represents a linear relationship	9
2	extend concepts learned in class to apply them in new contexts	10
3	solve application problems involving slope-intercept form	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may incorrectly substitute values into the slope-intercept form of a line. Remind students that m is the slope and b is the y-intercept.



Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

Identify the slope and y-intercept of the graph of each equation. (Example 1)

1. $y = \frac{1}{2}x - 5$
slope: $\frac{1}{2}$; y-intercept: -5

2. $y = 3x - 1$
slope: 3; y-intercept: -1

Write the equation of a line in slope-intercept form with each slope and y-intercept. (Example 2)

3. slope $= -\frac{1}{3}$, y-intercept $= 4$
$y = -\frac{1}{3}x + 4$

4. slope $= \frac{3}{2}$, y-intercept $= -3$
$y = \frac{3}{2}x - 3$

5. slope $= 4$, y-intercept $= -2$
$y = 4x - 2$

6. slope $= -1$, y-intercept $= 6$
$y = -x + 6$

7. Write an equation in slope-intercept form for the graph shown. (Example 3)
$y = 2x - 3$

8. The Augello family is driving from Columbus to St. Louis at a constant rate of 65 miles per hour. The distance between the two cities is 420 miles. Write an equation in slope-intercept form to represent the distance y in miles remaining after driving x hours. (Example 4)
$y = -65x + 420$

9. The table shows the costs for art show participants, including the $30 registration fee. Write an equation in slope-intercept form that represents the data in the table. (Example 5) $y = 30x + 30$

Number of Pieces of Art	Cost ($)
0	30
2	90
4	150
6	210
8	270

Test Practice

10. **Multiselect** Select all of the statements that are true about the equation $y = -\frac{3}{5}x + 8$.
- [x] The slope of the line is negative.
- [ ] The slope of the line is 8.
- [x] The y-intercept of the line is 8.
- [ ] The y-intercept of the line is $-\frac{3}{5}$.
- [x] The slope of the line is $-\frac{3}{5}$.

Lesson 8-5 • Slope-Intercept Form 545

8.EE.B.6

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Apply *indicates multi-step problem

*11. While on vacation, Santiago wants to go snorkeling. Coral Snorkeling charges $25 to rent equipment and $30 per hour of boat rental. Sea Water Adventures' costs are shown in the graph. Santiago wants to snorkel for 3 hours. Which company costs less for 3 hours? How much less?

Coral Snorkeling; $5 less

*12. Green's Flowers delivers standard flower arrangements for a $15 delivery fee. Each standard flower arrangement costs $45. The table for Binder's Bouquets shows the costs for different numbers of flower arrangements. Which florist charges less for delivering 8 standard flower arrangements? How much less?

Binder's Bouquets; $35.08 less

Binder's Bouquets

Number of Flower Arrangements, x	Total Cost ($), y
0	20.00
1	59.99
2	99.98
3	139.97
4	179.96

Higher-Order Thinking Problems

13. Write an equation of a line that does not have an x-intercept.

Sample answer: $y = -3$

14. **Make a Conjecture** Describe what happens to the graph of a line if the slope is doubled.

The line is twice as steep.

15. The equation of a line is $y = 3.6x + 2$. What is the rise and run of the slope?

Sample answer: rise = 18, run = 5

16. **Find the Error** Thomas paid a one-time registration fee of $15 and $10 for each cycling class he took. He determined the equation for this relationship is $y = 15x + 10$. Find his error and correct it.

Thomas paid a one-time registration fee. That fee is the y-intercept. The $10 per class cost is the rate of change, or slope, of the equation. So, the equation is $y = 10x + 15$.

546 Module 8 • Linear Relationships and Slope

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will describe what happens to the graph of a line if the slope is doubled. Encourage students to support their conjecture with an example or supporting details.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 16, students will find the mistake in the problem and correct it. Encourage students to determine the error and explain how they could correct it**.**

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 11–12 Have students work in pairs. Have students individually read Exercise 11 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 12.

Be sure everyone understands.

Use with Exercises 13–14 Have students work in groups of 3-4 to solve the problem in Exercise 13. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 14.

Lesson 8-6

Graph Linear Equations

8.EE.B.6

LESSON GOAL

Students will graph lines in slope-intercept form, vertical lines, and horizontal lines.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Graph Equations in Slope-Intercept Form
Example 1: Graph Lines Using Slope-Intercept Form
Example 2: Graph Lines Using Slope-Intercept Form
Learn: Graphs of Horizontal Lines
Example 3: Graph Horizontal Lines
Learn: Graphs of Vertical Lines
Example 4: Graph Vertical Lines
Apply: Travel

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Graph Linear Equations in Point-Slope Form		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 51 of the *Language Development Handbook* to help your students build mathematical language related to graphing linear equations.

ELL You can use the tips and suggestions on page T51 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Expressions and Equations
Major Cluster(s): In this lesson, students address the major cluster **8.EE.C** by graphing lines in slope-intercept form and by graphing vertical and horizontal lines.
Standards for Mathematical Content: **8.E E.B.6**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students wrote nonproportional linear relationships in the form $y = mx + b$.
8.EE.B.6

Now
Students graph lines in slope-intercept form, vertical lines, and horizontal lines.
8.EE.B.6

Next
Students will use precise terminology to classify the likelihood of simple events.
7.SP.C.5

Rigor

The Three Pillars of Rigor

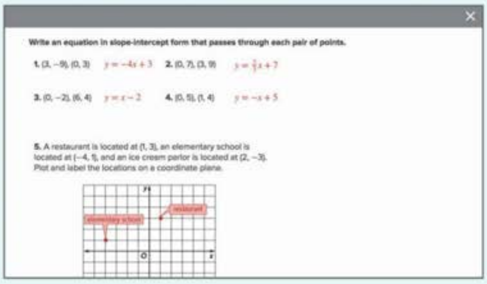
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to develop *understanding* of nonproportional linear relationships. They learn how to graph equations written in slope-intercept form, and come to understand that vertical lines ($x = a$) and horizontal lines ($y = b$) have specific equations used to represent the lines. They *apply* this understanding to solve real-world problems.

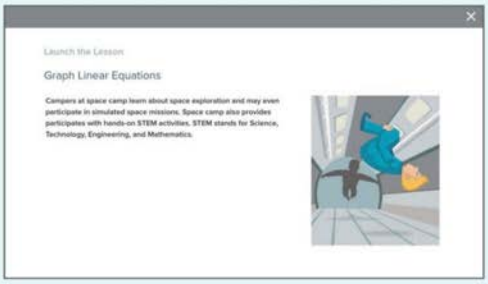
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- writing equations in slope-intercept form (Exercises 1–4)
- graphing on the coordinate plane (Exercise 5)

1–5. See Warm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about graphing the equation that represents the cost to attend a week-long space camp.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What does the *slope* of a line describe? In what other real-world contexts have you heard the term *slope* used? Sample answer: Slope describes the steepness of the line. I've also heard the term slope when describing a ski slope, or the slope of a roof or hill.
- What does it mean to *intercept* an object? How can you use this to describe the *y-intercept* of a line? Sample answer: To intercept an object means to obstruct it, or prevent it from continuing. The *y*-intercept of a line is the point at which the line intercepts the *y*-axis.

Learn Graph Equations in Slope-Intercept Form

Objective

Students will learn how to graph an equation in slope-intercept form by using the slope and y-intercept.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Have students watch the animation on Slide 1. The animation illustrates graphing an equation in slope-intercept form.
- Find sample answers for the *Talk About It!* questions.

Example 1 Graph Lines Using Slope-Intercept Form

Objective

Students will graph an equation in slope-intercept form by using the slope and y-intercept.

Questions for Mathematical Discourse

SLIDE 2

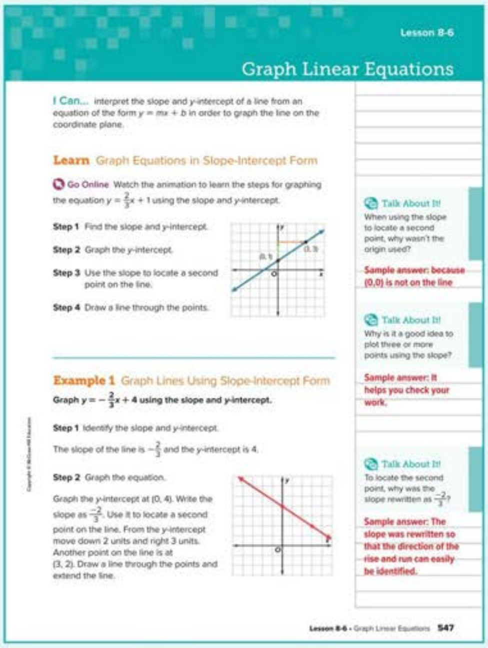
AL Is the equation given in slope-intercept form? Explain. yes; Sample answer: It is written in the form $y = mx + b$.

OL How do you know $-\frac{2}{3}$ is the slope and 4 is the y-intercept? Sample answer: $-\frac{2}{3}$ is the slope because it replaces m, the slope, in the equation $y = mx + b$. The y-intercept is 4, because it replaces b, the y-intercept, in the equation $y = mx + b$.

BL If the equation was written as $y = 4 - \frac{2}{3}x$, does the slope and y-intercept change? Explain. no; Sample answer: The slope still remains $-\frac{2}{3}$ and the y-intercept still remains 4, because addition is commutative.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Lesson 8-6

Graph Linear Equations

I Can... interpret the slope and y-intercept of a line from an equation of the form $y = mx + b$ in order to graph the line on the coordinate plane.

Learn Graph Equations in Slope-Intercept Form

Go Online Watch the animation to learn the steps for graphing the equation $y = \frac{2}{3}x + 1$ using the slope and y-intercept.

Step 1 Find the slope and y-intercept.

Step 2 Graph the y-intercept.

Step 3 Use the slope to locate a second point on the line.

Step 4 Draw a line through the points.

Talk About It! When using the slope to locate a second point, why wasn't the origin used? Sample answer: because (0,0) is not on the line

Talk About It! Why is it a good idea to plot three or more points using the slope? Sample answer: It helps you check your work.

Example 1 Graph Lines Using Slope-Intercept Form

Graph $y = -\frac{2}{3}x + 4$ using the slope and y-intercept.

Step 1 Identify the slope and y-intercept.

The slope of the line is $-\frac{2}{3}$ and the y-intercept is 4.

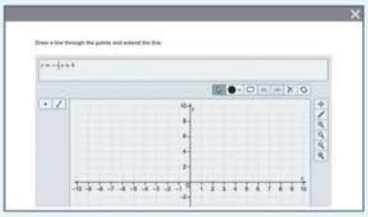
Step 2 Graph the equation.

Graph the y-intercept at (0, 4). Write the slope as $\frac{-2}{3}$. Use it to locate a second point on the line. From the y-intercept move down 2 units and right 3 units. Another point on the line is at (3, 2). Draw a line through the points and extend the line.

Talk About It! To locate the second point, why was the slope rewritten as $\frac{-2}{3}$? Sample answer: The slope was rewritten so that the direction of the rise and run can easily be identified.

Lesson 8-6 • Graph Linear Equations 547

Interactive Presentation


Example 1, Graph Lines Using Slope-Intercept Form, Slide 3 of 5

DRAG & DROP

On Slide 2, students drag to indicate the slope and y-intercept.

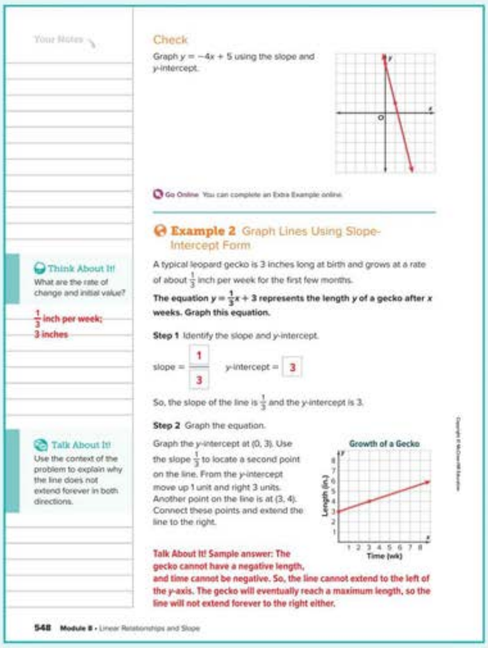
eTOOL

On Slide 3, students use the Coordinate Graphing eTool to graph the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Your Notes

Check

Graph $y = -4x + 5$ using the slope and y-intercept.

Go Online You can complete an Extra Example online.

Example 2 Graph Lines Using Slope-Intercept Form

A typical leopard gecko is 3 inches long at birth and grows at a rate of about $\frac{1}{3}$ inch per week for the first few months.

The equation $y = \frac{1}{3}x + 3$ represents the length y of a gecko after x weeks. Graph this equation.

Think About It! What are the rate of change and initial value? $\frac{1}{3}$ inch per week; 3 inches

Step 1 Identify the slope and y-intercept.

slope = $\frac{1}{3}$ y-intercept = 3

So, the slope of the line is $\frac{1}{3}$ and the y-intercept is 3.

Step 2 Graph the equation.

Graph the y-intercept at (0, 3). Use the slope $\frac{1}{3}$ to locate a second point on the line. From the y-intercept move up 1 unit and right 3 units. Another point on the line is at (3, 4). Connect these points and extend the line to the right.

Talk About It! Use the context of the problem to explain why the line does not extend forever in both directions.

Talk About It! Sample answer: The gecko cannot have a negative length, and time cannot be negative. So, the line cannot extend to the left of the y-axis. The gecko will eventually reach a maximum length, so the line will not extend forever to the right either.

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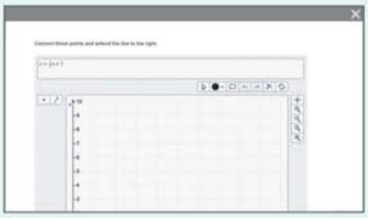
Interactive Presentation



Example 2, Graph Lines Using Slope Intercept Form, Slide 3 of 5

DRAG & DROP

On Slide 2, students drag to indicate the slope and y-intercept.

eTOOL

On Slide 3, students use the Coordinate Graphing eTool to graph the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Graph Lines Using Slope Intercept Form

Objective

Students will graph an equation in slope-intercept form by using the slope and y-intercept.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 4, encourage them to reason about why it would not make sense, within the context of this problem, for a gecko to have a negative length, or for time to be negative. Students should also be able to reason why the line will not extend forever to the right either, because a gecko will eventually reach a maximum length.

5 Use Appropriate Tools Strategically Students will use the Coordinate Graphing eTool to graph the line on thecoordinate plane.

Questions for Mathematical Discourse

SLIDE 2

AL Is the equation given in slope-intercept form? Explain. yes; Sample answer: It is written in the form $y = mx + b$.

OL How do you know $\frac{1}{3}$ is the slope and 3 is the y-intercept? Sample answer: $\frac{1}{3}$ is the slope because it replaces m, the slope, in the equation $y = mx + b$. The y-intercept is 3 because it replaces b, the y-intercept, in the equation $y = mx + b$.

BL Will the line pass through the point (3, 5)? Explain. no; Sample answer: The point (3, 5) does not work in the equation $y = \frac{1}{3}x + 3$, since $5 \neq \frac{1}{3}(3) + 3$. So, the line will not pass through the point (3, 5).

SLIDE 3

AL Why is (3, 0) *not* the y-intercept? Sample answer: (3, 0) is a point on the x-axis. A y-intercept has 0 as the x-coordinate and the y-intercept from the equation as the y-coordinate.

OL How can you verify that you graphed the correct equation? Sample answer: I can use the slope to locate more points on the line and verify they all fall on the same line.

BL Use the graph to determine when the gecko will reach a length of 5 inches. At 6 weeks, the gecko will reach a length of 5 inches.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Graphs of Horizontal Lines

Objective

Students will understand how the graph of a horizontal line is related to its equation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of what the equation $y = 3$ means. The y-coordinate of every point on the line $y = 3$ will be equal to 3.

Teaching Notes

SLIDE 1

All points on a horizontal line have the same y-coordinate. Have students use the markers on the interactive tool to verify that each of the selected ordered pairs that fall on the line $y = 3$ has the same y-coordinate. Have students use the slope-intercept form of a line to derive the equation for any horizontal line, since they know that the slope of a horizontal line is 0.

Talk About It!

SLIDE 2

Mathematical Discourse

Explain why it makes sense that the equation of the line is $y = 3$.
Sample answer: No matter what point falls on the line, the y-coordinate of that point will always be equal to 3.

DIFFERENTIATE

Enrichment Activity BL

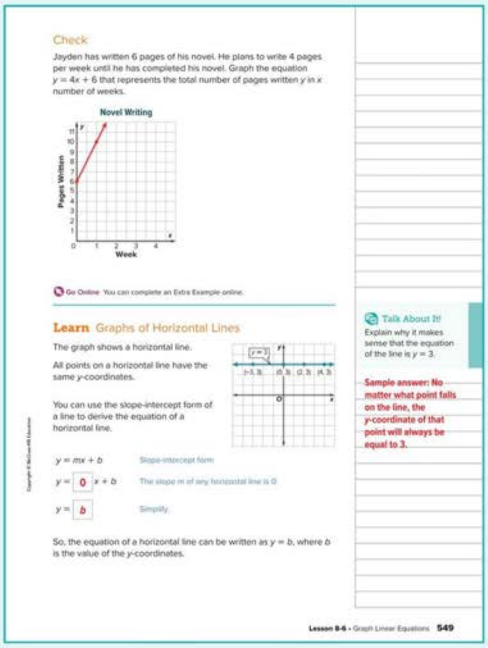
To further your students' understanding of the equations of horizontal lines, have students work with a partner to state whether each of the following equations represent horizontal lines. Have them be able to justify their responses.

A. $y = -5$ yes; This equation is in the form $y = b$.

B. $y + 2 = -1$ yes; This equation can be written in the form $y = b$ by subtracting 2 from each side.

C. $4y = -12$ yes; This equation can be written in the form $y = b$ by dividing each side of the equation by 4.

D. $x + y = -6$ no; This equation cannot be written in the form $y = b$.



Check

Jayden has written 6 pages of his novel. He plans to write 4 pages per week until he has completed his novel. Graph the equation $y = 4x + 6$ that represents the total number of pages written y in x number of weeks.

Go Online You can complete an Extra Example online.

Learn Graphs of Horizontal Lines

The graph shows a horizontal line.

All points on a horizontal line have the same y-coordinates.

You can use the slope-intercept form of a line to derive the equation of a horizontal line.

$y = mx + b$ Slope-intercept form

$y = \boxed{0}x + b$ The slope m of any horizontal line is 0

$y = \boxed{b}$ Simplify.

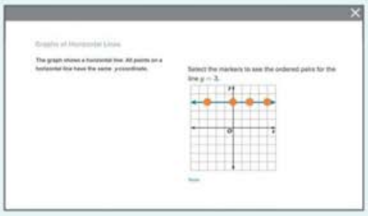
So, the equation of a horizontal line can be written as $y = b$, where b is the value of the y-coordinates.

Talk About It!
Explain why it makes sense that the equation of the line is $y = 3$.

Sample answer: No matter what point falls on the line, the y-coordinate of that point will always be equal to 3.

Lesson 8-6 • Graph Linear Equations 549

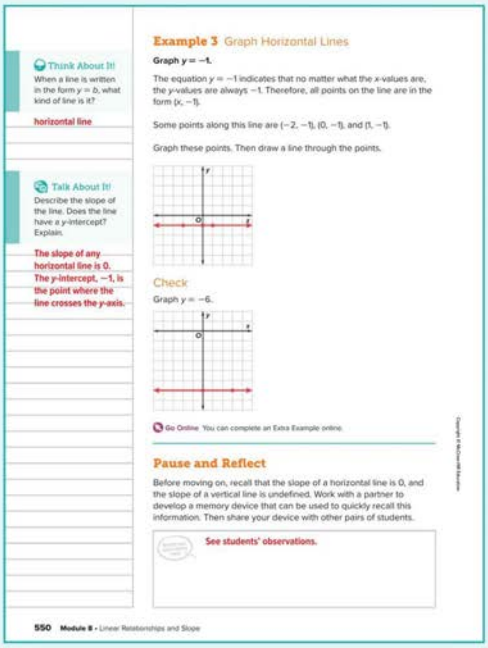
Interactive Presentation



Learn, Graphs of Horizontal Lines, Slide 1 of 2

CLICK

On Slide 1, students select the markers to see the ordered pairs for the line $y = 3$.



Think About It! When a line is written in the form $y = b$, what kind of line is it?

horizontal line

Talk About It! Describe the slope of the line. Does the line have a y-intercept? Explain.

The slope of any horizontal line is 0. The y-intercept, −1, is the point where the line crosses the y-axis.

Example 3 Graph Horizontal Lines

Graph $y = -1$.

The equation $y = -1$ indicates that no matter what the x-values are, the y-values are always −1. Therefore, all points on the line are in the form $(x, -1)$.

Some points along this line are (−2, −1), (0, −1), and (1, −1).

Graph these points. Then draw a line through the points.

Check

Graph $y = -6$.

Go Online You can complete an Extra Example online.

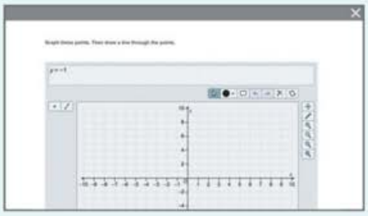
Pause and Reflect

Before moving on, recall that the slope of a horizontal line is 0, and the slope of a vertical line is undefined. Work with a partner to develop a memory device that can be used to quickly recall this information. Then share your device with other pairs of students.

See students' observations.

550 Module 8 • Linear Relationships and Slope

Interactive Presentation



Example 3, Graph Horizontal Lines, Slide 2 of 4

eTOOL

On Slide 2, students use the Coordinate Graphing eTool to graph the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

8.EE.B.6

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Graph Horizontal Lines

Objective

Students will graph a horizontal line given an equation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to use reasoning to understand why the equation represents the graph of a horizontal line.

5 Use Appropriate Tools Strategically Students will use the Coordinate Graphing eTool to graph the line on the coordinate plane.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to accurately describe the slope of this horizontal line and to use clear language to explain whether the line has a y-intercept.

7 Look For and Make Use of Structure Encourage students to make sense of the structure of the equation to determine that every point that falls on this line will have a y-coordinate of −1.

Questions for Mathematical Discourse

SLIDE 2

AL Other than (−2, −1), (0, −1), and (1, −1), what are some other points that lie on this line? Sample answer: (−4, −1), (3, −1), (5, −1)

OL What axes, if any, will this line intercept? Since the line is horizontal, it will not intercept the x-axis. It will intercept the y-axis at (0, −1).

OL Write the equation of a line that runs parallel to this line. Sample answer: $y = 3$

BL Write the equation of a line that is perpendicular to this line. Sample answer: $x = 3$

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Graphs of Vertical Lines

Objective

Students will understand how the graph of a vertical line is related to its equation.

Go Online to find additional teaching notes.

Example 4 Graph Vertical Lines

Objective

Students will graph a vertical line given an equation.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically St udents will use the Coordinate Graphing eTool to graph the line on the coordinate plane.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to accurately describe the fact that the slope of any vertical line is undefined, and to use clear language to explain why the line has no y-intercept.

7 Look For and Make Use of Structure Encourage students to make sense of the structure of the equation to determine that every point that falls on this line will have a x-coordinate of 4.

Questions for Mathematical Discourse

SLIDE 2

AL What points can you use to graph the line? Explain. Sample answer: $(4, -2)$, $(4, 0)$, and $(4, 1)$; the x-coordinates will all be 4 and the y-coordinates can be any number.

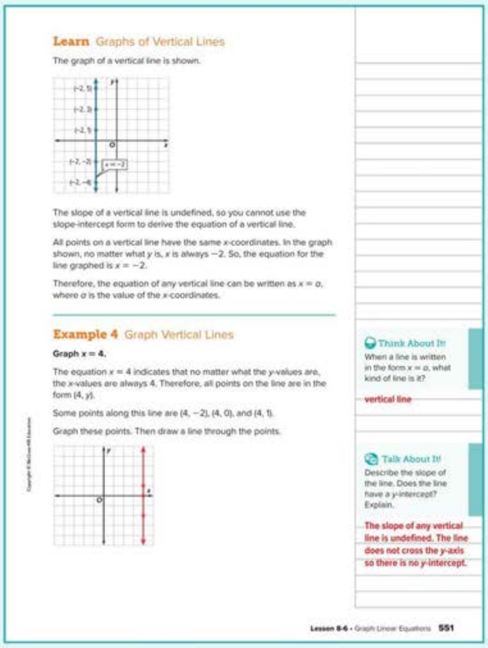
OL The general form of points on the line $x = 4$ is $(4, y)$. Why is this true? Sample answer: Any points that lie on this vertical line will always have an x-coordinate of 4. Since the line will go on forever in each direction, the y-coordinate can be any number.

OL Write the equation for a vertical line that is parallel to this line. Sample answer: $x = 7$

BL Will two vertical lines ever intersect one another? Sample answer: No, unless they are the same line, two vertical lines will never intersect one another because they are parallel.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Graphs of Vertical Lines

The graph of a vertical line is shown.

The slope of a vertical line is undefined, so you cannot use the slope-intercept form to derive the equation of a vertical line.

All points on a vertical line have the same x-coordinates. In the graph shown, no matter what y is, x is always -2. So, the equation for the line graphed is $x = -2$.

Therefore, the equation of any vertical line can be written as $x = a$, where a is the value of the x-coordinates.

Example 4 Graph Vertical Lines

Graph $x = 4$.

The equation $x = 4$ indicates that no matter what the y-values are, the x-values are always 4. Therefore, all points on the line are in the form $(4, y)$.

Some points along this line are $(4, -2)$, $(4, 0)$, and $(4, 1)$.

Graph these points. Then draw a line through the points.

Think About It! When a line is written in the form $x = a$, what kind of line is it?

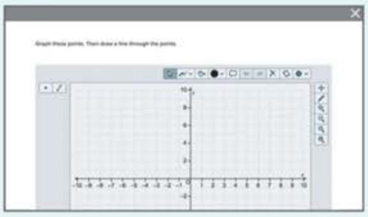
vertical line

Talk About It! Describe the slope of the line. Does the line have a y-intercept? Explain.

The slope of any vertical line is undefined. The line does not cross the y-axis so there is no y-intercept.

Lesson 8-6 • Graph Linear Equations 551

Interactive Presentation



Example 4, Graphs of Vertical Lines, Slide 2 of 4

eTOOL

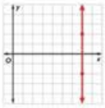
On Slide 2 of Example 4, students use the Coordinate Graphing eTool to graph the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Graph $x = 7$.



Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that will help you study the concepts you learned today in class.

See students' observations.

552 Module 8 • Linear Relationships and Slope

Apply Travel

Objective

Students will come up with their own strategy to solve an application problem involving travel times and distances.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

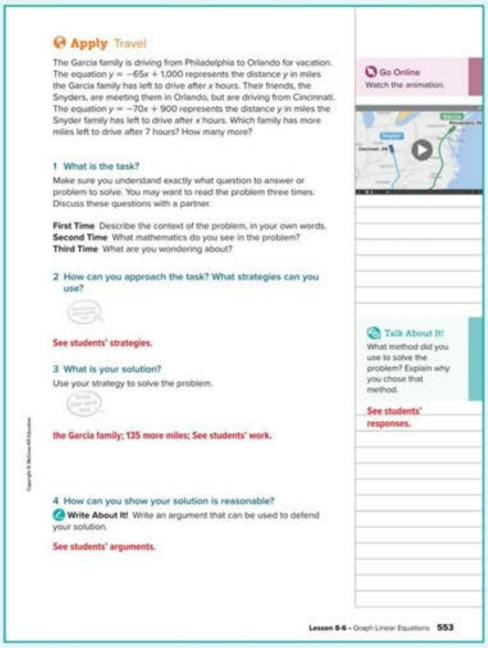
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What representation(s) could you use to help compare the distances?
- What does each equation tell you about each family's rates?
- Why are the rates negative?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Travel

The Garcia family is driving from Philadelphia to Orlando for vacation. The equation $y = -65x + 1{,}000$ represents the distance y in miles the Garcia family has left to drive after x hours. Their friends, the Snyders, are meeting them in Orlando, but are driving from Cincinnati. The equation $y = -70x + 900$ represents the distance y in miles the Snyder family has left to drive after x hours. Which family has more miles left to drive after 7 hours? How many more?

Go Online
Watch the animation.

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

the Garcia family; 135 more miles; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!
What method did you use to solve the problem? Explain why you chose that method.

See students' responses.

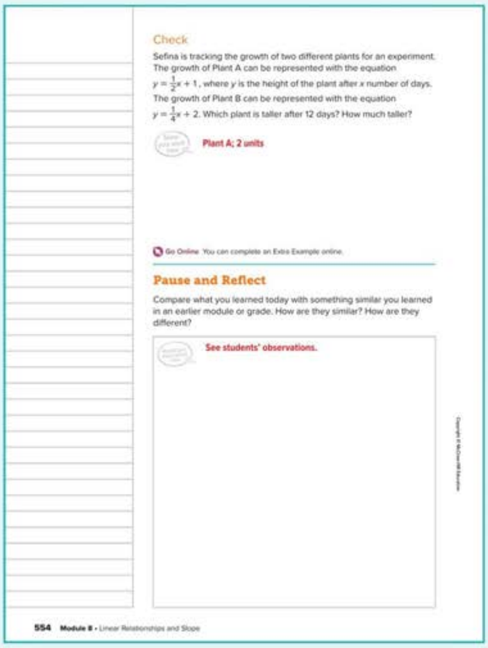
Lesson 8-6 • Graph Linear Equations 553

Interactive Presentation

Apply, Travel

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Sefina is tracking the growth of two different plants for an experiment. The growth of Plant A can be represented with the equation $y = \frac{1}{2}x + 1$, where y is the height of the plant after x number of days. The growth of Plant B can be represented with the equation $y = \frac{1}{4}x + 2$. Which plant is taller after 12 days? How much taller?

Plant A; 2 units

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you learned today with something similar you learned in an earlier module or grade. How are they similar? How are they different?

See students' observations.

554 Module 8 • Linear Relationships and Slope

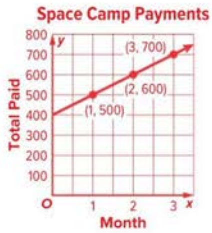
Interactive Presentation

Exit Ticket

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | **3 APPLICATION**

Exit Ticket

Refer to the Exit Ticket slide. Suppose a week-long space camp costs $800. You paid an initial $400 deposit and then paid the rest in monthly payments of $100. The situation can be represented with the equation $y = 100x + 400$. Graph the line on the coordinate plane.



ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 3, 7, 9–12
- Extension: Graph Linear Equations in Point-Slope Form
- ALEKS Tables and Graphs of Lines

IF students score 66-89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8, 11
- Extension: Graph Linear Equations in Point-Slope Form
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Ordered Pairs

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS Ordered Pairs

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

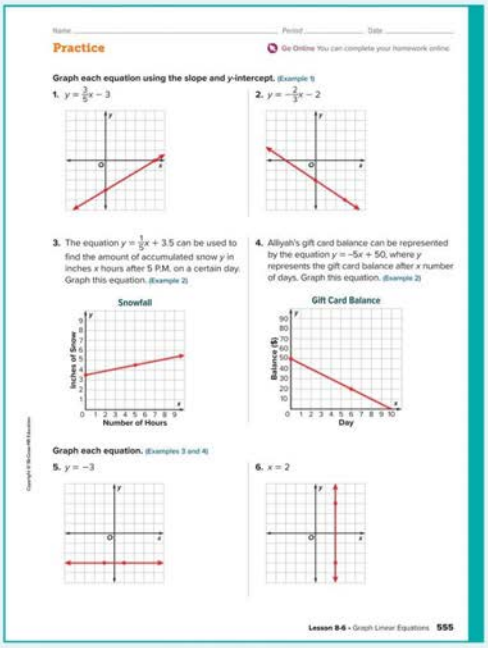
Suggested Assignments

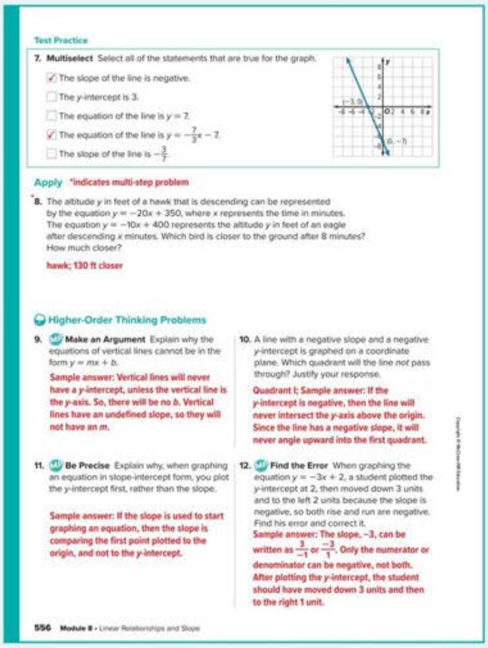
Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	graph equations in slope-intercept form by using the slope and y-intercept	1, 2
2	graph equations in slope-intercept form by using the slope and y-intercept	3, 4
1	graph a horizontal line given an equation	5
1	graph a vertical line given an equation	6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems that involve graphing equations in slope-intercept form	8
3	higher-order and critical thinking skills	9–12

Common Misconception

Some students may incorrectly use the coefficient of x, or m, to graph the y-intercept, and use the constant, or b, as the slope. Remind them that the constant in the equation is the y-intercept, and it should be graphed first. Then they should use the coefficient of x to graph more points on the coordinate plane.





Test Practice

7. **Multiselect** Select all of the statements that are true for the graph.

- [x] The slope of the line is negative.
- [ ] The y-intercept is 3.
- [ ] The equation of the line is $y = 7$.
- [x] The equation of the line is $y = -\frac{7}{3}x - 7$.
- [ ] The slope of the line is $-\frac{3}{7}$.

Apply *indicates multi-step problem

*8. The altitude y in feet of a hawk that is descending can be represented by the equation $y = -20x + 350$, where x represents the time in minutes. The equation $y = -10x + 400$ represents the altitude y in feet of an eagle after descending x minutes. Which bird is closer to the ground after 8 minutes? How much closer?

hawk; 130 ft closer

Higher-Order Thinking Problems

9. **Make an Argument** Explain why the equations of vertical lines cannot be in the form $y = mx + b$.

Sample answer: Vertical lines will never have a y-intercept, unless the vertical line is the y-axis. So, there will be no b. Vertical lines have an undefined slope, so they will not have an m.

10. A line with a negative slope and a negative y-intercept is graphed on a coordinate plane. Which quadrant will the line *not* pass through? Justify your response.

Quadrant I; Sample answer: If the y-intercept is negative, then the line will never intersect the y-axis above the origin. Since the line has a negative slope, it will never angle upward into the first quadrant.

11. **Be Precise** Explain why, when graphing an equation in slope-intercept form, you plot the y-intercept first, rather than the slope.

Sample answer: If the slope is used to start graphing an equation, then the slope is comparing the first point plotted to the origin, and not to the y-intercept.

12. **Find the Error** When graphing the equation $y = -3x + 2$, a student plotted the y-intercept at 2, then moved down 3 units and to the left 2 units because the slope is negative, so both rise and run are negative. Find his error and correct it.

Sample answer: The slope, −3, can be written as $\frac{3}{-1}$ or $\frac{-3}{1}$. Only the numerator or denominator can be negative, not both. After plotting the y-intercept, the student should have moved down 3 units and then to the right 1 unit.

556 Module 8 • Linear Relationships and Slope

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of others In Exercise 9, students will explain why the equations of vertical lines cannot take the form $y = mx + b$. Encourage students to use the structure of the equation to explain why vertical lines cannot be of the form $y = mx + b$.

6 Attend to Precision In Exercise 11, students will explain why, when graphing an equation in slope-intercept form, you plot the y-intercept first. Encourage students to explain why plotting the y-intercept first is crucial to plotting the equation of a line correctly.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students will find the mistake in the problem and correct it. Encourage students to determine the error and explain how they could fix it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 8 Have students work together to prepare a brief demonstration that illustrates why this problem may require multiple steps to solve. For example, before they can identify which bird is closer to the ground, they can first graph each bird's equation on the same coordinate plane. Have each pair or group of students present their response to the class.

Clearly and precisely explain.
Use with Exercise 11 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as using a graph in their responses.

Module 8 • Linear Relationships and Slope

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of linear relationships, represented as tables, graphs, and equations. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

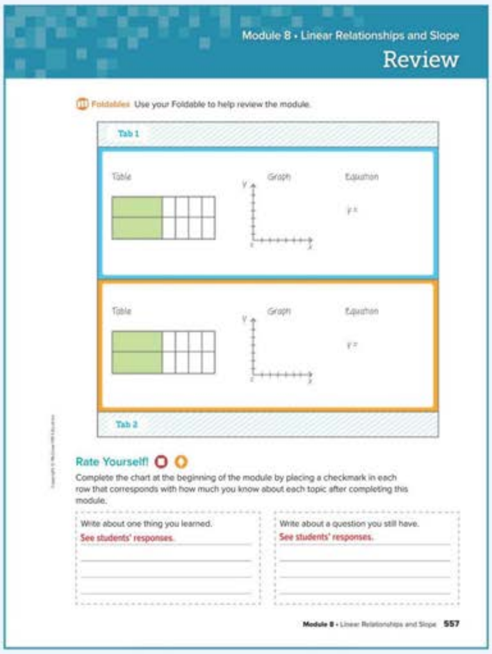
Assessment Resources

Put It All Together: Lessons 8-1, 8-2, 8-3, and 8-4
Put It All Together: Lessons 8-5 and 8-6
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Expressions and Equations**.

- Graph Proportional Relationships
- Slope of a Line and Rate of Change


Module 8 • Linear Relationships and Slope

Review

Foldables Use your Foldable to help review the module.

Tab 1

Table Graph Equation

y =

Table Graph Equation

y =

Tab 2

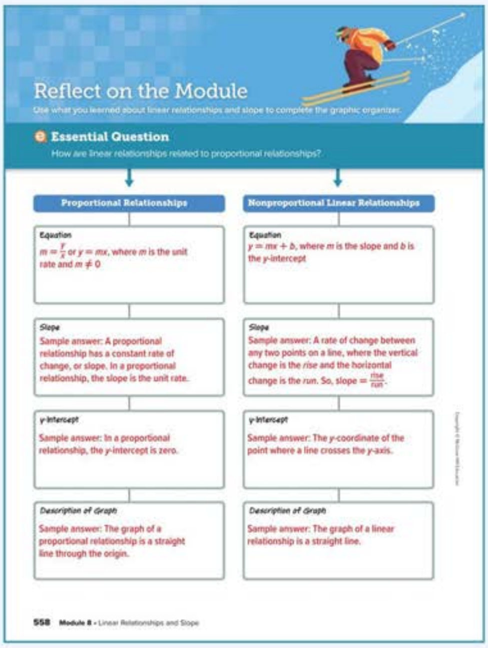
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 8 • Linear Relationships and Slope 557


Reflect on the Module

Use what you learned about linear relationships and slope to complete the graphic organizer.

Essential Question

How are linear relationships related to proportional relationships?

Proportional Relationships	Nonproportional Linear Relationships
Equation $m = \frac{y}{x}$ or $y = mx$, where m is the unit rate and $m \neq 0$	**Equation** $y = mx + b$, where m is the slope and b is the y-intercept
Slope Sample answer: A proportional relationship has a constant rate of change, or slope. In a proportional relationship, the slope is the unit rate.	**Slope** Sample answer: A rate of change between any two points on a line, where the vertical change is the *rise* and the horizontal change is the *run*. So, slope = $\frac{\text{rise}}{\text{run}}$.
y-Intercept Sample answer: In a proportional relationship, the y-intercept is zero.	**y-Intercept** Sample answer: The y-coordinate of the point where a line crosses the y-axis.
Description of Graph Sample answer: The graph of a proportional relationship is a straight line through the origin.	**Description of Graph** Sample answer: The graph of a linear relationship is a straight line.

558 Module 8 • Linear Relationships and Slope

Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How are linear relationships related to proportional relationships?
See students' graphic organizers.

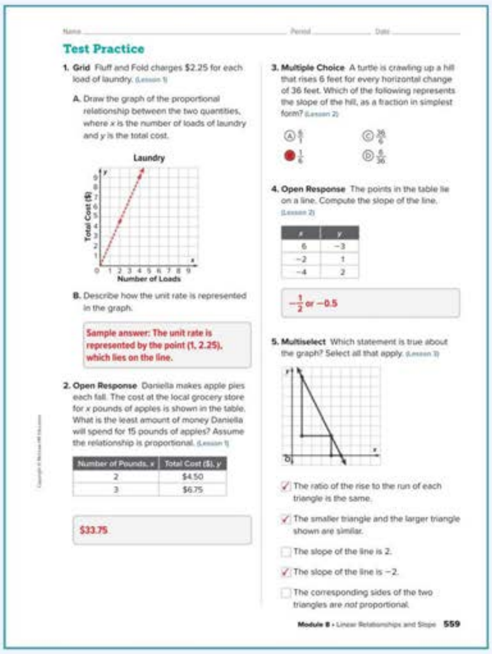
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–12 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	3
Multiselect	Multiple answers may be correct. Students must select all correct answers.	5
Table Item	Students complete a table by entering in the correct values.	8
Grid	Students create a graph on an online coordinate plane.	1, 11, 12
Open Response	Students construct their own response in the area provided.	2, 4, 6, 7, 9, 10

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
Foundational for 8.EE.B.6	8-2	3, 4
8.EE.B.5	8-1	1, 2
8.EE.B.6	8-3, 8-4, 8-5, 8-6	5–12


Name ______ Period ______ Date ______

Test Practice

1. **Grid** Fluff and Fold charges $2.25 for each load of laundry. (Lesson 1)

A. Draw the graph of the proportional relationship between the two quantities, where x is the number of loads of laundry and y is the total cost.

B. Describe how the unit rate is represented in the graph.

Sample answer: The unit rate is represented by the point (1, 2.25), which lies on the line.

2. **Open Response** Daniella makes apple pies each fall. The cost at the local grocery store for x pounds of apples is shown in the table. What is the least amount of money Daniella will spend for 15 pounds of apples? Assume the relationship is proportional. (Lesson 1)

Number of Pounds, x	Total Cost ($), y
2	$4.50
3	$6.75

$33.75

3. **Multiple Choice** A turtle is crawling up a hill that rises 6 feet for every horizontal change of 36 feet. Which of the following represents the slope of the hill, as a fraction in simplest form? (Lesson 2)

(A) $\frac{6}{1}$ (B) $\frac{1}{6}$ (C) $\frac{36}{6}$ (D) $\frac{6}{36}$

4. **Open Response** The points in the table lie on a line. Compute the slope of the line. (Lesson 2)

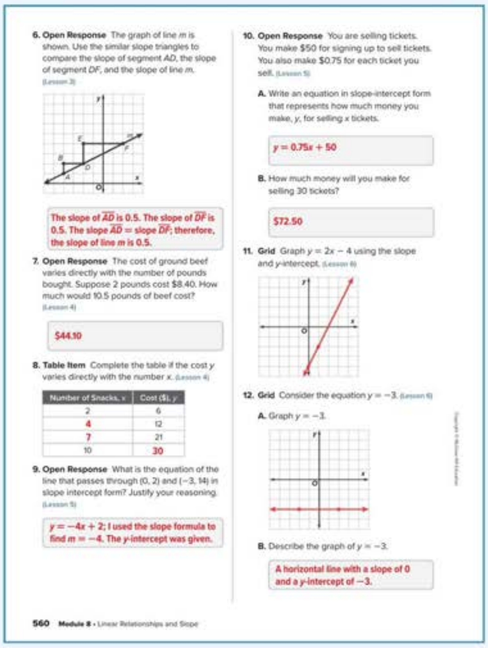
x	y
6	−3
−2	1
−4	2

$-\frac{1}{2}$ or -0.5

5. **Multiselect** Which statement is true about the graph? Select all that apply. (Lesson 3)

- [x] The ratio of the rise to the run of each triangle is the same.
- [x] The smaller triangle and the larger triangle shown are similar.
- [ ] The slope of the line is 2.
- [x] The slope of the line is −2.
- [ ] The corresponding sides of the two triangles are *not* proportional.

Module 8 • Linear Relationships and Slope **559**



6. **Open Response** The graph of line *m* is shown. Use the similar slope triangles to compare the slope of segment *AD*, the slope of segment *DF*, and the slope of line *m*. (Lesson 3)

The slope of $\overline{AD}$ is 0.5. The slope of $\overline{DF}$ is 0.5. The slope $\overline{AD}$ = slope $\overline{DF}$; therefore, the slope of line *m* is 0.5.

7. **Open Response** The cost of ground beef varies directly with the number of pounds bought. Suppose 2 pounds cost $8.40. How much would 10.5 pounds of beef cost? (Lesson 4)

$44.10

8. **Table Item** Complete the table if the cost *y* varies directly with the number *x*. (Lesson 4)

Number of Snacks, *x*	Cost ($), *y*
2	6
4	12
7	21
10	30

9. **Open Response** What is the equation of the line that passes through (0, 2) and (−3, 14) in slope intercept form? Justify your reasoning. (Lesson 5)

$y = -4x + 2$; I used the slope formula to find $m = -4$. The *y*-intercept was given.

10. **Open Response** You are selling tickets. You make $50 for signing up to sell tickets. You also make $0.75 for each ticket you sell. (Lesson 5)

A. Write an equation in slope-intercept form that represents how much money you make, *y*, for selling *x* tickets.

$y = 0.75x + 50$

B. How much money will you make for selling 30 tickets?

$72.50

11. **Grid** Graph $y = 2x - 4$ using the slope and *y*-intercept. (Lesson 6)

12. **Grid** Consider the equation $y = -3$. (Lesson 6)

A. Graph $y = -3$.

B. Describe the graph of $y = -3$.

A horizontal line with a slope of 0 and a *y*-intercept of −3.

560 **Module 8** • Linear Relationships and Slope

Module 9

Probability

Module Goal

Understand probability, find the probability of simple events and compound events, and design simulations.

Focus

Domain: Statistics and Probability

Supporting Cluster(s): **7.SP.C** Investigate chance processes and develop, use, and evaluate probability models.

Standards for Mathematical Content:

7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Also addresses 7.SP.C.5, 7.SP.C.7.A, 7.SP.C.7.B, 7.SP.C.8, 7.SP.C.8.A, 7.SP.C.8.B, and 7.SP.C.8.C.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- express equivalent forms of fractions, decimals, and percents
- solve proportions

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students understood ratios and used them to solve problems.
6.RP.A.1, 6.RP.A.3

Now
Students find the probability of simple events and compound events.
7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8

Next
Students will understand independence and conditional probability.
HSS.CP.A.2, HSS.CP.A.3

Rigor

The Three Pillars of Rigor

In this module, students will develop an *understanding* of probability of simple and compound events. They will use this understanding to develop *fluency* in finding likelihoods, relative frequencies, and determining the sample space for compound events. They will also compare probabilities, design simulations, and *apply* their understanding of probability to solve real-world problems.

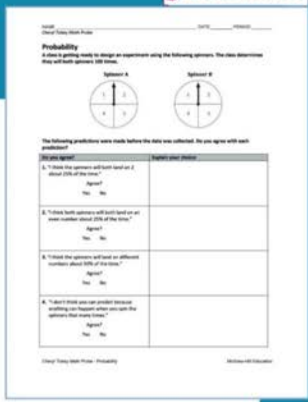
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson	Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
9-1 Find Likelihoods	7.SP.C.5	1	0.5
9-2 Relative Frequency of Simple Events	7.SP.C.6, 7.SP.C.7, 7.SP.C.7.B	1	0.5
Put It All Together 1: Lessons 9-1 and 9-2		0.5	0.25
9-3 Theoretical Probability of Simple Events	7.SP.C.7, 7.SP.C.7.A	1	0.5
9-4 Compare Probabilities of Simple Events	7.SP.C.6, 7.SP.C.7, 7.SP.C.7.A, 7.SP.C.7.B	1	0.5
Put It All Together 2: Lessons 9-1 through 9-4		0.5	0.25
9-5 Probability of Compound Events	7.SP.C.8, 7.SP.C.8.A, 7.SP.C.8.B	2	1
9-6 Simulate Chance Events	7.SP.C.8, 7.SP.C.8.C	1	0.5
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	11	5.5

Formative Assessment Math Probe
Probability

Module Resource



Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will state whether they agree with each prediction, and explain their choices.

Targeted Concepts Understand the relationship between theoretical and experimental probability and determine probability by noticing the number of possible occurrences of the wanted outcome divided by the number of all possible outcomes.

Targeted Misconceptions

- Students may not have a strategy for determining the number of desired outcomes out of all possible outcomes
- Students may incorrectly determine the probability as the number of possible occurrences of the wanted outcome divided by the remaining possible occurrences.
- Students may apply additive reasoning by counting, combining, and/or finding the difference.
- Students may incorrectly apply strategies for simple probability in compound situations.

Assign the probe after Lesson 5.

Correct Answers:
1. No
2. Yes
3. No
4. No

Collect and Assess Student Work

If the student selects...	Then the student likely...
1. Yes **2.** No **3.** Yes	used an absolute comparison of individual outcomes (as in 2 out of 8 for Exercise 1).
4. Yes	did not understand that the number of desired outcomes out of possible outcomes allows them to make predictions with a reasonable amount of accuracy.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Data Analysis and Probability
- Lesson 5, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can probability be used to predict future events? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

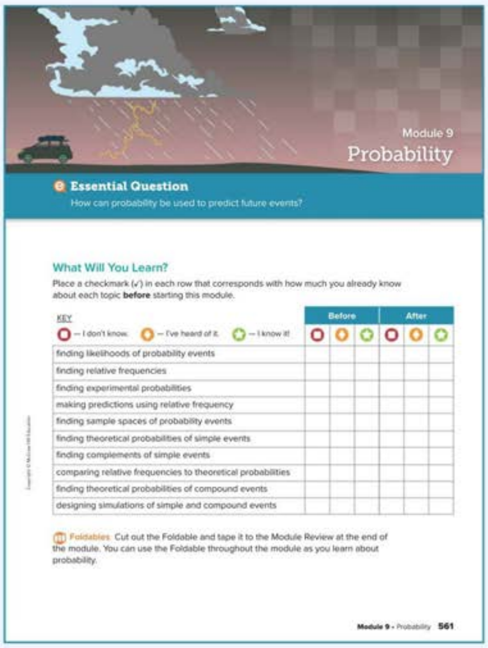
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of weather forecasting and sports statistics to introduce the idea of probability. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.


Module 9
Probability

Essential Question
How can probability be used to predict future events?

What Will You Learn?
Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

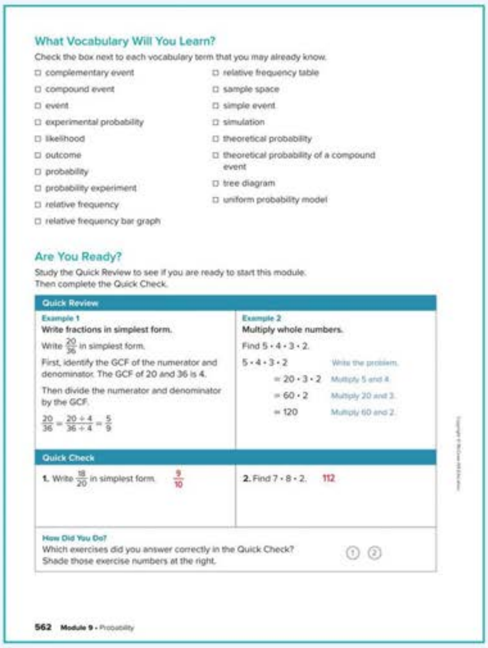
KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
finding likelihoods of probability events						
finding relative frequencies						
finding experimental probabilities						
making predictions using relative frequency						
finding sample spaces of probability events						
finding theoretical probabilities of simple events						
finding complements of simple events						
comparing relative frequencies to theoretical probabilities						
finding theoretical probabilities of compound events						
designing simulations of simple and compound events						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about probability.

Module 9 • Probability 561

Interactive Student Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ complementary event	☐ relative frequency table
☐ compound event	☐ sample space
☐ event	☐ simple event
☐ experimental probability	☐ simulation
☐ likelihood	☐ theoretical probability
☐ outcome	☐ theoretical probability of a compound event
☐ probability	☐ tree diagram
☐ probability experiment	☐ uniform probability model
☐ relative frequency	
☐ relative frequency bar graph	

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Write fractions in simplest form.
Write $\frac{20}{36}$ in simplest form.
First, identify the GCF of the numerator and denominator. The GCF of 20 and 36 is 4.
Then divide the numerator and denominator by the GCF.
$\frac{20}{36} = \frac{20 \div 4}{36 \div 4} = \frac{5}{9}$

Example 2
Multiply whole numbers.
Find $5 \cdot 4 \cdot 3 \cdot 2$.
$5 \cdot 4 \cdot 3 \cdot 2$ Write the problem.
$= 20 \cdot 3 \cdot 2$ Multiply 5 and 4.
$= 60 \cdot 2$ Multiply 20 and 3.
$= 120$ Multiply 60 and 2.

Quick Check

1. Write $\frac{18}{20}$ in simplest form. $\frac{9}{10}$

2. Find $7 \cdot 8 \cdot 2$. 112

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine.

Define Probability is the chance that an event will happen. It is the ratio of the number of favorable outcomes to the number of possible outcomes.

Example Jackson has a blue, a black, and a red pen in his book bag. He randomly selects one pen from his book bag. The probability that he will select a blue pen is $\frac{1}{3}$.

Ask What is the probability that a coin, when tossed, will land on tails? $\frac{1}{2}$, 0.5, or 50%

Are You Ready?

Students may need to review the following prerequisite skills to succeed.

- solving word problems involving simplifying and multiplying fractions
- converting among fractions, decimals, and percents
- finding equivalent ratios
- writing ratios as fractions

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Data Analysis and Probability** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Build Habits of Mind by Modeling Them

In mathematics, habits of mind are developed as students engage in solving problems. While not all of the problems need to be challenging, you can challenge students to think and talk about the mathematics in the problems and the strategies that can be used to solve them. It is important for you to model these habits of mind for your students.

How Can I Apply It?

Model problem solving for your students. Facilitate a class discussion about the problem, prior to jumping into a solution attempt – even though you may already know exactly how to solve it, at first glance. Some questions to help facilitate classroom discussion are listed below.

- What is the mathematics behind this problem?
- How can mathematics help me solve this problem?
- Are there any assumptions or variables? Will solving for the variable also solve the problem? Why or why not?
- What are strategies that might get us closer to solving the problem?

Lesson 9-1

Find Likelihoods

7.SP.C.5

LESSON GOAL

Students will solve problems that classify the likelihood of simple events.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Chance Events

Learn: Likelihood of Events

Example 1: Classify Likelihoods

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 52 of the *Language Development Handbook* to help your students build mathematical language related to likelihoods and chance events.

ELL You can use the tips and suggestions on page T52 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability

Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.C** by solving problems that classify the likelihood of simple events.

Standards for Mathematical Content: **7. SP.C.5**

Standards for Mathematical Practice: **MP 1, MP2, MP5, MP6**

Coherence

Vertical Alignment

Previous

Students understood ratios and used them to solve problems.
6.RP.A.1, 6.RP.A.3

Now

Students use precise terminology to classify the likelihood of simple events.
7.SP.C.5

Next

Students find the relative frequency of simple events and compare relative frequency to experimental probability.
7.SP.C.6, 7.SP.C.7.B

Rigor

The Three Pillars of Rigor

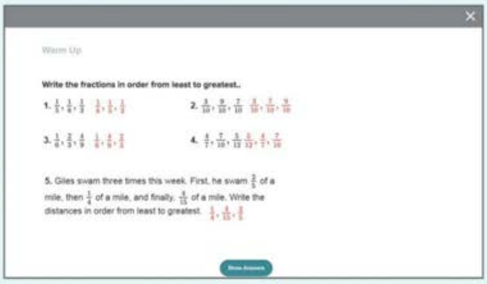
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop their *understanding* of outcomes and likelihoods of events. They build mathematical language to describe the likelihood of events and use the terminology to communicate about chance events.

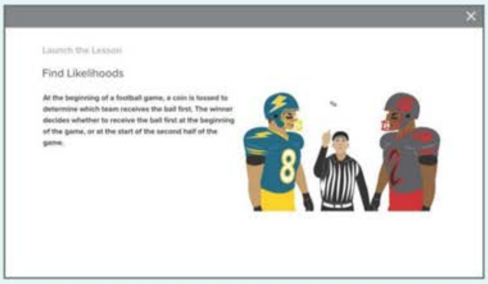
Mathematical Background

An *outcome* is a possible result. The desired outcome or set of outcomes is called an *event*. Many events cannot be predicted with total certainty. The best we can predict is the *likelihood* they are to happen.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- ordering fractions (Exercises 1–5)

Answers

1. $\frac{1}{8}, \frac{1}{5}, \frac{1}{3}$
2. $\frac{3}{10}, \frac{7}{10}, \frac{9}{10}$
3. $\frac{1}{6}, \frac{4}{9}, \frac{2}{3}$
4. $\frac{5}{12}, \frac{4}{7}, \frac{7}{10}$
5. $\frac{1}{4}, \frac{4}{15}, \frac{2}{5}$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the likelihood of winning a coin toss at the beginning of a football game.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Give an example of an *event* in everyday life. Sample answer: A football game is an example of an event in everyday life.
- What does the word *likely* mean? Sample answer: Likely means having a high chance of happening.
- Give an example of how the term *outcome* is used in everyday language. Sample answer: The outcome of an election for student body president is that one of the candidates either wins or loses.

Explore Chance Events

Objective

Students will use Web Sketchpad to explore how to describe the likelihood of events.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with different events, including events that are certain or impossible. Throughout this activity, students will use words to describe the chances of the events happening.

Inquiry Question

How can words be used to describe the chance of an event happening? Sample answer: The words *impossible*, *unlikely*, *likely*, and *certain* can be used to describe the chance of an event happening. The phrase *equally likely* is also used to describe the chance of an event happening.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

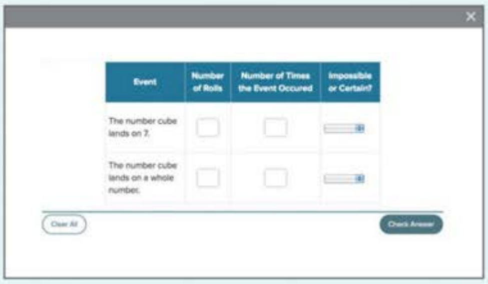
Mathematical Discourse

Which event is impossible? Certain? Explain. Sample answer: The first event is impossible. Because a number cube has numbers labeled from 1-6, it is impossible to land on 7. The second event is certain. All numbers on the number cube are whole numbers.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 9



Explore, Slide 3 of 9

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how numbers can be used to describe the chance of an event happening.

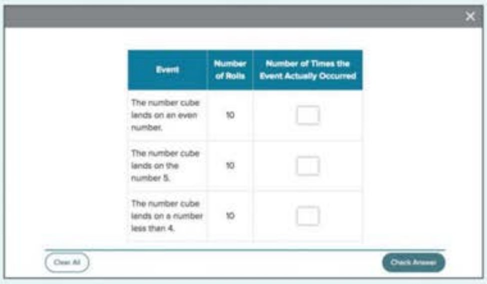
TYPE

On Slide 3, students complete a table to record the results of rolling number cube.

TYPE

On Slide 4, students summarize which numbers can be used to describe the chances of events happening that are either impossible or certain.

Interactive Presentation



Explore, Slide 7 of 9

TYPE

On Slide 7, students complete a table to record the number of occurrences of events.

TYPE

On Slide 8, students summarize which numbers can be used to describe the chances of events happening that are neither impossible nor certain.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Chance Events *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to point out the mathematical terms and numerical representation of terms such as *certain* and *impossible*.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how numbers can be used to describe the chance of an event occurring.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 7 are shown.

Talk About It!

SLIDE 7

Mathematical Discourse

How did the actual results compare to your prediction? See students' responses.

Which of the three events has the same chance of happening as not happening? the number cube landing on an even number

Between the last two events, which event has a better chance of success? Explain. Sample answer: The number cube landing on a number less than 4 has a better chance of success. The chances of landing on numbers less than 4 are more likely than landing on the number 5 because there are three numbers that result in the event actually occurring.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Likelihood of Events

Objective

Students will understand how to describe the likelihood of events using precise vocabulary.

Go Online to find additional teaching notes and Teaching the Mathematical practices.

Teaching Notes

SLIDE 2

Have students explore the interactive activity to show how words can be used to describe the likelihoods.

While they are exploring the activity, you may wish to ask them to describe a situation for each likelihood. Some samples answers are:

- impossible: The temperature tomorrow will be 150°F.
- unlikely: I will drive to school tomorrow.
- equally likely: If I toss a coin, it will land on heads.
- likely: I will complete my homework this evening.
- certain: The sun will rise tomorrow.

Talk About It!

SLIDE 3

Mathematical Discourse

Describe an event in everyday life that is unlikely to happen. Then describe an event that is likely to happen. See students' responses.

DIFFERENTIATE

Language Development Activity ELL

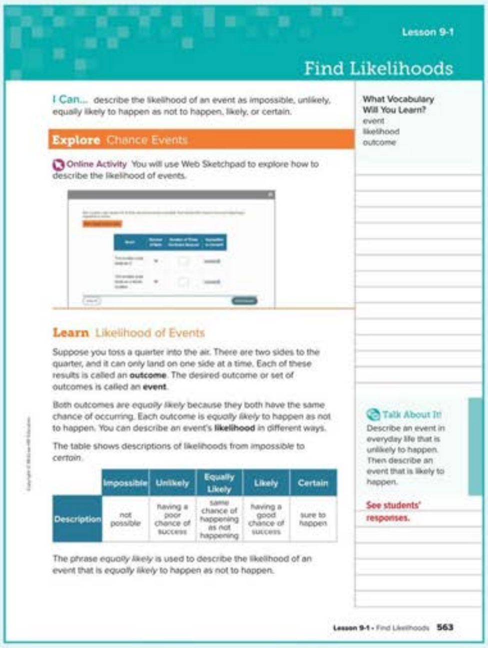
To help students better understand likelihoods and the terminology presented in the Learn, have them determine whether or not each of the following event-likelihood pairs are *true* or *false*.

It is impossible that the sun will rise in the east tomorrow.

It is equally likely that the temperature on this day next year will be greater than the temperature this year.

It is likely that you will roll the same number twice with a 6-sided number cube.

On one side of five notecards have students write the likelihoods shown in the table. On the other side of the notecard, students write a situation to match each likelihood. Students should share their cards with a partner, and discuss any differences about the situations listed.



Lesson 9-1

Find Likelihoods

I Can... describe the likelihood of an event as impossible, unlikely, equally likely to happen as not to happen, likely, or certain.

What Vocabulary Will You Learn?
event
likelihood
outcome

Explore Chance Events

Online Activity You will use Web Sketchpad to explore how to describe the likelihood of events.

Learn Likelihood of Events

Suppose you toss a quarter into the air. There are two sides to the quarter, and it can only land on one side at a time. Each of these results is called an **outcome**. The desired outcome or set of outcomes is called an **event**.

Both outcomes are *equally likely* because they both have the same chance of occurring. Each outcome is *equally likely* to happen as not to happen. You can describe an event's **likelihood** in different ways.

The table shows descriptions of likelihoods from impossible to certain.

	Impossible	Unlikely	Equally Likely	Likely	Certain
Description	not possible	having a poor chance of success	same chance of happening as not happening	having a good chance of success	sure to happen

The phrase *equally likely* is used to describe the likelihood of an event that is *equally likely* to happen as not to happen.

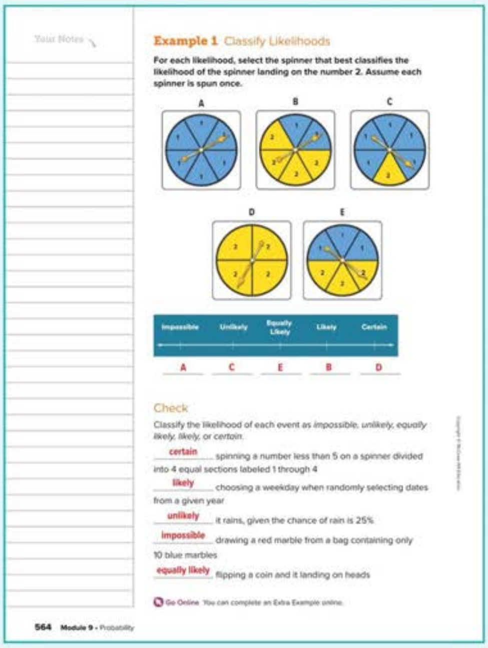
Talk About It! Describe an event in everyday life that is unlikely to happen. Then describe an event that is likely to happen.

See students' responses.

Lesson 9-1 • Find Likelihoods 563

Interactive Presentation

Learn, Likelihood of Events, Slide 2 of 3


Your Notes

Example 1 Classify Likelihoods

For each likelihood, select the spinner that best classifies the likelihood of the spinner landing on the number 2. Assume each spinner is spun once.

Check

Classify the likelihood of each event as *impossible, unlikely, equally likely, likely,* or *certain.*

certain spinning a number less than 5 on a spinner divided into 4 equal sections labeled 1 through 4

likely choosing a weekday when randomly selecting dates from a given year

unlikely it rains, given the chance of rain is 25%

impossible drawing a red marble from a bag containing only 10 blue marbles

equally likely flipping a coin and it landing on heads

Go Online You can complete an Extra Example online.

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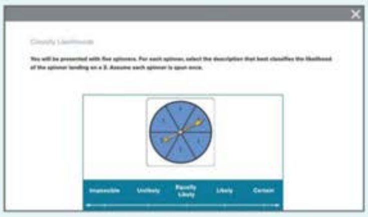
Interactive Presentation



Example 1, Classify Likelihoods, Slide 1 of 2

CLICK

On Slide 1, students will select the descriptions that best classify the likelihoods of the spinner landing on a 2.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Classify Likelihoods

Objective

Students will classify the likelihood of simple events.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enco urage students to refer to the *likelihood of events* number line for the description of each type of event as they examine each spinner presented.

6 Attend to Precision Students should precisely classify the likelihoods based on the sizes, distribution, and characteristics of the sections.

Questions for Mathematical Discourse

SLIDE 1

AL Which spinner's likelihood of landing on a 2 is classified as *impossible*? the spinner with all sections labeled with a 1

AL Which spinner's likelihood of landing on a 2 is classified as *certain*? the spinner with all sections labeled with a 2

OL Describe a method you can use for classifying each spinner's likelihood. Sample answer: Because the sections are all of equal size, compare the number of sections labeled with a 2 to the total number of sections. If this ratio is equal to one half, then the likelihood is *equally likely*. If this ratio is less than one half, then the likelihood is *unlikely*. If this ratio is greater than one half, then the likelihood is *likely*.

BL Describe a different spinner in which the likelihood of landing on red is *likely*. Sample answer: a spinner with 6 total equal-size sections in which 4 or 5 sections are labeled red

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Exit Ticket

Refer to the Exit Ticket slide. Suppose one of the teams calls heads. Classify this event as *impossible, unlikely, equally likely, likely,* or *certain*. Write a mathematical argument that can be used to defend your solution. equally likely; Sample answer: A coin can turn up either heads or tails. It is equally likely that the coin will turn up heads.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

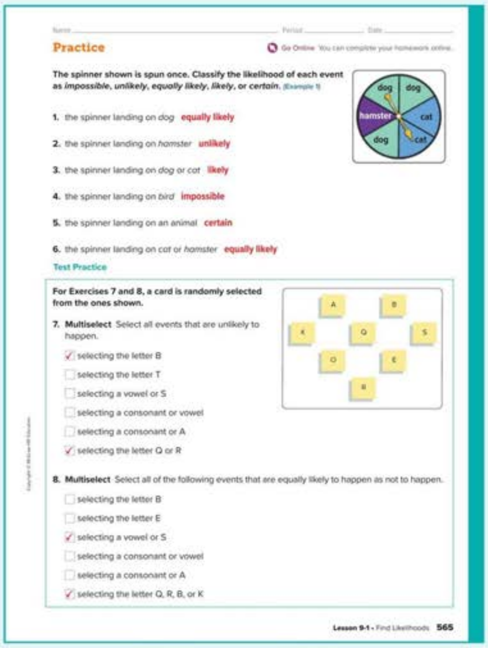
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	classify the likelihood of simple events	1–6
2	extend concepts learned in class to apply them in new contexts	7, 8
3	solve application problems involving finding likelihood	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may not understand that *or* means either outcome. For example, in Exercise 3, *dog or cat* refers to either a dog or a cat. So, both outcomes need considered when classifying the likelihood.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

The spinner shown is spun once. Classify the likelihood of each event as *impossible, unlikely, equally likely, likely,* or *certain*. (Example 1)

1. the spinner landing on *dog* **equally likely**
2. the spinner landing on *hamster* **unlikely**
3. the spinner landing on *dog or cat* **likely**
4. the spinner landing on *bird* **impossible**
5. the spinner landing on *an animal* **certain**
6. the spinner landing on *cat or hamster* **equally likely**

Test Practice

For Exercises 7 and 8, a card is randomly selected from the ones shown.

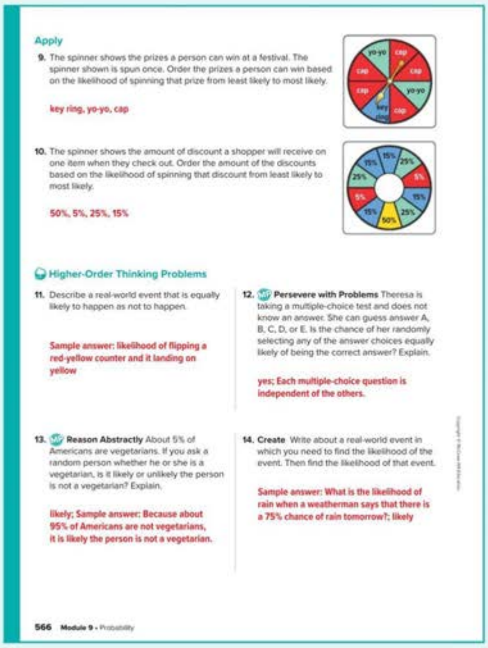
7. **Multiselect** Select all events that are unlikely to happen.
   - [x] selecting the letter B
   - [ ] selecting the letter T
   - [ ] selecting a vowel or S
   - [ ] selecting a consonant or vowel
   - [ ] selecting a consonant or A
   - [x] selecting the letter Q or R

8. **Multiselect** Select all of the following events that are equally likely to happen as not to happen.
   - [ ] selecting the letter B
   - [ ] selecting the letter E
   - [x] selecting a vowel or S
   - [ ] selecting a consonant or vowel
   - [ ] selecting a consonant or A
   - [x] selecting the letter Q, R, B, or K

Lesson 9-1 • Find Likelihoods 565

Interactive Presentation

Exit Ticket



Apply

9. The spinner shows the prizes a person can win at a festival. The spinner shown is spun once. Order the prizes a person can win based on the likelihood of spinning that prize from least likely to most likely.

key ring, yo-yo, cap

10. The spinner shows the amount of discount a shopper will receive on one item when they check out. Order the amount of the discounts based on the likelihood of spinning that discount from least likely to most likely.

50%, 5%, 25%, 15%

Higher-Order Thinking Problems

11. Describe a real-world event that is equally likely to happen as not to happen.

Sample answer: likelihood of flipping a red-yellow counter and it landing on yellow

12. MP **Persevere with Problems** Theresa is taking a multiple-choice test and does not know an answer. She can guess answer A, B, C, D, or E. Is the chance of her randomly selecting any of the answer choices equally likely of being the correct answer? Explain.

yes; Each multiple-choice question is independent of the others.

13. MP **Reason Abstractly** About 5% of Americans are vegetarians. If you ask a random person whether he or she is a vegetarian, is it likely or unlikely the person is not a vegetarian? Explain.

likely; Sample answer: Because about 95% of Americans are not vegetarians, it is likely the person is not a vegetarian.

14. Create Write about a real-world event in which you need to find the likelihood of the event. Then find the likelihood of that event.

Sample answer: What is the likelihood of rain when a weatherman says that there is a 75% chance of rain tomorrow?; likely

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Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 12, students explain if each option of a multiple-choice question is equally likely.

2 Reason Abstractly and Quantitatively In Exercise 13, students use reasoning about the size of the given percent to explain the likelihood of a random person being vegetarian given a population statistic.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Create your own higher-order thinking problem.
Use with Exercises 11–14 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7, 9, 11–14
- ALEKS® Frequency Tables

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 10, 12
- Remediation: Review Resources
- Personal Tutor
- Extra Example 1
- ALEKS® Frequency Tables

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Frequency Tables

Lesson 9-2

Relative Frequency of Simple Events

7.SP.C.6, 7.SP.C.7.B

LESSON GOAL

Students will find the relative frequency of simple events and compare relative frequency to experimental probability.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Experiments and Likelihood

Learn: Relative Frequency
Example 1: Find Relative Frequencies
Example 2: Find Relative Frequencies from Tables
Example 3: Find Relative Frequencies from Graphs
Learn: Relative Frequency Tables and Bar Graphs
Learn: Experimental Probability from Relative Frequency
Example 4: Find Experimental Probabilities
Example 5: Estimate to Make Predictions
Apply: Sales

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Arrive**MATH** Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 53 of the *Language Development Handbook* to help your students build mathematical language related to the relative frequency of simple events.

ELL You can use the tips and suggestions on page T53 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability
Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.C** by solving problems involving relative frequency and experimental probability of simple events.
Standards for Mathematical Content: **7 .SP.C.6, 7.SP.C.7, 7.SP.C.7.B**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students classified the likelihood of simple events.
7.SP.C.5

Now
Students find the relative frequency of simple events and compare relative frequency to experimental probability.
7.SP.C.6, 7.SP.C.7.B

Next
Students will solve problems involving theoretical probability of simple events and their complements.
7.SP.C.7.A

Rigor

The Three Pillars of Rigor

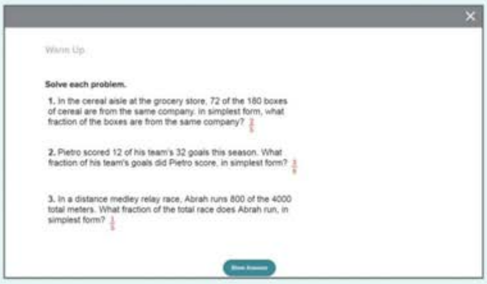
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will use their knowledge of outcomes and likelihood to develop an *understanding* of relative frequency of simple events and making predictions using relative frequency. They will use this understanding to develop *fluency* in finding relative frequencies of simple events.

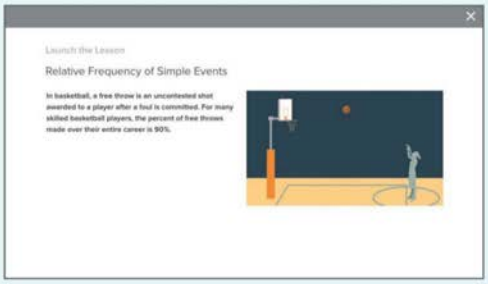
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

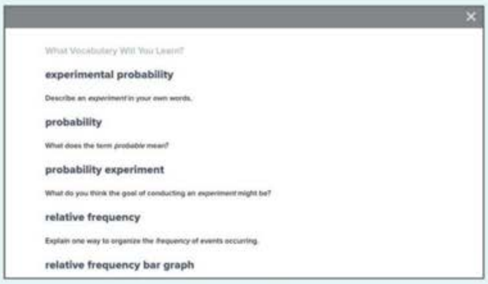
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- solving word problems involving simplifying fractions (Exercises 1–3)

Answers

1. $\frac{2}{5}$
2. $\frac{3}{8}$
3. $\frac{1}{5}$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about expected free throw shots made by basketball players.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- Describe an *experiment* in your own words. Sample answer: An experiment is the process of testing under controlled conditions.
- What does the term *probable* mean? Sample answer: Probable means likely to be true.
- What do you think the goal of conducting an *experiment* might be? Sample answer: to make inferences about the outcome of events, or to test a theory or hypothesis
- Explain one way to organize the *frequency* of events occurring. Sample answer: A tally chart can be used to organize the frequency of events occurring.
- What is a bar graph? Sample answer: a graph using bars to display data

Explore Experiments and Likelihood

Objective

Students will use Web Sketchpad to explore how running an experiment helps to classify the likelihood of an event.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch that simulates tossing darts at a target. Throughout this activity, students will use the sketch to describe the likelihood of hitting the target and to predict the number of darts that will hit the target out of 100 tosses. They will then test their predictions by performing experiments using the sketch.

Inquiry Question

How does running an experiment help you find the likelihood of an event occurring? Sample answer: By running an experiment, I can find the ratio of the number of successes to the total attempts and then classify that event's likelihood.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* questions on Slide 2 is shown.

Talk About It!

SLIDE 2

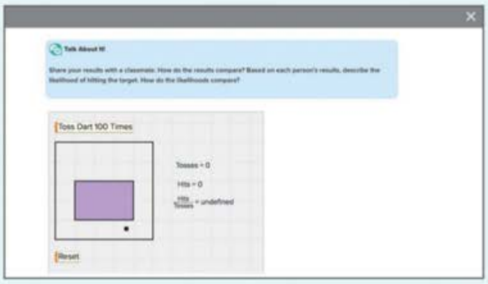
Mathematical Discourse

Share your results with a classmate. How do the results compare? Based on each person's results, describe the likelihood of hitting the target. How do the likelihoods compare? See students' responses.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

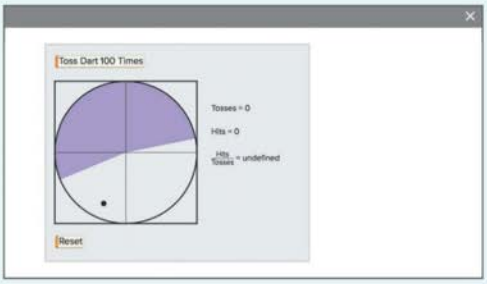


Explore, Slide 2 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how running an experiment helps to find the likelihood of an event occurring.

Interactive Presentation



Explore, Slide 6 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Experiments and Likelihood *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to use the interactive software to gain insight into the benefit of using an experiment to determine the likelihood of events.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how they can run experiments to help determine the likelihood of events.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* questions on Slide 6 is shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Share your results with a classmate. How do the results compare? Based on each person's results, describe the likelihood of hitting the target. How do the likelihoods compare? See students' responses.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Relative Frequency

Objective

Students will understand what relative frequency means and how to find the relative frequency of an event.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to refer to the definitions of the terms *probability* and *relative frequency* before providing their examples.

Go Online

Have students watch the animation on Slide 2. The animation illustrates how to find the relative frequency of an event.

Teaching Notes

SLIDE 1

Prior to having students select the flashcards, you may wish to have a class discussion about the term *probability*. Ask students to think about when they have used probability or chance in everyday life. Some students may say when they hear the weather report, the meteorologist reports on a "chance of rain". Other students may say that a player's batting average is like probability as it can predict how the batter will perform in the future. Then have students select the *Words* and *Ratio* flashcards to view more about relative frequency. Point out to students that a batting average is an example of relative frequency.

(continued on next page)

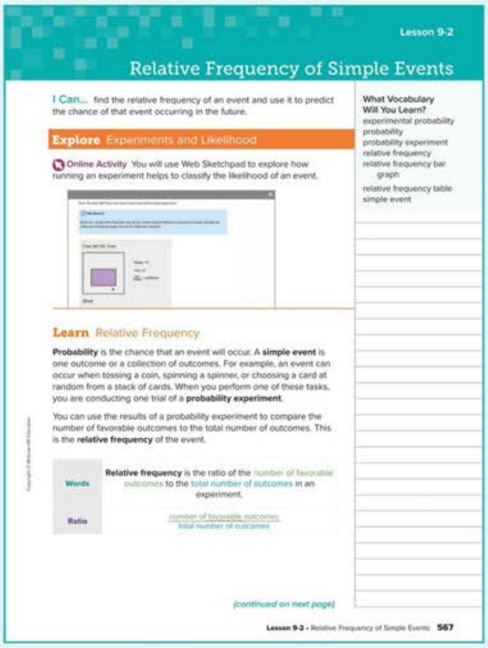
DIFFERENTIATE

Reteaching Activity AL

To help students better understand relative frequency, use the following activity with students in groups of four or five.

Give each group a small container. Assign one student in each group to be the recorder. Have them make a table like the one shown. Have each student write their name on a slip of paper, fold it up, and place it in the group's container. Instruct a student from each group to draw one slip of paper out of the container, read the name, and return the paper to the container. The recorder should place a tally mark next to the name that was drawn. Repeat the process until the experiment has been run at least 10 times. Students should then work together to write the relative frequency of each student's name being drawn.

Name	Tally



Lesson 9-2

Relative Frequency of Simple Events

I Can... find the relative frequency of an event and use it to predict the chance of that event occurring in the future.

What Vocabulary Will You Learn?
experimental probability
probability
probability experiment
relative frequency
relative frequency bar graph
relative frequency table
simple event

Explore Experiments and Likelihood

Online Activity You will use Web Sketchpad to explore how running an experiment helps to classify the likelihood of an event.

Learn Relative Frequency

Probability is the chance that an event will occur. A **simple event** is one outcome or a collection of outcomes. For example, an event can occur when tossing a coin, spinning a spinner, or choosing a card at random from a stack of cards. When you perform one of these tasks, you are conducting one trial of a **probability experiment**.

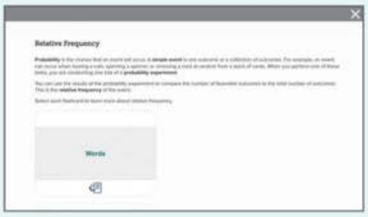
You can use the results of a probability experiment to compare the number of favorable outcomes to the total number of outcomes. This is the **relative frequency** of the event.

Words	**Relative frequency** is the ratio of the number of favorable outcomes to the total number of outcomes in an experiment.
Ratio	$\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

(continued on next page)

Lesson 9-2 • Relative Frequency of Simple Events 567

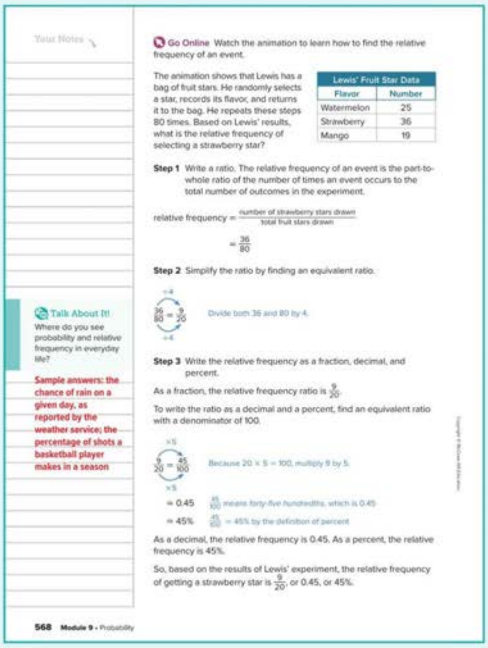
Interactive Presentation



Learn, Relative Frequency, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view different representations of relative frequency.



Your Notes

Go Online Watch the animation to learn how to find the relative frequency of an event.

The animation shows that Lewis has a bag of fruit stars. He randomly selects a star, records its flavor, and returns it to the bag. He repeats these steps 80 times. Based on Lewis' results, what is the relative frequency of selecting a strawberry star?

Lewis' Fruit Star Data	
Flavor	Number
Watermelon	25
Strawberry	36
Mango	19

Step 1 Write a ratio. The relative frequency of an event is the part-to-whole ratio of the number of times an event occurs to the total number of outcomes in the experiment.

$$\text{relative frequency} = \frac{\text{number of strawberry stars drawn}}{\text{total fruit stars drawn}} = \frac{36}{80}$$

Step 2 Simplify the ratio by finding an equivalent ratio.

$$\frac{36}{80} = \frac{9}{20}$$ (÷4) Divide both 36 and 80 by 4.

Step 3 Write the relative frequency as a fraction, decimal, and percent.

As a fraction, the relative frequency ratio is $\frac{9}{20}$.

To write the ratio as a decimal and a percent, find an equivalent ratio with a denominator of 100.

$$\frac{9}{20} = \frac{45}{100}$$ (×5) Because 20 × 5 = 100, multiply 9 by 5.

$= 0.45$ — $\frac{45}{100}$ means forty-five hundredths, which is 0.45

$= 45\%$ — $\frac{45}{100} = 45\%$ by the definition of percent

As a decimal, the relative frequency is 0.45. As a percent, the relative frequency is 45%.

So, based on the results of Lewis' experiment, the relative frequency of getting a strawberry star is $\frac{9}{20}$, or 0.45, or 45%.

Talk About It! Where do you see probability and relative frequency in everyday life?

Sample answers: the chance of rain on a given day, as reported by the weather service; the percentage of shots a basketball player makes in a season

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Interactive Presentation

Learn, Relative Frequency, Slide 2 of 3

WATCH

On Slide 2, students watch an animation that explains how to find the relative frequency of an event.

Learn Relative Frequency *(continued)*

Teaching Notes

SLIDE 2

Play the animation for the class. You may wish to pause the animation when the audio "...part-to-whole ratio of the number of times an event occurs to the number of trials in an experiment." concludes. Ask students for the number of times the desired event occurs and for the number of trials in the experiment. Some students may struggle with finding the total number of trials in the experiment. Point out that they can find the number by adding the numbers in the second column in the table.

During Step 2 of the animation, remind students that, in order to find the relative frequency as a percent, they will need to scale backward the ratio $\frac{36}{80}$ to $\frac{9}{20}$, then scale forward to $\frac{45}{100}$. Then, students can write the relative frequency as a percent.

After the animation has finished playing, ask students which form of the answer they prefer. Some students may say they prefer the percent because it is clear from looking at the percent that almost half of the fruit stars are strawberry.

Talk About It!

SLIDE 3

Mathematical Discourse

Where do you see probability and relative frequency in everyday life?

Sample answers: the chance of rain on a given day, as reported by the weather service; the percentage of shots a basketball player makes in a season

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 1 Find Relative Frequencies

Objective

Students will find relative frequencies from a verbal situation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enco urage students to understand that relative frequency is a ratio that compares the number of favorable outcomes to the total number of outcomes.

6 Attend to Precision Students should be able to make sense of the term *relative frequency*, meaning the frequency of the desired number occurring relative to the total number of rolls.

As students discuss the *Talk About It!* question on Slide 3, they should make sense of an event with a relative frequency of 20% as it relates to how likely it will occur.

Questions for Mathematical Discourse

SLIDE 2

AL How many times was the number cube rolled? 20 times

AL How many times was the number 5 rolled? 4 times

OL Explain the meaning of the relative frequency $\frac{1}{5}$. Sample answer: 1 out of 5 rolls resulted in a 5.

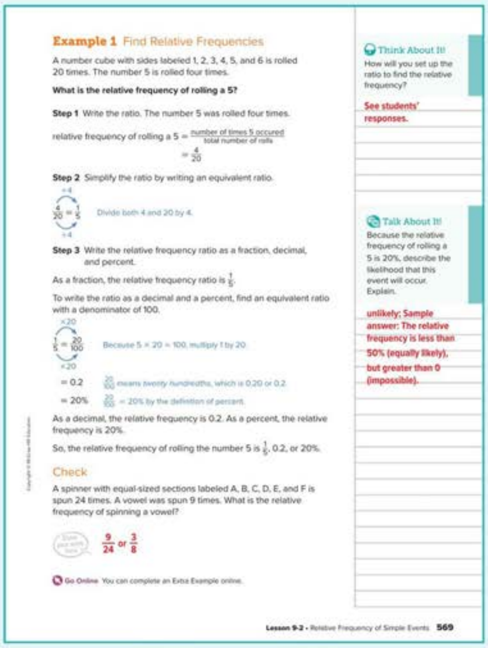
OL Find the relative frequency of *not* rolling a 5. Express your answer as a percent. Explain your reasoning. 80%; Sample answer: $100\% - 20\% = 80\%$

OL Why is simplifying the fraction beneficial? Sample answer: It is easier to think about 1 in 5 rolls resulting in the desired outcome than to think about 4 in 20 rolls

BL Suppose the relative frequency of rolling a 5 on the number cube when it is rolled a total of 60 times is 25%. How many times did the number cube land on 5? 15 times

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Find Relative Frequencies

A number cube with sides labeled 1, 2, 3, 4, 5, and 6 is rolled 20 times. The number 5 is rolled four times.

What is the relative frequency of rolling a 5?

Step 1 Write the ratio. The number 5 was rolled four times.

relative frequency of rolling a 5 $= \frac{\text{number of times 5 occured}}{\text{total number of rolls}}$

$= \frac{4}{20}$

Step 2 Simplify the ratio by writing an equivalent ratio.

$\frac{4}{20} = \frac{1}{5}$ (÷4) Divide both 4 and 20 by 4.

Step 3 Write the relative frequency ratio as a fraction, decimal, and percent.

As a fraction, the relative frequency ratio is $\frac{1}{5}$.

To write the ratio as a decimal and a percent, find an equivalent ratio with a denominator of 100.

$\frac{1}{5} = \frac{20}{100}$ (×20) Because 5 × 20 = 100, multiply 1 by 20.

$= 0.2$ $\frac{20}{100}$ means twenty hundredths, which is 0.20 or 0.2.

$= 20\%$ $\frac{20}{100} = 20\%$ by the definition of percent.

As a decimal, the relative frequency is 0.2. As a percent, the relative frequency is 20%.

So, the relative frequency of rolling the number 5 is $\frac{1}{5}$, 0.2, or 20%.

Check

A spinner with equal-sized sections labeled A, B, C, D, E, and F is spun 24 times. A vowel was spun 9 times. What is the relative frequency of spinning a vowel?

$\frac{9}{24}$ or $\frac{3}{8}$

Go Online You can complete an Extra Example online.

Think About It! How will you set up the ratio to find the relative frequency? See students' responses.

Talk About It! Because the relative frequency of rolling a 5 is 20%, describe the likelihood that this event will occur. Explain. unlikely; Sample answer: The relative frequency is less than 50% (equally likely), but greater than 0 (impossible).

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Interactive Presentation

Example 1, Find Relative Frequencies, Slide 2 of 4

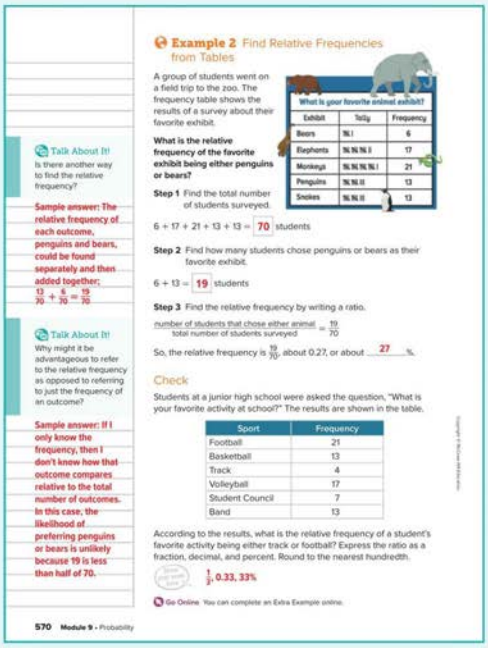
CLICK On Slide 2, students move through the steps to find the relative frequency of rolling a 5.

TYPE On Slide 2, students determine the relative frequency of rolling a 5.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

7.SP.C.6, 7.SP.C.7.B

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Example 2 Find Relative Frequencies from Tables

A group of students went on a field trip to the zoo. The frequency table shows the results of a survey about their favorite exhibit.

What is your favorite animal exhibit?		
Exhibit	Tally	Frequency
Bears	[illegible]	6
Elephants	[illegible]	17
Monkeys	[illegible]	21
Penguins	[illegible]	13
Snakes	[illegible]	13

What is the relative frequency of the favorite exhibit being either penguins or bears?

Step 1 Find the total number of students surveyed.

6 + 17 + 21 + 13 + 13 = 70 students

Step 2 Find how many students chose penguins or bears as their favorite exhibit.

6 + 13 = 19 students

Step 3 Find the relative frequency by writing a ratio.

$\frac{\text{number of students that chose either animal}}{\text{total number of students surveyed}} = \frac{19}{70}$

So, the relative frequency is $\frac{19}{70}$, about 0.27, or about 27 %.

Talk About It! Is there another way to find the relative frequency?

Sample answer: The relative frequency of each outcome, penguins and bears, could be found separately and then added together; $\frac{13}{70} + \frac{6}{70} = \frac{19}{70}$

Talk About It! Why might it be advantageous to refer to the relative frequency as opposed to referring to just the frequency of an outcome?

Sample answer: If I only know the frequency, then I don't know how that outcome compares relative to the total number of outcomes. In this case, the likelihood of preferring penguins or bears is unlikely because 19 is less than half of 70.

Check

Students at a junior high school were asked the question, "What is your favorite activity at school?" The results are shown in the table.

Sport	Frequency
Football	21
Basketball	13
Track	4
Volleyball	17
Student Council	7
Band	13

According to the results, what is the relative frequency of a student's favorite activity being either track or football? Express the ratio as a fraction, decimal, and percent. Round to the nearest hundredth.

$\frac{1}{3}$, 0.33, 33%

Go Online You can complete an Extra Example online.

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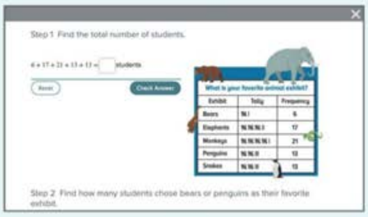
Interactive Presentation



Example 2, Find Relative Frequencies from Tables, Slide 2 of 5

TYPE

On Slide 2, students determine the totals.

DRAG & DROP

On Slide 3, students drag numbers to write a ratio.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find Relative Frequencies from Tables

Objective

Students will find relative frequencies from data in frequency tables.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to read the question carefully in order to know they need to find the relative frequency that *either* animal was preferred.

Questions for Mathematical Discourse

SLIDE 2

AL How will you find the total number of students surveyed? Add all of the frequencies for each animal.

AL How many students chose penguins or bears as their favorite exhibit? What operation does the term *or* indicate? 19 students; The term *or* indicates addition.

OL If you were asked how many more students chose penguins over bears, which key words would indicate the operation you should use? The key words *how many more* indicates subtraction; 13 – 6, or 7 more students chose penguins over bears.

BL Were there any students who chose *none of the above* as their favorite animal exhibit? Explain. no; Sample answer: If this was the case, the frequency table would indicate this as a new row with the number of students who chose none of the above.

SLIDE 3

AL Why is the denominator of the ratio 70, and not 19? The relative frequency ratio is the number of favorable outcomes to the total number of outcomes. In this case, the total number of outcomes corresponds to 70 students.

OL Is it easier to comprehend the relative frequency expressed as a fraction, decimal, or percent? Explain. Sample answer: In this case, it is easier to comprehend the relative frequency as a decimal or percent because it might not be as easy to comprehend the relationship between the numbers 19 and 70 in the fraction $\frac{19}{70}$.

BL A classmate stated the relative frequency is close to $\frac{1}{4}$. How would you respond? Sample answer: The classmate's statement is reasonable because $\frac{19}{70}$ is about 0.27, which is close to $\frac{1}{4}$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Find Relative Frequencies from Graphs

Objective

Students will find relative frequencies from frequency bar graphs.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to read the question carefully in order to know they need to find the relative frequency of *either* blue or green.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to use reasoning about the equal-size sections of the spinner to make an argument about what kind of results they might expect to see.

Questions for Mathematical Discourse

SLIDE 2

AL How many times was green spun? blue? 24 times; 22 times

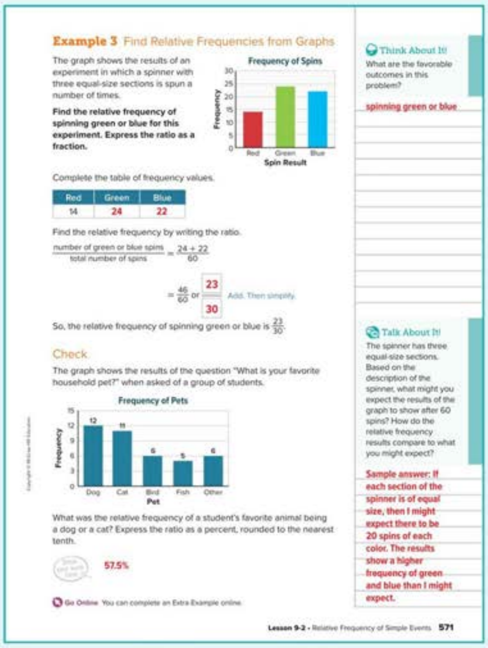
OL A classmate stated the total number of outcomes is 46. What is the likely error? Sample answer: The classmate only added the frequencies for green and blue. They also need to add the frequencies for red as part of the total.

OL If you did not know that the spinner had three equal-size sections, what prediction might someone make about a spinner based solely on these results? Sample answer: The sections that correspond to green and blue might be of equal size, because their results are similar. The section that corresponds to red might be smaller in size, because the number of spins that land on red are smaller than green or blue.

BL Is the relative frequency of spinning green or blue the same as the relative frequency of spinning *not* red? Explain, without calculating. yes; Sample answer: The only colors are red, green, and blue. So, spinning *not* red means you either spin green or blue.

Go Online

- Find additional teaching notes and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Find Relative Frequencies from Graphs

The graph shows the results of an experiment in which a spinner with three equal-size sections is spun a number of times.

Find the relative frequency of spinning green or blue for this experiment. Express the ratio as a fraction.

Complete the table of frequency values.

Red	Green	Blue
14	24	22

Find the relative frequency by writing the ratio.

$\frac{\text{number of green or blue spins}}{\text{total number of spins}} = \frac{24 + 22}{60}$

$= \frac{46}{60}$ or $\frac{23}{30}$ Add. Then simplify.

So, the relative frequency of spinning green or blue is $\frac{23}{30}$.

Check

The graph shows the results of the question "What is your favorite household pet?" when asked of a group of students.

What was the relative frequency of a student's favorite animal being a dog or a cat? Express the ratio as a percent, rounded to the nearest tenth.

57.5%

Go Online You can complete an Extra Example online.

Think About It! What are the favorable outcomes in this problem?

spinning green or blue

Talk About It! The spinner has three equal-size sections. Based on the description of the spinner, what might you expect the results of the graph to show after 60 spins? How do the relative frequency results compare to what you might expect?

Sample answer: If each section of the spinner is of equal size, then I might expect there to be 20 spins of each color. The results show a higher frequency of green and blue than I might expect.

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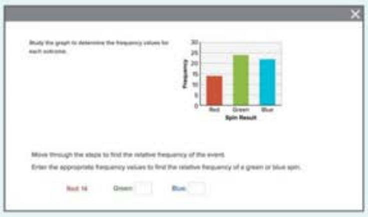
Interactive Presentation



Example 3, Find Relative Frequencies from Graphs, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the relative frequency of the event.

TYPE On Slide 2, students determine missing values to write a ratio representing the relative frequency.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Learn Relative Frequency Tables and Bar Graphs

Suppose 100 randomly selected people are asked their blood type. The results are shown.

Blood Type	Frequency
A	40
B	10
AB	5
O	45

To find the relative frequency ratio for each blood type, find the ratio of the frequency (number of people) for each blood type to the total number of people surveyed, 100.

Blood Type A: $\frac{40}{100}$ Blood Type AB: $\frac{5}{100}$

Blood Type B: $\frac{10}{100}$ Blood Type O: $\frac{45}{100}$

Talk About It!
Compare and contrast the relative frequency table with the frequency bar graph.

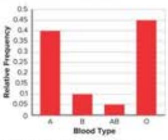
Sample answer: Both the table and graph show the same data. The graph shows how the relative frequencies for each blood type visually compare to one another. The visual comparisons are easier to see in the graph.

You can show the relative frequency of data in a **relative frequency table**. This kind of table lists both the frequency and relative frequency of data.

Blood Type	Frequency	Relative Frequency
A	40	$\frac{40}{100}=0.4$
B	10	$\frac{10}{100}=0.1$
AB	5	$\frac{5}{100}=0.05$
O	45	$\frac{45}{100}=0.45$

You can also graph the relative frequency values from the table on a **relative frequency bar graph**. This kind of bar graph shows the relative frequency of the data. Graph the relative frequencies.

Use a scale of 0 to 0.5 for the relative frequency values, because the greatest value is 0.45, which is less than 0.5.



The graph provides a visual representation for how the relative frequencies compare. The graph shows that the relative frequencies for Blood Types A and O are about the same, and much greater than the relative frequencies for Blood Types B and AB.

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Interactive Presentation

Learn, Relative Frequency Tables and Bar Graphs, Slide 1 of 3

TYPE

On Slide 1, students determine the missing values needed to complete a table of relative frequencies.

CLICK

On Slide 2, students use square shading to graph relative frequency.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Relative Frequency T ables and Bar Graphs

Objective

Students will understand how to create relative frequency tables and bar graphs from a set of data.

Go Online to find additional teaching notes and Teaching the Mathematical practices.

Teaching Notes

SLIDE 2

Students will learn how to graph the data from a relative frequency table into a *relative frequency bar graph*. Point out to students that both the table and the graph show the same information. The graph is a visual representation of the table. When creating the graph from the interactive activity, some students may just shade the top section. Point out that to accurately represent each relative frequency value in the bar graph, they need to shade the correct number of sections.

Talk About It!

SLIDE 3

Mathematical Discourse

Compare and contrast the relative frequency table with the frequency bar graph. Sample answer: Both the table and graph show the same data. The graph shows how the relative frequencies for each blood type visually compare to one another. The visual comparisons are easier to see in the graph.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of relative frequency tables, have students consider the following problem.

Rhett surveyed his classmates to determine how many siblings each had. Based on the results, he calculated the following relative frequencies.

0 siblings: 0.05
1 sibling: 0.2
2 siblings: 0.5
3 siblings: 0.25
4 or more siblings: 0.1

Explain why these relative frequencies are not possible. Sample answer: The relative frequencies add to more than 1.

Learn Experimental Probability from Relative Frequency

Objective

Students will understand how experimental probability is related to relative frequency.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to craft a clear and precise explanation that communicates each mathematical similarity and difference.

Teaching Notes

SLIDE 1

Students will learn the term *experimental probability*. Have them select the *Words* and *Ratio* flashcards to see how experimental probability can be described using these multiple representations.

Point out to students that relative frequency refers to events that have already happened. Experimental probability is a prediction of what can happen in the future based on the relative frequency. While the two ratios are the same, the meaning behind them is different.

SLIDE 2

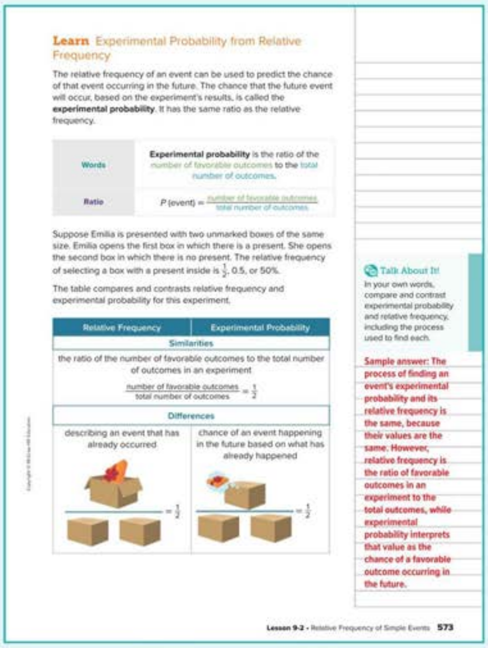
Students will compare and contrast relative frequency and experimental probability. You may wish to have them generate their own examples of situa tions that describe an experiment where they find the relative frequency of an event, and then how they can interpret the relative frequency as experimental probability. Sample answer: I pull a building block from a bin of blocks 25 times. The relative frequency of choosing a red block is $\frac{6}{25}$. The experimental probability of choosing a red block in a future experiment is $\frac{6}{25}$.

Talk About It!

SLIDE 3

Mathematical Discourse

In your own words, compare and contrast experimental probability and relative frequency, including the process used to find each. Sample answer: The process of finding an event's experimental probability and its relative frequency is the same because their values are the same. However, relative frequency is the ratio of favorable outcomes in an experiment to the total trials, while experimental probability interprets that value as the chance of a favorable outcome occurring in the future.



Learn Experimental Probability from Relative Frequency

The relative frequency of an event can be used to predict the chance of that event occurring in the future. The chance that the future event will occur, based on the experiment's results, is called the **experimental probability**. It has the same ratio as the relative frequency.

Words	**Experimental probability** is the ratio of the number of favorable outcomes to the total number of outcomes.
Ratio	$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

Suppose Emilia is presented with two unmarked boxes of the same size. Emilia opens the first box in which there is a present. She opens the second box in which there is no present. The relative frequency of selecting a box with a present inside is $\frac{1}{2}$, 0.5, or 50%.

The table compares and contrasts relative frequency and experimental probability for this experiment.

Relative Frequency	Experimental Probability
Similarities	
the ratio of the number of favorable outcomes to the total number of outcomes in an experiment $\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}} = \frac{1}{2}$	
Differences	
describing an event that has already occurred $= \frac{1}{2}$	chance of an event happening in the future based on what has already happened $= \frac{1}{2}$

Talk About It!

In your own words, compare and contrast experimental probability and relative frequency, including the process used to find each.

Sample answer: The process of finding an event's experimental probability and its relative frequency is the same, because their values are the same. However, relative frequency is the ratio of favorable outcomes in an experiment to the total outcomes, while experimental probability interprets that value as the chance of a favorable outcome occurring in the future.

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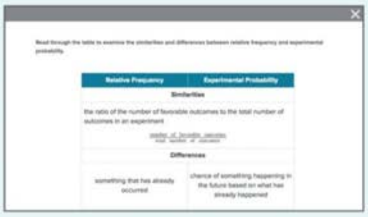
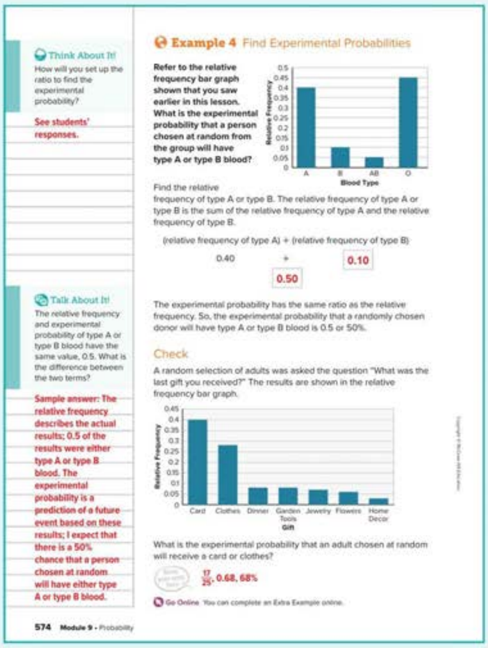
Interactive Presentation



Learn, Experimental Probability from Relative Frequency, Slide 2 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of experimental probability.


Think About It!
How will you set up the ratio to find the experimental probability?
See students' responses.

Example 4 Find Experimental Probabilities

Refer to the relative frequency bar graph shown that you saw earlier in this lesson. What is the experimental probability that a person chosen at random from the group will have type A or type B blood?

Find the relative frequency of type A or type B. The relative frequency of type A or type B is the sum of the relative frequency of type A and the relative frequency of type B.

(relative frequency of type A) + (relative frequency of type B)

0.40 + 0.10

0.50

The experimental probability has the same ratio as the relative frequency. So, the experimental probability that a randomly chosen donor will have type A or type B blood is 0.5 or 50%.

Talk About It!
The relative frequency and experimental probability of type A or type B blood have the same value, 0.5. What is the difference between the two terms?
Sample answer: The relative frequency describes the actual results; 0.5 of the results were either type A or type B blood. The experimental probability is a prediction of a future event based on these results; I expect that there is a 50% chance that a person chosen at random will have either type A or type B blood.

Check
A random selection of adults was asked the question "What was the last gift you received?" The results are shown in the relative frequency bar graph.

What is the experimental probability that an adult chosen at random will receive a card or clothes?

$\frac{17}{25}$, 0.68, 68%

Go Online You can complete an Extra Example online.

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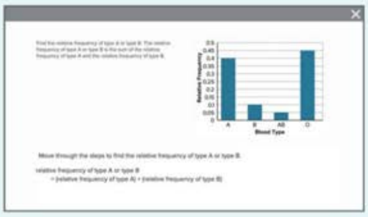
Interactive Presentation



Example 4, Find Experimental Probabilities, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the relative frequency of type A or type B.

TYPE On Slide 2, students determine the experimental probability.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find Experimental Probabilities

Objective

Students will find the experimental probability of an event from a relative frequency bar graph.

Teaching the Mathematical Practices

6 Attend to Precision Encour age students to read the question and analyze the graph carefully in order to know they need to find the experimental probability of a person chosen at random having either type A or type B blood.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language in their explanations for the difference between these two terms.

Questions for Mathematical Discourse

SLIDE 2

AL What is experimental probability? Experimental probability is a prediction of how likely an event is to happen in the future, based on the relative frequency of that event from an experiment. It has the same value as the relative frequency.

AL What is the relative frequency for type A? type B? 0.4; 0.1

OL Compare and contrast the relative frequency to the experimental probability. Sample answer: They have the same value. However, the relative frequency is a ratio describing the actual results. The experimental probability is a ratio describing the prediction of results that will happen in the future, based on the relative frequency results.

OL What is the sum of all of the relative frequencies? Why does this make sense? 1; Sample answer: These are the only blood types. Every person in the experiment had one of these blood types.

BL What is the experimental probability, expressed as a percent, that a person chosen at random will have *neither* type A *nor* type B blood? 50%

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 5 Estimate to Make Predictions

Objective

Students will make predictions using relative frequency and proportional reasoning.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enco urage students to analyze the problem and make sure their answer makes sense within the context of the problem.

As students discuss the *Talk About It!* question on Slide 4, encourage them to understand that the prediction is based on the actual relative frequency, and is not a guaranteed statement of what will actually happen.

Questions for Mathematical Discourse

SLIDE 2

AL What does a relative frequency ratio mean? It is the ratio of the number of favorable outcomes to the total number of outcomes.

AL Why is the denominator of the ratio 11,434 and not 4,189? The player had 4,189 hits in 11,434 at-bats, so the total is 11,434.

OL In order to predict the number of hits for 500 at-bats, why do you find the relative frequency first? Sample answer: I need to use the relative frequency ratio to set up and solve a proportion because the two quantities (hits, at-bats) are assumed to be proportional.

OL Estimate the relative frequency percentage. Sample answer: 4,189 ≈ 4,000 and 11,434 is close to 12,000. Because 4,000 out of 12,000 is about one third, the relative frequency percentage is about 33%.

BL In Major League Baseball, the relative frequency of hits to at-bats over a season is typically between 0.250 and 0.275. A relative frequency over 0.300 is considered to be very good, and a relative frequency of 0.400 or greater is rarely achieved. How many hits would this player need to achieve a batting average of 0.400 in 11,434 at-bats? about 4,574 hits

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 5 Estimate to Make Predictions

In baseball, a player's batting average is found by writing the ratio of a player's hits to their total at-bats, and then writing the ratio as a decimal. The player with the highest career batting average in history had 4,189 hits in 11,434 at-bats.

Based on this relative frequency, how many hits can be expected in a season where the player has 500 total at-bats?

Step 1 Find the relative frequency ratio of getting a hit.

$\frac{\text{number of hits}}{\text{number of at-bats}} = \frac{4,189}{11,434}$

Step 2 Use the relative frequency to make a prediction.

Use the relative frequency to find an equivalent ratio in order to predict the number of hits for 500 at-bats. Let h represent the number of hits for 500 at-bats.

$\frac{4,189}{11,434} \approx \frac{h}{500}$ (÷23) Round 11,434 to 11,500. 11,500 ÷ 23 = 500

$\frac{4,189}{11,434} \approx \frac{182}{500}$ 4,189 ÷ 23 ≈ 182

In each step, the values were rounded in order to find an approximate equivalent ratio. By using ratio reasoning, another possible equivalent ratio could be $\frac{183}{500}$.

So, in a season where the player has 500 total at-bats, it can be expected that he will have about 182 or 183 hits.

Think About It! How can you use the relative frequency to make the prediction?

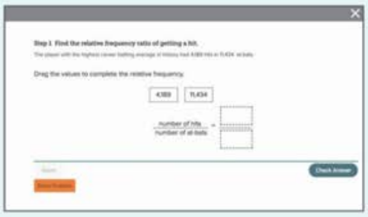
See students' responses.

Talk About It! Suppose the player had 200 hits in 500 at-bats. Does this mean your prediction was not accurate? Explain.

no; Sample answer: My prediction was based on the relative frequency results. It is a prediction, not a definitive statement of what will actually occur.

Lesson 9-2 • Relative Frequency of Simple Events 575

Interactive Presentation



Example 5, Estimate to Make Predictions, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag values to complete the relative frequency.

CLICK

On Slide 3, students move through the steps to estimate the number of hits.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Over the past five years, it rained on $\frac{1}{5}$ of the days in April. How many days can you expect it to rain in the upcoming April if the weather is expected to be consistent with the past five years?

6

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you learned today with something similar you learned in an earlier module or grade. How are they similar? How are they different?

See students' observations.

Apply Sales

Objective

Students will come up with their own strategy to solve an application problem involving DVD sales.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How many DVDs were sold altogether last year?
- How does the quantity of comedy DVDs sold last year compare to the other types of DVDs sold last year?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Sales

Last year, a DVD store sold 670 action DVDs, 580 comedy DVDs, 450 drama DVDs, and 300 science fiction DVDs. The store owner makes $3.31 profit on each DVD sold and expects to sell 5,000 DVDs this year. Based on last year's results, how much profit can she expect to make on comedy DVDs for this year?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$4,799.50; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

If more than 1,125 drama DVDs are sold this year, how does that compare to the store owner's prediction of the total number of DVDs sold?

Sample answer: The total number of DVDs will likely be more than the store owner's prediction of 5,000.

Lesson 9-2 • Relative Frequency of Simple Events 577

Interactive Presentation

Apply, Sales

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

A local pizza shop sold 100 pizzas last week. The number of each type of pizza sold is shown in the table. The store makes $5.15 profit on each sausage pizza sold. Based on the relative frequency, how much profit can they expect to make on sausage pizzas next month if they plan to sell 525 total pizzas in that month?

Type of Pizza	Frequency
Pepperoni	43
Sausage	28
Mushroom	22
Onion	7

$757.05

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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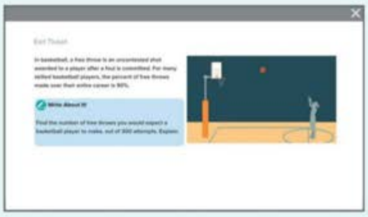
Interactive Presentation


Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about simple events. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Find the number of free throws you would expect a basketball player to make, out of 300 attempts. Write a mathematical argument that can be used to defend your solution. 270 free throws; Sample answer: Use the relative frequency of the basketball player making a free throw to set up and solve a proportion to predict the number of free throws made out of 300 attempts: $\frac{90}{100} = \frac{x}{300}$, so $x = 270$ free throws.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 3, 5, 7, 8–11
- ALEKS Frequency Tables

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–5, 7, 9
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS Frequency Tables

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Frequency Tables

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

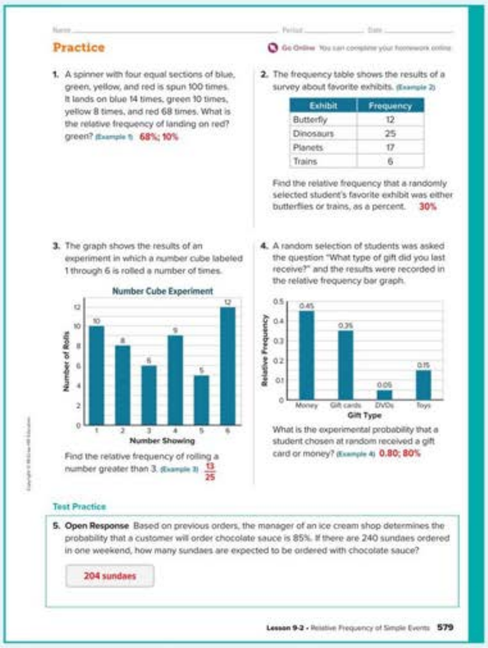
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find relative frequencies from word problems	1
2	find relative frequencies from data in frequency tables	2
2	find relative frequencies from frequency bar graphs	3
2	find the experimental probability of an event from a relative frequency bar graph	4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving relative frequency of simple events	6, 7
3	higher-order and critical thinking skills	8–11

Common Misconception

Some students may not understand that *or* refers to either outcome occurring. For example, in Exercise 2, in order to find the relative frequency for either the butterfly exhibit or the train exhibit, the two relative frequencies must be added.


Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

1. A spinner with four equal sections of blue, green, yellow, and red is spun 100 times. It lands on blue 14 times, green 10 times, yellow 8 times, and red 68 times. What is the relative frequency of landing on red? green? (Example 1) 68%; 10%

2. The frequency table shows the results of a survey about favorite exhibits. (Example 2)

Exhibit	Frequency
Butterfly	12
Dinosaurs	25
Planets	17
Trains	6

Find the relative frequency that a randomly selected student's favorite exhibit was either butterflies or trains, as a percent. 30%

3. The graph shows the results of an experiment in which a number cube labeled 1 through 6 is rolled a number of times.

Find the relative frequency of rolling a number greater than 3. (Example 3) $\frac{13}{25}$

4. A random selection of students was asked the question "What type of gift did you last receive?" and the results were recorded in the relative frequency bar graph.

What is the experimental probability that a student chosen at random received a gift card or money? (Example 4) 0.80; 80%

Test Practice

5. **Open Response** Based on previous orders, the manager of an ice cream shop determines the probability that a customer will order chocolate sauce is 85%. If there are 240 sundaes ordered in one weekend, how many sundaes are expected to be ordered with chocolate sauce?

204 sundaes

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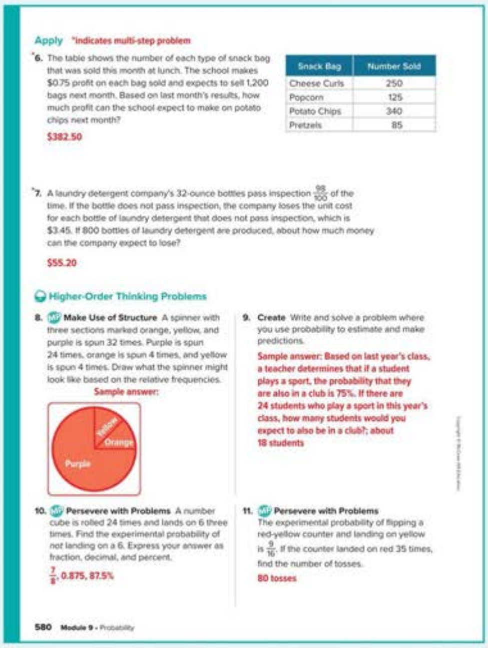

Apply *indicates multi-step problem

*6. The table shows the number of each type of snack bag that was sold this month at lunch. The school makes $0.75 profit on each bag sold and expects to sell 1,200 bags next month. Based on last month's results, how much profit can the school expect to make on potato chips next month?

Snack Bag	Number Sold
Cheese Curls	250
Popcorn	125
Potato Chips	340
Pretzels	85

$382.50

*7. A laundry detergent company's 32-ounce bottles pass inspection $\frac{98}{100}$ of the time. If the bottle does not pass inspection, the company loses the unit cost for each bottle of laundry detergent that does not pass inspection, which is $3.45. If 800 bottles of laundry detergent are produced, about how much money can the company expect to lose?

$55.20

Higher-Order Thinking Problems

8. **MP Make Use of Structure** A spinner with three sections marked orange, yellow, and purple is spun 32 times. Purple is spun 24 times, orange is spun 4 times, and yellow is spun 4 times. Draw what the spinner might look like based on the relative frequencies.

Sample answer:

9. **Create** Write and solve a problem where you use probability to estimate and make predictions.

Sample answer: Based on last year's class, a teacher determines that if a student plays a sport, the probability that they are also in a club is 75%. If there are 24 students who play a sport in this year's class, how many students would you expect to also be in a club?; about 18 students

10. **MP Persevere with Problems** A number cube is rolled 24 times and lands on 6 three times. Find the experimental probability of *not* landing on a 6. Express your answer as fraction, decimal, and percent.

$\frac{7}{8}$, 0.875, 87.5%

11. **MP Persevere with Problems** The experimental probability of flipping a red-yellow counter and landing on yellow is $\frac{9}{16}$. If the counter landed on red 35 times, find the number of tosses.

80 tosses

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Teaching the Mathematical Practices

7 Look For and Make Use of Structure In Exercise 8, students draw a spinner based on relative frequencies.

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 10, students determine experimental probability of an event not occuring and express it as a fraction, decimal, and percent.

In Exercise 11, students find the total number of tosses given an experimental probability and the number of times the counter landed on red.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.
Use with Exercise 6 Have students work together to prepare a brief demonstration that illustrates why this problem requires multiple steps to solve. For example, before they can determine the profit, they must first determine how many bags of potato chips the school expects to sell. Have each pair or group of students present their response to the class.

Solve the problem another way.
Use with Exercise 10 Have students work in groups of 3–4. After completing Exercise 10, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Lesson 9-3

7.SP.C.7, 7.SP.C.7.A

Theoretical Probability of Simple Events

LESSON GOAL

Students will solve problems involving theoretical probability of simple events and their complements.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Long-Run Relative Frequencies

Learn: Sample Space of Simple Events
Example 1: Find the Sample Space of Simple Events
Learn: Theoretical Probability of Simple Events
Example 2: Find Theoretical Probabilities of Simple Events
Learn: Complements of Simple Events
Example 3: Find Complements of Simple Events
Apply: Probability

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 54 of the *Language Development Handbook* to help your students build mathematical language related to the theoretical probability of simple events.

ELL You can use the tips and suggestions on page T54 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability
Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.C** by solving problems involving theoretical probability of simple events and their complements.
Standards for Mathematical Content: **7. SP.C.7, 7.SP.C.7.A**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students found the relative frequency of simple events and compared relative frequency to experimental probability.
7.SP.C.6, 7.SP.C.7.B

Now
Students solve problems involving theoretical probability of simple events and their complements.
7.SP.C.7, 7.SP.C.7.A

Next
Students will solve problems that compare probabilities and relative frequencies of simple events.
7.SP.C.6, 7.SP.C.7, 7.SP.C.7.A, 7.SP.C.7.B

Rigor

The Three Pillars of Rigor

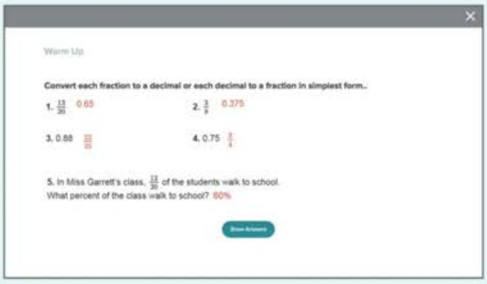
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop their *understanding* of the sample space for simple events and how it relates to probability. They will use their knowledge of relative frequency to develop an *understanding* of theoretical probability of simple events and the complement of a simple event. They will use this understanding to gain *fluency* in finding theoretical probabilities of simple events.

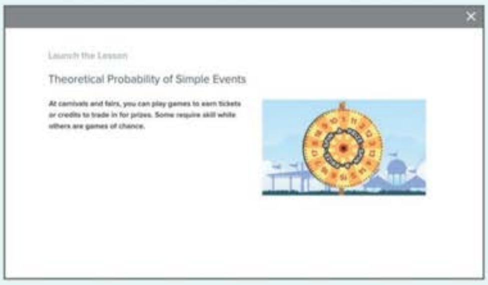
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- converting among fractions, decimals, and percents (Exercises 1–5)

Answers

1. 0.65 **2.** 0.375

3. $\frac{22}{25}$ **4.** $\frac{3}{4}$

5. 60%

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about games of skill and chance at a carnival.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- In everyday life, if two things are *complementary*, what does that mean? Give an example. Sample answer: If two things are complementary, it means that they correspond with each other in a harmonious way.
- Describe a *sample* in your own words. Sample answer: A sample is a small part or quantity intended to show what the whole is like.
- How would you describe a *theoretical* idea? Sample answer: An idea that is possible, assumed, and created from conjecture.
- What does the adjective *uniform* mean? Sample answer: Uniform means remaining the same in all cases and at all times.

Explore Long-Run Relative Frequencies

Objective

Students will use Web Sketchpad to explore the relationship between long-run relative frequency and theoretical probability.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch that simulates spinning a spinner. Throughout this activity, students will make predictions about spinner outcomes. They will test their predictions and find relative frequencies of events by spinning digital spinners 10 times and 100 times.

Inquiry Question

How can you predict relative frequency without performing an experiment? Sample answer: Over many trials, the relative frequency should get closer to the ratio of favorable outcomes to the total possible outcomes in the experiment. This ratio can be used to predict the relative frequency without performing an experiment.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

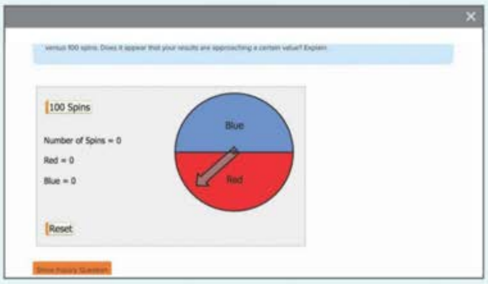
Compare your results with a classmate. How do the results compare with your predictions? See students' responses.

Consider your results after 10 spins versus 100 spins. Does it appear that your results are approaching a certain value? Explain. Students should notice that, for 100 spins, the relative frequency of landing on red is likely close to $\frac{1}{2}$, 0.5, or 50%.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

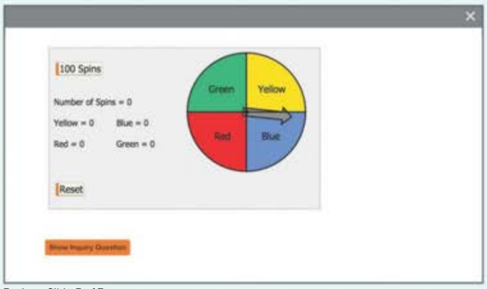


Explore, Slide 3 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationship between long-run relative frequency and theoretical probability.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 6, students make a prediction for how many spins out of 1,000 will land in a section.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Long-Run Relative Frequencies *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore long-run relative frequencies. Encourage students to use the interactive software to gain insight into finding how likely an event is, without conducting an experiment.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Compare your results with a classmate. How do the results compare with your predictions? See students' responses.

Consider your results after 10 spins versus 100 spins. Does it appear that your results are approaching a certain value? Explain. Sample answer: Students should notice that, for 100 spins, the relative frequency of landing on red is likely close to $\frac{1}{4}$, 0.25, or 25%.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Sample Space of Simple Events

Objective

Students will understand how to find the sample space of simple events.

Teaching Notes

SLIDE 1

Students will learn how to find the sample space of a simple event. You may wish to give them other examples of simple events, such as tossing a coin or spinning a spinner labeled 1-10, and have them list the sample space of each event.

SLIDE 2

Point out to students that all possible outcomes are included in the sample space and that the sample space is not dependent on the results of a probability experiment. This may become more relevant as students learn about theoretical probability.

DIFFERENTIATE

Enrichment Activity BL

If students need more of a challenge involving sample spaces of events, have them think about the scenario and answer the questions that follow.

Game 1: You simultaneously flip two coins and win if both land showing tails.

Game 2: You roll two standard number cubes and win if the sum of the numbers showing is divisible by 3.

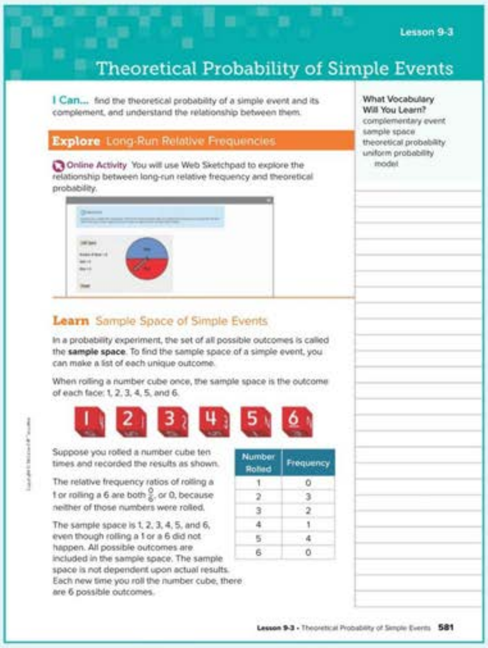
What is the sample space of Game 1? heads, tails; heads, heads; tails, tails

What is the sample space of Game 2? 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Out of the 3 possible outcomes, how many ways could you win Game 1? 1

Out of 11 possible outcomes, how many ways could you win Game 2? 4

Which game would you choose to play? Explain your reasoning. Sample answer: Game 2; I set up ratios comparing the number of ways to win compared to total outcomes and found the percent chance of winning. In Game 1, I have about 33% chance of winning. In Game 2, I have about 36% chance of winning.



Lesson 9-3

Theoretical Probability of Simple Events

I Can... find the theoretical probability of a simple event and its complement, and understand the relationship between them.

What Vocabulary Will You Learn?
complementary event
sample space
theoretical probability
uniform probability model

Explore Long-Run Relative Frequencies

Online Activity You will use Web Sketchpad to explore the relationship between long-run relative frequency and theoretical probability.

Learn Sample Space of Simple Events

In a probability experiment, the set of all possible outcomes is called the **sample space**. To find the sample space of a simple event, you can make a list of each unique outcome.

When rolling a number cube once, the sample space is the outcome of each face: 1, 2, 3, 4, 5, and 6.

Suppose you rolled a number cube ten times and recorded the results as shown.

Number Rolled	Frequency
1	0
2	3
3	2
4	1
5	4
6	0

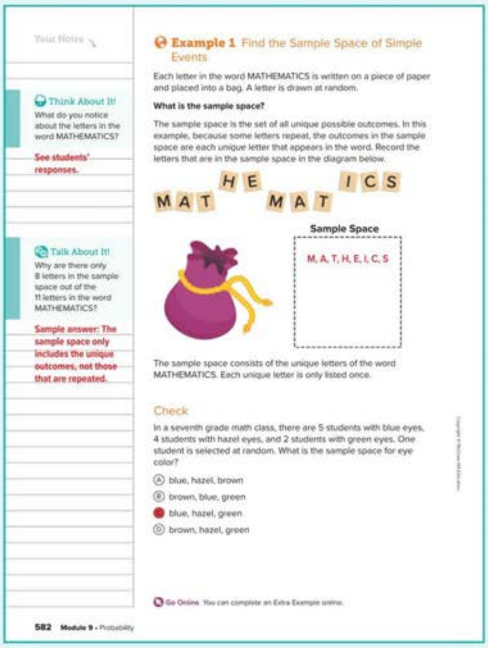
The relative frequency ratios of rolling a 1 or rolling a 6 are both $\frac{0}{6}$, or 0, because neither of those numbers were rolled.

The sample space is 1, 2, 3, 4, 5, and 6, even though rolling a 1 or a 6 did not happen. All possible outcomes are included in the sample space. The sample space is not dependent upon actual results. Each new time you roll the number cube, there are 6 possible outcomes.

Lesson 9-3 • Theoretical Probability of Simple Events 581

Interactive Presentation

Learn, Sample Space of Simple Events, Slide 1 of 2


Your Notes

Think About It!
What do you notice about the letters in the word MATHEMATICS?
See students' responses.

Talk About It!
Why are there only 8 letters in the sample space out of the 11 letters in the word MATHEMATICS?
Sample answer: The sample space only includes the unique outcomes, not those that are repeated.

Example 1 Find the Sample Space of Simple Events

Each letter in the word MATHEMATICS is written on a piece of paper and placed into a bag. A letter is drawn at random.

What is the sample space?

The sample space is the set of all unique possible outcomes. In this example, because some letters repeat, the outcomes in the sample space are each *unique* letter that appears in the word. Record the letters that are in the sample space in the diagram below.

The sample space consists of the unique letters of the word MATHEMATICS. Each unique letter is only listed once.

Check
In a seventh grade math class, there are 5 students with blue eyes, 4 students with hazel eyes, and 2 students with green eyes. One student is selected at random. What is the sample space for eye color?
(A) blue, hazel, brown
(B) brown, blue, green
(C) blue, hazel, green
(D) brown, hazel, green

Go Online You can complete an Extra Example online.

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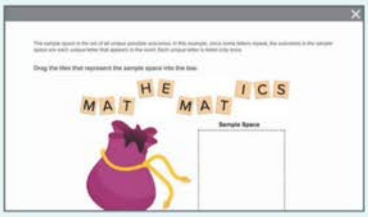
Interactive Presentation



Example 1, Find Sample Space of Simple Events, Slide 2 of 4

DRAG & DROP

On Slide 2, students drag the tiles that represent the sample space.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Find Sample Space of Simple Events

Objective

Students will find the sample space of simple events.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enco urage students to use the drag and drop activity to find the sample space.

6 Attend to Precision Students should use precision when finding the number of *unique* outcomes in the sample space.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language to explain the discrepancy.

Questions for Mathematical Discourse

SLIDE 2

AL How many letters are in the word MATHEMATICS? 11 letters

OL Do any of the letters repeat? If so, which ones? yes; M, A, and T each repeat

OL How many *unique* letters are in the sample space? 8 letters

OL Can the letter O be drawn from the bag? Explain. no; The letter O is not in the sample space.

BL Generate another word that would have a different number of letters in its sample space than total number of letters in the word. Explain your choice. Sample answer: BIOLOGY has 7 letters total, but only 6 letters in its sample space because the letter O is repeated.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Theoretical Probability of Simple Events

Objective

Students will understand how to find the theoretical probability of simple events.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to think about what it means for probability to be unlikely and how that translates to a numerical probability.

Teaching Notes

SLIDE 1

Students will learn the terms *uniform probability model* and *theoretical probability*. You may wish to have students describe how the everyday meaning of the word *uniform* can help them understand the meaning of *uniform probability*. Since the term *uniform* means remaining the same in all cases and all times, a uniform probability model means that each probability is the same, or the outcomes are equally likely.

Have students select the *Words* and *Ratio* flashcards to view how *theoretical* probability can be expressed using these multiple representations.

To help students remember the meaning of theoretical probability, have them use their understanding of the term *theoretical*, which means based or calculated through theory, as opposed to experiment or practice.

SLIDE 2

Students will compare the relative frequency of a completed probability experiment to the theoretical probability of each event in the experiment happening. Point out to students that theoretical probability is what you expect to happen when conducting an experiment, while the relative frequency ratio describes what actually happened.

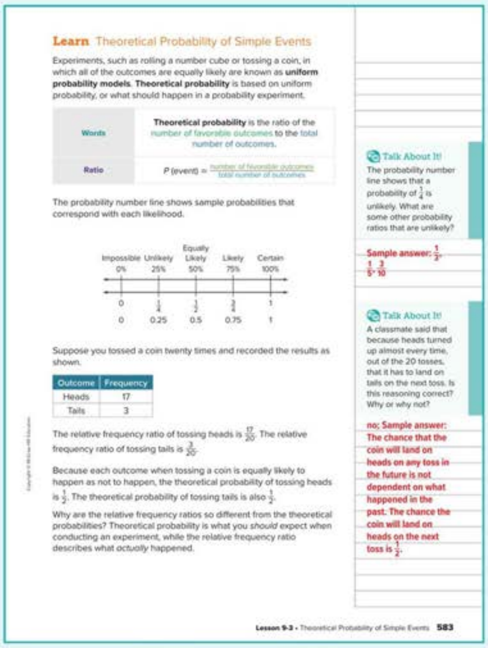
Talk About It!

SLIDE 3

Mathematical Discourse

The probability number line shows that a probability of $\frac{1}{4}$ is unlikely. What are some other probability ratios that are unlikely? Sample answer: $\frac{1}{3}, \frac{1}{5}, \frac{3}{10}$

A classmate said that because heads turned up almost every time, out of the 20 tosses, that it has to land on tails on the next toss. Is this reasoning correct? Why or why not? no; Sample answer: The chance that the coin will land on tails on any toss in the future is not dependent on what happened in the past. The chance the coin will land on tails on the next toss is $\frac{1}{2}$.



Learn Theoretical Probability of Simple Events

Experiments, such as rolling a number cube or tossing a coin, in which all of the outcomes are equally likely are known as **uniform probability models**. **Theoretical probability** is based on uniform probability, or what should happen in a probability experiment.

Words	**Theoretical probability** is the ratio of the number of favorable outcomes to the total number of outcomes.
Ratio	$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

The probability number line shows sample probabilities that correspond with each likelihood.

Suppose you tossed a coin twenty times and recorded the results as shown.

Outcome	Frequency
Heads	17
Tails	3

The relative frequency ratio of tossing heads is $\frac{17}{20}$. The relative frequency ratio of tossing tails is $\frac{3}{20}$.

Because each outcome when tossing a coin is equally likely to happen as not to happen, the theoretical probability of tossing heads is $\frac{1}{2}$. The theoretical probability of tossing tails is also $\frac{1}{2}$.

Why are the relative frequency ratios so different from the theoretical probabilities? Theoretical probability is what you *should* expect when conducting an experiment, while the relative frequency ratio describes what *actually* happened.

Talk About It! The probability number line shows that a probability of $\frac{1}{4}$ is unlikely. What are some other probability ratios that are unlikely?

Sample answer: $\frac{1}{2}, \frac{1}{5}, \frac{3}{10}$

Talk About It! A classmate said that because heads turned up almost every time, out of the 20 tosses, that it has to land on tails on the next toss. Is this reasoning correct? Why or why not?

no; Sample answer: The chance that the coin will land on heads on any toss in the future is not dependent on what happened in the past. The chance the coin will land on heads on the next toss is $\frac{1}{2}$.

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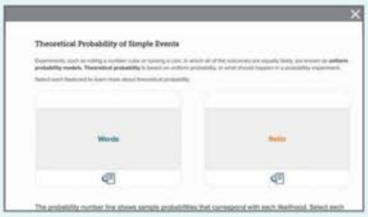
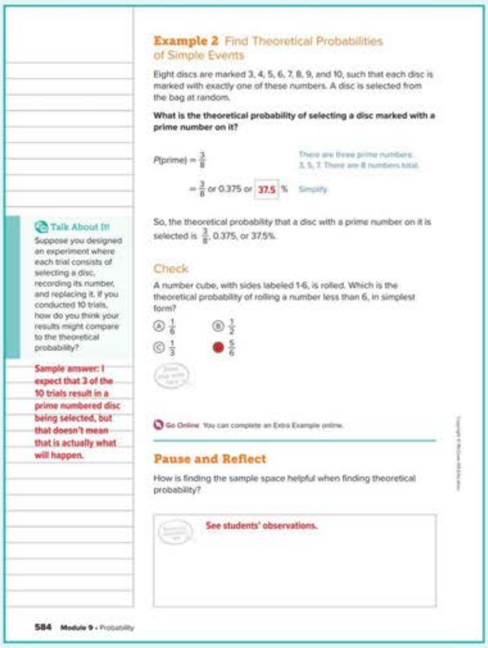
Interactive Presentation



Learn, Theoretical Probability of Simple Events, Slide 1 of 3

FLASHCARDS

On Slide 1, students use Flashcards to view different representations of theoretical probability.



Example 2 Find Theoretical Probabilities of Simple Events

Eight discs are marked 3, 4, 5, 6, 7, 8, 9, and 10, such that each disc is marked with exactly one of these numbers. A disc is selected from the bag at random.

What is the theoretical probability of selecting a disc marked with a prime number on it?

$P(\text{prime}) = \frac{3}{8}$ There are three prime numbers: 3, 5, 7. There are 8 numbers total.

$= \frac{3}{8}$ or 0.375 or 37.5 % Simplify.

So, the theoretical probability that a disc with a prime number on it is selected is $\frac{3}{8}$, 0.375, or 37.5%.

Talk About It!
Suppose you designed an experiment where each trial consists of selecting a disc, recording its number, and replacing it. If you conducted 10 trials, how do you think your results might compare to the theoretical probability?

Sample answer: I expect that 3 of the 10 trials result in a prime numbered disc being selected, but that doesn't mean that is actually what will happen.

Check
A number cube, with sides labeled 1-6, is rolled. Which is the theoretical probability of rolling a number less than 6, in simplest form?

Ⓐ $\frac{1}{6}$ Ⓑ $\frac{1}{2}$
Ⓒ $\frac{1}{3}$ ● $\frac{5}{6}$

Go Online You can complete an Extra Example online.

Pause and Reflect
How is finding the sample space helpful when finding theoretical probability?

See students' observations.

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Interactive Presentation

Example 2, Find Theoretical Probabilities of Simple Events, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the theoretical probability.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find Theoretical Probabilities of Simple Events

Objective

Students will find the theoretical probability of simple events.

Questions for Mathematical Discourse

SLIDE 2

AL How many total outcomes are there? List them. 8 outcomes; The outcomes are 3, 4, 5, 6, 7, 8, 9, and 10.

AL Describe the favorable outcome. The favorable outcome is a disc with a prime number on it.

OL How many favorable outcomes are there? List them. 3 outcomes; The favorable outcomes are 3, 5, and 7.

OL Is the theoretical probability of selecting a disc with a non-prime number on it the same as the theoretical probability of selecting a disc with a prime number on it? Explain. no; There are 3 discs with a prime number on them out of 8 total discs, so 5 discs out of 8 would have non-prime numbers on them. One of the probabilites is $\frac{3}{8}$ and the other is $\frac{5}{8}$.

BL How can you use the theoretical probability to predict the number of times a disc with a prime number on it will be selected if a disc is selected 500 times? Assume it is replaced after each selection. Then make a prediction. Sample answer: Find $\frac{3}{8}$ of 500; I predict that a disc with a prime number will be selected about 188 times.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Complements of Simple Events

Objective

Students will understand how to find the complements of simple events.

Go Online to find additional teaching notes and Teaching the Mathematical practices.

Teaching Notes

SLIDE 1

Before students choose the *Words*, *Symbols*, *Equation* flashcards, you may wish to ask them to explain complementary events in their own words. Some students may say that complementary events consist of the desired outcome and every thing *except* for the desired outcome. Remind students that the probability of an event that is certain is 100% or 1. It is certain that an event will either happen or not happen, so the sum of the probabilities of complementary events must be 100% or 1.

Have students select the *Words*, *Symbols*, *Equation* flashcards to view how com plementary events can be described using multiple representations.

Talk About It!

SLIDE 5

Mathematical Discourse

What do you notice about the relationship between the probability of an event and its complement? Sample answer: The probability of an event occurring or its complement make up all the possible outcomes of an event and their values add to 1.

DIFFERENTIATE

Reteaching Activity AL

Some students may better be able to identify complements by first writing out the sample space. Explain to students that the complement of an event is every event in the sample space other than the desired outcome. Have students identify the complement of the following event by listing the sample space and eliminating the desired outcome of the event.

A spinner is divided into 15 equal sections, numbered 1-15. The desired outcome is a multiple of 2 or a multiple of 3.

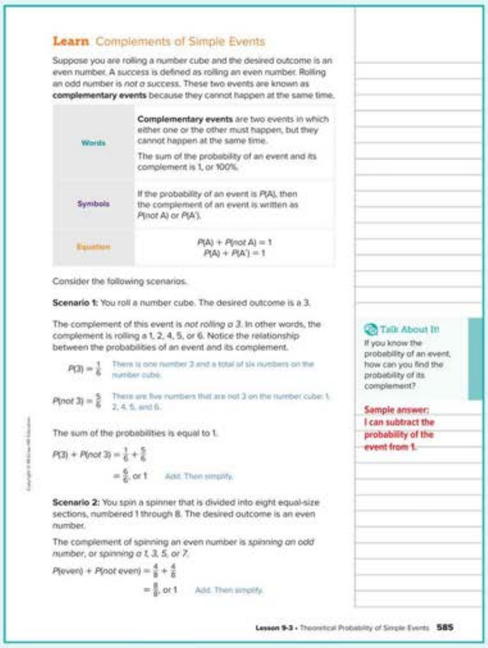
What is the sample space? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

What numbers in the sample space are multiples of 2? 2, 4, 6, 8, 10, 12, 14

What numbers in the sample space are multiples of 3? 3, 6, 9, 12, 15

Eliminate the multiples of 2 and 3 from the sample space. What numbers remain? 1, 5, 7, 11, 13

What is the complement of spinning a multiple of 2 or a multiple of 3? spinning a 1, 5, 7, 11, or 13



Learn Complements of Simple Events

Suppose you are rolling a number cube and the desired outcome is an even number. A success is defined as rolling an even number. Rolling an odd number is *not a success*. These two events are known as **complementary events** because they cannot happen at the same time.

Words	**Complementary events** are two events in which either one or the other must happen, but they cannot happen at the same time. The sum of the probability of an event and its complement is 1, or 100%.
Symbols	If the probability of an event is $P(A)$, then the complement of an event is written as $P(\text{not } A)$ or $P(A')$.
Equation	$P(A) + P(\text{not } A) = 1$ $P(A) + P(A') = 1$

Consider the following scenarios.

Scenario 1: You roll a number cube. The desired outcome is a 3.

The complement of this event is *not rolling a 3*. In other words, the complement is rolling a 1, 2, 4, 5, or 6. Notice the relationship between the probabilities of an event and its complement.

$P(3) = \frac{1}{6}$ There is one number 3 and a total of six numbers on the number cube.

$P(\text{not } 3) = \frac{5}{6}$ There are five numbers that are not 3 on the number cube: 1, 2, 4, 5, and 6.

The sum of the probabilities is equal to 1.

$P(3) + P(\text{not } 3) = \frac{1}{6} + \frac{5}{6}$

$= \frac{6}{6}$, or 1 Add. Then simplify.

Scenario 2: You spin a spinner that is divided into eight equal-size sections, numbered 1 through 8. The desired outcome is an even number.

The complement of spinning an even number is *spinning an odd number, or spinning a 1, 3, 5, or 7*.

$P(\text{even}) + P(\text{not even}) = \frac{4}{8} + \frac{4}{8}$

$= \frac{8}{8}$, or 1 Add. Then simplify.

Talk About It! If you know the probability of an event, how can you find the probability of its complement?

Sample answer: I can subtract the probability of the event from 1.

Lesson 9-3 • Theoretical Probability of Simple Events 585

Interactive Presentation

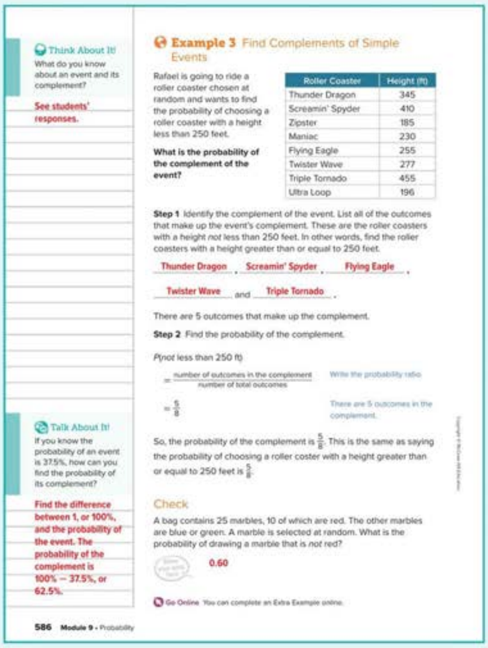
Learn, Complements of Simple Events, Slide 1 of 5

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of complementary events.

DRAG & DROP

On Slides 2-4, students drag and drop complements of scenarios.



Think About It! What do you know about an event and its complement?

See students' responses.

Example 3 Find Complements of Simple Events

Rafael is going to ride a roller coaster chosen at random and wants to find the probability of choosing a roller coaster with a height less than 250 feet.

What is the probability of the complement of the event?

Roller Coaster	Height (ft)
Thunder Dragon	345
Screamin' Spyder	410
Zipster	185
Maniac	230
Flying Eagle	255
Twister Wave	277
Triple Tornado	455
Ultra Loop	196

Step 1 Identify the complement of the event. List all of the outcomes that make up the event's complement. These are the roller coasters with a height *not* less than 250 feet. In other words, find the roller coasters with a height greater than or equal to 250 feet.

Thunder Dragon, Screamin' Spyder, Flying Eagle, Twister Wave and Triple Tornado.

There are 5 outcomes that make up the complement.

Step 2 Find the probability of the complement.

P(not less than 250 ft)

$= \frac{\text{number of outcomes in the complement}}{\text{number of total outcomes}}$ Write the probability ratio.

$= \frac{5}{8}$ There are 5 outcomes in the complement.

So, the probability of the complement is $\frac{5}{8}$. This is the same as saying the probability of choosing a roller coster with a height greater than or equal to 250 feet is $\frac{5}{8}$.

Talk About It! If you know the probability of an event is 37.5%, how can you find the probability of its complement?

Find the difference between 1, or 100%, and the probability of the event. The probability of the complement is 100% − 37.5%, or 62.5%.

Check

A bag contains 25 marbles, 10 of which are red. The other marbles are blue or green. A marble is selected at random. What is the probability of drawing a marble that is *not* red?

0.60

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 3, Find Complements of Simple Events, Slide 2 of 5

CLICK On Slide 2, students select all the outcomes that make up the event's complement.

CLICK On Slide 3, students move through the steps to find the probability of the complement.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Find Complements of Simple Events

Objective

Students will find complements of simple events.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the relationship between the probability of an event and its complement.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Students may confuse the terms *compliment* (everyday use), *complimentary* (everyday use), *complement* (in probability), and *complementary* (in probability or geometry). Have students create a table or graphic organizer to see when and how each term is used. A sample table is shown. Have them discuss with a partner how they can use the context of a conversation or topic to determine the meaning of these words within context.

Term	Use
compliment	**Everyday Use** To *compliment* someone is to acknowledge them with praise.
complimentary	**Everyday Use** An item that is given away for free is a *complimentary* item.
complement	**Math Use** In probability, the *complement* of an event occurring is the event *not* occurring.
complementary (angles)	**Math Use** In geometry, *complementary angles* are two angles whose measures add up to 90.
complementary (events)	**Math Use** In probability, *complementary events* are two events whose probabilities add to 1.

Apply Probability

Objective

Students will come up with their own strategy to solve an application problem involving probability.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

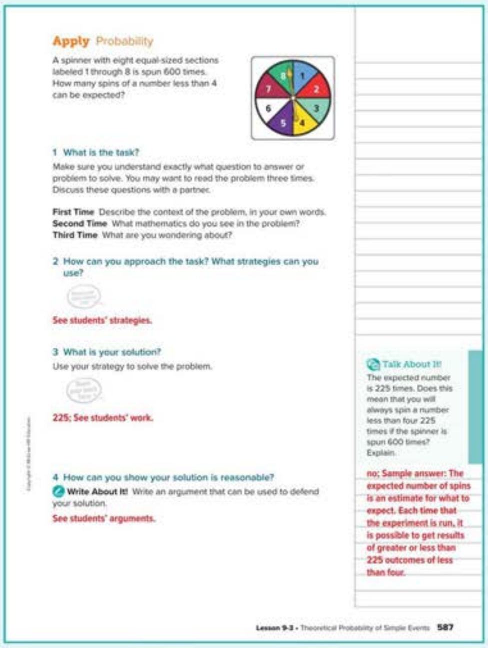
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How many possible outcomes are there?
- Write a fraction to represent the possibility of landing on one of the sections.
- Is each outcome equally likely? How do you know?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Probability

A spinner with eight equal-sized sections labeled 1 through 8 is spun 600 times. How many spins of a number less than 4 can be expected?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

225; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! The expected number is 225 times. Does this mean that you will always spin a number less than four 225 times if the spinner is spun 600 times? Explain.

no; Sample answer: The expected number of spins is an estimate for what to expect. Each time that the experiment is run, it is possible to get results of greater or less than 225 outcomes of less than four.

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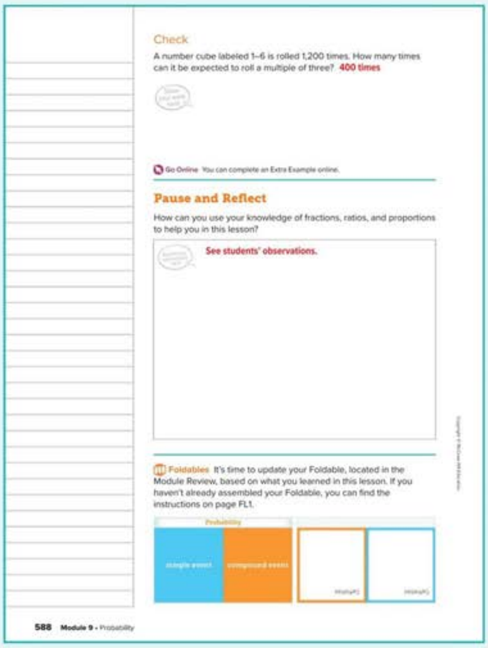
Interactive Presentation

Apply, Probability

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

A number cube labeled 1–6 is rolled 1,200 times. How many times can it be expected to roll a multiple of three? 400 times

Go Online You can complete an Extra Example online.

Pause and Reflect

How can you use your knowledge of fractions, ratios, and proportions to help you in this lesson?

See students' observations.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about simple events. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can probability be used to predict future events?

In this lesson, students learned how to find the theoretical probability of simple events. Encourage them to discuss with a partner how they can use the probability of rolling a 6 on a six-sided number cube to help them predict how likely it will be to roll a 6 when the same number cube is rolled in the future.

Exit Ticket

Refer to the Exit Ticket slide. The spinner shown has 18 equal-size sections. Suppose the spinner is spun once. Landing on a number greater than 15 means you win a prize. Explain how you can find the chance of winning before you play. Sample answer: Write a ratio of the number of favorable outcomes to the number of possible outcomes. There are 3 favorable outcomes: 16, 17, and 18. There are 18 total possible outcomes. The chance of winning is $\frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{3}{18} = \frac{1}{6}$, or about 16.7%.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7, 9, 10–13
- Extension: Extension Resources
- ALEKS® Probability of Simple Events

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 8, 10
- Extension: Extension Resources
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Frequency Tables

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Frequency Tables

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the sample space of simple events	1, 2
2	find the theoretical probability of simple events	3, 4
2	find complements of simple events	5, 6
2	extend concepts learned in class to apply them in new contexts	7
3	solve application problems involving theoretical probability of simple events	8–9
3	higher-order and critical thinking skills	10–13

Common Misconception

Some students may identify the sample space by listing all of the possible outcomes. In Exercise 1, students may list all the spinner spaces, including 5 four times, 4 two times, and so on. Explain to students that the sample space does not include duplicate events.

Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The spinner shown is spun once. What is the sample space? (Example 1)
1, 2, 3, 4, 5

2. Each letter in the word MISSISSIPPI is written on a piece of paper and placed into a bag. A letter is drawn at random. What is the sample space? (Example 1)
M, I, S, P

3. A teacher placed the letter cards E, L, O, R, U, and W in a bag. A card is drawn at random. Determine the theoretical probability for drawing a card that has a vowel on it. (Example 2)
$\frac{1}{2}$, 0.5, 50%

4. A player in a board game rolls a six-sided number cube labeled 1 through 6 once. Determine the theoretical probability of rolling a 1 or 2. (Example 2)
$\frac{1}{3}$, $0.\bar{3}$, $33\frac{1}{3}$%

5. The table shows the lengths of time for rides at a fair. Zane will choose a ride at random and wants to find the probability of choosing a ride that lasts less than 200 seconds. What is the probability of the complement of the event? Describe the complement. (Example 3)
$\frac{3}{8}$, 0.375, 37.5%; The complement is choosing a ride that lasts at least 200 seconds.

Ride	Time (seconds)
Barrel	150
Bumper Cars	195
Circus Carousel	210
Log Ride	120
Roller Coaster	55
Swings	225
Train	300
Zero Gravity Spinner	65

6. Red is spun on a spinner with five equal-size sections labeled red, yellow, blue, green, and purple. What is the probability of the complement of the event? Describe the complement. (Example 3)
$\frac{4}{5}$, 0.8, 80%; The complement is spinning yellow, blue, green, or purple.

Test Practice

7. **Multiselect** A sportscaster predicted that the local high school baseball team has a 75% chance of winning tonight. Select all of the values that represent the probability of the team *not* winning.

- [ ] 0.75
- [x] 0.25
- [ ] 75%
- [x] 25%
- [ ] $\frac{3}{4}$
- [x] $\frac{1}{4}$

Lesson 9-3 • Theoretical Probability of Simple Events 589

Apply *indicates multi-step problem

*8. A pet store is having a prize give-away. The spinner shows the type of toy a customer can win for their pet. If a customer spins the spinner and it lands on cat, they will win a free cat toy. If the spinner is spun 540 times throughout the day, about how many dog or cat toys are expected to be given away?

378 dog or cat toys

*9. The letters from the word FOOTBALL are written on 8 cards with one letter on each card. One card will be drawn randomly and then placed back into the stack. If this experiment is repeated 840 times, about how many times should you expect to draw a consonant?

525 times

Higher-Order Thinking Problems

10. Describe a real-world situation that involves a sample space. Then describe the sample space.

Sample answer: A teacher wrote each letter in the word BASEBALL on a piece of paper and placed the letters into a bag. What is the sample space?; B, A, S, E, L

11. **MP Find the Error** The spinner shown has 8 equal-size sections. A student said that the theoretical probability of spinning a multiple of 3 on the spinner is $\frac{5}{8}$. Find the student's error and correct it.

The student found the complement of spinning a multiple of 3. The correct probability is $\frac{3}{8}$.

12. **MP Reason Inductively** The weather reporter says that there is an 88% chance that it will not be windy tomorrow. Will tomorrow be a good day to fly a kite? Explain.

no; Sample answer: The complement of not windy is windy. So, there is a 100% − 88% or 12% chance of wind. A 12% chance means that it is unlikely to be windy and not a good day to fly a kite.

13. **Create** Write a real-world problem that involves finding the complement of the event. Then find the complement.

Sample answer: The weather reporter says that there is a 65% chance that it will rain tomorrow. What is the chance it will not rain tomorrow?; 35%

590 Module 9 • Probability

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students find and correct another student's mistake.

2 Reason Abstractly and Quantitatively In Exercise 12, students interpret probability of the weather by using reasoning about the relationship between the desired outcome and its complement.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercises 8–9 Have students work in groups of 3–4 to solve the problem in Exercise 8. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 9.

Make sense of the problem.

Use with Exercise 11 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise found the complement of the event rather than the probability of the event. Have each pair or group of students present their explanations to the class.

Lesson 9-4

7.SP.C.6, 7.SP.C.7

Compare Probabilities of Simple Events

LESSON GOAL

Students will solve problems that compare probabilities and relative frequencies of simple events.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Compare Relative Frequency to Theoretical Probability

Example 1: Compare Relative Frequencies to Probabilities

Apply: Experiments

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 55 of the *Language Development Handbook* to help your students build mathematical language related to comparing relative frequency and theoretical probability of simple events.

ELL You can use the tips and suggestions on page T55 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability

Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.C** by solving problems that compare probabilities and relative frequencies of simple events.

Standards for Mathematical Content: **7. SP.C.6, 7.SP.C.7, 7.SP.C.7.A 7.SP.C.7.B**

Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous

Students solved problems involving theoretical probability of simple events and their complements.

7.SP.C.7, 7.SP.C.7.A

Now

Students solve problems that compare probabilities and relative frequencies of simple events.

7.SP.C.6, 7.SP.C.7, 7.SP.C.7.A, 7.SP.C.7.B

Next

Students will solve problems involving the probability of compound events.

7.SP.C.8, 7.SP.C.8.A, 7.SP.C.8.B

Rigor

The Three Pillars of Rigor

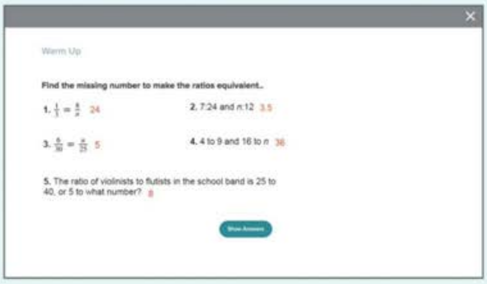
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will use their knowledge of relative frequency and theoretical probability to build an *understanding* of comparing probabilities of simple events. They will use this understanding to develop a *fluency* in comparing probabilities of simple events.

Mathematical Background

The relative frequency of an event is what actually happens in a probability experiment, while the theoretical probability is what should happen based on the experiment's design. As the number of trials increases, the theoretical probability and long-run relative frequency of an event become closer in value.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding equivalent ratios (Exercises 1–5)

Answers

1. 24 **2.** 3.5
3. 5 **4.** 36
5. 8

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about words used to describe the likelihood of an event.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Explain how the meaning of the word *relative* can help you understand the meaning of *relative frequency*. Sample answer: Relative means to consider something in relation or in proportion to something else. So, relative frequency is comparing frequencies in relation to other frequencies or to a whole.
- What does *theoretical* mean? Sample answer: Theoretical means something is based on theory rather than experience or practice.

Learn Compare Relative Frequency to Theoretical Probability

Objective

Students will understand how to compare relative frequency to theoretical probability.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, encourage them to use mathematical reasoning in their explanation.

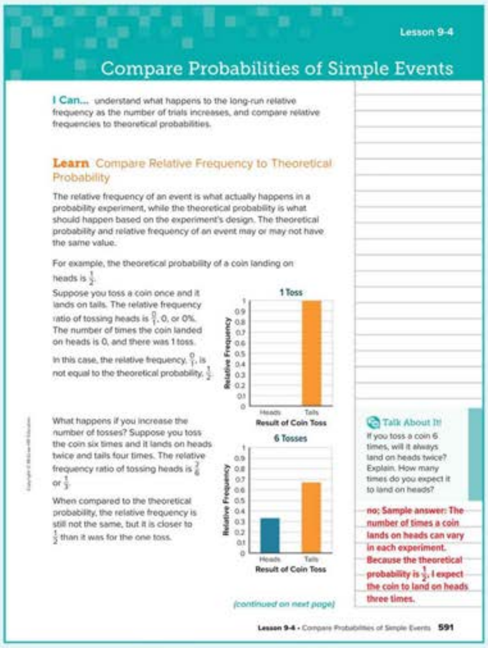
Talk About It!

SLIDE 2

Mathematical Discourse

If you toss a coin 6 times, will it always land on heads twice? Explain. How many times do you expect it to land on heads? no; Sample answer: The number of times a coin lands on heads can vary in each experiment. Because the theoretical probability is $\frac{1}{2}$, I expect the coin to land on heads three times.

(continued on next page)



Lesson 9-4

Compare Probabilities of Simple Events

I Can... understand what happens to the long-run relative frequency as the number of trials increases, and compare relative frequencies to theoretical probabilities.

Learn Compare Relative Frequency to Theoretical Probability

The relative frequency of an event is what actually happens in a probability experiment, while the theoretical probability is what should happen based on the experiment's design. The theoretical probability and relative frequency of an event may or may not have the same value.

For example, the theoretical probability of a coin landing on heads is $\frac{1}{2}$.

Suppose you toss a coin once and it lands on tails. The relative frequency ratio of tossing heads is $\frac{0}{1}$, 0, or 0%. The number of times the coin landed on heads is 0, and there was 1 toss.

In this case, the relative frequency, $\frac{0}{1}$, is not equal to the theoretical probability, $\frac{1}{2}$.

What happens if you increase the number of tosses? Suppose you toss the coin six times and it lands on heads twice and tails four times. The relative frequency ratio of tossing heads is $\frac{2}{6}$ or $\frac{1}{3}$.

When compared to the theoretical probability, the relative frequency is still not the same, but it is closer to $\frac{1}{2}$ than it was for the one toss.

Talk About It!

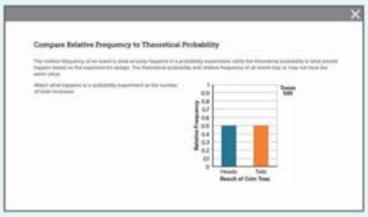
If you toss a coin 6 times, will it always land on heads twice? Explain. How many times do you expect it to land on heads?

no; Sample answer: The number of times a coin lands on heads can vary in each experiment. Because the theoretical probability is $\frac{1}{2}$, I expect the coin to land on heads three times.

(continued on next page)

Lesson 9-4 • Compare Probabilities of Simple Events 591

Interactive Presentation



Learn, Compare Relative Frequency to Theoretical Probability, Slide 1 of 2

DIFFERENTIATE

Reteaching Activity AL

If students are struggling to understand relative frequency and theoretical probability, have them work in groups to complete the following activity.

Give each group one standard number cube.

What is the sample space? 1, 2, 3, 4, 5, 6

What is the theoretical probability of rolling a 3 or a 4? $\frac{1}{3}$ or $0.\overline{3}$

Have each group roll the number cube 10 times and record the results in a table like the one shown. Then have them compare the relative frequency of the desired outcome to the theoretical probability, and discuss any differences.

Roll	Outcome	Ratio of Desired Outcomes to Total
1...		
...10		

Your Notes

What happens if you continue to increase the number of tosses? Suppose you toss the coin sixteen times and it lands on heads ten times and tails six times. The relative frequency ratio of tossing heads is $\frac{10}{16}$ or $\frac{5}{8}$.

When compared to the theoretical probability, the relative frequency is still not the same as the theoretical probability, but it is closer to $\frac{1}{2}$ than it was for the one toss or for the six tosses.

Suppose the number of tosses continues to increase as shown in the relative frequency bar graphs below.

As the number of tosses increase, the long-run relative frequency ratio of tossing heads becomes closer to the theoretical probability of $\frac{1}{2}$. The long-run relative frequency of an event will approach the value of the theoretical probability as the number of trials increases. This is known as the *Law of Large Numbers*.

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Interactive Presentation

Learn, Compare Relative Frequency to Theoretical Probability, Slide 2 of 2

WATCH

On Slide 2, students watch an animation that illustrates the effect of the number of trials in an experiment.

CLICK

On Slide 2, students select how the theoretical probability and the long-run relative frequency of an event relate.

7.SP.C.6, 7.SP.C.7

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Compare Relative Frequency to Theoretical Probability *(continued)*

Teaching Notes

SLIDE 1

Students will view a brief animation that illustrates how the number of trials in an experiment can affect how close the long-run relative frequency of an event comes to the theoretical probability of that event occurring.

Students should note that as the number of trials in a probability experiment increases, the long-run relative frequency of an event gets closer to the theoretical probability. This is known as the Law of Large Numbers.

Example 1 Compare Relative Frequencies to Probabilities

Objective

Students will compare relative frequency to the theoretical probability of a simple event.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enco urage students to make sense of what each frequency bar graph, the spinner, and the tetrahedron tells them about whether the outcomes are equally likely.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 3, encourage them to use clear and precise mathematical language, such as the comparison of relative frequency to theoretical probability when the number of trials is small, in their explanations.

7 Look For and Make Use of Structure Encourage students to study the structure of each frequency bar graph, the spinner, and the tetrahedron in order to make connections between each student's experiment and graph.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the frequency bar graphs for each student? Sample answer: The bars for results of 3 and 4 are much higher than 1 and 2 for Student 1's graph. In Student 2's graph, the bars are all around the same height.

AL What do you notice about the structure of the spinner? the tetrahedron? Sample answer: The spinner's sections are not all the same size. The tetradhedron's faces are all the same size.

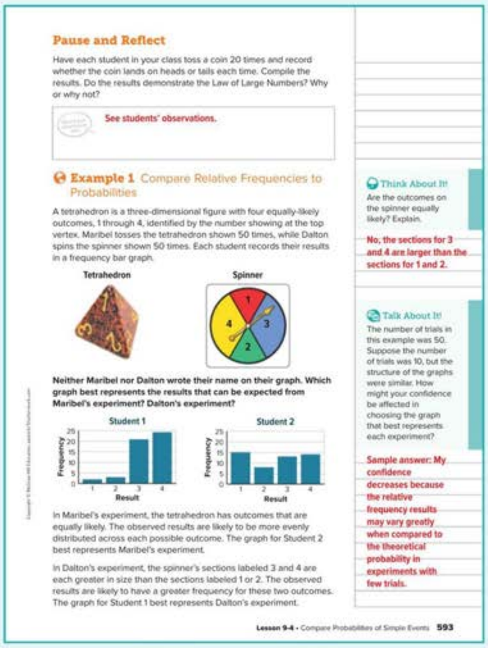
OL When the tetrahedron is tossed, what do you expect to happen? Sample answer: The results 1, 2, 3, and 4 are expected to occur with the same frequency.

OL Which graph do you think best represents tossing the tetrahedron? Explain. Sample answer: Student 2, because the bars for 1, 2, 3, and 4 are all about the same height.

BL Is it possible that Student 1's graph actually represents the results of the tetrahedron? Explain. Sample answer: Yes, it is possible, but unlikely.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Pause and Reflect

Have each student in your class toss a coin 20 times and record whether the coin lands on heads or tails each time. Compile the results. Do the results demonstrate the Law of Large Numbers? Why or why not?

See students' observations.

Example 1 Compare Relative Frequencies to Probabilities

A tetrahedron is a three-dimensional figure with four equally-likely outcomes, 1 through 4, identified by the number showing at the top vertex. Maribel tosses the tetrahedron shown 50 times, while Dalton spins the spinner shown 50 times. Each student records their results in a frequency bar graph.

Neither Maribel nor Dalton wrote their name on their graph. Which graph best represents the results that can be expected from Maribel's experiment? Dalton's experiment?

In Maribel's experiment, the tetrahedron has outcomes that are equally likely. The observed results are likely to be more evenly distributed across each possible outcome. The graph for Student 2 best represents Maribel's experiment.

In Dalton's experiment, the spinner's sections labeled 3 and 4 are each greater in size than the sections labeled 1 or 2. The observed results are likely to have a greater frequency for these two outcomes. The graph for Student 1 best represents Dalton's experiment.

Think About It! Are the outcomes on the spinner equally likely? Explain.

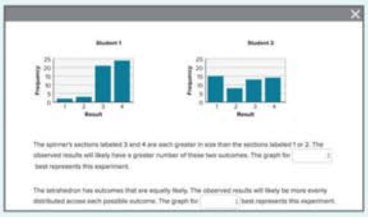
No, the sections for 3 and 4 are larger than the sections for 1 and 2.

Talk About It! The number of trials in this example was 50. Suppose the number of trials was 10, but the structure of the graphs were similar. How might your confidence be affected in choosing the graph that best represents each experiment?

Sample answer: My confidence decreases because the relative frequency results may vary greatly when compared to the theoretical probability in experiments with few trials.

Lesson 9-4 • Compare Probabilities of Simple Events 593

Interactive Presentation



Example 1, Compare Relative Frequencies to Probabilities, Slide 2 of 4

CLICK

On Slide 2, students select from drop-down menus the graphs that best represent given experiments.

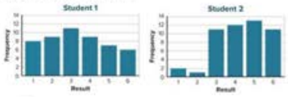
CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Jaden and Nikhil each spin their respective spinners shown 50 times. They each record their results in a frequency bar graph.

Neither Jaden nor Nikhil wrote their name on their graph. Which graph best represents the results that can be expected from Jaden's experiment? Nikhil's experiment?



Sample answer: In Jaden's experiment, the spinner's sections labeled 1 and 2 are each smaller in size than the other sections. The observed results are likely to have a lesser frequency for these two outcomes. The graph for Student 2 best represents Jaden's experiment. In Nikhil's experiment, the spinner is divided into 6 equal-size sections. The outcomes are all equally likely. The observed results are likely to be more evenly distributed across each possible outcome. The graph for Student 1 best represents Nikhil's experiment.

Go Online You can complete an Extra Example online.

Apply Experiments

Objective

Students will come up with their own strategy to solve an application problem involving probability experiments.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

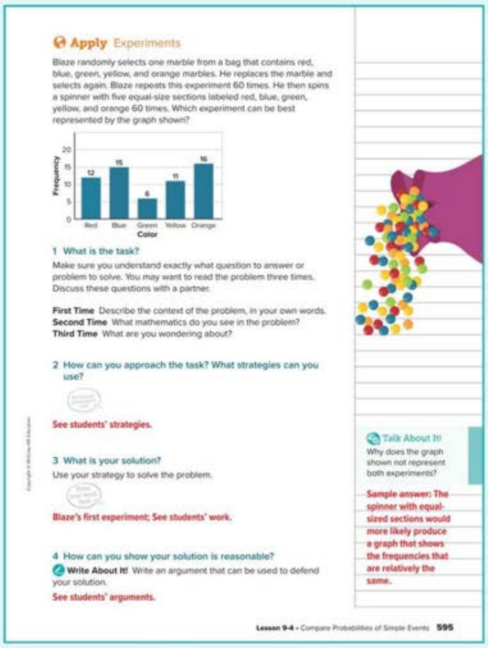
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What do you notice about the heights of the bars for each color?
- Are the probabilities of landing on or choosing a color the same for each experiment?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Experiments

Blaze randomly selects one marble from a bag that contains red, blue, green, yellow, and orange marbles. He replaces the marble and selects again. Blaze repeats this experiment 60 times. He then spins a spinner with five equal-size sections labeled red, blue, green, yellow, and orange 60 times. Which experiment can be best represented by the graph shown?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Blaze's first experiment; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

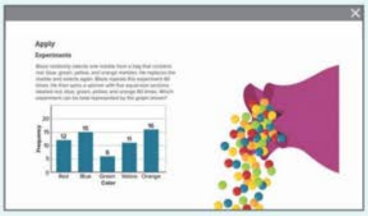
See students' arguments.

Talk About It!
Why does the graph shown not represent both experiments?

Sample answer: The spinner with equal-sized sections would more likely produce a graph that shows the frequencies that are relatively the same.

Lesson 9-4 • Compare Probabilities of Simple Events 595

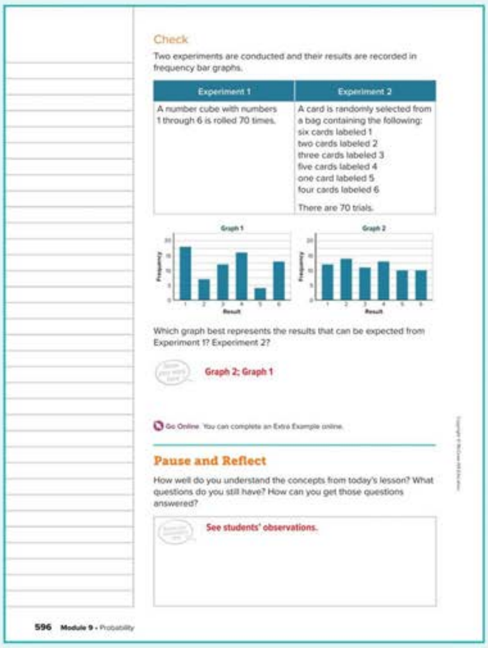
Interactive Presentation



Apply, Experiments

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Two experiments are conducted and their results are recorded in frequency bar graphs.

Experiment 1	Experiment 2
A number cube with numbers 1 through 6 is rolled 70 times.	A card is randomly selected from a bag containing the following: six cards labeled 1, two cards labeled 2, three cards labeled 3, five cards labeled 4, one card labeled 5, four cards labeled 6. There are 70 trials.

Which graph best represents the results that can be expected from Experiment 1? Experiment 2?

Graph 2; Graph 1

Go Online You can complete an Extra Example online.

Pause and Reflect

How well do you understand the concepts from today's lesson? What questions do you still have? How can you get those questions answered?

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can probability be used to predict future events?

In this lesson, students learned how to compare relative frequency to theoretical probability. Encourage them to discuss with a partner when the relative frequency of an event during an experiment might be close to the event's theoretical probability, and when it might be different.

Exit Ticket

Refer to the Exit Ticket slide. Explain when the relative frequency of an experiment might be different from its theoretical probability. Also, explain when the relative frequency of an experiment might be close to its theoretical probability. Sample answer: The relative frequency of an experiment might be different from its theoretical probability if there are few trials. The relative frequency of an experiment might be close to its theoretical probability if there are many trials.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 3, 4–7
- ALEKS® Probability of Simple Events

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–3, 5
- Remediation: Review Resources
- Personal Tutor
- Extra Example 1
- ALEKS® Probability of Simple Events

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ALEKS® Probability of Simple Events

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

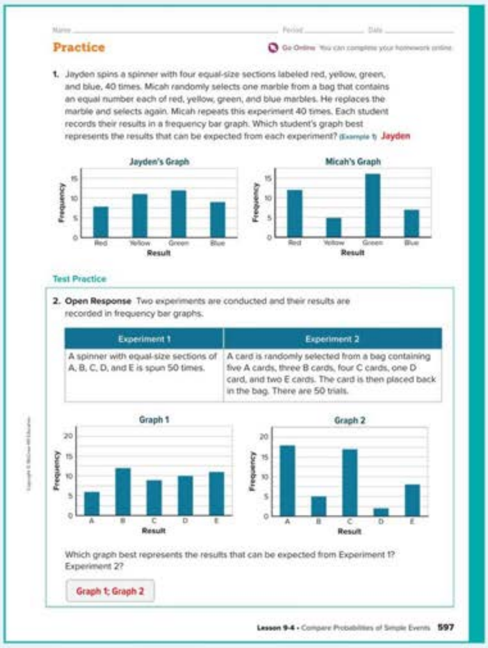
The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	compare relative frequency to the theoretical probability of a simple event	1
2	extend concepts learned in class to apply them in new contexts	2
3	solve application problems involving comparing relative frequency to theoretical probability of simple events	3
3	higher-order and critical thinking skills	4–7


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Jayden spins a spinner with four equal-size sections labeled red, yellow, green, and blue, 40 times. Micah randomly selects one marble from a bag that contains an equal number each of red, yellow, green, and blue marbles. He replaces the marble and selects again. Micah repeats this experiment 40 times. Each student records their results in a frequency bar graph. Which student's graph best represents the results that can be expected from each experiment? (Example 1) **Jayden**

Test Practice

2. **Open Response** Two experiments are conducted and their results are recorded in frequency bar graphs.

Experiment 1	Experiment 2
A spinner with equal-size sections of A, B, C, D, and E is spun 50 times.	A card is randomly selected from a bag containing five A cards, three B cards, four C cards, one D card, and two E cards. The card is then placed back in the bag. There are 50 trials.

Which graph best represents the results that can be expected from Experiment 1? Experiment 2?

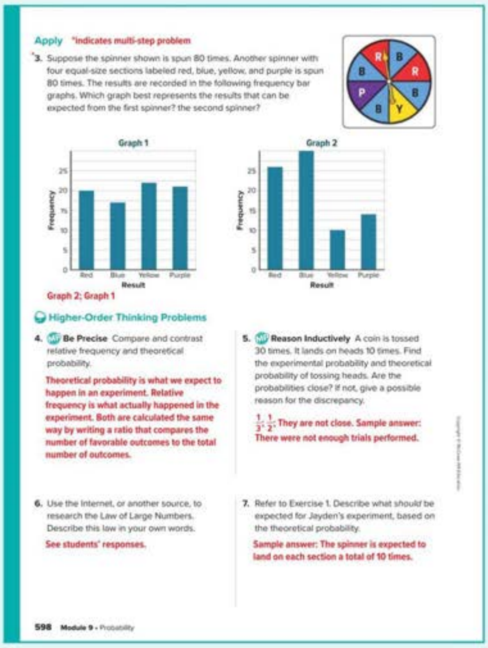
Graph 1; Graph 2

Lesson 9-4 • Compare Probabilities of Simple Events 597

3 REFLECT AND PRACTICE

7.SP.C.6, 7.SP.C.7

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**


Apply *indicates multi-step problem

*3. Suppose the spinner shown is spun 80 times. Another spinner with four equal-size sections labeled red, blue, yellow, and purple is spun 80 times. The results are recorded in the following frequency bar graphs. Which graph best represents the results that can be expected from the first spinner? the second spinner?

Graph 2; Graph 1

Higher-Order Thinking Problems

4. **Be Precise** Compare and contrast relative frequency and theoretical probability.

Theoretical probability is what we expect to happen in an experiment. Relative frequency is what actually happened in the experiment. Both are calculated the same way by writing a ratio that compares the number of favorable outcomes to the total number of outcomes.

5. **Reason Inductively** A coin is tossed 30 times. It lands on heads 10 times. Find the experimental probability and theoretical probability of tossing heads. Are the probabilities close? If not, give a possible reason for the discrepancy.

$\frac{1}{3}$; $\frac{1}{2}$; They are not close. Sample answer: There were not enough trials performed.

6. Use the Internet, or another source, to research the Law of Large Numbers. Describe this law in your own words.

See students' responses.

7. Refer to Exercise 1. Describe what should be expected for Jayden's experiment, based on the theoretical probability.

Sample answer: The spinner is expected to land on each section a total of 10 times.

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Teaching the Mathematical Practices

6 Attend to Precision In E xercise 4, students use precise mathematical language to compare and contrast relative frequency and theoretical probability.

2 Reason Abstractly and Quantitatively In Exercise 5, students use reasoning to compare the experimental and theoretical probabilities of an event and explain possible reasons for any discrepancies.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.
Use with Exercise 3 Have students work in pairs. Have students individually read Exercise 3 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection.

Clearly and precisely explain.
Use with Exercise 4 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as giving examples with their explanations.

Lesson 9-5

Probability of Compound Events

7.SP.C.8.A, 7.SP.C.8.B

LESSON GOAL

Students will solve problems involving the probability of compound events.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Sample Space of Repeated Simple Events

Learn: Sample Space for Compound Events
Example 1: Find Sample Space of Compound Events
Example 2: Find Sample Space of Compound Events
Learn: Theoretical Probability of Compound Events
Example 3: Find Probabilities of Compound Events
Example 4: Find Probabilities of Compound Events
Apply: Outcomes

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Probability With and Without Replacement		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 56 of the *Language Development Handbook* to help your students build mathematical language related to probability of compound events.

ELL You can use the tips and suggestions on page T56 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Statistics and Probability
Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.C** by solving problems involving the probability of compound events.
Standards for Mathematical Content: **7 .SP.C.8, 7.SP.C.8.A, 7.SP.C.8.B**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students solved problems that compare probabilities and relative frequencies of simple events.
7.SP.C.6, 7.SP.C.7, 7.SP.C.7.A, 7.SP.C.7.B

Now
Students solve problems involving the probability of compound events.
7.SP.C.8, 7.SP.C.8.A, 7.SP.C.8.B

Next
Students will solve problems by simulating compound probability events.
7.SP.C.8, 7.SP.C.8.C

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

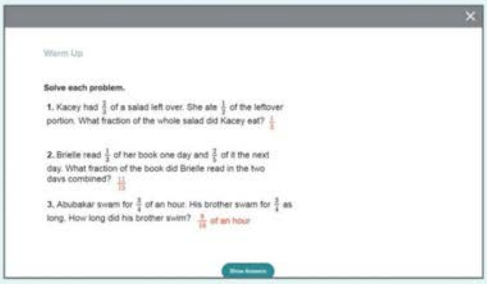
Conceptual Bridge In this lesson, students will use their knowledge of sample space of simple events to develop an *understanding* of finding the sample space of compound events. They will use their knowledege of the probability of simple events to develop *fluency* in finding the probability of compound events. They will *apply* their knowledge to solve real-world problems involving compound events.

Mathematical Background

A *compound event* consists of two or more simple events. For a compound event, the sample space is the set of all possible outcomes. You can use a table, a list, or a *tree diagram* to represent the sample space.

The *theoretical probability of a compound event* is the ratio of the number of favorable outcomes in the sample space to the total possible outcomes in the sample space.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- solving word problems involving multiplying and adding fractions (Exercises 1–3)

Answers

1. $\frac{1}{3}$
2. $\frac{11}{15}$
3. $\frac{9}{16}$ of an hour

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about possible outcomes of a multiple-choice quiz.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Explain the meaning of the term *compound* in your own words. Sample answer: A compound object is composed of two or more parts.
- Use what you know about the *theoretical probability of a simple event* to make a prediction as to what you think the *theoretical probability of a compound event* might mean. Sample answer: The theoretical probability of a compound event might mean the probability that a compound event will happen, based on theory as opposed to actual experimental results.
- Use what you know about a *tree* and a *diagram* to make a conjecture as to the meaning of a *tree diagram*. Sample answer: A tree has branches. A diagram is a model. So, a tree diagram is a model with many branches.

Explore Sample Space of Repeated Simple Events

Objective

Students will use Web Sketchpad to explore how to find the sample space of repeated simple events.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with the description of a coin that has a different color on each side. Throughout this activity, students will use tables to show the possible outcomes when the coin is tossed once, twice, and three times.

Inquiry Question

How can you use a table or organized list to represent all possible outcomes from repeated simple events? Sample answer: By using a table or organized list I can keep track of the possible outcomes for each trial in repeated simple events by showing each combination available in the sample space.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

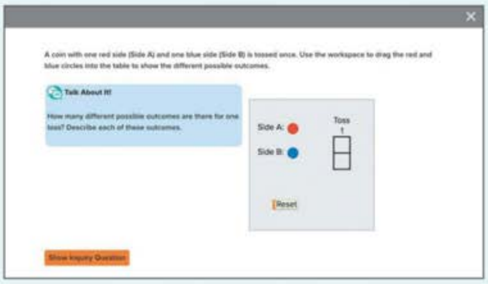
Mathematical Discourse

How many different possible outcomes are there for one toss? Describe each of these outcomes. Sample answer: There are two possible outcomes for one toss: red or blue.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6

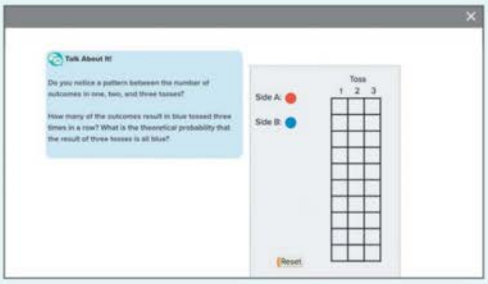


Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the sample space of repeated events.

Interactive Presentation



Explore, Slide 4 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Sample Space of Repeated Simple Events *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore the compound event of tossing a coin multiple times. Encourage students to use the interactive software to help them see the benefit of using a table or organized list to represent all the outcomes of a repeated simple event.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Do you notice a pattern between the number of outcomes in one, two, and three tosses? Sample answer: The number of outcomes is the product of the number of possible outcomes in each event that makes up the compound event.

How many of the outcomes result in blue tossed three times in a row? What is the theoretical probability that the result of three tosses is all blue? one; $\frac{1}{8}$, 0.125, or 12.5%

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Sample Space of Compound Events

Objective

Students will understand how to find the sample space of compound events.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to create a plausible argument, with justification, for whether or not they think the order of the events affects the sample space.

Teaching Notes

SLIDE 1

After defining the phrase *compound event*, you may wish to provide some examples of compound events, such as rolling a number cube twice or spinning a spinner with five different color sections three times. Ask students what one of the outcomes could be from that event. Sample answer: rolling a 3 and then a 5 or spinning blue, red, and red. Then ask them how they could go about finding all of the possible outcomes, or the sample space. Some students may say they would make an organized list or a table.

(continued on next page)

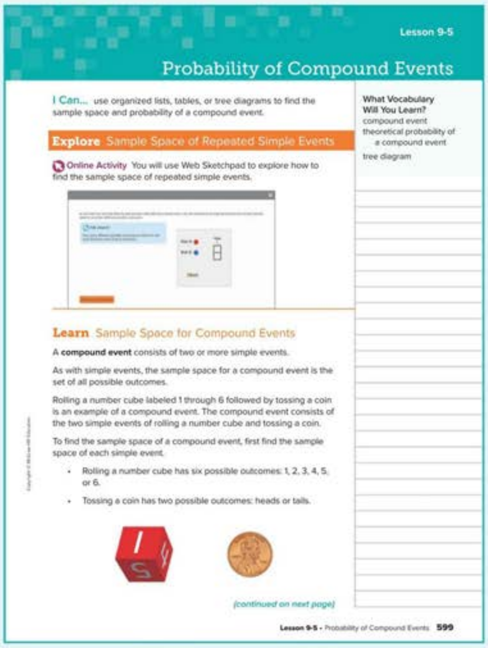
DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of sample spaces of compound events, have them create a tree diagram to count the size of the sample space for the following compound event.

spinning a spinner with four equal sections, rolling a 6-sided number cube, and tossing a coin 48 possible outcomes

Then have students determine another method they could use to find sample space of compound events without using a diagram or writing all possible outcomes.
Sample answer: I can find the sample space for each individual event and multiply them. For example, $4 \times 6 \times 2 = 48$.


Lesson 9-5

Probability of Compound Events

I Can... use organized lists, tables, or tree diagrams to find the sample space and probability of a compound event.

Explore Sample Space of Repeated Simple Events

Online Activity You will use Web Sketchpad to explore how to find the sample space of repeated simple events.

Learn Sample Space for Compound Events

A **compound event** consists of two or more simple events.

As with simple events, the sample space for a compound event is the set of all possible outcomes.

Rolling a number cube labeled 1 through 6 followed by tossing a coin is an example of a compound event. The compound event consists of the two simple events of rolling a number cube and tossing a coin.

To find the sample space of a compound event, first find the sample space of each simple event.

- Rolling a number cube has six possible outcomes: 1, 2, 3, 4, 5, or 6.
- Tossing a coin has two possible outcomes: heads or tails.

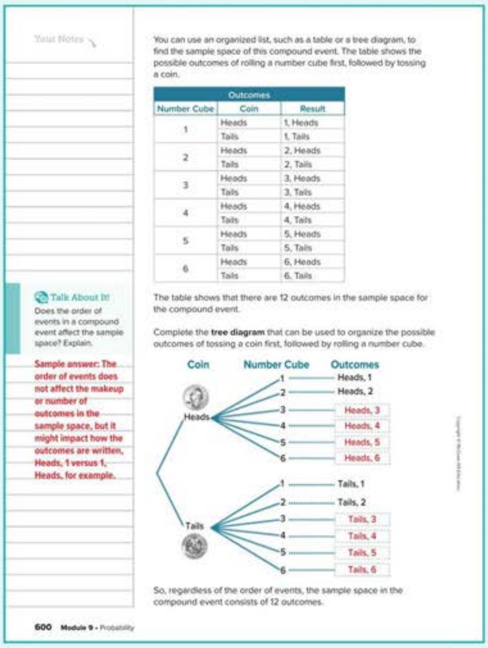
(continued on next page)

What Vocabulary Will You Learn?
compound event
theoretical probability of a compound event
tree diagram

Lesson 9-5 • Probability of Compound Events 599

Interactive Presentation

Learn, Sample Space of Compound Events, Slide 1 of 4



Your Notes

You can use an organized list, such as a table or a tree diagram, to find the sample space of this compound event. The table shows the possible outcomes of rolling a number cube first, followed by tossing a coin.

Outcomes		
Number Cube	**Coin**	**Result**
1	Heads	1, Heads
	Tails	1, Tails
2	Heads	2, Heads
	Tails	2, Tails
3	Heads	3, Heads
	Tails	3, Tails
4	Heads	4, Heads
	Tails	4, Tails
5	Heads	5, Heads
	Tails	5, Tails
6	Heads	6, Heads
	Tails	6, Tails

Talk About It! Does the order of events in a compound event affect the sample space? Explain.

Sample answer: The order of events does not affect the makeup or number of outcomes in the sample space, but it might impact how the outcomes are written, Heads, 1 versus 1, Heads, for example.

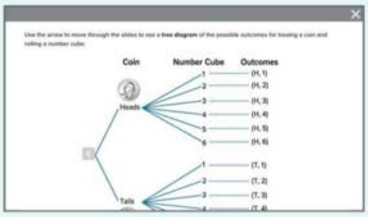
The table shows that there are 12 outcomes in the sample space for the compound event.

Complete the **tree diagram** that can be used to organize the possible outcomes of tossing a coin first, followed by rolling a number cube.

So, regardless of the order of events, the sample space in the compound event consists of 12 outcomes.

600 Module 9 • Probability

Interactive Presentation



Learn, Sample Space of Compound Events, Slide 3 of 4

CLICK

On Slide 3, students select from a drop-down menu the number of outcomes in the sample space.

Learn Sample Space of Compound Events *(continued)*

Teaching Notes

SLIDE 2

Point out to students that if your method of listing the sample space isn't organized, you may miss some combinations. Ask students how the structure of this table guarantees that no combinations will be left out. Some students may say that the numbers were written in order and the same order was used when listing the coin outcomes for every roll of the number cube. Explain to students that if there are six possible outcomes for rolling a number cube and 2 possible outcomes for tossing a coin, there are $6 \cdot 2$ or 12 total possible outcomes of the compound event.

SLIDE 3

Students will learn how a tree diagram can help them organize the possible outcomes for a compound event. Have students explore the interactive tree diagram to see how the list of outcomes is generated. You may wish to have students discuss when they would use each one to find the sample space for a compound event.

Talk About It!

SLIDE 4

Mathematical Discourse

Does the order of the events in a compound event affect the sample space? Explain. Sample answer: The order of events does not affect the makeup or number of outcomes in the sample space, but it might impact how the outcomes are written, *Heads, 1* vs *1, Heads*, for example.

Example 1 Find Sample Space of Compound Events

Objective

Students will find the sample space of compound events using a table or a tree diagram.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the relationship between the two quantities.

5 Use Appropriate Tools Strategically Encourage students to use an organized list and a tree diagram to generate the sample space for the compound event. Have them compare and contrast each method, noting how each illustrates the sample space.

Questions for Mathematical Discourse

SLIDE 2

AL Explain why this is a compound event. Sample answer: A penny *and* a nickel are tossed.

OL Explain why the outcomes *Heads, Tails* and *Tails, Heads* are unique outcomes. Sample answer: *Heads, Tails* might refer to heads turning up on the nickel, and tails turning up on the penny, while *Tails, Heads* would refer to tails turning up on the nickel and heads turning up on the penny.

BL Use the sample space to describe an unlikely outcome. Then describe a likely outcome. Sample answer: An unlikely outcome is tossing both heads (25%). A likely outcome is tossing at least one tail (75%).

SLIDE 3

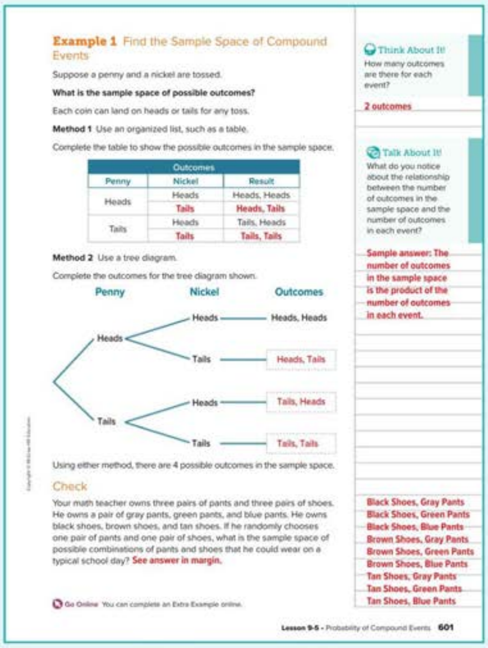
AL How many possible outcomes are there for each coin? 2; heads or tails

OL A classmate stated there is a 50% chance of either both Heads or both Tails turning up. Is this correct? Explain. yes; Sample answer: 2 of the 4 outcomes (50%) are either *Heads, Heads* or *Tails, Tails*.

BL How many outcomes would there be if you also tossed a dime? Explain. 8; Sample answer: Each of the current four branches would have an additional two branches for the dime; $4 \cdot 2 = 8$.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Find the Sample Space of Compound Events

Suppose a penny and a nickel are tossed.

What is the sample space of possible outcomes?

Each coin can land on heads or tails for any toss.

Method 1 Use an organized list, such as a table.

Complete the table to show the possible outcomes in the sample space.

Outcomes		
Penny	Nickel	Result
Heads	Heads	Heads, Heads
	Tails	Heads, Tails
Tails	Heads	Tails, Heads
	Tails	Tails, Tails

Method 2 Use a tree diagram.

Complete the outcomes for the tree diagram shown.

Using either method, there are 4 possible outcomes in the sample space.

Check

Your math teacher owns three pairs of pants and three pairs of shoes. He owns a pair of gray pants, green pants, and blue pants. He owns black shoes, brown shoes, and tan shoes. If he randomly chooses one pair of pants and one pair of shoes, what is the sample space of possible combinations of pants and shoes that he could wear on a typical school day? See answer in margin.

Go Online You can complete an Extra Example online.

Think About It! How many outcomes are there for each event?

2 outcomes

Talk About It! What do you notice about the relationship between the number of outcomes in the sample space and the number of outcomes in each event?

Sample answer: The number of outcomes in the sample space is the product of the number of outcomes in each event.

Black Shoes, Gray Pants
Black Shoes, Green Pants
Black Shoes, Blue Pants
Brown Shoes, Gray Pants
Brown Shoes, Green Pants
Brown Shoes, Blue Pants
Tan Shoes, Gray Pants
Tan Shoes, Green Pants
Tan Shoes, Blue Pants

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Interactive Presentation

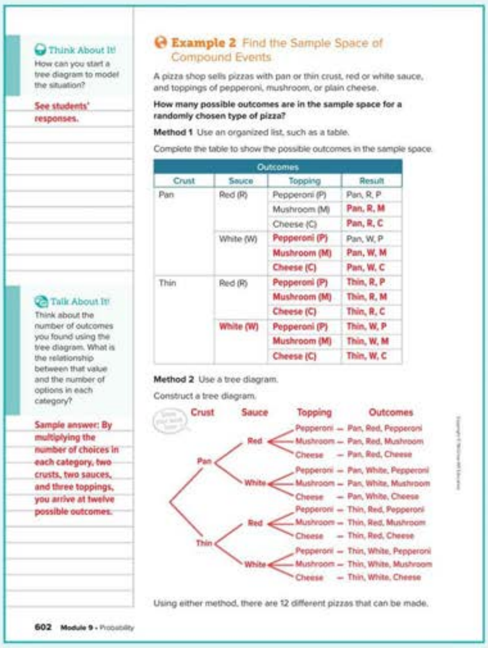
Example 1, Find Sample Space of Compound Events, Slide 2 of 5

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.SP.C.8.A, 7.SP.C.8.B

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Think About It!
How can you start a tree diagram to model the situation?
See students' responses.

Example 2 Find the Sample Space of Compound Events

A pizza shop sells pizzas with pan or thin crust, red or white sauce, and toppings of pepperoni, mushroom, or plain cheese.

How many possible outcomes are in the sample space for a randomly chosen type of pizza?

Method 1 Use an organized list, such as a table.

Complete the table to show the possible outcomes in the sample space.

Outcomes			
Crust	Sauce	Topping	Result
Pan	Red (R)	Pepperoni (P)	Pan, R, P
		Mushroom (M)	Pan, R, M
		Cheese (C)	Pan, R, C
	White (W)	Pepperoni (P)	Pan, W, P
		Mushroom (M)	Pan, W, M
		Cheese (C)	Pan, W, C
Thin	Red (R)	Pepperoni (P)	Thin, R, P
		Mushroom (M)	Thin, R, M
		Cheese (C)	Thin, R, C
	White (W)	Pepperoni (P)	Thin, W, P
		Mushroom (M)	Thin, W, M
		Cheese (C)	Thin, W, C

Method 2 Use a tree diagram.

Construct a tree diagram.

Using either method, there are 12 different pizzas that can be made.

Talk About It!
Think about the number of outcomes you found using the tree diagram. What is the relationship between that value and the number of options in each category?
Sample answer: By multiplying the number of choices in each category, two crusts, two sauces, and three toppings, you arrive at twelve possible outcomes.

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Interactive Presentation

Example 2, Find Sample Space of Compound Events, Slide 1 of 5

TYPE

On Slide 3, students determine the number of pizzas that can be made.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find Sample Space of Compound Events

Objective

Students will find the sample space of compound events using an organized list or a tree diagram.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 4, encourage them to make sense of the relationship between the two quantities. This will prepare them for learning about the Fundamental Counting Principle in high school probability courses.

5 Use Appropriate Tools Strategically Encourage students to use an organized list and a tree diagram to generate the sample space for the compound event. Have them compare and contrast each method, noting how each illustrates the relationship between the outcomes and the sample space.

Questions for Mathematical Discourse

SLIDE 2

AL How many possible choices are there for the crusts? sauce? toppings? 2 choices; 2 choices; 3 choices

OL Generate one possible type of pizza using these choices. Sample answer: Thin crust, red sauce, mushroom

BL Without making the tree diagram, explain how to find the number of possible outcomes that are in the sample space. 12 outcomes; Sample answer: For every crust, there are 2 sauces. This means there are four choices for crust and sauce alone (pan, red; pan, white; thin, red; thin; white). For each of these four choices for crust and sauce, there are three choices of toppings. So, there are 12 total possible outcomes.

SLIDE 3

AL How many total outcomes are listed in the tree diagram? 12 outcomes

OL How many of the outcomes listed have red sauce? 6 outcomes

OL How many of the outcomes have pepperoni? 4 outcomes

BL Suppose a customer can choose from small, medium, or large size of pizza. If this choice is added to the tree diagram, how many total outcomes would there be? 36 outcomes

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Theoretical Probability of Compound Events

Objective

Students will understand how to find the theoretical probability of compound events.

Teaching Notes

SLIDE 1

Have students select the *Words* and *Ratio* flashcards to view how to describe theoretical probability using these multiple representations. Ask students to compare the theoretical probability ratios for a compound event and a simple event. Students should note that both probabilities are written as a ratio comparing the number of favorable outcomes to the total number of outcomes and both describe what is expected to happen.

SLIDE 2

Students use the drag and drop activity to learn how to find the theoretical probability of a compound event, such as tossing a coin and then rolling a number cube. Point out to students that they can find the favorable outcome on a tree diagram by first locating a success (T) for tossing the coin, and then from that set of branches find a success for rolling a number cube (6). Some students may say there are two favorable outcomes because a 6 appears twice in the tree diagram. Remind them that both simple events must be successful outcomes for the compound event to be a successful outcome.

DIFFERENTIATE

Reteaching Activity AL

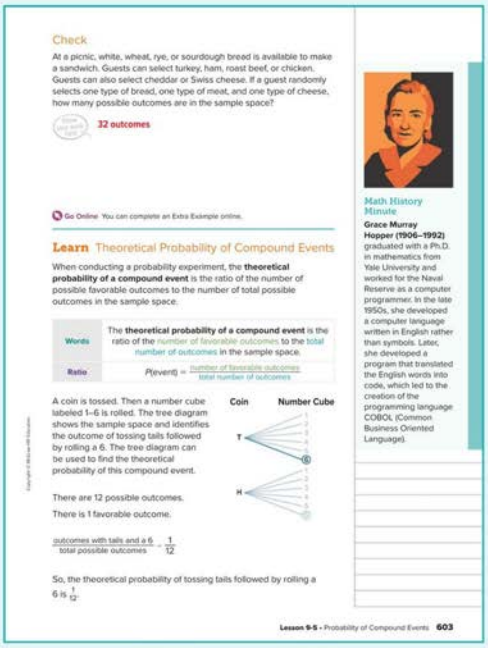
If students are struggling to understand theoretical probability of compound events, have them consider the following.

You previously determined that the sample space for rolling a standard number cube and flipping a coin was 12.

How many possible ways could you have an outcome of rolling an even number and the coin landing on tails? Explain your reasoning. 3; Sample answer: There are 3 ways to roll an even number: 2, 4, or 6. There is 1 way to land on heads.

Write the ratio of favorable outcomes to total possible outcomes. $\frac{3}{12}$ or $\frac{1}{4}$

How does this compare with finding the theoretical probability of a simple event? Sample answer: the ratio is written the same, number of favorable outcomes to total number of outcomes.



Check

At a picnic, white, wheat, rye, or sourdough bread is available to make a sandwich. Guests can select turkey, ham, roast beef, or chicken. Guests can also select cheddar or Swiss cheese. If a guest randomly selects one type of bread, one type of meat, and one type of cheese, how many possible outcomes are in the sample space?

32 outcomes

Go Online You can complete an Extra Example online.

Learn Theoretical Probability of Compound Events

When conducting a probability experiment, the **theoretical probability of a compound event** is the ratio of the number of possible favorable outcomes to the number of total possible outcomes in the sample space.

Words	The **theoretical probability of a compound event** is the ratio of the number of favorable outcomes to the total number of outcomes in the sample space.
Ratio	$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

A coin is tossed. Then a number cube labeled 1–6 is rolled. The tree diagram shows the sample space and identifies the outcome of tossing tails followed by rolling a 6. The tree diagram can be used to find the theoretical probability of this compound event.

There are 12 possible outcomes.

There is 1 favorable outcome.

$\frac{\text{outcomes with tails and a 6}}{\text{total possible outcomes}} = \frac{1}{12}$

So, the theoretical probability of tossing tails followed by rolling a 6 is $\frac{1}{12}$.

Math History Minute

Grace Murray Hopper (1906–1992) graduated with a Ph.D. in mathematics from Yale University and worked for the Naval Reserve as a computer programmer. In the late 1950s, she developed a computer language written in English rather than symbols. Later, she developed a program that translated the English words into code, which led to the creation of the programming language COBOL (Common Business Oriented Language).

Lesson 9-5 • Probability of Compound Events 603

Interactive Presentation

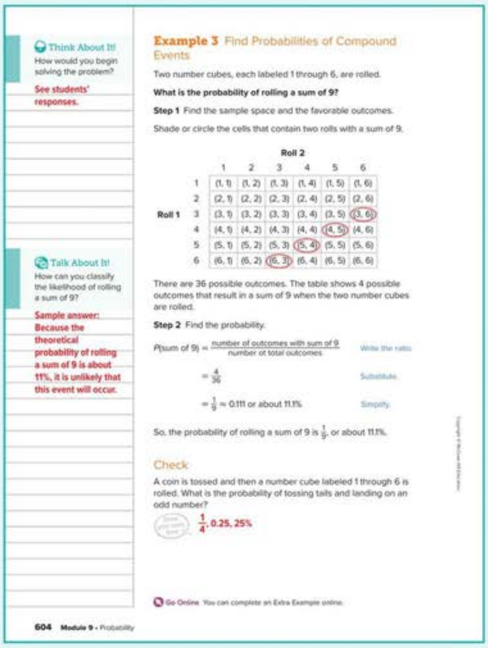
Learn, Theoretical Probability of Compound Events, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of theoretical probability.

DRAG & DROP

On Slide 2, students determine the ratio of favorable outcomes to total outcomes.



Think About It!
How would you begin solving the problem?
See students' responses.

Talk About It!
How can you classify the likelihood of rolling a sum of 9?
Sample answer: Because the theoretical probability of rolling a sum of 9 is about 11%, it is unlikely that this event will occur.

Example 3 Find Probabilities of Compound Events

Two number cubes, each labeled 1 through 6, are rolled.

What is the probability of rolling a sum of 9?

Step 1 Find the sample space and the favorable outcomes.

Shade or circle the cells that contain two rolls with a sum of 9.

		Roll 2					
		1	2	3	4	5	6
	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
Roll 1	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

There are 36 possible outcomes. The table shows 4 possible outcomes that result in a sum of 9 when the two number cubes are rolled.

Step 2 Find the probability.

$P(\text{sum of 9}) = \frac{\text{number of outcomes with sum of 9}}{\text{number of total outcomes}}$ Write the ratio.

$= \frac{4}{36}$ Substitute.

$= \frac{1}{9} \approx 0.111$ or about 11.1% Simplify.

So, the probability of rolling a sum of 9 is $\frac{1}{9}$, or about 11.1%.

Check

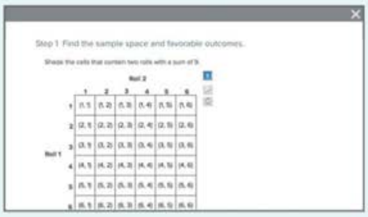
A coin is tossed and then a number cube labeled 1 through 6 is rolled. What is the probability of tossing tails and landing on an odd number?

$\frac{1}{4}$, 0.25, 25%

Go Online You can complete an Extra Example online.

604 Module 9 • Probability

Interactive Presentation



Example 3, Find Probabilities of Compound Events, Slide 2 of 5

CLICK

On Slide 2, students shade cells of a table.

CLICK

On Slide 3, students move through the steps to find the probability of rolling a sum of 9.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Find Probabilities of Compound Events

Objective

Students will find the theoretical probability of compound events using a table or list.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically St udents will use the interactive shading tool to identify the favorable outcomes.

6 Attend to Precision Encourage students to adhere to the definition of theoretical probability to find the ratio of favorable outcomes to total number of outcomes.

Questions for Mathematical Discourse

SLIDE 2

AL Explain why this is a compound event. Sample answer: Two number cubes are rolled.

AL Identify the cells that show the sum of the outcomes being 9. (3, 6), (4, 5), (5, 4), and (6, 3)

OL A classmate stated that (2, 7) and (7, 2) also have a sum of 9. Explain why these are not correct outcomes. Sample answer: These outcomes are not in the sample space because the number 7 does not appear on either of the number cubes.

OL What is the greatest sum you can have when rolling two number cubes labeled 1-6? The least? The greatest sum is 12 (6, 6). The least sum is 2 (1, 1).

BL How many outcomes, when two number cubes labeled 1-6 are rolled, have a sum less than 5? 6 outcomes

SLIDE 3

AL What is the number of outcomes with a sum of 9? 4 outcomes

AL How many total possible outcomes are there? 36 outcomes

OL Classify the likelihood of rolling a sum of 9. unlikely

OL Find the probability expressed as a percent for the complement of rolling a sum of 9. Then classify its likelihood. about 89%; likely

BL Describe an event, in this scenario, in which the likelihood is likely. Sample answer: rolling a sum that is a composite number

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Find Probabilities of Compound Events

Objective

Students will find the theoretical probability of compound events using a tree diagram.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically In th is example, a tree diagram was used to find the number of outcomes and the number of favorable outcomes. Students may use other tools, such as a table or organized list. Have them choose a tool and explain their choice.

6 Attend to Precision Students should adhere to the definition of *theoretical probability* to find the ratio of favorable outcomes to total number of outcomes, as a fraction and as a percent.

Questions for Mathematical Discourse

SLIDE 2

AL Explain why this is a compound event. Sample answer: Two coins are tossed, and then a number cube is rolled.

AL How many possible outcomes are there for each event? 2 for the first coin toss, 2 for the second coin toss, 6 for the number cube

OL How many total possible outcomes are there? 24 outcomes

OL How many outcomes result in at least one heads tossed and an even number rolled? 9 outcomes

BL Describe an event, in this scenario, in which the likelihood is impossible. Sample answer: (Heads, Heads, 7)

SLIDE 3

AL What is the number of favorable outcomes? 9 outcomes

AL How many total possible outcomes are there? 24 outcomes

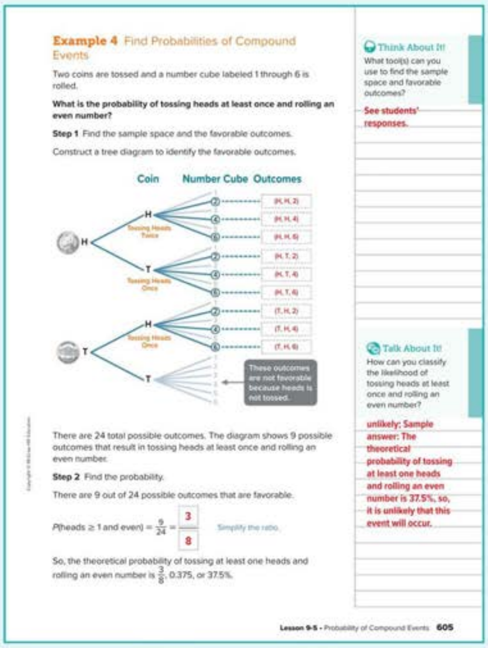
OL Classify the likelihood of tossing at least one heads and rolling an even number. unlikely

OL Find the probability expressed as a percent for the complement of this event. Then classify its likelihood. 62.5%; likely

BL Describe an event, in this scenario, in which the probability is 50%. Sample answer: Flipping a heads once and rolling a number less than 7.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Find Probabilities of Compound Events

Two coins are tossed and a number cube labeled 1 through 6 is rolled.

What is the probability of tossing heads at least once and rolling an even number?

Step 1 Find the sample space and the favorable outcomes.

Construct a tree diagram to identify the favorable outcomes.

There are 24 total possible outcomes. The diagram shows 9 possible outcomes that result in tossing heads at least once and rolling an even number.

Step 2 Find the probability.

There are 9 out of 24 possible outcomes that are favorable.

$P(\text{heads} \geq 1 \text{ and even}) = \frac{9}{24} = \frac{3}{8}$ Simplify the ratio.

So, the theoretical probability of tossing at least one heads and rolling an even number is $\frac{3}{8}$, 0.375, or 37.5%.

Think About It! What tool(s) can you use to find the sample space and favorable outcomes?

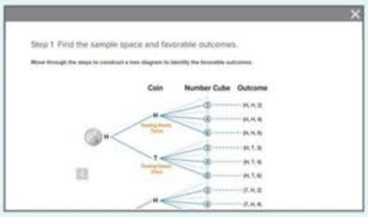
See students' responses.

Talk About It! How can you classify the likelihood of tossing heads at least once and rolling an even number?

unlikely; Sample answer: The theoretical probability of tossing at least one heads and rolling an even number is 37.5%, so, it is unlikely that this event will occur.

Lesson 9-5 • Probability of Compound Events 605

Interactive Presentation



Example 4, Find Probabilities of Compound Events, Slide 2 of 5

CLICK

On Slide 2, students move through the steps to construct a tree diagram.

TYPE

On Slide 3, students determine the theoretical probability.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A spinner with three equal-size sections labeled red, green, and yellow is spun once. Then a coin is tossed and one of two cards labeled with a 1 or a 2 is selected. What is the probability of spinning yellow, tossing heads, and selecting the number 2?

$\frac{1}{12}$, $0.08\overline{3}$, 8.3%

Go Online You can complete an Extra Example online.

Pause and Reflect

When finding probability, the language used is important. Describe the difference between *rolling a 3 or 6* in a simple event, and *rolling a 3 and 6* in a compound event.

See students' observations.

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Apply Outcomes

Objective

Students will come up with their own strategy to solve an application problem involving the outcomes of rolling two number cubes.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

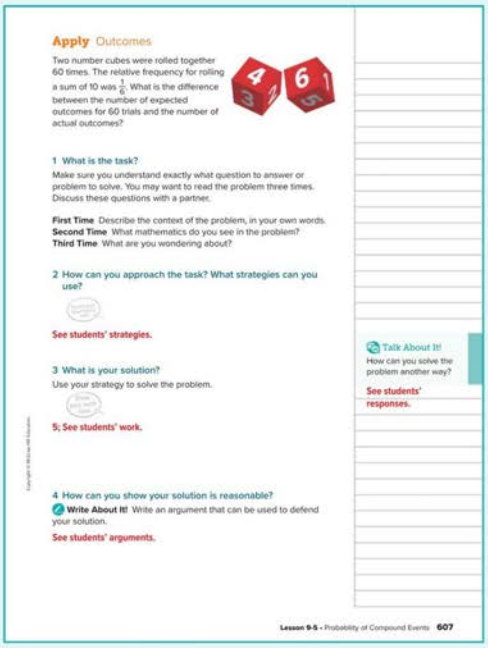
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What are the possible ways to roll a sum of 10?
- What does the relative frequency of $\frac{1}{6}$ tell you?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Outcomes

Two number cubes were rolled together 60 times. The relative frequency for rolling a sum of 10 was $\frac{1}{6}$. What is the difference between the number of expected outcomes for 60 trials and the number of actual outcomes?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

5; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How can you solve the problem another way?

See students' responses.

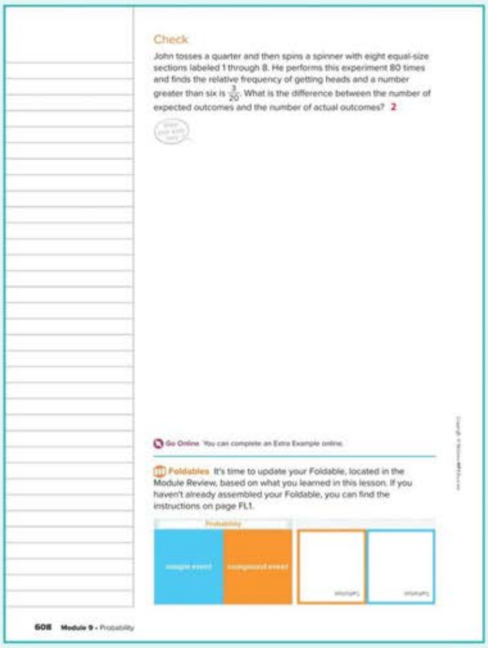
Lesson 9-5 • Probability of Compound Events 607

Interactive Presentation

Apply, Outcomes

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

John tosses a quarter and then spins a spinner with eight equal-size sections labeled 1 through 8. He performs this experiment 80 times and finds the relative frequency of getting heads and a number greater than six is $\frac{3}{20}$. What is the difference between the number of expected outcomes and the number of actual outcomes? 2

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about compound events. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can probability be used to predict future events?

In this lesson, students learned how to find the probability of compound events using lists, tables, and tree diagrams. Encourage them to work with a partner to brainstorm a compound event, determine the probability of it occurring, and how that probability helps them predict the likelihood of that event occurring the next time the compound event occurs.

Exit Ticket

Suppose there were two multiple-choice questions, each with answer choices A, B, C, D, and E. Each question had a single correct answer. Explain how to find the probability of guessing the correct answer to *both* questions. Sample answer: Make a tree diagram to find the sample space, of possible outcomes. Then use the sample space to identify the favorable outcome(s). Find the probability by writing a ratio of favorable outcomes, 1, to possible outcomes, 25. The probability is 1 out of 25.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1, 3, 5, 7–11
- Extension: Probability With and Without Replacement
- ALEKS® Probability of Compound Events

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8
- Extension: Probability With and Without Replacement
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Probability of Simple Events

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Probability of Simple Events

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

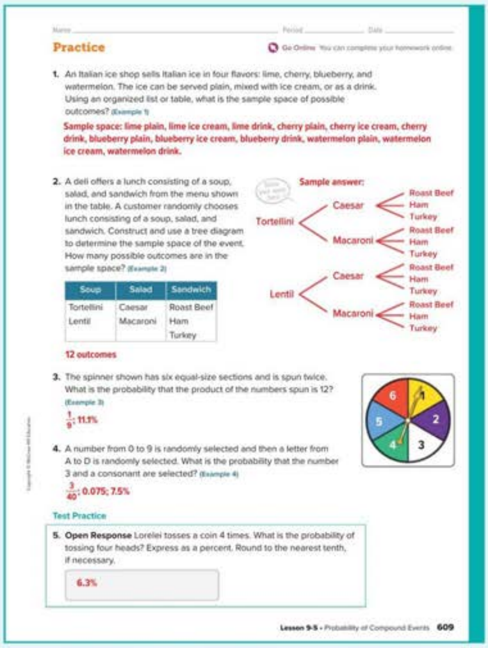
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find the sample space of compound events using a table or list	1
2	find the sample space of compound events using a tree diagram	2
2	find the theoretical probability of compound events using a table or list	3
2	find the theoretical probability of compound events	4, 5
3	solve application problems involving probability of compound events	6, 7
3	higher-order and critical thinking skills	8–11

Common Misconception

Some students may find sample spaces using incorrect criteria. In Exercise 3, students may consider the sample space as the set of possible products rather than the set of two-spin number pairs. Some students may also identify spinning a 3 and then a 4 as the same as spinning a 4 and then a 3.



Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

1. An Italian ice shop sells Italian ice in four flavors: lime, cherry, blueberry, and watermelon. The ice can be served plain, mixed with ice cream, or as a drink. Using an organized list or table, what is the sample space of possible outcomes? (Example 1)
Sample space: lime plain, lime ice cream, lime drink, cherry plain, cherry ice cream, cherry drink, blueberry plain, blueberry ice cream, blueberry drink, watermelon plain, watermelon ice cream, watermelon drink.

2. A deli offers a lunch consisting of a soup, salad, and sandwich from the menu shown in the table. A customer randomly chooses lunch consisting of a soup, salad, and sandwich. Construct and use a tree diagram to determine the sample space of the event. How many possible outcomes are in the sample space? (Example 2)

Soup	Salad	Sandwich
Tortellini	Caesar	Roast Beef
Lentil	Macaroni	Ham
		Turkey

Sample answer:

12 outcomes

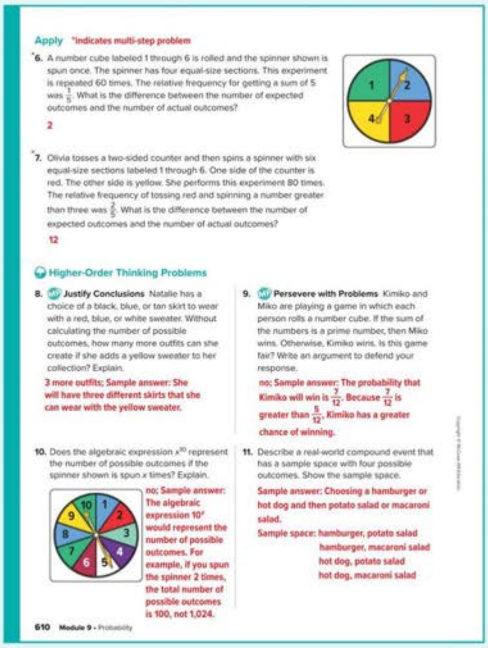
3. The spinner shown has six equal-size sections and is spun twice. What is the probability that the product of the numbers spun is 12? (Example 3)
$\frac{1}{9}$; 11.1%

4. A number from 0 to 9 is randomly selected and then a letter from A to D is randomly selected. What is the probability that the number 3 and a consonant are selected? (Example 4)
$\frac{3}{40}$; 0.075; 7.5%

Test Practice

5. **Open Response** Lorelei tosses a coin 4 times. What is the probability of tossing four heads? Express as a percent. Round to the nearest tenth, if necessary.
6.3%

Lesson 9-5 • Probability of Compound Events 609



Apply *indicates multi-step problem

*6. A number cube labeled 1 through 6 is rolled and the spinner shown is spun once. The spinner has four equal-size sections. This experiment is repeated 60 times. The relative frequency for getting a sum of 5 was $\frac{1}{5}$. What is the difference between the number of expected outcomes and the number of actual outcomes?

2

*7. Olivia tosses a two-sided counter and then spins a spinner with six equal-size sections labeled 1 through 6. One side of the counter is red. The other side is yellow. She performs this experiment 80 times. The relative frequency of tossing red and spinning a number greater than three was $\frac{2}{5}$. What is the difference between the number of expected outcomes and the number of actual outcomes?

12

Higher-Order Thinking Problems

8. **Justify Conclusions** Natalie has a choice of a black, blue, or tan skirt to wear with a red, blue, or white sweater. Without calculating the number of possible outcomes, how many more outfits can she create if she adds a yellow sweater to her collection? Explain.

3 more outfits; Sample answer: She will have three different skirts that she can wear with the yellow sweater.

9. **Persevere with Problems** Kimiko and Miko are playing a game in which each person rolls a number cube. If the sum of the numbers is a prime number, then Miko wins. Otherwise, Kimiko wins. Is this game fair? Write an argument to defend your response.

no; Sample answer: The probability that Kimiko will win is $\frac{7}{12}$. Because $\frac{7}{12}$ is greater than $\frac{5}{12}$, Kimiko has a greater chance of winning.

10. Does the algebraic expression x^{10} represent the number of possible outcomes if the spinner shown is spun x times? Explain.

no; Sample answer: The algebraic expression 10^x would represent the number of possible outcomes. For example, if you spun the spinner 2 times, the total number of possible outcomes is 100, not 1,024.

11. Describe a real-world compound event that has a sample space with four possible outcomes. Show the sample space.

Sample answer: Choosing a hamburger or hot dog and then potato salad or macaroni salad.

Sample space: hamburger, potato salad
hamburger, macaroni salad
hot dog, potato salad
hot dog, macaroni salad

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 8, students explain how to find how many more outfits can be created if an additional sweater is added to the wardrobe, without calculating.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 9, students analyze a game and determine whether or not it is fair, based on probabilities.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 6–7 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 6 might be "What is the sample space of the compound event?"

Be sure everyone understands.
Use with Exercises 8–9 Have students work in groups of 3–4 to solve the problem in Exercise 8. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 9.

Lesson 9-6

Simulate Chance Events

7.SP.C.8.C

LESSON GOAL

Students will solve problems by simulating compound probability events.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Simulations

Learn: Simulate Simple Events
Learn: Simulate Compound Events
Example 1: Simulate Compound Events
Example 2: Interpret Simulations of Compound Events

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 57 of the *Language Development Handbook* to help your students build mathematical language related to simulations.

ELL You can use the tips and suggestions on page T57 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability
Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.C** by solving problems by simulating compound probability events.
Standards for Mathematical Content: **7 .SP.C.8, 7.SP.C.8.C**
Standards for Mathematical Practice: **MP 2, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students solved problems involving the probability of compound events.
7.SP.C.8, 7.SP.C.8.A, 7.SP.C.8.B

Now
Students solve problems by simulating compound probability events.
7.SP.C.8, 7.SP.C.8.C

Next
Students will understand independence and conditional probability.
HSS.CP.A.2, HSS.CP.A.3

Rigor

The Three Pillars of Rigor

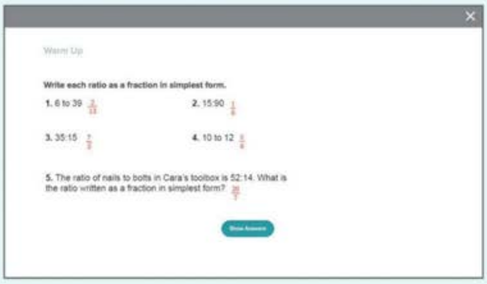
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will use their knowledge of simple and compound probability to develop an *understanding* of creating a simulation for a simple or compound event. They will use this understanding to gain *fluency* in simulating events and interpreting the results. They will *apply* their understandings to solve real-world problems.

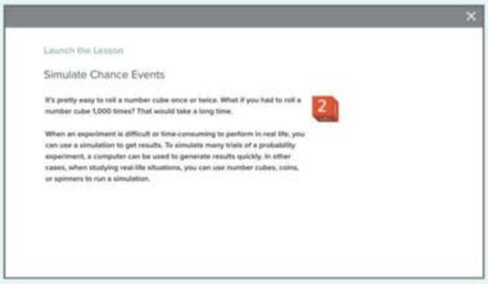
Mathematical Background

A *simulation* is an experiment that is designed to model a given situation. Simulations often model events that would be difficult, time consuming, or impractical to perform in real life.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing ratios as fractions (Exercises 1–5)

Answers

1. $\frac{2}{13}$ **2.** $\frac{1}{6}$

3. $\frac{7}{3}$ **4.** $\frac{5}{6}$

5. $\frac{26}{7}$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about possible ways to run a simulation of an event.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- Describe what the term *simulate* means in your own words. Sample answer: Simulate means to act out or mimic an actual or probable real-life condition, event, or situation.

Explore Simulations

Objective

Students will use Web Sketchpad to explore simulating events.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a random number generator sketch that simulates rolling number cubes. Throughout this activity, students will use the random number generator to model rolling one or two number cubes. They will use the results of the probability experiments to find the relative frequencies of events.

Inquiry Question

How can you use a random number generator to model a probability experiment? Sample answer: I can determine the number of different outcomes in the experiment and assign each outcome a different number for the generator to produce.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

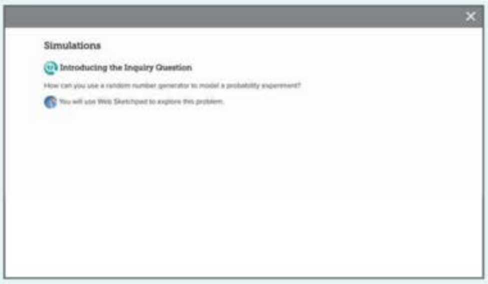
Mathematical Discourse

What is the relative frequency of rolling a 4? See students' responses.

How does the relative frequency compare to the theoretical probability of rolling a 4? See students' responses.

(continued on next page)

Interactive Presentation



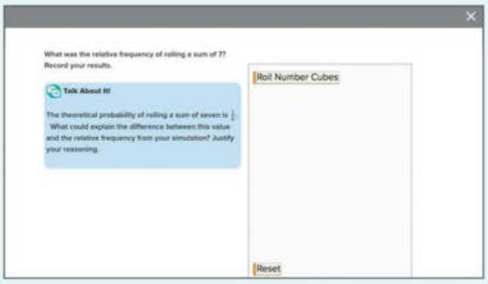
Explore, Slide 1 of 7

Explore, Slide 2 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore simulating events.

Interactive Presentation



Explore, Slide 4 of 7

TYPE

On Slide 5, students explain how using a random number generator helps them conduct the probability experiments.

TYPE

On Slide 7, students respond to the Inquiry Question and can view a sample answer.

Explore Simulations *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use the Web Sketchpad random number generator to gain insight into the benefit of a random number generator for modeling certain probability experiments.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

The theoretical probability of rolling a sum of seven is $\frac{1}{6}$. What could explain the difference between this value and the relative frequency from your simulation? Justify your reasoning. Sample answer: Because there were only 10 trials in the experiment it is possible that the theoretical probability and relative frequency have different values. As the number of trials grows, these values should become closer.

Learn Simulate Simple Events

Objective

Students will understand how to simulate simple events.

Teaching the Mathematical Practices

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to use the definitions of *likely, unlikely*, and how success and failure is interpreted for the first event.

Teaching Notes

SLIDE 1

Ask students if they have heard the word *simulation* in other situations and what it means. Some students may say they have heard of simulations as part of video games and that they model situations that are not easy to do in real life. Explain that a probability simulation is similar in that they model events that are difficult to perform in real life.

Have students analyze the scenario involving cereal box prizes and explain why it is impractical to recreate this situation in real life.

SLIDE 2

Ask students the following questions:

- Why are there three sections to the spinner? Sample answer: There are three sections because one out of every three boxes has a prize.
- Why is a success counted as landing on a blue section? Sample answer: There is one success in every three boxes.
- Why is a failure counted as landing on a red section? Sample answer: If one box out of three has a prize (a success), then two boxes out of three do not have a prize (a failure).

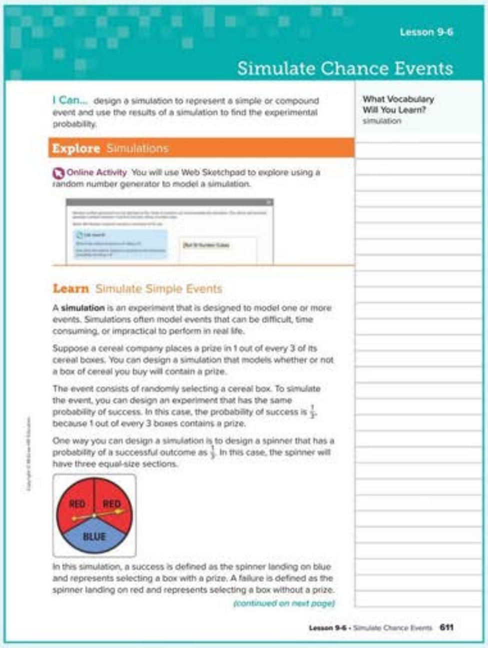
(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

For students who need more of a challenge, have them design a simulation for the given scenario.

Every person who walks through the main entrance of a department store is given the opportunity to draw from six envelopes. One of those six envelopes contains a gift card. If 75 people are given the opportunity to draw an envelope, estimate how many people will draw the envelope with a gift card. Explain how you designed your simulation.



Lesson 9-6

Simulate Chance Events

I Can... design a simulation to represent a simple or compound event and use the results of a simulation to find the experimental probability.

What Vocabulary Will You Learn?
simulation

Explore Simulations

Online Activity You will use Web Sketchpad to explore using a random number generator to model a simulation.

Learn Simulate Simple Events

A **simulation** is an experiment that is designed to model one or more events. Simulations often model events that can be difficult, time consuming, or impractical to perform in real life.

Suppose a cereal company places a prize in 1 out of every 3 of its cereal boxes. You can design a simulation that models whether or not a box of cereal you buy will contain a prize.

The event consists of randomly selecting a cereal box. To simulate the event, you can design an experiment that has the same probability of success. In this case, the probability of success is $\frac{1}{3}$, because 1 out of every 3 boxes contains a prize.

One way you can design a simulation is to design a spinner that has a probability of a successful outcome as $\frac{1}{3}$. In this case, the spinner will have three equal-size sections.

In this simulation, a success is defined as the spinner landing on blue and represents selecting a box with a prize. A failure is defined as the spinner landing on red and represents selecting a box without a prize.

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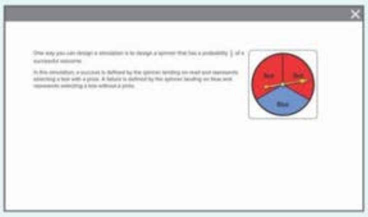
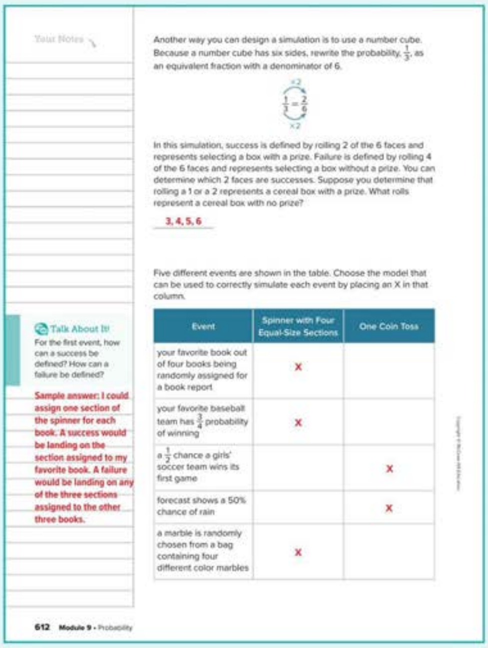
Lesson 9-6 • Simulate Chance Events 611

Interactive Presentation



Learn, Simulate Simple Events, Slide 2 of 4



Your Notes

Another way you can design a simulation is to use a number cube. Because a number cube has six sides, rewrite the probability, $\frac{1}{3}$, as an equivalent fraction with a denominator of 6.

$$\frac{1}{3} = \frac{2}{6}$$ (×2)

In this simulation, success is defined by rolling 2 of the 6 faces and represents selecting a box with a prize. Failure is defined by rolling 4 of the 6 faces and represents selecting a box without a prize. You can determine which 2 faces are successes. Suppose you determine that rolling a 1 or a 2 represents a cereal box with a prize. What rolls represent a cereal box with no prize?

3, 4, 5, 6

Five different events are shown in the table. Choose the model that can be used to correctly simulate each event by placing an X in that column.

Event	Spinner with Four Equal-Size Sections	One Coin Toss
your favorite book out of four books being randomly assigned for a book report	X	
your favorite baseball team has $\frac{3}{4}$ probability of winning	X	
a $\frac{1}{2}$ chance a girls' soccer team wins its first game		X
forecast shows a 50% chance of rain		X
a marble is randomly chosen from a bag containing four different color marbles	X	

Talk About It!
For the first event, how can a success be defined? How can a failure be defined?

Sample answer: I could assign one section of the spinner for each book. A success would be landing on the section assigned to my favorite book. A failure would be landing on any of the three sections assigned to the other three books.

612 Module 9 • Probability

Interactive Presentation

Learn, Simulate Simple Events, Slide 4 of 4

DRAG & DROP

On Slide 4, students match the event to the model that can be used to simulate it.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

7.SP.C.8.C

Learn Simulate Simple Events *(continued)*

Teaching Notes

SLIDE 3

Point out to students that different tools can be used to simulate events. Before students analyze the number cube model for the simulation, ask them if they could use a four-section spinner to simulate the event. Some students may say they cannot because the probability ratio has a denominator of three, you should use multiples of three choices for the simulation.

SLIDE 4

You may wish to have student volunteers come up to the board to drag each icon representing an event to its appropriate bin and explain their reasoning.

Talk About It!

SLIDE 4

Mathematical Discourse

For the first event, how can a success be defined? How can a failure be defined? Sample answer: I could assign one section of the spinner to each book. A success would be landing on the section assigned to my favorite book. A failure would be landing on any of the three sections assigned to the other three books.

Learn Simulate Compound Events

Objective

Students will learn about simulating compound events.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 1, encourage them to present a clear, concise comparison that shows that they understand the meaning of *simulated probability* and *theoretical probability*.

Go Online

- Find additional notes.
- Have students watch the animation on Slide 1. The animation illustrates how to use a simulation to estimate the probability of a compound event.

Teaching Notes

SLIDE 1

Play the animation for the class. You may wish to pause the animation at the end of Step 1 when all of the coins have been labeled and ask the following questions:

- Why was a coin used as the object for the simulation? Sample answer: Because the cub is either a male or a female, there are two possible outcomes and a tossing a coin has two possible outcomes.
- Why were three coins used? Sample answer: There are three cubs so three coins should be tossed.
- How many trials do you think should be done? Student responses will vary. Remind students that when looking at simulations, the greater the number of trials, the closer the relative frequency will be to the theoretical probability.

Talk About It!

SLIDE 1

Mathematical Discourse

Use a tree diagram to find the theoretical probability of having three female cubs. How does the simulated probability compare to the theoretical probability? Explain. Sample answer: The theoretical probability of three female cubs is $\frac{1}{8}$ or 12.5%, while the simulated probability is 14%. These probabilities are very close to each other and would likely become closer as more trials were performed in the simulation.

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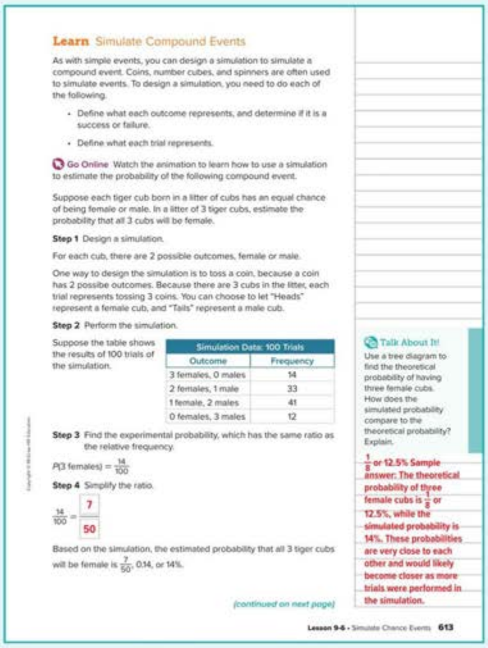


Learn Simulate Compound Events

As with simple events, you can design a simulation to simulate a compound event. Coins, number cubes, and spinners are often used to simulate events. To design a simulation, you need to do each of the following.

- Define what each outcome represents, and determine if it is a success or failure.
- Define what each trial represents.

Go Online Watch the animation to learn how to use a simulation to estimate the probability of the following compound event.

Suppose each tiger cub born in a litter of cubs has an equal chance of being female or male. In a litter of 3 tiger cubs, estimate the probability that all 3 cubs will be female.

Step 1 Design a simulation.

For each cub, there are 2 possible outcomes, female or male.

One way to design the simulation is to toss a coin, because a coin has 2 possibe outcomes. Because there are 3 cubs in the litter, each trial represents tossing 3 coins. You can choose to let "Heads" represent a female cub, and "Tails" represent a male cub.

Step 2 Perform the simulation.

Suppose the table shows the results of 100 trials of the simulation.

Simulation Data: 100 Trials	
Outcome	Frequency
3 females, 0 males	14
2 females, 1 male	33
1 female, 2 males	41
0 females, 3 males	12

Step 3 Find the experimental probability, which has the same ratio as the relative frequency.

$P(3\text{ females}) = \frac{14}{100}$

Step 4 Simplify the ratio.

$\frac{14}{100} = \frac{7}{50}$

Based on the simulation, the estimated probability that all 3 tiger cubs will be female is $\frac{7}{50}$, 0.14, or 14%.

Talk About It! Use a tree diagram to find the theoretical probability of having three female cubs. How does the simulated probability compare to the theoretical probability? Explain.

$\frac{1}{8}$ or 12.5% Sample answer: The theoretical probability of three female cubs is $\frac{1}{8}$ or 12.5%, while the simulated probability is 14%. These probabilities are very close to each other and would likely become closer as more trials were performed in the simulation.

(continued on next page)

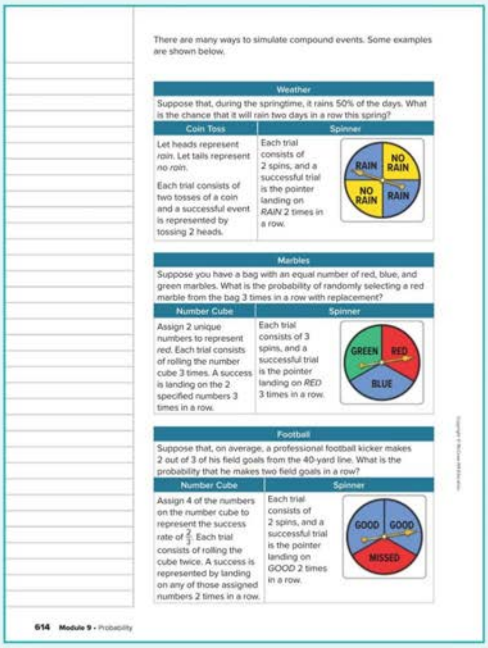
Lesson 9-6 • Simulate Chance Events 613

Interactive Presentation

Learn, Simulate Compound Events, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that shows how to use a simulation.



There are many ways to simulate compound events. Some examples are shown below.

Weather	
Suppose that, during the springtime, it rains 50% of the days. What is the chance that it will rain two days in a row this spring?	
Coin Toss	**Spinner**
Let heads represent *rain*. Let tails represent *no rain*. Each trial consists of two tosses of a coin and a successful event is represented by tossing 2 heads.	Each trial consists of 2 spins, and a successful trial is the pointer landing on *RAIN* 2 times in a row. (RAIN, NO RAIN, NO RAIN, RAIN)

Marbles	
Suppose you have a bag with an equal number of red, blue, and green marbles. What is the probability of randomly selecting a red marble from the bag 3 times in a row with replacement?	
Number Cube	**Spinner**
Assign 2 unique numbers to represent *red*. Each trial consists of rolling the number cube 3 times. A success is landing on the 2 specified numbers 3 times in a row.	Each trial consists of 3 spins, and a successful trial is the pointer landing on *RED* 3 times in a row. (GREEN, RED, BLUE)

Football	
Suppose that, on average, a professional football kicker makes 2 out of 3 of his field goals from the 40-yard line. What is the probability that he makes two field goals in a row?	
Number Cube	**Spinner**
Assign 4 of the numbers on the number cube to represent the success rate of $\frac{2}{3}$. Each trial consists of rolling the cube twice. A success is represented by landing on any of those assigned numbers 2 times in a row.	Each trial consists of 2 spins, and a successful trial is the pointer landing on *GOOD* 2 times in a row. (GOOD, GOOD, MISSED)

614 Module 9 • Probability

Interactive Presentation

Learn, Simulate Compound Events, Slide 2 of 2

CLICK

On Slide 2, students select topics to learn about simulation tools for probability events related to that topic.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Simulate Compound Events *(continued)*

Teaching Notes

SLIDE 2

Have students select each topic to learn about simulation tools for probability events related to that topic. You may wish to discuss each topic and the two simulation presented with the class. Ask the following questions to clarify each simulation:

- How many possible outcomes for each simple event are there?
- How many simple events are in each compound event?
- What represents a success? a failure?
- How many trials should I perform?
- What are some potential errors that might occur during the simulation?

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Simulate Compound Events

Objective

Students will design a simulation of a compound event and analyze the results.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* questions on Slide 5, encourage them to make sense of the simulated results within the context of the problem.

5 Use Appropriate Tools Strategically Encourage students to strategically choose appropriate tools, such as spinners and number cubes, to simulate the event.

Questions for Mathematical Discourse

SLIDE 2

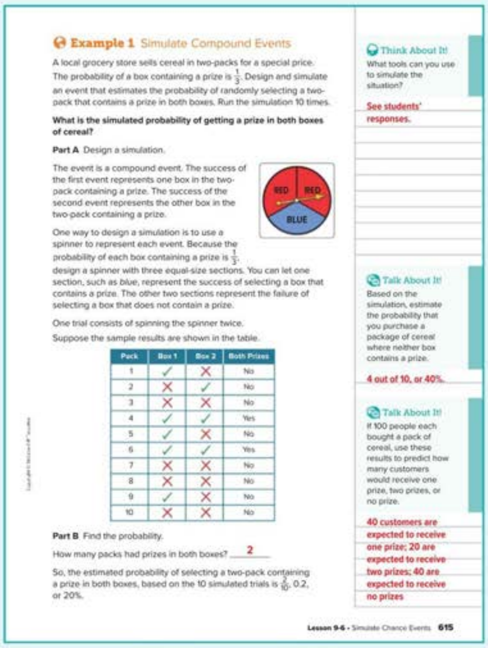
AL To model the probability of success of $\frac{1}{3}$, how many sections out of three should be labeled *Blue*? Explain. one section; $\frac{1}{3}$ means 1 out of 3

OL What else, besides the spinner, can you use to simulate choosing a box that contains a prize? Sample answer: I can use a number cube. If I roll the numbers 1 or 2, that represents a success, or the box contains a prize. If I roll the numbers 3, 4, 5, or 6, that represents a failure, or the box does not contain a prize.

BL Describe a different simulation you can use to find the probability of getting two prizes. Sample answer: Using a spinner with three equal-sized sections, two sections labeled "NP" for no prize and one section labeled "P" for prize. The spinner is spun twice.

Go Online

- Find additional teaching notes, discussion questions, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Simulate Compound Events

A local grocery store sells cereal in two-packs for a special price. The probability of a box containing a prize is $\frac{1}{3}$. Design and simulate an event that estimates the probability of randomly selecting a two-pack that contains a prize in both boxes. Run the simulation 10 times.

What is the simulated probability of getting a prize in both boxes of cereal?

Part A Design a simulation.

The event is a compound event. The success of the first event represents one box in the two-pack containing a prize. The success of the second event represents the other box in the two-pack containing a prize.

One way to design a simulation is to use a spinner to represent each event. Because the probability of each box containing a prize is $\frac{1}{3}$, design a spinner with three equal-size sections. You can let one section, such as blue, represent the success of selecting a box that contains a prize. The other two sections represent the failure of selecting a box that does not contain a prize.

One trial consists of spinning the spinner twice.

Suppose the sample results are shown in the table.

Pack	Box 1	Box 2	Both Prizes
1	✓	✗	No
2	✗	✓	No
3	✗	✗	No
4	✓	✓	Yes
5	✓	✗	No
6	✓	✓	Yes
7	✗	✗	No
8	✗	✗	No
9	✓	✗	No
10	✗	✗	No

Part B Find the probability.

How many packs had prizes in both boxes? 2

So, the estimated probability of selecting a two-pack containing a prize in both boxes, based on the 10 simulated trials is $\frac{2}{10}$, 0.2, or 20%.

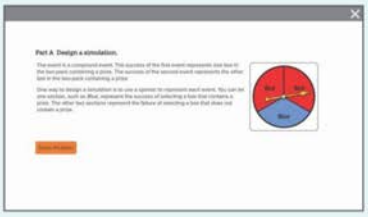
Think About It! What tools can you use to simulate the situation? See students' responses.

Talk About It! Based on the simulation, estimate the probability that you purchase a package of cereal where neither box contains a prize. 4 out of 10, or 40%.

Talk About It! If 100 people each bought a pack of cereal, use these results to predict how many customers would receive one prize, two prizes, or no prize. 40 customers are expected to receive one prize; 20 are expected to receive two prizes; 40 are expected to receive no prizes

Lesson 9-6 • Simulate Chance Events 615

Interactive Presentation



Example 1, Simulate Compound Events, Slide 2 of 6

CLICK

On Slide 3, students move through the slides to see sample results.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A store randomly gives a gift card to 5 out of every 8 customers that enter the store for a weekend grand opening event. The store owner wants to estimate the probability that a repeat customer receives a gift card two days in a row.

Part A

Select the appropriate way you can simulate the event, where a success is receiving a gift card and a failure is not receiving a gift card.

Ⓐ Each trial consists of rolling a number cube twice. A success represents both tosses landing on any of the numbers 1 through 4. A failure represents one or both tosses landing on the numbers 5 or 6.

Ⓑ Each trial consists of tossing a coin twice. A success represents both tosses landing on heads. A failure represents one or both tosses landing on tails.

● Each trial consists of spinning a spinner with 8 equal-size sections twice. Label 5 of the sections with a "C", and label the 3 remaining sections with an "X". A success represents both spins landing on "C". A failure represents one or both spins landing on "X".

Ⓓ Each trial consists of spinning a spinner with 8 equal-size sections twice. Label 5 of the sections with a "C", and label the 3 remaining sections with an "X". A failure represents both spins landing on "C". A success represents one or both spins landing on "X".

Part B

The table shows the results of simulating 10 trials of the compound event. A "C" represents that they received a gift card that day and an "X" represents that they did not receive a gift card that day. According to the results, what is the experimental probability that a repeat customer receives gift cards two days in a row?

Trial	1	2	3	4	5	6	7	8	9	10
Day 1	C	C	C	X	C	X	X	X	X	C
Day 2	C	C	X	C	C	C	X	X	C	X

$\frac{3}{10}$, 0.3, 30%

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Example 2 Interpret Simulations of Compound Events

Objective

Students will interpret a relative frequency bar graph that shows the results of a simulated compound event.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to adhere to the precise meaning of *eight or fewer rolls* when determining the simulated probability.

As students discuss the *Talk About It!* question on Slide 3, encourage them to relate this problem to finding the percent of a number, and use clear and precise mathematical language in their explanations.

7 Look For and Make Use of Structure Encourage students to analyze the structure of the relative frequency bar graph, in order to determine that they need to find the sum of the bar heights for 6, 7, and 8 rolls.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the probability that eight or fewer rolls are needed to obtain all of the even numbers on a number cube

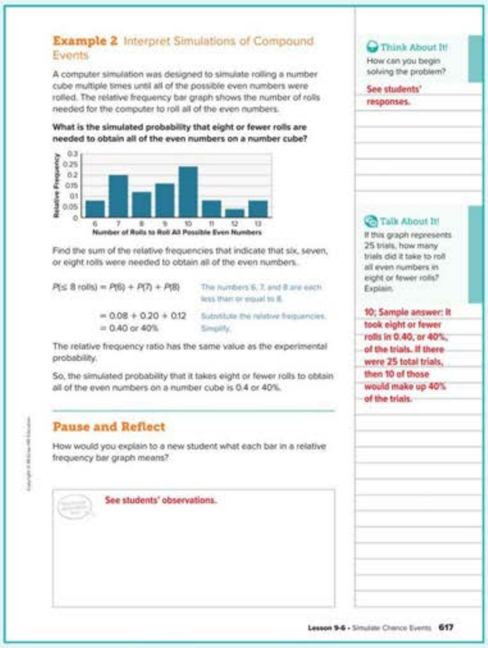
OL What are the relative frequency values for six, seven and eight rolls being needed to obtain all of the even numbers? 0.08, 0.20, 0.12, respectively

OL Based on these results, do you think it is more likely that 7 rolls are needed or 13 rolls? Explain. Sample answer: Because the bar representing 7 rolls is taller than the bar representing 13 rolls, it is more likely that 7 rolls are needed than 13 rolls.

BL What is the probability that twelve or fewer rolls are needed to obtain all of the even numbers on a number cube? 92%

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Example 2 Interpret Simulations of Compound Events

A computer simulation was designed to simulate rolling a number cube multiple times until all of the possible even numbers were rolled. The relative frequency bar graph shows the number of rolls needed for the computer to roll all of the even numbers.

What is the simulated probability that eight or fewer rolls are needed to obtain all of the even numbers on a number cube?

Find the sum of the relative frequencies that indicate that six, seven, or eight rolls were needed to obtain all of the even numbers.

$P(\leq 8 \text{ rolls}) = P(6) + P(7) + P(8)$ — The numbers 6, 7, and 8 are each less than or equal to 8.

$= 0.08 + 0.20 + 0.12$ — Substitute the relative frequencies.

$= 0.40$ or 40% — Simplify.

The relative frequency ratio has the same value as the experimental probability.

So, the simulated probability that it takes eight or fewer rolls to obtain all of the even numbers on a number cube is 0.4 or 40%.

Pause and Reflect

How would you explain to a new student what each bar in a relative frequency bar graph means?

See students' observations.

Think About It! How can you begin solving the problem? See students' responses.

Talk About It! If this graph represents 25 trials, how many trials did it take to roll all even numbers in eight or fewer rolls? Explain. 10; Sample answer: It took eight or fewer rolls in 0.40, or 40%, of the trials. If there were 25 total trials, then 10 of those would make up 40% of the trials.

Lesson 9-6 • Simulate Chance Events 617

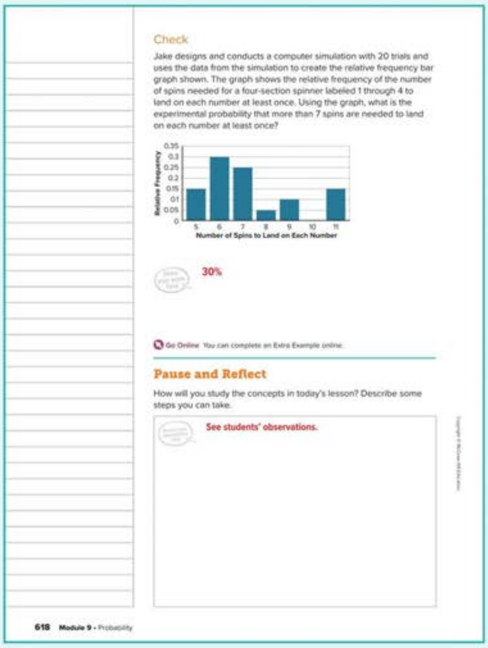
Interactive Presentation

Example 2, Interpret Simulations of Compound Events, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the solution.

TYPE On Slide 2, students determine the probability.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Check

Jake designs and conducts a computer simulation with 20 trials and uses the data from the simulation to create the relative frequency bar graph shown. The graph shows the relative frequency of the number of spins needed for a four-section spinner labeled 1 through 4 to land on each number at least once. Using the graph, what is the experimental probability that more than 7 spins are needed to land on each number at least once?

30%

Go Online You can complete an Extra Example online.

Pause and Reflect

How will you study the concepts in today's lesson? Describe some steps you can take.

See students' observations.

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Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Give an example where a coin can be used to simulate the probability of an event occurring. Explain how to design and interpret the simulation. Sample answer: Suppose there are five true/false questions on a sports quiz in a magazine. Let heads represent guessing a question correctly and tails represent guessing a question incorrectly. Flip a coin five times to simulate the outcome of guessing each of the five true/false questions on the quiz correctly.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 3, 5–8
- ALEKS Simulations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–3, 5
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Probability of Simple Events, Probability of Compound Events

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Probablity of Simple Events, Probability of Compound Events

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

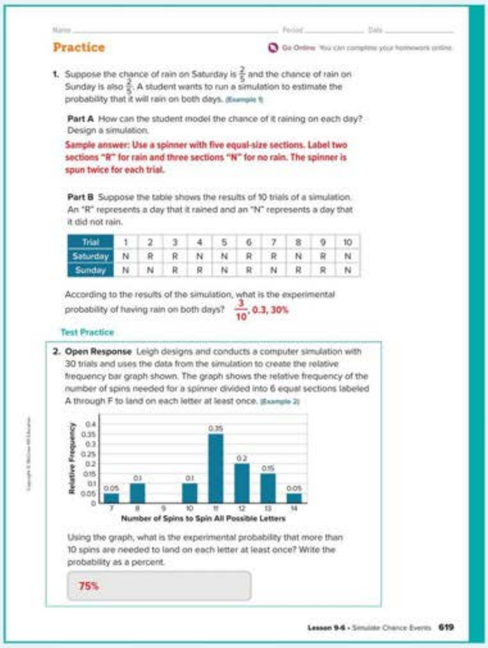
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	design a simulation of a compound event and analyze the results	1
2	interpret a relative frequency bar graph that shows the results of a simulated compound event	2
2	extend concepts learned in class to apply them in new contexts	3–4
3	higher-order and critical thinking skills	5–8

Common Misconception

Some students may misinterpret a relative frequency bar graph. For example, in Exercise 2, help students understand that each bar represents a relative frequency. The bar for 8, for example, shows a relative frequency of 0.1. This means that 0.1, or 10%, of the time, it took 8 spins to spin all of the letters.



Practice

1. Suppose the chance of rain on Saturday is $\frac{2}{5}$ and the chance of rain on Sunday is also $\frac{2}{5}$. A student wants to run a simulation to estimate the probability that it will rain on both days. (Example 1)

Part A How can the student model the chance of it raining on each day? Design a simulation.

Sample answer: Use a spinner with five equal-size sections. Label two sections "R" for rain and three sections "N" for no rain. The spinner is spun twice for each trial.

Part B Suppose the table shows the results of 10 trials of a simulation. An "R" represents a day that it rained and an "N" represents a day that it did not rain.

Trial	1	2	3	4	5	6	7	8	9	10
Saturday	N	R	R	N	N	R	R	N	R	N
Sunday	N	N	R	R	N	R	N	R	R	N

According to the results of the simulation, what is the experimental probability of having rain on both days? **$\frac{3}{10}$, 0.3, 30%**

Test Practice

2. **Open Response** Leigh designs and conducts a computer simulation with 30 trials and uses the data from the simulation to create the relative frequency bar graph shown. The graph shows the relative frequency of the number of spins needed for a spinner divided into 6 equal sections labeled A through F to land on each letter at least once. (Example 2)

Using the graph, what is the experimental probability that more than 10 spins are needed to land on each letter at least once? Write the probability as a percent.

75%

Lesson 9-6 • Simulate Chance Events 619

7.SP.C.8.C

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Apply

For Exercises 3 and 4, refer to the following information.

Nelly designs and conducts a computer simulation with 50 trials and uses the data from the simulation to create the frequency bar graph shown. The graph shows the relative frequency of the number of rolls needed for a number cube labeled 7 through 12 to roll all of the possible odd numbers.

3. How much greater is the probability that 7 or 11 rolls are needed than 13 rolls?
$\frac{1}{20}$, 0.05, 5%

4. Is the probability that 7 or 12 rolls are needed greater than the probability that all of the other rolls are needed? Explain.
no; The probability that it takes 7 or 12 rolls is 15% + 30% or 45%. The probability of all other rolls is 100% − 45% or 55%. 55% is greater than 45%.

Higher-Order Thinking Problems

5. Use the Internet, or another source, to look up the term *fair game*. Describe a real-world scenario in which a game is fair. Then describe a real-world scenario in which a game is not fair.
Sample answer: A fair game could consist of tossing a coin, and winning the game is represented by tossing heads. A game that is not fair could consist of rolling a number cube labeled 1–6, and winning the game is represented by landing on the numbers 1 or 2.

6. MP **Model with Mathematics** Describe a real-world situation that can be simulated by tossing a coin and rolling a number cube. Be sure to include the number of outcomes in your description.
Sample answer: An ice cream shop offers 2 types of cones and 6 flavors of ice cream. To determine the probability that a customer will choose one type of cone and one flavor of ice cream, assign each one of the 12 outcomes a certain combination.

7. Why is it important to define each of the following when designing a simulation?
what each trial represents
what each outcome represents
what a success and a failure each represent
Sample answer: If you do not clearly define each of these, it can be difficult to interpret the results of the simulation in order to find the experimental probability of a favorable outcome.

8. MP **Use Math Tools** Suppose the players at a certain carnival game win about 40% of the time. Describe a model that can be used to simulate the outcomes of playing this game.
Sample answer: Place 2 red marbles representing wins and 3 green marbles representing losses and randomly pick one marble from the bag.

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Teaching the Mathematical Practices

4 Model with Mathematics In E xercise 6, students describe a real-world situation that can be simulated by tossing a coin and rolling a number cube.

5 Use Appropriate Tools Strategically In Exercise 8, students describe a tool that can be used to simulate the outcomes of playing a game.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 4 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Create your own higher-order thinking problem.
Use with Exercises 5–8 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include a review of probability including simple and compound events. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

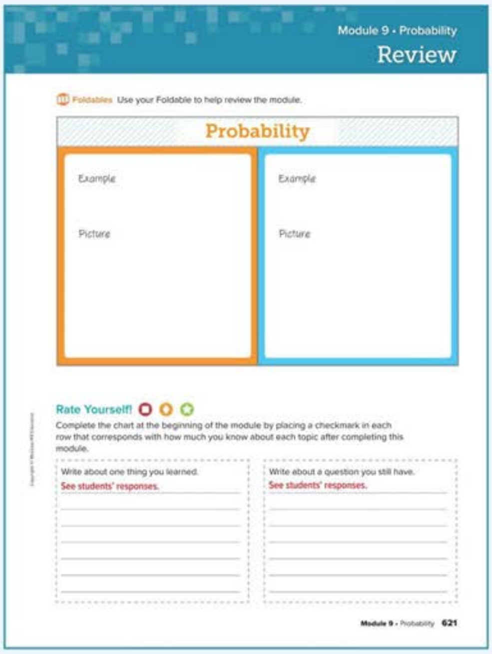
Assessment Resources

Put It All Together: Lessons 9-1 and 9-2
Put It All Together: Lessons 9-1, 9-2, 9-3, and 9-4
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with this topic for **Statistics and Probability**.

- Probability


Module 9 • Probability

Review

Foldables Use your Foldable to help review the module.

Probability

Example

Picture

Example

Picture

Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

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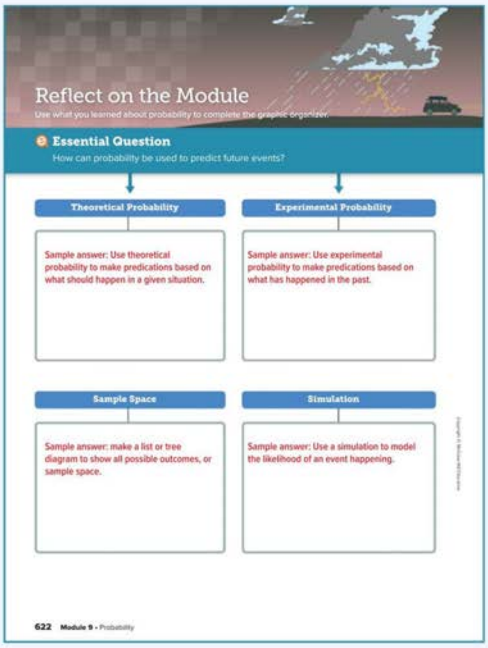

Reflect on the Module

Use what you learned about probability to complete the graphic organizer.

Essential Question

How can probability be used to predict future events?

Theoretical Probability

Sample answer: Use theoretical probability to make predications based on what should happen in a given situation.

Experimental Probability

Sample answer: Use experimental probability to make predications based on what has happened in the past.

Sample Space

Sample answer: make a list or tree diagram to show all possible outcomes, or sample space.

Simulation

Sample answer: Use a simulation to model the likelihood of an event happening.

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can probability be used to predict future events? See students' graphic organizers.

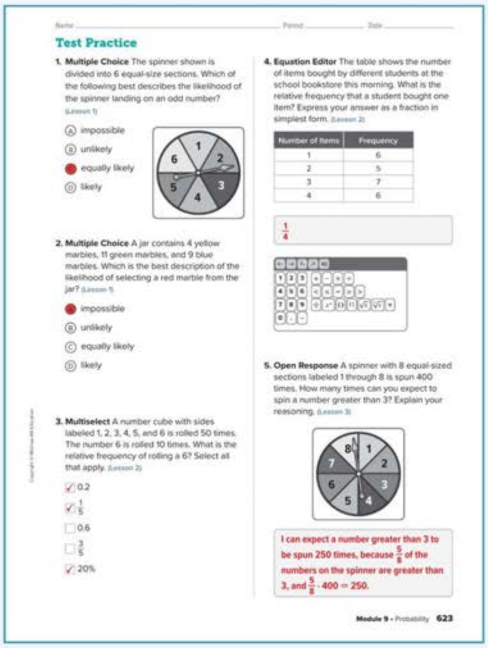
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–10 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	1, 2
Multiselect	Multiple answers may be correct. Students must select all correct answers.	3, 6
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	4, 8
Open Response	Students construct their own response in the area provided.	5, 7, 9, 10

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.SP.C.5	9-1	1, 2
7.SP.C.6	9-2, 9-4	3, 4, 7
7.SP.C.7	9-2, 9-3, 9-4	3–7
7.SP.C.7.A	9-3, 9-4	5, 6
7.SP.C.7.B	9-2, 9-4	3, 4, 7
7.SP.C.8	9-5, 9-6	8–10
7.SP.C.8.A	9-5	8, 9
7.SP.C.8.B	9-5	8, 9
7.SP.C.8.C	9-6	10


Name ______ Period ______ Date ______

Test Practice

1. **Multiple Choice** The spinner shown is divided into 6 equal-size sections. Which of the following best describes the likelihood of the spinner landing on an odd number? (Lesson 1)

Ⓐ impossible
Ⓑ unlikely
● equally likely
Ⓓ likely

2. **Multiple Choice** A jar contains 4 yellow marbles, 11 green marbles, and 9 blue marbles. Which is the best description of the likelihood of selecting a red marble from the jar? (Lesson 1)

● impossible
Ⓑ unlikely
Ⓒ equally likely
Ⓓ likely

3. **Multiselect** A number cube with sides labeled 1, 2, 3, 4, 5, and 6 is rolled 50 times. The number 6 is rolled 10 times. What is the relative frequency of rolling a 6? Select all that apply. (Lesson 2)

☑ 0.2
☑ $\frac{1}{5}$
☐ 0.6
☐ $\frac{3}{5}$
☑ 20%

4. **Equation Editor** The table shows the number of items bought by different students at the school bookstore this morning. What is the relative frequency that a student bought one item? Express your answer as a fraction in simplest form. (Lesson 2)

Number of Items	Frequency
1	6
2	5
3	7
4	6

$\frac{1}{4}$

5. **Open Response** A spinner with 8 equal-sized sections labeled 1 through 8 is spun 400 times. How many times can you expect to spin a number greater than 3? Explain your reasoning. (Lesson 3)

I can expect a number greater than 3 to be spun 250 times, because $\frac{5}{8}$ of the numbers on the spinner are greater than 3, and $\frac{5}{8} \cdot 400 = 250$.

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6. **Multiselect** The number 4 is rolled on a number cube with sides numbered 1, 2, 3, 4, 5, and 6. (Lesson 3)

A. Which outcomes make up the complement of the event? Select all that apply.

- [x] rolling a 1
- [x] rolling a 2
- [x] rolling a 3
- [ ] rolling a 4
- [x] rolling a 5
- [x] rolling a 6

B. What is the probability of the complement?

- [ ] $\frac{1}{6}$
- [x] $\frac{5}{6}$
- [ ] $\frac{1}{3}$
- [ ] $\frac{1}{2}$
- [ ] $\frac{2}{3}$
- [ ] 0

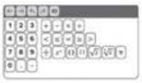
7. **Open Response** The table shows the number of each color pen in Mrs. Devon's desk drawer. Suppose she selects a pen at random from the drawer. How much greater is the theoretical probability of selecting a black pen than selecting a red or a green pen? Express your answer as a percent. (Lesson 4)

Color	Number of Pens
Black	9
Blue	6
Green	1
Red	4

20%

8. **Equation Editor** Suppose Julio tosses a coin four times. What is the theoretical probability of tossing heads at least two times? Express your answer as a fraction in simplest form. (Lesson 5)

$\frac{11}{16}$



9. **Open Response** Olivia flips a coin and rolls a number cube with sides labeled 1, 2, 3, 4, 5, and 6. After 90 trials of the experiment, the relative frequency of flipping heads and rolling a number less than 3 is $\frac{2}{15}$. What is the difference between the number of expected outcomes and the number of actual outcomes? (Lesson 5)

3

10. **Open Response** A weather forecast calls for a 60% chance of rain today and a 60% chance of rain tomorrow. The table shows the results of 10 simulated trials, where "R" represents rain and "N" represents no rain. (Lesson 6)

Trial	1	2	3	4	5	6	7	8	9	10
Today	N	R	R	N	R	R	N	R	R	R
Tomorrow	N	R	R	N	R	R	R	N	N	R

According to the results of the simulation, what is the experimental probability of having rain on both days?

0.5

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Module 10

Sampling and Statistics

Module Goal

Analyze samples and interpret the data.

Focus

Domain: Statistics and Probability

Supporting and Additional Cluster(s):

7.SP.A Use random sampling to draw inferences about a population.

7.SP.B Draw informal comparative inferences about two populations.

Standards for Mathematical Content:

7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Also addresses 7.RP.A.2, 7.RP.A.3, 7.SP.B.3, and 7.SP.B.4.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- write fractions in simplest form
- express equivalent forms of fractions, decimals, and percents
- find the percent of a number
- find the mean and mean absolute deviation of a set of data

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students developed an understanding of statistical variability.
6.SP.A.1, 6.SP.A.2, 6.SP.A.3

Now
Students analyze samples and interpret the data.
7.SP.A.1, 7.SP.A.2

Next
Students will make inferences and justify conclusions from sample experiments.
HSS.IC.B.3, HSS.IC.B.4, HSS.IC.B.5, HSS.IC.B.6

Rigor

The Three Pillars of Rigor

In this module, students draw upon their knowledge of measures of center, measures of variation, and ratios to develop *understanding* about statistical sampling and making inferences and predictions. Students come to *understand* that taking multiple samples can help them gauge the variation in their predictions. Students build *fluency* in using ratio reasoning to make predictions about a population and in using the measures of center and variation to compare two sample distributions. They *apply* their *understanding* of the mean and mean absolute deviation to informally assess the degree of visual overlap between two distributions to infer how close the population means might be.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

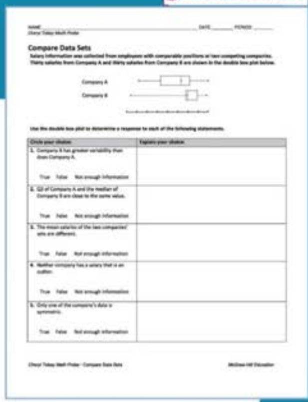
EXPLORE → LEARN → EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
10-1	Biased and Unbiased Samples	7.SP.A.1, 7.SP.A.2	1	0.5
10-2	Make Predictions	7.SP.A.2, *Also addresses 7.RP.A.2, 7.RP.A.3*	1	0.5
10-3	Generate Multiple Samples	7.SP.A.2, *Also addresses 7.RP.A.2*	2	1
Put It All Together: Lessons 10-1 through 10-3			0.5	0.25
10-4	Compare Two Populations	7.SP.B.4	2	1
10-5	Assess Visual Overlap	7.SP.B.3	1	0.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	10.5	5.25

Formative Assessment Math Probe
Compare Data Sets

Module Resource



Correct Answers: 1. False; **2.** True; **3.** Not enough information; **4.** False; **5.** False

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine if each statement is true or false, and explain their choices.

Targeted Concept Sets of data summarized as box plots can be analyzed and compared (minimum, lower quartile, median, upper quartile, maximum) even without a specific scale.

Targeted Misconceptions

- Students may incorrectly view the quartiles as the values of the data.
- Students may incorrectly view the middle line as representing the mean and not the median (middle number) of the data.
- Students may not know what an outlier is and/or how to find one without a given scale of the graph.

Assign the probe after Lesson 4.

Collect and Assess Student Work

If the student selects...	Then the student likely...
Various Incorrect responses	misunderstood or confused the terms *variability, median, mean, outlier, symmetric* as they relate to boxplots.
1. True	used the range rather than interquartile range.
3. True	confused the mean and the median.
Chooses *not enough information* for Exercises 1, 2, 4, or 5	believed comparisons cannot be made without a known scale.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Data Analysis
- Lesson 4, Examples 1–2

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can you use a sample to gain information about a population? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topic of surveying people about their favorite breakfast food to introduce the idea of sampling and statistics. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.


Module 10
Sampling and Statistics

Essential Question
How can you use a sample to gain information about a population?

What Will You Learn?
Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
identifying valid sampling methods						
identifying biased samples						
identifying valid inferences						
making predictions using sample data						
understanding the benefit of taking multiple samples						
making comparative inferences about two populations						
making inferences about the variability between two populations						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about sampling and statistics.

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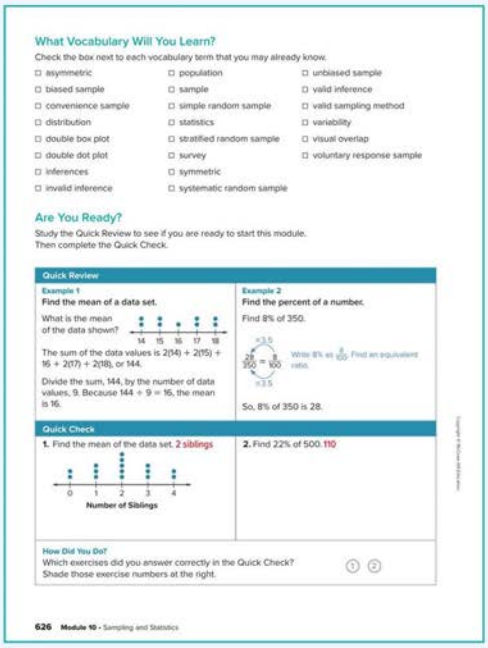
Interactive Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ asymmetric
- ☐ biased sample
- ☐ convenience sample
- ☐ distribution
- ☐ double box plot
- ☐ double dot plot
- ☐ inferences
- ☐ invalid inference
- ☐ population
- ☐ sample
- ☐ simple random sample
- ☐ statistics
- ☐ stratified random sample
- ☐ survey
- ☐ symmetric
- ☐ systematic random sample
- ☐ unbiased sample
- ☐ valid inference
- ☐ valid sampling method
- ☐ variability
- ☐ visual overlap
- ☐ voluntary response sample

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Find the mean of a data set.
What is the mean of the data shown?
The sum of the data values is 2(14) + 2(15) + 16 + 2(17) + 2(18), or 144.
Divide the sum, 144, by the number of data values, 9. Because 144 ÷ 9 = 16, the mean is 16.

Example 2
Find the percent of a number.
Find 8% of 350.
$\frac{28}{350} = \frac{8}{100}$ Write 8% as $\frac{8}{100}$. Find an equivalent ratio.
So, 8% of 350 is 28.

Quick Check

1. Find the mean of the data set. 2 siblings

2. Find 22% of 500. 110

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

626 Module 10 • Sampling and Statistics

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define A **sample** is a randomly selected group chosen for the purpose of collecting data.

Example The sixth-grade students are a sample of all of the students in a school.

Ask Which of all the following would be an appropriate sample for the cars in a city: all of the cars in the city, or all of the cars in a parking garage located in the city? the cars in the parking garage

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- expressing equivalent forms of fractions, decimals, and percents
- finding the percent of a number
- summarizing numerical data using the mean
- finding the mean absolute deviation of a data set

ALEKS®

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Data Analysis** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Attitude Ownership

Part of developing a growth mindset involves acknowledging progress in growth thinking and sharing it with others. It's important for a student to own his or her mindset, attitude, and beliefs and be proud of the growth. Students should view themselves as people who have a growth mentality—not just in math, but with learning, in general.

How Can I Apply It?

Have students complete a math mindset project to share how they have grown throughout the year. They might choose their own delivery method, such as a poster, blog post, video, or podcast. Encourage them to give specific examples from their journey, such as times when they made a mistake and learned from it, times when they took a risk to solve a challenging problem, or times when they engaged in reflection. Students can share their mindset journey with their classmates, or might post their projects for others to see.

Biased and Unbiased Samples

7.SP.A.1, 7.SP.A.2

LESSON GOAL

Students will identify samples as biased or unbiased and determine whether inferences from the samples are valid.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Populations and Samples
Learn: Valid Sampling Methods
Example 1: Identify Valid Sampling Methods
Learn: Biased Samples
Example 2: Identify Biased Sampling Methods
Learn: Valid Inferences
Example 3: Identify Valid Inferences
Example 4: Identify Valid Inferences

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Margin of Sampling Error		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 58 of the *Language Development Handbook* to help your students build mathematical language related to biased and unbiased samples.

ELL You can use the tips and suggestions on page T58 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability

Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.A** by identifying samples as biased or unbiased and whether inferences from the samples are valid.

Standards for Mathematical Content: **7. SP.A.1, 7.SP.A.2**

Standards for Mathematical Practice: **MP 2, MP3, MP6**

Coherence

Vertical Alignment

Previous

Students understood that a statistical question anticipates a variety of responses. **6.SP.A.1**

Now

Students identify samples as biased or unbiased and determine whether inferences from the samples are valid.
7.SP.A.1

Next

Students will make predictions based on data gathered using a valid sampling method.
7.SP.A.2

Rigor

The Three Pillars of Rigor

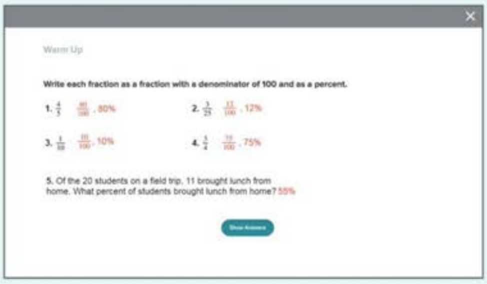
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw upon their knowledge of statistics and statistical questions to *understand* that inferences made about a population are only valid if the sampling method used was unbiased. Students build *fluency* in identifying unbiased and biased sampling methods, and determining whether inferences made are valid or invalid.

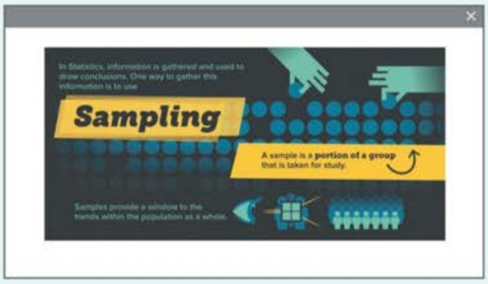
Mathematical Background

A *sample* is often used to study a desired characteristic of a population. Valid sampling methods should be used to make valid inferences about the population being studied. Valid (unbiased) samples are representative of the population and selected at random, where each member has an equal chance of being selected. A biased sample usually favors one or more parts of the population over another, and thus is not representative of the population.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing fractions as percents (Exercises 1–5)

Answers

1. $\frac{80}{100}$, 80%
2. $\frac{12}{100}$, 12%
3. $\frac{10}{100}$, 10%
4. $\frac{75}{100}$, 75%
5. 55%

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about sampling using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Students will learn more terms than these three in the lesson, but starting with a discussion around these terms can help them begin thinking about biased samples.

Ask:

- What are some synonyms for the term *bias*? Sample answers: favoritism, unfairness, one-sidedness
- Define *convenience* in your own words. Sample answer: being able to do something with relative ease
- What does *voluntary* mean? Sample answer: to be able to do or not do something based on one's own choice

Learn Populations and Samples

Objective

Students will learn about populations and samples.

Teaching Notes

SLIDE 1

Students will learn the terms *statistics*, *population*, and *sample*. You may wish to ask students to generate several examples of samples, if the population consists of all of the students in their school.

SLIDE 2

Students will learn about *surveys*. Have students use the interactive tool to determine whether each phrase describes a population, or a sample, for each survey topic.

DIFFERENTIATE

Reteaching Activity AL

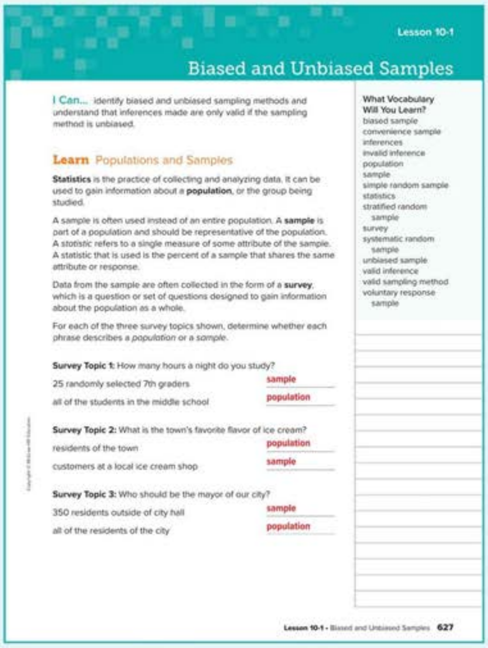
For students that may be struggling to understand populations and samples, explain that a sample is always a part of the population, and the population is often much larger than the sample. Have them describe a possible sample from each of the given populations.

Population: the students in the classroom Sample answer: the students in the classroom wearing blue

Population: the teachers in the school Sample answer: the seventh-grade teachers

Population: the people that live in the United States Sample answer: people that live in Florida

Population: the professional athletes in the world Sample answer: the professional athletes who play soccer



Lesson 10-1

Biased and Unbiased Samples

I Can... identify biased and unbiased sampling methods and understand that inferences made are only valid if the sampling method is unbiased.

Learn Populations and Samples

Statistics is the practice of collecting and analyzing data. It can be used to gain information about a **population**, or the group being studied.

A sample is often used instead of an entire population. A **sample** is part of a population and should be representative of the population. A statistic refers to a single measure of some attribute of the sample. A statistic that is used is the percent of a sample that shares the same attribute or response.

Data from the sample are often collected in the form of a **survey**, which is a question or set of questions designed to gain information about the population as a whole.

For each of the three survey topics shown, determine whether each phrase describes a *population* or a *sample*.

Survey Topic 1: How many hours a night do you study?

25 randomly selected 7th graders sample

all of the students in the middle school population

Survey Topic 2: What is the town's favorite flavor of ice cream?

residents of the town population

customers at a local ice cream shop sample

Survey Topic 3: Who should be the mayor of our city?

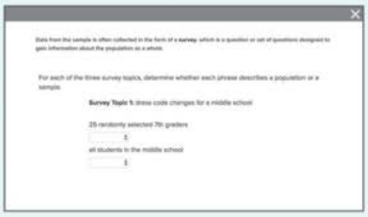
350 residents outside of city hall sample

all of the residents of the city population

What Vocabulary Will You Learn?
biased sample
convenience sample
inferences
invalid inference
population
sample
simple random sample
statistics
stratified random sample
survey
systematic random sample
unbiased sample
valid inference
valid sampling method
voluntary response sample

Lesson 10-1 • Biased and Unbiased Samples 627

Interactive Presentation



Learn, Populations and Samples, Slide 2 of 2

CLICK

On Slide 2, students determine if each phrase represents a population or a sample.

2 EXPLORE AND DEVELOP

7.SP.A.1, 7.SP.A.2

Your Notes

Learn Valid Sampling Methods

When choosing a sample from a population, it is important to use a valid sample method. A **valid sampling method** is one that is:

- representative of the population
- selected at random, where each member has an equal chance of being selected, and
- large enough to provide accurate data

The table shows various valid sampling methods.

	Definition	Example
Simple Random Sample	Each item or person in the population is as likely to be chosen as any other.	Twenty-five student names are written on slips of paper and placed in a basket. One name is randomly selected.
Stratified Random Sample	The population is divided into groups with similar traits that do not overlap. A simple random sample is then selected from each group.	Students of a school are divided into 6th, 7th, and 8th grade. A random sample of 25 students from each grade is chosen.
Systematic Random Sample	The sample is selected from the population according to a specific item or time interval.	Every 10th customer at a store is given a survey, or a customer is chosen to complete a survey every half hour.

Talk About It!
Why do the three names of the valid sampling methods all contain the word *random*?

Sample answer: It is important that the selection of the sample is not planned so each member of the sample has an equal chance of being selected.

Pause and Reflect

Where have you seen surveys in everyday life? What type of sample was the survey using?

See students' observations.

628 **Module 10** • Sampling and Statistics

Interactive Presentation

Learn, Valid Sampling Methods, Slide 3 of 6

CLICK
On Slide 2, students compare and contrast several valid sampling methods.

WEB SKETCHPAD
On Slides 3-5, students use a sketch to explore each type of valid sampling method.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Valid Sampling Methods

Objective

Students will learn about valid sampling methods.

Teaching Notes

SLIDE 1

Students will learn the characteristics of a *valid sampling method*. You may wish to have students explain why a sampling method might not be valid if it does not meet these characteristics.

SLIDE 2

Students will learn about types of valid sampling methods. Have them use the interactive tool to view the definition and an example of each type of sample: *stratified random sample*, *systematic random sample*, and *simple random sample*.

SLIDE 3

On Slides 3-5, students will be presented with one of the three types of valid sampling methods they have just learned. Have them follow the instructions to view how each sampling method can be used to obtain a random sample. You may wish to have students compare and contrast these three types of valid sampling methods, and describe each one in their own words.

Talk About It!

SLIDE 6

Mathematical Discourse

Why do the three names of the valid sampling methods all contain the word *random*? Sample answer: It is important that the selection of the sample is not planned so each member of the sample has an equal chance of being selected.

DIFFERENTIATE

Language Development Activity ELL

To support students' vocabulary development, have them work with a partner to use a dictionary, thesaurus, the Internet or another source to look up the meanings of the terms *stratified* and *systematic*. Have them explain the meanings in their own words and how understanding these terms helps them understand the meanings of *stratified random sample* and *systematic random sample*. Sample responses are shown.

Stratified means arranged, classified, or organized. So, a stratified random sample is one in which the population is arranged into groups before taking the random sample.

Systematic means following a certain method or pre-determined routine. So, a systematic random sample is one in which the random sample is selected according to a pre-determined method, such as every hour or every 10th person.

Example 1 Identify Valid Sampling Methods

Objective

Students will identify valid sampling methods that best represent survey descriptions.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to consider how the results of the survey might be affected if a sample was taken that was not taken at random.

6 Attend to Precision Encourage students to use precision in selecting the correct sampling methods for each description, paying careful attention to the names for each method.

Questions for Mathematical Discourse

SLIDE 2

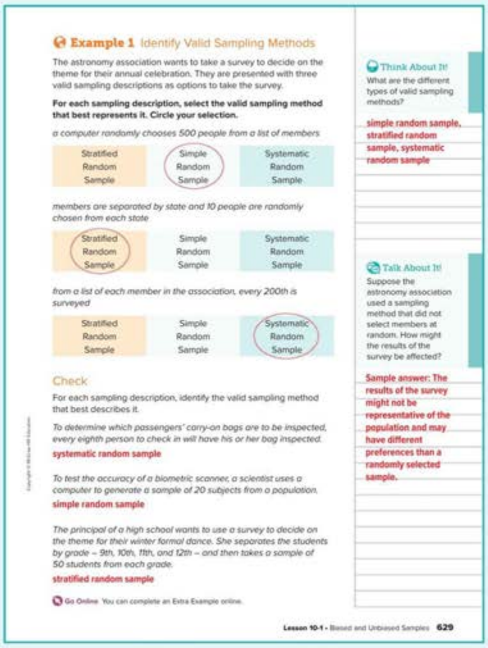
AL In which method is each item or person in the population as likely to be chosen as any other? Does this fit the first description? simple random sample; yes

OL Describe, in your own words, the second description. What key phrases can help you classify the correct sampling method? Sample answer: The members are organized by state and then 10 members are randomly selected from each state. The key phrases "separated by state" helps me know this is a stratified random sample, because members are selected from nonoverlapping groups.

BL Describe another way that a systematic random sample can be taken, in this scenario. Sample answer: From the list of all members, every 50th member is surveyed.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Identify Valid Sampling Methods

The astronomy association wants to take a survey to decide on the theme for their annual celebration. They are presented with three valid sampling descriptions as options to take the survey.

For each sampling description, select the valid sampling method that best represents it. Circle your selection.

a computer randomly chooses 500 people from a list of members

Stratified Random Sample | Simple Random Sample (circled) | Systematic Random Sample

members are separated by state and 10 people are randomly chosen from each state

Stratified Random Sample (circled) | Simple Random Sample | Systematic Random Sample

from a list of each member in the association, every 200th is surveyed

Stratified Random Sample | Simple Random Sample | Systematic Random Sample (circled)

Check

For each sampling description, identify the valid sampling method that best describes it.

To determine which passengers' carry-on bags are to be inspected, every eighth person to check in will have his or her bag inspected.
systematic random sample

To test the accuracy of a biometric scanner, a scientist uses a computer to generate a sample of 20 subjects from a population.
simple random sample

The principal of a high school wants to use a survey to decide on the theme for their winter formal dance. She separates the students by grade – 9th, 10th, 11th, and 12th – and then takes a sample of 50 students from each grade.
stratified random sample

Go Online You can complete an Extra Example online.

Think About It! What are the different types of valid sampling methods?
simple random sample, stratified random sample, systematic random sample

Talk About It! Suppose the astronomy association used a sampling method that did not select members at random. How might the results of the survey be affected?
Sample answer: The results of the survey might not be representative of the population and may have different preferences than a randomly selected sample.

Lesson 10-1 • Biased and Unbiased Samples 629

Interactive Presentation

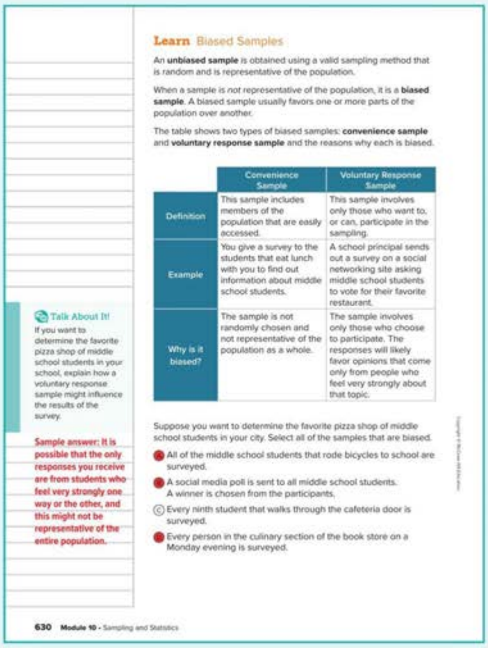
Example 1, Identify Valid Sampling Methods, Slide 2 of 4

CLICK

On Slide 2, students select the valid sampling method for each description.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Biased Samples

An **unbiased sample** is obtained using a valid sampling method that is random and is representative of the population.

When a sample is *not* representative of the population, it is a **biased sample**. A biased sample usually favors one or more parts of the population over another.

The table shows two types of biased samples: **convenience sample** and **voluntary response sample** and the reasons why each is biased.

	Convenience Sample	Voluntary Response Sample
Definition	This sample includes members of the population that are easily accessed.	This sample involves only those who want to, or can, participate in the sampling.
Example	You give a survey to the students that eat lunch with you to find out information about middle school students.	A school principal sends out a survey on a social networking site asking middle school students to vote for their favorite restaurant.
Why is it biased?	The sample is not randomly chosen and not representative of the population as a whole.	The sample involves only those who choose to participate. The responses will likely favor opinions that come only from people who feel very strongly about that topic.

Talk About It!
If you want to determine the favorite pizza shop of middle school students in your school, explain how a voluntary response sample might influence the results of the survey.

Sample answer: It is possible that the only responses you receive are from students who feel very strongly one way or the other, and this might not be representative of the entire population.

Suppose you want to determine the favorite pizza shop of middle school students in your city. Select all of the samples that are biased.

- (A) All of the middle school students that rode bicycles to school are surveyed.
- (B) A social media poll is sent to all middle school students. A winner is chosen from the participants.
- (C) Every ninth student that walks through the cafeteria door is surveyed.
- (D) Every person in the culinary section of the book store on a Monday evening is surveyed.

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Interactive Presentation

Learn, Biased Samples, Slide 1 of 3

CLICK

On Slide 1, students use the interactive tool to compare and constrast types of samples.

Learn Biased Samples

Objective

Students will learn about biased sampling methods.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 3, encourage them to adhere to the meaning of the term *voluntary response sample* in their response.

Teaching Notes

SLIDE 1

Facilitate a class discussion about *biased* and *unbiased* samples. Have students discuss the two types of biased samples presented: *convenience samples* and *voluntary response samples*. Have them use the interactive tool to view a definition and an example of each type of biased sample, and the reason why each sample is biased. You may wish to ask them to explain each type of sample in their own words, and why each is biased.

Talk About It!

SLIDE 3

Mathematical Discourse

If you want to determine the favorite pizza shop of middle school students in your school, explain how a voluntary response sample might influence the results of the survey. Sample answer: It is possible that the only responses you receive are from students who feel very strongly one way or the other, and this might not be representative of the entire population.

DIFFERENTIATE

Language Development Activity ELL

To further students' understanding of biased samples, have students give an example of a *simple random sample*, a *voluntary response sample*, and a *convenience sample* when the population represents the city residents that own a dog.

Simple Random Sample Sample answer: the names of the city residents that own a dog are entered in a computer which randomly selects 50 of them

Voluntary Response Sample Sample answer: city residents that own a dog are emailed and asked to complete a survey

Convenience Sample Sample answer: dog owners are surveyed at one of the veterinary clinics in town

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Identify Biased Sampling Methods

Objective

Students will classify biased samples by type.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to understand the difference between a voluntary response sample and convenience sample, and use precision in classifying the correct sampling method.

Questions for Mathematical Discourse

SLIDE 1

AL What is a biased sample? a sample that is not representative of the entire population

AL What is a convenience sample? a sample that includes members of the population that are easily accessed

OL What key aspect of the first scenario indicates the type of biased sample? Sample answer: The student surveys everyone in his class. This is convenient for the student, but may produce invalid results.

OL What key aspect of the second scenario indicates the type of biased sample? Sample answer: The survey is posted on the website and readers may choose to respond or not respond.

BL Describe a different real-world situation in which a voluntary response sample might be used. Sample answer: A grocery store manager wants to know about customer satisfaction. The store posts an online survey on their website and asks for customers to take the survey.

Go Online

- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Identify Biased Sampling Methods

Identify the type of biased sample for each situation.

To gather information about the favorite animal of middle school students, a student surveys everyone in his 7th grade language arts class.

Because the sample is everyone that is available in one class, it is a convenience sample.

A news organization posts a survey on its website that asks its readers to vote on a current topic.

Because only those people that choose to respond will, the sample is a voluntary response sample.

Check

Identify the type of biased sample for each situation.

You survey a sample of 10 households on your street to determine how often households in the state cut their grass.
convenience sample

The manager of a retail store mails out a survey to 500 potential customers. She uses the results of the surveys that are mailed back to determine which items to keep in stock for the spring season.
voluntary response sample

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe some biased sampling methods you may have encountered in your everyday life. Why were they biased?

See students' observations.

Math History Minute

Victor Neumann-Lara (1933-2004) was a Mexican mathematician and a pioneer in the field of graph theory. In 1982, he introduced the idea of a digraph. A digraph, also known as a directed graph, is a graph consisting of a set of vertices connected by edges, in which the edges have a direction associated with them.

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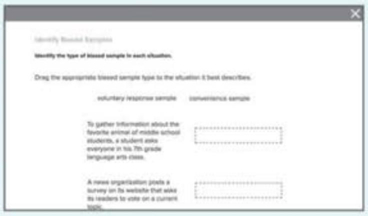
Interactive Presentation



Example 2, Identify Biased Samples, Slide 1 of 2

DRAG & DROP

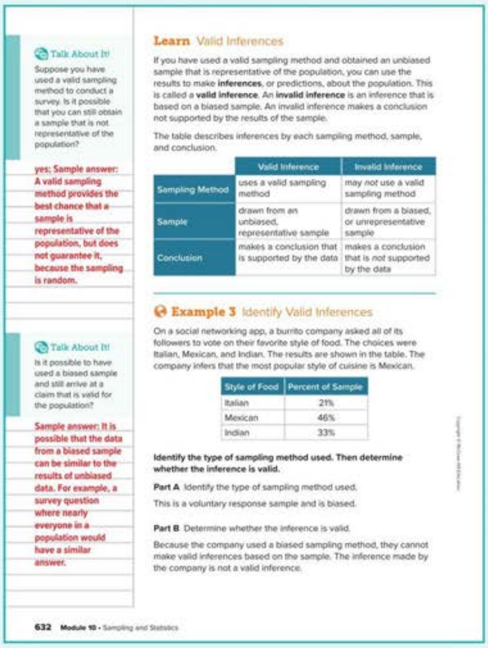
On Slide 1, students drag the type of biased sample to the situation it best describes.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.SP.A.1, 7.SP.A.2

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION



Talk About It!

Suppose you have used a valid sampling method to conduct a survey. Is it possible that you can still obtain a sample that is not representative of the population?

yes; Sample answer: A valid sampling method provides the best chance that a sample is representative of the population, but does not guarantee it, because the sampling is random.

Learn Valid Inferences

If you have used a valid sampling method and obtained an unbiased sample that is representative of the population, you can use the results to make **inferences**, or predictions, about the population. This is called a **valid inference**. An **invalid inference** is an inference that is based on a biased sample. An invalid inference makes a conclusion not supported by the results of the sample.

The table describes inferences by each sampling method, sample, and conclusion.

	Valid Inference	Invalid Inference
Sampling Method	uses a valid sampling method	may *not* use a valid sampling method
Sample	drawn from an unbiased, representative sample	drawn from a biased, or unrepresentative sample
Conclusion	makes a conclusion that is supported by the data	makes a conclusion that is *not* supported by the data

Example 3 Identify Valid Inferences

On a social networking app, a burrito company asked all of its followers to vote on their favorite style of food. The choices were Italian, Mexican, and Indian. The results are shown in the table. The company infers that the most popular style of cuisine is Mexican.

Style of Food	Percent of Sample
Italian	21%
Mexican	46%
Indian	33%

Identify the type of sampling method used. Then determine whether the inference is valid.

Part A Identify the type of sampling method used.

This is a voluntary response sample and is biased.

Part B Determine whether the inference is valid.

Because the company used a biased sampling method, they cannot make valid inferences based on the sample. The inference made by the company is not a valid inference.

Talk About It!

Is it possible to have used a biased sample and still arrive at a claim that is valid for the population?

Sample answer: It is possible that the data from a biased sample can be similar to the results of unbiased data. For example, a survey question where nearly everyone in a population would have a similar answer.

632 Module 10 • Sampling and Statistics

Interactive Presentation

Learn, Valid Inferences, Slide 1 of 2

CLICK

On Slide 1, students use the interactive tool to compare and contrast valid and invalid inferences.

Learn Valid Inferences

Objective

Students will learn about the differences between valid and invalid inferences.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

Suppose you have used a valid sampling method to conduct a survey. Is it possible that you can still obtain a sample that is not representative of the population? yes; Sample answer: A valid sampling method provides the best chance that a sample is representative of the population, but does not guarantee it, because the sampling is random.

Example 3 Identify Valid Inferences

Objective

Students will identify the sampling method used in order to determine that an invalid inference was made.

Questions for Mathematical Discourse

SLIDE 3

AL What might be true about the participants who choose to respond to the survey? What might that mean for the results of the survey? Sample answer: They might prefer Mexican food. The results of the survey may be biased.

OL How might you alter the sampling method in order to ensure the sampling method is unbiased? Sample answer: Use a systematic random sampling method, or a stratified random sampling method.

BL Suppose you are the manager of the burrito company. Design your own sampling method to determine the favorite style of food for a large city. Share your sampling method with a classmate, and have each student determine if the sampling methods are biased or unbiased. See students' responses.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Identify Valid Inferences

Objective

Students will identify the sampling method used in order to determine that a valid inference was made.

Teaching the Mathematical Practices

3 Construct Valid Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to consider the context from which the data arose as they justify their conclusion.

6 Attend to Precision Encourage students to use the correct terminology in identifying the type of sampling method used and whether the inference is valid or invalid.

Questions for Mathematical Discourse

SLIDE 2

AL In your own words, describe this sample. Sample answer: The sample consists of responses from every third customer that walks into the shoe store.

OL What type of sampling method was used? systematic random sample

OL Is the sample biased? Explain. no; Sample answer: The sample was obtained using an unbiased sampling method.

BL Generate a biased sampling method that might be used in this situation. Sample answer: If the manager of the shoe store asked customers to respond to an online survey, this would be a voluntary response sample, which could produce biased results.

SLIDE 3

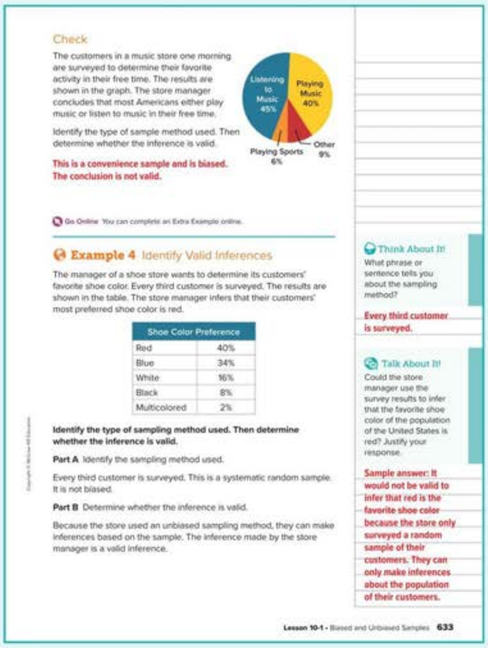
AL Make another inference about the survey results. Sample answer: Blue is the second favorite shoe color of the store's customers.

OL What is another unbiased sampling method that can be used? Sample answer: Simple random sample; the store manager can assign a random number to each customer between 1 and 3 and then survey every customer assigned with the number 3.

BL Suppose you are the manager of the shoe store. Design your own sampling method to determine the favorite shoe color for a large city. Share your sampling method with a classmate, and have each student determine if the sampling methods are biased or unbiased. See students' responses.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

The customers in a music store one morning are surveyed to determine their favorite activity in their free time. The results are shown in the graph. The store manager concludes that most Americans either play music or listen to music in their free time.

Identify the type of sample method used. Then determine whether the inference is valid.

This is a convenience sample and is biased. The conclusion is not valid.

Go Online You can complete an Extra Example online.

Example 4 Identify Valid Inferences

The manager of a shoe store wants to determine its customers' favorite shoe color. Every third customer is surveyed. The results are shown in the table. The store manager infers that their customers' most preferred shoe color is red.

Shoe Color Preference	
Red	40%
Blue	34%
White	16%
Black	8%
Multicolored	2%

Identify the type of sampling method used. Then determine whether the inference is valid.

Part A Identify the sampling method used.

Every third customer is surveyed. This is a systematic random sample. It is not biased.

Part B Determine whether the inference is valid.

Because the store used an unbiased sampling method, they can make inferences based on the sample. The inference made by the store manager is a valid inference.

Think About It! What phrase or sentence tells you about the sampling method?

Every third customer is surveyed.

Talk About It! Could the store manager use the survey results to infer that the favorite shoe color of the population of the United States is red? Justify your response.

Sample answer: It would not be valid to infer that red is the favorite shoe color because the store only surveyed a random sample of their customers. They can only make inferences about the population of their customers.

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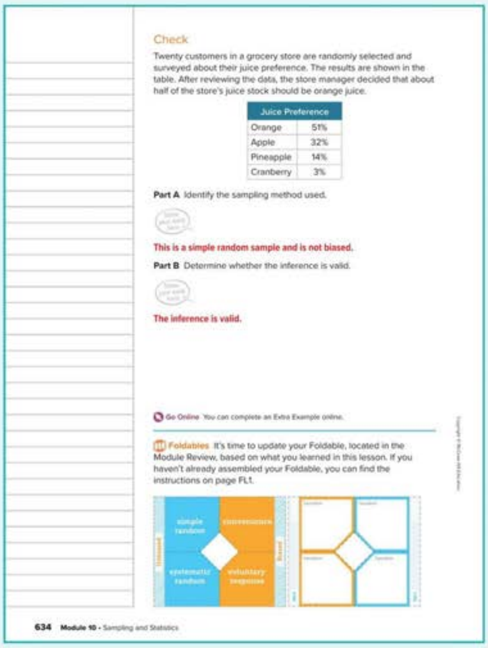
Interactive Presentation

Example 4, Identify Valid Inferences, Slide 1 of 5

CLICK — On Slide 2, students select from drop-down menus to describe a sample.

CLICK — On Slide 3, students select from drop-down menus to describe an inference made from a sample.

CLICK — Students complete the Check exercise online to determine if they are ready to move on.



Check

Twenty customers in a grocery store are randomly selected and surveyed about their juice preference. The results are shown in the table. After reviewing the data, the store manager decided that about half of the store's juice stock should be orange juice.

Juice Preference	
Orange	51%
Apple	32%
Pineapple	14%
Cranberry	3%

Part A Identify the sampling method used.

This is a simple random sample and is not biased.

Part B Determine whether the inference is valid.

The inference is valid.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about unbiased and biased sampling methods. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How can you use a sample to gain information about a population?
In this lesson, students learned how to determine whether a sample is biased or unbiased. Encourage them to discuss with a partner how taking a biased or an unbiased sample can affect the inferences they may be able to make about a population.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you wanted to determine the number of students in your entire school who prefer having a certain type of pet (cat, dog, or other). Design an unbiased sampling method that you can use and explain why your sampling method is unbiased. Sample answer: Survey every tenth student as they enter the school about their preferred pet. This is a systematic random sample, so it is unbiased.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1–7 odd, 8–11
- Extension: Margin of Sampling Error
- ALEKS® Collecting Data

IF students score 66-89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 9
- Extension: Margin of Sampling Error
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1-4
- ALEKS® Collecting Data

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Collecting Data

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	classify valid sampling methods from a situation	1
2	classify biased samples by type	2
2	interpret valid and invalid inferences made from a sample	3, 4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving biased and unbiased samples	6, 7
3	higher-order and critical thinking skills	8–11

Common Misconception

As students identify sampling methods being used, encourage them to consider whether people who complete a survey volunteered to do so. This is most often a voluntary response sample and is considered biased. The inferences made from this type of sample are considered not valid.

Name ______ Period ______ Date ______

Practice Go Online You can complete your homework online.

1. For each sampling description, identify the valid sampling method that best describes it. Choose from *simple random sample, stratified random sample,* or *systematic random sample.* (Example 1)
   a. To determine if a candidate for state senator is popular with voters, 25% of voters in 160 counties are surveyed. **stratified random sample**
   b. To determine whether students think a new school library is needed, a computer generates a list of 100 random students and they are surveyed. **simple random sample**
   c. To determine the freshness of doughnuts, a baker selects a doughnut every 30 minutes and checks it. **systematic random sample**
2. Identify the type of biased sample for each situation. Choose from convenience *sample* or *voluntary response sample.* (Example 2)
   a. A physical education teacher posts an online survey about whether students would be interested in a 5K race. The responses received determine whether there will be a 5K race. **voluntary response sample**
   b. To determine the theme of the school dance, the student council president surveys his homeroom class. **convenience sample**

Identify the sample method used and whether it is biased or unbiased. Then determine whether the inference is valid. (Examples 3 and 4)

3. To evaluate customer satisfaction, a grocery store manager gives double coupons to anyone who completes a survey as they enter the store. The store manager determines that customers are very satisfied with their shopping experience in his store. **voluntary response sample; biased; The inference is not valid.**
4. A member of the cafeteria staff asks every fifth student leaving the cafeteria to rank 5 entrees from most favorite to least favorite. She finds that pizza is one of the favorite entrees. **systematic random sample; unbiased; The inference is valid.**

Test Practice

5. **Multiselect** To evaluate the defect rate of its lenses, a camera lens manufacturer tests every 100th lens off the production line. Out of 1,000 lenses tested, one lens is found to be defective. The manufacturer concludes that 3 lenses out of 3,000 will be defective. Select all of the statements that are true about the sampling method.
   - [x] This scenario is a systematic random sample.
   - [ ] The sampling method is biased.
   - [x] The inference is valid.
   - [ ] This scenario is a convenience sample.
   - [x] The sampling method is unbiased.

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Apply *indicates multi-step problem

*6. Members of the drama club plan to sell popcorn as a fundraiser for their spring play. To determine what flavor to sell, the members survey every 15th student from an alphabetical listing of all students. The table shows the results of their survey. Was the sample obtained using a valid sampling method? If so, find what percent of students prefer each type of popcorn. If not, explain why.

Flavor	Number
Butter	33
Cheese	15
Caramel	27

yes; butter: 44%, cheese: 20%, caramel: 36%

*7. As people leave a restaurant one evening, 20 people are surveyed at random. Eight people say they usually order dessert when they eat out. Was the sample obtained using a valid sampling method? If so, what percent of those surveyed say they usually do not order dessert when they eat out? If not, explain why.

yes; 60%

Higher-Order Thinking Problems

8. Give an example of a convenience sample.
Sample answer: A real estate agent surveys people about their housing preferences at an open house.

9. MP **Reason Abstractly** Marc wants to determine how many students plan to attend the school's walk-a-thon. He decides to post an online survey. Of the survey responses, 80% plan to attend the walk-a-thon. Marc infers most students will attend the walk-a-thon. Is Marc's inference valid? Explain.
no; Sample answer: Marc used a voluntary response sample. The results are biased and therefore invalid because only those that wanted to respond were included.

10. MP **Justify Conclusions** Determine if the statement is *true* or *false*. Explain.

A stratified random sample's results are never valid.
false; Sample answer: A stratified random sample's results are usually valid because the sampling method is representative of the entire population, selected at random, and large enough to provide accurate data.

11. Suppose you wanted to know how many students brought their lunch to school. Describe a valid sampling method you could use.
Sample answer: Every 25th student is chosen from an alphabetical listing of all students. The chosen students are then surveyed.

636 **Module 10** • Sampling and Statistics

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 9, students use reasoning to determine if another student's inference is valid and justify their response.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students determine if a stratified random sample's results can be valid and justify their conclusion.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 6–7 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 9 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise used a voluntary response sample, which could lead to biased results. Have each pair or group of students present their explanations to the class.

Lesson 10-2

Make Predictions

7.SP.A.2

LESSON GOAL

Students will make predictions based on data gathered using a valid sampling method.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Make Predictions
Example 1: Make Predictions
Example 2: Make Predictions
Apply: Profit

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 59 of the *Language Development Handbook* to help your students build mathematical language related to making predictions.

ELL You can use the tips and suggestions on page T59 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min: 0.5 day
45 min: 1 day

Focus

Domain: Statistics and Probability
Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.A** by making predictions based on data gathered using a valid sampling method.
Standards for Mathematical Content: **7. SP.A.2,** *Also addresses 7.RP.A.2, 7.RP.A.3*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6**

Coherence

Vertical Alignment

Previous
Students identified samples as biased or unbiased and determined whether inferences from the samples were valid. **7.SP.A.1, 7.SP.A.2**

Now
Students use ratio reasoning to make predictions based on data gathered using a valid sampling method. **7.SP.A.2**

Next
Students will understand that taking multiple samples can help them gauge the variation in their predictions. **7.SP.A.2**

Rigor

The Three Pillars of Rigor

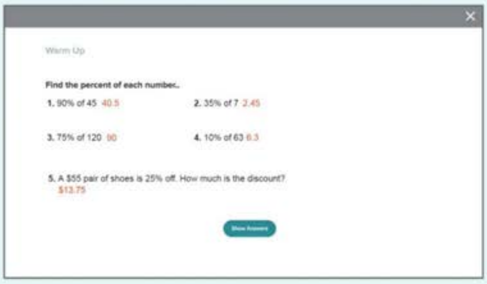
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw upon their knowledge of ratios and percents and sampling from the prior lesson to develop *understanding* about how valid statistical sampling methods can be used to make inferences and predictions about a population of interest. Students build *fluency* in using ratio reasoning and *apply* their knowledge and skills to make predictions about a population based on a random sample of data.

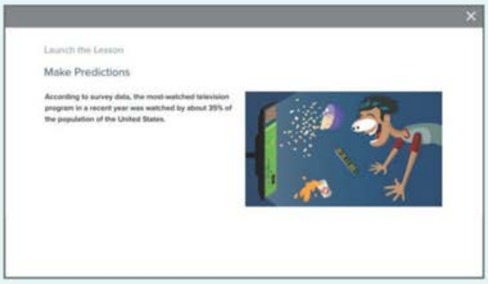
Mathematical Background

If unbiased sampling methods are used to obtain sample data, valid predictions can be made about the population if the prediction is supported by the sample data. Reasoning about ratios and percents can be used to make these inferences.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding the percent of a number (Exercises 1–5)

Answers

1. 40.5 **4.** 6.3

2. 2.45 **5.** $13.75

3. 90

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about making predictions about television program viewing.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The term predict comes from the Latin term *praedict* which means *to make known*, or *declare, beforehand*. What are some real-world contexts in which you might make a prediction of an event? Sample answers: predict the weather forecast for the next day, predict what move your opponent might make in a game of chess or a sports event, predict how long it will take you to complete a task

Learn Make Predictions

Objective

Students will understand that they can make predictions about a population by using information from a survey, provided the survey used an unbiased sample.

Teaching the Mathematical Practices

6 Attend to Precison A s students discuss the *Talk About It!* question on Slide 1, encourage them to think about what types of samples are unbiased and how they can use that method to design a survey.

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to use ratio reasoning to predict the percent of fans inside the stadium that are fans of the green team.

Teaching Notes

SLIDES 1–3

Students previously learned about valid (unbiased) sampling methods and biased sampling methods. If the sampling method is valid, you can make predictions about the population. Present the sporting scenario from the Learn. For each sample, the ratio of fans of the blue team to fans of the green team is 2 : 8. Ask students why this is a *part-to-part ratio*. They learned this term in a prior grade. Then have them write the *part-to-whole ratio* that compares the fans of the blue team to the total number of fans in the sample. Ask students to explain why the ratio 2 : 10 is a *part-to-whole ratio*. They also learned this term in a prior grade.

Present some of the inferences that can be made about the total number of fans attending the event, based on these ratios. Then have students generate other inferences that can be made. Some sample inferences are shown. Ask them which ratio they used to make each inference.

For every fan of the blue team, there are four fans of the green team. (part-to-part ratio of 2 : 8 which is equivalent to 1 : 4)

There are four times as many fans of the green team than of the blue team. (part-to-part ratio of 8 : 2)

Four out of every five fans attending the event are fans of the green team. (part-to-whole ratio of 8 : 10 which is equivalent to 4 : 5)

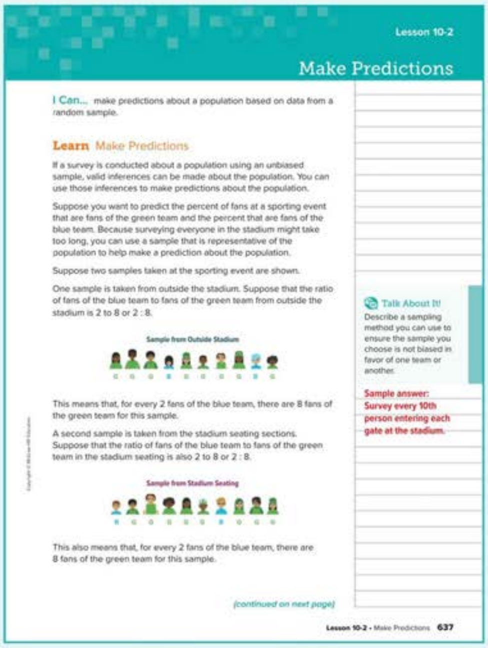
Talk About It!

SLIDE 1

Mathematical Discourse

Describe a sampling method you can use to ensure the sample you choose is not biased in favor of one team or another. Sample answer: Survey every 10th person entering each gate at the stadium.

(continued on next page)


Lesson 10-2

Make Predictions

I Can... make predictions about a population based on data from a random sample.

Learn Make Predictions

If a survey is conducted about a population using an unbiased sample, valid inferences can be made about the population. You can use those inferences to make predictions about the population.

Suppose you want to predict the percent of fans at a sporting event that are fans of the green team and the percent that are fans of the blue team. Because surveying everyone in the stadium might take too long, you can use a sample that is representative of the population to help make a prediction about the population.

Suppose two samples taken at the sporting event are shown.

One sample is taken from outside the stadium. Suppose that the ratio of fans of the blue team to fans of the green team from outside the stadium is 2 to 8 or 2 : 8.

Sample from Outside Stadium

This means that, for every 2 fans of the blue team, there are 8 fans of the green team for this sample.

A second sample is taken from the stadium seating sections. Suppose that the ratio of fans of the blue team to fans of the green team in the stadium seating is also 2 to 8 or 2 : 8.

Sample from Stadium Seating

This also means that, for every 2 fans of the blue team, there are 8 fans of the green team for this sample.

Talk About It! Describe a sampling method you can use to ensure the sample you choose is not biased in favor of one team or another.

Sample answer: Survey every 10th person entering each gate at the stadium.

(continued on next page)

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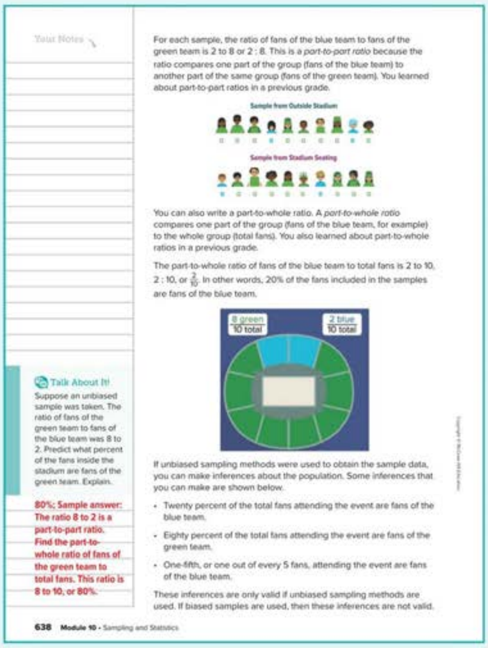
Interactive Presentation


Learn, Make Predictions, Slide 1 of 4



Your Notes

For each sample, the ratio of fans of the blue team to fans of the green team is 2 to 8 or 2 : 8. This is a *part-to-part ratio* because the ratio compares one part of the group (fans of the blue team) to another part of the same group (fans of the green team). You learned about part-to-part ratios in a previous grade.

Sample from Outside Stadium

Sample from Stadium Seating

You can also write a part-to-whole ratio. A *part-to-whole ratio* compares one part of the group (fans of the blue team, for example) to the whole group (total fans). You also learned about part-to-whole ratios in a previous grade.

The part-to-whole ratio of fans of the blue team to total fans is 2 to 10, 2 : 10, or $\frac{2}{10}$. In other words, 20% of the fans included in the samples are fans of the blue team.

If unbiased sampling methods were used to obtain the sample data, you can make inferences about the population. Some inferences that you can make are shown below.

- Twenty percent of the total fans attending the event are fans of the blue team.
- Eighty percent of the total fans attending the event are fans of the green team.
- One-fifth, or one out of every 5 fans, attending the event are fans of the blue team.

These inferences are only valid if unbiased sampling methods are used. If biased samples are used, then these inferences are not valid.

Talk About It!

Suppose an unbiased sample was taken. The ratio of fans of the green team to fans of the blue team was 8 to 2. Predict what percent of the fans inside the stadium are fans of the green team. Explain.

80%; Sample answer: The ratio 8 to 2 is a part-to-part ratio. Find the part-to-whole ratio of fans of the green team to total fans. This ratio is 8 to 10, or 80%.

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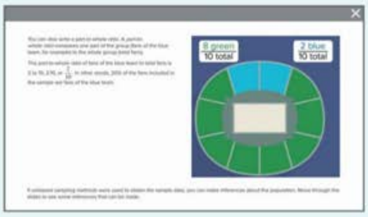
Interactive Presentation



Learn, Make Predictions, Slide 3 of 4

CLICK

On Slide 3, students select from a drop-down menu to determine of which team 20% of total fans are fans.

Learn Make Predictions *(continued)*

Talk About It!

SLIDE 4

Mathematical Discourse

Suppose an unbiased sample was taken. The ratio of fans of the green team to fans of the blue team was 8 to 2. Predict what percent of the fans inside the stadium are fans of the green team. Explain. 80%; Sample answer: The ratio 8 to 2 is a part-to-part ratio. Find the part-to-whole ratio of fans of the green team to total fans. This ratio is 8 to 10, or 80%.

DIFFERENTIATE

Reteaching Activity AL

To help students that may be struggling to understand how to make predictions from samples, explain to students that the statistics describing unbiased samples are expected to be similar to the statistics describing the population. For each of the following unbiased sample statistics, have students make predictions about the population using percents.

1. Population: students in the school

Sample: 3 out of 5 students in a simple random sample have a pet

Prediction: 60% of students in the school have a pet

2. Population: seventh grade students in the United States

Sample: of 1,000 randomly sampled students, 325 participate in school sports

Prediction: 32.5% of seventh grade students in the United States participate in school sports

Example 1 Make Predictions

Objective

Students will use proportional reasoning to make a prediction about a population from a valid sample.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to apply the mathematics they know about ratios and proportional reasoning to make a prediction. They should reason about the ratios $\frac{9}{150}$ and $\frac{30}{500}$ to determine if their prediction is reasonable.

Questions for Mathematical Discourse

SLIDE 2

AL Explain why the information in the table represents the *sample*. Sample answer: The table shows the results of the 150 students who were surveyed. This is the sample.

AL How many students plan to play volleyball? 9 students

OL Compare the number of students who plan to play volleyball to the number of students who plan to play each of the other sports. Sample answer: The number of students who plan to play volleyball is significantly less than the number of students who plan to play the other sports.

OL Why is the total number of students in the ratio not 500? Sample answer: The ratio represents the sample. The total number of students in the sample is 150, not 500.

BL What percent of students surveyed do not plan to play volleyball? 94%

SLIDE 3

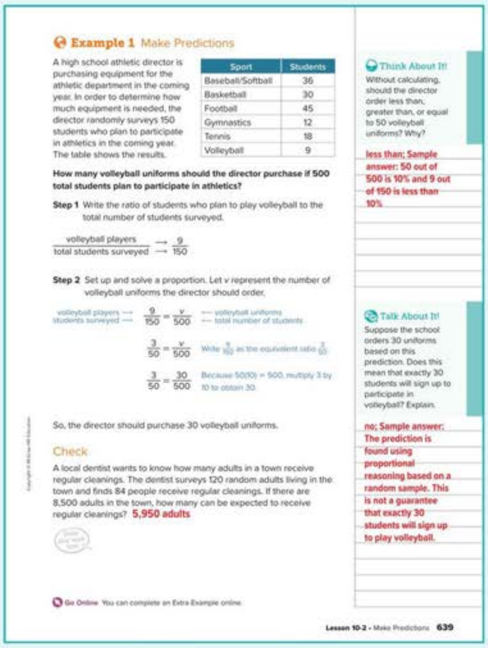
AL How many students in the school plan to participate in athletics? 500 students

OL Explain how to estimate the solution. Sample answer: 150 is less than one-third of 500, because 150(3) = 450. So, 9 is less than one-third the value of x. Because 9(3) = 27, the value of x must be greater than 27.

BL How can you solve this problem another way? Sample answer: Find the percent of students who plan to play volleyball and then multiply the percent expressed as a decimal by 500.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Make Predictions

A high school athletic director is purchasing equipment for the athletic department in the coming year. In order to determine how much equipment is needed, the director randomly surveys 150 students who plan to participate in athletics in the coming year. The table shows the results.

Sport	Students
Baseball/Softball	36
Basketball	30
Football	45
Gymnastics	12
Tennis	18
Volleyball	9

How many volleyball uniforms should the director purchase if 500 total students plan to participate in athletics?

Step 1 Write the ratio of students who plan to play volleyball to the total number of students surveyed.

$\frac{\text{volleyball players}}{\text{total students surveyed}} \rightarrow \frac{9}{150}$

Step 2 Set up and solve a proportion. Let v represent the number of volleyball uniforms the director should order.

volleyball players → / students surveyed → $\frac{9}{150} = \frac{v}{500}$ ← volleyball uniforms / ← total number of students

$\frac{3}{50} = \frac{v}{500}$ Write $\frac{9}{150}$ as the equivalent ratio $\frac{3}{50}$.

$\frac{3}{50} = \frac{30}{500}$ Because 50(10) = 500, multiply 3 by 10 to obtain 30.

So, the director should purchase 30 volleyball uniforms.

Check

A local dentist wants to know how many adults in a town receive regular cleanings. The dentist surveys 120 random adults living in the town and finds 84 people receive regular cleanings. If there are 8,500 adults in the town, how many can be expected to receive regular cleanings? 5,950 adults

Go Online You can complete an Extra Example online.

Think About It! Without calculating, should the director order less than, greater than, or equal to 50 volleyball uniforms? Why?

less than; Sample answer: 50 out of 500 is 10% and 9 out of 150 is less than 10%.

Talk About It! Suppose the school orders 30 uniforms based on this prediction. Does this mean that exactly 30 students will sign up to participate in volleyball? Explain.

no; Sample answer: The prediction is found using proportional reasoning based on a random sample. This is not a guarantee that exactly 30 students will sign up to play volleyball.

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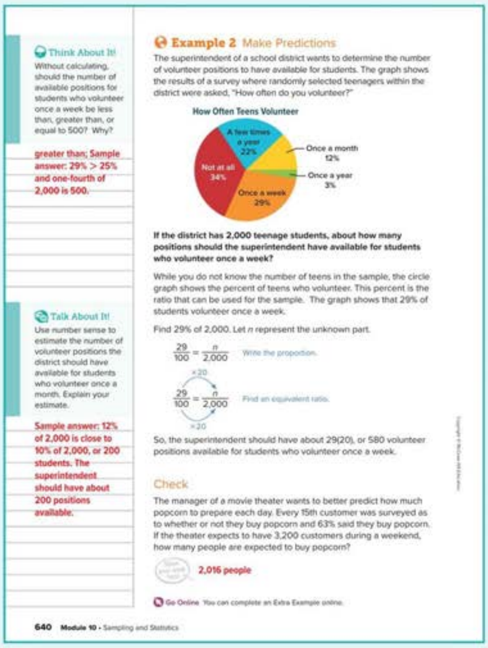
Interactive Presentation

Example 1, Make Predictions, Slide 2 of 5

DRAG & DROP On Slide 2, students drag numbers to create the ratio of participants from the sample.

TYPE On Slide 3, students determine the number of volleyball uniforms that should be purchased.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It! Without calculating, should the number of available positions for students who volunteer once a week be less than, greater than, or equal to 500? Why?

greater than; Sample answer: 29% > 25% and one-fourth of 2,000 is 500.

Talk About It! Use number sense to estimate the number of volunteer positions the district should have available for students who volunteer once a month. Explain your estimate.

Sample answer: 12% of 2,000 is close to 10% of 2,000, or 200 students. The superintendent should have about 200 positions available.

Example 2 Make Predictions

The superintendent of a school district wants to determine the number of volunteer positions to have available for students. The graph shows the results of a survey where randomly selected teenagers within the district were asked, "How often do you volunteer?"

How Often Teens Volunteer

If the district has 2,000 teenage students, about how many positions should the superintendent have available for students who volunteer once a week?

While you do not know the number of teens in the sample, the circle graph shows the percent of teens who volunteer. This percent is the ratio that can be used for the sample. The graph shows that 29% of students volunteer once a week.

Find 29% of 2,000. Let n represent the unknown part.

$\frac{29}{100} = \frac{n}{2{,}000}$ Write the proportion.

$\frac{29}{100} = \frac{n}{2{,}000}$ (×20) Find an equivalent ratio.

So, the superintendent should have about 29(20), or 580 volunteer positions available for students who volunteer once a week.

Check

The manager of a movie theater wants to better predict how much popcorn to prepare each day. Every 15th customer was surveyed as to whether or not they buy popcorn and 63% said they buy popcorn. If the theater expects to have 3,200 customers during a weekend, how many people are expected to buy popcorn?

2,016 people

Go Online You can complete an Extra Example online.

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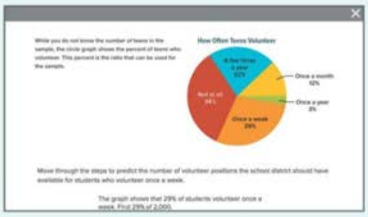
Interactive Presentation



Example 2, Make Predictions, Slide 2 of 4

CLICK On Slide 2, students move through the steps to predict the number of positions the district should have available.

TYPE On Slide 2, students determine the number of volunteer positions the school should have available.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

7.SP.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Make Predictions

Objective

Students will use proportional reasoning to make a prediction about a population from a valid sample.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students should be able to understand that the percent given in the circle graph for the teens who volunteer once a week (29%) represents the ratio of the sample. Have them use proportional reasoning to generate an equivalent ratio that represents a reasonable prediction.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use estimation as a strategy to make a reasonable prediction.

Questions for Mathematical Discourse

SLIDE 2

AL Do you know the number of teens who participated in the survey? Do you need to know this number? Explain. no; Sample answer: I do not know the number of teens who participated in the survey, but I do not need to know this number because I am given the ratio of the sample, expressed as a percent.

OL Describe the ratio of the sample for the teens who volunteer once a week. Sample answer: The ratio is expressed as a percent, 29%. In other words, the ratio is 29 to 100.

OL Explain how to solve this problem mentally. Sample answer: Because 100 multiplied by 20 is 2,000, multiply 29 by 20 to obtain 580. So, 580 students can be expected to volunteer once a week out of 2,000 total students.

BL Based on a population of 2,000 students, how many times might students be expected to volunteer either once a month for an entire year? Sample answer: 12% of 2,000, or 240, students are expected to volunteer once a month. This means that there are expected to be 240(12), or 2,880 times that students will volunteer during an entire year (12 months).

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Profit

Objective

Students will come up with their own strategy to solve an application problem involving using surveys to predict profit.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the ratio of jeans to total responses?
- How many pairs of jeans are expected to be sold?
- What is the expected profit from jeans?
- How can you continue this process for each type of pants?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Profit

A store sells 3 types of pants: jeans, capris, and athletic pants. The store employees randomly survey every 10th customer about their preferred type of pants. A total of 50 customers are surveyed. Their responses are shown in the table.

Type of Pants	Survey Response Frequency	Profit per Pair Sold ($)
Jeans	27	9.00
Capris	9	10.50
Athletic Pants	14	8.25

A total of 1,500 pairs of pants are expected to be sold in one month. The store manager uses the results of the survey to determine how many of each type are expected to be sold that month. What is the profit the store manager can expect to make?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$13,590; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! About what percent of the profit is expected to come from jeans?

about 54%

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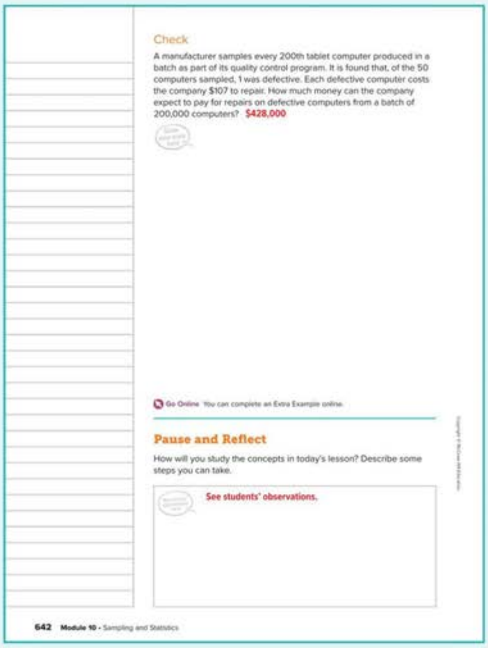
Interactive Presentation

Apply, Profit

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

A manufacturer samples every 200th tablet computer produced in a batch as part of its quality control program. It is found that, of the 50 computers sampled, 1 was defective. Each defective computer costs the company \$107 to repair. How much money can the company expect to pay for repairs on defective computers from a batch of 200,000 computers? \$428,000

Go Online You can complete an Extra Example online.

Pause and Reflect

How will you study the concepts in today's lesson? Describe some steps you can take.

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you use a sample to gain information about a population?

In this lesson, students learned how to make predictions about a population based on data from a random sample. Encourage them to discuss with a partner how their understanding of ratios, proportions, and percent can help them make these predictions.

Exit Ticket

Refer to the Exit ticket slide. In a recent year, a television network randomly surveyed 800 viewers in the United States and reported that 264 viewers had watched the most-watched television program at least once that year. Suppose the following year, there are approximately 320 million people in the United States. About how many people can be expected to watch the same television program at least once? Write a mathematical argument that can be used to defend your solution.

105,600,000 viewers; Sample answer: Write a ratio of program watchers to viewers surveyed: $\frac{264}{800}$. Next, set up a proportion: $\frac{264}{800} = \frac{x}{320{,}000{,}000}$. Finally, solve the proportion: $x = 105{,}600{,}000$ viewers.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–7 odd, 9–12
- ALEKS Collecting Data

IF students score 66-89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–6, 8, 10
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Collecting Data

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Collecting Data

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Independent Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

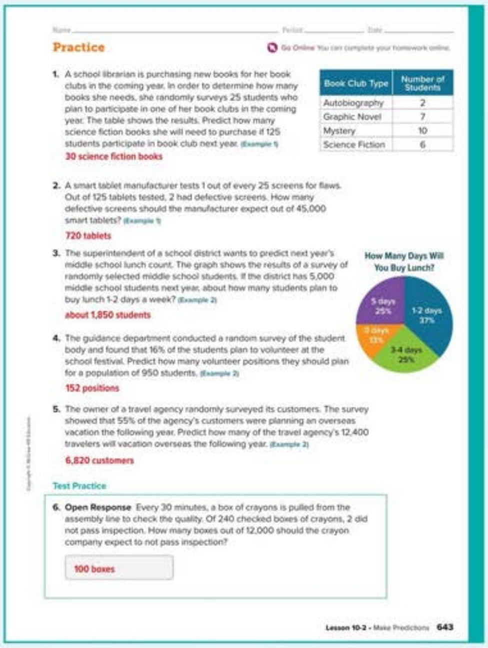
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	use proportional reasoning to make predictions about a population from a valid sample	1–5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving making predictions	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

Some students may incorrectly set up a proportion to find a missing value. In Exercise 2, students may use the testing ratio of 1 to 25 rather than the flaw ratio of 2 to 125 to make a prediction. Encourage students to carefully read each problem to identify what information is presented. The testing ratio indicates the sampling method was a systematic random sample; 1 out of every 25 screens was tested. To find the expected number of *defective* screens, students must use the ratio of *defective* screens.


Name ______ Period ______ Date ______

Practice Go Online You can complete your homework online.

1. A school librarian is purchasing new books for her book clubs in the coming year. In order to determine how many books she needs, she randomly surveys 25 students who plan to participate in one of her book clubs in the coming year. The table shows the results. Predict how many science fiction books she will need to purchase if 125 students participate in book club next year. (Example 1)
30 science fiction books

Book Club Type	Number of Students
Autobiography	2
Graphic Novel	7
Mystery	10
Science Fiction	6

2. A smart tablet manufacturer tests 1 out of every 25 screens for flaws. Out of 125 tablets tested, 2 had defective screens. How many defective screens should the manufacturer expect out of 45,000 smart tablets? (Example 1)
720 tablets

3. The superintendent of a school district wants to predict next year's middle school lunch count. The graph shows the results of a survey of randomly selected middle school students. If the district has 5,000 middle school students next year, about how many students plan to buy lunch 1-2 days a week? (Example 2)
about 1,850 students

4. The guidance department conducted a random survey of the student body and found that 16% of the students plan to volunteer at the school festival. Predict how many volunteer positions they should plan for a population of 950 students. (Example 2)
152 positions

5. The owner of a travel agency randomly surveyed its customers. The survey showed that 55% of the agency's customers were planning an overseas vacation the following year. Predict how many of the travel agency's 12,400 travelers will vacation overseas the following year. (Example 2)
6,820 customers

Test Practice

6. **Open Response** Every 30 minutes, a box of crayons is pulled from the assembly line to check the quality. Of 240 checked boxes of crayons, 2 did not pass inspection. How many boxes out of 12,000 should the crayon company expect to not pass inspection?
100 boxes

Lesson 10-2 • Make Predictions 643

3 REFLECT AND PRACTICE

7.SP.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Apply *indicates multi-step problem

*7. A bike shop surveys every 20th customer about future bike purchases. The responses of 50 customers are shown in the table. If the store uses the results of the survey to determine how many of each type of bike are purchased in an order of 600, how much profit can the store expect to make on comfort bikes?

Bicycle Type	Survey Response	Profit per Bike Sold ($)
Mountain	11	87.98
Touring	8	66.45
Comfort	9	32.50
Youth	19	34.50
Road	3	29.95

$3,510

*8. For an upcoming field trip to the science center, the school will allow students to select one extra activity. The school surveys a random sample of 25 students to determine about how many tickets of each kind they will need to buy. If there are 1,200 students going on the field trip, how much should the school expect to spend on all the activities?

Activity Type	Survey Response	Cost ($)
Movie	14	1.55
Planetarium	7	1.05
Backstage Tour	4	1.10

$1,605.60

Higher-Order Thinking Problems

9. **Create** Write and solve a real-world problem where you use survey results to make a prediction.

Sample answer: A random survey of high school students with jobs were asked whether they saved some of the money they earned. 82% of the students said they saved some money. Out of 340 students, predict how many would save some of their earnings; about 279 students

10. **Find the Error** A student was solving the problem below and found the answer to be 92,500 customers. Find the student's error and correct it.

An unbiased survey showed that 74% of a pet supply's online customers spent at least $100 on their pets each year. Predict how many of the 125,000 online customers will spend less than $100 on their pets next year.

Sample answer: The student found the number of customers who spent over $100 instead of less than $100. The correct answer is 32,500 customers.

11. **Justify Conclusions** Determine if the following statement is *true* or *false*. Explain.

Survey results can always be used to make predictions.

false; The survey's sample must be unbiased.

12. **Reason Abstractly** When making predictions from valid survey results, are the predictions exact answers or estimates? Explain your reasoning.

estimates; Sample answer: The predictions are based on what is expected but the actual results may vary.

644 Module 10 • Sampling and Statistics

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students find and correct a student's error.

In Exercise 11, students determine if survey results can always be used to make predictions, and justify their conclusion.

2 Reason Abstractly and Quantitatively In Exercise 12, students use reasoning to determine whether predictions are exact or estimates, and explain their response.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 7 Have students work together to prepare a brief demonstration that illustrates why this problem requires multiple steps to solve. For example, before they can determine profit, they must first predict how many comfort bikes will be purchased. Have each pair or group of students present their response to the class.

Listen and ask clarifying questions.

Use with Exercises 11–12 Have students work in pairs. Have students individually read Exercise 11 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 12.

Lesson 10-3

Generate Multiple Samples

7.SP.A.2

LESSON GOAL

Students will understand that taking multiple samples can help them gauge the variation in their predictions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Generate Multiple Samples

Learn: Analyze Means of Multiple Samples
Example 1: Analyze Means of Multiple Samples

Explore: Sample Size in Multiple Samples

Apply: Animal Science

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

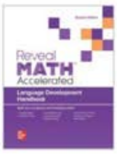
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 60 of the *Language Development Handbook* to help your students build mathematical language related to taking multiple samples.



ELL You can use the tips and suggestions on page T60 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Statistics and Probability

Supporting Cluster(s): In this lesson, students address supporting cluster **7.SP.A** by generating and analyzing the results of data obtained from multiple samples.

Standards for Mathematical Content: **7.SP.A.2,** *Also addresses 7.RP.A.2*

Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students used ratio reasoning to make predictions based on data gathered using a valid sampling method. **7.SP.A.2**

Now
Students understand that taking multiple samples can help them gauge the variation in their predictions. **7.SP.A.2**

Next
Students will make comparative inferences about two populations based on the data from random samples. **7.SP.B.4**

Rigor

The Three Pillars of Rigor

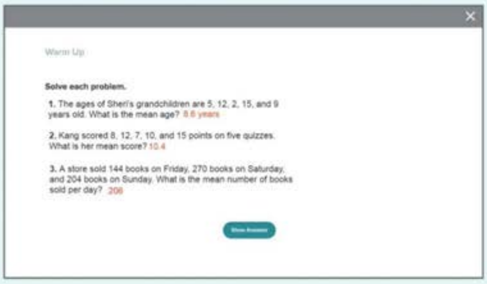
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw upon their knowledge of sampling, the mean, and the mean absolute deviation to develop an *understanding* that taking multiple samples can help them gauge the variation in their predictions. Students build *fluency* in calculating the mean and mean absolute deviation to describe the variability in a sample distribution. They *apply* their *understanding* of multiple samples and variability to make reasonable estimates about a population mean.

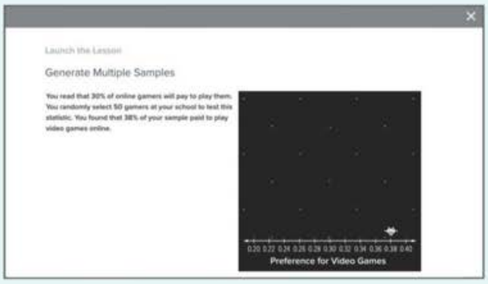
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- summarizing numerical data using the mean (Exercises 1–3)

Answers

1. 8.6 years

2. 10.4

3. 206

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the percentage of online gamers who pay to play the games.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What does the prefix *vari-* mean? What are some other terms that begin with this prefix? Sample answer: The prefix *vari-* means variation or difference; variation, variety, various.

Explore Generate Multiple Samples

Objective

Students will explore how taking multiple samples can help them when making inferences about a population.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with three random samples of words from a dictionary. Throughout this activity, students will use the samples to make inferences about the frequency of vowels for all words in the dictionary.

Inquiry Question

How can taking multiple samples help you when making inferences about a population? Sample answer: It is important to analyze multiple samples of data because all samples have the possibility of showing different results than those in the population. By looking at more than one sample, you can see how the samples vary. If there is low variation among the samples, you can be more confident in your inference.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

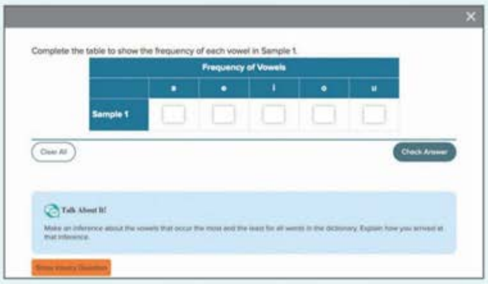
Mathematical Discourse

Make an inference about the vowels that occur the most and the least for all words in the dictionary. Explain how you arrived at that inference. See students' responses.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 9

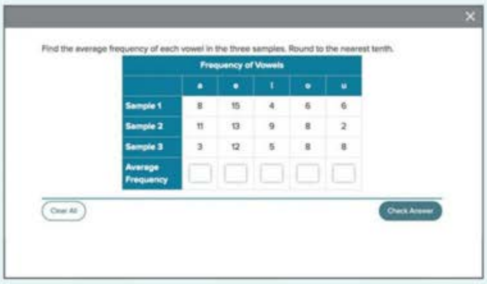


Explore, Slide 3 of 9

TYPE

On Slides 3 through 5, students complete tables to show vowel frequencies in each sample.

Interactive Presentation



Explore, Slide 7 of 9

TYPE

On Slide 7, students complete a table to show the average frequency of each vowel.

TYPE

On Slide 8, students explain what they could do so that their results more closely match the actual occurrences.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Generate Multiple Samples *(continued)*

Teaching the Mathematical Practices

8 Look For and Express Regularity in Repeated Reasoning
Encourage students to look for patterns among the frequency of vowels they note for each sample in the activity, in order to make a conjecture about the frequency of vowels in the English language.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 8 are shown.

Talk About It!

SLIDE 8

Mathematical Discourse

The number of words in the English Language is approximately 1,025,110. The vowel that occurs least often in the English language is the Letter *u*. What are some reasons why the results from the Explore didn't clearly indicate this? Sample answer: The letter *u* was the least frequently occurring vowel in Sample 2 and was never the most frequently occurring vowel in a sample. If more samples were taken, the letter *u* could appear to be the least frequent vowel in many of the samples.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Analyze Means of Multiple Samples

Objective

Students will understand that, by analyzing the means of multiple samples, they can gain more insight into the true mean of the population.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 1, encourage them to be able to clearly and precisely explain how to find the mean of a set of data.

7 Look For and Make Use of Structure As students discuss the *Talk About It!* questions, be sure they understand the difference between using multiple samples as opposed to one sample when making inferences.

Teaching Notes

SLIDES 1-2

Be sure students understand the difference between a *sample mean* and a *population mean*. A *sample mean* is the mean of one sample. The *population mean* is the mean of the entire population being studied. This is sometimes called the *true mean* or the *actual mean*. Just because a sample was collected using a valid sampling method, the sample mean is rarely equal to the population mean, but it should be close.

Have students watch the animation to see how collecting multiple samples of a given size can help them determine how "far off" a sample mean might be from the actual, population mean. Be sure to stress that each data value in the graph represents the mean of one sample. The first sample that was collected had a mean of 5.3 letters per word. After collecting multiple samples, the population mean is around 4.8 letters per word. Be sure students understand that collecting multiple samples helps them to see that their first sample mean was not very close to the population mean. The point of taking multiple samples is to assess the variation, so that you know how "far off" the mean of a sample you collected might be.

Go Online Have students watch the animation on Slide 2. The animation illustrates how collecting multiple samples of a given size can help them determine how "far off" a sample mean might be from the actual population mean.

Talk About It!

SLIDE 1

Mathematical Discourse

Explain to a partner how to find the mean of a set of data. You learned about the mean in a previous grade. Sample answer: To find the mean, find the sum of the data values. Then divide by the number of data values. The mean is also called the average.

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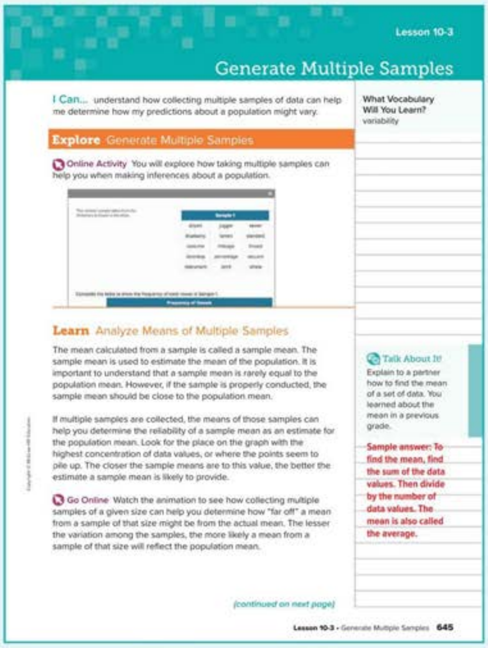

Lesson 10-3

Generate Multiple Samples

I Can... understand how collecting multiple samples of data can help me determine how my predictions about a population might vary.

What Vocabulary Will You Learn?
variability

Explore Generate Multiple Samples

Online Activity You will explore how taking multiple samples can help you when making inferences about a population.

Learn Analyze Means of Multiple Samples

The mean calculated from a sample is called a sample mean. The sample mean is used to estimate the mean of the population. It is important to understand that a sample mean is rarely equal to the population mean. However, if the sample is properly conducted, the sample mean should be close to the population mean.

If multiple samples are collected, the means of those samples can help you determine the reliability of a sample mean as an estimate for the population mean. Look for the place on the graph with the highest concentration of data values, or where the points seem to pile up. The closer the sample means are to this value, the better the estimate a sample mean is likely to provide.

Go Online Watch the animation to see how collecting multiple samples of a given size can help you determine how "far off" a mean from a sample of that size might be from the actual mean. The lesser the variation among the samples, the more likely a mean from a sample of that size will reflect the population mean.

Talk About It! Explain to a partner how to find the mean of a set of data. You learned about the mean in a previous grade.

Sample answer: To find the mean, find the sum of the data values. Then divide by the number of data values. The mean is also called the average.

(continued on next page)

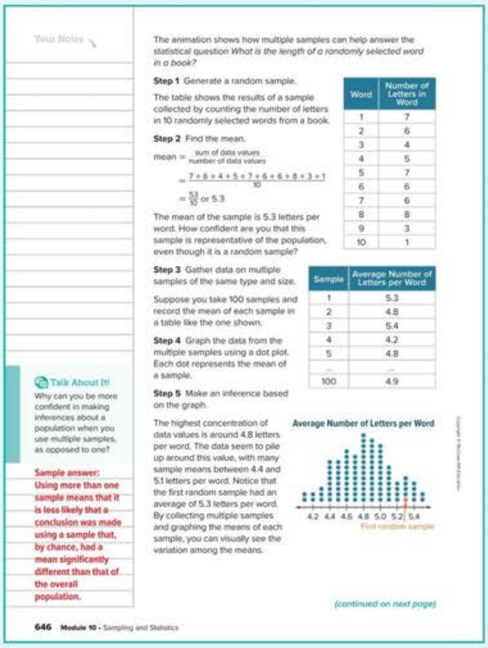
Lesson 10-3 • Generate Multiple Samples 645

Interactive Presentation

Learn, Analyze Means of Multiple Samples, Slide 1 of 9

WATCH

On Slide 2, students watch an animation that explains how to analyze the means of multiple samples.



Your Notes

The animation shows how multiple samples can help answer the statistical question *What is the length of a randomly selected word in a book?*

Step 1 Generate a random sample.

The table shows the results of a sample collected by counting the number of letters in 10 randomly selected words from a book.

Word	Number of Letters in Word
1	7
2	6
3	4
4	5
5	7
6	6
7	6
8	8
9	3
10	1

Step 2 Find the mean.

$$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$
$$= \frac{7+6+4+5+7+6+6+8+3+1}{10}$$
$$= \frac{53}{10} \text{ or } 5.3$$

The mean of the sample is 5.3 letters per word. How confident are you that this sample is representative of the population, even though it is a random sample?

Step 3 Gather data on multiple samples of the same type and size.

Suppose you take 100 samples and record the mean of each sample in a table like the one shown.

Sample	Average Number of Letters per Word
1	5.3
2	4.8
3	5.4
4	4.2
5	4.8
...	...
100	4.9

Step 4 Graph the data from the multiple samples using a dot plot. Each dot represents the mean of a sample.

Step 5 Make an inference based on the graph.

The highest concentration of data values is around 4.8 letters per word. The data seem to pile up around this value, with many sample means between 4.4 and 5.1 letters per word. Notice that the first random sample had an average of 5.3 letters per word. By collecting multiple samples and graphing the means of each sample, you can visually see the variation among the means.

Talk About It!

Why can you be more confident in making inferences about a population when you use multiple samples, as opposed to one?

Sample answer: Using more than one sample means that it is less likely that a conclusion was made using a sample that, by chance, had a mean significantly different than that of the overall population.

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Interactive Presentation

Learn, Analyze Means of Multiple Samples, Slide 2 of 9

Learn Analyze Means of Multiple Samples *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

Why can you be more confident in making inferences about a population when you use multiple samples, as opposed to one? Sample answer: Using more than one sample means that it is less likely that a conclusion was made using a sample that, by chance, had a mean significantly different than that of the overall population.

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DIFFERENTIATE

Enrichment Activity BL

Have students work with a partner to study the distribution from the animation. Even though the population mean is around 4.8 letters per word, it is not uncommon to see sample means as low as 4.4 letters per word or as high as 5.1 letters per word. The data are largely clustered between 4.4 and 5.1 letters per word. Ask them to work together to respond to these questions.

- How “far off” are the sample means of 4.4 letters per word? How did you determine this? 0.4 letters per word; Find the distance from 4.4 to 4.8.
- How “far off” are the sample means of 5.1 letters per word? How did you determine this? 0.3 letters per word; Find the distance from 5.1 to 4.8.
- Suppose you collected an additional random sample of the same size. Describe the mean of that sample that you should be able to expect. Construct an argument to justify your response. Sample answer: I would expect the mean of that sample to be within 0.3-0.4 letters per word of 4.8 letters per word, so somewhere between 4.4 and 5.1 letters per word.

Learn Analyze Means of Multiple Samples *(continued)*

Teaching Notes

SLIDES 4-5

Students will learn the term *variability* and view examples of distributions showing high, low, and no variability. Point out to students that there is usually variability in samples, even if the samples are random. Another term for variability that is often used is *spread*, such as the spread of the data.

Students may have trouble remembering that each data point on a graph of multiple samples represents the mean of each sample taken. You may wish to use one of the dot plots shown and have a volunteer point to each data point (dot) and say aloud “This point represents the mean of one of the samples.” Repeat for other data points.

You may wish to ask students if they can make any inferences about the variability of a single sample, if the graph shows a distribution of multiple samples. They should note that if the graph shows a distribution of multiple samples, they can only make inferences about the variability of the means of the samples. They do not know the variability of the data within each sample.

Talk About It!

SLIDE 6

Mathematical Discourse

How can you know the mean is 25 without calculating it? How can you know the mean absolute deviation is 0 without calculating it? Sample answer: Each data value is the same, so the mean value will be the same as each data value. Because the data do not vary at all, there is no variability. This means the MAD is 0.

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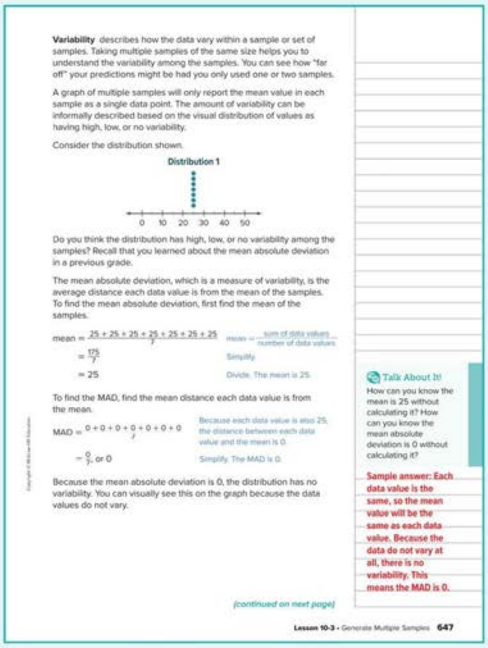

Variability describes how the data vary within a sample or set of samples. Taking multiple samples of the same size helps you to understand the variability among the samples. You can see how “far off” your predictions might be had you only used one or two samples.

A graph of multiple samples will only report the mean value in each sample as a single data point. The amount of variability can be informally described based on the visual distribution of values as having high, low, or no variability.

Consider the distribution shown.

Distribution 1

0 10 20 30 40 50

Do you think the distribution has high, low, or no variability among the samples? Recall that you learned about the mean absolute deviation in a previous grade.

The mean absolute deviation, which is a measure of variability, is the average distance each data value is from the mean of the samples. To find the mean absolute deviation, first find the mean of the samples.

$\text{mean} = \frac{25 + 25 + 25 + 25 + 25 + 25 + 25}{7}$ — mean = $\frac{\text{sum of data values}}{\text{number of data values}}$

$= \frac{175}{7}$ — Simplify.

$= 25$ — Divide. The mean is 25.

To find the MAD, find the mean distance each data value is from the mean.

$\text{MAD} = \frac{0 + 0 + 0 + 0 + 0 + 0 + 0}{7}$ — Because each data value is also 25, the distance between each data value and the mean is 0.

$= \frac{0}{7}$, or 0 — Simplify. The MAD is 0.

Because the mean absolute deviation is 0, the distribution has no variability. You can visually see this on the graph because the data values do not vary.

Talk About It!

How can you know the mean is 25 without calculating it? How can you know the mean absolute deviation is 0 without calculating it?

Sample answer: Each data value is the same, so the mean value will be the same as each data value. Because the data do not vary at all, there is no variability. This means the MAD is 0.

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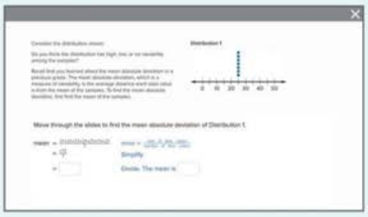
Lesson 10-3 • Generate Multiple Samples 647

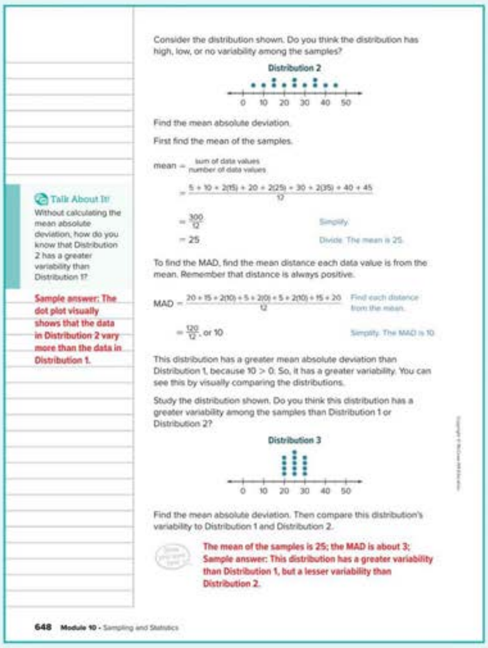
Interactive Presentation



Learn, Analyze Means of Multiple Samples, Slide 5 of 9

TYPE

On Slide 7, students determine the mean of the samples.



Consider the distribution shown. Do you think the distribution has high, low, or no variability among the samples?

Find the mean absolute deviation.

First find the mean of the samples.

$$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

$$= \frac{5 + 10 + 2(15) + 20 + 2(25) + 30 + 2(35) + 40 + 45}{12}$$

$$= \frac{300}{12} \quad \text{Simplify.}$$

$$= 25 \quad \text{Divide. The mean is 25.}$$

To find the MAD, find the mean distance each data value is from the mean. Remember that distance is always positive.

$$\text{MAD} = \frac{20 + 15 + 2(10) + 5 + 2(0) + 5 + 2(10) + 15 + 20}{12} \quad \text{Find each distance from the mean.}$$

$$= \frac{120}{12}, \text{ or } 10 \quad \text{Simplify. The MAD is 10.}$$

This distribution has a greater mean absolute deviation than Distribution 1, because 10 > 0. So, it has a greater variability. You can see this by visually comparing the distributions.

Study the distribution shown. Do you think this distribution has a greater variability among the samples than Distribution 1 or Distribution 2?

Find the mean absolute deviation. Then compare this distribution's variability to Distribution 1 and Distribution 2.

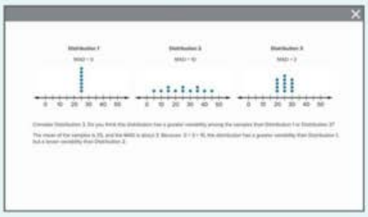
The mean of the samples is 25; the MAD is about 3; Sample answer: This distribution has a greater variability than Distribution 1, but a lesser variability than Distribution 2.

Talk About It! Without calculating the mean absolute deviation, how do you know that Distribution 2 has a greater variability than Distribution 1?

Sample answer: The dot plot visually shows that the data in Distribution 2 vary more than the data in Distribution 1.

648 Module 10 • Sampling and Statistics

Interactive Presentation



Learn, Analyze Means of Multiple Samples, Slide 9 of 9

Learn Analyze Means of Multiple Samples *(continued)*

Teaching Notes

SLIDE 7

In Distribution 2, encourage students to notice how varied the data are. The data values are spread out over a wide range of values. By finding the mean absolute deviation, students should see that variability represented by a greater MAD value than Distribution 1. The greater the mean absolute deviation, the greater the variability of the sample.

SLIDE 9

The values in Distribution 3 are not as spread out as Distribution 2. After calculating the mean and mean absolute deviation, help students to analyze the values and compare them to Distributions 1 and 2. Distribution 3 has a greater variability (and MAD) than Distribution 1, but a lesser variability (and MAD) than Distribution 3.

Talk About It!

SLIDE 8

Mathematical Discourse

Without calculating the mean absolute deviation, how do you know that Distribution 2 has a greater variability than Distribution 1? Sample answer: The dot plot visually shows that the data in Distribution 2 vary more than the data in Distribution 1.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Analyze Means of Multiple Samples

Objective

Students will analyze the means of multiple samples of data to predict the population mean, and describe the variability of the distribution.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively S tudents should use reasoning about where the highest concentration of data points lie in order to find the value that best represents the data.

As students discuss the *Talk About It!* question on Slide 4, encourage them to justify their response using sound reasoning and mathematical terminology.

7 Look For and Make Use of Structure Encourage students to study the structure of the graph in order to make sense that each data point represents the mean of one of the samples.

Questions for Mathematical Discourse

SLIDE 2

AL Study the structure of the distribution. What does each data point represent? Each data point represents the mean of each sample. There were 24 samples taken, and 24 data points.

AL What does the single data point above the number 8 mean? The mean of this sample of 20 runners is 8 minutes.

AL What do the two data points above the number 8.5 mean? There were two samples that each had a mean of 8.5 minutes.

OL Describe an inference that you cannot make based on this distribution. Explain. Sample answer: The lowest time to run a mile was 5 minutes. I cannot make this inference because the graph only shows the means of each sample, not the individual data values within each sample.

BL If each sample consists of 20 distinct students, how many students overall are represented to some degree in this dot plot? Explain. 480; Sample answer: There are 24 samples. If each sample consists of 20 distinct students, then there are 24(20), or 480 students represented to some degree in the dot plot.

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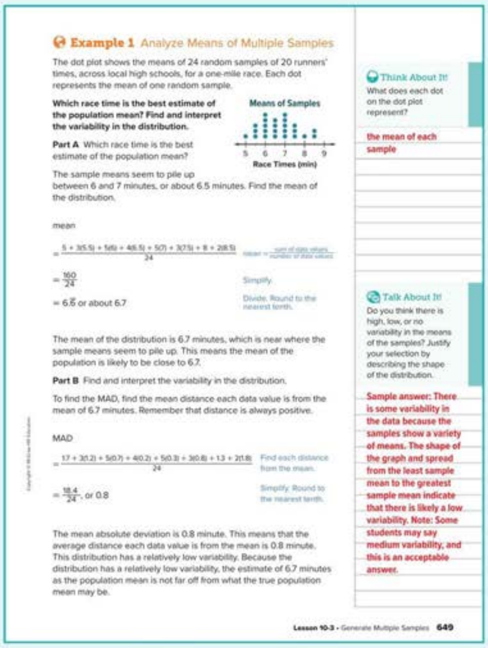


Example 1 Analyze Means of Multiple Samples

The dot plot shows the means of 24 random samples of 20 runners' times, across local high schools, for a one-mile race. Each dot represents the mean of one random sample.

Which race time is the best estimate of the population mean? Find and interpret the variability in the distribution.

Means of Samples

Race Times (min)

Part A Which race time is the best estimate of the population mean?

The sample means seem to pile up between 6 and 7 minutes, or about 6.5 minutes. Find the mean of the distribution.

mean

$= \frac{5 + 3(5.5) + 5(6) + 4(6.5) + 5(7) + 3(7.5) + 8 + 2(8.5)}{24}$ mean = sum of data values / number of data values

$= \frac{160}{24}$ Simplify.

$= 6.\overline{6}$ or about 6.7 Divide. Round to the nearest tenth.

The mean of the distribution is 6.7 minutes, which is near where the sample means seem to pile up. This means the mean of the population is likely to be close to 6.7.

Part B Find and interpret the variability in the distribution.

To find the MAD, find the mean distance each data value is from the mean of 6.7 minutes. Remember that distance is always positive.

MAD

$= \frac{1.7 + 3(1.2) + 5(0.7) + 4(0.2) + 5(0.3) + 3(0.8) + 1.3 + 2(1.8)}{24}$ Find each distance from the mean.

$= \frac{18.4}{24}$, or 0.8 Simplify. Round to the nearest tenth.

The mean absolute deviation is 0.8 minute. This means that the average distance each data value is from the mean is 0.8 minute. This distribution has a relatively low variability. Because the distribution has a relatively low variability, the estimate of 6.7 minutes as the population mean is not far off from what the true population mean may be.

Think About It! What does each dot on the dot plot represent?

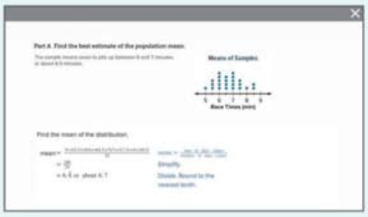
the mean of each sample

Talk About It! Do you think there is high, low, or no variability in the means of the samples? Justify your selection by describing the shape of the distribution.

Sample answer: There is some variability in the data because the samples show a variety of means. The shape of the graph and spread from the least sample mean to the greatest sample mean indicate that there is likely a low variability. Note: Some students may say medium variability, and this is an acceptable answer.

Lesson 10-3 • Generate Multiple Samples 649

Interactive Presentation



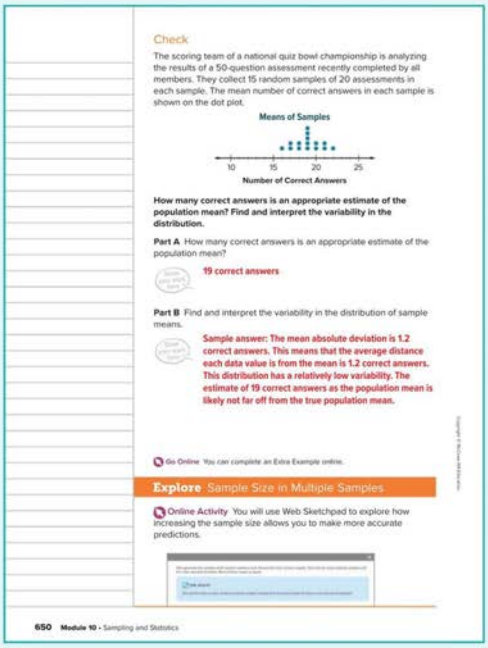
Example 1, Analyze Means of Multiple Samples, Slide 2 of 5

CLICK

On Slide 2, students find the value that best represents the mean in the population.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

The scoring team of a national quiz bowl championship is analyzing the results of a 50-question assessment recently completed by all members. They collect 15 random samples of 20 assessments in each sample. The mean number of correct answers in each sample is shown on the dot plot.

How many correct answers is an appropriate estimate of the population mean? Find and interpret the variability in the distribution.

Part A How many correct answers is an appropriate estimate of the population mean?

19 correct answers

Part B Find and interpret the variability in the distribution of sample means.

Sample answer: The mean absolute deviation is 1.2 correct answers. This means that the average distance each data value is from the mean is 1.2 correct answers. This distribution has a relatively low variability. The estimate of 19 correct answers as the population mean is likely not far off from the true population mean.

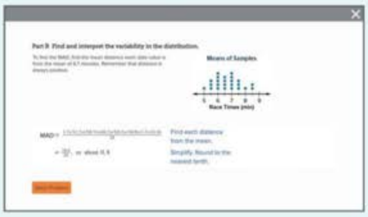
Go Online You can complete an Extra Example online.

Explore Sample Size in Multiple Samples

Online Activity You will use Web Sketchpad to explore how increasing the sample size allows you to make more accurate predictions.

650 Module 10 • Sampling and Statistics

Interactive Presentation



Example 1, Analyze Means of Multiple Samples, Slide 3 of 5

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 1 Analyze Means of Multiple Samples *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL When we say the data vary as low as 5, does this mean the shortest time was 5 minutes? Explain. no; Sample answer: Each data point represents the mean of one sample. When we say the data vary as low as 5, this means that 5 minutes was the least mean of all of the samples. It does not mean that the least individual running time was 5 minutes.

OL Why do you use the absolute value when finding the variability from the center? Sample answer: I need to find the distance from the center to each extreme, and distance is never negative.

OL Suppose each sample consisted of 50 runners' times. Predict how the dot plot would change. Sample answer: The dots that represent the sample means would cluster closer to the population mean, but maybe not exactly *at* the population mean.

BL Why is finding the variability from the center helpful? Sample answer: It is one way to describe variability. Knowing that the distance from the center is similar on both sides of the center indicates that the data is roughly symmetrical.

BL Suppose each sample consisted of 50 runners' times. Would you expect your estimate for the population mean to be closer, or further away, than the estimate using samples of size 20? Explain. Sample answer: The greater the sample size, the greater the confidence that the estimate of the population mean is close to the true mean.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Sample Size in Multiple Samples

Objective

Students will use Web Sketchpad to explore how increasing the sample size allows you to make more accurate predictions.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a random number generator in Web Sketchpad to generate different-sized samples of random numbers. Throughout this activity, students will observe how increasing the sample size is likely to reduce the amount of variability among the samples.

Inquiry Question

How does increasing the sample size allow you to make more accurate predictions? Sample answer: As the sample size within samples increases the amount of variability between the results of those samples is expected to decrease. This will allow you to make better predictions about a population if you use a large sample size.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

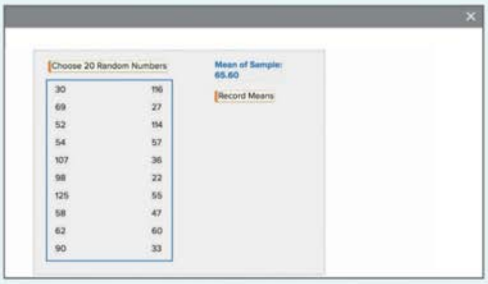
How did the total number of values in these samples change from the total number of values in the first set of samples? Sample answer: The mean of each sample is now the mean from 20 randomly selected values rather than the mean of 5 randomly selected values as in the first set of samples.

What do you notice about the MAD of these samples, compared to the MAD of the first set of samples? What does this mean? Sample answer: The MAD of these samples is less than the MAD of the first set of samples. It likely means that the variability has decreased.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 3 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how increasing the sample size allows you to make better predictions.

Interactive Presentation

Explore, Slide 4 of 6

TYPE

On Slide 5, students complete a table to record the results of experiments.

CLICK

On Slide 5, students select from a drop-down menu to indicate which sample size best represents the mean and MAD of the population.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Sample Size in Multiple Samples *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Web Sketchpad random number generator to generate samples of randomly selected numbers and calculate the mean of the samples.

8 Look For and Express Regularity in Repeated Reasoning Encourage students to look for any patterns in these values based on the sample size.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

How did the total number of values in these samples change from the previous samples? Do you think the mean of each sample is more, or less, representative of the total population of numbers than the previous sets of numbers? Explain. Sample answer: The mean of each sample contains 50 randomly generated values, a greater number of values than previous sets. The mean of the sample should be more representative of the mean of the population than previous sets. When the sample contains fewer values it is more likely to be affected by values that are much lower or higher than the true mean.

What do you notice about the MAD of these samples, compared to when the sample size was 5 or 20 numbers? What does this mean? Sample answer: The MAD of these samples is less than the MADs of the first two sets of samples. It likely means that the variability has decreased.

Apply Animal Science

Objective

Students will come up with their own strategy to solve an application problem involving how to infer manatee weights given data.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

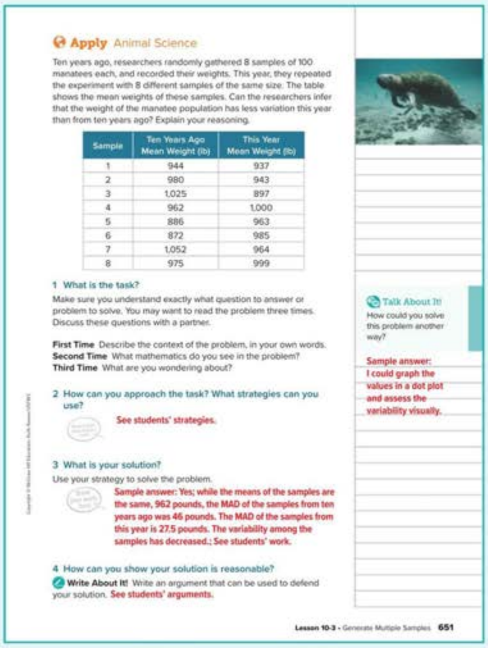
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the mean weight from 10 years ago?
- How can you find the variation for each set of samples?
- What do you notice about the means of each set of samples?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Animal Science

Ten years ago, researchers randomly gathered 8 samples of 100 manatees each, and recorded their weights. This year, they repeated the experiment with 8 different samples of the same size. The table shows the mean weights of these samples. Can the researchers infer that the weight of the manatee population has less variation this year than from ten years ago? Explain your reasoning.

Sample	Ten Years Ago Mean Weight (lb)	This Year Mean Weight (lb)
1	944	937
2	980	943
3	1,025	897
4	962	1,000
5	886	963
6	872	985
7	1,052	964
8	975	999

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Sample answer: Yes; while the means of the samples are the same, 962 pounds, the MAD of the samples from ten years ago was 46 pounds. The MAD of the samples from this year is 27.5 pounds. The variability among the samples has decreased.; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution. See students' arguments.

Talk About It!
How could you solve this problem another way?

Sample answer: I could graph the values in a dot plot and assess the variability visually.

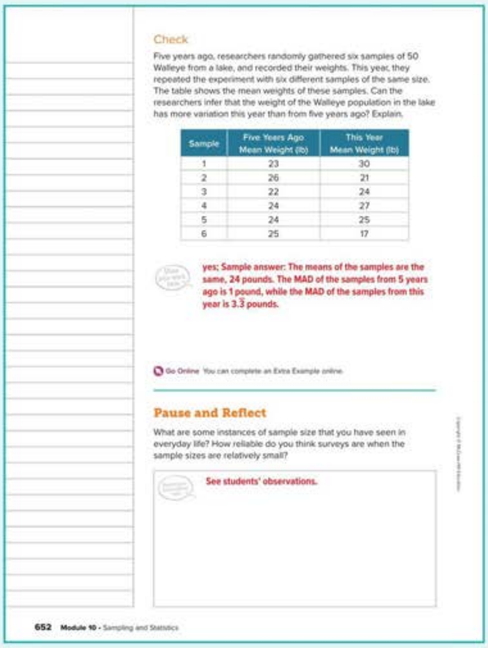
Lesson 10-3 • Generate Multiple Samples 651

Interactive Presentation

Apply, Animal Science

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Five years ago, researchers randomly gathered six samples of 50 Walleye from a lake, and recorded their weights. This year, they repeated the experiment with six different samples of the same size. The table shows the mean weights of these samples. Can the researchers infer that the weight of the Walleye population in the lake has more variation this year than from five years ago? Explain.

Sample	Five Years Ago Mean Weight (lb)	This Year Mean Weight (lb)
1	23	30
2	26	21
3	22	24
4	24	27
5	24	25
6	25	17

yes; Sample answer: The means of the samples are the same, 24 pounds. The MAD of the samples from 5 years ago is 1 pound, while the MAD of the samples from this year is $3.\overline{3}$ pounds.

Go Online You can complete an Extra Example online.

Pause and Reflect

What are some instances of sample size that you have seen in everyday life? How reliable do you think surveys are when the sample sizes are relatively small?

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you use a sample to gain information about a population?

In this lesson, students learned how taking multiple samples can help them make more accurate inferences about a population. Encourage them to discuss with a partner how both the number of samples that are taken, and the size of each sample, can affect the accuracy of the inferences they may be able to make about a population.

Exit Ticket

Refer to the Exit Ticket slide. A statewide survey found that 25% of students who are online gamers will pay to play them. You randomly select 50 gamers at your school. Your data showed that 35% of the gamers you surveyed pay to play online games. Describe at least one reason that might explain why these percents are different. Which percent might you trust more? Write a mathematical argument that can be used to defend your solution. Sample answer: The gamers at your school may be more interested in online gaming in general than students statewide. I might trust the statewide percent more because it likely took into consideration multiple schools in the state, not just one school.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 1, 3, 5–7
- ALEKS® Graphs of Data, Measures of Variation

IF students score 66-89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–4, 6
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Mean, Median, and Mode

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Mean, Median, and Mode

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

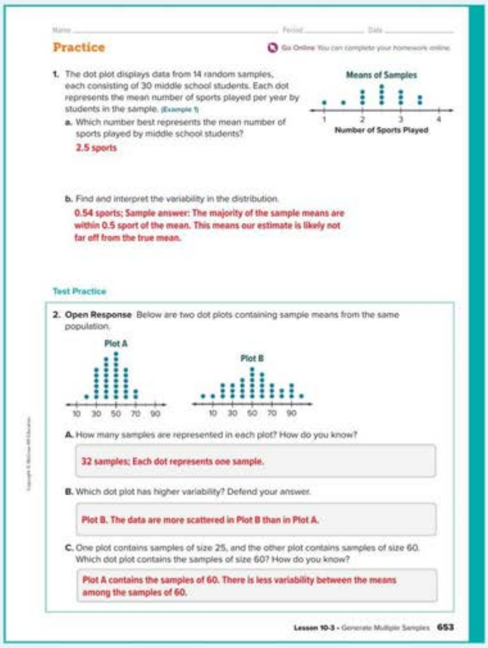
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	analyze multiple samples of data involving means of samples to gauge variation and make predictions	1
2	extend concepts learned in class to apply them in new contexts	2
3	solve application problems involving generating multiple samples	3
3	higher-order and critical thinking skills	4, 5

Common Misconception

Some students may describe the variability of a distribution using only the range of values. In Exercise 1, students may only describe that the means range from 2 to 3.5. Encourage students to include how the variability of the distribution compares to the center by using the mean absolute deviation.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The dot plot displays data from 14 random samples, each consisting of 30 middle school students. Each dot represents the mean number of sports played per year by students in the sample. (Example 1)

a. Which number best represents the mean number of sports played by middle school students?
2.5 sports

b. Find and interpret the variability in the distribution.
0.54 sports; Sample answer: The majority of the sample means are within 0.5 sport of the mean. This means our estimate is likely not far off from the true mean.

Test Practice

2. **Open Response** Below are two dot plots containing sample means from the same population.

A. How many samples are represented in each plot? How do you know?
32 samples; Each dot represents one sample.

B. Which dot plot has higher variability? Defend your answer.
Plot B. The data are more scattered in Plot B than in Plot A.

C. One plot contains samples of size 25, and the other plot contains samples of size 60. Which dot plot contains the samples of size 60? How do you know?
Plot A contains the samples of 60. There is less variability between the means among the samples of 60.

Lesson 10-3 • Generate Multiple Samples 653

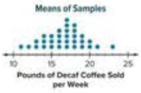
3 REFLECT AND PRACTICE

7.SP.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Apply *Indicates multi-step problem

*3. A large company is trying to determine the mean number of pounds of decaf coffee sold per week in its stores. The dot plot shows the mean pounds of decaf coffee sold per week from 32 samples of 50 stores each.



a. Describe the variability of the dot plot.

The majority of the data are clustered between 14 and 19 pounds.

b. How might the dot plot be different if each of the 32 samples contained data from 200 stores?

Sample answer: The data would be more tightly clustered between 15 and 18 pounds.

c. The company randomly samples 50 of its stores and records the pounds of decaf sold per week for each store. A mean sale of 18 pounds of decaf coffee per week is calculated from this sample. Based on the sample mean of 18 and the variability observed in the dot plot, what range of values could be used to describe the population mean?

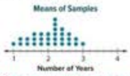
Sample answer: The majority of the data appear to be within 3 pounds of the center, so the company can expect the sample mean of 18 pounds to be within 3 pounds of the population mean. The mean decaf sales for stores in this company is likely to be between 15 and 21 pounds per week.

d. The company samples 200 stores and finds a mean of 17 pounds of decaf coffee sold per week. Based on your answer to Part B, what range of values might describe the mean for all stores in the company? Justify your answer.

Sample answer: Due to the increased sample size, there will be less discrepancy between the sample mean and the population mean. The store might expect to sell between 16 and 18 pounds of decaf coffee.

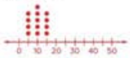
Higher-Order Thinking Problems

4. MP **Find the Error** A student examines the dot plot below and states that it contains samples of size 30. Find the student's mistake and correct it.



Sample answer: The student confused the number of samples with the sample size. There are 30 samples. The size of the samples cannot be determined from the dot plot.

5. Draw a dot pot with low variability. Write an argument to support why your dot plot has low variability.



Sample answer: The mean is 10; the MAD is about 3, which means that the average distance each data value is from the mean is 3.

654 **Module 10** • Sampling and Statistics

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 4, students find and correct a student's error.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 3 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.

Use with Exercise 4 Have students work in groups of 3–4 to solve the problem in Exercise 4. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class.

Lesson 10-4

Compare Two Populations

7.SP.B.4

LESSON GOAL

Students will make comparative inferences about two populations based on the data from random samples.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Shape of Data Distributions
Learn: Compare Two Populations
Example 1: Compare Two Populations
Example 2: Compare Two Populations

Explore: Compare Means of Two Populations

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Select an Appropriate Display, Standard Deviation		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 61 of the *Language Development Handbook* to help your students build mathematical language related to comparing populations.

ELL You can use the tips and suggestions on page T61 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Statistics and Probability
Additional Cluster(s): In this lesson, students address the additional cluster **7.SP.B** by comparing two populations based on the data from two random samples.
Standards for Mathematical Content: **7 .SP.B.4**
Standards for Mathematical Practice: **MP 2, MP3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students understood that taking multiple samples can help them gauge the variation in their predictions.
7.SP.A.2

Now
Students make comparative inferences about two populations based on the data from random samples.
7.SP.B.4

Next
Students will informally assess the degree of visual overlap between two distributions.
7.SP.B.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of how the shape of a data distribution, including symmetry, indicates which measure of center and variation to use to describe the data. They build *fluency* in calculating these measures and *apply* their knowledge and skills to make comparative inferences about two populations based on sample data.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- dividing decimals (Exercises 1–5)

Answers

1. 0.625
2. 3.1
3. 0.15
4. 7.5
5. 1,032

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about comparing two samples of movie running times.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- What does the prefix *a*- mean? What do you think *asymmetric* means? Sample answer: The prefix *a*- means "not". Asymmetric means not having symmetry.
- What does it mean to *distribute* something? Sample answer: to give, share, or deal out
- Based on the meaning of the word *double*, what do you think a *double box plot* might be? Sample answer: Two box plots represented on one number line
- Based on the meaning of the word *double*, what do you think a *double dot plot* might be? Sample answer: Two dot plots represented on one number line

Learn Shape of Data Distributions

Objective

Students will understand which measures of center and variability best represent asymmetric and symmetric distributions of data.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 3, encourage them to use precise mathematical language, such as *distance* and *absolute value* to explain how to calculate these measures. Have them explain to a partner while the partner listens and asks clarifying questions.

Teaching Notes

SLIDES 1–2

Point out that the shape of a graph representing data is called a *distribution*, because it visually shows how the data are *distributed*. Present the terms *symmetric* and *asymmetric*, as they relate to distributions. Students may or may not be familiar with the term *symmetry*. You may wish to have them use a dictionary or the Internet to look up the term. Ask them if they have seen examples of objects that display *symmetry* – such as the face of a person or animal, or a tire on a car.

Have students work with a partner to study the examples of box plots and dot plots (both symmetric and asymmetric) presented in the Learn. Have them discuss how a box plot displaying symmetry is different from a dot plot displaying symmetry. Encourage students to reason about what the lengths of the boxes and whiskers indicate about the data. A longer box or whisker indicates the data are more spread out. If the boxes and whiskers are not all the same length, the data are not evenly distributed. Thus, the distribution is not symmetric.

(continued on next page)

DIFFERENTIATE

Reteaching Activity AL

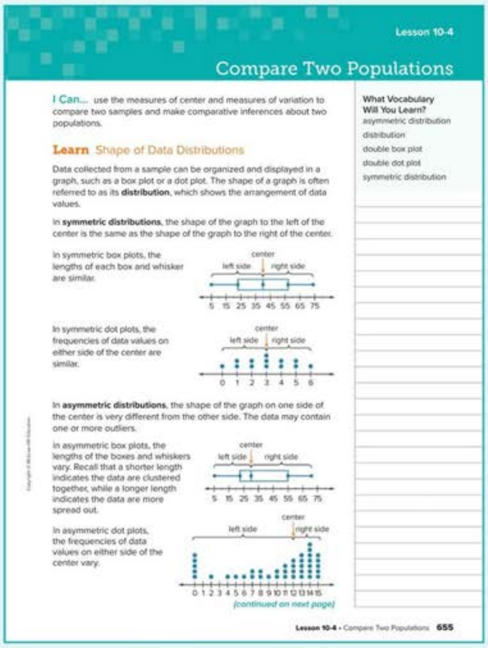
To help students better understand shapes of distributions, explain that they can first identify the center of the distribution. If the data on either side of the center match, the distribution is symmetric. Have students graph the following data sets on a number line and determine if each distribution is symmetric or asymmetric.

2, 3, 5, 3, 4, 4, 2, 1, 3 symmetric

1, 1, 3, 2, 1, 4, 2, 1 asymmetric

15, 11, 11, 9, 13, 7 symmetric

12, 12, 12, 8, 10, 12, 6, 10 asymmetric



Lesson 10-4

Compare Two Populations

I Can... use the measures of center and measures of variation to compare two samples and make comparative inferences about two populations.

Learn Shape of Data Distributions

Data collected from a sample can be organized and displayed in a graph, such as a box plot or a dot plot. The shape of a graph is often referred to as its **distribution**, which shows the arrangement of data values.

In **symmetric distributions**, the shape of the graph to the left of the center is the same as the shape of the graph to the right of the center.

In symmetric box plots, the lengths of each box and whisker are similar.

In symmetric dot plots, the frequencies of data values on either side of the center are similar.

In **asymmetric distributions**, the shape of the graph on one side of the center is very different from the other side. The data may contain one or more outliers.

In asymmetric box plots, the lengths of the boxes and whiskers vary. Recall that a shorter length indicates the data are clustered together, while a longer length indicates the data are more spread out.

In asymmetric dot plots, the frequencies of data values on either side of the center vary.

(continued on next page)

What Vocabulary Will You Learn?
asymmetric distribution
distribution
double box plot
double dot plot
symmetric distribution

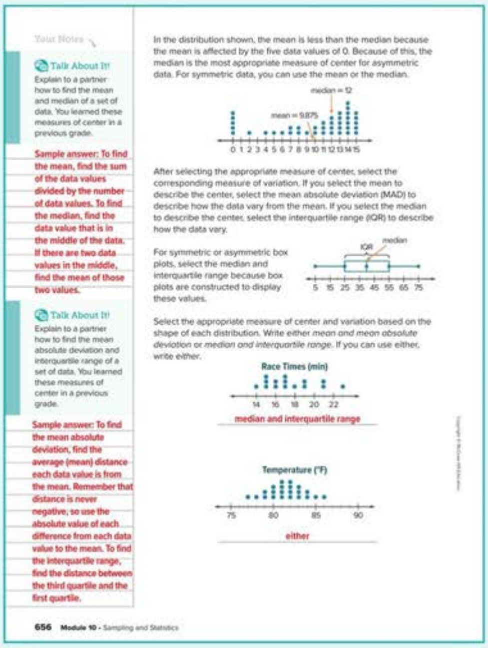
Lesson 10-4 • Compare Two Populations 655

Interactive Presentation

Learn, Shape of Data Distributions, Slide 1 of 4

CLICK

On Slide 1, students compare symmetric distributions in box plots and dotplots.



Your Notes

Talk About It!
Explain to a partner how to find the mean and median of a set of data. You learned these measures of center in a previous grade.

Sample answer: To find the mean, find the sum of the data values divided by the number of data values. To find the median, find the data value that is in the middle of the data. If there are two data values in the middle, find the mean of those two values.

Talk About It!
Explain to a partner how to find the mean absolute deviation and interquartile range of a set of data. You learned these measures of center in a previous grade.

Sample answer: To find the mean absolute deviation, find the average (mean) distance each data value is from the mean. Remember that distance is never negative, so use the absolute value of each difference from each data value to the mean. To find the interquartile range, find the distance between the third quartile and the first quartile.

In the distribution shown, the mean is less than the median because the mean is affected by the five data values of 0. Because of this, the median is the most appropriate measure of center for asymmetric data. For symmetric data, you can use the mean or the median.

After selecting the appropriate measure of center, select the corresponding measure of variation. If you select the mean to describe the center, select the mean absolute deviation (MAD) to describe how the data vary from the mean. If you select the median to describe the center, select the interquartile range (IQR) to describe how the data vary.

For symmetric or asymmetric box plots, select the median and interquartile range because box plots are constructed to display these values.

Select the appropriate measure of center and variation based on the shape of each distribution. Write either *mean and mean absolute deviation* or *median and interquartile range*. If you can use either, write *either*.

median and interquartile range

either

656 Module 10 • Sampling and Statistics

Interactive Presentation

Learn, Shape of Data Distributions, Slide 3 of 4

CLICK

On Slide 4, students select the best measure of center and variability based on the shape of each distribution.

Learn Shape of Data Distributions *(continued)*

Teaching Notes

SLIDES 3–4

Present the asymmetric dot plot with the five data values of 0. To support students' procedural skill and fluency, you may wish to have them calculate the mean and median of the data set. Ask them why the mean is less than the median.

Be sure students understand how analyzing a distribution's symmetry indicates which measure of center and variation should be used to describe the data. You may wish to have them create a graphic organizer or table to help them organize their understanding. A sample table is shown. Have them explain why they should use the median and interquartile range for symmetric box plots – students should be able to reason that box plots are constructed using those values.

	Measure of Center	Measure of Variation
Symmetric Dot Plots	mean	mean absolute deviation
Symmetric Box Plots	median	interquartile range
Asymmetric Dot Plots	median	interquartile range
Asymmetric Box Plots	median	interquartile range

Talk About It!

SLIDE 3

Mathematical Discourse

Explain to a partner how to find the mean absolute deviation and interquartile range of a set of data. You learned these measures of center in a previous grade. Sample answer: To find the mean absolute deviation, find the average (mean) distance each data value is from the mean. Remember that distance is never negative, so use the absolute value of each difference from each data value to the mean. To find the interquartile range, find the distance between the third quartile and the first quartile.

Learn Compare Two Populations

Objective

Students will understand that they can make comparative inferences about two populations by comparing their centers and variations.

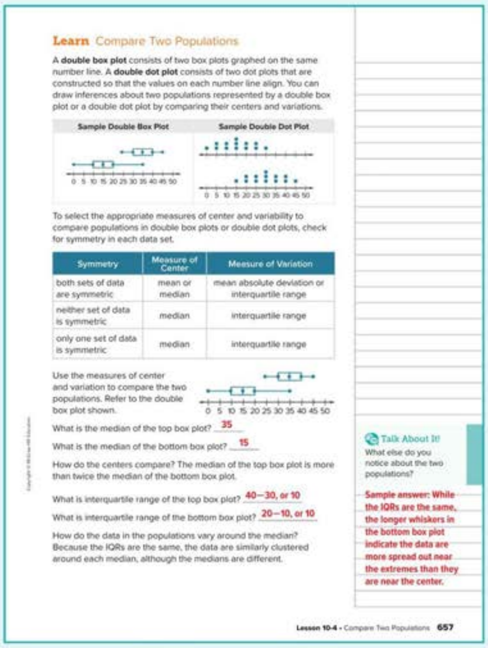
Teaching Notes

SLIDE 1

Students have previously learned how to analyze data presented in a box plot or dot plot. If necessary, remind students that a box plot uses five values to show the distribution of data (minimum, lower quartile, median, upper quartile, and maximum) and a dot plot shows the frequency of the values in a data set. Present the sample double box plot and sample double dot plot. Ask students if they can make any inferences about the two distributions shown in each double plot. For example, one student might say that the median in the top box plot is greater than the median in the bottom box plot. Another student might say that the data are more spread out in the bottom box plot because the whiskers are longer.

SLIDE 2

Have students explore the interactive tool to show which measure of center and variation to use when both data sets are symmetric, when neither data set is symmetric, or when only one data set is symmetric.



Learn Compare Two Populations

A **double box plot** consists of two box plots graphed on the same number line. A **double dot plot** consists of two dot plots that are constructed so that the values on each number line align. You can draw inferences about two populations represented by a double box plot or a double dot plot by comparing their centers and variations.

Sample Double Box Plot | Sample Double Dot Plot

0 5 10 15 20 25 30 35 40 45 50

To select the appropriate measures of center and variability to compare populations in double box plots or double dot plots, check for symmetry in each data set.

Symmetry	Measure of Center	Measure of Variation
both sets of data are symmetric	mean or median	mean absolute deviation or interquartile range
neither set of data is symmetric	median	interquartile range
only one set of data is symmetric	median	interquartile range

Use the measures of center and variation to compare the two populations. Refer to the double box plot shown.

0 5 10 15 20 25 30 35 40 45 50

What is the median of the top box plot? 35

What is the median of the bottom box plot? 15

How do the centers compare? The median of the top box plot is more than twice the median of the bottom box plot.

What is interquartile range of the top box plot? 40−30, or 10

What is interquartile range of the bottom box plot? 20−10, or 10

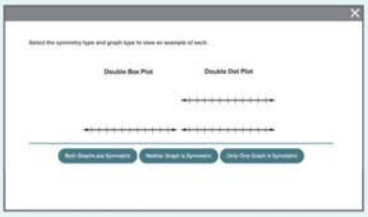
How do the data in the populations vary around the median? Because the IQRs are the same, the data are similarly clustered around each median, although the medians are different.

Talk About It! What else do you notice about the two populations?

Sample answer: While the IQRs are the same, the longer whiskers in the bottom box plot indicate the data are more spread out near the extremes than they are near the center.

Lesson 10-4 • Compare Two Populations 657

Interactive Presentation



Learn, Compare Two Populations, Slide 2 of 4

CLICK

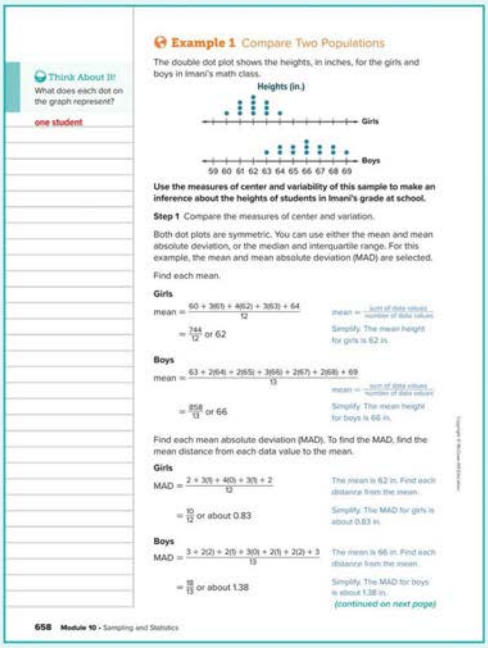
On Slide 2, students select the symmetry type and graph type to view an example of each.

DIFFERENTIATE

Language Development Activity ELL

Encourage student's vocabulary development by consistently using precise and correct mathematical terminology, such as *symmetric*, *asymmetric*, *measures of center*, and *measures of variation*. When explaining symmetry, you can ask students if the left side of the data distribution looks like the right side – but then encourage them to use the correct terminology, *symmetric* or *asymmetric* to describe the distribution. You may wish to provide these collaborative conversation starters to support all students as they use symmetry to determine the appropriate measure of center and variation to use. Have students use these questions to engage in conversation with a partner when presented with a double box plot or double dot plot.

- How many sets of data are symmetric? How can you tell?
- What does this tell you about which measure of center(s) are appropriate to use? measure(s) of variation? Why?



Think About It!
What does each dot on the graph represent?
one student

Example 1 Compare Two Populations

The double dot plot shows the heights, in inches, for the girls and boys in Imani's math class.

Heights (in.)
Girls
Boys
59 60 61 62 63 64 65 66 67 68 69

Use the measures of center and variability of this sample to make an inference about the heights of students in Imani's grade at school.

Step 1 Compare the measures of center and variation.

Both dot plots are symmetric. You can use either the mean and mean absolute deviation, or the median and interquartile range. For this example, the mean and mean absolute deviation (MAD) are selected.

Find each mean.

Girls

$\text{mean} = \frac{60 + 3(61) + 4(62) + 3(63) + 64}{12}$ — $\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$

$= \frac{744}{12}$ or 62 — Simplify. The mean height for girls is 62 in.

Boys

$\text{mean} = \frac{63 + 2(64) + 2(65) + 3(66) + 2(67) + 2(68) + 69}{13}$ — $\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$

$= \frac{858}{13}$ or 66 — Simplify. The mean height for boys is 66 in.

Find each mean absolute deviation (MAD). To find the MAD, find the mean distance from each data value to the mean.

Girls

$\text{MAD} = \frac{2 + 3(1) + 4(0) + 3(1) + 2}{12}$ — The mean is 62 in. Find each distance from the mean.

$= \frac{10}{12}$ or about 0.83 — Simplify. The MAD for girls is about 0.83 in.

Boys

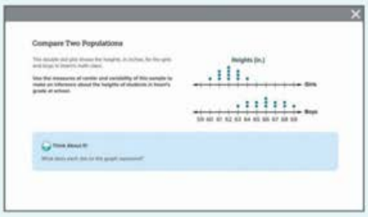
$\text{MAD} = \frac{3 + 2(2) + 2(1) + 3(0) + 2(1) + 2(2) + 3}{13}$ — The mean is 66 in. Find each distance from the mean.

$= \frac{18}{13}$ or about 1.38 — Simplify. The MAD for boys is about 1.38 in.

(continued on next page)

658 Module 10 • Sampling and Statistics

Interactive Presentation



Example 1, Compare Two Populations, Slide 1 of 5

CLICK

On Slide 2, students move through the slides to analyze the distributions.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Compare Two Populations

Objective

Students will make informal comparative inferences about two populations using a double dot plot with symmetric distributions.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Students will reason about symmetry and appropriate measures of center and variation to compare the data within the double dot plot.

As students discuss the *Talk About It!* question on Slide 4, encourage them to study the samples in order to make some other inferences about the heights of students in Imani's grade.

7 Look For and Make Use of Structure Encourage students to study the structure of the double dot plot in order to determine that both distributions are symmetric.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the symmetry or asymmetry of the two dot plots, in your own words. Sample answer: Both dot plots are symmetric.

OL Compare the mean heights of the samples. Sample answer: The mean height for girls, 62 inches, is 4 inches less than the mean height for boys, 66 inches.

OL Compare the variabilities of the samples. Sample answer: The MAD for girls, about 0.83 inch, is 0.55 inch less than the MAD for boys, about 1.38 inches.

BL Are any girls taller than any boys in the sample? Explain. yes; One of the girls had a height of 64 inches, which is taller than the boy with the least height, 63 inches.

(continued on next page)

Example 1 Compare Two Populations *(continued)*

Questions for Mathematical Discourse

SLIDE 3

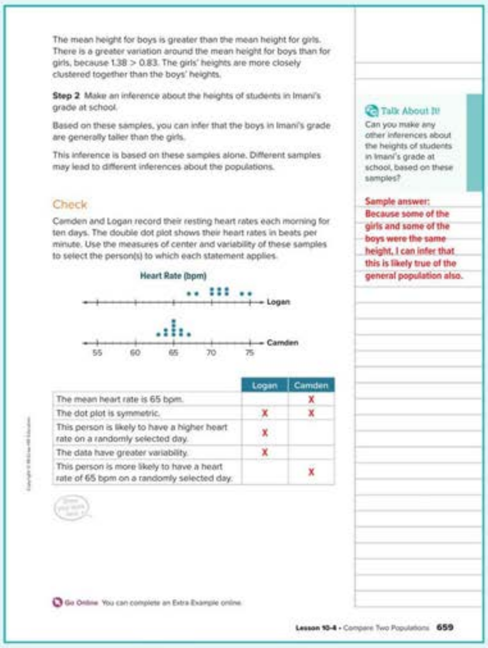
AL Does this inference mean that there are no girls that are taller than boys in Imani's grade at her school? Explain. no; Sample answer: These samples show that boys are generally taller than girls, but that does not mean that every boy is taller than every girl.

OL Why is it important to say that your inference is based on these samples? Sample answer: Different samples may lead to different inferences about the entire population.

BL A classmate stated that these samples show that the girls' heights are more consistent with each other than the boys' heights. How would you respond to this claim? Sample answer: Based on these samples alone, that claim seems reasonable because there is less variability in the data for the girls' heights. However, that is just based on looking at these samples.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


The mean height for boys is greater than the mean height for girls. There is a greater variation around the mean height for boys than for girls, because 1.38 > 0.83. The girls' heights are more closely clustered together than the boys' heights.

Step 2 Make an inference about the heights of students in Imani's grade at school.

Based on these samples, you can infer that the boys in Imani's grade are generally taller than the girls.

This inference is based on these samples alone. Different samples may lead to different inferences about the populations.

Talk About It! Can you make any other inferences about the heights of students in Imani's grade at school, based on these samples?

Sample answer: Because some of the girls and some of the boys were the same height, I can infer that this is likely true of the general population also.

Check

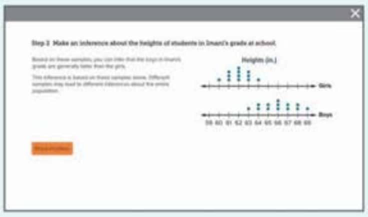
Camden and Logan record their resting heart rates each morning for ten days. The double dot plot shows their heart rates in beats per minute. Use the measures of center and variability of these samples to select the person(s) to which each statement applies.

	Logan	Camden
The mean heart rate is 65 bpm.		X
The dot plot is symmetric.	X	X
This person is likely to have a higher heart rate on a randomly selected day.	X	
The data have greater variability.	X	
This person is more likely to have a heart rate of 65 bpm on a randomly selected day.		X

Go Online You can complete an Extra Example online.

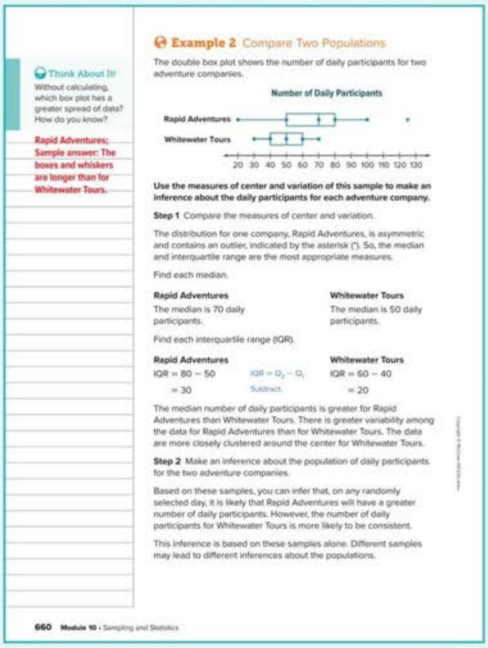
Lesson 10-4 • Compare Two Populations 659

Interactive Presentation



Example 1, Compare Two Populations, Slide 3 of 5

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Think About It!
Without calculating, which box plot has a greater spread of data? How do you know?

Rapid Adventures; Sample answer: The boxes and whiskers are longer than for Whitewater Tours.

Example 2 Compare Two Populations

The double box plot shows the number of daily participants for two adventure companies.

Use the measures of center and variation of this sample to make an inference about the daily participants for each adventure company.

Step 1 Compare the measures of center and variation.

The distribution for one company, Rapid Adventures, is asymmetric and contains an outlier, indicated by the asterisk (*). So, the median and interquartile range are the most appropriate measures.

Find each median.

Rapid Adventures
The median is 70 daily participants.

Whitewater Tours
The median is 50 daily participants.

Find each interquartile range (IQR).

Rapid Adventures
IQR = 80 − 50
= 30

$IQR = Q_3 - Q_1$
Subtract.

Whitewater Tours
IQR = 60 − 40
= 20

The median number of daily participants is greater for Rapid Adventures than Whitewater Tours. There is greater variability among the data for Rapid Adventures than for Whitewater Tours. The data are more closely clustered around the center for Whitewater Tours.

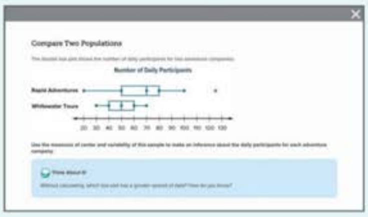
Step 2 Make an inference about the population of daily participants for the two adventure companies.

Based on these samples, you can infer that, on any randomly selected day, it is likely that Rapid Adventures will have a greater number of daily participants. However, the number of daily participants for Whitewater Tours is more likely to be consistent.

This inference is based on these samples alone. Different samples may lead to different inferences about the populations.

660 Module 10 • Sampling and Statistics

Interactive Presentation



Example 2, Compare Two Populations, Slide 1 of 5

CLICK

On Slide 2, students select markers to compare the measures and variability of the samples.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Compare Two Populations

Objective

Students will make informal comparative inferences about two populations using a double box plot with asymmetric distributions.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively S tudents will reason about symmetry and appropriate measures of center and variation to compare the data within the double box plot.

7 Look For and Make Use of Structure Encourage students to study the structure of the double box plot in order to determine the symmetry of each distribution.

Questions for Mathematical Discourse

SLIDE 2

AL How can you tell if a plot is symmetric? The plot is symmetric if the length of the box and whiskers on either side of the median are similar.

AL Which data distribution is symmetric? Whitewater Tours

OL What do the medians tell you about the data? Sample answer: Rapid Adventures has a greater number of daily participants overall, as a measure of center.

OL What do the interquartile ranges tell you about the data? Sample answer: Rapid Adventures has a greater spread, or variation. It is more difficult to predict how many participants they may have each day.

BL What does the asterisk (*) mean? It is an outlier.

BL How does an outlier affect data? By including it in calculations, it will affect the mean. In this case it will increase the mean.

BL Can you determine the mean of either data set? Explain. Sample answer: I cannot determine the mean of the data for Rapid Adventures, because the data are not symmetric. I can assume the mean of the data for Whitewater Tours is the same as the median, because the data are symmetric.

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 2 Compare Two Populations *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Does this inference mean that Rapid Adventures will always have more participants on any randomly selected day? Explain. no; Sample answer: These samples show that it is likely that Rapid Adventures will have a greater number of participants. However, because Whitewater Tours is likely to be more consistent, Rapid Adventures could also have a lesser number of participants.

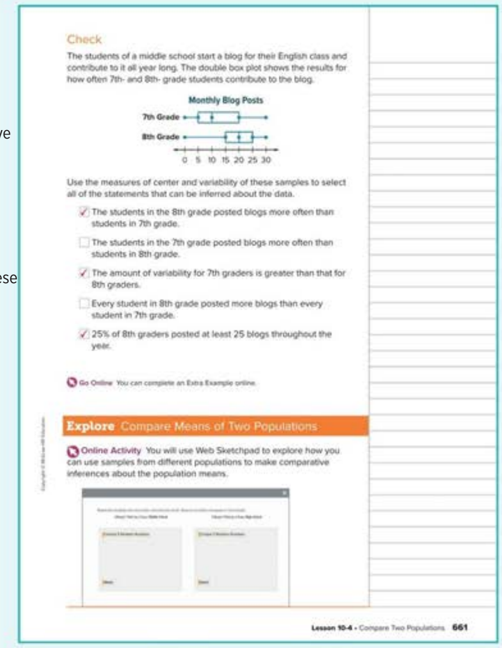
OL Why is it important to say that your inference is based on these samples? Sample answer: Different samples may lead to different inferences about the entire population.

OL If you were to financially support one of these adventure companies, which would you choose, and why? See students' responses. Some students may choose Rapid Adventures because of the higher median, while others may choose Whitewater Tours because the data are more consistent. Students should support their preference using logical reasoning.

BL Why do you think the right whisker on the box plot for Rapid Adventures does not extend all the way to the asterisk? Sample answer: The right whisker represents 25% of the data. If the right whisker extended to the asterisk, then 25% of the data would have to be between 80 and 125 participants. The asterisk indicates the data point 125 is an outlier.

Go Online

- Find additional teaching notes and discussion questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

The students of a middle school start a blog for their English class and contribute to it all year long. The double box plot shows the results for how often 7th- and 8th- grade students contribute to the blog.

Monthly Blog Posts

7th Grade

8th Grade

0 5 10 15 20 25 30

Use the measures of center and variability of these samples to select all of the statements that can be inferred about the data.

- [x] The students in the 8th grade posted blogs more often than students in 7th grade.
- [ ] The students in the 7th grade posted blogs more often than students in 8th grade.
- [x] The amount of variability for 7th graders is greater than that for 8th graders.
- [ ] Every student in 8th grade posted more blogs than every student in 7th grade.
- [x] 25% of 8th graders posted at least 25 blogs throughout the year.

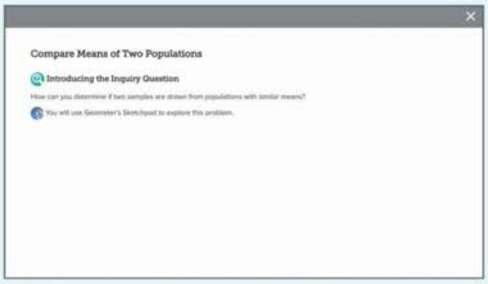
Go Online You can complete an Extra Example online.

Explore Compare Means of Two Populations

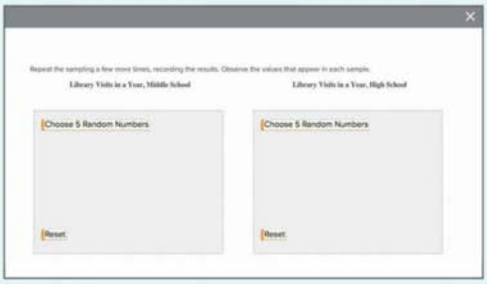
Online Activity You will use Web Sketchpad to explore how you can use samples from different populations to make comparative inferences about the population means.

Lesson 10-4 • Compare Two Populations 661

Interactive Presentation



Explore, Slide 1 of 8



Explore, Slide 4 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to determine if two samples are drawn from populations with similar means.

TYPE

On Slide 4, students make a prediction for the means of each population using Samples 1 and 2.

Explore Compare Means of Two Populations

Objective

Students will use Web Sketchpad to explore whether samples drawn from different populations have similar means.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a random number generator to generate two samples. They will compare the means of the samples and use them to predict the means of the corresponding populations. Throughout this activity, students will repeat the sampling with greater sample sizes.

Inquiry Question

How can you determine if two samples are drawn from populations with similar means? Sample answer: If the means of the samples are similar, it is likely that the means of the populations will be similar. The lesser the difference between the two means in the samples, the greater the likelihood that the means in the population are similar.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Do you think that the mean of each sample is similar to the mean of the population of numbers that they are being drawn from? Explain your reasoning. Yes; Sample answer: Since the sample is randomly drawn from the population there should be a relationship between their means.

(continued on next page)

Explore Compare Means of Two Populations *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Web Sketchpad random number generator to generate two samples, compare them, and use the comparisons to make predictions.

7 Look For and Make Use of Structure Encourage students to look for any patterns in these values based on the sample size.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the Talk About It! question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Do the samples appear to be coming from populations with similar means? Justify your reasoning. Answers will vary. Students should notice that means from the samples of middle school students appear to be lesser than the means from the samples of high school students as the sample size increases.

Interactive Presentation

Explore, Slide 6 of 8

TYPE

On Slide 5, students explain if their predictions have changed after observing a sample of size 10.

TYPE

On Slide 6, students write a final prediction for the mean of each population.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.


Pause and Reflect

Create a graphic organizer that will help you study the concepts you learned today in class.

See students' observations.

662 Module 10 • Sampling and Statistics

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can you use a sample to gain information about a population?
In this lesson, students learned how to compare two populations by analyzing sample data of each. Encourage them to discuss with a partner how the symmetry of two sample distributions can help them make inferences comparing the two populations.

Exit Ticket

Refer to the Exit Ticket slide. Sketch a double box plot of the data for each movie genre. Compare the two populations of movie running times. What inferences can you make based on the double box plot? See students' double box plots. Sample answer: The median for the comedy movies is 96.5 minutes with an IQR of 13 minutes. The median for the action movies is 123.5 minutes with an IQR of 20 minutes. The data for the comedy movies varies less than the data for the action movies. Based on these samples, the running time for comedies is generally less than the running time for action movies.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 2–7
- Extension: Select an Appropriate Display, Standard Deviation
- ALEKS Graphs of Data, Measures of Variation

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 5
- Extension: Select an Appropriate Display, Standard Deviation
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Mean, Median, and Mode

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Mean, Median, and Mode

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Independent Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

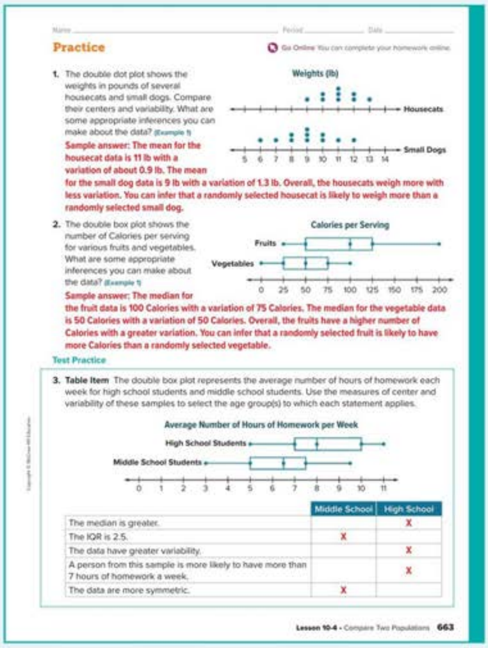
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	make informal inferences about two populations using a double dot plot with symmetric distributions	1
2	make informal inferences about two populations using a double box plot with asymmetric distributions	2
2	extend concepts learned in class to apply them in new contexts	3
3	solve application problems involving comparing populations	4, 5
3	higher-order and critical thinking skills	6–8

Common Misconception

Some students may continue to carry a common misconception about the shape of box plots in that they mistakenly think a longer box or whisker indicates there are more data values within that interval than in a shorter box or whisker. For example, in Exercise 2, students may think that there are more data values that are between 125–200 than there are between 25–50. They may make correct comparative inferences in this particular exercise, but still carry the misconception. Encourage students to use reasoning that each box and each whisker represents 25% of the data. Each box and each whisker contain the same number of data values. Shorter boxes and whiskers indicate the data have less variation in those intervals, while longer boxes and whiskers indicate the data are more spread out.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The double dot plot shows the weights in pounds of several housecats and small dogs. Compare their centers and variability. What are some appropriate inferences you can make about the data? (Example 1)

Sample answer: The mean for the housecat data is 11 lb with a variation of about 0.9 lb. The mean for the small dog data is 9 lb with a variation of 1.3 lb. Overall, the housecats weigh more with less variation. You can infer that a randomly selected housecat is likely to weigh more than a randomly selected small dog.

2. The double box plot shows the number of Calories per serving for various fruits and vegetables. What are some appropriate inferences you can make about the data? (Example 1)

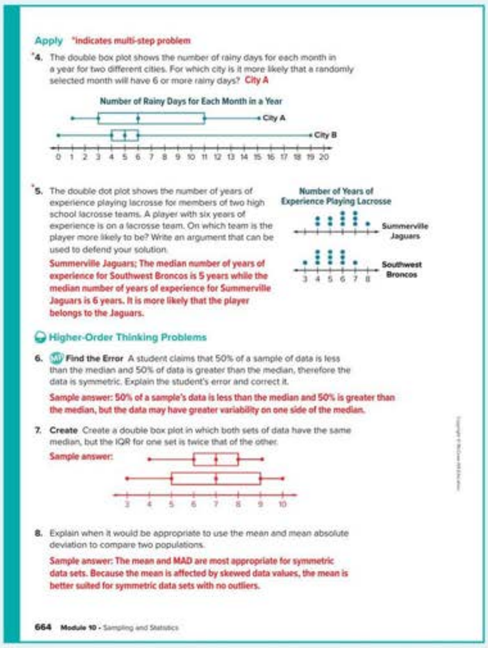
Sample answer: The median for the fruit data is 100 Calories with a variation of 75 Calories. The median for the vegetable data is 50 Calories with a variation of 50 Calories. Overall, the fruits have a higher number of Calories with a greater variation. You can infer that a randomly selected fruit is likely to have more Calories than a randomly selected vegetable.

Test Practice

3. **Table Item** The double box plot represents the average number of hours of homework each week for high school students and middle school students. Use the measures of center and variability of these samples to select the age group(s) to which each statement applies.

	Middle School	High School
The median is greater.		X
The IQR is 2.5.	X	
The data have greater variability.		X
A person from this sample is more likely to have more than 7 hours of homework a week.		X
The data are more symmetric.	X	

Lesson 10-4 • Compare Two Populations 663



Apply *indicates multi-step problem

*4. The double box plot shows the number of rainy days for each month in a year for two different cities. For which city is it more likely that a randomly selected month will have 6 or more rainy days? **City A**

*5. The double dot plot shows the number of years of experience playing lacrosse for members of two high school lacrosse teams. A player with six years of experience is on a lacrosse team. On which team is the player more likely to be? Write an argument that can be used to defend your solution.

Summerville Jaguars; The median number of years of experience for Southwest Broncos is 5 years while the median number of years of experience for Summerville Jaguars is 6 years. It is more likely that the player belongs to the Jaguars.

Higher-Order Thinking Problems

6. **Find the Error** A student claims that 50% of a sample of data is less than the median and 50% of data is greater than the median, therefore the data is symmetric. Explain the student's error and correct it.

Sample answer: 50% of a sample's data is less than the median and 50% is greater than the median, but the data may have greater variability on one side of the median.

7. **Create** Create a double box plot in which both sets of data have the same median, but the IQR for one set is twice that of the other.

Sample answer:

8. Explain when it would be appropriate to use the mean and mean absolute deviation to compare two populations.

Sample answer: The mean and MAD are most appropriate for symmetric data sets. Because the mean is affected by skewed data values, the mean is better suited for symmetric data sets with no outliers.

664 Module 10 • Sampling and Statistics

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 6, students find the error in a student's reasoning about the structure of a box plot and correct it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.

Use with Exercise 4 Have pairs of students interview each other as they complete this application problem. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 4 might be "How can you use asymmetry to determine which measure of center to analyze?"

Clearly and precisely explain.

Use with Exercise 6 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as giving examples of data sets.

Lesson 10-5

Assess Visual Overlap

7.SP.B.3

LESSON GOAL

Students will informally assess the degree of visual overlap between two distributions.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Interpret Visual Overlap

Example 1: Measure Variability Between Populations

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 62 of the *Language Development Handbook* to help your students build mathematical language related to assessing visual overlap between two populations.

ELL You can use the tips and suggestions on page T62 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Statistics and Probability

Additional Cluster(s): In this lesson, students address additional cluster **7.SP.B** by making inferences about the distribution of data in a sample.

Standards for Mathematical Content: **7 .SP.B.3**

Standards for Mathematical Practice: **MP 2, MP3, MP7**

Coherence

Vertical Alignment

Previous

Students made comparative inferences about two populations based on the data from random samples. **7.SP.B.4**

Now

Students informally assess the degree of visual overlap between two distributions. **7.SP.B.3**

Next

Students will make inferences and justify conclusions from sample experiments. **HSS.IC.B.3, HSS.IC.B.4, HSS.IC.B.5, HSS.IC.B.6**

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of the visual overlap between two distributions with similar variability and how this can be informally measured using a ratio comparing the difference in the means of the samples to the variability. They build *fluency* in calculating this ratio and *apply* their understanding of its meaning to make inferences as to how likely the population means are similar or different.

Mathematical Background

When two sample distributions have similar variability and graphed on the same number line, you can informally assess the visual overlap between them to make an inference as to how likely the means of the two populations are similar or different. In this lesson, the ratio $\frac{\text{difference in means}}{\text{MAD}}$ is used to assess the overlap.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- writing ratios (fractions) as decimals (Exercises 1–5)

Answers

1. 4	**4.** 2.24
2. 3.53	**5.** 2.94
3. 7.75	

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about comparing two samples of heights of Olympic athletes.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What are some synonyms of the word *overlap*? Sample answers: intersection, connection, cover, coincide

Learn Interpret Visual Overlap

Objective

Students will learn how they can use a ratio to assess the degree of visual overlap between two samples in order to make an inference as to how likely the population means are similar or different.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 1, they should be able to use reasoning to explain that the greater the visual overlap, the more likely it is that the populations could have the same mean or similar means.

Teaching Notes

SLIDES 1–5

Present students with the double dot plot comparing the weights of male and female adult dogs. Be sure students understand that these distributions represent *samples* of the two populations.

Before introducing the ratio, have students make inferences based on the visual overlap shown on the double dot plot. Some students may say that it is likely the population means are different (for example, male adult dogs will have a greater mean weight than female adult dogs) because more data values are greater for the sample representing *males* than are for the sample representing *females*. Some students may say that it is likely the population means are similar, because the data overlap. Encourage students to wrestle with these questions and ask them which measure(s) they can use to help them make these comparative inferences.

Present students with the ratio $\frac{\text{difference in means}}{\text{MAD}}$. Point out that calculating the mean alone of the two samples only gives an indication of the centers. By using this ratio, the variability among the samples is also considered. The difference in the mean weights of the samples is 3 pounds. But this does not tell you whether the variability is low or high. Because the MAD is about 1 pound, the variability among the samples is relatively low. You can have a higher confidence in making an inference that the population means are likely to be different – because the sample means are different and the variability is low.

Provide students with the conventions they will use in the lesson in interpreting the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

Talk About It!

SLIDE 1

Mathematical Discourse

Just by looking at the samples in the double dot plot, how likely do you think it is that the population means are different? Explain. Sample answer: I think it is likely that the population means are different because both sets of data appear to have low variability, and the data overlap.

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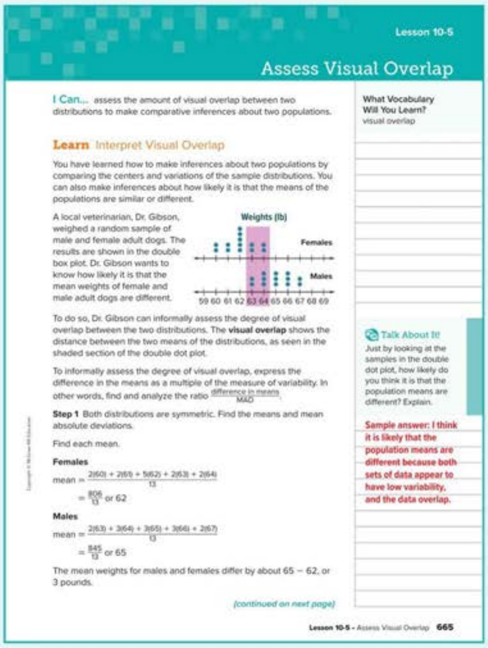
Lesson 10-5

Assess Visual Overlap

I Can... assess the amount of visual overlap between two distributions to make comparative inferences about two populations.

What Vocabulary Will You Learn?
visual overlap

Learn Interpret Visual Overlap

You have learned how to make inferences about two populations by comparing the centers and variations of the sample distributions. You can also make inferences about how likely it is that the means of the populations are similar or different.

A local veterinarian, Dr. Gibson, weighed a random sample of male and female adult dogs. The results are shown in the double box plot. Dr. Gibson wants to know how likely it is that the mean weights of female and male adult dogs are different.

To do so, Dr. Gibson can informally assess the degree of visual overlap between the two distributions. The **visual overlap** shows the distance between the two means of the distributions, as seen in the shaded section of the double dot plot.

To informally assess the degree of visual overlap, express the difference in the means as a multiple of the measure of variability. In other words, find and analyze the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

Step 1 Both distributions are symmetric. Find the means and mean absolute deviations.

Find each mean.

Females

$\text{mean} = \frac{2(60) + 2(61) + 5(62) + 2(63) + 2(64)}{13}$

$= \frac{806}{13}$ or 62

Males

$\text{mean} = \frac{2(63) + 3(64) + 3(65) + 3(66) + 2(67)}{13}$

$= \frac{845}{13}$ or 65

The mean weights for males and females differ by about 65 − 62, or 3 pounds.

Talk About It! Just by looking at the samples in the double dot plot, how likely do you think it is that the population means are different? Explain.

Sample answer: I think it is likely that the population means are different because both sets of data appear to have low variability, and the data overlap.

(continued on next page)

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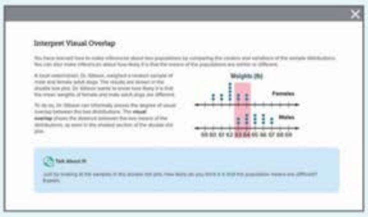
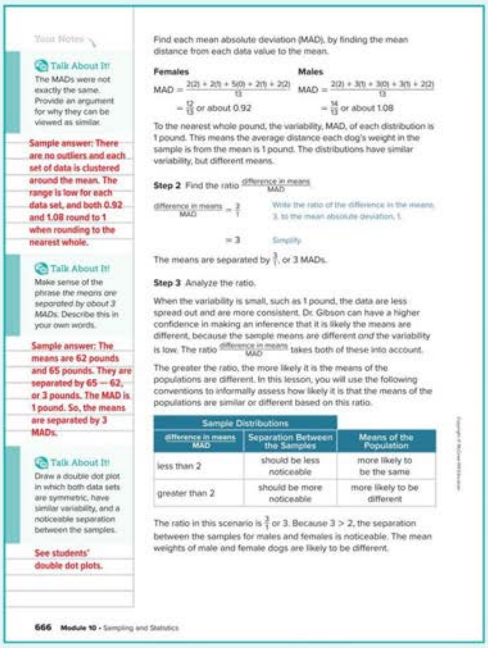
Interactive Presentation



Learn, Interpret Visual Overlap, Slide 1 of 6

FLASHCARDS

On Slide 3, students use flashcards to find the means and MADs.



Your Notes

Talk About It!
The MADs were not exactly the same. Provide an argument for why they can be viewed as similar.

Sample answer: There are no outliers and each set of data is clustered around the mean. The range is low for each data set, and both 0.92 and 1.08 round to 1 when rounding to the nearest whole.

Talk About It!
Make sense of the phrase *the means are separated by about 3 MADs*. Describe this in your own words.

Sample answer: The means are 62 pounds and 65 pounds. They are separated by 65 − 62, or 3 pounds. The MAD is 1 pound. So, the means are separated by 3 MADs.

Talk About It!
Draw a double dot plot in which both data sets are symmetric, have similar variability, and a noticeable separation between the samples.

See students' double dot plots.

Find each mean absolute deviation (MAD), by finding the mean distance from each data value to the mean.

Females

$\text{MAD} = \frac{2(2) + 2(1) + 5(0) + 2(1) + 2(2)}{13} = \frac{12}{13}$ or about 0.92

Males

$\text{MAD} = \frac{2(2) + 3(1) + 3(0) + 3(1) + 2(2)}{13} = \frac{14}{13}$ or about 1.08

To the nearest whole pound, the variability, MAD, of each distribution is 1 pound. This means the average distance each dog's weight in the sample is from the mean is 1 pound. The distributions have similar variability, but different means.

Step 2 Find the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

$\frac{\text{difference in means}}{\text{MAD}} = \frac{3}{1}$ Write the ratio of the difference in the means, 3, to the mean absolute deviation, 1.

$= 3$ Simplify.

The means are separated by $\frac{3}{1}$, or 3 MADs.

Step 3 Analyze the ratio.

When the variability is small, such as 1 pound, the data are less spread out and are more consistent. Dr. Gibson can have a higher confidence in making an inference that it is likely the means are different, because the sample means are different *and* the variability is low. The ratio $\frac{\text{difference in means}}{\text{MAD}}$ takes both of these into account.

The greater the ratio, the more likely it is the means of the populations are different. In this lesson, you will use the following conventions to informally assess how likely it is that the means of the populations are similar or different based on this ratio.

Sample Distributions		
$\frac{\text{difference in means}}{\text{MAD}}$	Separation Between the Samples	Means of the Population
less than 2	should be less noticeable	more likely to be the same
greater than 2	should be more noticeable	more likely to be different

The ratio in this scenario is $\frac{3}{1}$ or 3. Because 3 > 2, the separation between the samples for males and females is noticeable. The mean weights of male and female dogs are likely to be different.

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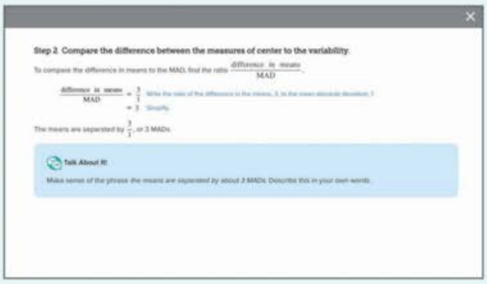
Interactive Presentation



Step 2 Compare the difference between the measures of center to the variability.

To compare the difference in means to the MAD, find the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

$\frac{\text{difference in means}}{\text{MAD}} = \frac{3}{1}$ Write the ratio of the difference in the means, 3, to the mean absolute deviation, 1.

$= 3$ Simplify.

The means are separated by $\frac{3}{1}$, or 3 MADs.

Talk About It!
Make sense of the phrase *the means are separated by about 3 MADs*. Describe this in your own words.

Learn, Interpret Visual Overlap, Slide 4 of 5

Learn Interpret Visual Overlap *(continued)*

Talk About It!

SLIDE 3

Mathematical Discourse

The MADs were not exactly the same. Provide an argument for why they can be viewed as similar. Sample answer: There are no outliers and each set of data is clustered around the mean. The range is low for each data set, and both 0.92 and 1.08 round to 1 when rounding to the nearest whole.

SLIDE 4

Mathematical Discourse

Make sense of the phrase *the means are separated by about 3 MADs*. Describe this in your own words. Sample answer: The means are 62 pounds and 65 pounds. They are separated by 65 — 62, or 3 pounds. The MAD is 1 pound. So, the means are separated by 3 MADs.

SLIDE 6

Mathematical Discourse

Draw a double dot plot in which both data sets are symmetric, have similar variability, and a noticeable separation between the samples. See students' double dot plots.

DIFFERENTIATE

Language Development Activity ELL

To support students' use of precise mathematical language, have them work with a partner to discuss the *Talk About It!* question *Make sense of the phrase the means are separated by about 3 MADs. Describe this in your own words.* You may wish to provide them with these sentence frames to help get them started. Some students may benefit from a discussion of the term *separated* as it is used in this mathematical context. On the number line, the means 65 pounds and 62 pounds are 3 pounds *apart*. Because the *distance* between them is 3 pounds, they are *separated* by 3 pounds.

The means of each sample are ______ and ______. They are separated by ______.

Because the MAD is ______, the means are separated by ______ MADs.

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Example 1 Measure Variability Between Populations

Objective

Students will find the number of measures of variability that separate the means of two samples with similar variability, and make an inference about the population means.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Par t A, students should use reasoning to explain how this ratio can help them make an inference comparing the two population means.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to create a plausible argument for the possibility that two population means are different, even if their dot plots overlap.

7 Look For and Make Use of Structure Encourage students to study the structure of the sample distributions in order to be able to informally assess the degree of visual overlap.

Questions for Mathematical Discourse

SLIDE 2

AL Just by looking at the graphs, how would you describe the degree of visual overlap between the two data sets? Sample answer: There is a large amount of visual overlap.

AL What is true about the variability for each city? Sample answer: The variabilities are the same, about 1.4.

OL Describe what the variability means, in your own words. Sample answer: The average distance each data value is from the mean temperature is about 1.4 degrees Fahrenheit.

OL Describe what the value 1.43 means, in your own words. Sample answer: The means are separated by about 1.43 measures of variability. Because this value is less than 2, it is likely that the population means are similar, or the same.

BL If the two sample distributions were separated farther apart, yet retained the same variability, how would this affect the ratio of the difference in means to the MAD? The ratio would increase. It would mean that the sample means are farther apart, because the variability would remain the same. The numerator of the ratio would increase, while the denominator would stay the same.

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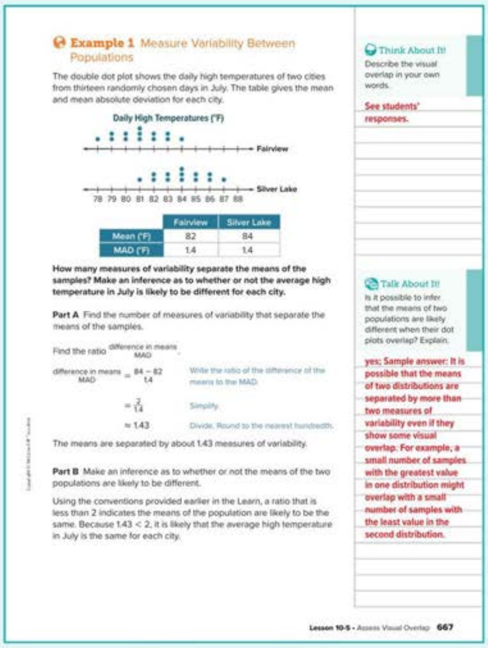

Example 1 Measure Variability Between Populations

The double dot plot shows the daily high temperatures of two cities from thirteen randomly chosen days in July. The table gives the mean and mean absolute deviation for each city.

	Fairview	Silver Lake
Mean (°F)	82	84
MAD (°F)	1.4	1.4

How many measures of variability separate the means of the samples? Make an inference as to whether or not the average high temperature in July is likely to be different for each city.

Part A Find the number of measures of variability that separate the means of the samples.

Find the ratio $\frac{\text{difference in means}}{\text{MAD}}$.

$\frac{\text{difference in means}}{\text{MAD}} = \frac{84 - 82}{1.4}$ Write the ratio of the difference of the means to the MAD.

$= \frac{2}{1.4}$ Simplify.

≈ 1.43 Divide. Round to the nearest hundredth.

The means are separated by about 1.43 measures of variability.

Part B Make an inference as to whether or not the means of the two populations are likely to be different.

Using the conventions provided earlier in the Learn, a ratio that is less than 2 indicates the means of the population are likely to be the same. Because 1.43 < 2, it is likely that the average high temperature in July is the same for each city.

Think About It! Describe the visual overlap in your own words.

See students' responses.

Talk About It! Is it possible to infer that the means of two populations are likely different when their dot plots overlap? Explain.

yes; Sample answer: It is possible that the means of two distributions are separated by more than two measures of variability even if they show some visual overlap. For example, a small number of samples with the greatest value in one distribution might overlap with a small number of samples with the least value in the second distribution.

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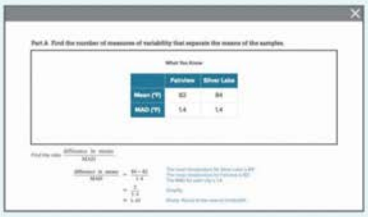
Interactive Presentation

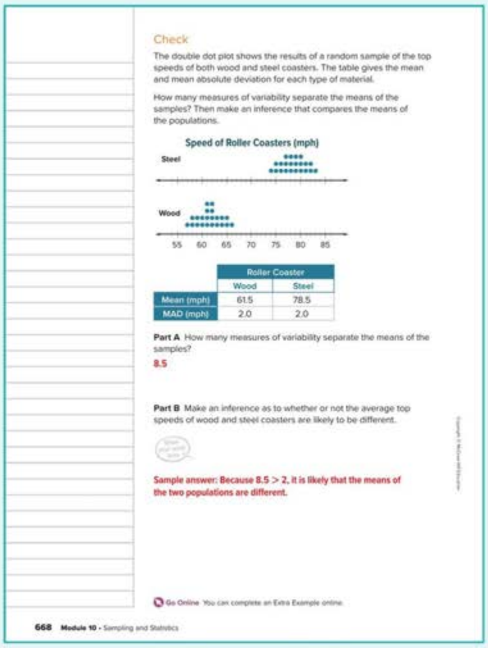


Example 1, Measure Variability Between Populations, Slide 2 of 5

CLICK

On Slide 3, students select from a drop-down menu the likelihood that the average high temperature in July is the same for each city.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Check

The double dot plot shows the results of a random sample of the top speeds of both wood and steel coasters. The table gives the mean and mean absolute deviation for each type of material.

How many measures of variability separate the means of the samples? Then make an inference that compares the means of the populations.

Speed of Roller Coasters (mph)

	Roller Coaster	
	Wood	Steel
Mean (mph)	61.5	78.5
MAD (mph)	2.0	2.0

Part A How many measures of variability separate the means of the samples?

8.5

Part B Make an inference as to whether or not the average top speeds of wood and steel coasters are likely to be different.

Sample answer: Because 8.5 > 2, it is likely that the means of the two populations are different.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 1, Measure Variability Between Populations, Slide 3 of 5

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Measure Variability Between Populations *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL Compare the ratio you found, about 1.43, to the conventions we are using throughout this lesson. What do you notice? Sample answer: The ratio I found, about 1.43, is less than the ratio 2. This means that it is likely the means of the populations are similar.

OL How does the degree of visual overlap on the two dot plots compare to the claim that the population means are likely similar? Sample answer: There is a large amount of visual overlap. This supports the claim that the population means are likely similar.

BL Describe what the visual overlap might look like if the ratio of the difference in means to the MAD for two data distributions was 5. Then explain what inference you could make about the population means. Sample answer: If the ratio was 5, then the amount of visual overlap would be small, or non-existent. Itis likely that the population means would be significantly different.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Practice and Homework

The Independent Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

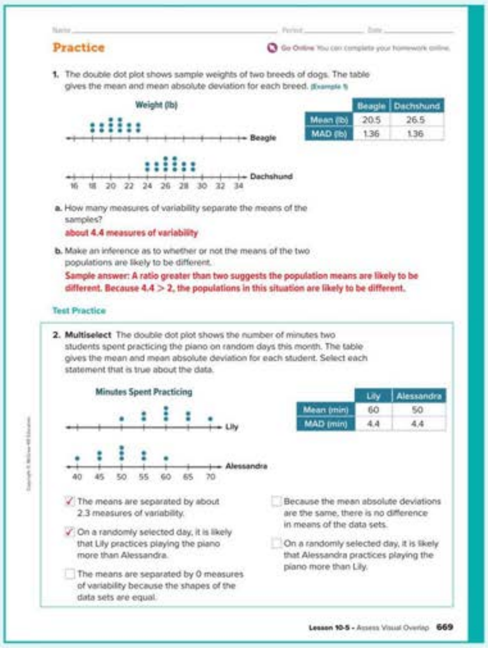
DOK	Topic	Exercises
2	make informal inferences about two populations based on their visual overlap	1
2	extend concepts learned in class to apply them in new contexts	2
3	solve application problems involving visual overlap	3, 4
3	higher-order and critical thinking skills	5–7

Common Misconception

Students may confuse the mean and mean absolute deviation when assessing the degree of visual overlap. For example, in Exercise 2, students may incorrectly select the last statement because they believe that since both distributions are symmetric and have the same variability, then the means are also the same. Remind them that the mean is a measure of center, not variability. Encourage them to adhere to the precise definitions and terminology for *mean* and *mean absolute deviation*. The term *deviation* in *mean absolute deviation* should help them understand that the MAD is a measure of *deviation*, or *variance*.

Exit Ticket

Refer to the Exit Ticket slide. Sketch a double dot plot that might show significant visual overlap between these two samples. Explain what this might indicate about the two populations. Then sketch a double dot plot that might show little, or no, overlap between the two samples. Explain what this might indicate about the two populations. See students' double dot plots and explanations; For the double dot plot with significant visual overlap, students' sketches should clearly show several data values that overlap. Students' responses should indicate that it is likely the mean ages of the two populations are similar. For the double dot plot with little, or no overlap, students' sketches should show few or no data values that overlap. Students' responses should indicate that it is likely the mean ages of the two populations are different.



Name ______ Period ______ Date ______

Practice Go Online You can complete your homework online.

1. The double dot plot shows sample weights of two breeds of dogs. The table gives the mean and mean absolute deviation for each breed. (Example 1)

	Beagle	Dachshund
Mean (lb)	20.5	26.5
MAD (lb)	1.36	1.36

a. How many measures of variability separate the means of the samples?
about 4.4 measures of variability

b. Make an inference as to whether or not the means of the two populations are likely to be different.
Sample answer: A ratio greater than two suggests the population means are likely to be different. Because $4.4 > 2$, the populations in this situation are likely to be different.

Test Practice

2. **Multiselect** The double dot plot shows the number of minutes two students spent practicing the piano on random days this month. The table gives the mean and mean absolute deviation for each student. Select each statement that is true about the data.

	Lily	Alessandra
Mean (min)	60	50
MAD (min)	4.4	4.4

- [x] The means are separated by about 2.3 measures of variability.
- [x] On a randomly selected day, it is likely that Lily practices playing the piano more than Alessandra.
- [ ] The means are separated by 0 measures of variability because the shapes of the data sets are equal.
- [ ] Because the mean absolute deviations are the same, there is no difference in means of the data sets.
- [ ] On a randomly selected day, it is likely that Alessandra practices playing the piano more than Lily.

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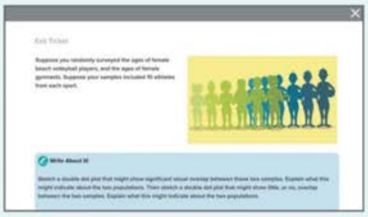
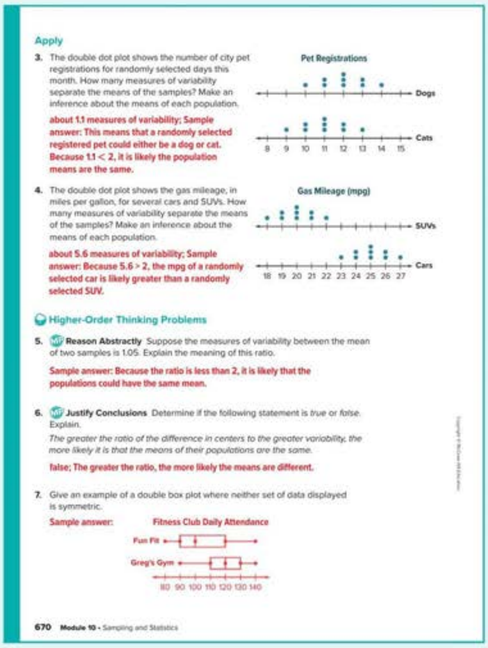
Interactive Presentation



Exit Ticket



Apply

3. The double dot plot shows the number of city pet registrations for randomly selected days this month. How many measures of variability separate the means of the samples? Make an inference about the means of each population.

about 1.1 measures of variability; Sample answer: This means that a randomly selected registered pet could either be a dog or cat. Because 1.1 < 2, it is likely the population means are the same.

4. The double dot plot shows the gas mileage, in miles per gallon, for several cars and SUVs. How many measures of variability separate the means of the samples? Make an inference about the means of each population.

about 5.6 measures of variability; Sample answer: Because 5.6 > 2, the mpg of a randomly selected car is likely greater than a randomly selected SUV.

Higher-Order Thinking Problems

5. **Reason Abstractly** Suppose the measures of variability between the mean of two samples is 1.05. Explain the meaning of this ratio.

Sample answer: Because the ratio is less than 2, it is likely that the populations could have the same mean.

6. **Justify Conclusions** Determine if the following statement is *true* or *false*. Explain.

The greater the ratio of the difference in centers to the greater variability, the more likely it is that the means of their populations are the same.

false; The greater the ratio, the more likely the means are different.

7. Give an example of a double box plot where neither set of data displayed is symmetric.

Sample answer:

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Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 5, students explain what the ratio of two measures of variability means.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 6, students determine if a statement is true or false about the ratio of difference in centers to variability, and justify their conclusion.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Create your own application problem.

Use with Exercise 3 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 2–7
- ALEKS® Graphs of Data, Measures of Variation

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1, 2, 4, 6
- Remediate: Review Resources
- Personal Tutor
- Extra Example 1
- ALEKS® Mean, Median, and Mode

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Mean, Median, and Mode

Module 10 • Sampling and Statistics

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include a review of biased and unbiased samples, with examples of each. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

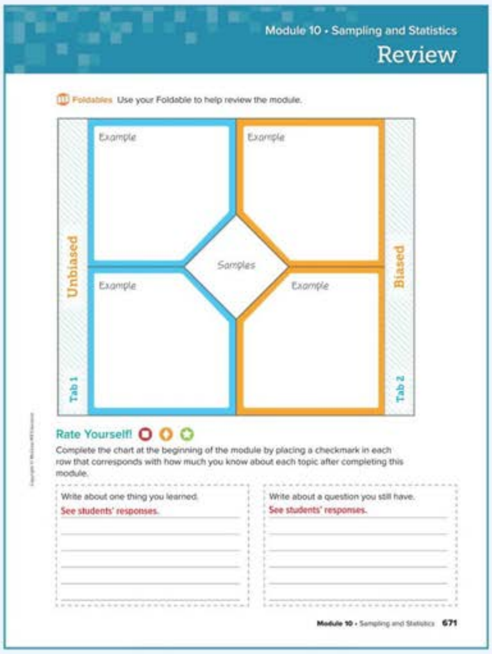
Assessment Resources

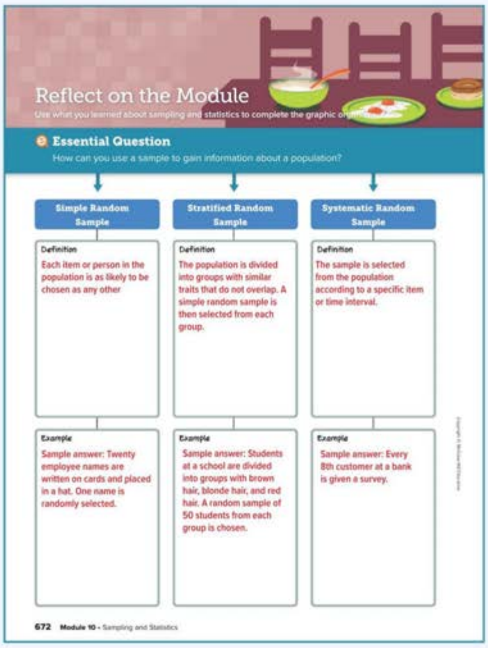
Put It All Together: Lessons 10-1, 10-2, and 10-3
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Statistics and Probability**.

- Statistics
- Compare Populations





Reflect on the Module

Use what you learned about sampling and statistics to complete the graphic organizer.

Essential Question

How can you use a sample to gain information about a population?

Simple Random Sample	Stratified Random Sample	Systematic Random Sample
Definition Each item or person in the population is as likely to be chosen as any other	Definition The population is divided into groups with similar traits that do not overlap. A simple random sample is then selected from each group.	Definition The sample is selected from the population according to a specific item or time interval.
Example Sample answer: Twenty employee names are written on cards and placed in a hat. One name is randomly selected.	Example Sample answer: Students at a school are divided into groups with brown hair, blonde hair, and red hair. A random sample of 50 students from each group is chosen.	Example Sample answer: Every 8th customer at a bank is given a survey.

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can you use a sample to gain information about a population?
See students' graphic organizers.

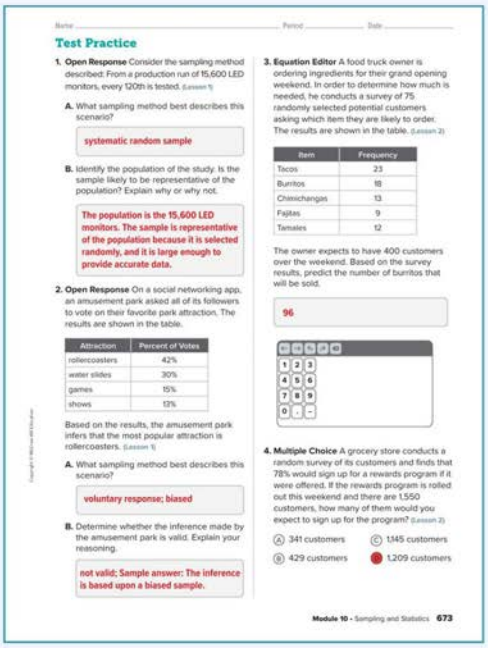
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–8 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	4, 6, 7
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	3
Table Item	Students complete a table.	8
Open Response	Students construct their own response in the area provided.	1, 2, 5

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.SP.A.1	10-1	1, 2
7.SP.A.2	10-1, 10-2, 10-3	2–6
7.SP.B.3	10-5	7
7.SP.B.4	10-4	8


Name ______ Period ______ Date ______

Test Practice

1. **Open Response** Consider the sampling method described: From a production run of 15,600 LED monitors, every 120th is tested. (Lesson 1)

   A. What sampling method best describes this scenario?

   systematic random sample

   B. Identify the population of the study. Is the sample likely to be representative of the population? Explain why or why not.

   The population is the 15,600 LED monitors. The sample is representative of the population because it is selected randomly, and it is large enough to provide accurate data.

2. **Open Response** On a social networking app, an amusement park asked all of its followers to vote on their favorite park attraction. The results are shown in the table.

Attraction	Percent of Votes
rollercoasters	42%
water slides	30%
games	15%
shows	13%

Based on the results, the amusement park infers that the most popular attraction is rollercoasters. (Lesson 1)

   A. What sampling method best describes this scenario?

   voluntary response; biased

   B. Determine whether the inference made by the amusement park is valid. Explain your reasoning.

   not valid; Sample answer: The inference is based upon a biased sample.

3. **Equation Editor** A food truck owner is ordering ingredients for their grand opening weekend. In order to determine how much is needed, he conducts a survey of 75 randomly selected potential customers asking which item they are likely to order. The results are shown in the table. (Lesson 2)

Item	Frequency
Tacos	23
Burritos	18
Chimichangas	13
Fajitas	9
Tamales	12

The owner expects to have 400 customers over the weekend. Based on the survey results, predict the number of burritos that will be sold.

96

4. **Multiple Choice** A grocery store conducts a random survey of its customers and finds that 78% would sign up for a rewards program if it were offered. If the rewards program is rolled out this weekend and there are 1,550 customers, how many of them would you expect to sign up for the program? (Lesson 2)

   (A) 341 customers
   (B) 429 customers
   (C) 1,145 customers
   (D) 1,209 customers

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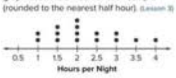
5. Open Response In order to analyze how much time students typically spend doing homework each night, a teacher takes 15 random samples of 20 students each. The graph shows the mean of each sample (rounded to the nearest half hour). (Lesson 3)



A. What is the best estimate of the mean number of hours spent per night doing homework of the population?

2

B. Describe the variability in the distribution.

The data vary as low as 1 and as high as 4. To the left, the data vary 1 off of the center, and to the right, the data vary 2 off of the center.

6. Multiple Choice Wildlife conservationists want to estimate the deer population in a region. They place an identifying tag on 220 deer. Several months later, researchers investigate 8 samples of 50 deer and record how many have a tag. Based on these results, how many deer should they estimate live in the region? (Lesson 3)

Sample	1	2	3	4	5	6	7	8
Number of Tagged Deer	3	1	5	4	2	2	1	4

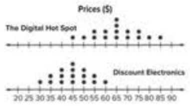
(A) 2,750 deer

(B) 3,175 deer

(C) 3,500 deer

(D) 4,000 deer

7. Multiple Choice The double dot plot shows the costs, in dollars, of MP3 players at two different stores. (Lesson 5)



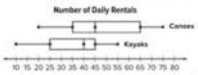
Which of the following are appropriate inferences that can be made about the data?

(A) Both distributions are asymmetric.

(B) The mean price at the Digital Hot Spot is higher than the mean price at Discount Electronics.

(C) The prices at Discount Electronics have a higher variability than the prices at the Digital Hot Spot.

8. Table Item The double box plot shows the number of daily canoe and kayak rentals at Riverside Adventures. Based on the data, identify the type of equipment rental to which each statement applies. (Lesson 4)



	Canoes	Kayaks
This data set has an interquartile range of 20.		✓
This data set has the day with the highest number of rentals.	✓	
This data set has a smaller displayed measure of variation.		✓

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Module 11

Geometric Figures

Module Goal

Draw, describe, and solve problems involving geometric figures.

Focus

Domain: Geometry

Additional Cluster(s):

7.G.A D raw, construct and describe geometrical figures and describe the relationships between them.

7.G.B Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Standards for Mathematical Content:

7.G.B.5 Use fac ts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Also addresses 7.G.A.1, 8.G.A.5.

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- use a protractor to draw an angle of a specified measure
- solve one-step and two-step equations
- convert measurement units within the customary and metric systems

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students solved real-world and mathematical problems involving area, surface area, and volume.
6.G.A.1, 6.G.A.2, 6.G.A.3, 6.G.A.4

Now
Students draw, describe, and solve problems involving geometric figures.
7.G.A.1, 7.G.A.2, 7.G.A.3, 7.G.B.5, 8.G.A.5

Next
Students will find the circumference and area of circles.
7.G.B.4

Rigor

The Three Pillars of Rigor

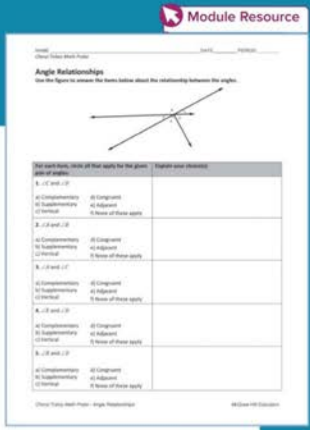
In this module, students will draw on their knowledge of lines and angles, equivalent ratios, and three-dimensional figures to gain *understanding* of angles, parallel lines, triangles, and scale drawings. They will use this understanding to develop *fluency* with vertical, adjacent, complementary and supplementary angles, angle relationships and triangles, classifying and drawing triangles, scale drawings and three-dimensional figures. They will *apply* their fluency to solve real-world problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
11-1	Vertical and Adjacent Angles	7.G.B.5, *Also addresses 7.EE.B.3, 7.EE.B.4.A*	1	0.5
11-2	Complementary and Supplementary Angles	7.G.B.5, *Also addresses 7.EE.B.3, 7.EE.B.4.A*	1	0.5
Put It All Together 1: Lessons 11-1 and 11-2			0.5	0.25
11-3	Angle Relationships and Parallel Lines	8.G.A.5	2	1
11-4	Triangles	7.G.A.2	2	1
11-5	Angle Relationships and Triangles	8.G.A.5	2	1
Put It All Together 2: Lessons 11-3 through 11-5			0.5	0.25
11-6	Scale Drawings	7.G.A.1, *Also addresses 7.RP.A.2, 7.RP.A.2.B, 7.RP.A.3, 7.NS.A.3, 7.EE.B.3*	1	0.5
11-7	Three-Dimensional Figures	7.G.A.3	1	0.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	14	7

Formative Assessment Math Probe
Angle Relationships



Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will classify selected pairs of angles, and explain their choices.

Targeted Concept Understand the terms *complementary, supplementary, vertical, congruent,* and *adjacent* and use reasoning about angles to analyze relationships.

Targeted Misconceptions

- Students may use intuitive rules to incorrectly assume congruency.
- Students may have inaccurate notions about the relationships of angles and their measurements.

Assign the probe after Lesson 2.

Correct Answers: 1. a, e; **2.** b, e; **3.** c, d; **4.** e; **5.** f

Collect and Assess Student Work

If the student selects...	Then the student likely...
1. an answer other than a	did not recognize ∠*C* and ∠*D* as forming a 90-degree angle.
1. an answer other than e	used intuitive rules based on what the angles look like, answered based on a perception of the measures instead of what is actually known.
5. c	viewed vertical as “across from.”
Various incorrect responses	confused the meanings of the various terms.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Angles, Lines, and Polygons
- Lesson 1, Examples 1–5
- Lesson 2, Examples 1–4

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

![IGNITE!]

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How does geometry help to describe objects? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

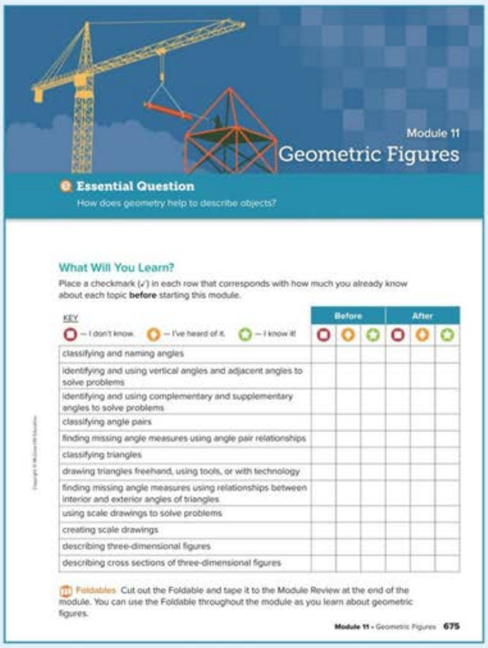
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of bridges, roof trusses, boxes, and paint containers to introduce the idea of geometric figures. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 11

Geometric Figures

Essential Question

How does geometry help to describe objects?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know.	I've heard of it.	I know it!	I don't know.	I've heard of it.	I know it!
classifying and naming angles						
identifying and using vertical angles and adjacent angles to solve problems						
identifying and using complementary and supplementary angles to solve problems						
classifying angle pairs						
finding missing angle measures using angle pair relationships						
classifying triangles						
drawing triangles freehand, using tools, or with technology						
finding missing angle measures using relationships between interior and exterior angles of triangles						
using scale drawings to solve problems						
creating scale drawings						
describing three-dimensional figures						
describing cross sections of three-dimensional figures						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about geometric figures.

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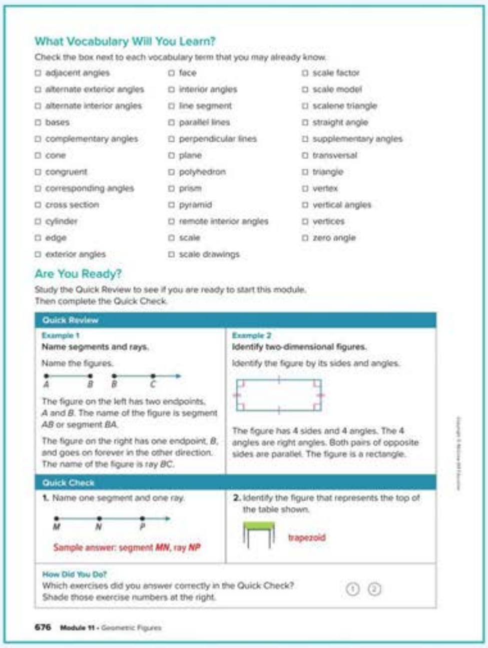
Interactive Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ adjacent angles
- ☐ alternate exterior angles
- ☐ alternate interior angles
- ☐ bases
- ☐ complementary angles
- ☐ cone
- ☐ congruent
- ☐ corresponding angles
- ☐ cross section
- ☐ cylinder
- ☐ edge
- ☐ exterior angles
- ☐ face
- ☐ interior angles
- ☐ line segment
- ☐ parallel lines
- ☐ perpendicular lines
- ☐ plane
- ☐ polyhedron
- ☐ prism
- ☐ pyramid
- ☐ remote interior angles
- ☐ scale
- ☐ scale drawings
- ☐ scale factor
- ☐ scale model
- ☐ scalene triangle
- ☐ straight angle
- ☐ supplementary angles
- ☐ transversal
- ☐ triangle
- ☐ vertex
- ☐ vertical angles
- ☐ vertices
- ☐ zero angle

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Name segments and rays.

Name the figures.

The figure on the left has two endpoints, *A* and *B*. The name of the figure is segment *AB* or segment *BA*.

The figure on the right has one endpoint, *B*, and goes on forever in the other direction. The name of the figure is ray *BC*.

Example 2
Identify two-dimensional figures.

Identify the figure by its sides and angles.

The figure has 4 sides and 4 angles. The 4 angles are right angles. Both pairs of opposite sides are parallel. The figure is a rectangle.

Quick Check

1. Name one segment and one ray. Sample answer: segment *MN*, ray *NP*
2. Identify the figure that represents the top of the table shown. trapezoid

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define Supplementary angles are two angles with measures that have a sum of 180°.

Example Suppose $m\angle A = 120°$ and $m\angle B = 60°$. Because the sum of their angle measures is 180°, they are supplementary angles.

Ask What is the measure of an angle supplementary to an angle with a measure of 56°? 124°

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- drawing angles
- drawing line segments
- converting measures of length
- identifying two-dimensional figures

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Angles, Lines, and Polygons** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Promote Process Over Results

The process that a student takes as he or she encounters a new problem is just as important—if not more important—than the results achieved.

How Can I Apply It?

Encourage students to consider the **Think About It!** prompts that precede many of the Examples. These prompts often ask students how they might begin to solve the problem, or have them digest the information they are given in attempts to understand what they might do next. Have students discuss their strategies with a partner and/or engage students in a whole-class discussion. Be sure to support the process and reward student effort as they explore and work through problems instead of merely rewarding the correct answer.

Lesson 11-1

Vertical and Adjacent Angles

7.G.B.5

LESSON GOAL

Students will identify vertical and adjacent angles and use what they know to find missing values.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Angles

Learn: Name Angles
Example 1: Name Angles

Explore: Vertical and Adjacent Angle Pairs

Learn: Identify Vertical Angles
Example 2: Identify Vertical Angles
Learn: Use Vertical Angles to Find Missing Values
Example 3: Use Vertical Angles to Find Missing Values
Learn: Identify Adjacent Angles
Example 4: Identify Adjacent Angles
Learn: Use Adjacent Angles to Find Missing Values
Example 5: Use Adjacent Angles to Find Missing Values
Apply: Art

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 63 of the *Language Development Handbook* to help your students build mathematical language related to vertical and adjacent angles.

ELL You can use the tips and suggestions on page T63 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.B** by identifying vertical and adjacent angles and finding missing values.
Standards for Mathematical Content: **7. G.B.5,** *Also addresses 7.EE.B.3, 7.EE.B.4.A*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students solved real-world and mathematical problems involving area, surface area, and volume.
6.G.A.1, 6.G.A.2, 6.G.A.3, 6.G.A.4

Now
Students identify vertical and adjacent angles and write and solve equations to find missing values.
7.G.B.5

Next
Students will identify complementary and supplementary angles and write and solve equations to find missing values.
7.G.B.5

Rigor

The Three Pillars of Rigor

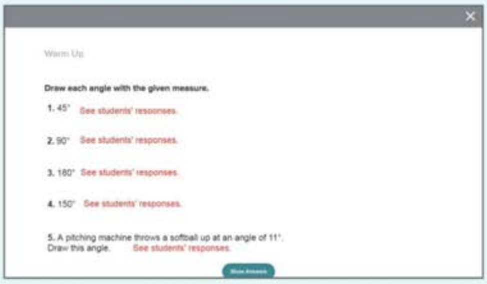
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of lines and angles to develop *understanding* of naming angles, identifying vertical angles, and identifying adjacent angles. They use this understanding to build *fluency* in naming angles, using vertical angles, and using adjacent angles.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- drawing angles (Exercises 1–5)

Answers

1–5. See students' responses.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about engineers using different combinations of vertical and adjacent angles to ensure the safety of roller coasters.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate class discussion. Additional questions are available online.

Ask:

- Y ou have previously learned the term *acute angle*. Describe an acute angle in your own words. Sample answer: An acute angle has a measure that is greater than 0 degrees, but less than 90 degrees.
- Use the meaning of the term *adjacent* to make a conjecture as to what *adjacent angles* might be. Sample answer: Adjacent means next to, so adjacent angles may be angles that are next to one another.
- The term *congruent* comes from the Latin *congruere* which means *to be in agreement, or in harmony*. What do you think it might mean for two angles to be *congruent?* Sample answer: Two angles that are congruent may agree on the size and shape, which means they may have the same size and shape.

Learn Angles

Objective

Students will understand how to classify angles by their measures.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 2, encourage them to use the face of a clock to make sense of what it might mean for an angle to have a measure greater than 180°.

6 Attend to Precision As students complete the drag and drop activity on Slide 1, encourage them to learn and appropriately use the mathematical terms.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Is it possible for an angle to have a measure greater than 180°? Explain. yes; Sample answer: At 8:00 on a clock, the angle shown from 12:00 to 8:00 going clockwise is greater than 180°.

Learn Name Angles

Objective

Students will understand the different ways in which to name angles.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to reference the rules for naming angles in their explanation for why the classmate is incorrect.

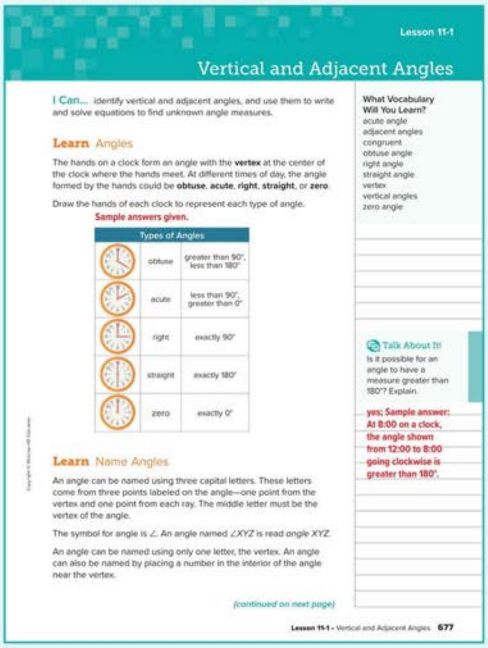
Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

A classmate states that the angle is named $\angle RTS$. Explain why this is incorrect. Sample answer: The letter for the vertex must be the middle letter in the name.



Lesson 11-1

Vertical and Adjacent Angles

I Can... identify vertical and adjacent angles, and use them to write and solve equations to find unknown angle measures.

What Vocabulary Will You Learn?
acute angle
adjacent angles
congruent
obtuse angle
right angle
straight angle
vertex
vertical angles
zero angle

Learn Angles

The hands on a clock form an angle with the **vertex** at the center of the clock where the hands meet. At different times of day, the angle formed by the hands could be **obtuse**, **acute**, **right**, **straight**, or **zero**.

Draw the hands of each clock to represent each type of angle.

Sample answers given.

Types of Angles		
[clock]	obtuse	greater than 90°, less than 180°
[clock]	acute	less than 90°, greater than 0°
[clock]	right	exactly 90°
[clock]	straight	exactly 180°
[clock]	zero	exactly 0°

Talk About It! Is it possible for an angle to have a measure greater than 180°? Explain.

yes; Sample answer: At 8:00 on a clock, the angle shown from 12:00 to 8:00 going clockwise is greater than 180°.

Learn Name Angles

An angle can be named using three capital letters. These letters come from three points labeled on the angle—one point from the vertex and one point from each ray. The middle letter must be the vertex of the angle.

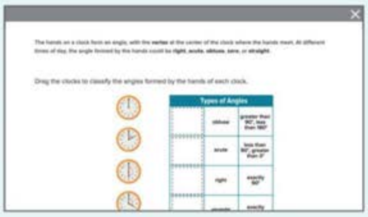
The symbol for angle is ∠. An angle named ∠XYZ is read *angle XYZ*.

An angle can be named using only one letter, the vertex. An angle can also be named by placing a number in the interior of the angle near the vertex.

(continued on next page)

Lesson 11-1 • Vertical and Adjacent Angles 677

Interactive Presentation



Learn, Angles, Slide 1 of 2

DRAG & DROP

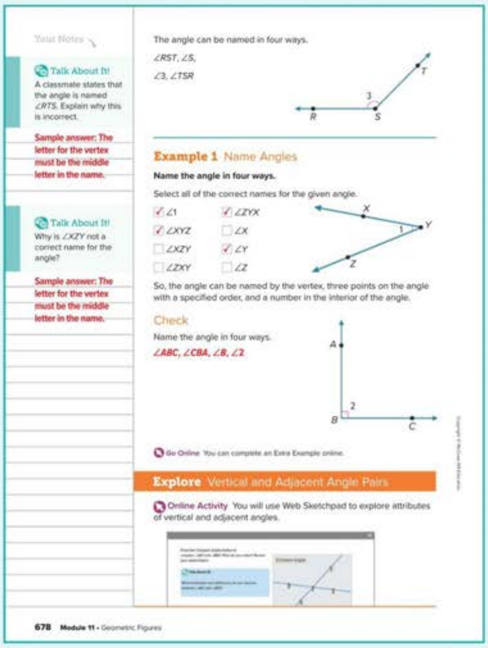
On Slide 1 of Learn, Angles, students drag clocks to classify angles formed by the hands of the clocks.

CLICK

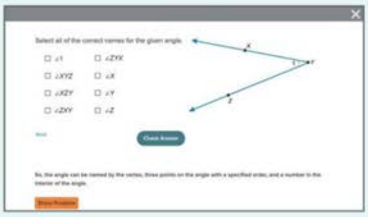
On Slide 1 of Learn, Name Angles, students select starting points to see the ways an angle can be named.

CLICK

On Slide 2 of Learn, Name Angles, students select points on the angle to see the various ways the angle can be named.



Interactive Presentation



Example 1, Name Angles, Slide 2 of 4

CLICK

On Slide 2, students select the correct names for the given angle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Name Angles

Objective

Students will name angles using different notations.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to identify and clearly explain why the name is not a correct name for the given angle.

6 Attend to Precision Encourage students to carefully follow the naming conventions for angles, and be able to explain why it is important to have naming conventions.

Questions for Mathematical Discourse

SLIDE 2

AL How can you name the angle using a number? Explain. ∠1; Sample answer: The interior of the angle, near the vertex, is labeled with the number 1.

AL How can you name the angle using only its vertex? ∠*Y*

OL Make a conjecture as to why we have naming conventions for angles. Sample answer: If there were no naming conventions, then we might not know which angle is being referred to, if there are multiple angles in a diagram.

OL Why is it not correct to name the angle as ∠*Z* ? Sample answer: *Z* is not the vertex of the angle. The only way to name an angle with a single letter is by using its vertex.

BL Suppose there is another point on ray *YX*, and this point is labeled *P*. Give two other possibilities for naming this angle. ∠*PYZ* or ∠*ZYP*

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Vertical and Adjacent Angle Pairs

Objective

Students will use Web Sketchpad to explore attributes of vertical and adjacent angle pairs.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a sketch of two intersecting lines that create four angles. Throughout this activity, students will use the sketch to compare different angle pairs that are formed and observe similarities and differences between the angles. Students will use their observations to make conjectures about the relationships between the angle measures in each pair.

Inquiry Question

What are some relationships between pairs of angles created by two intersecting lines? Sample answer: Vertical angles have the same angle measure. Adjacent angles have an angle measure sum of 180°.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

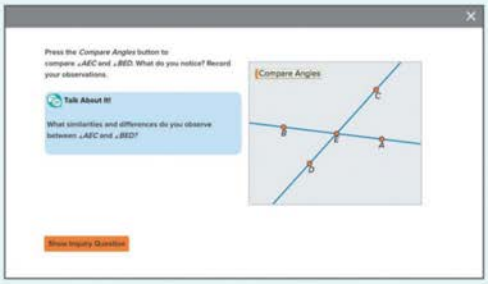
Mathematical Discourse

What similarities and differences do you observe between ∠*AEC* and ∠*BED*? Sample answer: They appear to have the same angle measure but are on opposite sides of the vertex.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 3 of 8

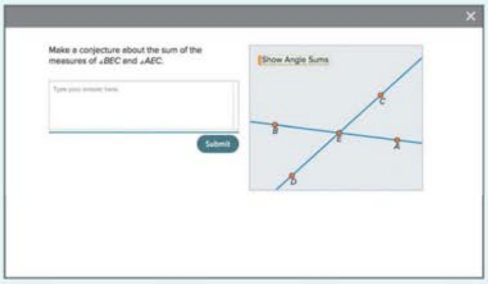
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore relationships between pairs of angles created by intersecting lines.

DRAG & DROP

On Slide 2, students drag angle names to identify angles created by intersecting lines.

Interactive Presentation



Explore, Slide 7 of 8

TYPE

On Slides 4 and 6, students make conjectures about angle relationships.

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Vertical and Adjacent Angle Pairs *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore the relationships between angles that are formed by intersecting lines.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Press the *Show Angle Sums* button. Drag points *C* and *A* to change the angle measures. Does your conjecture hold true? Sample answer: Yes, the sums of the measures of the adjacent angle pairs are 180°.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Identify Vertical Angles

Objective

Students will understand the relationship between vertical angles.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

Vertical angles share a common point. How can you name or describe that point to a classmate? Sample answer: It is called the vertex.

Example 2 Identify Vertical Angles

Objective

Students will identify vertical angle pairs.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the diagram, in your own words. Sample answer: There are three intersecting lines that form six angles.

AL How can you identify vertical angles? Sample answer: Look for angles that are opposite sides of the vertex formed by any two intersecting lines.

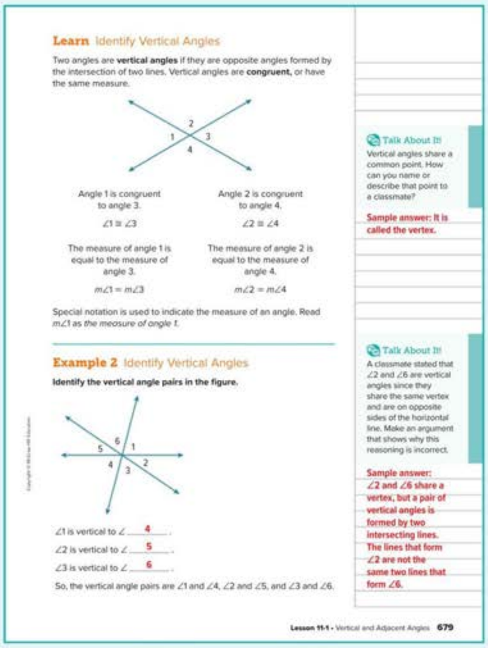
OL How many pairs of vertical angles are formed by these three intersecting lines? 3 angle pairs

OL Is it possible to have three angles that are vertical angles? Explain. no; Sample answer: Vertical angles are formed when two lines intersect. Only two angles can be opposite the same vertex, not three.

BL Will there always be three pairs of vertical angles formed if three lines intersect? Explain. no; Sample answer: There will be three pairs of vertical angles if the three lines intersect in one point. But if they intersect at different points, then there won't necessarily be three pairs of vertical angles formed.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Identify Vertical Angles

Two angles are **vertical angles** if they are opposite angles formed by the intersection of two lines. Vertical angles are **congruent,** or have the same measure.

Angle 1 is congruent to angle 3.

$\angle 1 \cong \angle 3$

Angle 2 is congruent to angle 4.

$\angle 2 \cong \angle 4$

The measure of angle 1 is equal to the measure of angle 3.

$m\angle 1 = m\angle 3$

The measure of angle 2 is equal to the measure of angle 4.

$m\angle 2 = m\angle 4$

Special notation is used to indicate the measure of an angle. Read $m\angle 1$ as *the measure of angle 1.*

Talk About It! Vertical angles share a common point. How can you name or describe that point to a classmate?

Sample answer: It is called the vertex.

Example 2 Identify Vertical Angles

Identify the vertical angle pairs in the figure.

$\angle 1$ is vertical to $\angle$ 4.

$\angle 2$ is vertical to $\angle$ 5.

$\angle 3$ is vertical to $\angle$ 6.

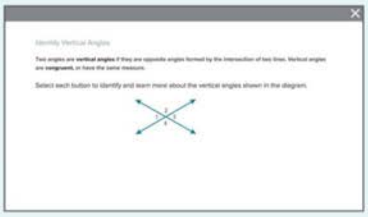
So, the vertical angle pairs are $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 5$, and $\angle 3$ and $\angle 6$.

Talk About It! A classmate stated that $\angle 2$ and $\angle 6$ are vertical angles since they share the same vertex and are on opposite sides of the horizontal line. Make an argument that shows why this reasoning is incorrect.

Sample answer: $\angle 2$ and $\angle 6$ share a vertex, but a pair of vertical angles is formed by two intersecting lines. The lines that form $\angle 2$ are not the same two lines that form $\angle 6$.

Lesson 11-1 • Vertical and Adjacent Angles 679

Interactive Presentation



Learn, Identify Vertical Angles, Slide 1 of 2

CLICK

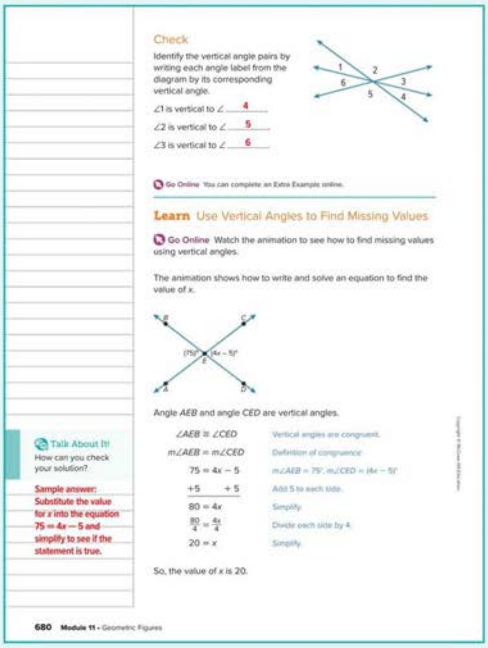
On Slide 1 of the Learn, students select buttons to identify and learn more about the vertical angles shown in the diagram.

DRAG & DROP

On Slide 2 of Example 2, students drag angle labels to identify vertical angle pairs.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Identify the vertical angle pairs by writing each angle label from the diagram by its corresponding vertical angle.

∠1 is vertical to ∠ 4

∠2 is vertical to ∠ 5

∠3 is vertical to ∠ 6

Go Online You can complete an Extra Example online.

Learn Use Vertical Angles to Find Missing Values

Go Online Watch the animation to see how to find missing values using vertical angles.

The animation shows how to write and solve an equation to find the value of *x*.

Angle *AEB* and angle *CED* are vertical angles.

$\angle AEB \cong \angle CED$	Vertical angles are congruent.
$m\angle AEB = m\angle CED$	Definition of congruence
$75 = 4x - 5$	$m\angle AEB = 75°$, $m\angle CED = (4x - 5)°$
$+5 \quad +5$	Add 5 to each side.
$80 = 4x$	Simplify.
$\frac{80}{4} = \frac{4x}{4}$	Divide each side by 4.
$20 = x$	Simplify.

So, the value of *x* is 20.

Talk About It!
How can you check your solution?

Sample answer: Substitute the value for *x* into the equation 75 = 4*x* − 5 and simplify to see if the statement is true.

680 Module 11 • Geometric Figures

Interactive Presentation

Learn, Use Vertical Angles to Find Missing Values

WATCH

Students watch an animation that explains how to find missing values using vertical angles.

Learn Use Vertical Angles to Find Missing Values

Objective

Students will understand how to use the properties of vertical angles to find missing values.

Go Online

Have students watch the animation on Slide 1. The animation illustrates how to find missing values using vertical angles.

Teaching Notes

SLIDE 1

Play the animation for the class. You may wish to pause the animation when the notation *∠AEB and ∠CED are vertical angles* first appears. Ask students what they know about the measures of vertical angles. Some students may say that vertical angles are congruent so the measures are equal.

Continue playing the animation. You may wish to pause the animation again when the notation $m\angle AEB = m\angle CED$ first appears. Point out to students that the first line is a *congruency statement* and expresses the fact that two figures have the same size and shape. The second line is an equation and expresses that the measures of the two angles are equal. Students commonly mistake writing a congruency statement using the equals sign, but a congruency statement must be written using the congruence symbol ≅. Students can then use the algebraic expressions that represent the congruent angles to write and solve an equation using an equals sign.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Use Vertical Angles to Find Missing Values

Objective

Students will use the properties of vertical angles to find missing values.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enco urage students to make sense of the solution within the context of the diagram.

6 Attend to Precision Encourage students to apply their knowledge of vertical angles to set up and solve a correct two-step equation.

Questions for Mathematical Discourse

SLIDE 2

AL What kind of relationship do the two labeled angles have? They are vertical angles.

OL What is true about the measures of vertical angles? They are equal.

OL What equation can you use to find the value of x? $2x + 2 = 130$

BL What is the measure of each of the other two unlabeled angles? Explain. 50°; Because they are also vertical, they are congruent to each other. Each one has a measure of 50° because the angle labeled 130° and one of the unlabeled angles form a straight angle, which has a measure of 180°.

SLIDE 3

AL How many steps will it take to solve this equation? Explain. two steps; Sample answer: This is a two-step equation. The two operations are multiplication and addition.

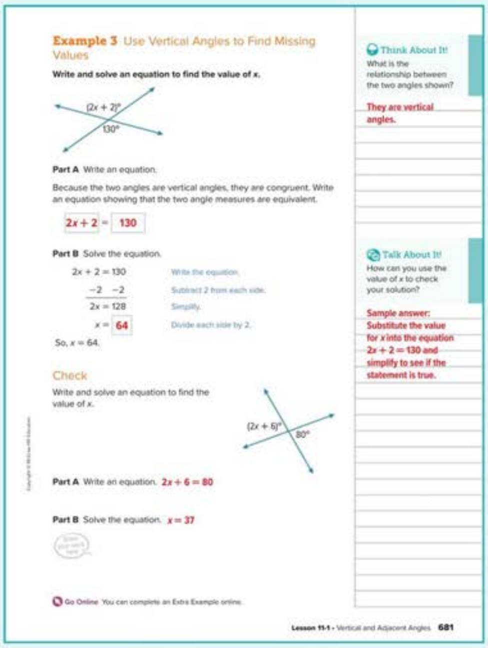
OL When solving for x, does it mean that one of the angle measures is 64°? Explain. no; Sample answer: I need to replace x with 64 in the expression $(2x + 2)°$ to find the measure of the angle.

OL Without solving for x, what is the measure of the angle labeled $(2x + 2)°$? Explain. 130°; Vertical angles are congruent.

BL Describe how to solve this equation using the Distributive Property. Sample answer: Write the equation as $2(x + 1) = 130$. Divide each side by 2 to obtain the equation $x + 1 = 65$. Then subtract 1 from each side to obtain $x = 64$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Example 3 Use Vertical Angles to Find Missing Values

Write and solve an equation to find the value of x.

Part A Write an equation.

Because the two angles are vertical angles, they are congruent. Write an equation showing that the two angle measures are equivalent.

$2x + 2 = 130$

Part B Solve the equation.

$2x + 2 = 130$ Write the equation.
$-2 \quad -2$ Subtract 2 from each side.
$2x = 128$ Simplify.
$x = 64$ Divide each side by 2.

So, $x = 64$.

Check

Write and solve an equation to find the value of x.

Part A Write an equation. $2x + 6 = 80$

Part B Solve the equation. $x = 37$

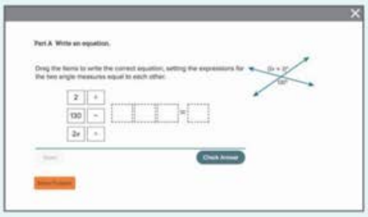
Go Online You can complete an Extra Example online.

Think About It! What is the relationship between the two angles shown? They are vertical angles.

Talk About It! How can you use the value of x to check your solution? Sample answer: Substitute the value for x into the equation $2x + 2 = 130$ and simplify to see if the statement is true.

Lesson 11-1 • Vertical and Adjacent Angles 681

Interactive Presentation



Example 3, Use Vertical Angles to Find Missing Values, Slide 2 of 5

DRAG & DROP

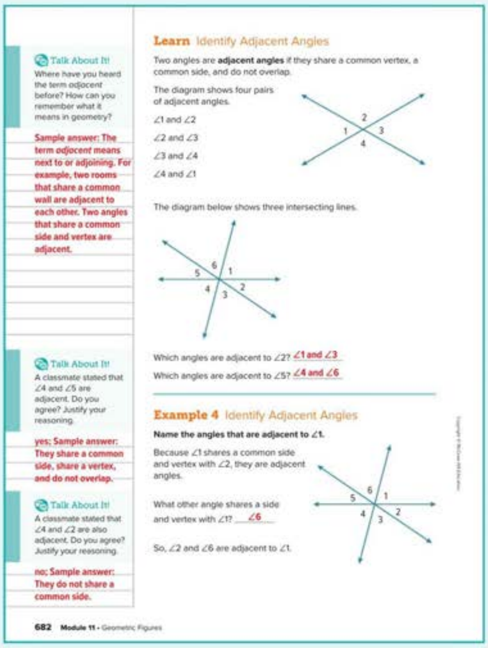
On Slide 2, students drag items to write the correct equation.

TYPE

On Slide 3, students determine the solution to the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Talk About It! Where have you heard the term *adjacent* before? How can you remember what it means in geometry?

Sample answer: The term *adjacent* means next to or adjoining. For example, two rooms that share a common wall are adjacent to each other. Two angles that share a common side and vertex are adjacent.

Learn Identify Adjacent Angles

Two angles are **adjacent angles** if they share a common vertex, a common side, and do not overlap.

The diagram shows four pairs of adjacent angles.

$\angle 1$ and $\angle 2$

$\angle 2$ and $\angle 3$

$\angle 3$ and $\angle 4$

$\angle 4$ and $\angle 1$

The diagram below shows three intersecting lines.

Which angles are adjacent to $\angle 2$? $\angle 1$ and $\angle 3$

Which angles are adjacent to $\angle 5$? $\angle 4$ and $\angle 6$

Talk About It! A classmate stated that $\angle 4$ and $\angle 5$ are adjacent. Do you agree? Justify your reasoning.

yes; Sample answer: They share a common side, share a vertex, and do not overlap.

Example 4 Identify Adjacent Angles

Name the angles that are adjacent to $\angle 1$.

Because $\angle 1$ shares a common side and vertex with $\angle 2$, they are adjacent angles.

What other angle shares a side and vertex with $\angle 1$? $\angle 6$

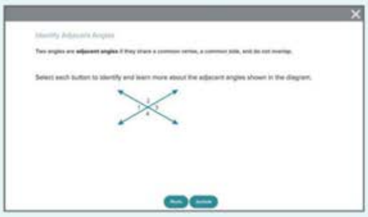
So, $\angle 2$ and $\angle 6$ are adjacent to $\angle 1$.

Talk About It! A classmate stated that $\angle 4$ and $\angle 2$ are also adjacent. Do you agree? Justify your reasoning.

no; Sample answer: They do not share a common side.

682 Module 11 • Geometric Figures

Interactive Presentation



Learn, Identify Adjacent Angles, Slide 1 of 3

CLICK

On Slide 1 of the Learn, students select buttons to identify and learn more about the adjacent angles shown in the diagram.

CLICK

On Slide 2 of Example 4, students select from a drop-down menu the angle that is adjacent to a given angle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Identify Adjacent Angles

Objective

Students will understand the relationship between adjacent angles.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 3

Mathematical Discourse

Where have you heard the term *adjacent* before? How can you remember what it means in geometry? Sample answer: The term *adjacent* means next to or adjoining. For example, two rooms that share a common wall are adjacent to each other. Two angles that share a common side and vertex are adjacent.

Example 4 Identify Adjacent Angles

Objective

Students will identify adjacent angle pairs.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to use the precise meaning of adjacent angles to identify the adjacent angle pairs in the diagram.

Questions for Mathematical Discourse

SLIDE 2

AL What do adjacent angles share? a common side and vertex

OL Is there more than one angle that shares a common side and vertex with $\angle 1$? Explain. yes; $\angle 2$ and $\angle 6$ each share a common side with $\angle 1$.

OL Name two other pairs of adjacent angles in the diagram. Sample answer: $\angle 5$ and $\angle 6$ are adjacent angles, and $\angle 2$ and $\angle 3$ are adjacent angles.

BL How many pairs of adjacent angles are in the diagram? 6 angle pairs

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Use Adjacent Angles to Find Missing Values

Objective

Students will understand how to use the properties of adjacent angles to find missing values.

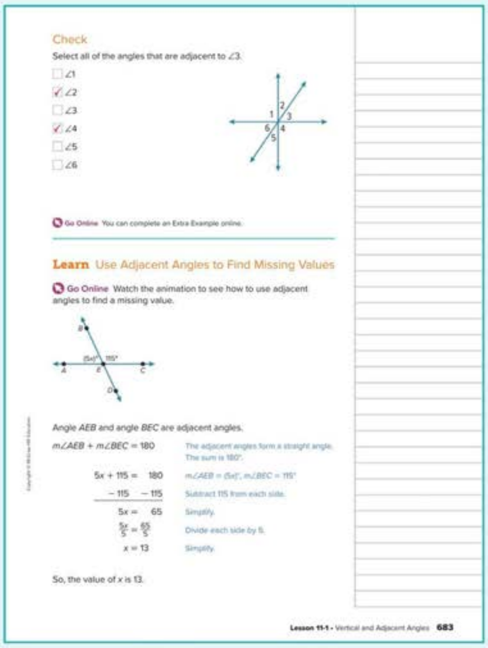
Teaching Notes

SLIDE 1

Point out to students that not every pair of adjacent angles forms a straight line. You may wish to ask students to draw their own diagrams that illustrate adjacent angles in which pairs of adjacent angles do not form straight lines. When a pair of adjacent angles does form a straight line, the sum of their angle measures is 180 degrees. Students can then use the algebraic expressions from the diagram to write and solve an equation.

Go Online

Have students watch the animation on Slide 1. The animation illustrates how adjacent angles can be used to find a missing value.



Check

Select all of the angles that are adjacent to ∠3.

- [ ] ∠1
- [x] ∠2
- [ ] ∠3
- [x] ∠4
- [ ] ∠5
- [ ] ∠6

Go Online You can complete an Extra Example online.

Learn Use Adjacent Angles to Find Missing Values

Go Online Watch the animation to see how to use adjacent angles to find a missing value.

Angle *AEB* and angle *BEC* are adjacent angles.

$m\angle AEB + m\angle BEC = 180$ — The adjacent angles form a straight angle. The sum is 180°.

$5x + 115 = 180$ — $m\angle AEB = (5x)°$, $m\angle BEC = 115°$

$-115 \quad -115$ — Subtract 115 from each side.

$5x = 65$ — Simplify.

$\frac{5x}{5} = \frac{65}{5}$ — Divide each side by 5.

$x = 13$ — Simplify.

So, the value of *x* is 13.

Lesson 11-1 • Vertical and Adjacent Angles 683

Interactive Presentation

Learn, Use Adjacent Angles to Find Missing Values

WATCH

Students watch an animation to see how to use adjacent angles to find a missing value.

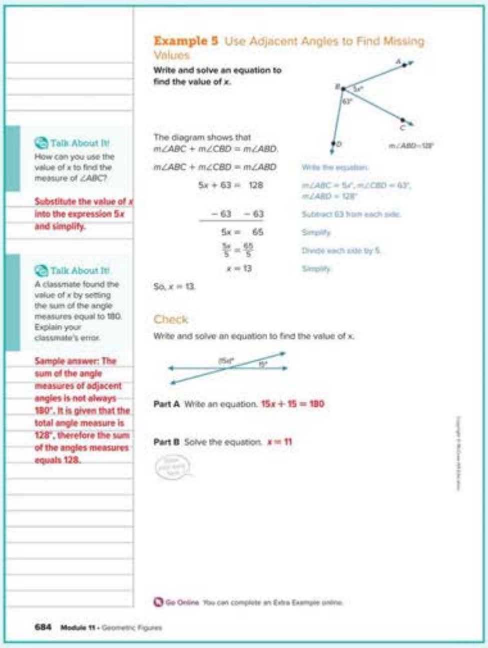
DIFFERENTIATE

Language Development Activity ELL

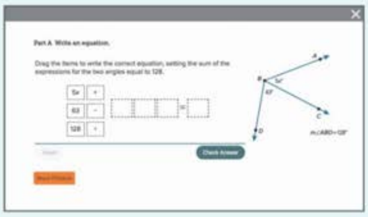
If any of your students are struggling with differentiating between the different types of angles they encounter, have them work with a partner to create a poster or graphic organizer that illustrates the different angles in this lesson. They should include examples of vertical angles and adjacent angles. They should include descriptions and/or properties of the angles and how to find missing angle measures in a diagram. Some sample properties are shown.

- Vertical angles are formed when two lines intersect.
- Vertical angles are on opposite sides of the point of intersection.
- Vertical angles have the same measure.
- Adjacent angles share a common side and vertex, but do not overlap.
- Adjacent angles may form a straight line, so the sum of their measures is 180°.

Students can add other types of angles to the poster or graphic organizer as they move through the module. They can present their work to the class, or you can hang the posters or graphic organizers around the classroom.



Interactive Presentation



Example 5, Use Adjacent Angles to Find Missing Values, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag items to write the correct equation.

TYPE

On Slide 3, students determine the solution to the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Use Adjacent Angles to Find Missing Values

Objective

Students will use the properties of adjacent angles to find missing values.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the solution within the context of the diagram.

6 Attend to Precision Encourage students to apply their knowledge of adjacent angles to set up and solve a correct two-step equation.

Questions for Mathematical Discourse

SLIDE 2

AL What type of angles are $\angle DBC$ and $\angle ABC$? Explain. adjacent angles; They share a vertex and a common side.

OL A classmate states that $5x = 63$ because $\angle DBC$ and $\angle ABC$ are congruent. Is this correct? Explain. no; $\angle DBC$ and $\angle ABC$ are not vertical angles, therefore, they may not be congruent.

OL What equation can be used to find the value of x? Explain. $63 + 5x = 128$; Sample answer: The diagram indicates that $m\angle ABD = 128°$ and $m\angle DBC + m\angle ABC = m\angle ABD$.

BL Without solving the equation, what must be true about $5x$? Explain. $5x = 65$; Sample answer: The sum of the two adjacent angles is 128°. So, the angle labeled $5x$ must have a measure of $128° - 63°$, or 65°.

SLIDE 3

AL Explain how to solve the equation. Subtract 63 from each side. Then divide each side by 5.

OL How can you check your solution? Sample answer: Replace x with 13 in the equation $5x + 63 = 128$ to verify it is a true statement.

OL Once you know that $x = 13$, does this mean that $m\angle ABC = 13°$? Explain. no; Sample answer: $m\angle ABC = 5x°$, not $x°$. To find $m\angle ABC$, evaluate the expression $5x$ when $x = 13$; $m\angle ABC = 65°$.

BL A classmate wrote the equation $63 = 128 - 5x$. Is this equation correct? Explain. yes; Sample answer: $m\angle DBC$ is equal to $m\angle ABD$ minus $m\angle ABC$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Art

Objective

Students will come up with their own strategy to solve an application problem involving vertical and adjacent angles.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the relationship between $\angle A$ and $\angle B$?
- What is the relationship between the 42° angle and $\angle B$?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they used to defend their solution.

Apply Art

Tamika is using lines and angles to create abstract art. She needs to find the measure of $\angle A$ to continue the pattern in the art. What is the measure of $\angle A$?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

48°; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It! How can you solve the problem another way?

See students' responses.

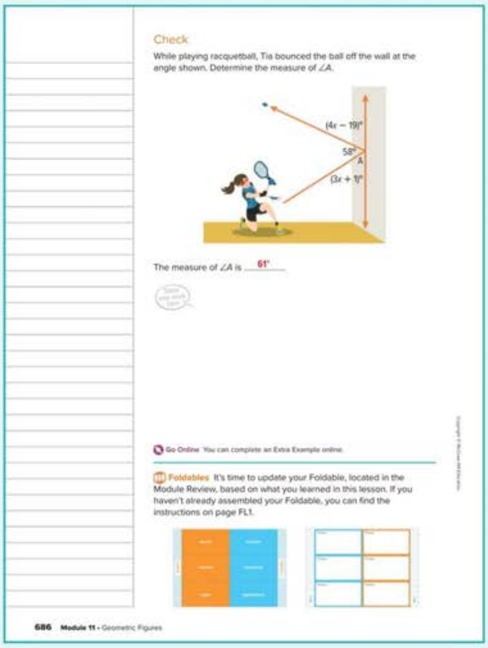
Lesson 11-1 • Vertical and Adjacent Angles 685

Interactive Presentation

Apply, Art

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

While playing racquetball, Tia bounced the ball off the wall at the angle shown. Determine the measure of ∠A.

The measure of ∠A is 61°.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

686 Module 11 • Geometric Figures

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about different types of angles. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How does geometry help to describe objects?

In this lesson, students learned how to classify vertical and adjacent angles. Encourage them to discuss with a partner how they can use this terminology to describe real-world objects. For example, they may state that the angle pairs formed by two intersecting roads can be described as vertical and adjacent.

Exit Ticket

Refer to the Exit Ticket slide. Explain the difference between vertical angles and adjacent angles. Sample answer: Vertical angles are opposite angles formed by the intersection of two lines. Adjacent angles share a common vertex, a common side, and do not overlap.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11–14
- ALEKS® Angle Relationships

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 10, 13
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Classifying and Measuring Angles

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS® Classifying and Measuring Angles

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

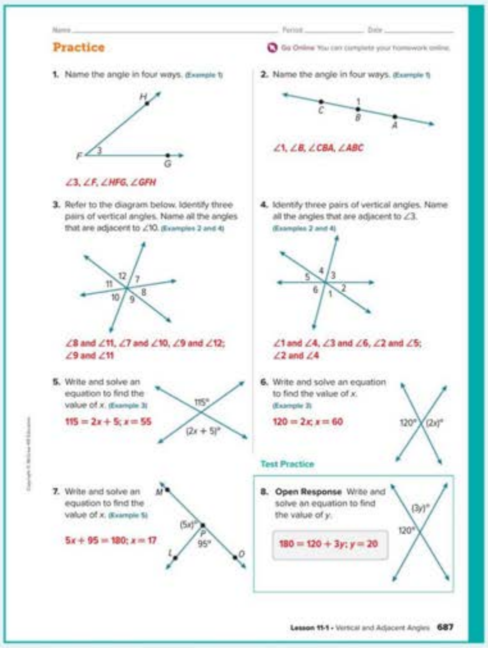
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	name angles	1, 2
1	identify vertical and adjacent angle pairs	3, 4
1	use vertical angles to find missing values	5, 6
1	use adjacent angles to find missing values	7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving adjacent angles	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may identify vertical angles incorrectly by finding non-adjacent angles located on opposite sides of a line. In Exercise 4, students may identify ∠2 and ∠4 as vertical angles. Explain to students that vertical angles are formed by the same two intersecting lines.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Name the angle in four ways. (Example 1)

∠3, ∠F, ∠HFG, ∠GFH

2. Name the angle in four ways. (Example 1)

∠1, ∠B, ∠CBA, ∠ABC

3. Refer to the diagram below. Identify three pairs of vertical angles. Name all the angles that are adjacent to ∠10. (Examples 2 and 4)

∠8 and ∠11, ∠7 and ∠10, ∠9 and ∠12; ∠9 and ∠11

4. Identify three pairs of vertical angles. Name all the angles that are adjacent to ∠3. (Examples 2 and 4)

∠1 and ∠4, ∠3 and ∠6, ∠2 and ∠5; ∠2 and ∠4

5. Write and solve an equation to find the value of x. (Example 3)

$115 = 2x + 5; x = 55$

6. Write and solve an equation to find the value of x. (Example 3)

$120 = 2x; x = 60$

7. Write and solve an equation to find the value of x. (Example 5)

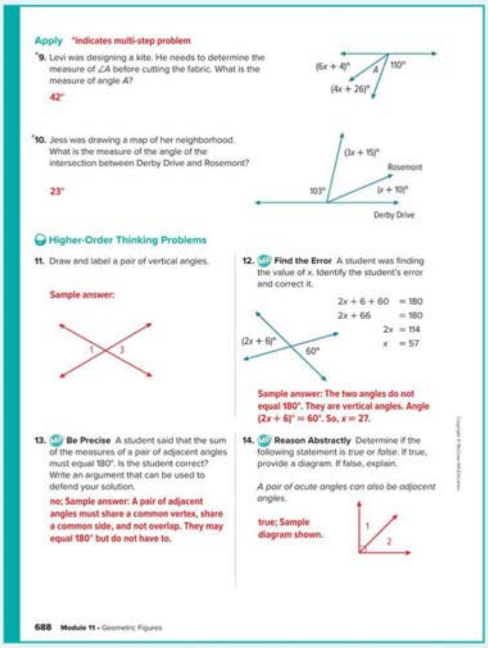
$5x + 95 = 180; x = 17$

Test Practice

8. **Open Response** Write and solve an equation to find the value of y.

$180 = 120 + 3y; y = 20$

Lesson 11-1 • Vertical and Adjacent Angles 687



Apply *indicates multi-step problem

*9. Levi was designing a kite. He needs to determine the measure of ∠A before cutting the fabric. What is the measure of angle A?

42°

*10. Jess was drawing a map of her neighborhood. What is the measure of the angle of the intersection between Derby Drive and Rosemont?

23°

Higher-Order Thinking Problems

11. Draw and label a pair of vertical angles.

Sample answer:

12. MP Find the Error A student was finding the value of x. Identify the student's error and correct it.

$2x + 6 + 60 = 180$
$2x + 66 = 180$
$2x = 114$
$x = 57$

Sample answer: The two angles do not equal 180°. They are vertical angles. Angle $(2x + 6)° = 60°$. So, $x = 27$.

13. MP Be Precise A student said that the sum of the measures of a pair of adjacent angles must equal 180°. Is the student correct? Write an argument that can be used to defend your solution.

no; Sample answer: A pair of adjacent angles must share a common vertex, share a common side, and not overlap. They may equal 180° but do not have to.

14. MP Reason Abstractly Determine if the following statement is *true* or *false*. If true, provide a diagram. If false, explain.

A pair of acute angles can also be adjacent angles.

true; Sample diagram shown.

688 Module 11 • Geometric Figures

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

7.G.B.5

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students find and correct another student's mistake.

6 Attend to Precision In Exercise 13, students use their knowledge of adjacent angles to explain why they may or may not have a sum of 180 degrees.

2 Reason Abstractly and Quantitatively In Exercise 14, students determine if a statement is true or false and support their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 10 Have students work together to prepare a brief demonstration that illustrates how they solved the problem. For example, before they can determine the measure of the angle, they must first write and solve an equation to find the value of *x*. Have each pair or group of students present their response to the class.

Be sure everyone understands.

Use with Exercises 13–14 Have students work in groups of 3–4 to solve the problem in Exercise 13. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution with the class. Repeat the process for Exercise 14.

Lesson 11-2

7.G.B.5

Complementary and Supplementary Angles

LESSON GOAL

Students will identify complementary and supplementary angles and use what they know to find missing values.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Complementary and Supplementary Angle Pairs

Learn: Identify Complementary Angles
Example 1: Identify Complementary Angles
Learn: Use Complementary Angles to Find Missing Values
Example 2: Use Complementary Angles to Find Missing Values
Learn: Identify Supplementary Angles
Example 3: Identify Supplementary Angles
Learn: Use Supplementary Angles to Find Missing Values
Example 4: Use Supplementary Angles to Find Missing Values
Apply: Engineering

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Solve Complementary and Supplementary Angle Problems		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 64 of the *Language Development Handbook* to help your students build mathematical language related to complementary and supplementary angles.

ELL You can use the tips and suggestions on page T64 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.B** by identifying complementary and supplementary angles and finding missing values.
Standards for Mathematical Content: **7. G.B.5,** *Also addresses 7.EE.B.3, 7.EE.B.4.A*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students identified vertical and adjacent angles and wrote and solved equations to find missing values.
7.G.B.5

Now
Students identify complementary and supplementary angles and write and solve equations to find missing values.
7.G.B.5

Next
Students will examine relationships of angles formed by parallel lines cut by a transversal.
8.G.A.5

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

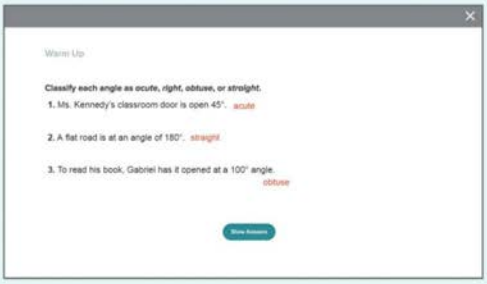
Conceptual Bridge In this lesson, students draw on their knowledge of lines and angles to develop *understanding* of complementary and supplementary angles. They use this understanding to build *fluency* in using complementary and supplementary angles. They *apply* their fluency to solve real-world problems dealing with complementary and supplementary angles.

Mathematical Background

Complementary angles are two angles in which the sum of their measures is 90°.

Supplementary angles are two angles in which the sum of their measures is 180°.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- classifying angles (Exercises 1–3)

Answers

1. acute

2. straight

3. obtuse

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about different angle measures used in the construction of a bridge, affecting the load that a bridge can support.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate class discussion.

Ask:

- How have you used the term *complement* in everyday life? Sample answer: Two colors or hues may complement each other in clothing or paint samples.
- How have you used the terms *supplement* or *supplemental* in everyday life? Sample answer: I supplement my diet with vitamins, my part-time job provides me with supplemental income.

Explore Complementary and Supplementary Angle Pairs

Objective

Students will use Web Sketchpad to explore the properties of complementary and supplementary angle pairs.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with sketches of lines and rays that intersect to form different angles. Throughout this activity, students will use Web Sketchpad to investigate and make conjectures about the sums of the measures of angles that are complementary and of angles that are supplementary.

Inquiry Question

What does it mean for angle pairs to be complementary or supplementary? Sample answer: Two angles are complementary if the sum of their angle measures is 90°; two angles are supplementary if the sum of their angle measures is 180°.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* question on Slide 2 are shown.

Talk About It!

SLIDE 2

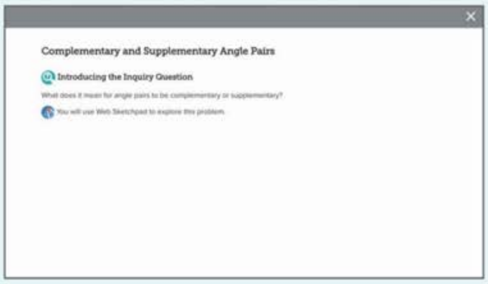
Mathematical Discourse

Press the *Show Angle Measurements* button to test your conjecture. Experiment with dragging point *D* to test your conjecture. Does your conjecture hold true?

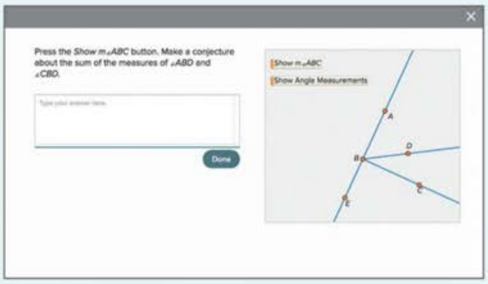
Sample answer: Yes, the sum of the angle measures is always 90°.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 6



Explore, Slide 2 of 7

WEBSKETCH PAD

Throughout the Explore, students use Web Sketchpad to explore complementary and supplementary angles.

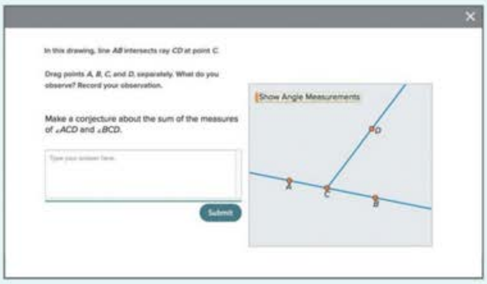
TYPE

On Slide 2, students make a conjecture about an angle measure.

TYPE

On Slide 3, students use another resource to look up the term *complementary*, and then connect that definition to this activity.

Interactive Presentation



Explore, Slide 4 of 6

TYPE

On Slide 4, students make a conjecture about the sum of two angle measures.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Complementary and Supplementary Angle Pairs *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use Web Sketchpad to explore the relationships between complementary and supplementary angles.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Press the *Show Angle Measurements* button. Experiment with dragging points *C* and *D*, separately, to test your conjecture. Does your conjecture hold true? Sample answer: Yes, the sum of the angle measures is always 180°.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Identify Complementary Angles

Objective

Students will understand the properties of complementary angles.

Teaching Notes

SLIDE 1

Have students select the *Words* and *Symbols* buttons to illustrate the relationship between two complementary angles using words and symbols. Ask students to discuss with a partner why the sum of 90° is significant, and if complementary angles must be adjacent in order to be complementary. Some students may say that the definition does not require that they are adjacent. You may wish to have students draw and label other examples of complementary angles that are not adjacent in order to demonstrate that the angles do not need to be adjacent to be complementary.

Example 1 Identify Complementary Angles

Objective

Students will identify complementary angle pairs.

Questions for Mathematical Discourse

SLIDE 2

AL What is true about two complementary angles? The sum of their angle measures is 90°.

AL What is the measure of the given angle? 60°

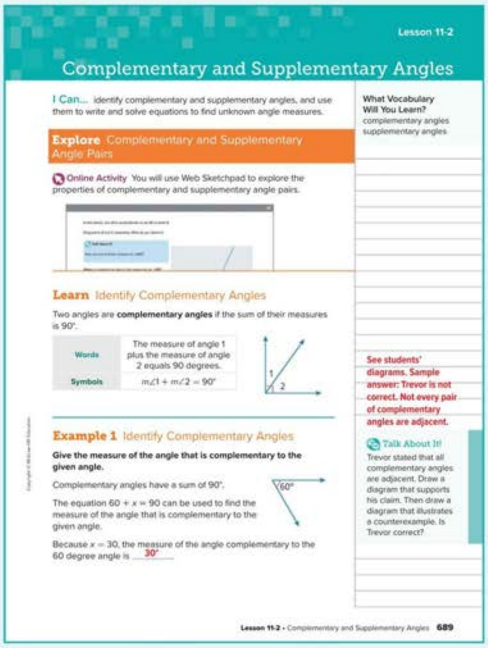
OL Explain why the equation $60 + x = 90$ can be used to find x, the measure of the angle complementary to the given angle. Sample answer: The sum of the measures of the given angle, 60 degrees, and the angle complementary to this angle, x degrees, is 90 degrees.

OL In this case, does the value of x represent the measure of the angle you need to find? Explain. yes; Sample answer: x represents the measure of the complementary angle

BL Suppose two angles are complementary. The measure of one angle is represented by the expression $15 + y$. Write and simplify an expression that represents the measure of the angle complementary to this angle. Sample answer: $90 - (15 + y)$, or $75 - y$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 11-2

Complementary and Supplementary Angles

I Can... identify complementary and supplementary angles, and use them to write and solve equations to find unknown angle measures.

What Vocabulary Will You Learn?
complementary angles
supplementary angles

Explore Complementary and Supplementary Angle Pairs

Online Activity You will use Web Sketchpad to explore the properties of complementary and supplementary angle pairs.

Learn Identify Complementary Angles

Two angles are **complementary angles** if the sum of their measures is 90°.

Words	The measure of angle 1 plus the measure of angle 2 equals 90 degrees.
Symbols	$m\angle 1 + m\angle 2 = 90°$

See students' diagrams. Sample answer: Trevor is not correct. Not every pair of complementary angles are adjacent.

Example 1 Identify Complementary Angles

Give the measure of the angle that is complementary to the given angle.

Complementary angles have a sum of 90°.

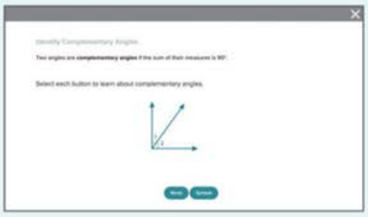
The equation $60 + x = 90$ can be used to find the measure of the angle that is complementary to the given angle.

Because $x = 30$, the measure of the angle complementary to the 60 degree angle is 30°.

Talk About It! Trevor stated that all complementary angles are adjacent. Draw a diagram that supports his claim. Then draw a diagram that illustrates a counterexample. Is Trevor correct?

Lesson 11-2 • Complementary and Supplementary Angles 689

Interactive Presentation

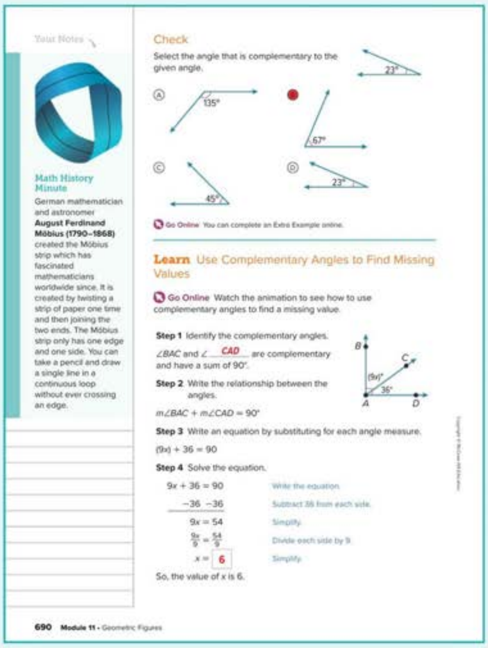


Learn, Identify Complementary Angles

CLICK On Slide 1 of the Learn, students select buttons to learn about complementary angles.

TYPE On Slide 2 of Example 1, students determine the sum of the angle measures of two complementary angles.

CHECK Students complete the Check exercise online to determine if they are ready to move on.


Your Notes

Math History Minute

German mathematician and astronomer **August Ferdinand Möbius (1790–1868)** created the Möbius strip which has fascinated mathematicians worldwide since. It is created by twisting a strip of paper one time and then joining the two ends. The Möbius strip only has one edge and one side. You can take a pencil and draw a single line in a continuous loop without ever crossing an edge.

Check

Select the angle that is complementary to the given angle.

Go Online You can complete an Extra Example online.

Learn Use Complementary Angles to Find Missing Values

Go Online Watch the animation to see how to use complementary angles to find a missing value.

Step 1 Identify the complementary angles.

$\angle BAC$ and $\angle$ CAD are complementary and have a sum of 90°.

Step 2 Write the relationship between the angles.

$m\angle BAC + m\angle CAD = 90°$

Step 3 Write an equation by substituting for each angle measure.

$(9x) + 36 = 90$

Step 4 Solve the equation.

$9x + 36 = 90$ Write the equation.

$-36 \quad -36$ Subtract 36 from each side.

$9x = 54$ Simplify.

$\frac{9x}{9} = \frac{54}{9}$ Divide each side by 9.

$x = 6$ Simplify.

So, the value of x is 6.

690 Module 11 • Geometric Figures

Interactive Presentation

Learn, Use Complementary Angles to Find Missing Values

WATCH

Students watch an animation that explains how to use complementary angles to find a missing value.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Use Complementary Angles to Find Missing Values

Objective

Students will understand how to use the properties of complementary angles to find missing values.

Go Online

Have students watch the animation on Slide 1. The animation illustrates how to use complementary angles to find missing values.

Teaching Notes

SLIDE 1

Play the animation for the class. You may wish to pause the animation when the notation *Identify complementary angles* first appears. Ask students to identify the two angles that are complementary and explain why they are complementary.

Continue playing the animation. When the animation has finished, you may wish to ask students how they can use the value for x to find $m\angle BAC$. Ask students if there is another way to find $m\angle BAC$. Some students may say that they can write the expression 90 – 36 to find $m\angle BAC$. Point out to students that this expression will give the measure of the angle, but not the value for x.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' reasoning about complementary angles, have them solve the following problem.

Suppose two angles are complementary. One angle is three times the measure of the other angle. Write and solve an equation to find the measures of the two angles.

22.5°; 67.5°

Example 2 Use Complementary Angles to Find Missing Values

Objective

Students will use the properties of complementary angles to find missing values.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively, 6 Attend to Precision Encourage students to adhere to the precise definition of complementary angles to represent the given information symbolically with a correct two-step equation. They should make sense of the solution within the context of the diagram.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that these two angles are complementary? Sample answer: They form a right angle, indicated by the right angle symbol. So, the sum of their angle measures is 90°.

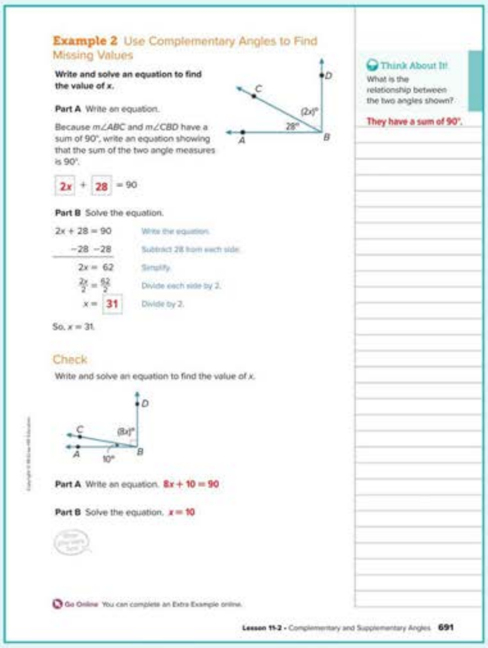
OL A classmate wrote the equation $2x = 28$. Explain the error that might have been made. Sample answer: The classmate may have thought the angles were vertical angles, and thus congruent. The angles are complementary, not vertical.

OL What kind of equation is $28 + 2x = 90$? Explain. Sample answer: It is a two-step equation, because the variable x is paired with two operations, multiplication and addition.

BL A classmate reasoned that if $2x + 28 = 90$, then $x + 14 = 45$, because each term can be divided by 2. Is this reasoning correct? Explain. yes; Sample answer: Dividing each term by 2 is an application of the Division Property of Equality.

Go Online

- Find additional teaching notes and discussion questions.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Use Complementary Angles to Find Missing Values

Write and solve an equation to find the value of x.

Part A Write an equation.

Because $m\angle ABC$ and $m\angle CBD$ have a sum of 90°, write an equation showing that the sum of the two angle measures is 90°.

$2x + 28 = 90$

Part B Solve the equation.

$2x + 28 = 90$ Write the equation.
$-28 \quad -28$ Subtract 28 from each side.
$2x = 62$ Simplify.
$\frac{2x}{2} = \frac{62}{2}$ Divide each side by 2.
$x = 31$ Divide by 2.

So, $x = 31$.

Think About It! What is the relationship between the two angles shown? They have a sum of 90°.

Check

Write and solve an equation to find the value of x.

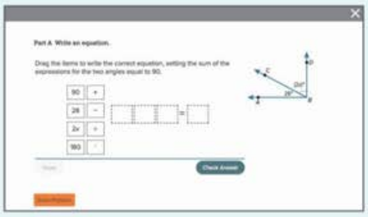
Part A Write an equation. $8x + 10 = 90$

Part B Solve the equation. $x = 10$

Go Online You can complete an Extra Example online.

Lesson 11-2 • Complementary and Supplementary Angles 691

Interactive Presentation



Example 2, Use Complementary Angles to Find Missing Values, Slide 2 of 4

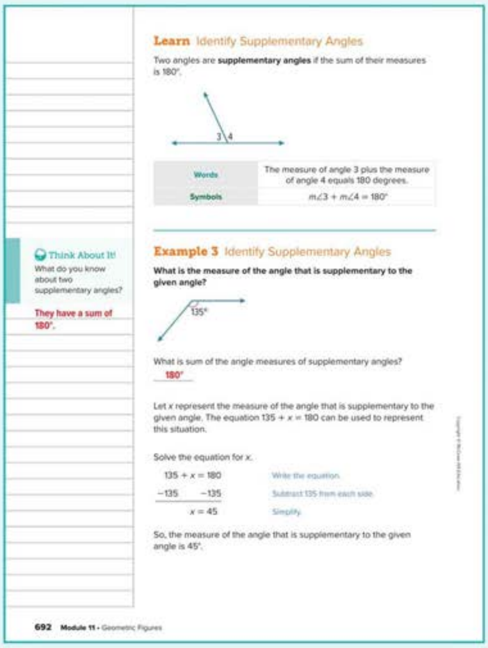
DRAG & DROP On Slide 2, students drag items to write the correct equation.

TYPE On Slide 3, students determine the solution to the equation.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

7.G.B.5

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Learn Identify Supplementary Angles

Two angles are **supplementary angles** if the sum of their measures is 180°.

Words	The measure of angle 3 plus the measure of angle 4 equals 180 degrees.
Symbols	$m\angle 3 + m\angle 4 = 180°$

Think About It! What do you know about two supplementary angles?

They have a sum of 180°.

Example 3 Identify Supplementary Angles

What is the measure of the angle that is supplementary to the given angle?

What is sum of the angle measures of supplementary angles? 180°

Let x represent the measure of the angle that is supplementary to the given angle. The equation $135 + x = 180$ can be used to represent this situation.

Solve the equation for x.

$135 + x = 180$ Write the equation.

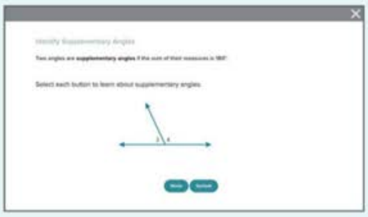
$-135 \quad -135$ Subtract 135 from each side.

$x = 45$ Simplify.

So, the measure of the angle that is supplementary to the given angle is 45°.

692 Module 11 • Geometric Figures

Interactive Presentation



Learn, Identify Supplementary Angles

CLICK

In the Learn, students select buttons to learn about supplementary angles.

TYPE

On Slide 2 of Example 3, students determine the measure of the supplementary angle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Identify Supplementary Angles

Objective

Students will understand the properties of supplementary angles.

Teaching Notes

SLIDE 1

Have students select the *Words* and *Symbols* buttons to illustrate the relationship between two supplementary angles using words and symbols. Ask students to discuss with a partner why the sum of 180° is significant, and if supplementary angles must be adjacent in order to be supplementary. Some students may say that the definition does not require that they are adjacent. You may wish to have students draw and label other examples of supplementary angles that are not adjacent in order to demonstrate that the angles do not need to be adjacent to be supplementary.

Example 3 Identify Supplementary Angles

Objective

Students will identify supplementary angle pairs.

Questions for Mathematical Discourse

SLIDE 2

AL What is true about two supplementary angles? The sum of their angle measures is 180°.

AL What is the measure of the given angle? 135°

OL Explain why the equation $135 + x = 180$ can be used to find x, the measure of the angle supplementary to the given angle. Sample answer: The sum of the measures of the given angle, 135 degrees, and the angle supplementary to this angle, x degrees, is 180 degrees.

OL In this case, does the value of x represent the measure of the angle you need to find? Explain. yes; Sample answer: x represents the measure of the supplementary angle

BL Suppose two angles are supplementary. The measure of one angle is represented by the expression $110 + y$. Write and simplify an expression that represents the measure of the angle supplementary to this angle. Sample answer: $180 - (110 + y)$, or $70 - y$

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Use Supplementary Angles to Find Missing Values

Objective

Students will understand how to use the properties of supplementary angles to find missing values.

Go Online

Have students watch the animation on Slide 1. The animation illustrates how to use supplementary angles to find missing values.

Teaching Notes

SLIDE 1

Play the animation for the class. You may wish to pause the animation when the notation *Identify supplementary angles* first appears. Ask students to identify the two angles that are supplementary and explain why they are supplementary.

Continue playing the animation. When the animation has finished, you may wish to ask students how they can find $m\angle ADB$ and to explain their reasoning. Some students may say since the sum of the measures of two supplementary angles is 180, they can find $m\angle ADB$ by using the expression $180° - 108°$. Point out to students that this expression will give the measure of the angle, but not the value for x. They can solve the equation $4x = 72$ to find the value of x.

DIFFERENTIATE

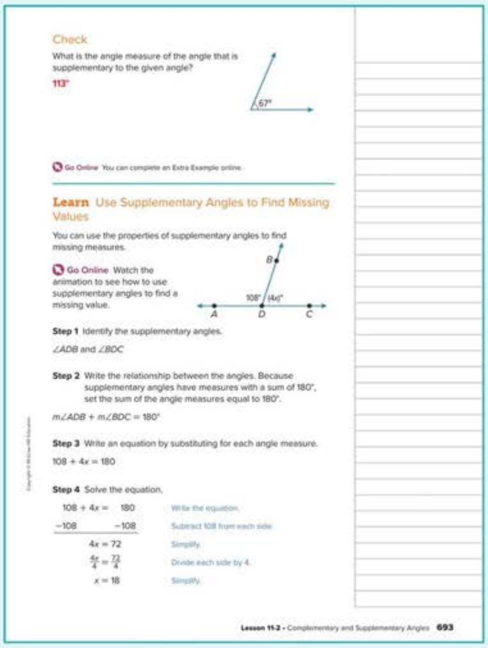
Reteaching Activity L

If any of your students are struggling with finding missing values when complementary and supplementary angles are involved, have them create a flow chart or outline that walks through the steps needed to find the missing value. They should list all that they know about complementary and supplementary angles that will help them write and solve the equation. A sample outline is shown.

1. Complementary angles
- **a.** Form a right angle shown by the right angle symbol.
- **b.** The measures of complementary angles add up to 90°.
- **c.** Write an equation: $m\angle 1 + m\angle 2 = 90$.
- **d.** Substitute what I know.
- **e.** Solve the equation.

2. Supplementary angles
- **a.** Form a straight line.
- **b.** The measures of supplementary angles add up to 180°.
- **c.** Write an equation: $m\angle 1 + m\angle 2 = 180$.
- **d.** Substitute what I know.
- **e.** Solve the equation.

If students created a poster or graphic organizer from Lesson 1 listing the properties and characteristics of vertical and adjacent angles, they can add information about complementary and supplementary angles to the poster.



Check

What is the angle measure of the angle that is supplementary to the given angle?

113°

Go Online You can complete an Extra Example online.

Learn Use Supplementary Angles to Find Missing Values

You can use the properties of supplementary angles to find missing measures.

Go Online Watch the animation to see how to use supplementary angles to find a missing value.

Step 1 Identify the supplementary angles.

$\angle ADB$ and $\angle BDC$

Step 2 Write the relationship between the angles. Because supplementary angles have measures with a sum of 180°, set the sum of the angle measures equal to 180°.

$m\angle ADB + m\angle BDC = 180°$

Step 3 Write an equation by substituting for each angle measure.

$108 + 4x = 180$

Step 4 Solve the equation.

$108 + 4x = 180$	Write the equation.
$-108 \quad -108$	Subtract 108 from each side.
$4x = 72$	Simplify.
$\frac{4x}{4} = \frac{72}{4}$	Divide each side by 4.
$x = 18$	Simplify.

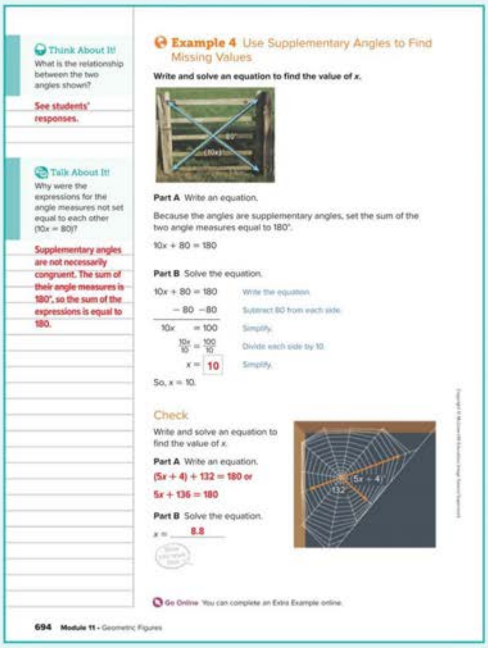
Lesson 11-2 • Complementary and Supplementary Angles 693

Interactive Presentation

Learn, Use Supplementary Angles to Find Missing Values

WATCH

Students watch an animation that explains how to use supplementary angles to find a missing value.



Think About It! What is the relationship between the two angles shown?

See students' responses.

Talk About It! Why were the expressions for the angle measures not set equal to each other $(10x = 80)$?

Supplementary angles are not necessarily congruent. The sum of their angle measures is 180°, so the sum of the expressions is equal to 180.

Example 4 Use Supplementary Angles to Find Missing Values

Write and solve an equation to find the value of *x*.

Part A Write an equation.

Because the angles are supplementary angles, set the sum of the two angle measures equal to 180°.

$10x + 80 = 180$

Part B Solve the equation.

$10x + 80 = 180$ Write the equation.

$-80 \quad -80$ Subtract 80 from each side.

$10x = 100$ Simplify.

$\frac{10x}{10} = \frac{100}{10}$ Divide each side by 10.

$x = 10$ Simplify.

So, $x = 10$.

Check

Write and solve an equation to find the value of *x*.

Part A Write an equation.

$(5x + 4) + 132 = 180$ or

$5x + 136 = 180$

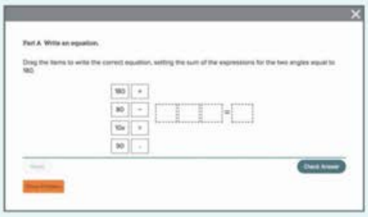
Part B Solve the equation.

$x =$ 8.8

Go Online You can complete an Extra Example online.

694 Module 11 • Geometric Figures

Interactive Presentation



Example 4, Use Supplementary Angles to Find Missing Values, Slide 2 of 5

DRAG & DROP

On Slide 2, students drag items to write the correct equation.

TYPE

On Slide 3, students determine the solution to the equation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Use Supplementary Angles to Find Missing Values

Objective

Students will use the properties of supplementary angles to find missing values.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively, 6 Attend to Precision
Encourage students to adhere to the precise definition of supplementary angles to represent the given information symbolically with a correct two-step equation.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that these two angles are supplementary? Sample answer: They form a straight angle. So, the sum of their angle measures is 180°.

OL A classmate wrote the equation $10x = 80$. Explain the error that might have been made. Sample answer: The classmate may have thought the angles were congruent angles. The angles are supplementary, not necessarily congruent.

OL What kind of equation is $80 + 10x = 180$? Explain. Sample answer: It is a two-step equation, because the variable *x* is paired with two operations, multiplication and addition.

BL A classmate reasoned that if $80 + 10x = 180$, then $8 + x = 18$, since each term can be divided by 10. Is this reasoning correct? Explain. yes; Sample answer: Dividing each term by 10 is an application of the Division Property of Equality.

SLIDE 3

AL Describe how to solve the equation. Subtract 80 from each side. Then divide each side by 10.

OL If the value of *x* is 10, does this mean that one of the angles in the diagram has a measure of 10°? Explain. no; Sample answer: One angle is known, with an angle measure of 80°. The other angle is labeled $10x$. Replace *x* with 10 in this expression. Since $10(10) = 100$, the other angle has a measure of 100°.

BL Explain how to solve the equation by using the Distributive Property. Sample answer: Write the equation as $10(x + 8) = 180$. Divide each side by 10 to obtain the equation $x + 8 = 18$. Then subtract 8 from each side. So, $x = 10$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Engineering

Objective

Students will come up with their own strategy to solve an application problem involving the engineering of a space shuttle scaffold.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students time to make sure they understand the task, think of possible strategies, and work to solve the problem.

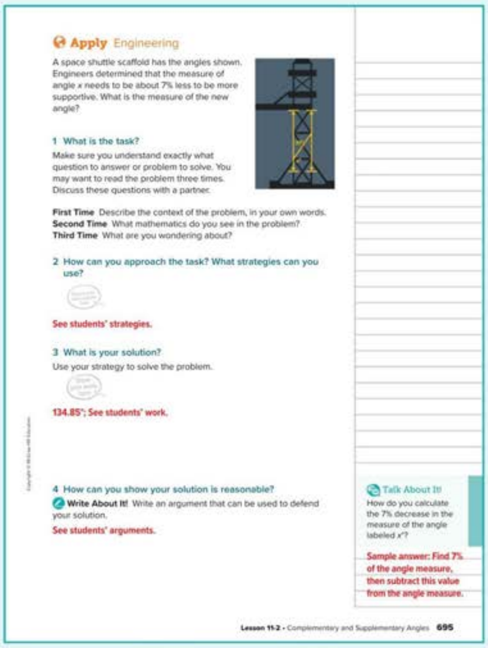
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the relationship between the 35° angle and the $x°$ angle?
- How do you find 7% of a number?
- Once you find 7% of the angle measure, is that the measure of the new angle?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Engineering

A space shuttle scaffold has the angles shown. Engineers determined that the measure of angle x needs to be about 7% less to be more supportive. What is the measure of the new angle?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

134.85°; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.
See students' arguments.

Talk About It!
How do you calculate the 7% decrease in the measure of the angle labeled $x°$?

Sample answer: Find 7% of the angle measure, then subtract this value from the angle measure.

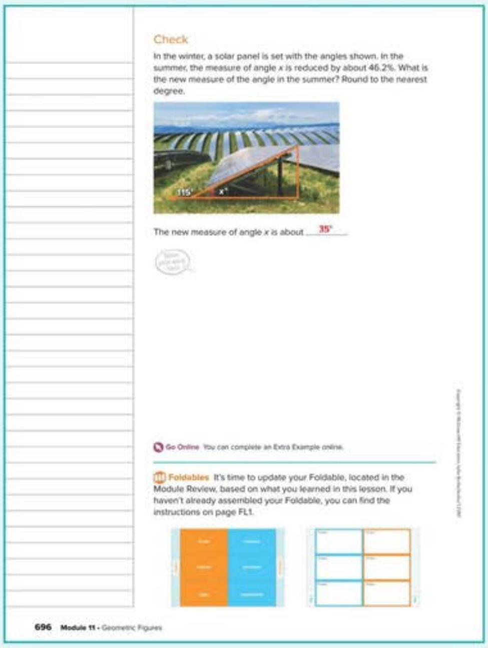
Lesson 11-2 • Complementary and Supplementary Angles 695

Interactive Presentation

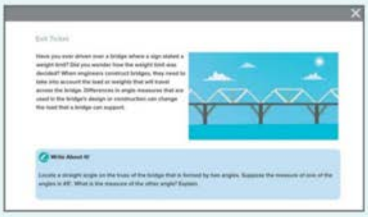
Apply, Engineering

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students can record information about pairs of angles. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How does geometry help to describe objects?
In this lesson, students learned how to classify complementary and supplementary angles. Encourage them to discuss with a partner how they can use this terminology to describe real-world objects. For example, they may describe some of the angles that form the truss of a bridge as complementary and/or supplementary.

Exit Ticket

Refer to the Exit Ticket slide. Locate a straight angle on the truss of the bridge that is formed by two angles. Suppose the measure of one of the angles is 45°. What is the measure of the other angle? Write a mathematical argument that can be used to defend your solution. 135°; Sample answer: If the two angles form a straight angle, then they are supplementary. The sum of the measures of two supplementary angles is 180°.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 13, 15–18
- Extension: Solve Complementary and Supplementary Angle Problems
- ALEKS® Angle Relationships

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises Exercises 1–12, 14, 16
- Extension: Solve Complementary and Supplementary Angle Problems
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Classifying and Measuring Angles

IF students score 65% or below on the Checks, **AL**
THEN assign:

- ArriveMATH Take Another Look
- ALEKS® Classifying and Measuring Angles

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

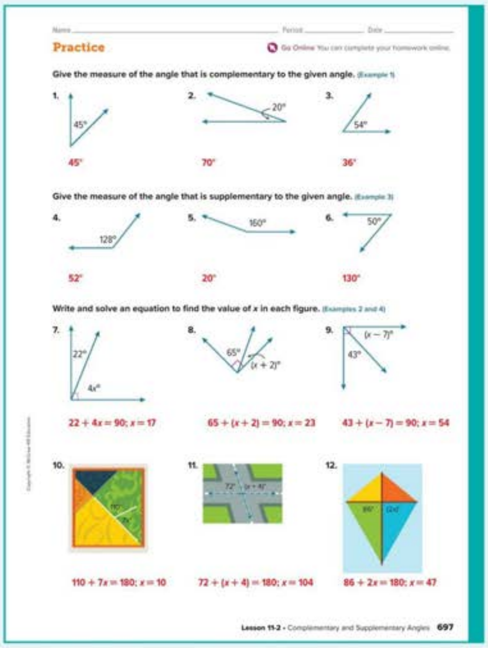
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	identify the complement of angles	1–3
1	identify supplementary angles	4–6
1	use complementary and supplementary angles to find missing values	7–12
3	solve application problems involving complementary and supplementary angles	13, 14
3	higher-order and critical thinking skills	15–18

Common Misconception

In diagrams, students may assume that angles are the same measure if they appear to be. For example, in Exercise 9, students may incorrectly assume that both angles are 43 degrees because they look the same. Encourage them to use properties of pairs of angles when determining missing measures.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Give the measure of the angle that is complementary to the given angle. (Example 1)

1. 45° — 45°
2. 20° — 70°
3. 54° — 36°

Give the measure of the angle that is supplementary to the given angle. (Example 3)

4. 128° — 52°
5. 160° — 20°
6. 50° — 130°

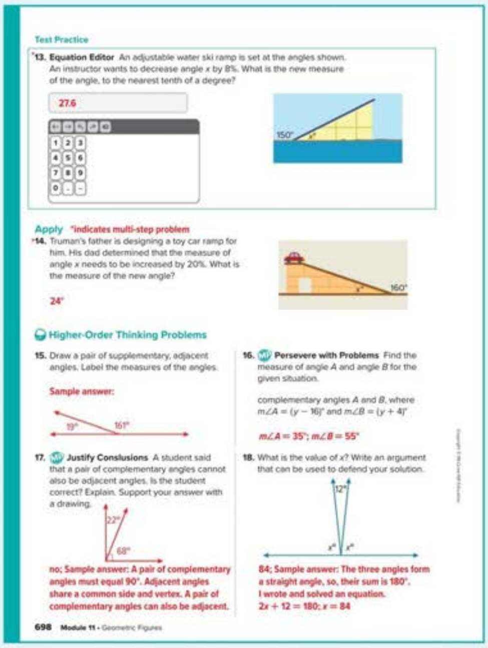
Write and solve an equation to find the value of x in each figure. (Examples 2 and 4)

7. 22°, $4x°$ — $22 + 4x = 90$; $x = 17$
8. 65°, $(x + 2)°$ — $65 + (x + 2) = 90$; $x = 23$
9. 43°, $(x - 7)°$ — $43 + (x - 7) = 90$; $x = 54$
10. 110°, $7x°$ — $110 + 7x = 180$; $x = 10$
11. 72°, $(x + 4)°$ — $72 + (x + 4) = 180$; $x = 104$
12. 86°, $(2x)°$ — $86 + 2x = 180$; $x = 47$

Lesson 11-2 • Complementary and Supplementary Angles 697

7.G.B.5

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**


Test Practice

*13. **Equation Editor** An adjustable water ski ramp is set at the angles shown. An instructor wants to decrease angle x by 8%. What is the new measure of the angle, to the nearest tenth of a degree?

27.6

Apply *indicates multi-step problem

*14. Truman's father is designing a toy car ramp for him. His dad determined that the measure of angle x needs to be increased by 20%. What is the measure of the new angle?

24°

Higher-Order Thinking Problems

15. Draw a pair of supplementary, adjacent angles. Label the measures of the angles.

Sample answer:

16. **Persevere with Problems** Find the measure of angle A and angle B for the given situation.

complementary angles A and B, where $m\angle A = (y - 16)°$ and $m\angle B = (y + 4)°$

$m\angle A = 35°$; $m\angle B = 55°$

17. **Justify Conslusions** A student said that a pair of complementary angles cannot also be adjacent angles. Is the student correct? Explain. Support your answer with a drawing.

no; Sample answer: A pair of complementary angles must equal 90°. Adjacent angles share a common side and vertex. A pair of complementary angles can also be adjacent.

18. What is the value of x? Write an argument that can be used to defend your solution.

84; Sample answer: The three angles form a straight angle, so, their sum is 180°. I wrote and solved an equation. $2x + 12 = 180$; $x = 84$

698 Module 11 • Geometric Figures

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 16, students use multiple steps to find missing angle measures by writing and solving an equation.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 17, students explain if a pair of complementary angles can also be adjacent angles and support their reasoning with a drawing.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercise 13 Have pairs of students interview each other as they complete this application problem. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 13 might be, "How would you describe the pair of angles that measure 150 degrees and x degrees in the diagram?"

Solve the problem another way.
Use with Exercise 18 Have students work in groups of 3–4. After completing Exercise 18, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Lesson 11-3

Angle Relationships and Parallel Lines

8.G.A.5

LESSON GOAL

Students will examine relationships of angles formed by parallel lines cut by a transversal.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Parallel Lines and Transversals

Learn: Lines, Angles, and Transversals
Example 1: Classify Angle Pairs
Example 2: Classify Angle Pairs
Learn: Find Missing Angle Measures
Example 3: Find Missing Angle Measures
Example 4: Find Missing Angle Measures
Apply: Construction

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

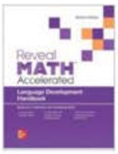
DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 65 of the *Language Development Handbook* to help your students build mathematical language related to angle relationships and parallel lines.



ELL You can use the tips and suggestions on page T65 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by examining relationships of angles formed by parallel lines cut by a transversal.
Standards for Mathematical Content: **8. G.A.5**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students identified supplementary and complementary angles and write and solve equations to find missing values.
7.G.B.5

Now
Students examine relationships of angles formed by parallel lines cut by a transversal.
8.G.A.5

Next
Students will draw triangles freehand, with ruler and protractor, and with technology.
7.G.A.2

Rigor

The Three Pillars of Rigor

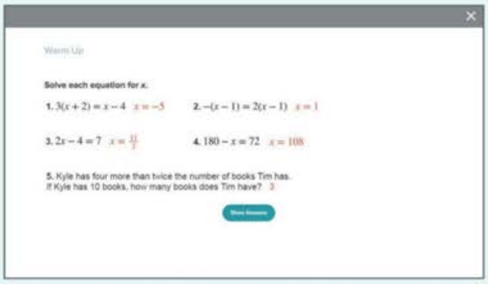
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of special angle pairs to develop *understanding* of angle pairs formed when two parallel lines are cut by a transversal. They come to understand how to identify the type of angle pair and their measures.

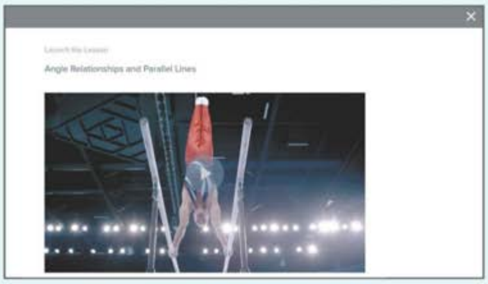
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

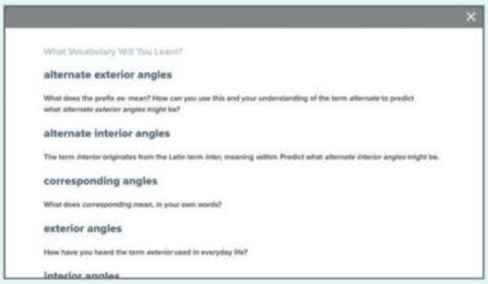
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- solving equations (Exercises 1–5)

Answers

1. $x = -5$
2. $x = 1$
3. $x = \frac{11}{2}$
4. $x = 108$
5. 3

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the gymnastics events involving parallel bars.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- What does the prefix *ex-* mean? How can you use this and your understanding of the term *alternate* to predict what *alternate exterior angles* might be? Sample answer: *ex-* means *out of* or *not*. Alternate exterior angles might be angles that are outside of a figure on opposite sides.
- The term *interior* originates from the Latin term *inter*, meaning *within*. Predict what *alternate interior angles* might be. Sample answer: Alternate interior angles might be angles that are inside of a figure on opposite sides.
- What does *corresponding* mean, in your own words? Sample answer: Corresponding means having the same characteristics as something else; similar in position.
- How have you heard the term *exterior* used in everyday life? Sample answer: exterior door, exterior paint

Explore Parallel Lines and T ransversals

Objective

Students will use Web Sketchpad to explore the relationships between angles created by parallel lines and transversals.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a set of parallel lines crossed by a transversal. Throughout this activity, students will move the lines around to observe what happens to the angles. Students will use properties of vertical and supplementary angles to demonstrate various relationships between angles.

Inquiry Question

What are the angle relationships formed when a line intersects two parallel lines? Sample answer: Eight angles are formed; four angles are congruent to each other and the other four angles are congruent to each other. There are four pairs of vertical angles and eight pairs of supplementary angles formed by the intersecting lines.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Experiment with the sketch by dragging any or all of the points to different locations. Do your answers to the questions above change? Why or why not? no; Sample answer: The location of any of the points does not affect lines *DE* and *GA* appearing to be parallel, eight angles are formed around the intersections. Pairs of vertical angles are congruent and the angles that are in the same position in relation to the transversal and parallel lines appear to be congruent.

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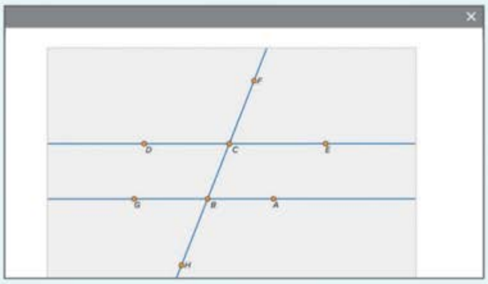
Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationships between angles createdby parallel lines and transversals.

Interactive Presentation

Explore, Slide 5 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Parallel Lines and T ransversals *(continued)*

Teaching the Mathematical Practices

3 Construct Viable Arguments S tudents should be able to justify their conclusions and make sense of their findings.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore the relationships between angles created by parallel lines and transversals.

7 Look for and Make Use of Structure Encourage students to use the sketch to examine the structure of the parallel lines and transversals in order to make predictions about the angle measures that are formed.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What must be true about the three lines in order for the relationships among the eight angles formed to have the relationship you discovered in this Explore? Sample answer: Two of the lines must be parallel and a third line must intersect both of the parallel lines.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Lines, Angles, and T ransversals

Objective

Students will learn about perpendicular lines, parallel lines, and the angles formed when parallel lines are cut by a transversal.

Teaching Notes

SLIDE 1

Students should be familiar with the concept of perpendicular lines from prior grades. Be sure students understand the notation used to indicate perpendicular lines and why this notation might visually help them remember the meaning of perpendicular lines.

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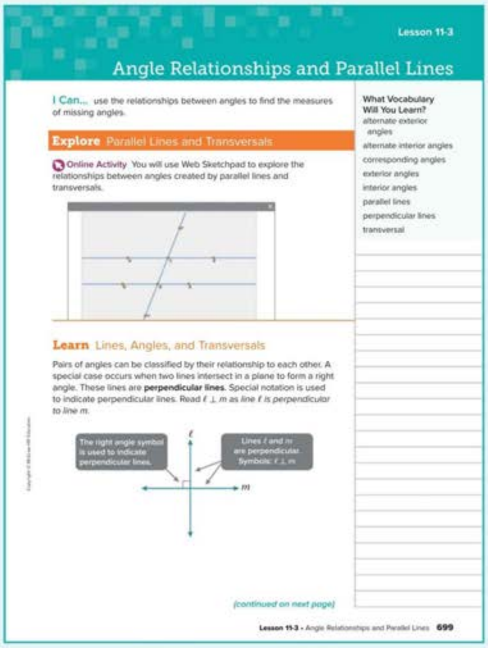

Lesson 11-3

Angle Relationships and Parallel Lines

I Can... use the relationships between angles to find the measures of missing angles.

Explore Parallel Lines and Transversals

Online Activity You will use Web Sketchpad to explore the relationships between angles created by parallel lines and transversals.

Learn Lines, Angles, and Transversals

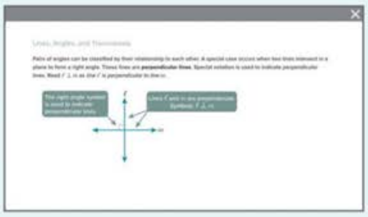
Pairs of angles can be classified by their relationship to each other. A special case occurs when two lines intersect in a plane to form a right angle. These lines are **perpendicular lines**. Special notation is used to indicate perpendicular lines. Read $\ell \perp m$ as *line ℓ is perpendicular to line m*.

What Vocabulary Will You Learn?
alternate exterior angles
alternate interior angles
corresponding angles
exterior angles
interior angles
parallel lines
perpendicular lines
transversal

(continued on next page)

Lesson 11-3 • Angle Relationships and Parallel Lines 699

Interactive Presentation



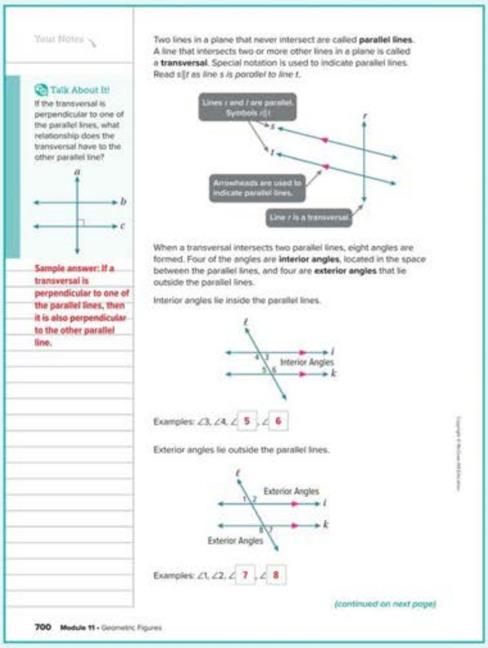
Learn, Lines, Angles, and Transversals, Slide 1 of 5

DIFFERENTIATE

Language Development Activity ELL

If students are struggling with the vocabulary presented in the Learn, use the following activity.

Supply students with a ruler and blank sheet of paper. Have students define a right angle, perpendicular lines, parallel lines, and a transversal. Then encourage students to draw a mathematical diagram and a real-world picture depicting each term.



Your Notes

Talk About It!

If the transversal is perpendicular to one of the parallel lines, what relationship does the transversal have to the other parallel line?

Sample answer: If a transversal is perpendicular to one of the parallel lines, then it is also perpendicular to the other parallel line.

Two lines in a plane that never intersect are called **parallel lines**. A line that intersects two or more other lines in a plane is called a **transversal**. Special notation is used to indicate parallel lines. Read s∥t as *line s is parallel to line t.*

When a transversal intersects two parallel lines, eight angles are formed. Four of the angles are **interior angles**, located in the space between the parallel lines, and four are **exterior angles** that lie outside the parallel lines.

Interior angles lie inside the parallel lines.

Examples: ∠3, ∠4, ∠ 5, ∠ 6

Exterior angles lie outside the parallel lines.

Examples: ∠1, ∠2, ∠ 7, ∠ 8

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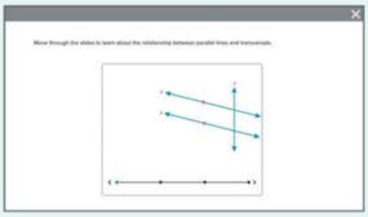
700 Module 11 • Geometric Figures

Interactive Presentation



Learn, Lines, Angles, and Transversals, Slide 2 of 5

CLICK

On Slide 2, students move through the slides to learn about the relationship between parallel lines and transversals.

CLICK

On Slide 4, students select to view the interior and exterior angles in a diagram.

Learn Lines, Angles, and T ransversals *(continued)*

Teaching Notes

SLIDE 2

Students may be familiar with the concept of parallel lines from everyday life. Be sure they understand the notation used to indicate parallel lines. Prior to having them move through the slides in the interactive tool, ask them to study the diagram in order to determine which lines are parallel, and which line represents the transversal. Point out that the arrowheads are used to indicate parallel lines in diagrams. Ask them to explain why line *r* is the transversal. Students should be able to explain that line *r* intersects lines *s* and *t*.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively W hile discussing the *Talk About It!* question on Slide 3, encourage students to use the image in order to make sense of the relationship between the lines.

Talk About It!

SLIDE 3

Mathematical Discourse

If the transversal is perpendicular to one of the parallel lines, what relationship does the transversal have to the other parallel line? Sample answer: If a transversal is perpendicular to one of the parallel lines, then it is also perpendicular to the other parallel line.

Teaching Notes

SLIDE 4

Students will learn how to identify *interior* and *exterior* angles. You may wish to have student volunteers come up to the board to select each of the buttons. Have students predict before selecting the buttons which angles will be interior and which ones will be exterior, based on what they know about these terms in everyday use.

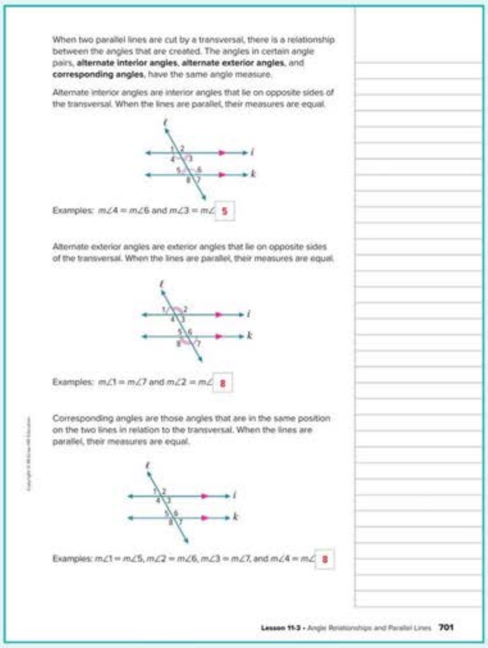
(continued on next page)

Learn Lines, Angles, and Transversals *(continued)*

Teaching Notes

SLIDE 5

Students will learn how to identify *alternate interior angles, alternate exterior angles*, and *corresponding angles*, and will learn that these angle pairs have the same measure when parallel lines are cut by a transversal. Be sure that students understand and can use the notation used to denote the measure of an angle. Prior to having students use the interactive tool to identify these types of angles on the diagram, have them make a conjecture as to which angles might be *alternate interior* and *alternate exterior* based on what these terms *alternate, interior*, and *exterior* mean in everyday use.



When two parallel lines are cut by a transversal, there is a relationship between the angles that are created. The angles in certain angle pairs, **alternate interior angles, alternate exterior angles**, and **corresponding angles**, have the same angle measure.

Alternate interior angles are interior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal.

Examples: $m\angle 4 = m\angle 6$ and $m\angle 3 = m\angle$ 5

Alternate exterior angles are exterior angles that lie on opposite sides of the transversal. When the lines are parallel, their measures are equal.

Examples: $m\angle 1 = m\angle 7$ and $m\angle 2 = m\angle$ 8

Corresponding angles are those angles that are in the same position on the two lines in relation to the transversal. When the lines are parallel, their measures are equal.

Examples: $m\angle 1 = m\angle 5$, $m\angle 2 = m\angle 6$, $m\angle 3 = m\angle 7$, and $m\angle 4 = m\angle$ 8

Lesson 11-3 • Angle Relationships and Parallel Lines 701

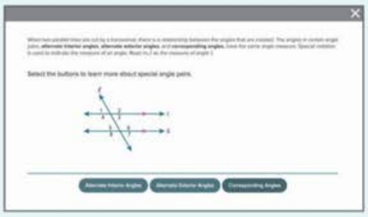
DIFFERENTIATE

Enrichment Activity

To further students' understanding of the angles formed when two lines are cut by a transversal, have students work with a partner to draw diagrams that satisfy each of the following conditions. Ask students to draw several different diagrams to satisfy each of the given conditions. If more than one diagram can be drawn, ask students if the number of angles that are formed is affected by how the diagram is drawn.

- Condition A: Two parallel lines intersected by one transversal Sample answer: No matter how the diagram is drawn, eight angles are always formed.
- Condition B: Two parallel lines intersected by two transversals, in which the two transversals are also parallel Sample answer: No matter how the diagram is drawn, sixteen angles are always formed.
- Condition C: Two parallel lines intersected by two transversals, in which the two transversals are not parallel Sample answer: It depends on how the transversals intersect each other. Some students' diagrams may show 14 angles or 20 angles that are formed.

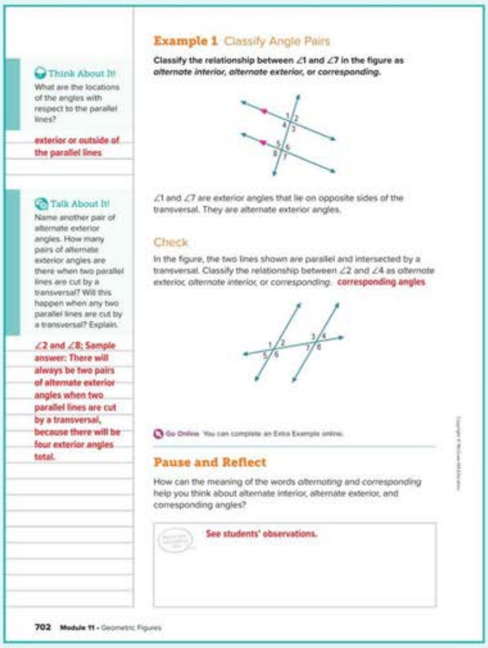
Interactive Presentation



Learn, Lines, Angles, and Transversals, Slide 5 of 5

CLICK

On Slide 5, students select to view alternate interior, alternate exterior, and corresponding angles in a diagram.


Think About It!
What are the locations of the angles with respect to the parallel lines?
exterior or outside of the parallel lines

Talk About It!
Name another pair of alternate exterior angles. How many pairs of alternate exterior angles are there when two parallel lines are cut by a transversal? Will this happen when any two parallel lines are cut by a transversal? Explain.
∠2 and ∠8; Sample answer: There will always be two pairs of alternate exterior angles when two parallel lines are cut by a transversal, because there will be four exterior angles total.

Example 1 Classify Angle Pairs

Classify the relationship between ∠1 and ∠7 in the figure as *alternate interior, alternate exterior, or corresponding.*

∠1 and ∠7 are exterior angles that lie on opposite sides of the transversal. They are alternate exterior angles.

Check

In the figure, the two lines shown are parallel and intersected by a transversal. Classify the relationship between ∠2 and ∠4 as *alternate exterior, alternate interior, or corresponding.* corresponding angles

Go Online You can complete an Extra Example online.

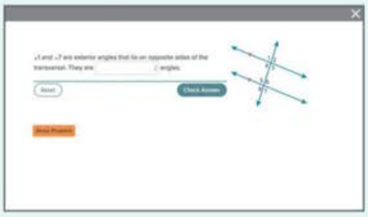
Pause and Reflect

How can the meaning of the words *alternating* and *corresponding* help you think about alternate interior, alternate exterior, and corresponding angles?

See students' observations.

702 Module 11 • Geometric Figures

Interactive Presentation



Example 1, Classify Angle Pairs, Slide 2 of 4

CLICK

On Slide 2, students determine the relationship between angles 1 and 7.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

8.G.A.5

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 1 Classify Angle Pairs

Objective

Students will classify angle pairs created when parallel lines are cut by a transversal.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to use the proper terminology when referring to and classifying the angle pairs.

7 Look For and Make Use of Structure While discussing the *Talk About It!* questions on Slide 3, encourage students to use the structure of the diagram illustrating the two parallel lines cut by a transversal, in order to generalize how many pairs of alternate exterior angles there will be.

Questions for Mathematical Discourse

SLIDE 2

AL Why are ∠1 and ∠7 called *exterior* angles? Sample answer: These two angles are on the outside of the two parallel lines.

OL Why does the term *alternate* apply in this case? Sample answer: The two angles are on opposite sides of the transversal.

OL If a student labels the two angles as alternate interior angles, how would you convince the student that this is incorrect? Sample answer: Since the angles are outside of the parallel lines, they are exterior angles. If the angles were alternate interior angles, then they would have to be inside the parallel lines.

BL Study the figure. How do you think the two angles will compare, in terms of their measures? Sample answer: In the figure, the two angles appear to be the same size. So, they appear to have the same measure.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Classify Angle Pairs

Objective

Students will classify angle pairs created when parallel lines are cut by a transversal.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to use the proper terminology when referring to and classifying the angle pairs.

7 Look For and Make Use of Structure While discussing the *Talk About It!* questions on Slide 3, encourage students to use the structure of the diagram illustrating the two parallel lines cut by a transversal, in order to generalize how many pairs of corresponding angles there will be.

Questions for Mathematical Discourse

SLIDE 2

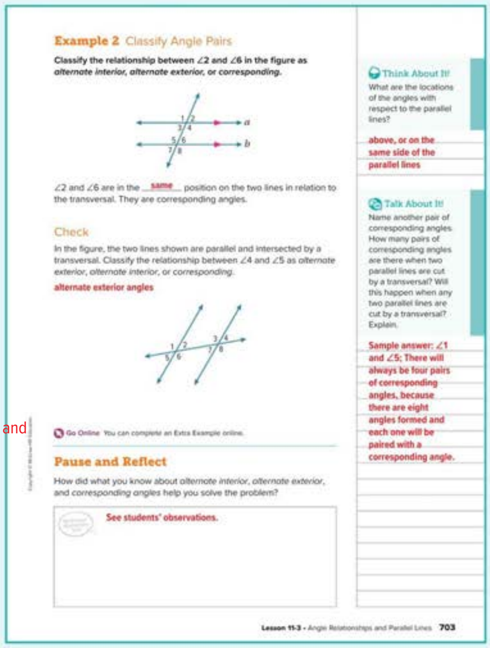
AL What are the parallel lines and which line is the transversal? *a* and *b* are the parallel lines and the line crossing these two lines is the transversal.

OL Why do you know right away that these two angles cannot be alternate interior or alternate exterior? Sample answer: The angles are not both interior, nor are they both exterior. Additionally, they are not on opposite sides of the transversal, so they are not alternate.

BL A student claims that ∠2 and ∠8 are also corresponding angles because they are on the same side of the transversal and both are outside of the parallel lines. Why is this student incorrect? Sample answer: Corresponding angles must be in the same position on the two lines in relation to the transversal. ∠2 sits “on top” of line *a*, while ∠8 sits “below” line *b*, so they are not in the same position.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Classify Angle Pairs

Classify the relationship between ∠2 and ∠6 in the figure as *alternate interior, alternate exterior, or corresponding.*

∠2 and ∠6 are in the __same__ position on the two lines in relation to the transversal. They are corresponding angles.

Think About It! What are the locations of the angles with respect to the parallel lines?

above, or on the same side of the parallel lines

Check

In the figure, the two lines shown are parallel and intersected by a transversal. Classify the relationship between ∠4 and ∠5 as *alternate exterior, alternate interior, or corresponding.*

alternate exterior angles

Talk About It! Name another pair of corresponding angles. How many pairs of corresponding angles are there when two parallel lines are cut by a transversal? Will this happen when any two parallel lines are cut by a transversal? Explain.

Sample answer: ∠1 and ∠5; There will always be four pairs of corresponding angles, because there are eight angles formed and each one will be paired with a corresponding angle.

Go Online You can complete an Extra Example online.

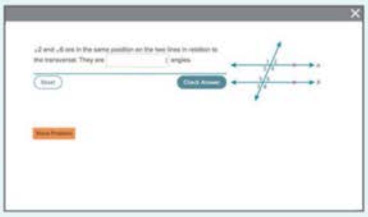
Pause and Reflect

How did what you know about *alternate interior, alternate exterior,* and *corresponding angles* help you solve the problem?

See students' observations.

Lesson 11-3 • Angle Relationships and Parallel Lines 703

Interactive Presentation



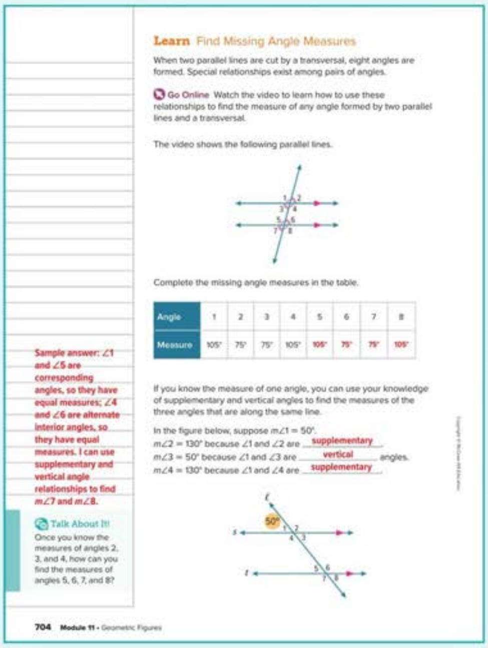
Example 2, Classify Angle Pairs, Slide 2 of 4

CLICK

On Slide 2, students determine the relationship between angles 2 and 6.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Find Missing Angle Measures

When two parallel lines are cut by a transversal, eight angles are formed. Special relationships exist among pairs of angles.

Go Online Watch the video to learn how to use these relationships to find the measure of any angle formed by two parallel lines and a transversal.

The video shows the following parallel lines.

Complete the missing angle measures in the table.

Angle	1	2	3	4	5	6	7	8
Measure	105°	75°	75°	105°	105°	75°	75°	105°

If you know the measure of one angle, you can use your knowledge of supplementary and vertical angles to find the measures of the three angles that are along the same line.

In the figure below, suppose $m\angle 1 = 50°$.
$m\angle 2 = 130°$ because $\angle 1$ and $\angle 2$ are supplementary.
$m\angle 3 = 50°$ because $\angle 1$ and $\angle 3$ are vertical angles.
$m\angle 4 = 130°$ because $\angle 1$ and $\angle 4$ are supplementary.

Sample answer: $\angle 1$ and $\angle 5$ are corresponding angles, so they have equal measures; $\angle 4$ and $\angle 6$ are alternate interior angles, so they have equal measures. I can use supplementary and vertical angle relationships to find $m\angle 7$ and $m\angle 8$.

Talk About It! Once you know the measures of angles 2, 3, and 4, how can you find the measures of angles 5, 6, 7, and 8?

704 Module 11 • Geometric Figures

Interactive Presentation

Learn, Find Missing Angle Measures, Slide 1 of 3

WATCH On Slide 1, students watch the video to learn how to find the measures of angles formed by a pair of parallel lines cut by a transversal.

CLICK On Slide 2, students move through the slides to find the measures of angles 2, 3, and 4.

Learn Find Missing Angle Measures

Objective

Students will understand that they can use angle relationships to find missing angle measures, when two parallel lines are cut by transversals.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discussing the *Talk About It!* question on Slide 3, encourage students to use reasoning and their knowledge of supplementary and vertical angle relationships to find the missing angle measures.

6 Attend to Precision While discussing the *Talk About It!* question on Slide 3, students should use precise mathematical terminology in their explanations.

Go Online to have students watch the video on Slide 1. The video illustrates the angle relationships formed when two parallel lines are cut by a transversal.

Teaching Notes

SLIDE 1

You may wish to pause the video after the first diagram is shown. Have students draw their own diagram that illustrates two parallel lines cut by a transversal. Have them label the angles 1 through 8, and create a similar table to record the angle measures. Have them use a protractor to measure each angle and record its measure. When completed, ask them to describe what they notice about the angle measures. If they measured accurately, students should note that four of the angles are congruent, and the remaining four angles are also congruent. Have them describe what they notice about pairs of vertical angles, corresponding angles, alternate interior angles, and alternate exterior angles. Replay the video and have students compare what they noticed about the angle relationships with what is shown in the video.

SLIDE 2

You may wish to have students work with a partner to come up with possible strategies they can use to find the measures of angles 2, 3, and 4, prior to progressing through the interactive tool. Have students share their strategies with the class. Be sure that each pair can justify their reasoning using correct mathematical vocabulary.

Talk About It!

SLIDE 3

Mathematical Discourse

Once you know the measures of angles 2, 3, and 4, how can you find the measures of angles 5, 6, 7, and 8? Sample answer: $\angle 1$ and $\angle 5$ are corresponding angles, so they have equal measures; $\angle 4$ and $\angle 6$ are alternate interior angles, so they have equal measures. I can use supplementary and vertical angle relationships to find $m\angle 7$ and $m\angle 8$.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 3 Find Missing Angle Measures

Objective

Students will find missing angle measures when two parallel lines are cut by a transversal.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you think we are finding $m\angle 6$ first? Sample answer: The measure of $\angle 2$ is known and $\angle 6$ is supplementary to $\angle 2$. It is not clear yet how $\angle 3$ relates to $\angle 2$.

AL What are supplementary angles? How can you use this information to find $m\angle 6$? Sample answer: Two angles are supplementary angles if the sum of their measures is 180 degrees. Subtract 105 from 180.

OL Since $m\angle 2$ is given, what other angles in this diagram have measures equal to $m\angle 2$? Sample answer: $\angle 2$ and $\angle 5$ are vertical angles, so their measures are equal. $\angle 2$ and $\angle 4$ are corresponding angles, so their measures are equal. $\angle 2$ and $\angle 7$ are alternate interior angles, so their measures are equal.

BL If the transversal on the bookshelf was repositioned so that the measure of $\angle 2$ increased, what would happen to the measure of $\angle 6$? How do you know? Sample answer: The measure of $\angle 6$ would decrease. This is because the sum of the angle measures is 180 degrees, so if one increases, the other must decrease, since the sum must remain the same, 180 degrees.

SLIDE 3

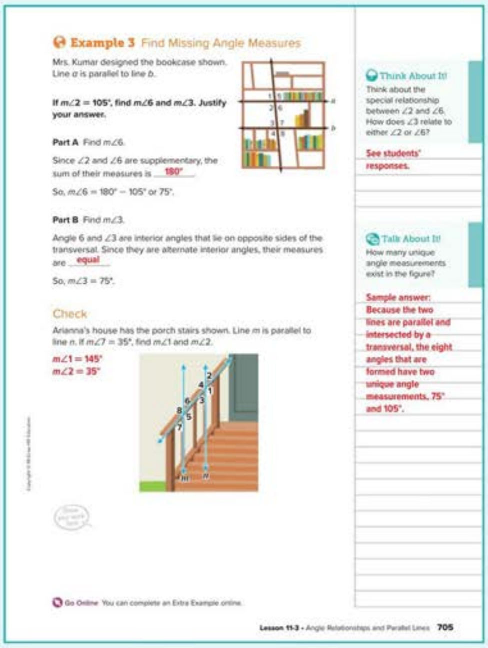
AL What kind of angles are $\angle 3$ and $\angle 6$? What does this mean? alternate interior angles; They have the same measure.

OL Now that you know $m\angle 6$, what other angles in this diagram have measures equal to $m\angle 6$? Sample answer: $\angle 1$ and $\angle 6$ are vertical angles, so their measures are equal. $\angle 6$ and $\angle 8$ are corresponding angles, so their measures are equal.

BL Describe two reasons why you know that $m\angle 1 = 75°$. Sample answer: One reason is that $\angle 1$ and $\angle 3$ are corresponding angles, which have equal measures. The second reason is that $\angle 1$ and $\angle 6$ are vertical angles, which have equal measures.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Find Missing Angle Measures

Mrs. Kumar designed the bookcase shown. Line a is parallel to line b.

If $m\angle 2 = 105°$, find $m\angle 6$ and $m\angle 3$. Justify your answer.

Part A Find $m\angle 6$.

Since $\angle 2$ and $\angle 6$ are supplementary, the sum of their measures is 180°.

So, $m\angle 6 = 180° - 105°$ or $75°$.

Part B Find $m\angle 3$.

Angle 6 and $\angle 3$ are interior angles that lie on opposite sides of the transversal. Since they are alternate interior angles, their measures are equal.

So, $m\angle 3 = 75°$.

Check

Arianna's house has the porch stairs shown. Line m is parallel to line n. If $m\angle 7 = 35°$, find $m\angle 1$ and $m\angle 2$.

$m\angle 1 = 145°$
$m\angle 2 = 35°$

Think About It! Think about the special relationship between $\angle 2$ and $\angle 6$. How does $\angle 3$ relate to either $\angle 2$ or $\angle 6$?

See students' responses.

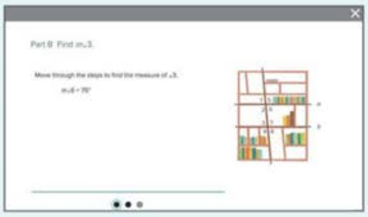
Talk About It! How many unique angle measurements exist in the figure?

Sample answer: Because the two lines are parallel and intersected by a transversal, the eight angles that are formed have two unique angle measurements, 75° and 105°.

Go Online You can complete an Extra Example online.

Lesson 11-3 • Angle Relationships and Parallel Lines 705

Interactive Presentation



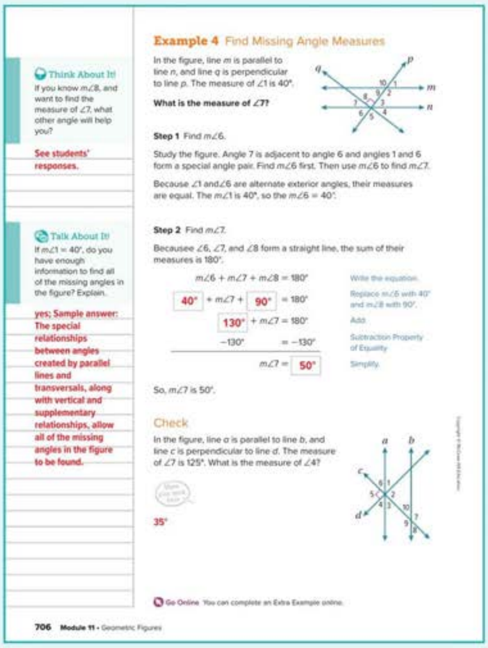
Example 3, Find Missing Angle Measures, Slide 3 of 5

CLICK

On Slide 3, students move through the steps to find the measure of angle 3.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 4 Find Missing Angle Measures

In the figure, line m is parallel to line n, and line q is perpendicular to line p. The measure of $\angle 1$ is 40°.

What is the measure of $\angle 7$?

Step 1 Find $m\angle 6$.

Study the figure. Angle 7 is adjacent to angle 6 and angles 1 and 6 form a special angle pair. Find $m\angle 6$ first. Then use $m\angle 6$ to find $m\angle 7$.

Because $\angle 1$ and $\angle 6$ are alternate exterior angles, their measures are equal. The $m\angle 1$ is 40°, so the $m\angle 6 = 40°$.

Step 2 Find $m\angle 7$.

Becausee $\angle 6$, $\angle 7$, and $\angle 8$ form a straight line, the sum of their measures is 180°.

$m\angle 6 + m\angle 7 + m\angle 8 = 180°$	Write the equation.
$40° + m\angle 7 + 90° = 180°$	Replace $m\angle 6$ with 40° and $m\angle 8$ with 90°.
$130° + m\angle 7 = 180°$	Add.
$-130° \quad = -130°$	Subtraction Property of Equality
$m\angle 7 = 50°$	Simplify.

So, $m\angle 7$ is 50°.

Think About It! If you know $m\angle 8$, and want to find the measure of $\angle 7$, what other angle will help you?

See students' responses.

Talk About It! If $m\angle 1 = 40°$, do you have enough information to find all of the missing angles in the figure? Explain.

yes; Sample answer: The special relationships between angles created by parallel lines and transversals, along with vertical and supplementary relationships, allow all of the missing angles in the figure to be found.

Check

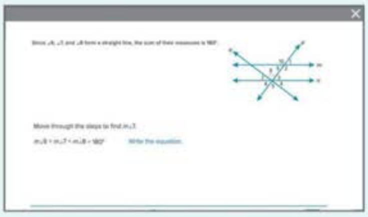
In the figure, line a is parallel to line b, and line c is perpendicular to line d. The measure of $\angle 7$ is 125°. What is the measure of $\angle 4$?

35°

Go Online You can complete an Extra Example online.

706 Module 11 • Geometric Figures

Interactive Presentation



Example 4, Find Missing Angle Measures, Slide 3 of 5

CLICK On Slide 3, students move through the steps to find the measure of angle 7.

TYPE On Slide 3, students determine the correct measure of angle 7.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find Missing Angle Measures

Objective

Students will find missing angle measures when two parallel lines are cut by more than one transversal.

Questions for Mathematical Discourse

SLIDE 2

AL Identify the two parallel lines in the diagram. Lines m and n are parallel.

AL Use the structure of the diagram to identify an angle that has a special relationship with $\angle 1$. Describe that relationship. Sample answer: $\angle 1$ and $\angle 6$ are alternate exterior angles, so their measures are equal.

OL Why might it be more efficient to try to find $m\angle 6$ as opposed to angles that are near $\angle 1$? Sample answer: $\angle 6$ is near $\angle 7$, which is the angle measure we need to find.

OL What other angles in the diagram have measures equal to $m\angle 1$? Sample answer: $\angle 1$ and $\angle 9$ are vertical angles, so $m\angle 9 = 40°$; $\angle 1$ and $\angle 3$ are corresponding so $m\angle 3 = 40°$.

BL A classmate claims that $\angle 1$ and $\angle 5$ are also alternate exterior angles because they are outside of the parallel lines and on different sides of line q. Why is this incorrect? Sample answer: $\angle 5$ is not formed by a transversal and one of the parallel lines.

SLIDE 3

AL Now that you know $m\angle 6$, what are some angles that are near $\angle 6$ of which you know the measure? Sample answer: $\angle 8$ is near $\angle 6$ and the right angle symbol indicates that $m\angle 8 = 90°$.

AL What kind of relationship is formed by $\angle 6$, $\angle 7$, and $\angle 8$? Sample answer: They form a straight line.

OL Knowing $m\angle 6$ and $m\angle 8$, why is this enough information to find $m\angle 7$? Sample answer: The three angles form a straight line, so the sum of their measures is 180°. Knowing two of the three is enough to solve for the third.

BL How else could you have solved this problem? Sample answer: Since $\angle 1$ and $\angle 3$ are corresponding angles, they have the same measure. $\angle 3$, $\angle 7$, and $\angle 8$ form a straight line, so the sum of their measures is 180°. I can subtract $m\angle 3$ and $m\angle 8$ from 180° to find $m\angle 7$.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Construction

Objective

Students will come up with their own strategy to solve an application problem involving angles found in bridge construction.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

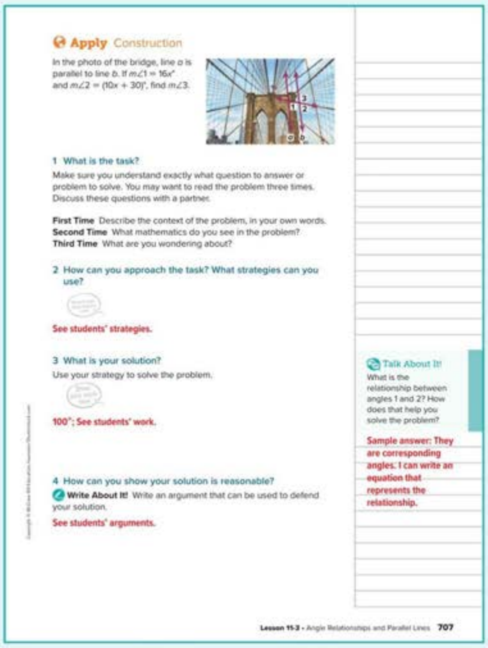
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What do you know about corresponding angles?
- What is the relationship between angles 2 and 3?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Construction

In the photo of the bridge, line a is parallel to line b. If $m\angle 1 = 16x°$ and $m\angle 2 = (10x + 30)°$, find $m\angle 3$.

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

100°; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! What is the relationship between angles 1 and 2? How does that help you solve the problem?

Sample answer: They are corresponding angles. I can write an equation that represents the relationship.

Lesson 11-3 • Angle Relationships and Parallel Lines 707

Interactive Presentation

Apply, Construction

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

In the painting, line *a* is parallel to line *b*. The measure of angle 1 is $(5x + 24)°$ and the measure of angle 2 is $7x°$. Find $m\angle 1$. 84°

Go Online You can complete an Extra Example online.

Pause and Reflect

What have you learned about the angles formed by *parallel lines* and *transversals*? Can you name the angles that are formed? Can you determine which angles have the same measure?

See students' observations.

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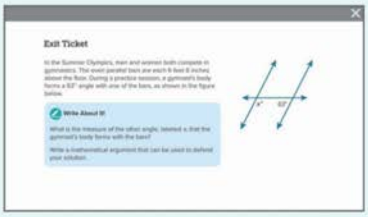
Interactive Presentation



Exit Ticket

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | **3 APPLICATION**

Exit Ticket

Refer to the Exit Ticket slide. What is the measure of the other angle, labeled x, that the gymnast's arms form with the bars? Write a mathematical argument that can be used to defend your solution. 117°; Sample answer: $x° + 63° = 180°$, so $m\angle x = 180° - 63°$ or 117°.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 9, 11–14
- ALEKS Angle Relationships, Parallel Lines

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 9, 13
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Classifying and Measuring Angles

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Classifying and Measuring Angles

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

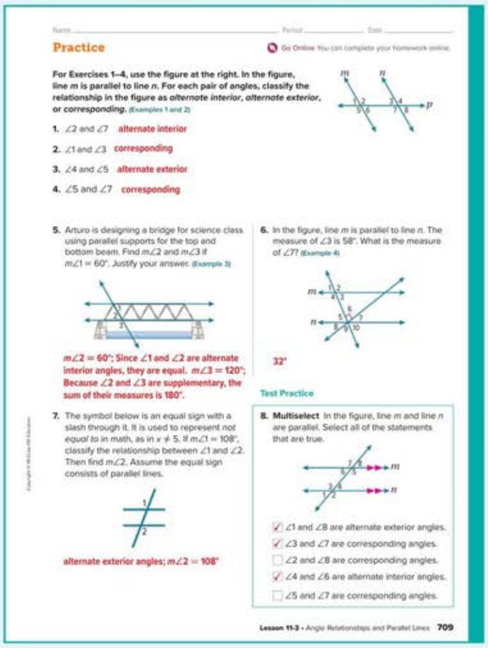
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	classify angle pairs created when parallel lines are cut by a transversal	1–4
2	find missing angle measures when two parallel lines are cut by a transversal	5
1	find missing angle measures when two parallel lines are cut by more than one transversal	6
2	extend concepts learned in class to apply them in new contexts	7, 8
3	solve application problems that involve finding missing angle measures formed by parallel lines and transversals	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Students may apply the congruence properties of *alternate interior, alternate exterior*, and *corresponding angles* introduced in this lesson to nonparallel lines. Remind students that these angle properties are only true for parallel lines. For deeper understanding, have students draw two nonparallel lines crossed by a transversal and examine the measures of the *alternate interior* and *alternate exterior* angles formed by the three lines.


Name ________ Period ______ Date ______

Practice

Go Online You can complete your homework online.

For Exercises 1–4, use the figure at the right. In the figure, line *m* is parallel to line *n*. For each pair of angles, classify the relationship in the figure as *alternate interior, alternate exterior,* or *corresponding.* (Examples 1 and 2)

1. ∠2 and ∠7 **alternate interior**
2. ∠1 and ∠3 **corresponding**
3. ∠4 and ∠5 **alternate exterior**
4. ∠5 and ∠7 **corresponding**

5. Arturo is designing a bridge for science class using parallel supports for the top and bottom beam. Find $m\angle 2$ and $m\angle 3$ if $m\angle 1 = 60°$. Justify your answer. (Example 3)

$m\angle 2 = 60°$; Since ∠1 and ∠2 are alternate interior angles, they are equal. $m\angle 3 = 120°$; Because ∠2 and ∠3 are supplementary, the sum of their measures is 180°.

6. In the figure, line *m* is parallel to line *n*. The measure of ∠3 is 58°. What is the measure of ∠7? (Example 4)

32°

7. The symbol below is an equal sign with a slash through it. It is used to represent *not equal to* in math, as in $x \neq 5$. If $m\angle 1 = 108°$, classify the relationship between ∠1 and ∠2. Then find $m\angle 2$. Assume the equal sign consists of parallel lines.

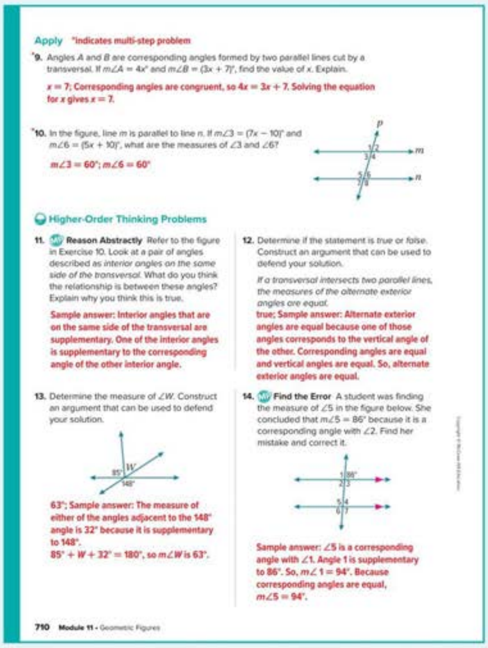
alternate exterior angles; $m\angle 2 = 108°$

Test Practice

8. **Multiselect** In the figure, line *m* and line *n* are parallel. Select all of the statements that are true.

- [x] ∠1 and ∠8 are alternate exterior angles.
- [x] ∠3 and ∠7 are corresponding angles.
- [ ] ∠2 and ∠8 are corresponding angles.
- [x] ∠4 and ∠6 are alternate interior angles.
- [ ] ∠5 and ∠7 are corresponding angles.

Lesson 11-3 • Angle Relationships and Parallel Lines 709



Apply *indicates multi-step problem

*9. Angles A and B are corresponding angles formed by two parallel lines cut by a transversal. If $m\angle A = 4x°$ and $m\angle B = (3x + 7)°$, find the value of x. Explain.

$x = 7$; Corresponding angles are congruent, so $4x = 3x + 7$. Solving the equation for x gives $x = 7$.

*10. In the figure, line m is parallel to line n. If $m\angle 3 = (7x - 10)°$ and $m\angle 6 = (5x + 10)°$, what are the measures of $\angle 3$ and $\angle 6$?

$m\angle 3 = 60°$; $m\angle 6 = 60°$

Higher-Order Thinking Problems

11. **Reason Abstractly** Refer to the figure in Exercise 10. Look at a pair of angles described as *interior angles on the same side of the transversal*. What do you think the relationship is between these angles? Explain why you think this is true.

Sample answer: Interior angles that are on the same side of the transversal are supplementary. One of the interior angles is supplementary to the corresponding angle of the other interior angle.

12. Determine if the statement is *true* or *false*. Construct an argument that can be used to defend your solution.

If a transversal intersects two parallel lines, the measures of the alternate exterior angles are equal.

true; Sample answer: Alternate exterior angles are equal because one of those angles corresponds to the vertical angle of the other. Corresponding angles are equal and vertical angles are equal. So, alternate exterior angles are equal.

13. Determine the measure of $\angle W$. Construct an argument that can be used to defend your solution.

63°; Sample answer: The measure of either of the angles adjacent to the 148° angle is 32° because it is supplementary to 148°.
$85° + W + 32° = 180°$, so $m\angle W$ is 63°.

14. **Find the Error** A student was finding the measure of $\angle 5$ in the figure below. She concluded that $m\angle 5 = 86°$ because it is a corresponding angle with $\angle 2$. Find her mistake and correct it.

Sample answer: $\angle 5$ is a corresponding angle with $\angle 1$. Angle 1 is supplementary to 86°. So, $m\angle 1 = 94°$. Because corresponding angles are equal, $m\angle 5 = 94°$.

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Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 11, students will describe the relationship between interior angles on the same side of a transversal. Encourage students to use reasoning to explain the relationship between the described angles.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students will identify the mistake and correct it. Encourage students to explain where the mistake was made and how to correct it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercises 9–10 Have students work in pairs. After completing the application problems, have students write two true statements and one false statement about each situation. An example of a true statement for Exercise 10 might be, “The measure of ∠2 is 60°.” An example of a false statement might be, “The measure of ∠4 is 60°.” Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Create your own higher-order thinking problem.
Use with Exercises 11–14 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other’s work, and discuss and resolve any differences.

Lesson 11-4

Triangles

7.G.A.2

LESSON GOAL

Students will draw triangles with and without tools.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Create Triangles

Learn: Classify Triangles
Learn: Draw Triangles Freehand
Example 1: Draw Triangles Freehand
Learn: Draw Triangles Using Tools
Example 2: Draw Triangles Using Tools
Example 3: Draw Triangles Using Tools
Learn: Draw Triangles with Technology
Example 4: Draw Triangles with Technology

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 66 of the *Language Development Handbook* to help your students build mathematical language related to classifying and drawing triangles given certain conditions.

ELL You can use the tips and suggestions on page T66 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.A** by drawing triangles with and without tools.
Standards for Mathematical Content: **7. G.A.2**
Standards for Mathematical Practice: **MP 2, MP3, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students examined relationships of angles formed by parallel lines cut by a transversal.
8.G.A.5

Now
Students draw triangles freehand, with ruler and protractor, and with technology.
7.G.A.2

Next
Students will examine relationships among the angles in a triangle.
8.G.A.5

Rigor

The Three Pillars of Rigor

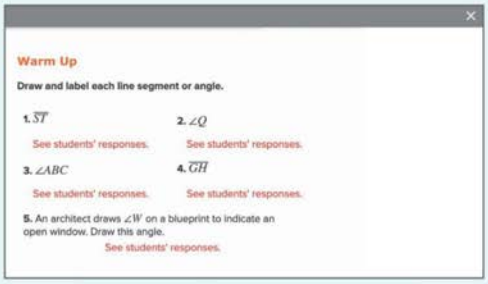
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of angles to develop an *understanding* of classifying and drawing triangles. They will use this understanding to build *fluency* in drawing triangles freehand, with tools, and with technology.

Mathematical Background

A triangle is a figure formed by three line segments that intersect only at their endpoints. The points where the segments intersect are called vertices. Triangles can be classified by their angles and their sides. Triangles can be drawn without tools, given angle and side length descriptions. They also can be drawn precisely if you use tools like a ruler, a protractor, or technology.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- drawing line segments and angles (Exercises 1–5)

Answers

1–5. See students ' responses.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the use of triangular sails on sailboats.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate class discussion. Additional questions are available online.

Ask:

- Use your understanding of acute angles to make a conjecture about what might be true about an *acute triangle*. Sample answer: An acute triangle may have angles that measure less than 90°.
- What does the prefix *equi*- mean? What does the term *lateral* mean? Using these meanings, what do you think the term *equilateral* means? Sample answer: The prefix *equi*- means equal and the term *lateral* means *side*. *Equilateral* means having equal sides.
- The term *isosceles* comes from the prefix *iso*- meaning *equal* and *skelos* meaning *leg*. Make a conjecture about what might be true about an *isosceles triangle*. Sample answer: Two of the sides of an isosceles triangle have equal length.

Explore Create Triangles

Objective

Students will use Web Sketchpad to explore the relationships among the side lengths or angle measures in a triangle.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with demonstrations of how to use sketches to create triangles. Throughout this activity, students will use sketches to try to create triangles when given three side lengths or three angle measures. They will use the patterns they observe to make conjectures about when it is possible to create triangles with those conditions.

Inquiry Question

How do you know whether or not it is possible to create a triangle given any three side lengths or any three angle measures? Sample answer: I know I can create a triangle given three side lengths if the third side length is less than the sum of the other two sides. I know I can create a triangle given three angle measures if the sum of the angle measures is exactly 180°.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Compare your results with a classmate. What patterns among the side lengths do you observe with the triangles you created? What patterns do you observe when triangles could not be created? Sample answer: If the triangle is created, the sum of two side lengths is always greater than the length of the third side. If the third side is longer than the sum of the other two sides, it does not create a triangle.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 11

Explore, Slide 4 of 11

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to create triangles.

CLICK

On Slides 2 and 3, students select from a drop-down menu the number of unique triangles they were able to make.

TYPE

On Slide 5, students make a conjecture about the relationship among the three side lengths of a triangle.

Interactive Presentation

Explore, Slide 8 of 11

TYPE

On Slide 6, students explain if they can form a triangle using the side lengths 7, 8, and 18 units.

CLICK

On Slides 7 and 8, students select from a drop-down menu the number of unique triangles they were able to make.

TYPE

On Slide 11, students respond to the Inquiry Question and view a sample answer.

Explore Create T riangles *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the measures they are given in order to determine whether triangles can be created from these measures.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore the relationships among possible side lengths and angle measures and whether or not triangles can be created from these measures.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 9 are shown.

Talk About It!

SLIDE 9

Mathematical Discourse

Compare your results with a classmate. What patterns among the angle measures do you observe with the triangles you created? What patterns do you observe when triangles could not be created? Sample answer: Triangles can only be created when the sum of the angle measures is 180°.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Classify Triangles

Objective

Students will understand how to classify triangles by angle measures and by side lengths.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, recommend that they use the definitions of the types of triangles in constructing their explanation. Students should be able to construct a plausible argument for why it is not possible to have these two types of triangles.

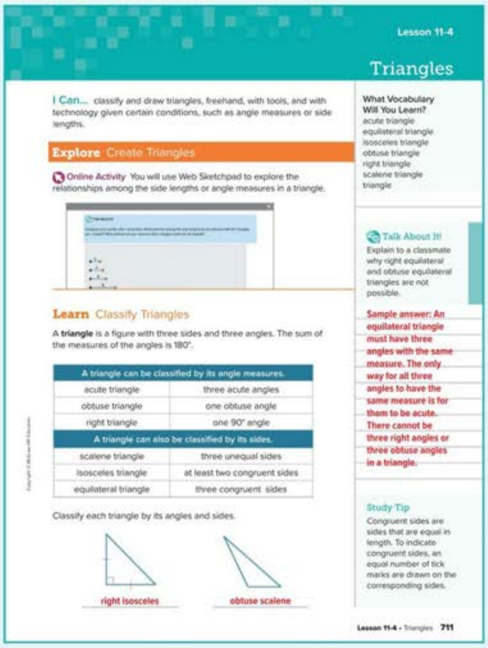
Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

Explain to a classmate why right equilateral and obtuse equilateral triangles are not possible. Sample answer: An equilateral triangle must have three angles with the same measure. The only way for all three angles to have the same measure is for them to be acute. There cannot be three right angles or three obtuse angles in a triangle.



Lesson 11-4

Triangles

I Can... classify and draw triangles, freehand, with tools, and with technology given certain conditions, such as angle measures or side lengths.

What Vocabulary Will You Learn?
acute triangle
equilateral triangle
isosceles triangle
obtuse triangle
right triangle
scalene triangle
triangle

Explore Create Triangles

Online Activity You will use Web Sketchpad to explore the relationships among the side lengths or angle measures in a triangle.

Learn Classify Triangles

A **triangle** is a figure with three sides and three angles. The sum of the measures of the angles is 180°.

A triangle can be classified by its angle measures.	
acute triangle	three acute angles
obtuse triangle	one obtuse angle
right triangle	one 90° angle
A triangle can also be classified by its sides.	
scalene triangle	three unequal sides
isosceles triangle	at least two congruent sides
equilateral triangle	three congruent sides

Classify each triangle by its angles and sides.

right isosceles — obtuse scalene

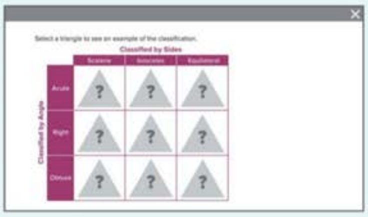
Talk About It! Explain to a classmate why right equilateral and obtuse equilateral triangles are not possible.

Sample answer: An equilateral triangle must have three angles with the same measure. The only way for all three angles to have the same measure is for them to be acute. There cannot be three right angles or three obtuse angles in a triangle.

Study Tip Congruent sides are sides that are equal in length. To indicate congruent sides, an equal number of tick marks are drawn on the corresponding sides.

Lesson 11-4 • Triangles 711

Interactive Presentation



Learn, Classify Triangles, Slide 2 of 3

CLICK

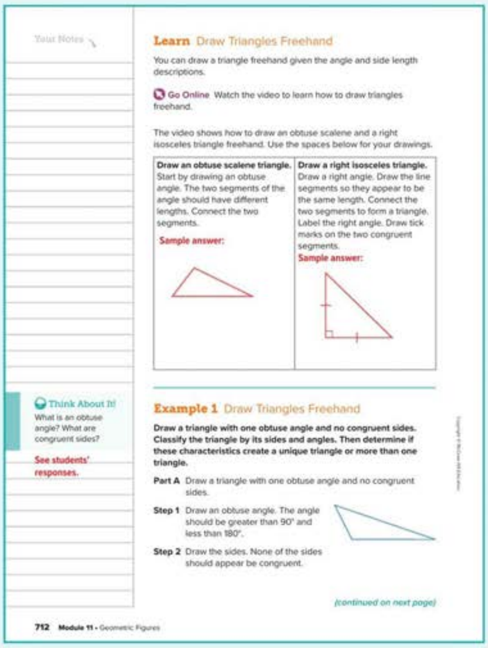
On Slide 2, students select triangles to see examples of the classifications by sides and angles.

DIFFERENTIATE

Language Development Activity ELL

If students are struggling with the different classifications of triangles, have them work with a partner to complete the following activity.

Each student should write the words *isosceles*, *scalene*, *equilateral*, *acute*, *obtuse*, and *right* on separate notecards and separate the cards into two piles: classified by side and classified by angle. The first student chooses a card from each pile. Both students draw the triangle described by the cards, share their drawings with their partners, and discuss any differences. If the triangle cannot be drawn (i.e. an obtuse equilateral triangle) each student should explain why. Students continue choosing cards and drawing triangles until the piles are empty.



Your Notes

Learn Draw Triangles Freehand

You can draw a triangle freehand given the angle and side length descriptions.

Go Online Watch the video to learn how to draw triangles freehand.

The video shows how to draw an obtuse scalene and a right isosceles triangle freehand. Use the spaces below for your drawings.

Draw an obtuse scalene triangle. Start by drawing an obtuse angle. The two segments of the angle should have different lengths. Connect the two segments. Sample answer:	Draw a right isosceles triangle. Draw a right angle. Draw the line segments so they appear to be the same length. Connect the two segments to form a triangle. Label the right angle. Draw tick marks on the two congruent segments. Sample answer:

Think About It! What is an obtuse angle? What are congruent sides?

See students' responses.

Example 1 Draw Triangles Freehand

Draw a triangle with one obtuse angle and no congruent sides. Classify the triangle by its sides and angles. Then determine if these characteristics create a unique triangle or more than one triangle.

Part A Draw a triangle with one obtuse angle and no congruent sides.

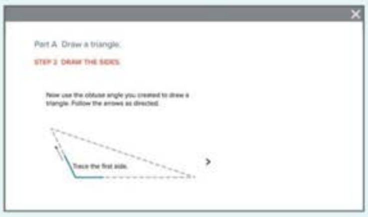
Step 1 Draw an obtuse angle. The angle should be greater than 90° and less than 180°.

Step 2 Draw the sides. None of the sides should appear be congruent.

(continued on next page)

712 Module 11 • Geometric Figures

Interactive Presentation



Example 1, Draw Triangles Freehand, Slide 3 of 7

WATCH

In the Learn, students watch a video that demonstrates how to draw a triangle without tools.

DRAG & DROP

On Slide 4 of Example 1, students drag names to classify the triangle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Draw Triangles Freehand

Objective

Students will understand how to draw triangles without tools.

Go Online

- Find additional teaching notes.
- Have students watch the video on Slide 1. The video illustrates how to draw a triangle freehand.

Example 1 Draw Triangles Freehand

Objective

Students will draw triangles without tools, classify the triangles by their sides and angles, and determine if the triangles are unique.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to adhere to the correct mathematical terminology when classifying the triangle that they drew and to clearly explain why more than one triangle can be drawn with these conditions.

Questions for Mathematical Discourse

SLIDE 2

AL What are the two criteria the triangle needs to have? It needs to have one obtuse angle and no congruent sides.

AL How can you draw an obtuse angle? Sample answer: Draw an angle that appears to be greater than 90°, but less than 180°.

OL If a triangle has one obtuse angle, what types of angles are the other two angles? Explain. acute angles; Sample answer: The sum of the angles in a triangle is 180°. If there is one angle that measures greater than 90°, then the other two need to each measure less than 90°.

BL Would it be possible to draw a triangle with one angle measuring 180°? Explain. no; Sample answer: Because the sum of the angle measures in a triangles is 180°, one angle cannot measure 180°. This would require the other two angles to each measure 0°. It is not possible to draw a triangle with these conditions.

(continued on next page)

Example 1 Draw Triangles Freehand *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL How can you make sure the triangle you draw has no congruent sides? Sample answer: Make sure all three side lengths appear to be different.

OL Can you draw other triangles that meet these same conditions of having one obtuse triangle and no congruent sides? If so, draw some examples. yes; See students' drawings.

OL What are some of the differences between the various triangles you drew? Sample answer: The measure of the obtuse angle can vary greatly. Some examples of its measurement can be 91°, 100°, 125°, 152°, and 179°.

BL Is it possible to draw a triangle with two obtuse angles? Explain. no; Sample answer: There can only be one obtuse angle in a triangle. If there were two obtuse angles, for example 91° and 92°, the sum of the angle measures would already exceed the sum of the angle measures of a triangle, 180°, without even considering the third angle measure.

SLIDE 4

AL What are the two ways to classify a triangle? by its angle measures and by its side lengths

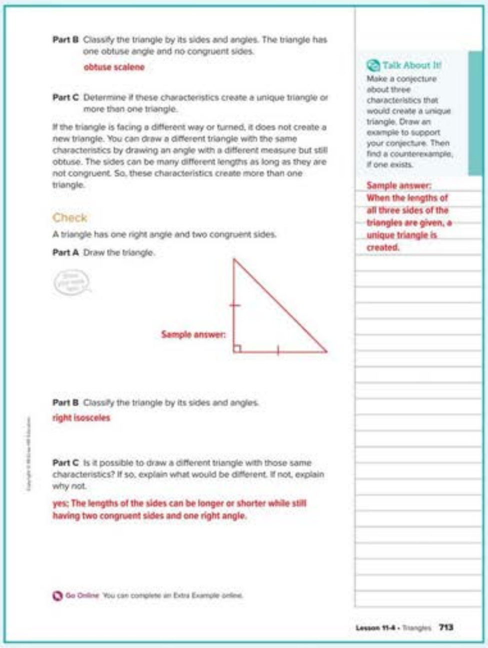
OL Which classification term indicates the triangle has no congruent sides? scalene

OL A classmate stated the triangle was both obtuse and acute because it has one obtuse angle and two acute angles. Is this reasoning correct? Explain. no; Sample answer: An acute triangle must have all three angles be acute. Every obtuse triangle will have one obtuse angle and two acute angles, so the only way to classify it by angle measures is as an obtuse triangle.

BL Can an obtuse triangle also be isosceles? Explain. yes; Sample answer: Draw the two sides that form the obtuse angle so that they have the same length.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Part B Classify the triangle by its sides and angles. The triangle has one obtuse angle and no congruent sides.

obtuse scalene

Part C Determine if these characteristics create a unique triangle or more than one triangle.

If the triangle is facing a different way or turned, it does not create a new triangle. You can draw a different triangle with the same characteristics by drawing an angle with a different measure but still obtuse. The sides can be many different lengths as long as they are not congruent. So, these characteristics create more than one triangle.

Talk About It! Make a conjecture about three characteristics that would create a unique triangle. Draw an example to support your conjecture. Then find a counterexample, if one exists.

Sample answer: When the lengths of all three sides of the triangles are given, a unique triangle is created.

Check

A triangle has one right angle and two congruent sides.

Part A Draw the triangle.

Sample answer:

Part B Classify the triangle by its sides and angles.

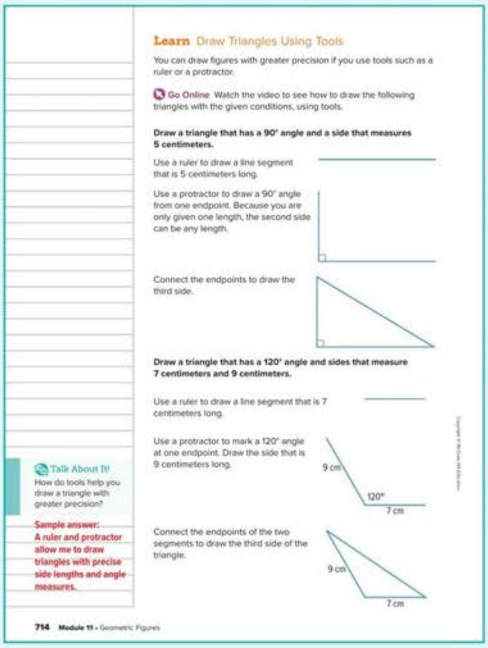
right isosceles

Part C Is it possible to draw a different triangle with those same characteristics? If so, explain what would be different. If not, explain why not.

yes; The lengths of the sides can be longer or shorter while still having two congruent sides and one right angle.

Go Online You can complete an Extra Example online.

Lesson 11-4 • Triangles 713



Learn Draw Triangles Using Tools

You can draw figures with greater precision if you use tools such as a ruler or a protractor.

Go Online Watch the video to see how to draw the following triangles with the given conditions, using tools.

Draw a triangle that has a 90° angle and a side that measures 5 centimeters.

Use a ruler to draw a line segment that is 5 centimeters long.

Use a protractor to draw a 90° angle from one endpoint. Because you are only given one length, the second side can be any length.

Connect the endpoints to draw the third side.

Draw a triangle that has a 120° angle and sides that measure 7 centimeters and 9 centimeters.

Use a ruler to draw a line segment that is 7 centimeters long.

Use a protractor to mark a 120° angle at one endpoint. Draw the side that is 9 centimeters long.

9 cm
120°
7 cm

Connect the endpoints of the two segments to draw the third side of the triangle.

9 cm
7 cm

Talk About It!
How do tools help you draw a triangle with greater precision?

Sample answer: A ruler and protractor allow me to draw triangles with precise side lengths and angle measures.

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Interactive Presentation

Learn, Draw Triangles Using Tools, Slide 1 of 2

WATCH

On Slide 1, students watch a video that demonstrates how to draw a triangles with tools.

Learn Draw T riangles Using Tools

Objective

Students will understand how to draw triangles using a ruler and protractor.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically A s students discuss the *Talk About It!* question on Slide 2, encourage them to recognize how using these tools can help draw triangles with greater precision of measurements.

Go Online

Have students watch the video on Slide 1. The video illustrates how to draw a triangle using a ruler and protractor.

Teaching Notes

SLIDE 1

Play the video for the class. You may wish to have students draw the figures at their desks during the video. Pause the video after each step to allow students to complete that step. Ask students to compare the triangles they drew with the ones in the video.

You may wish to ask students if there is a different triangle they can draw for each example. Some students may say they can draw a different triangle with a 90° angle and a 5 centimeter side since the second side can be any length, but they cannot draw a different triangle for the second example. Point out that in the example, the 120° angle was drawn between the two given sides. Encourage them to experiment with the placement of the angle to see if they can draw a different triangle.

Talk About It!

SLIDE 2

Mathematical Discourse

How do tools help you draw a triangle with greater precision? Sample answer: A ruler and protractor allow me to draw triangles with precise side lengths and angle measures.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of using tools to draw triangles, have students solve the following problem.

A triangle has sides of lengths 6 cm and 8 cm connected by an obtuse angle. What are the possible values for the length of the third side, x? Explain. 10 cm $< x < 14$ cm; Sample answer: If the sides of 6 cm and 8 cm were connected by a right angle, the third side would be 10 cm (using a ruler to measure). Because the angle is greater than 90°, $x > 10$ cm. As the obtuse angle gets closer to 180°, the length of the third side gets closer to 14 cm, but it can never be equal to 14 cm. This means that 10 cm $< x < 14$ cm.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Draw Triangles Using Tools

Objective

Students will draw triangles (given three angles) using a ruler and protractor.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 6, encourage them to use their understanding of the sum of the angle measures of a triangle to explain their reasoning, and to use reasoning to explain why a different triangle can be drawn to meet these conditions.

5 Use Appropriate Tools Strategically Encourage students to use a ruler and protractor to try to draw a triangle with the given conditions.

Questions for Mathematical Discourse

SLIDE 2

AL What are some sample side lengths you can use to draw the first line segment? Sample answers: 3 in., 5.25 in., 4 cm

OL Can the first line segment you draw have any length? Explain. yes; Sample answer: No side lengths are given in the conditions, so the segment can be any length.

BL Why do you think it may be helpful to label the endpoints? Sample answer: I will need to draw the angles next. It will be helpful to name the angles as I refer to them.

SLIDE 3

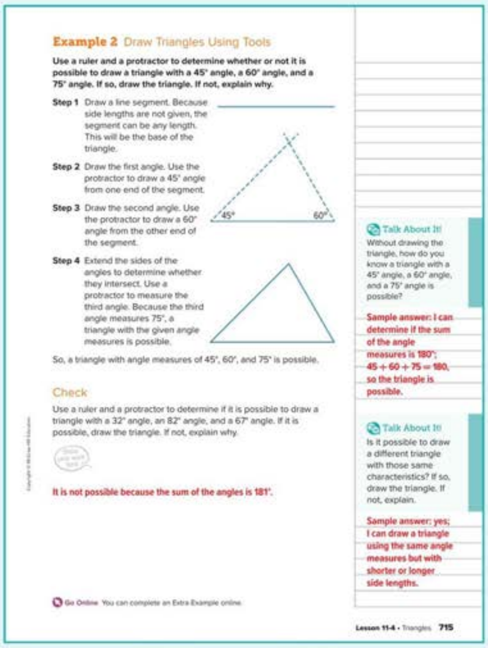
AL What angle measurements were given as the criteria for this triangle? 45°, 60°, and 75°

OL Does it matter which angle you draw first? Explain. no; Sample answer: As long as I eventually draw all three angles, it does not matter which one I draw first.

BL In this example, the 45° angle is drawn to represent angle *A*. Can the angle be drawn to represent angle *B* instead? Yes, the triangle will have the same conditions.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Draw Triangles Using Tools

Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 45° angle, a 60° angle, and a 75° angle. If so, draw the triangle. If not, explain why.

Step 1 Draw a line segment. Because side lengths are not given, the segment can be any length. This will be the base of the triangle.

Step 2 Draw the first angle. Use the protractor to draw a 45° angle from one end of the segment.

Step 3 Draw the second angle. Use the protractor to draw a 60° angle from the other end of the segment.

Step 4 Extend the sides of the angles to determine whether they intersect. Use a protractor to measure the third angle. Because the third angle measures 75°, a triangle with the given angle measures is possible.

So, a triangle with angle measures of 45°, 60°, and 75° is possible.

Check

Use a ruler and a protractor to determine if it is possible to draw a triangle with a 32° angle, an 82° angle, and a 67° angle. If it is possible, draw the triangle. If not, explain why.

It is not possible because the sum of the angles is 181°.

Go Online You can complete an Extra Example online.

Talk About It! Without drawing the triangle, how do you know a triangle with a 45° angle, a 60° angle, and a 75° angle is possible?

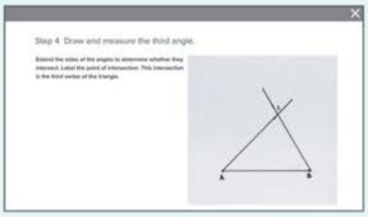
Sample answer: I can determine if the sum of the angle measures is 180°; $45 + 60 + 75 = 180$, so the triangle is possible.

Talk About It! Is it possible to draw a different triangle with those same characteristics? If so, draw the triangle. If not, explain.

Sample answer: yes; I can draw a triangle using the same angle measures but with shorter or longer side lengths.

Lesson 11-4 • Triangles 715

Interactive Presentation



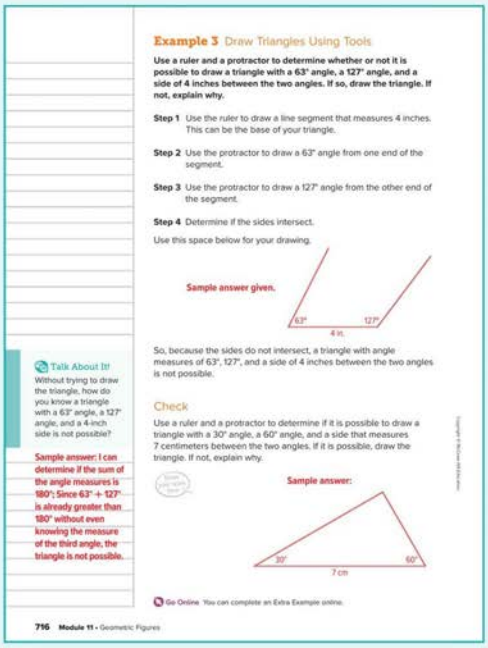
Example 2, Draw Triangles Using Tools, Slide 5 of 7

CLICK

On Slide 5, students select from a drop-down menu if a triangle is possible or not possible with given angle measures.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Draw Triangles Using Tools

Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 63° angle, a 127° angle, and a side of 4 inches between the two angles. If so, draw the triangle. If not, explain why.

Step 1 Use the ruler to draw a line segment that measures 4 inches. This can be the base of your triangle.

Step 2 Use the protractor to draw a 63° angle from one end of the segment.

Step 3 Use the protractor to draw a 127° angle from the other end of the segment.

Step 4 Determine if the sides intersect.

Use this space below for your drawing.

Sample answer given.

So, because the sides do not intersect, a triangle with angle measures of 63°, 127°, and a side of 4 inches between the two angles is not possible.

Talk About It!
Without trying to draw the triangle, how do you know a triangle with a 63° angle, a 127° angle, and a 4-inch side is not possible?

Sample answer: I can determine if the sum of the angle measures is 180°; Since 63° + 127° is already greater than 180° without even knowing the measure of the third angle, the triangle is not possible.

Check

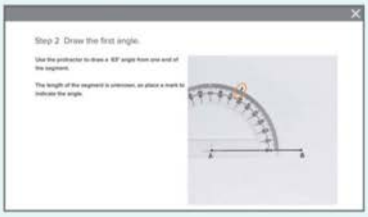
Use a ruler and a protractor to determine if it is possible to draw a triangle with a 30° angle, a 60° angle, and a side that measures 7 centimeters between the two angles. If it is possible, draw the triangle. If not, explain why.

Sample answer:

Go Online You can complete an Extra Example online.

716 Module 11 • Geometric Figures

Interactive Presentation



Example 3, Draw Triangles Using Tools, Slide 3 of 7

CLICK

On Slide 5, students select from a drop-down menu if a triangle with given angle measures is possible or not possible.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.G.A.2

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Draw T riangles Using Tools

Objective

Students will draw triangles (given two angles and the length of the included side) using a ruler and protractor.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use the indicated tools, ruler and protractor, to try to draw a triangle with the given conditions.

Questions for Mathematical Discourse

SLIDE 2

AL What criteria does this proposed triangle need to meet? It must have a 63° angle, a 127° angle, and a side of 4 inches between the two angles.

OL Why do you think it will be more helpful to draw the line segment first? Sample answer: The line segment is between the two angles. If I draw it first, then I can draw the two angles on either endpoint.

BL Would drawing a line segment of a different length possibly change the outcome of whether or not you could draw the triangle? Explain your reasoning. yes; Sample answer: The distance between the two endpoints would change. Because the angles will be formed from those endpoints, I may or may not be able to connect the other two line segments to form a triangle.

SLIDE 3

AL In this example, we chose to draw the 63° angle first. Can you draw the 127° angle first instead? Explain? yes; Sample answer: As long as both angles are eventually drawn, it does not matter which one is drawn first.

OL In this example, we chose to draw the 63° angle from endpoint *A*. Can you draw this angle from endpoint *B* instead? Explain. yes; Sample answer: As long as both angles are eventually drawn, it doesn't matter from which endpoint they are drawn.

BL Based on the drawing from Step 2, predict the outcome after drawing the 127° angle. Explain. Sample answer: It will not be possible to form the triangle because the line segments will not intersect.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Draw T riangles with Technology

Objective

Students will understand how to draw triangles using technology.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 2, encourage them to communicate precisely about how they can determine the number of possible triangles that can be drawn.

Teaching Notes

SLIDE 1

Before students select the *Animation* button on the software, you may wish to ask them if they think another triangle can be drawn with the three angle measures. Students familiar with scale factors may say that the angle measures can remain the same but the side lengths can change by the same scale factor.

Students can also select a point and move it on Web Sketchpad without changing the angle measures. You may wish to give them side lengths and ask them to create different triangles.

Talk About It!

SLIDE 2

Mathematical Discourse

Without using technology, is there another way to determine how many triangles can be drawn? Sample answer: When only angle measures are given, the triangle can be created using different side lengths. Therefore, more than one triangle can be created.

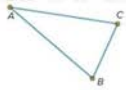
Learn Draw Triangles with Technology

You can create triangles with even greater precision by using technology. If you are given a set of conditions, you can use geometry software, such as Web Sketchpad, to determine if the conditions determine a unique triangle, more than one triangle, or no triangle.

Go Online Use Web Sketchpad to complete the activity.

Web Sketchpad was used to create a triangle with a 32° angle and two 74° angles. Do these conditions determine a unique triangle, more than one triangle, or no triangle?

$m\angle BAC = 32.0°$ $AB = 3.26$ in.
$m\angle CBA = 74.0°$ $AC = 3.26$ in.
$m\angle ACB = 74.0°$ $BC = 1.80$ in.



The side lengths can vary, but the angle measures stay the same. So, the given conditions create more than one triangle.

Talk About It!

Without using technology, is there another way to determine how many triangles can be drawn?

Sample answer: When only angle measures are given, the triangle can be created using different side lengths. Therefore, more than one triangle can be created.

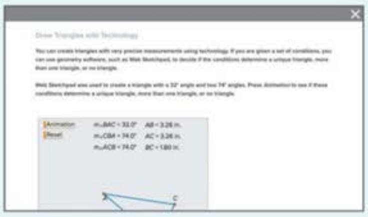
Pause and Reflect

Use Web Sketchpad to verify that more than one triangle can be drawn that satisfies the given conditions of having one 32° angle and two 74° angles. Sketch and label at least 2 triangles below that meet these conditions.

See students' observations.

Lesson 11-4 • Triangles 717

Interactive Presentation



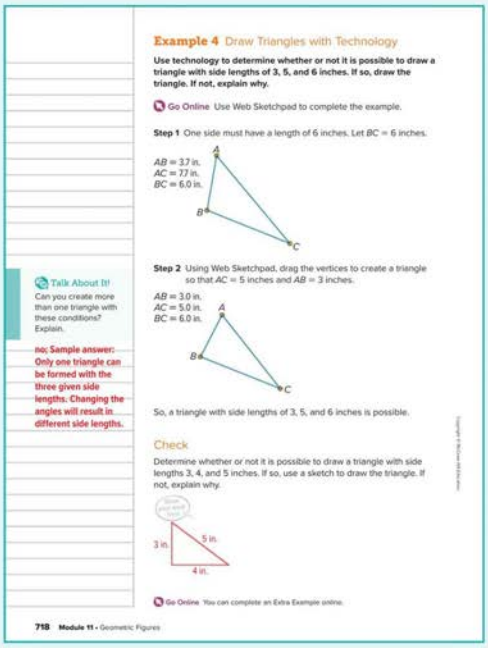
Learn, Draw Triangles with Technology, Slide 1 of 2

WEB SKETCHPAD

On Slide 1, students use Web Sketchpad to create a triangle given angles.

CLICK

On Slide 1, students select from a drop-down menu to indicate if the conditions determine a unique triangle.



Example 4 Draw Triangles with Technology

Use technology to determine whether or not it is possible to draw a triangle with side lengths of 3, 5, and 6 inches. If so, draw the triangle. If not, explain why.

Go Online Use Web Sketchpad to complete the example.

Step 1 One side must have a length of 6 inches. Let $BC = 6$ inches.

$AB = 3.7$ in.
$AC = 7.7$ in.
$BC = 6.0$ in.

Step 2 Using Web Sketchpad, drag the vertices to create a triangle so that $AC = 5$ inches and $AB = 3$ inches.

$AB = 3.0$ in.
$AC = 5.0$ in.
$BC = 6.0$ in.

So, a triangle with side lengths of 3, 5, and 6 inches is possible.

Talk About It! Can you create more than one triangle with these conditions? Explain.

no; Sample answer: Only one triangle can be formed with the three given side lengths. Changing the angles will result in different side lengths.

Check

Determine whether or not it is possible to draw a triangle with side lengths 3, 4, and 5 inches. If so, use a sketch to draw the triangle. If not, explain why.

3 in. 5 in. 4 in.

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4, Draw Triangles with Technology, Slide 2 of 4

WEB SKETCHPAD

On Slide 2, students use Web Sketchpad to draw a triangle to satisfy the given conditions.

CLICK

On Slide 2, students select from a drop-down menu to indicate if the side lengths form a triangle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Draw Triangles with Technology

Objective

Students will draw triangles (given three sides) using technology.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of how side lengths and angle measures compare as they construct their response.

5 Use Appropriate Tools Strategically Encourage students to use the indicated tool, Web Sketchpad, to try to draw a triangle with the given conditions.

Questions for Mathematical Discourse

SLIDE 2

AL What criteria does this proposed triangle need to meet? The triangle needs to have side lengths of 3 inches, 5 inches, and 6 inches.

OL What is the advantage of using technology to draw triangles? Sample answer: The triangles can be drawn with measurements that are very precise.

BL Use the sketch to explore different side length measures that will also create a triangle. What side lengths did you discover will also create a triangle? Sample answer: 5 inches, 5 inches, and 6 inches

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Essential Question Follow-Up

How does geometry help to describe objects?

In this lesson, students learned how to classify and draw triangles. Encourage them to discuss with a partner how their understanding of triangle terminology can help them describe real-world objects. For example, they may describe a bannister for a staircase as a scalene right triangle.

Exit Ticket

Refer to the Exit Ticket slide. Suppose you need to buy a sail for your sailboat, and you are told your sail needs to have side lengths of 7 feet and 24 feet, with a 90° angle between them. Do you have all the information you need to buy the correct size sail? Write a mathematical argument that can be used to you defend your solution. yes; Sample answer: If I draw the two side lengths with a right angle between them, there is only one unique triangle that can be formed with these criteria.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

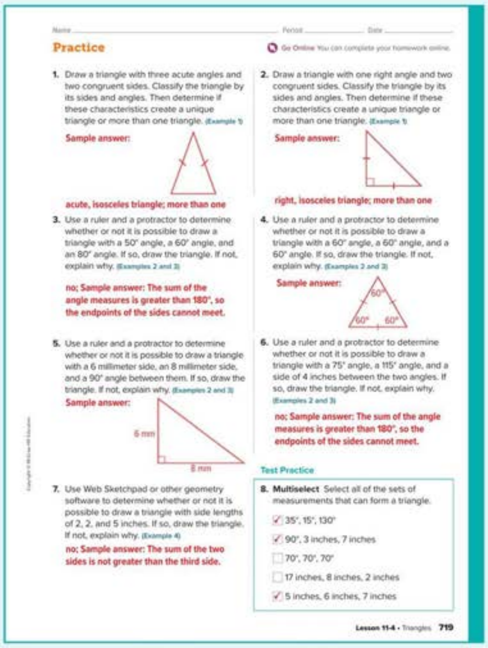
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	draw triangles without tools, classify the triangle by its sides and angles, and determine if it has the given characteristics	1, 2
2	draw triangles using a ruler and protractor	3–6
2	draw triangles using technology	7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving triangles	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Students may or may not draw right triangles with the right angle on the left-hand side. If so, encourage them to check the measurements and their placements.


Name _______ Period _______ Date _______

Practice

Go Online You can complete your homework online.

1. Draw a triangle with three acute angles and two congruent sides. Classify the triangle by its sides and angles. Then determine if these characteristics create a unique triangle or more than one triangle. (Example 1)
   Sample answer: acute, isosceles triangle; more than one
2. Draw a triangle with one right angle and two congruent sides. Classify the triangle by its sides and angles. Then determine if these characteristics create a unique triangle or more than one triangle. (Example 1)
   Sample answer: right, isosceles triangle; more than one
3. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 50° angle, a 60° angle, and an 80° angle. If so, draw the triangle. If not, explain why. (Examples 2 and 3)
   no; Sample answer: The sum of the angle measures is greater than 180°, so the endpoints of the sides cannot meet.
4. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 60° angle, a 60° angle, and a 60° angle. If so, draw the triangle. If not, explain why. (Examples 2 and 3)
   Sample answer: 60° 60° 60°
5. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 6 millimeter side, an 8 millimeter side, and a 90° angle between them. If so, draw the triangle. If not, explain why. (Examples 2 and 3)
   Sample answer: 6 mm 8 mm
6. Use a ruler and a protractor to determine whether or not it is possible to draw a triangle with a 75° angle, a 115° angle, and a side of 4 inches between the two angles. If so, draw the triangle. If not, explain why. (Examples 2 and 3)
   no; Sample answer: The sum of the angle measures is greater than 180°, so the endpoints of the sides cannot meet.
7. Use Web Sketchpad or other geometry software to determine whether or not it is possible to draw a triangle with side lengths of 2, 2, and 5 inches. If so, draw the triangle. If not, explain why. (Example 4)
   no; Sample answer: The sum of the two sides is not greater than the third side.

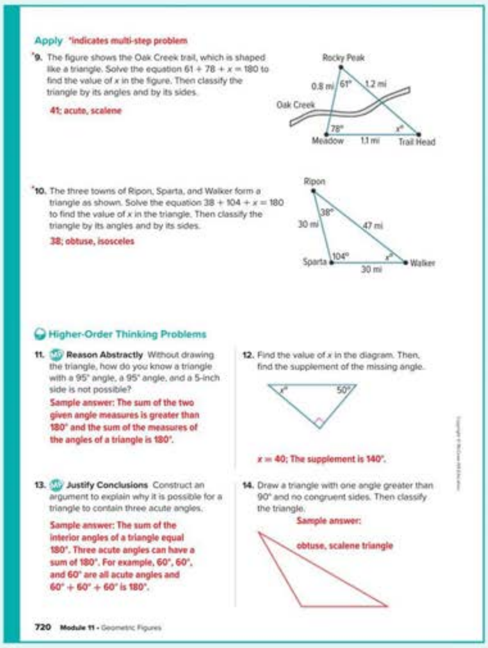
Test Practice

8. **Multiselect** Select all of the sets of measurements that can form a triangle.
   - [x] 35°, 15°, 130°
   - [x] 90°, 3 inches, 7 inches
   - [ ] 70°, 70°, 70°
   - [ ] 17 inches, 8 inches, 2 inches
   - [x] 5 inches, 6 inches, 7 inches

Lesson 11-4 • Triangles 719

Interactive Presentation


Exit Ticket



Apply *indicates multi-step problem

*9. The figure shows the Oak Creek trail, which is shaped like a triangle. Solve the equation $61 + 78 + x = 180$ to find the value of x in the figure. Then classify the triangle by its angles and by its sides.

41; acute, scalene

*10. The three towns of Ripon, Sparta, and Walker form a triangle as shown. Solve the equation $38 + 104 + x = 180$ to find the value of x in the triangle. Then classify the triangle by its angles and by its sides.

38; obtuse, isosceles

Higher-Order Thinking Problems

11. **Reason Abstractly** Without drawing the triangle, how do you know a triangle with a 95° angle, a 95° angle, and a 5-inch side is not possible?

Sample answer: The sum of the two given angle measures is greater than 180° and the sum of the measures of the angles of a triangle is 180°.

12. Find the value of x in the diagram. Then, find the supplement of the missing angle.

$x = 40$; The supplement is 140°.

13. **Justify Conclusions** Construct an argument to explain why it is possible for a triangle to contain three acute angles.

Sample answer: The sum of the interior angles of a triangle equal 180°. Three acute angles can have a sum of 180°. For example, 60°, 60°, and 60° are all acute angles and $60° + 60° + 60°$ is 180°.

14. Draw a triangle with one angle greater than 90° and no congruent sides. Then classify the triangle.

Sample answer:

obtuse, scalene triangle

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Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 11, use reasoning to explain why a triangle with given characteristics is not possible.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students explain why it is possible for a triangle to contain three acute angles.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Clearly and precisely explain.

Use with Exercise 13 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning. Encourage students to come up with a variety of responses, such as drawing a diagram.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 9, 11–14
- ALEKS® Triangle Construction and Triangle Inequalities

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 10, 12
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Classifying Triangles

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS® Classifying Triangles

Lesson 11-5

Angle Relationships and Triangles

8.G.A.5

LESSON GOAL

Students will examine relationships among the angles in a triangle.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Triangles

Explore: Angles of Triangles

Learn: Angle Sum of Triangles
Example 1: Find Missing Angle Measures
Example 2: Use Ratios to Find Angle Measures

Explore: Exterior Angles of Triangles

Learn: Exterior Angles of Triangles
Example 3: Find Exterior Angle Measures
Example 4: Use Exterior Angles to Find Missing Angle Measures
Apply: Geometry

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Interior Angle Sum Using Triangles		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 67 of the *Language Development Handbook* to help your students build mathematical language related to angle relationships and triangles.

ELL You can use the tips and suggestions on page T67 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by examining relationships of angles in a triangle.
Standards for Mathematical Content: **8. G.A.5**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students drew triangles freehand, with ruler and protractor, and with technology.
7.G.A.2

Now
Students examine relationships among the angles in a triangle.
8.G.A.5

Next
Students will solve problems involving scale drawings.
7.G.A.1

Rigor

The Three Pillars of Rigor

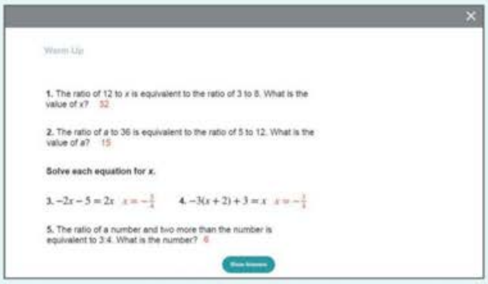
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of triangles by learning about the angle sum and exterior angles of triangles. They use their understanding to find missing angle measures of interior and exterior angles of triangles.

Mathematical Background

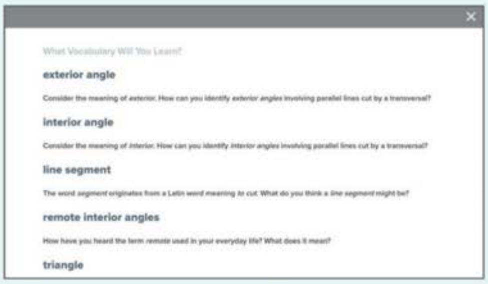
The sides of a triangle are called *line segments* and the points where the lines intersect are called *vertices.* The angles inside a triangle are referred to as *interior angles*. The sum of the measures of the interior angles of a triangle is 180 degrees. An *exterior angle* of a triangle is the angle formed by extending one of the sides past a vertex. As such, the exterior angle falls outside the triangle. The two interior angles opposite the exterior angle are called *remote interior angles.* The measure of an exterior angle is equal to the sum of the measures of its two remote interior angles.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding ratios (Exercises 1, 2, 5)
- solving equations (Exercises 3–4)

Answers

1. 32
2. 15
3. $x = -\frac{5}{4}$
4. $x = -\frac{3}{4}$
5. 6

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the use of triangles in the design of many structures.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- Consider the meaning of *exterior*. How can you identify *exterior angles* involving parallel lines cut by a transversal? Sample answer: Since the term *exterior* means outside of something, exterior angles lie on the outside of the two parallel lines.
- Consider the meaning of *interior*. How can you identify *interior angles* involving parallel lines cut by a transversal? Sample answer: Since the term *interior* means inside of something, interior angles lie on the inside of, or between, the two parallel lines.
- The word *segment* originates from a Latin word meaning *to cut*. What do you think a *line segment* might be? Sample answer: A portion of a line.
- How have you heard the term *remote* used in your everyday life? What does it mean? Sample answer: remote control, work remotely; Remote can mean far away.

Learn Triangles

Objective

Students will understand the parts of a triangle (sides, vertices, and angles), and how to name them.

Teaching Notes

SLIDE 1

Students will learn the definitions for *line segment, triangle, vertex*, and *interior angles* of triangles. In addition, students will learn how to name triangles using its vertices, and how to name line segments, or sides of triangles. Students have previously learned these terms, but may need a refresher. Be sure students understand that when naming an angle using three vertices, the middle letter must be the vertex of the angle.

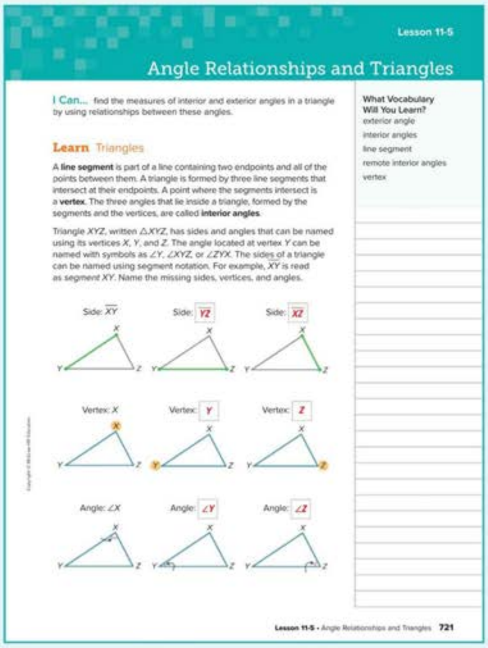
Have students use the interactive tool to see the given sides, vertices, and angles highlighted on the triangle. You may wish to ask a volunteer to come to the board and point to where they think the given side, vertex, or angle is based on its name. Then have them select to highlight the location and see if they were correct in their prediction. Ask students if there are any other ways to name the sides or angles. For example, the side with vertices X and Y can be named either $\overline{XY}$ or $\overline{YX}$. To name angle X using three letters, you can write $\angle YXZ$ or $\angle ZXY$. The vertex X must be in the middle.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling with how to name sides of a triangle, have them work with a partner to complete the following activity.

- Draw a triangle of any size. Circle the *vertices* and label them *A, B*, and *C*.
- A side is formed between *two* vertices. How many ways are there to write *two* of the letters *A, B*, and *C* together? List them. There are six ways of writing two of the letters *A, B*, and *C* together; *AB, BA, BC, CB, AC*, and *CA*.
- How many sides of a triangle are there? A triangle has three sides.
- Why do you think there are three sides, but six ways to write pairs of the vertices? Sample answer: The vertices can be written in any order, because the side between vertices *A* and *B* can thought of as connecting vertices *A* and *B*, or *B* and *A*.
- Why do you think we use a bar above the two vertices when naming the side of a triangle? Sample answer: The side of a triangle is not a line because it does not extend forever in either direction. Its endpoints are the vertices of the triangle.


Lesson 11-5

Angle Relationships and Triangles

I Can... find the measures of interior and exterior angles in a triangle by using relationships between these angles.

What Vocabulary Will You Learn?
exterior angle
interior angles
line segment
remote interior angles
vertex

Learn Triangles

A **line segment** is part of a line containing two endpoints and all of the points between them. A triangle is formed by three line segments that intersect at their endpoints. A point where the segments intersect is a **vertex**. The three angles that lie inside a triangle, formed by the segments and the vertices, are called **interior angles**.

Triangle XYZ, written $\triangle XYZ$, has sides and angles that can be named using its vertices X, Y, and Z. The angle located at vertex Y can be named with symbols as $\angle Y$, $\angle XYZ$, or $\angle ZYX$. The sides of a triangle can be named using segment notation. For example, $\overline{XY}$ is read as segment XY. Name the missing sides, vertices, and angles.

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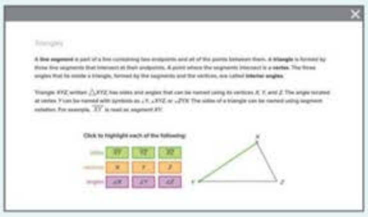
Interactive Presentation



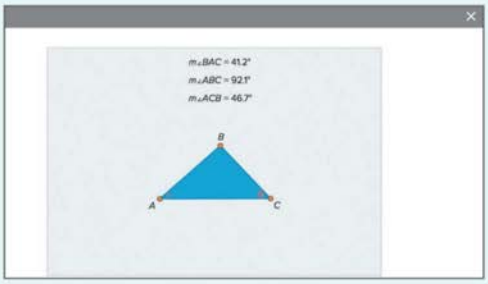
Learn, Triangles

CLICK

On Slide 1, students highlight the different sides, angles, and vertices of the triangle.

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationship among the angles in triangles.

Explore Angles of T riangles

Objective

Students will use Web Sketchpad to explore the relationship among the angles in triangles.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with a triangle drawn using the sketch. Students will be able to move the vertices around and asked to observe what happens to the angle measures. Students will use their observations to make conjectures about the sum of the measures in a triangle. Finally, students will be guided through the informal proof that the sum of the measures of the angles is 180 degrees.

Inquiry Question

What is the relationship among the measures of a triangle? Sample answer: The sum of the measures of the angles of a triangle is 180°.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

How do the measures of the interior angles change? How do they relate to one another? Sample answer: When one angle of the triangle is changed, the measures of the two other angles also change. The sum of the measures of the three angles is 180 degrees.

(continued on next page)

Explore Angles of Triangles *(continued)*

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others Encourage students to use what they discovered in the Explore activity to reason about the sum of the measures of a triangle before being guided through the informal proof.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how the measures of the interior angles of a triangle change as they drag the vertices to create new triangles.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

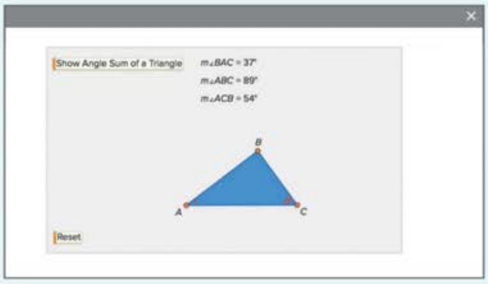
Talk About It!

SLIDE 4

Mathematical Discourse

Press the *Rotate Triangles* button. Drag point *B* and observe how the three triangles are related to each other and to the parallel line. What is true about the measures of $\angle BAC$ and $\angle ABC'$? $\angle ACB$ and $\angle A'BC$? Explain. Sample answer: They are equal because they are alternate interior angles.

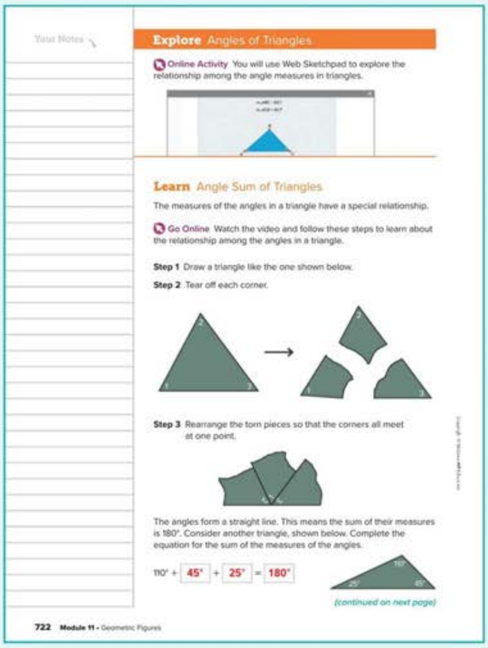
Interactive Presentation



Explore, Slide 4 of 6

TYPE

On Slide 6, students respond tothe Inquiry Question and view a sample answer.



Your Notes

Explore Angles of Triangles

Online Activity You will use Web Sketchpad to explore the relationship among the angle measures in triangles.

Learn Angle Sum of Triangles

The measures of the angles in a triangle have a special relationship.

Go Online Watch the video and follow these steps to learn about the relationship among the angles in a triangle.

Step 1 Draw a triangle like the one shown below.

Step 2 Tear off each corner.

Step 3 Rearrange the torn pieces so that the corners all meet at one point.

The angles form a straight line. This means the sum of their measures is 180°. Consider another triangle, shown below. Complete the equation for the sum of the measures of the angles.

110° + 45° + 25° = 180°

(continued on next page)

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Interactive Presentation

Learn, Angle Sum of Triangles, Slide 1 of 2

WATCH

On Slide 1, students watch a video that explains the angle sum relationship of triangles.

FLASHCARDS

On Slide 2, students use Flashcards to view the angle sum relationship expressed in multiple representations.

8.G.A.5

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Angle Sum of T riangles

Objective

Students will understand that the sum of the three interior angle measures of a triangle is 180 degrees.

Go Online to have students watch the video on Slide 1. The video illustrates the sum of the measures of the angles in a triangle.

Teaching Notes

SLIDE 1

You may wish to pause the video after the student has rearranged the three torn parts of the triangle to form a straight line. Ask students what they know about straight lines. Students should note that a straight line has an angle measure of 180 degrees. Thus, the sum of the three torn angles is also equal to 180 degrees. After students have watched the video, you may wish to have them recreate the activity by drawing a triangle on a piece of paper, cutting out the triangle, and then tearing off each angle. Have them rearrange the torn pieces so that they form a straight line to verify the sum of the angle measures is 180 degrees.

(continued on next page)

Learn Angle Sum of Triangles *(continued)*

Teaching Notes

SLIDE 2

Have students select the *Words, Variables*, and *Model* flashcards to connect the model of the angle sum of a triangle with an equation.

Example 1 Find Missing Angle Measures

Objective

Students will find missing angle measures in triangles.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to represent the relationship among the angles shown in the flag with the correct equation.

6 Attend to Precision Students should adhere to the angle sum formula of triangles to accurately calculate the value of *x*. Be sure students understand that while the value of *x* is 34 (no units), the measure of the angle is 34 degrees (with units).

Questions for Mathematical Discourse

SLIDE 1

AL What are the two known angle measures in the triangle? 56° and 90°

AL Study the triangle shown in the flag. How does the angle labeled *x* relate to the other two angles of the triangle? Sample answer: The measures of the angles *x*, 56°, and the right angle, have a sum of 180°.

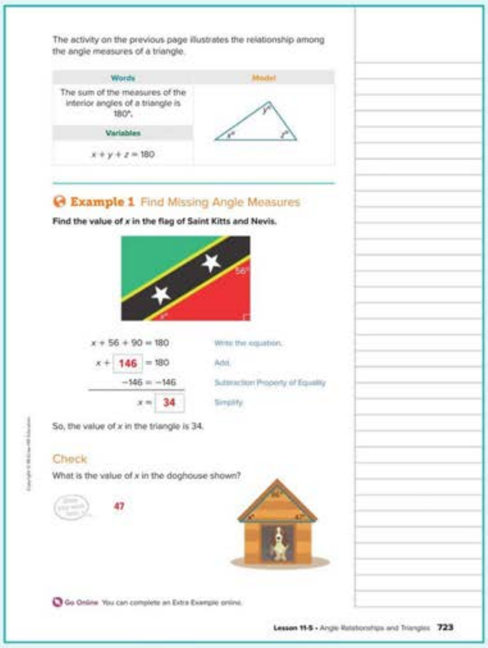
OL Explain why $x + 56 + 90 = 180$ models the situation. Sample answer: The sum of the three angle measures must be 180°. One angle is given as 56°. The other angle is a right angle, and so has a measure of 90°.

OL How can you check your answer? Sample answer: Find the sum of the three angle measures. Since $34° + 56° + 90° = 180°$, my answer is correct.

BL Suppose the flag of Saint Kitts and Nevis was scaled to be twice the size of the flag you see here. How will the angle measures compare? Explain. Sample answer: The side lengths of the triangle, and flag, will be greater, but the angle measures will still be the same.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



The activity on the previous page illustrates the relationship among the angle measures of a triangle.

Words	Model
The sum of the measures of the interior angles of a triangle is 180°.	
Variables	
$x + y + z = 180$	

Example 1 Find Missing Angle Measures

Find the value of *x* in the flag of Saint Kitts and Nevis.

$x + 56 + 90 = 180$ Write the equation.

$x + 146 = 180$ Add.

$-146 = -146$ Subtraction Property of Equality

$x = 34$ Simplify.

So, the value of *x* in the triangle is 34.

Check

What is the value of *x* in the doghouse shown?

47

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 1, Find Missing Angle Measures, Slide 1 of 2

TYPE

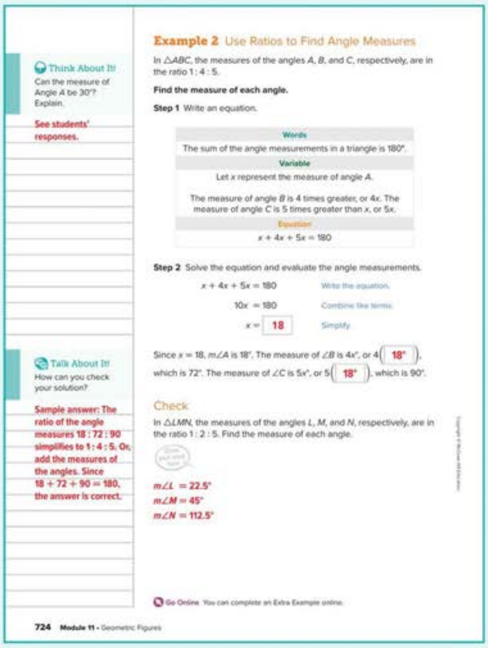
On Slide 1, students determine the value of *x*.

CLICK

On Slide 1, students move through the steps to find the measure of the missing angle in the triangle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 2 Use Ratios to Find Angle Measures

In $\triangle ABC$, the measures of the angles A, B, and C, respectively, are in the ratio 1 : 4 : 5.

Find the measure of each angle.

Step 1 Write an equation.

Words
The sum of the angle measurements in a triangle is 180°.
Variable
Let x represent the measure of angle A. The measure of angle B is 4 times greater, or $4x$. The measure of angle C is 5 times greater than x, or $5x$.
Equation
$x + 4x + 5x = 180$

Step 2 Solve the equation and evaluate the angle measurements.

$x + 4x + 5x = 180$ Write the equation.

$10x = 180$ Combine like terms.

$x = 18$ Simplify.

Since $x = 18$, $m\angle A$ is 18°. The measure of $\angle B$ is $4x°$, or 4(18°), which is 72°. The measure of $\angle C$ is $5x°$, or 5(18°), which is 90°.

Think About It! Can the measure of Angle A be 30°? Explain.

See students' responses.

Talk About It! How can you check your solution?

Sample answer: The ratio of the angle measures 18 : 72 : 90 simplifies to 1 : 4 : 5. Or, add the measures of the angles. Since 18 + 72 + 90 = 180, the answer is correct.

Check

In $\triangle LMN$, the measures of the angles L, M, and N, respectively, are in the ratio 1 : 2 : 5. Find the measure of each angle.

$m\angle L = 22.5°$
$m\angle M = 45°$
$m\angle N = 112.5°$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example, Use Ratios to Find Angle Measures, Slide 2 of 5

FLASHCARDS

On Slide 2, students use Flashcards to write the equation that models the ratio.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Use Ratios to Find Angle Measures

Objective

Students will find the angle measures in a triangle given the ratio between each of the angles.

Questions for Mathematical Discourse

SLIDE 2

AL What does it mean for two quantities to be in a 1:4 ratio? Sample answer: It means that the second quantity is four times as great as the first quantity.

AL What does it mean for three quantities to be in a 1:4:5 ratio? Sample answer: It means that the second quantity is four times as great as the first quantity, and the third quantity is five times as great as the first quantity.

OL Why might it be more helpful to represent the measure of angle A with x rather than the other two angles? Sample answer: Since $\angle A$ is the angle with the least measure, it makes sense to represent its measure by x rather than the other two angles. If I represent either $m\angle B$ or $m\angle C$ by x, then $\angle A$'s measure will be represented as a fraction of x.

OL Explain why $4x$ represents the measure of angle B. Sample answer: Since $m\angle A$ is represented by x, and $m\angle B$ is four times as great as $m\angle A$, $m\angle B$ is represented by $4x$.

BL If the ratio was 3:4:5, how can you set up an equation? Sample answer: Label $\angle A$ with the quantity $3x$, so the equation would be $3x + 4x + 5x = 180$.

SLIDE 3

AL What is the first step in solving this equation? combine like terms

OL Explain how the Distributive Property is used to combine like terms. Sample answer: $x + 4x + 5x = x(1 + 4 + 5)$ by the Distributive Property. Therefore, the expression equals $x(10)$, or $10x$.

BL What would be the measure of angle A if the ratio were 3:4:5? 45°

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Exterior Angles of Triangles

Objective

Students will use Web Sketchpad to explore the relationship between an exterior angle and two remote interior angles of a triangle.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore the relationship between an exterior angle of a triangle and its two remote interior angles. After making a conjecture, students will use the sketch to confirm the conjecture.

Inquiry Question

How is the measure of a triangle's exterior angle related to the measures of its remote interior angles? Sample answer: The sum of the measures of the two remote interior angles equals the measure of the exterior angle.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Which angle is the exterior angle of $\triangle ABC$? Which angles are its remote interior angles? *$\angle BCD$; $\angle CAB$* and *$\angle ABC$*

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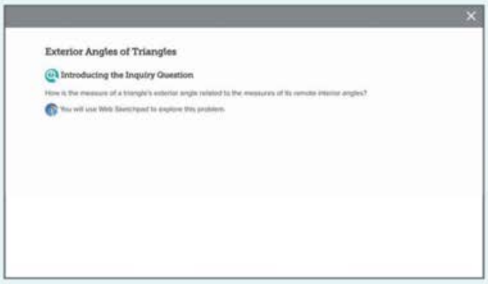
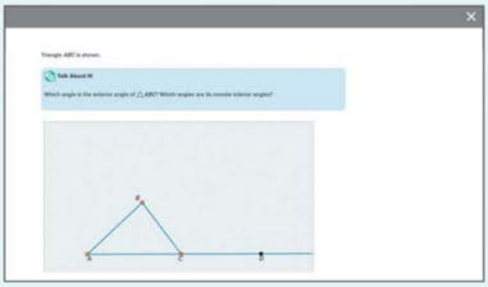
Interactive Presentation



Explore, Slide 1 of 5

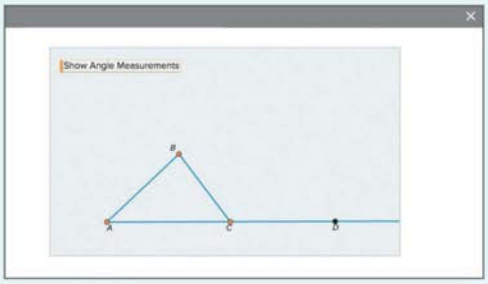


Explore, Slide 2 of 5

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the relationship between an exterior angle and two remote interior angles of a triangle.

Interactive Presentation



Explore, Slide 3 of 5

TYPE

On Slide 4, students type to make a conjecture about how angle measures are related.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.

Explore Exterior Angles of T riangles *(continued)*

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others Encourage students to use their observations to write a conjecture about the relationship between an exterior angle and its two remote interior angles.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore the relationship between an exterior angle and two remote interior angles of a triangle.

Go Online to find additional teaching notes.

Learn Exterior Angles of T riangles

Objective

Students will understand the relationship between an exterior angle and its two remote interior angles of a triangle.

Teaching Notes

SLIDE 1

Have students study the diagram. Point out that angles 4, 5, and 6 are exterior angles. Ask students to explain the relationship between each exterior angle and its adjacent interior angle. Students should note that since the pair of angles forms a straight line, the angles are supplementary and the sum of their measures is 180 degrees. You may wish to ask students the following question.

If $m\angle 4 + m\angle 1 = 180°$, and $m\angle 1 + m\angle 2 + m\angle 3 = 180°$, make a conjecture about the relationship between $m\angle 4$, $m\angle 2$, and $m\angle 3$. Justify your reasoning. Sample answer: Since both equations are equal to 180 degrees, and the left side of each equation includes the measure of angle 1, then I can conclude that the remaining angle measures on the left side ($m\angle 4$ and $m\angle 2 + m\angle 3$) must be equal to each other. So, $m\angle 4$ must be equal to the sum of $m\angle 2$ and $m\angle 3$.

SLIDE 2

Have students use the interactive tool to further their understanding about how each exterior angle of a triangle has two remote interior angles. You may wish to ask students why angle 1 is not a remote interior angle for angle 4. Students should note that angle 1 is adjacent to angle4, which by definition, means it is not a remote interior angle. You may also wish to ask students to make a conjecture as to how many exterior angles a triangle has. They should be able to reason that since a triangle has three interior angles, and each interior angle has an adjacent exterior angle, then every triangle has three exterior angles.

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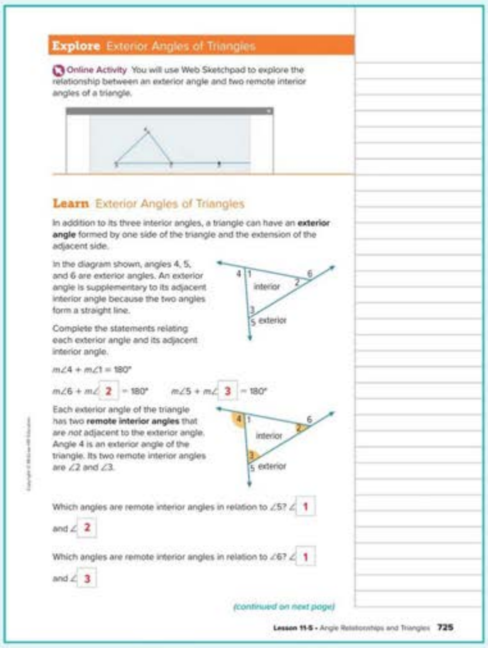

Explore Exterior Angles of Triangles

Online Activity You will use Web Sketchpad to explore the relationship between an exterior angle and two remote interior angles of a triangle.

Learn Exterior Angles of Triangles

In addition to its three interior angles, a triangle can have an **exterior angle** formed by one side of the triangle and the extension of the adjacent side.

In the diagram shown, angles 4, 5, and 6 are exterior angles. An exterior angle is supplementary to its adjacent interior angle because the two angles form a straight line.

Complete the statements relating each exterior angle and its adjacent interior angle.

$m\angle 4 + m\angle 1 = 180°$

$m\angle 6 + m\angle$ 2 $= 180°$ $m\angle 5 + m\angle$ 3 $= 180°$

Each exterior angle of the triangle has two **remote interior angles** that are *not* adjacent to the exterior angle. Angle 4 is an exterior angle of the triangle. Its two remote interior angles are $\angle 2$ and $\angle 3$.

Which angles are remote interior angles in relation to $\angle 5$? $\angle$ 1 and $\angle$ 2

Which angles are remote interior angles in relation to $\angle 6$? $\angle$ 1 and $\angle$ 3

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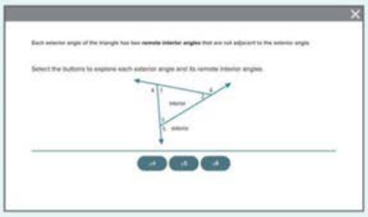
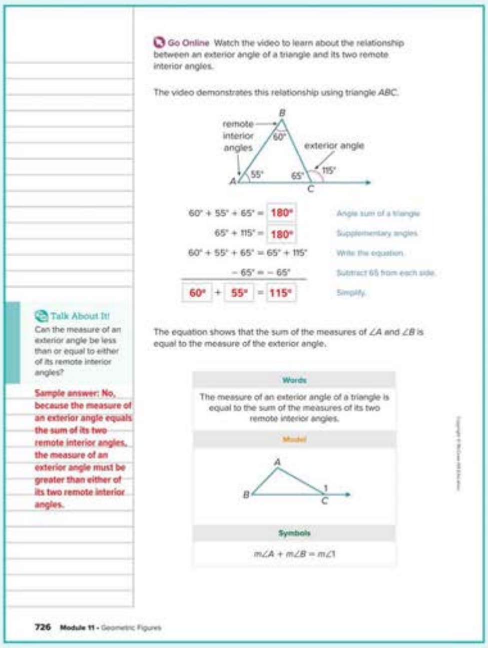
Interactive Presentation



Learn, Exterior Angles of Triangles, Slide 2 of 5

CLICK

On Slide 2, students select the buttons to view an exterior angle and its remote interior angles.


Go Online Watch the video to learn about the relationship between an exterior angle of a triangle and its two remote interior angles.

The video demonstrates this relationship using triangle ABC.

$60° + 55° + 65° = 180°$	Angle sum of a triangle
$65° + 115° = 180°$	Supplementary angles
$60° + 55° + 65° = 65° + 115°$	Write the equation.
$-65° = -65°$	Subtract 65 from each side.
$60° + 55° = 115°$	Simplify.

Talk About It! Can the measure of an exterior angle be less than or equal to either of its remote interior angles?

Sample answer: No, because the measure of an exterior angle equals the sum of its two remote interior angles, the measure of an exterior angle must be greater than either of its two remote interior angles.

The equation shows that the sum of the measures of $\angle A$ and $\angle B$ is equal to the measure of the exterior angle.

Words
The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Model

Symbols
$m\angle A + m\angle B = m\angle 1$

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Interactive Presentation

Learn, Exterior Angles of Triangles, Slide 3 of 5

WATCH

On Slide 3, students watch the video that illustrates the relationship between an exterior angle and its remote interior angles.

FLASHCARDS

On Slide 4, students use Flashcards to learn about the measure of exterior angles in a triangle.

Learn Exterior Angles of T riangles *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discus sing the *Talk About It!* question on Slide 5, encourage students to use an equation to interpret the situation and decide whether an exterior angle can be less than or equal to either of its remote interior angles.

Go Online to have students watch the video on Slide 3. The video illustrates the relationship between the exterior angle and its two remote interior angles.

Teaching Notes

SLIDE 3

You may wish to have students recreate the activity shown in the video by having them draw a triangle and use a protractor to measure and label the three interior angles and one of the exterior angles. Then have them write two equations, (1) an equation that represents the sum of the three interior angles, and (2) an equation that represents the relationship between the exterior angle and its adjacent interior angles. Have them study the equations to see if they can write one equation that represents the relationship between an exterior angle and its two remote interior angles. Have students share their triangles and equations with the class to show that this relationship is true for any triangle (the measure of its exterior angle is equal to the sum of the measures of its remote interior angles).

SLIDE 4

Have students select the *Words, Symbols*, and *Model* flashcards to summarize the relationship between an exterior angle and its remote interior angle using these multiple representations.

Talk About It!

SLIDE 5

Mathematical Discourse

Can the measure of an exterior angle be less than or equal to either of its remote interior angles? Sample answer: No, since the measure of an exterior angle equals the sum of its two remote interior angles, the measure of an exterior angle must be greater than either of its two remote interior angles.

Example 3 Find Exterior Angle Measures

Objective

Students will find the missing angle measure of an exterior angle using the relationship between an exterior angle and two remote interior angles of a triangle.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to use reasoning about the relationship between angles 1, 2, and 3 in order to determine how to represent that relationship algebraically.

3 Construct Viable Arguments and Critique the Reasoning of Others While discussing the *Talk About It!* question on Slide 3, encourage students to come up with an alternative way to find the measure of angle 1 and be able to justify their method mathematically.

6 Attend to Precision Students should be able to describe the relationship between angles 1, 2, and 3 using correct mathematical vocabulary, such as *the measure of an exterior angle is equal to the sum of its remote interior angle measures.*

Questions for Mathematical Discourse

SLIDE 2

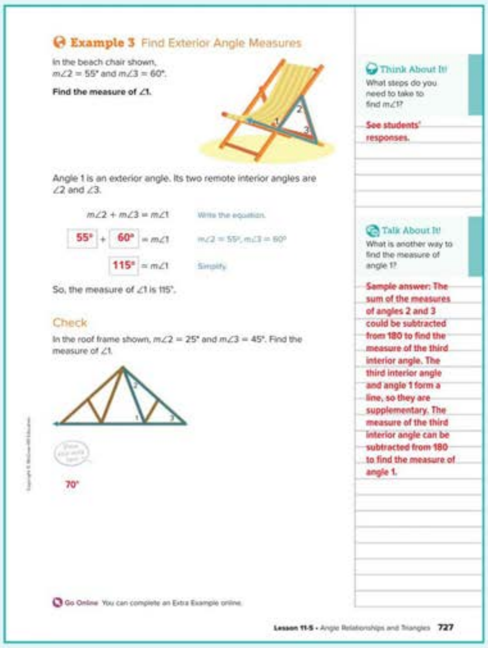
AL What is the relationship between an exterior angle and its two remote interior angles? Sample answer: The measure of the exterior angle is equal to the sum of the measures of its two remote interior angles.

OL Why are ∠2 and ∠3 remote interior angles to ∠1? Sample answer: ∠2 and ∠3 are both interior angles because they are inside the triangle. They are remote to ∠1 because they are not adjacent to ∠1.

BL What is the measure of the third angle of the triangle? 65°

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Find Exterior Angle Measures

In the beach chair shown, $m\angle 2 = 55°$ and $m\angle 3 = 60°$.

Find the measure of ∠1.

Angle 1 is an exterior angle. Its two remote interior angles are ∠2 and ∠3.

$m\angle 2 + m\angle 3 = m\angle 1$ — Write the equation.

$55° + 60° = m\angle 1$ — $m\angle 2 = 55°$, $m\angle 3 = 60°$

$115° = m\angle 1$ — Simplify.

So, the measure of ∠1 is 115°.

Check

In the roof frame shown, $m\angle 2 = 25°$ and $m\angle 3 = 45°$. Find the measure of ∠1.

70°

Go Online You can complete an Extra Example online.

Think About It! What steps do you need to take to find $m\angle 1$?

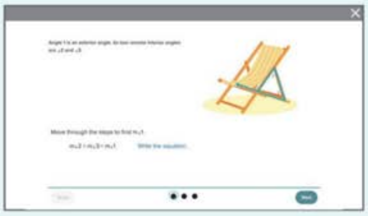
See students' responses.

Talk About It! What is another way to find the measure of angle 1?

Sample answer: The sum of the measures of angles 2 and 3 could be subtracted from 180 to find the measure of the third interior angle. The third interior angle and angle 1 form a line, so they are supplementary. The measure of the third interior angle can be subtracted from 180 to find the measure of angle 1.

Lesson 11-5 • Angle Relationships and Triangles 727

Interactive Presentation

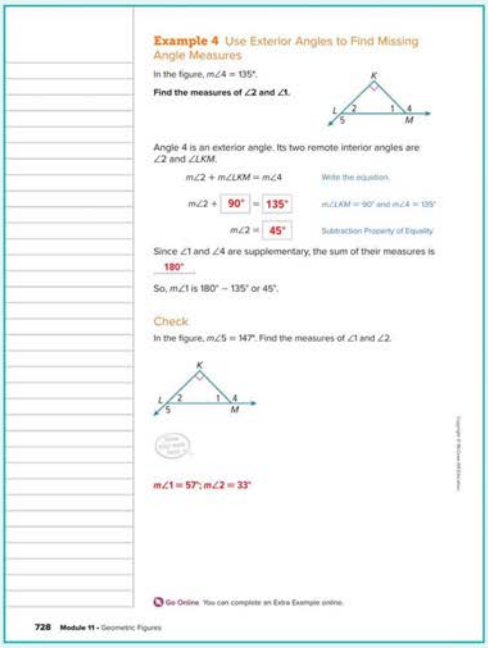


Example 3, Find Exterior Angle Measures, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the measure of angle 1.

TYPE On Slide 2, students determine the measure of angle 1.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Example 4 Use Exterior Angles to Find Missing Angle Measures

In the figure, $m\angle 4 = 135°$.

Find the measures of $\angle 2$ and $\angle 1$.

Angle 4 is an exterior angle. Its two remote interior angles are $\angle 2$ and $\angle LKM$.

$m\angle 2 + m\angle LKM = m\angle 4$ — Write the equation.

$m\angle 2 + 90° = 135°$ — $m\angle LKM = 90°$ and $m\angle 4 = 135°$

$m\angle 2 = 45°$ — Subtraction Property of Equality

Since $\angle 1$ and $\angle 4$ are supplementary, the sum of their measures is 180°.

So, $m\angle 1$ is $180° - 135°$ or 45°.

Check

In the figure, $m\angle 5 = 147°$. Find the measures of $\angle 1$ and $\angle 2$.

$m\angle 1 = 57°$; $m\angle 2 = 33°$

Go Online You can complete an Extra Example online.

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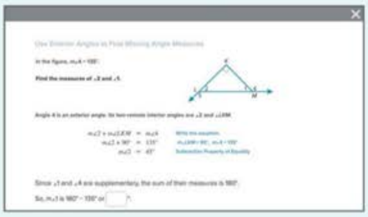
Interactive Presentation



Example 4, Use Exterior Angles to Find Missing Angle Measures, Slide 1 of 2

TYPE

On Slide 1, students determine the measure of angle 1.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Use Exterior Angles to Find Missing Angle Measures

Objective

Students will find the missing angle measures of two interior angles using the relationship between an exterior angle and two remote interior angles of a triangle.

Teaching the Mathematical Practices

6 Attend to Precision Students should be able to describe the relationship between angles 1, 2, and 4 using correct mathematical vocabulary, such as *the measure of an exterior angle is equal to the sum of its remote interior angle measures.*

7 Look For and Make Use of Structure Encourage students to study the structure of the diagram in order to plan a solution pathway to find the missing angle measures.

Questions for Mathematical Discourse

SLIDE 1

AL Knowing $m\angle 4$, which other angle measure can you immediately calculate? Sample answer: The measure of $\angle 1$, because it is supplementary to $\angle 4$.

OL Explain how to use $\angle 4$ and the right angle to find $m\angle 2$? Sample answer: Since the right angle and $\angle 2$ are remote interior angles to $\angle 4$, they must add to the measure of $\angle 4$. Since I know that a right angle is 90 degrees and I know $m\angle 4$, I can subtract 90 from 135 to get $m\angle 2$.

BL If you found $m\angle 1$ first, how could you use this, together with the right angle, to find $m\angle 2$? Sample answer: Since $\angle 1$ and $\angle 2$ and the right angle are the three angles in a triangle, the sum of the measures is 180°. I can subtract the measure of the right angle and $\angle 1$ from 180 to find $m\angle 2$.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Geometry

Objective

Students will come up with their own strategy to solve an application problem involving geometric figures.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

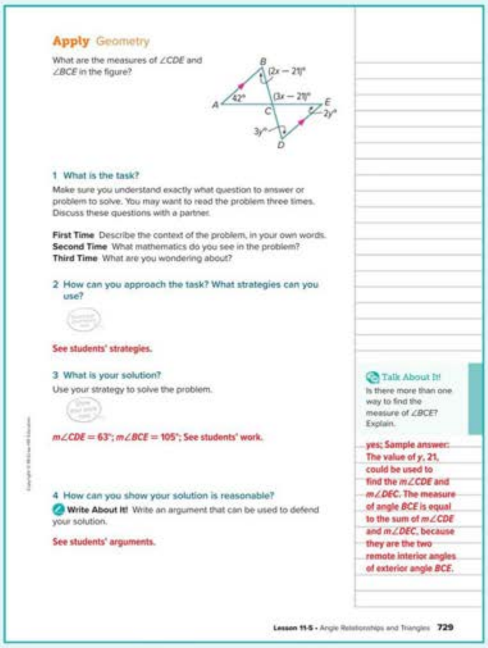
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What line segments are parallel?
- What line segment(s) represent the transversal?
- What is the relationship between $\angle BAC$ and $\angle DEC$?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Geometry

What are the measures of $\angle CDE$ and $\angle BCE$ in the figure?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$m\angle CDE = 63°$; $m\angle BCE = 105°$; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Is there more than one way to find the measure of $\angle BCE$? Explain.

yes; Sample answer: The value of y, 21, could be used to find the $m\angle CDE$ and $m\angle DEC$. The measure of angle BCE is equal to the sum of $m\angle CDE$ and $m\angle DEC$, because they are the two remote interior angles of exterior angle BCE.

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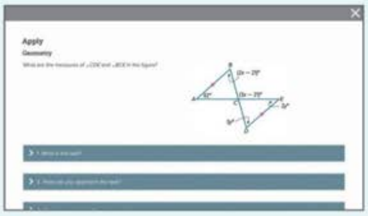
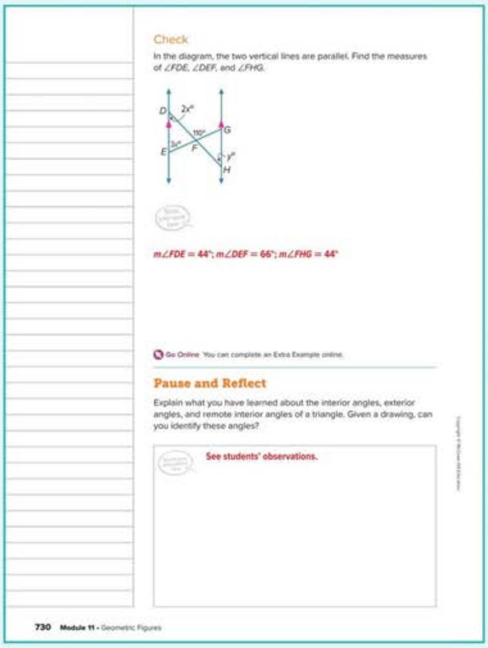
Interactive Presentation



Apply, Geometry

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

In the diagram, the two vertical lines are parallel. Find the measures of $\angle FDE$, $\angle DEF$, and $\angle FHG$.

$m\angle FDE = 44°$; $m\angle DEF = 66°$; $m\angle FHG = 44°$

Go Online You can complete an Extra Example online.

Pause and Reflect

Explain what you have learned about the interior angles, exterior angles, and remote interior angles of a triangle. Given a drawing, can you identify these angles?

See students' observations.

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Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. What is the measure of the angle labeled *x*? Write a mathematical argument that can be used to defend your solution. 135°; Sample answer: The angle labeled *x* is an exterior angle. The angles labeled 115° and 20° form a triangle with the third, unlabeled angle that is adjacent to the angle labeled *x*. This third, unlabeled angle has a measure of 45°, and is supplementary to the angle labeled *x*. This means the angle labeled *x* has a measure of 135°, which is also equal to the sum of the remote interior angles.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 6, 7, 9–12
- Extension: Interior Angle Sum Using Triangles
- ALEKS Classifying Triangles, Angles of Triangles

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 7, 9, 10
- Extension: Interior Angle Sum Using Triangles
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Classifying and Measuring Angles

IF students score less than 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Classifying and Measuring Angles

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

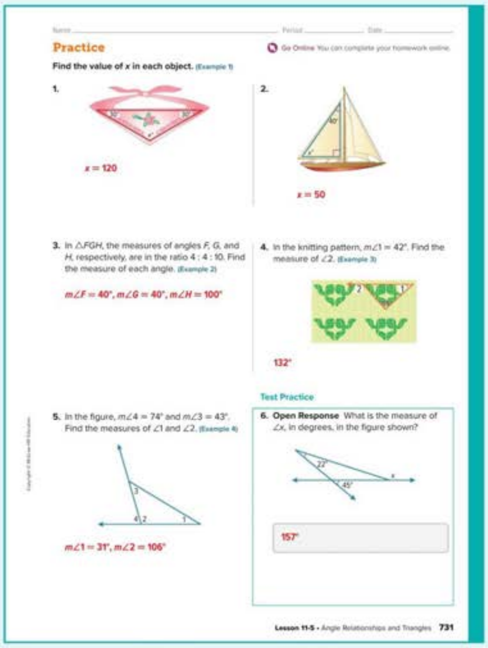
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find missing angle measures in triangles	1, 2
1	find missing angle measures in triangles using the ratio between each of the angles	3
1	find the missing angle measure of an exterior angle using the relationship between an exterior angle and two remote interior angles of a triangle	4
1	find the missing angle measures of two interior angles using the relationship between an exterior angle and two remote interior angles of a triangle	5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems that involve angle relationships and triangles	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

Some students may incorrectly find the value of the missing angle measure. In Exercises 1 and 2, students may not subtract the known angle measures from 180°. Remind students that the sum of the measures of the interior angles of a triangle is equal to 180°.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find the value of x in each object. (Example 1)

1. $x = 120$

2. $x = 50$

3. In $\triangle FGH$, the measures of angles F, G, and H, respectively, are in the ratio 4 : 4 : 10. Find the measure of each angle. (Example 2)

$m\angle F = 40°$, $m\angle G = 40°$, $m\angle H = 100°$

4. In the knitting pattern, $m\angle 1 = 42°$. Find the measure of $\angle 2$. (Example 3)

132°

5. In the figure, $m\angle 4 = 74°$ and $m\angle 3 = 43°$. Find the measures of $\angle 1$ and $\angle 2$. (Example 4)

$m\angle 1 = 31°$, $m\angle 2 = 106°$

Test Practice

6. **Open Response** What is the measure of $\angle x$, in degrees, in the figure shown?

157°

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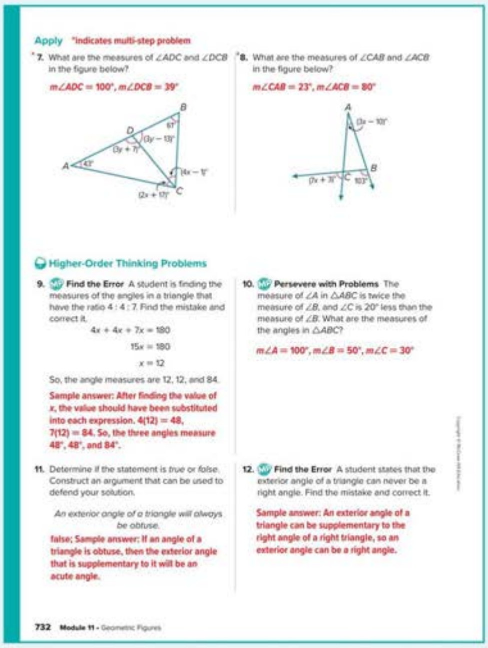

Apply *indicates multi-step problem

*7. What are the measures of ∠ADC and ∠DCB in the figure below?

$m\angle ADC = 100°$, $m\angle DCB = 39°$

*8. What are the measures of ∠CAB and ∠ACB in the figure below?

$m\angle CAB = 23°$, $m\angle ACB = 80°$

Higher-Order Thinking Problems

9. **Find the Error** A student is finding the measures of the angles in a triangle that have the ratio 4 : 4 : 7. Find the mistake and correct it.

$$4x + 4x + 7x = 180$$
$$15x = 180$$
$$x = 12$$

So, the angle measures are 12, 12, and 84.

Sample answer: After finding the value of x, the value should have been substituted into each expression. 4(12) = 48, 7(12) = 84. So, the three angles measure 48°, 48°, and 84°.

10. **Persevere with Problems** The measure of ∠A in △ABC is twice the measure of ∠B, and ∠C is 20° less than the measure of ∠B. What are the measures of the angles in △ABC?

$m\angle A = 100°$, $m\angle B = 50°$, $m\angle C = 30°$

11. Determine if the statement is *true* or *false*. Construct an argument that can be used to defend your solution.

An exterior angle of a triangle will always be obtuse.

false; Sample answer: If an angle of a triangle is obtuse, then the exterior angle that is supplementary to it will be an acute angle.

12. **Find the Error** A student states that the exterior angle of a triangle can never be a right angle. Find the mistake and correct it.

Sample answer: An exterior angle of a triangle can be supplementary to the right angle of a right triangle, so an exterior angle can be a right angle.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercises 9 and 12, students will find the mistake and correct it. Encourage students to construct a response that details the mistake and how to fix it.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 10, students will determine the measures of the angles in the triangle. Encourage students to identify the important pieces of information and plan a solution pathway that they can implement to solve the problem.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.

Use with Exercises 7–8 Have students work in groups of 3–4. After completing Exercise 7, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method. Repeat this process for Exercise 8.

Make sense of the problem.

Use with Exercise 9 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that the solution of the equation is the measure of two of the angles. Have each pair or group of students present their explanations to the class.

Lesson 11-6
Scale Drawings

7.G.A.1

LESSON GOAL

Students will solve problems involving scale drawings.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Use Scale Drawings to Find Length
Example 1: Use Scale Drawings to Find Length
Learn: Create Scale Drawings
Learn: Use Scale Drawings to Find Area
Example 2: Use Scale Drawings to Find Area
Learn: Reproduce Scale Drawings

Explore: Scale Drawings

Example 3: Reproduce Scale Drawings
Apply: Construction

Learn: Find a Scale Factor

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

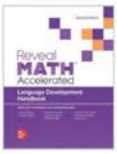
DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL BL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Dilations		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 68 of the *Language Development Handbook* to help your students build mathematical language related to scale drawings.



ELL You can use the tips and suggestions on page T68 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.A** by having students use scale drawings.
Standards for Mathematical Content: **7. G.A.1,** *Also addresses 7.RP.A.2, 7.RP.A.2.B, 7.RP.A.3, 7.NS.A.3, 7.EE.B.3*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students examined relationships among the angles in a triangle.
8.G.A.5

Now
Students solve problems involving scale drawings.
7.G.A.1

Next
Students will describe cross sections of three-dimensional figures.
7.G.A.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

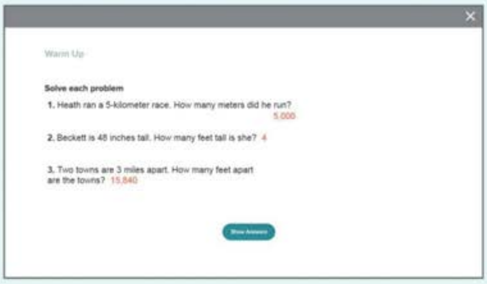
Conceptual Bridge In this lesson, students draw on their knowledge of equivalent ratios to develop an *understanding* of scale drawings. They will use this understanding to gain *fluency* in finding lengths using scale drawings, finding area using scale drawings, and reproducing scale drawings. They *apply* their fluency to solve real-world problems involving scale drawings.

Mathematical Background

In a *scale drawing* or *scale model*, the dimensions of the object being represented are reduced or enlarged. The *scale* is the ratio that compares the measurements of the drawing or model to the measurements of the real object. A scale written as a ratio without units in simplest form is called the *scale factor*. You can use a scale drawing to:

- find actual length of an object or the actual distance between two points.
- find the actual area of a space.
- reproduce a drawing at a different scale.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- converting measures of length (Exercises 1–3)

Answers

1. 5,000
2. 4
3. 15,840

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about blueprints as examples of scale drawings.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate class discussion.

Ask:

- Where have you seen the term *scale* used before, either in mathematics or in everyday life? Sample answer: In mathematics, a scale describes how a model length is related to the length of the actual object.
- The blueprint of a house is an example of a scale drawing. Give another real-world example of a *scale drawing*. Sample answer: a map
- What are *factors?* Sample answer: Factors are numbers that are multiplied together to form a product.
- If a model is a three-dimensional representation of a structure, what do you think a *scale model* might represent? Sample answer: A representation that is based on the actual dimensions of the structure.

Learn Use Scale Drawings to Find Length

Objective

Students will understand how to use scale drawings and the scale to find actual length.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 3, encourage them to be careful about specifying the units of measure, as they may choose a smaller unit of measure for the drawing and a larger unit of measure for the actual Eiffel Tower.

Teaching Notes

SLIDE 1

Have students select the buttons to view three different views of the Eiffel Tower. Ask students to give examples of scale drawings or scale models they have seen or worked with previously. Examples might include maps, blueprints, floor plans, or directions to build a product.

SLIDE 2

Before students select the buttons to change the scale, you may wish to have them estimate the distance between the two roads. Then, as they change the scale, ask them to estimate the distances again. Have students discuss with a partner which view made it easier to estimate the distances and explain why.

Talk About It!

SLIDE 3

Mathematical Discourse

What might be a good scale to use for the scale drawing of the Eiffel Tower, if the actual height of the tower is 324 meters? Sample answers: 1 cm = 5 m; 1 cm = 10 m

DIFFERENTIATE

Reteaching Activity AL

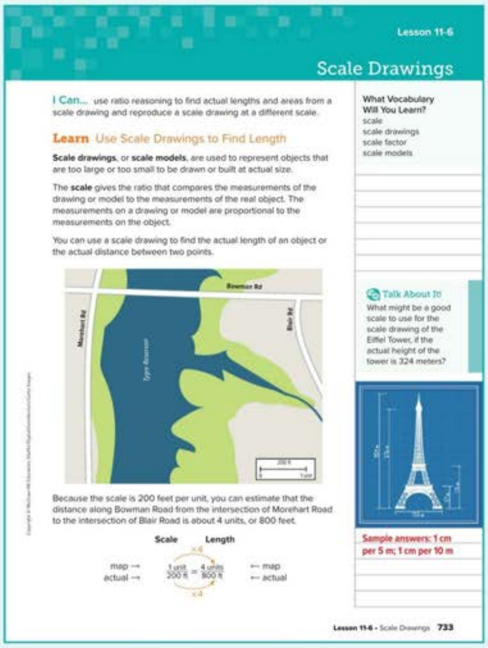
To help students that may be struggling to understand length on scale drawings, have them use the following scales to determine the lengths represented by 2, 3, and 4 units on the scale drawing.

1 inch per 100 miles 200 miles, 300 miles, 400 miles

1 cm per 10 m 20 m, 30 m, 40 m

1 inch per 50 feet 100 feet, 150 feet, 200 feet

1 cm per 75 cm 150 cm, 225 cm, 300 cm


Lesson 11-6

Scale Drawings

I Can... use ratio reasoning to find actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.

Learn Use Scale Drawings to Find Length

Scale drawings, or **scale models**, are used to represent objects that are too large or too small to be drawn or built at actual size.

The **scale** gives the ratio that compares the measurements of the drawing or model to the measurements of the real object. The measurements on a drawing or model are proportional to the measurements on the object.

You can use a scale drawing to find the actual length of an object or the actual distance between two points.

Bowman Rd, Morehart Rd, Blair Rd

Because the scale is 200 feet per unit, you can estimate that the distance along Bowman Road from the intersection of Morehart Road to the intersection of Blair Road is about 4 units, or 800 feet.

Scale Length

map → $\frac{1 \text{ unit}}{200 \text{ ft}} = \frac{4 \text{ units}}{800 \text{ ft}}$ ← map

actual → ← actual

×4

What Vocabulary Will You Learn?

scale

scale drawings

scale factor

scale models

Talk About It! What might be a good scale to use for the scale drawing of the Eiffel Tower, if the actual height of the tower is 324 meters?

Sample answers: 1 cm per 5 m; 1 cm per 10 m

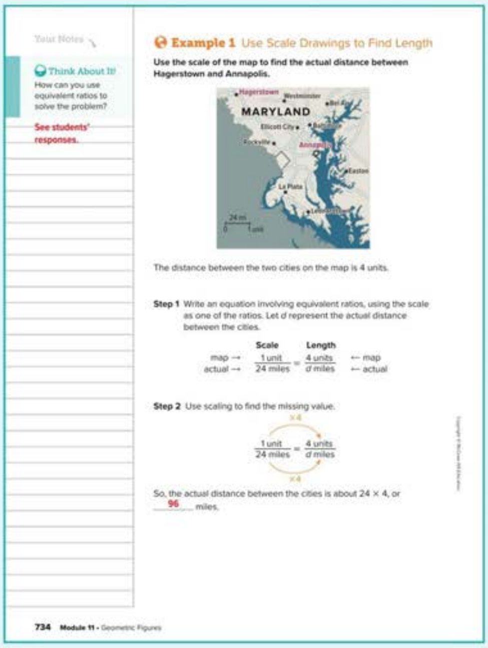
Lesson 11-6 • Scale Drawings 733

Interactive Presentation

Learn, Use Scale Drawings to Find Length, Slide 2 of 3

CLICK On Slide 1, students select buttons to see three views of the Eiffel Tower.

CLICK On Slide 2, students select buttons to zoom out or zoom in on a scale map.


Your Notes

Think About It!
How can you use equivalent ratios to solve the problem?

See students' responses.

Example 1 Use Scale Drawings to Find Length

Use the scale of the map to find the actual distance between Hagerstown and Annapolis.

The distance between the two cities on the map is 4 units.

Step 1 Write an equation involving equivalent ratios, using the scale as one of the ratios. Let d represent the actual distance between the cities.

	Scale		Length	
map → actual →	$\frac{1 \text{ unit}}{24 \text{ miles}}$	=	$\frac{4 \text{ units}}{d \text{ miles}}$	← map ← actual

Step 2 Use scaling to find the missing value.

$$\frac{1 \text{ unit}}{24 \text{ miles}} = \frac{4 \text{ units}}{d \text{ miles}} \quad (\times 4)$$

So, the actual distance between the cities is about 24 × 4, or 96 miles.

734 Module 11 • Geometric Figures

Interactive Presentation

Example 1, Use Scale Drawings to Find Length, Slide 3 of 4

TYPE

On Slide 3, students determine the actual distance between the cities.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.G.A.1

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 1 Use Scale Drawings to Find Length

Objective

Students will use the scale of a scale drawing to find actual length.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the information in the real-world problem by representing it symbolically with a correct equation.

Questions for Mathematical Discourse

SLIDE 2

AL What do we need to find? the actual distance between the cities

AL What is the scale? 1 unit = 24 miles

OL Explain how to set up an equation involving equivalent ratios. Sample answer: Use the scale to set up the first ratio. Then set up the second ratio by using the map distance of 4 units and the actual distance represented by the variable *d*.

BL What is another way you can set up the equation involving equivalent ratios?
$\frac{1 \text{ unit}}{4 \text{ units}} = \frac{24 \text{ miles}}{d \text{ miles}}$

SLIDE 3

AL How can you solve the equation? Sample answer: Use equivalent ratios.

OL Explain how to solve the equation mentally. Sample answer: Because 1 • 4 = 4, multiply 24 by 4 to obtain 96.

BL What is the approximate total distance traveled if you travel from Leonardtown to Annapolis, and then to Baltimore? about 84 miles

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Create Scale Drawings

Objective

Students will understand how to make a scale drawing.

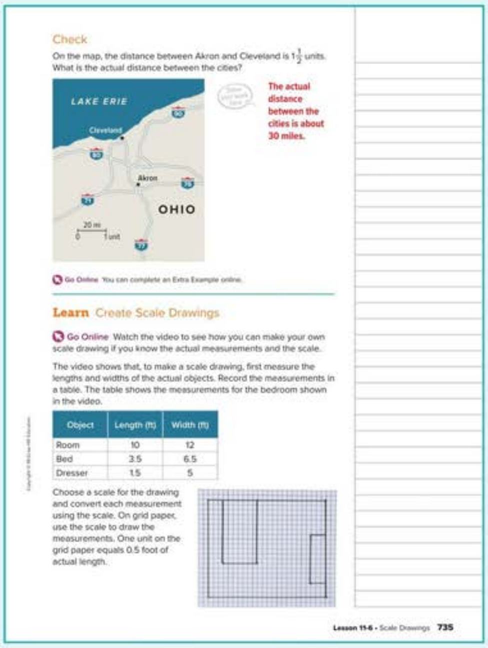
Teaching Notes

SLIDE 1

Before playing the video, you may wish to ask students if they have ever created their own scale drawing. Some students may have created a scale drawing of their bedroom. Ask them what information they would need to know before creating a scale drawing. Some students may say the shape of the actual figure, actual measurements, and the scale they will use in the drawing.

Go Online

Have students watch the video on Slide 1. The video illustrates how to make a scale drawing.



Check

On the map, the distance between Akron and Cleveland is $1\frac{1}{2}$ units. What is the actual distance between the cities?

The actual distance between the cities is about 30 miles.

Go Online You can complete an Extra Example online.

Learn Create Scale Drawings

Go Online Watch the video to see how you can make your own scale drawing if you know the actual measurements and the scale.

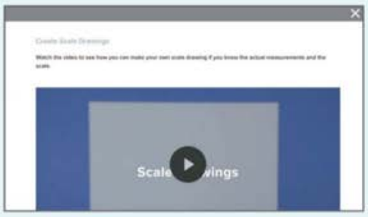
The video shows that, to make a scale drawing, first measure the lengths and widths of the actual objects. Record the measurements in a table. The table shows the measurements for the bedroom shown in the video.

Object	Length (ft)	Width (ft)
Room	10	12
Bed	3.5	6.5
Dresser	1.5	5

Choose a scale for the drawing and convert each measurement using the scale. On grid paper, use the scale to draw the measurements. One unit on the grid paper equals 0.5 foot of actual length.

Lesson 11-6 • Scale Drawings 735

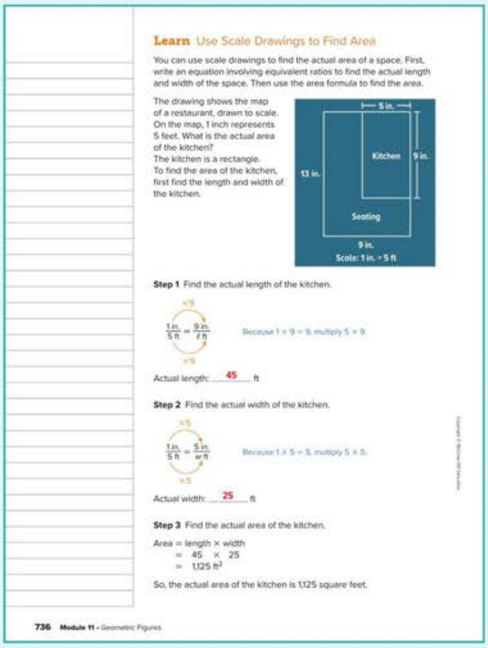
Interactive Presentation



Learn, Create Scale Drawings

WATCH

Students watch a video that demonstrates how to make a scale drawing given the actual measurements and the scale.



Learn Use Scale Drawings to Find Area

You can use scale drawings to find the actual area of a space. First, write an equation involving equivalent ratios to find the actual length and width of the space. Then use the area formula to find the area.

The drawing shows the map of a restaurant, drawn to scale. On the map, 1 inch represents 5 feet. What is the actual area of the kitchen?

The kitchen is a rectangle. To find the area of the kitchen, first find the length and width of the kitchen.

Step 1 Find the actual length of the kitchen.

$\frac{1 \text{ in.}}{5 \text{ ft}} = \frac{9 \text{ in.}}{\ell \text{ ft}}$ Because $1 \times 9 = 9$, multiply 5×9.

Actual length: 45 ft

Step 2 Find the actual width of the kitchen.

$\frac{1 \text{ in.}}{5 \text{ ft}} = \frac{5 \text{ in.}}{w \text{ ft}}$ Because $1 \times 5 = 5$, multiply 5×5.

Actual width: 25 ft

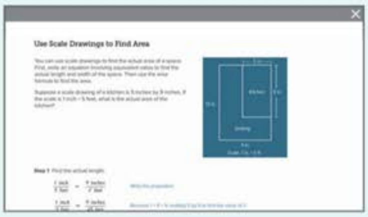
Step 3 Find the actual area of the kitchen.

Area = length × width
= 45 × 25
= 1,125 ft^2

So, the actual area of the kitchen is 1,125 square feet.

736 Module 11 • Geometric Figures

Interactive Presentation



Learn, Use Scale Drawings to Find Area

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Use Scale Drawings to Find Area

Objective

Students will understand how to use scale drawings to find area.

Teaching Notes

SLIDE 1

Ask students to give real-world examples of when they might need to find the area of something from a scale drawing. Sample responses might include the area of a bedroom floor for carpet, the area of a garden for mulch, or the area of a wall for paint.

Ask students what equation they could write to find the actual length of the kitchen. Some students may compare the actual length to the blueprint length. Point out that if both ratios compare the same things, the answers should be the same.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of scale and area, use the following activity.

Ask students to write the ratio of the area of the kitchen on the blueprint to the area of the actual kitchen. Then ask them to use that ratio to make a conjecture about how to find the area of an actual figure without calculating the actual side lengths. Have them test their conjecture by using different scales. Students can share their conjectures with a partner and discuss any differences they find.

Sample answer: You can find the area from a scale drawing by first finding the area on the scale drawing. When you write the ratio for the scale in the equation, use the square of the scale. The equation would be $\frac{1 \text{ in}^2}{25 \text{ ft}^2} = \frac{45 \text{ in}^2}{a \text{ ft}}$.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 2 Use Scale Drawings to Find Area

Objective

Students will use scale drawings to find area.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to decontextualize the information in the real-world problem by representing it symbolically with a correct equation.

6 Attend to Precision Students should use precision in specifying the units for the length, width, and area of Bedroom 3.

Questions for Mathematical Discourse

SLIDE 2

AL What is the scale? 1 inch = 3 feet

OL When finding the length of Bedroom 3, does it matter if you use the decimal or fraction form of the numerical value? Explain. no; Sample answer: They are equivalent.

BL What is the actual length of the Bathroom and Bedroom 3 combined? 21.75 ft

SLIDE 3

AL Will the scale change now that you need to find the actual width of Bedroom 3? Explain. no; Sample answer: The scale is the same for the entire scale drawing.

OL Explain how to solve the equation mentally. Sample answer: Because 1 • 3 = 3, multiply 3 by 3 to obtain 9.

BL What is the actual width of the house? 19.5 ft

SLIDE 4

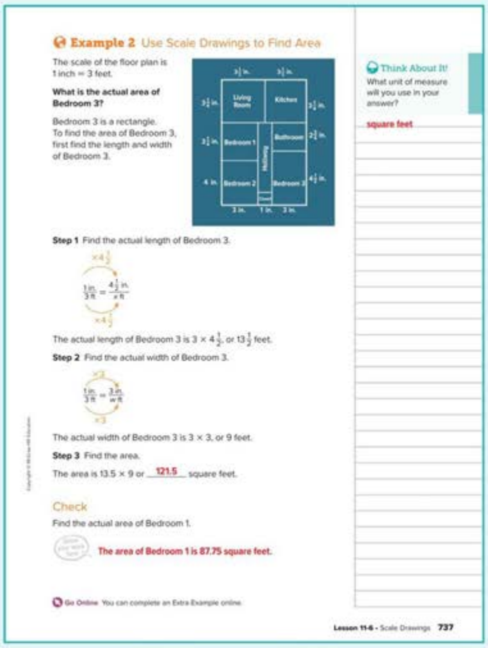
AL What is the formula for area of a rectangle? $A = \ell \cdot w$

OL Is Bedroom 3 the same size as the other bedrooms? Explain. Sample answer: No, it appears to be the largest bedroom.

BL What is the actual perimeter of Bedroom 3? 45 feet

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Use Scale Drawings to Find Area

The scale of the floor plan is 1 inch = 3 feet.

What is the actual area of Bedroom 3?

Bedroom 3 is a rectangle. To find the area of Bedroom 3, first find the length and width of Bedroom 3.

Step 1 Find the actual length of Bedroom 3.

$\frac{1 \text{ in.}}{3 \text{ ft}} = \frac{4\frac{1}{2} \text{ in.}}{x \text{ ft}}$ (×4½)

The actual length of Bedroom 3 is $3 \times 4\frac{1}{2}$, or $13\frac{1}{2}$ feet.

Step 2 Find the actual width of Bedroom 3.

$\frac{1 \text{ in.}}{3 \text{ ft}} = \frac{3 \text{ in.}}{w \text{ ft}}$ (×3)

The actual width of Bedroom 3 is 3 × 3, or 9 feet.

Step 3 Find the area.

The area is 13.5 × 9 or 121.5 square feet.

Check

Find the actual area of Bedroom 1.

The area of Bedroom 1 is 87.75 square feet.

Go Online You can complete an Extra Example online.

Think About It! What unit of measure will you use in your answer? square feet

Lesson 11-6 • Scale Drawings 737

Interactive Presentation

Example 2, Use Scale Drawings to Find Area, Slide 2 of 5

TYPE

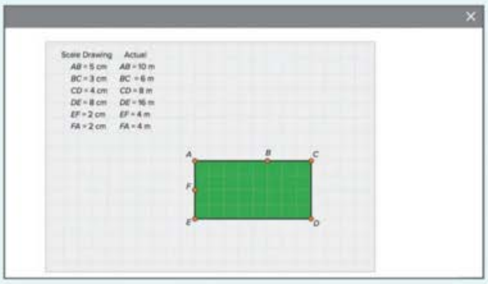
On Slide 4, students determine the area of Bedroom 3.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Interactive Presentation

Explore, Slide 1 of 5



Explore, Slide 2 of 5

WEB SKETCHPAD

Throughout the Explore, students will use Web Sketchpad to explore scale drawings.

TYPE

On Slide 2, students make a conjecture about the scale of a drawing compared to the actual dimensions.

Explore Scale Drawings

Objective

Students will use Web Sketchpad to explore reproducing scale drawings using different scales.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch that contains draggable points. Throughout this activity, students will drag the points to create scale drawings. They will make conjectures about identifying and using different scales.

Inquiry Question

How can I use the scale to create a scale drawing? Sample answer: I can divide the actual dimensions by the scale to find the dimensions of the scale drawing. Then I can use the scale drawing dimensions to draw the figure.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Drag the points to change the dimensions of the scale drawing. Does your conjecture hold true? Sample answer: Yes, every time a side length on the scale drawing changed one centimeter, the actual side length changed two meters.

(continued on next page)

Explore Scale Drawings *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Enc ourage students to use Web Sketchpad to create a scale drawing to model the new hole on the golf course.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

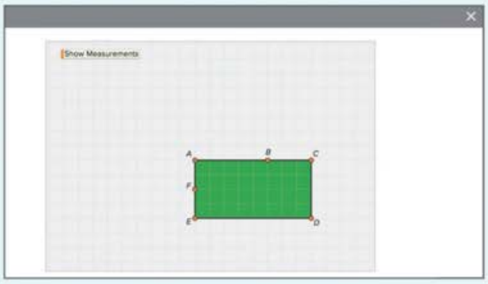
Talk About It!

SLIDE 3

Mathematical Discourse

How do you know the scale drawing is drawn at a scale of 1 centimeter = 3 meters? Sample answer: Every time a side length on the scale drawing changes one centimeter, the actual side length changes three meters.

Interactive Presentation



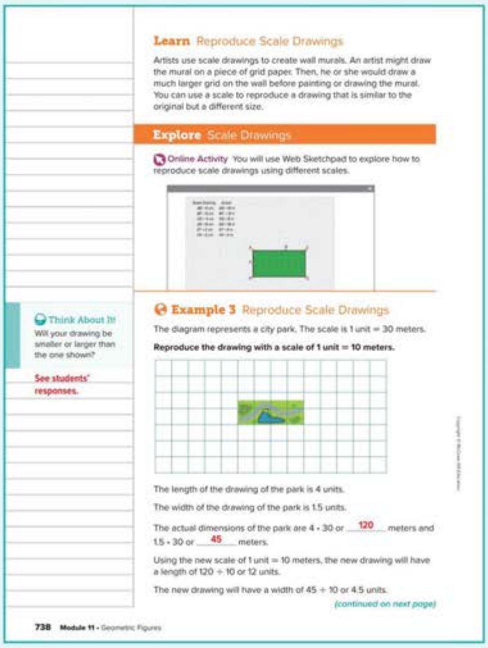
Explore, Slide 3 of 5

TYPE

On Slide 4, students explain how they could determine the dimensions of the scale drawing with a specific scale without using the sketch.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.



Learn Reproduce Scale Drawings

Artists use scale drawings to create wall murals. An artist might draw the mural on a piece of grid paper. Then, he or she would draw a much larger grid on the wall before painting or drawing the mural. You can use a scale to reproduce a drawing that is similar to the original but a different size.

Explore Scale Drawings

Online Activity You will use Web Sketchpad to explore how to reproduce scale drawings using different scales.

Example 3 Reproduce Scale Drawings

The diagram represents a city park. The scale is 1 unit = 30 meters.

Reproduce the drawing with a scale of 1 unit = 10 meters.

Think About It! Will your drawing be smaller or larger than the one shown? See students' responses.

The length of the drawing of the park is 4 units.

The width of the drawing of the park is 1.5 units.

The actual dimensions of the park are 4 · 30 or 120 meters and 1.5 · 30 or 45 meters.

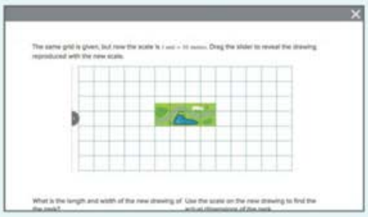
Using the new scale of 1 unit = 10 meters, the new drawing will have a length of 120 ÷ 10 or 12 units.

The new drawing will have a width of 45 ÷ 10 or 4.5 units.

(continued on next page)

738 Module 11 • Geometric Figures

Interactive Presentation



Example 3, Reproduce Scale Drawings, Slide 3 of 5

TYPE

On Slide 2, students determine the dimensions of the park in the drawing and the actual dimensions.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Reproduce Scale Drawings

Objective

Students will understand how to reproduce a scale drawing at a different scale.

Teaching Notes

SLIDE 1

Ask students to give examples of how other professions might use scale drawings in their work. Sample responses might include architects use drawings for buildings, interior designers use drawings of rooms, landscapers use drawings of lawns, and engineers use drawings of roller coasters.

Example 3 Reproduce Scale Drawings

Objective

Students will reproduce a scale drawing at a different scale.

Teaching the Mathematical Practices

6 Attend to Precision Enco urage students to use precision in reproducing the scale drawing with a different scale.

As students discuss the *Talk About It!* question on Slide 4, encourage them to use clear and precise mathematical vocabulary, such as *scale* and *scale drawing*, in their explanations.

Questions for Mathematical Discourse

SLIDE 2

AL What is the scale used in the diagram? 1 unit = 30 m

AL What is the new scale? 1 unit = 10 m

OL Describe how to use the current scale to find the actual dimensions of the park. Sample answer: The length of the drawing is 4 units and the width is 1.5 units. Since the scale is 1 unit = 30 m, multiply the length and width each by 30 to find the number of meters for each dimension.

BL Why might it be important or helpful to change scales? Sample answer: We might need a more detailed view of the park.

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 3 Reproduce Scale Drawings *(continued)*

Questions for Mathematical Discourse

SLIDE 3

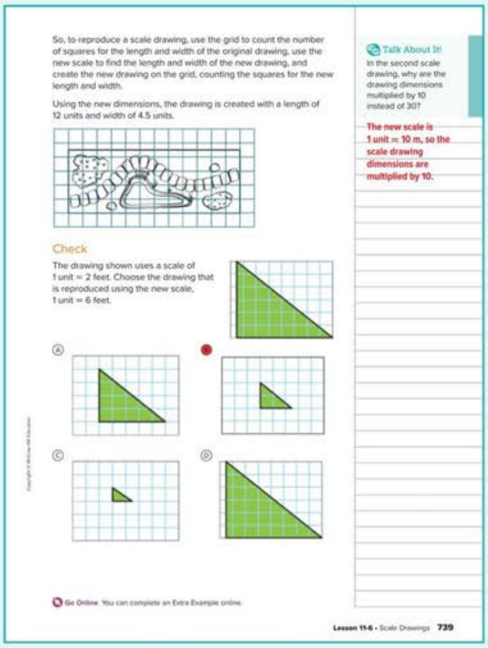
AL Do the park's actual dimensions change when the scale changes? Explain. no; Sample answer: Only the scale changes. The park's actual dimensions remain the same.

OL How can you check that the new diagram's length and width are reasonable? Sample answer: They should be greater than the original drawing's length and width.

BL What is the actual perimeter and area of the park? 330 m; 5,400 m^2

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



So, to reproduce a scale drawing, use the grid to count the number of squares for the length and width of the original drawing, use the new scale to find the length and width of the new drawing, and create the new drawing on the grid, counting the squares for the new length and width.

Using the new dimensions, the drawing is created with a length of 12 units and width of 4.5 units.

Check

The drawing shown uses a scale of 1 unit = 2 feet. Choose the drawing that is reproduced using the new scale, 1 unit = 6 feet.

(A) (B) (C) (D)

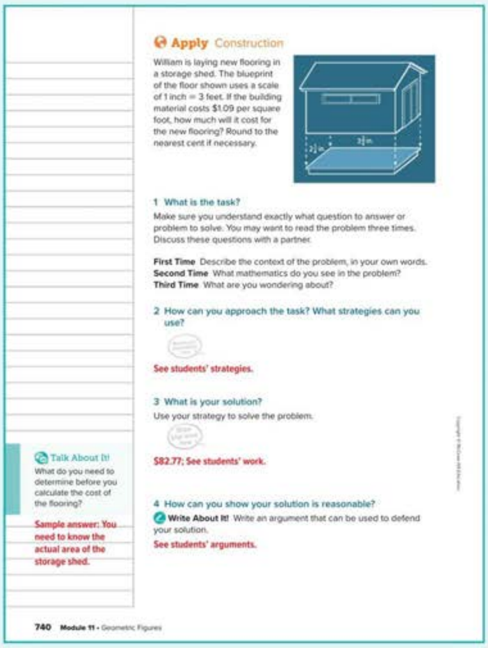
Go Online You can complete an Extra Example online.

Talk About It!

In the second scale drawing, why are the drawing dimensions multiplied by 10 instead of 30?

The new scale is 1 unit = 10 m, so the scale drawing dimensions are multiplied by 10.

Lesson 11-6 • Scale Drawings 739



Apply Construction

William is laying new flooring in a storage shed. The blueprint of the floor shown uses a scale of 1 inch = 3 feet. If the building material costs $1.09 per square foot, how much will it cost for the new flooring? Round to the nearest cent if necessary.

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

$82.77; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

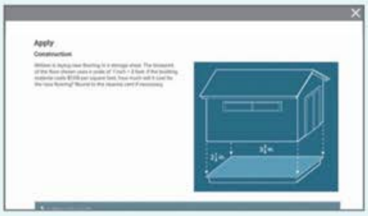
See students' arguments.

Talk About It!
What do you need to determine before you calculate the cost of the flooring?

Sample answer: You need to know the actual area of the storage shed.

740 Module 11 • Geometric Figures

Interactive Presentation



Apply, Construction

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | **3 APPLICATION**

Apply Construction

Objective

Students will come up with their own strategy to solve an application problem involving the cost to install flooring.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

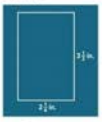
- What does the blueprint of the shed show?
- How do you find the actual dimensions of the shed floor?
- What formula do you need to use to find square footage of the shed floor?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Check

Chantele is buying wallpaper for one wall of her living room. The blueprint of the wall uses a scale of 1 inch = 4 feet. If the wallpaper costs $1.79 per square foot, how much will it cost to buy wallpaper for the actual wall of the living room? Round to the nearest cent if necessary.



$225.54

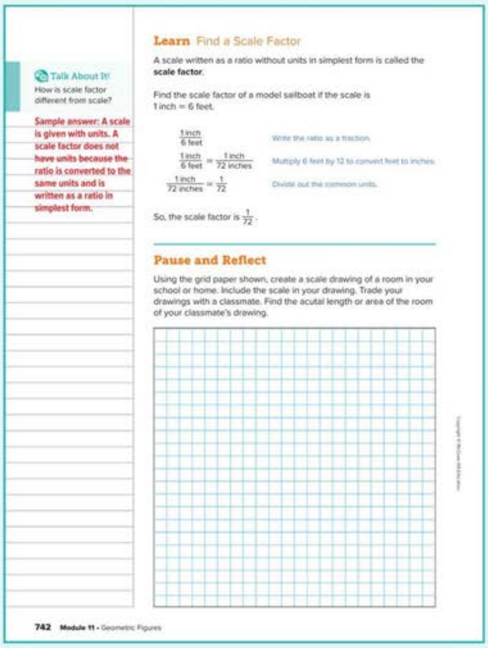
Go Online You can complete an Extra Example online.

Pause and Reflect

Compare what you learned about scales and scale drawings in this lesson to concepts you learned in an earlier module or grade. How did knowing those concepts help you in this lesson?

See students' observations.

Lesson 11-6 • Scale Drawings **741**



Talk About It!
How is scale factor different from scale?

Sample answer: A scale is given with units. A scale factor does not have units because the ratio is converted to the same units and is written as a ratio in simplest form.

Learn Find a Scale Factor

A scale written as a ratio without units in simplest form is called the **scale factor**.

Find the scale factor of a model sailboat if the scale is 1 inch = 6 feet.

$\frac{1 \text{ inch}}{6 \text{ feet}}$ Write the ratio as a fraction.

$\frac{1 \text{ inch}}{6 \text{ feet}} = \frac{1 \text{ inch}}{72 \text{ inches}}$ Multiply 6 feet by 12 to convert feet to inches.

$\frac{1 \text{ inch}}{72 \text{ inches}} = \frac{1}{72}$ Divide out the common units.

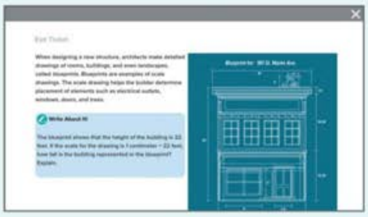
So, the scale factor is $\frac{1}{72}$.

Pause and Reflect

Using the grid paper shown, create a scale drawing of a room in your school or home. Include the scale in your drawing. Trade your drawings with a classmate. Find the acutal length or area of the room of your classmate's drawing.

742 Module 11 • Geometric Figures

Interactive Presentation



Exit Ticket

TYPE

On Slide 1 of the Learn, students determine the scale factor.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Find a Scale Factor

Objective

Students will understand how to find a scale factor.

Go Online to find additional teaching notes, Teaching the Mathematical Practices, and sample answers for the *Talk About It!* question.

Exit Ticket

Refer to the Exit Ticket slide. The blueprint shows that the height of the building is 22 feet. If the scale for the drawing is 1 centimeter = 2 feet, how tall is the building represented in the blueprint? Write a mathematical argument that can be used to defend your solution. 11 cm; Sample answer: Write an equation: $\frac{1 \text{ cm}}{2 \text{ ft}} = \frac{x \text{ cm}}{22 \text{ ft}}$. Solve the equation to find that $x = 11$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7, 9–12
- Extension: Dilations
- ALEKS® Scale Factors and Scale Drawings

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8, 9
- Extension: Dilations
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Proportions

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Proportions

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

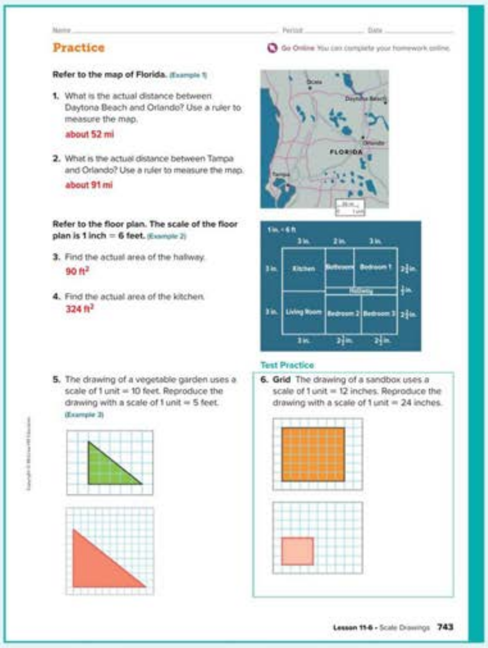
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use the scale of a map to find actual length	1, 2
1	use scale drawings to find area	3, 4
1	reproduce scale drawings at different sizes	5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving scale drawings	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

Some students may try to find the actual area from a scale drawing by finding the area on the scale drawing and then multiplying by the scale. However, this will produce an area that is off by one scale factor. Each scaled distance must be converted to actual distance before calculating actual area. In Exercise 3, students may incorrectly find the area of the hallway to be 18 square feet by finding the scaled area to be 3 square inches and multiplying the result by 6.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Refer to the map of Florida. (Example 1)

1. What is the actual distance between Daytona Beach and Orlando? Use a ruler to measure the map.
about 52 mi

2. What is the actual distance between Tampa and Orlando? Use a ruler to measure the map.
about 91 mi

Refer to the floor plan. The scale of the floor plan is 1 inch = 6 feet. (Example 2)

3. Find the actual area of the hallway.
90 ft^2

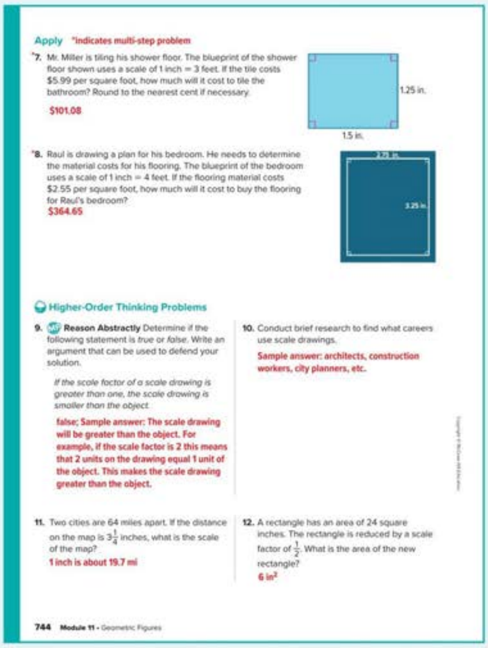
4. Find the actual area of the kitchen.
324 ft^2

5. The drawing of a vegetable garden uses a scale of 1 unit = 10 feet. Reproduce the drawing with a scale of 1 unit = 5 feet. (Example 3)

Test Practice

6. **Grid** The drawing of a sandbox uses a scale of 1 unit = 12 inches. Reproduce the drawing with a scale of 1 unit = 24 inches.

Lesson 11-6 • Scale Drawings 743



Apply *indicates multi-step problem

*7. Mr. Miller is tiling his shower floor. The blueprint of the shower floor shown uses a scale of 1 inch = 3 feet. If the tile costs $5.99 per square foot, how much will it cost to tile the bathroom? Round to the nearest cent if necessary.

$101.08

*8. Raul is drawing a plan for his bedroom. He needs to determine the material costs for his flooring. The blueprint of the bedroom uses a scale of 1 inch = 4 feet. If the flooring material costs $2.55 per square foot, how much will it cost to buy the flooring for Raul's bedroom?

$364.65

Higher-Order Thinking Problems

9. **Reason Abstractly** Determine if the following statement is *true* or *false*. Write an argument that can be used to defend your solution.

If the scale factor of a scale drawing is greater than one, the scale drawing is smaller than the object.

false; Sample answer: The scale drawing will be greater than the object. For example, if the scale factor is 2 this means that 2 units on the drawing equal 1 unit of the object. This makes the scale drawing greater than the object.

10. Conduct brief research to find what careers use scale drawings.

Sample answer: architects, construction workers, city planners, etc.

11. Two cities are 64 miles apart. If the distance on the map is $3\frac{1}{4}$ inches, what is the scale of the map?

1 inch is about 19.7 mi

12. A rectangle has an area of 24 square inches. The rectangle is reduced by a scale factor of $\frac{1}{2}$. What is the area of the new rectangle?

6 in^2

744 Module 11 • Geometric Figures

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 9, students determine if a statement is true or false and explain their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Be sure everyone understands.

Use with Exercises 7–8 Have students work in groups of 3–4 to solve the problem in Exercise 7. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's undersanding. Call on a randomly numbered student from one group to share their group's solution with the class. Repeat the process for Exercise 8.

Explore the truth of statements created by others.

Use with Exercises 9–10 Have students work in pairs. After completing the exercises, have students write two true statements about scale factors or scale drawings and one false statement. An example of a true statement might be "If the scale factor of a scale drawing is less than one, the drawing is smaller than the object." An example of a false statement might be "If the scale factor is 2, then the scale drawing is four times as great as the object." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. For each false statement, have them generate a counterexample. Have them discuss and resolve any differences.

Lesson 11-7

Three-Dimensional Figures

7.G.A.3

LESSON GOAL

Students will analyze three-dimensional figures.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Describe Three-Dimensional Figures
Example 1: Describe Three-Dimensional Figures
Learn: Describe Cross Sections of Three-Dimensional Figures
Example 2: Describe Cross Sections of Three-Dimensional Figures
Example 3: Describe Cross Sections of Three-Dimensional Figures

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
ArriveMATH Take Another Look	●		
Extension: Similar Solids		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 69 of the *Language Development Handbook* to help your students build mathematical language related to three-dimensional figures.

ELL You can use the tips and suggestions on page T69 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.A** by analyzing three-dimensional figures.
Standards for Mathematical Content: **7. G.A.3**
Standards for Mathematical Practice: **MP2, MP3, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students solved problems involving scale drawings.
7.G.A.1

Now
Students describe cross sections of three-dimensional figures.
7.G.A.3

Next
Students will find the circumference and area of circles.
7.G.B.4

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

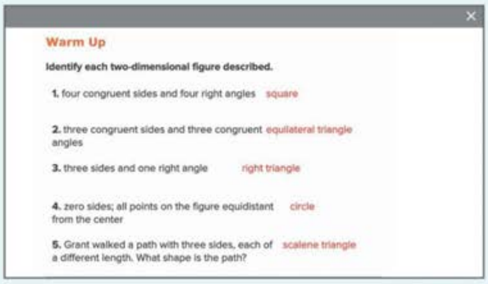
Conceptual Bridge In this lesson, students draw on their knowledge of three-dimensional figures to develop an *understanding* of naming three-dimensional figures and finding the cross sections of three-dimensional figures.

Mathematical Background

The parts of a three-dimensional figure are its *faces, edges,* and *vertices*. The shape and number of bases are used to name the figure.

A *plane* is a flat surface that goes on forever in all directions. The intersection of a solid and a plane is called a *cross* section of the solid.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- identifying two-dimensional figures (Exercises 1–5)

Answers

1. square
2. equilateral triangle
3. right triangle
4. circle
5. scalene triangle

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the pyramid-shaped entrance to the Rock and Roll Hall of Fame in Cleveland, Ohio.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion. Additional questions are available online.

Ask:

- What are some two-dimensional figures that have a *base*? Sample answers: triangle, parallelogram, trapezoid
- Give some real-world examples of *cones*. Sample answers: ice cream cone, traffic cone, funnel
- Make a prediction as to what a *cross-section* might be. Sample answer: To cut through a figure to reveal a section of it.
- Give some real-world examples of *cylinders*. Sample answers: soup can, a battery, candle
- Where might you find an *edge* in everyday objects? Sample answer: the edge of a table or counter
- Describe the two *faces* of a coin. Sample answer: heads and tails
- Where else in mathematics have you seen the term *plane*? Sample answer: the coordinate plane

Learn Describe Three-Dimensional Figures

Objective

Students will understand the attributes of polyhedron and non-polyhedron.

Teaching the Mathematical Practices

6 Attend to Precision A s students discuss the *Talk About It!* question on Slide 5, encourage them to use definitions and accurate mathematical terms when constructing their explanation.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 5

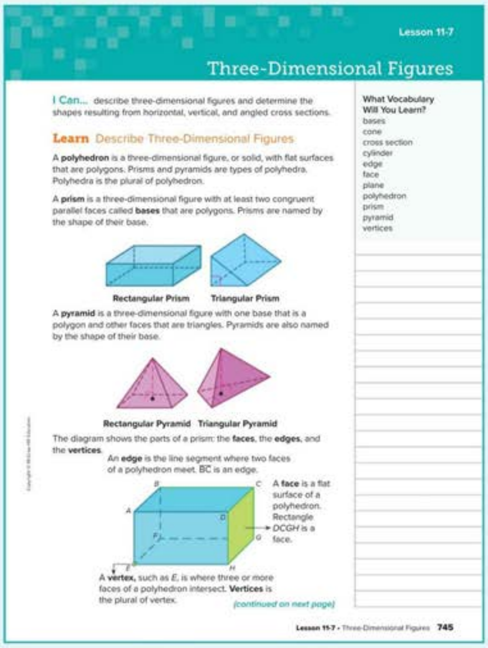
Mathematical Discourse

Why are cylinders and cones not polyhedra? Sample answer: Polyhedra are figures with flat surfaces and faces that are polygons. Both cylinders and cones have curved surfaces and circular faces that are not polygons. Therefore, they are not polyhedra.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling with naming the different parts of three-dimensional figures, you may wish to have them build rectangular prisms, triangular prisms, and various pyramids using nets. Students can cut out the net, tape it together, and label each vertex with the correct letter. This allows students to view each side of the three-dimensional figure to better understand how to count and name the vertices, edges, and faces.



Lesson 11-7

Three-Dimensional Figures

I Can... describe three-dimensional figures and determine the shapes resulting from horizontal, vertical, and angled cross sections.

Learn Describe Three-Dimensional Figures

A **polyhedron** is a three-dimensional figure, or solid, with flat surfaces that are polygons. Prisms and pyramids are types of polyhedra. Polyhedra is the plural of polyhedron.

A **prism** is a three-dimensional figure with at least two congruent parallel faces called **bases** that are polygons. Prisms are named by the shape of their base.

Rectangular Prism | Triangular Prism

A **pyramid** is a three-dimensional figure with one base that is a polygon and other faces that are triangles. Pyramids are also named by the shape of their base.

Rectangular Pyramid | Triangular Pyramid

The diagram shows the parts of a prism: the **faces**, the **edges**, and the **vertices**.

An **edge** is the line segment where two faces of a polyhedron meet. $\overline{BC}$ is an edge.

A **face** is a flat surface of a polyhedron. Rectangle *DCGH* is a face.

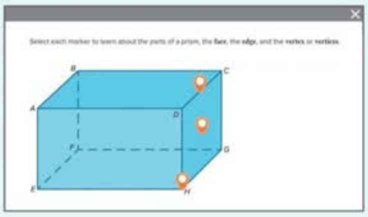
A **vertex,** such as *E*, is where three or more faces of a polyhedron intersect. **Vertices** is the plural of vertex.

(continued on next page)

What Vocabulary Will You Learn?
bases
cone
cross section
cylinder
edge
face
plane
polyhedron
prism
pyramid
vertices

Lesson 11-7 • Three-Dimensional Figures 745

Interactive Presentation



Learn, Describe Three-Dimensional Figures, Slide 2 of 5

CLICK

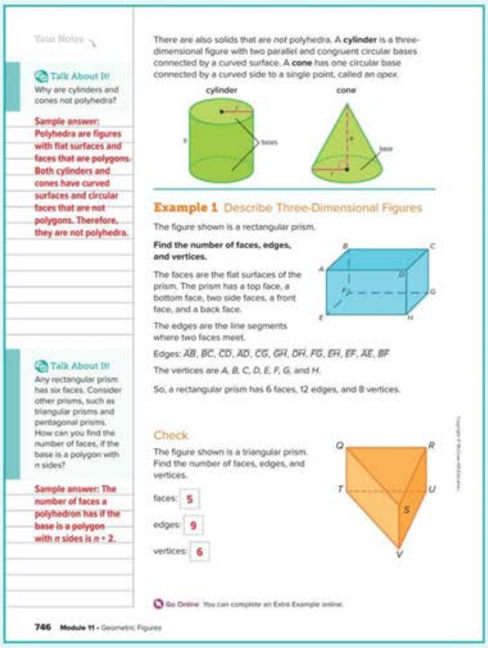
On Slide 2, students select markers to learn about the parts of a prism.

CLICK

On Slide 3, students select markers to learn about the difference between a cylinder and a cone.

CLICK

On Slide 4, students select different three-dimensional figures to compare and contrast their features.



Your Notes

Talk About It!
Why are cylinders and cones not polyhedra?

Sample answer: Polyhedra are figures with flat surfaces and faces that are polygons. Both cylinders and cones have curved surfaces and circular faces that are not polygons. Therefore, they are not polyhedra.

There are also solids that are *not* polyhedra. A **cylinder** is a three-dimensional figure with two parallel and congruent circular bases connected by a curved surface. A **cone** has one circular base connected by a curved side to a single point, called an *apex*.

cylinder — cone

Example 1 Describe Three-Dimensional Figures

The figure shown is a rectangular prism.

Find the number of faces, edges, and vertices.

The faces are the flat surfaces of the prism. The prism has a top face, a bottom face, two side faces, a front face, and a back face.

The edges are the line segments where two faces meet.

Edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{CG}$, $\overline{GH}$, $\overline{DH}$, $\overline{FG}$, $\overline{EH}$, $\overline{EF}$, $\overline{AE}$, $\overline{BF}$

The vertices are *A*, *B*, *C*, *D*, *E*, *F*, *G*, and *H*.

So, a rectangular prism has 6 faces, 12 edges, and 8 vertices.

Talk About It!
Any rectangular prism has six faces. Consider other prisms, such as triangular prisms and pentagonal prisms. How can you find the number of faces, if the base is a polygon with *n* sides?

Sample answer: The number of faces a polyhedron has if the base is a polygon with *n* sides is *n* + 2.

Check

The figure shown is a triangular prism. Find the number of faces, edges, and vertices.

faces: 5

edges: 9

vertices: 6

Go Online You can complete an Extra Example online.

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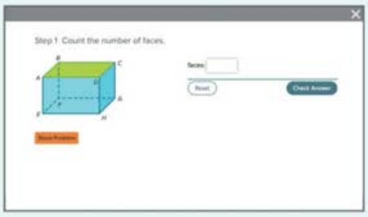
Interactive Presentation



Example 1, Describe Three-Dimensional Figures, Slide 2 of 6

TYPE

On Slides 2–4, students determine the number of sides, edges, and vertices.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Describe Three-Dimensional Figures

Objective

Students will use the number of faces, edges, and vertices to describe three dimensional figures.

Questions for Mathematical Discourse

SLIDE 2

AL How would you describe a face of a prism in your own words? Sample answer: a flat surface

OL How many faces does the figure have? Explain. 6 faces; Sample answer: The figure has a bottom face, a top face, a front face, a back face, a left face, and a right face.

BL Can a rectangular prism have more than 6 faces? less than 6 faces? Explain. Sample answer: No, a rectangular prism will always have 6 faces, because there are two congruent parallel bases that are faces, and four additional faces.

SLIDE 3

AL Name some of the edges you see in the figure. Sample answer: $\overline{AE}$, $\overline{AB}$, $\overline{EH}$, $\overline{CG}$

OL How many edges are there? Explain. 12 edges; Sample answer: Each face has 4 edges, but some of the edges are shared by the faces. Count the number of unique edges.

BL Will a rectangular prism always have 12 edges? Explain. yes; Sample answer: A rectangular prism will always have 12 edges because the number of faces and vertices does not change.

SLIDE 4

AL How many vertices are there? 8 vertices

OL Compare the number of faces, edges, and vertices in a rectangular prism. Sample answer: A rectangular prism has twice as many edges as faces and two more vertices than faces.

BL Determine whether the statement *a prism always has 6 faces* is *always*, *sometimes*, or *never* true. Explain. Sometimes; Sample answer: A rectangular prism always has 6 faces, but a triangular prism will have only 5 faces (two triangular bases plus three additional faces).

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Describe Cross Sections of Three Dimensional Figures

Objective

Students will understand horizontal, vertical, and angled cross sections of three-dimensional figures.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Have students watch the video on Slide 1. The video illustrates cross sections of different three-dimensional figures.

Talk About It!

SLIDE 2

Mathematical Discourse

Cross sections are sometimes used to show the interiors of buildings, cars, airplanes, and even bugs! Research to find examples of cross sections and explain how they might be used. Sample answer: A cross section of a building might be used to show the interiors of the rooms and furniture inside. A CT scan produces cross sectional images of specific areas of a human body to diagnose illnesses.

Example 2 Describe Cross Sections of Three-Dimensional Figures

Objective

Students will describe the shape resulting from vertical, horizontal, and angled cross sections of pyramids and cones.

Questions for Mathematical Discourse

SLIDE 2

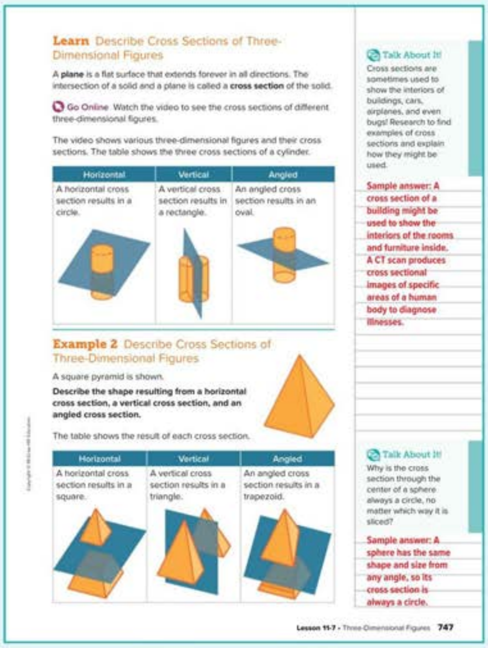
AL What shape results from a vertical cross section? triangle

OL Why is the more specific term for the shape resulting from a horizontal cross section a square, not a rectangle? Sample answer: The shape of the base of the pyramid is square, so the horizontal cross-section will also be square.

BL If the angled cross section were at a different angle (other than strictly horizontal or vertical), what effect would this have on the cross section? Sample answer: It would still be a trapezoid, but it may have different dimensions.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Describe Cross Sections of Three-Dimensional Figures

A **plane** is a flat surface that extends forever in all directions. The intersection of a solid and a plane is called a **cross section** of the solid.

Go Online Watch the video to see the cross sections of different three-dimensional figures.

The video shows various three-dimensional figures and their cross sections. The table shows the three cross sections of a cylinder.

Horizontal	Vertical	Angled
A horizontal cross section results in a circle.	A vertical cross section results in a rectangle.	An angled cross section results in an oval.

Talk About It! Cross sections are sometimes used to show the interiors of buildings, cars, airplanes, and even bugs! Research to find examples of cross sections and explain how they might be used.

Sample answer: A cross section of a building might be used to show the interiors of the rooms and furniture inside. A CT scan produces cross sectional images of specific areas of a human body to diagnose illnesses.

Example 2 Describe Cross Sections of Three-Dimensional Figures

A square pyramid is shown.

Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section.

The table shows the result of each cross section.

Horizontal	Vertical	Angled
A horizontal cross section results in a square.	A vertical cross section results in a triangle.	An angled cross section results in a trapezoid.

Talk About It! Why is the cross section through the center of a sphere always a circle, no matter which way it is sliced?

Sample answer: A sphere has the same shape and size from any angle, so its cross section is always a circle.

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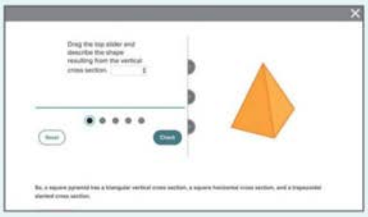
Interactive Presentation



Example 2, Describe Cross Sections of Three-Dimensional Figures, Slide 2 of 4

WATCH

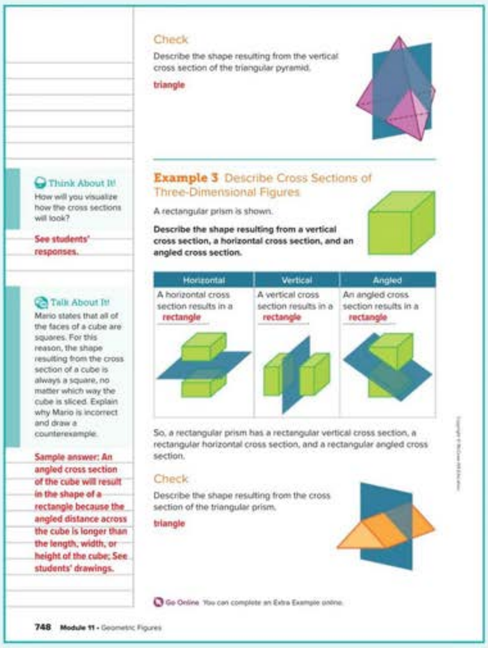
On Slide 1 of the Learn, students watch a video that shows the cross sections of different three-dimensional figures.

CLICK

On Slide 2 of Example 2, students select from a dropdown menu the shape of the cross section.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Describe the shape resulting from the vertical cross section of the triangular pyramid.

triangle

Think About It!

How will you visualize how the cross sections will look?

See students' responses.

Example 3 Describe Cross Sections of Three-Dimensional Figures

A rectangular prism is shown.

Describe the shape resulting from a vertical cross section, a horizontal cross section, and an angled cross section.

Horizontal	Vertical	Angled
A horizontal cross section results in a rectangle	A vertical cross section results in a rectangle	An angled cross section results in a rectangle

Talk About It!

Mario states that all of the faces of a cube are squares. For this reason, the shape resulting from the cross section of a cube is always a square, no matter which way the cube is sliced. Explain why Mario is incorrect and draw a counterexample.

Sample answer: An angled cross section of the cube will result in the shape of a rectangle because the angled distance across the cube is longer than the length, width, or height of the cube; See students' drawings.

So, a rectangular prism has a rectangular vertical cross section, a rectangular horizontal cross section, and a rectangular angled cross section.

Check

Describe the shape resulting from the cross section of the triangular prism.

triangle

Go Online You can complete an Extra Example online.

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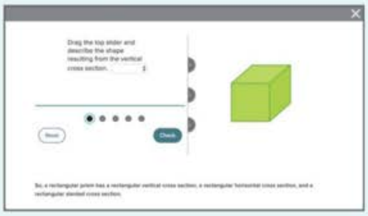
Interactive Presentation



Example 3, Describe Cross Sections of Three-Dimensional Figures, Slide 2 of 4

CLICK

On Slide 2, students select from a dropdown menu the shape of the cross section.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Describe Cross Sections of Three-Dimensional Figures

Objective

Students will describe the shape resulting from vertical, horizontal, and angled cross sections of prisms and cylinders.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 3, encourage them to draw a counterexample to illustrate the flaw in Mario's reasoning.

7 Look For and Make Use of Structure Encourage students to study the structure of the prism and make a conjecture about each cross section before they view the cross section.

Questions for Mathematical Discourse

SLIDE 2

AL What shape results from a vertical cross section? rectangle

OL How do you know that the horizontal cross section is a rectangle and not necessarily a square? Sample answer: The shape of the base of the prism is a rectangle, but we were not told that it is a square.

BL Describe a polyhedron whose horizontal cross section is a hexagon. Sample answer: hexagonal prism or hexagonal pyramid

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Essential Question Follow-Up

How does geometry help to describe objects?

In this lesson, students learned how to describe three-dimensional objects, such as prisms, pyramids, cylinders, and cones and the cross-sections of each. Encourage them to discuss with a partner how they can use this terminology to describe real-world objects. For example, they may state that the reason the top of a soup can is a circle is because it represents a horizontal cross-section of the cylindrical can.

Exit Ticket

Refer to the Exit Ticket slide. Suppose a skyscraper was shaped like a cylinder. What would be the shape of a horizontal cross section? circle

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

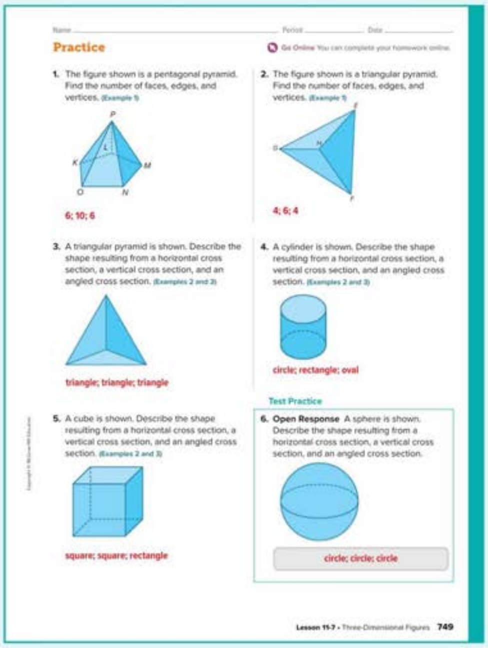
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	use the number of faces, edges, and vertices to describe three dimensional figures	1, 2
1	describe the shape resulting from vertical, horizontal, and angled cross sections of three-dimensional figures	3–5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving three-dimensional figures	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

For Exercise 2, students may have difficulty visualizing the faces of the pyramid. If possible, provide them with a triangular pyramid manipulative to use when counting faces.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The figure shown is a pentagonal pyramid. Find the number of faces, edges, and vertices. (Example 1)

6; 10; 6

2. The figure shown is a triangular pyramid. Find the number of faces, edges, and vertices. (Example 1)

4; 6; 4

3. A triangular pyramid is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section. (Examples 2 and 3)

triangle; triangle; triangle

4. A cylinder is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section. (Examples 2 and 3)

circle; rectangle; oval

5. A cube is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section. (Examples 2 and 3)

square; square; rectangle

Test Practice

6. **Open Response** A sphere is shown. Describe the shape resulting from a horizontal cross section, a vertical cross section, and an angled cross section.

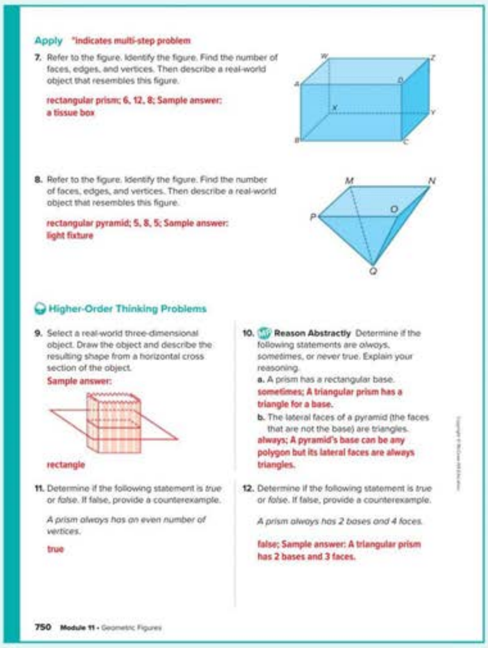
circle; circle; circle

Lesson 11-7 • Three-Dimensional Figures 749

Interactive Presentation


Exit Ticket

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

7. Refer to the figure. Identify the figure. Find the number of faces, edges, and vertices. Then describe a real-world object that resembles this figure.

rectangular prism; 6, 12, 8; Sample answer: a tissue box

8. Refer to the figure. Identify the figure. Find the number of faces, edges, and vertices. Then describe a real-world object that resembles this figure.

rectangular pyramid; 5, 8, 5; Sample answer: light fixture

Higher-Order Thinking Problems

9. Select a real-world three-dimensional object. Draw the object and describe the resulting shape from a horizontal cross section of the object.

Sample answer:

rectangle

10. **Reason Abstractly** Determine if the following statements are *always*, *sometimes*, or *never* true. Explain your reasoning.

a. A prism has a rectangular base.

sometimes; A triangular prism has a triangle for a base.

b. The lateral faces of a pyramid (the faces that are not the base) are triangles.

always; A pyramid's base can be any polygon but its lateral faces are always triangles.

11. Determine if the following statement is *true* or *false*. If false, provide a counterexample.

A prism always has an even number of vertices.

true

12. Determine if the following statement is *true* or *false*. If false, provide a counterexample.

A prism always has 2 bases and 4 faces.

false; Sample answer: A triangular prism has 2 bases and 3 faces.

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Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 10, students determine if statements are sometimes, always, or never true.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Create your own higher-order thinking problem.

Use with Exercises 9–12 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7, 9–12
- Extension: Similar Solids
- ALEKS® Three-Dimensional Figures

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8, 10
- Extension: Similar Solids
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Three-Dimensional Figures

IF students score 65% or below on the Checks, **THEN** assign: AL

- ArriveMATH Take Another Look
- ALEKS® Three-Dimensional Figures

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples and definitions for different types of angles and triangles. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

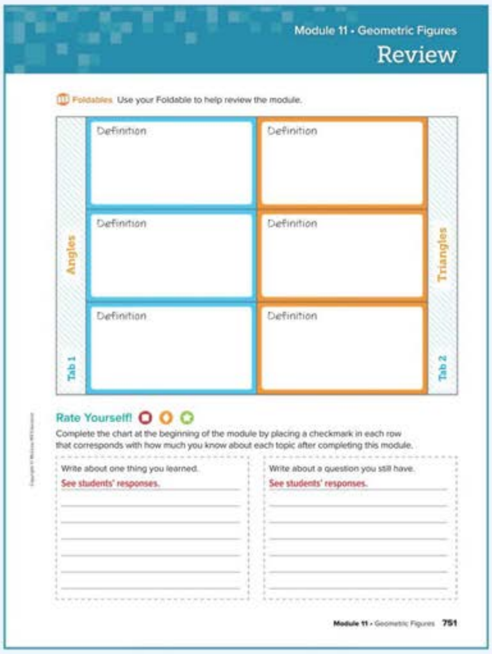
Assessment Resources

Put It All Together 1: Lessons 11-1 and 11-2
Put It All Together 2: Lessons 11-3 through 11-5
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Geometry**.

- Scale Drawings
- Constructing Geometric Shapes
- Cross-Sections
- Circles
- Angles
- Lines, Angles, and Triangles


Module 11 • Geometric Figures
Review

Foldables Use your Foldable to help review the module.

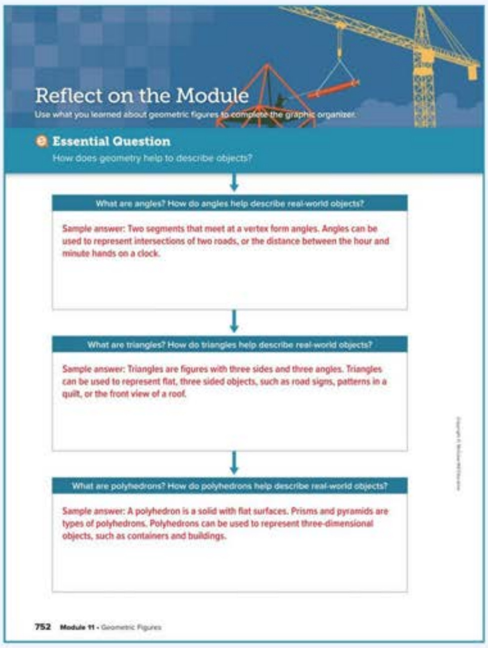
Definition
Definition
Definition
Definition
Definition
Definition
Angles
Triangles
Tab 1
Tab 2

Rate Yourself!
Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 11 • Geometric Figures 751



Reflect on the Module

Use what you learned about geometric figures to complete the graphic organizer.

Essential Question

How does geometry help to describe objects?

What are angles? How do angles help describe real-world objects?

Sample answer: Two segments that meet at a vertex form angles. Angles can be used to represent intersections of two roads, or the distance between the hour and minute hands on a clock.

What are triangles? How do triangles help describe real-world objects?

Sample answer: Triangles are figures with three sides and three angles. Triangles can be used to represent flat, three sided objects, such as road signs, patterns in a quilt, or the front view of a roof.

What are polyhedrons? How do polyhedrons help describe real-world objects?

Sample answer: A polyhedron is a solid with flat surfaces. Prisms and pyramids are types of polyhedrons. Polyhedrons can be used to represent three-dimensional objects, such as containers and buildings.

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How does geometry help to describe objects? See students' graphic organizers.

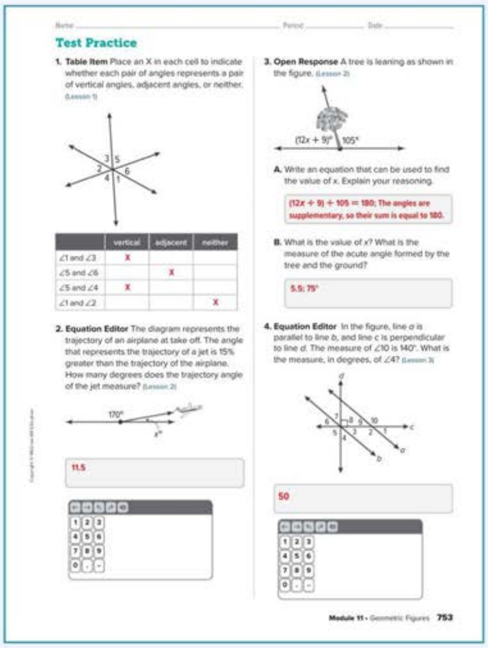
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–9 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	5, 8, 9
Multiselect	Multiple answers may be correct. Students must select all correct answers.	6
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	2, 4
Table Item	Students complete a table by correctly classifying the information.	1
Open Response	Students construct their own response in the area provided.	3, 7

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.G.A.1	11-6	7
7.G.A.2	11-4	5
7.G.A.3	11-7	8, 9
7.G.B.5	11-1, 11-2	1–3
8.G.A.5	11-3, 11-5	4, 6


Name ______ Period ______ Date ______

Test Practice

1. **Table Item** Place an X in each cell to indicate whether each pair of angles represents a pair of vertical angles, adjacent angles, or neither. (Lesson 1)

	vertical	adjacent	neither
∠1 and ∠3	X		
∠5 and ∠6		X	
∠5 and ∠4	X		
∠1 and ∠2			X

2. **Equation Editor** The diagram represents the trajectory of an airplane at take off. The angle that represents the trajectory of a jet is 15% greater than the trajectory of the airplane. How many degrees does the trajectory angle of the jet measure? (Lesson 2)

11.5

3. **Open Response** A tree is leaning as shown in the figure. (Lesson 2)

A. Write an equation that can be used to find the value of x. Explain your reasoning.

$(12x + 9) + 105 = 180$; The angles are supplementary, so their sum is equal to 180.

B. What is the value of x? What is the measure of the acute angle formed by the tree and the ground?

5.5; 75°

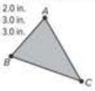
4. **Equation Editor** In the figure, line a is parallel to line b, and line c is perpendicular to line d. The measure of ∠10 is 140°. What is the measure, in degrees, of ∠4? (Lesson 3)

50

Module 11 • Geometric Figures **753**

5. Multiple Choice Is it possible to draw a triangle with side lengths of 2, 3, and 5 inches? If yes, select the triangle that meets the given conditions. If not, select the answer that explains why it is not possible. (Lesson 4)

Ⓐ $AB = 2.0$ in.
$AC = 3.0$ in.
$BC = 3.0$ in.



Ⓑ $AB = 2.0$ in.
$AC = 3.0$ in.
$BC = 4.0$ in.

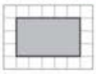
Ⓒ It is not possible to draw the triangle because the sum of two of the side lengths *is* greater than the third side.

● It is not possible to draw the triangle because the sum of two of the side lengths *is not* greater than the third side.

6. Multiselect In $\triangle JKL$, the measures of the angles J, K, and L, respectively, are in the ratio 3 : 3 : 6. Which of the following statements are accurate regarding the angle measures? Select all that apply. (Lesson 5)

- [x] Angles J and K have equal measures.
- [ ] The measure of $\angle L$ is half the measure of $\angle J$.
- [ ] $m\angle L = 45°$
- [ ] $m\angle K = 90°$
- [x] The angles form an isosceles right triangle.
- [x] The measure of $\angle L$ is twice the measure of $\angle K$.

7. Open Response The shaded figure in the diagram below represents a rectangular parking lot. The scale of the drawing is 1 unit = 40 yards. (Lesson 6)



A. What is the length and width of the parking lot in units?

5, 3

B. Suppose the scale drawing is reproduced using a scale of 1 unit = 20 yards. What is the new length and width, in units, of the drawing of the parking lot?

10, 6

8. Multiple Choice Which of the following describes the cross section of a cylinder and a vertical plane as shown? (Lesson 7)

Ⓐ circle
Ⓑ ellipse
● rectangle
Ⓓ square

9. Multiple Choice Which three-dimensional figure has 6 faces, 12 edges, and 8 vertices? (Lesson 7)

● rectangular prism
Ⓑ square pyramid
Ⓒ triangular prism
Ⓓ triangular pyramid

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Module 12

Area, Surface Area, and Volume

Module Goal

Solve real-world and mathematical problems involving area, surface area, and volume.

Focus

Domain: Geometry
Additional Cluster(s): **7.G.B** S olve real-life and mathematical problems involving angle measure, area, surface area, and volume.
Standards for Mathematical Content:
7.G.B.6 S olve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. *Also addresses 7.NS.A.3, 7.EE.B.4, 7.EE.B.4.A, 7.G.A.1, 7.G.B.4 and 8.G.C.9.*
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- solve one-step and two-step equations
- evaluate powers and exponents
- find the area of triangles and quadrilaterals

Use the Module Pretest to diagnose readiness. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students found the area of two-dimensional figures and the volume of rectangular prisms. **6.G.A. 1, 6.G.A.2**

Now
Students solve real-world and mathematical problems involving area, volume, and surface area. **7.G.B.4, 7.G.B.6, 8.G.C.9**

Next
Students will analyze transformations and use similar and congruentfigures using transformations. **8.G.A.1, 8.G.A.3**

Rigor

The Three Pillars of Rigor

In this module, students will develop an *understanding* of radius and diameter, and finding the circumference and area of circles. They will also gain *fluency* in finding the area of composite figures, volume, and surface area. They will use this knowledge to gain fluency in finding the volume and surface area of composite three-dimensional solids. They will also *apply* their fluency to solve real-world problems.

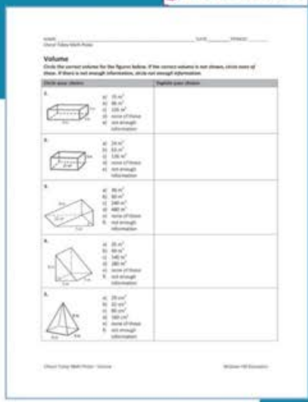
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
12-1	Circumference of Circles	7.G.B.4	1	0.5
12-2	Area of Circles	7.G.B.4	1	0.5
12-3	Area of Composite Figures	7.G.B.6	1	0.5
Put It All Together 1: Lessons 12-1 through 12-3			0.5	0.25
12-4	Volume of Prisms and Pyramids	7.G.B.6	1	0.5
12-5	Surface Area of Prisms and Pyramids	7.G.B.6	1	0.5
12-6	Volume of Cylinders	8.G.C.9	1	0.5
12-7	Volume of Cones	8.G.C.9	1	0.5
12-8	Volume of Spheres	8.G.C.9	1	0.5
12-9	Volume and Surface Area of Composite Solids	7.G.B.6	1	0.5
Put It All Together 2: Lessons 12-4 through 12-9			0.5	0.25
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	13	6.5

Formative Assessment Math Probe
Volume

Module Resource



Correct Answers: 1. b; **2.** b; **3.** a; **4.** c; **5.** f

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine the correct volume for each three-dimensional figure, and explain their choices.

Targeted Concept Understand what information is necessary and sufficient to determine the volume of a figure, and accurately identify the information in a figure.

Targeted Misconceptions

- Students may incorrectly follow the procedural steps of using the formula for determining volume.
- Students may have incomplete understanding of how the area of the base and the height are related to finding the volume of a prism.

Assign the probe after Lesson 4.

Collect and Assess Student Work

If the student selects...	Then the student likely...
2. not enough information	did not understand that the volume is the product of the area of the base and the height.
3. c	found half of the product of the 3 measurements provided.
3. b **4.** d	multiplied the base by the height without dividing the result in half.
5. any incorrect choice	confused the side length 8 meters for the height of the pyramid.
Various incorrect responses	does not have a conceptual understanding of volume and is using the formulas incorrectly. Because the height of the pyramid is not given in Exercise 5, the volume cannot be found.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS Perimeters, Areas, and Volumes
- Lesson 4, Examples 1–5

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

How can we measure objects to solve problems? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition.* They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

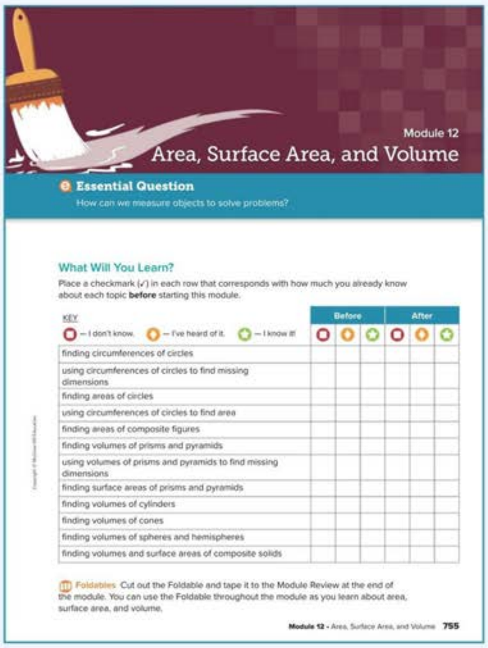
When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of bicycles, rugs, and paint to introduce the idea of measuring figures. Use the video to engage students before starting the module.

Pause and Reflect

Encourage your students to engage in the habit of reflection. As they progress through the module, they will be encouraged to pause and think about what they just learned. These moments of reflection are indicated by the *Pause and Reflect* questions that appear in the *Interactive Student Edition*. You may wish to have your students share their responses with a partner or use these questions to facilitate a whole-class discussion.



Module 12

Area, Surface Area, and Volume

Essential Question

How can we measure objects to solve problems?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

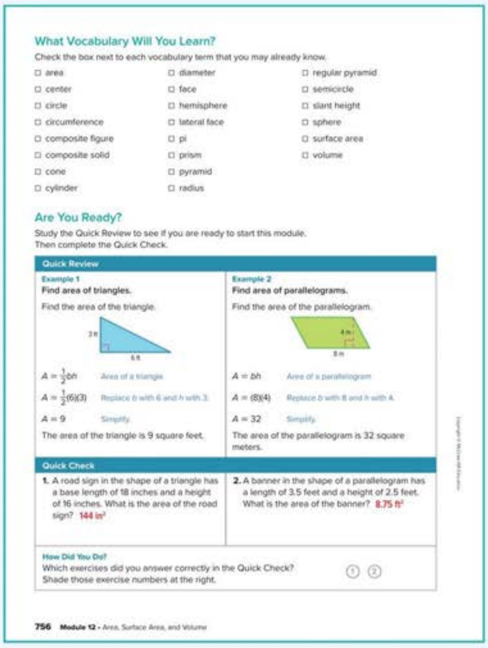
KEY: ○ — I don't know. ○ — I've heard of it. ○ — I know it!

	Before			After		
	I don't know.	I've heard of it.	I know it!	I don't know.	I've heard of it.	I know it!
finding circumferences of circles						
using circumferences of circles to find missing dimensions						
finding areas of circles						
using circumferences of circles to find area						
finding areas of composite figures						
finding volumes of prisms and pyramids						
using volumes of prisms and pyramids to find missing dimensions						
finding surface areas of prisms and pyramids						
finding volumes of cylinders						
finding volumes of cones						
finding volumes of spheres and hemispheres						
finding volumes and surface areas of composite solids						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about area, surface area, and volume.

Module 12 • Area, Surface Area, and Volume 755

Interactive Student Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

☐ area	☐ diameter	☐ regular pyramid
☐ center	☐ face	☐ semicircle
☐ circle	☐ hemisphere	☐ slant height
☐ circumference	☐ lateral face	☐ sphere
☐ composite figure	☐ pi	☐ surface area
☐ composite solid	☐ prism	☐ volume
☐ cone	☐ pyramid	
☐ cylinder	☐ radius	

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1
Find area of triangles.

Find the area of the triangle.

$A = \frac{1}{2}bh$ Area of a triangle

$A = \frac{1}{2}(6)(3)$ Replace b with 6 and h with 3.

$A = 9$ Simplify.

The area of the triangle is 9 square feet.

Example 2
Find area of parallelograms.

Find the area of the parallelogram.

$A = bh$ Area of a parallelogram

$A = (8)(4)$ Replace b with 8 and h with 4.

$A = 32$ Simplify.

The area of the parallelogram is 32 square meters.

Quick Check

1. A road sign in the shape of a triangle has a base length of 18 inches and a height of 16 inches. What is the area of the road sign? 144 in^2

2. A banner in the shape of a parallelogram has a length of 3.5 feet and a height of 2.5 feet. What is the area of the banner? 8.75 ft^2

How Did You Do?
Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

756 Module 12 • Area, Surface Area, and Volume

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define Volume is the amount of space inside a three-dimensional figure.

Example A rectangular prism has a length of $2\frac{1}{2}$ inches, a width of $3\frac{1}{2}$ inches, and a height of 7 inches. The volume of the prism is found by multiplying the length, width, and height, which is $61\frac{1}{4}$ cubic inches.

Ask Why do you think that volume is measured in cubic units? Sample answer: There are three dimensions (length, width, and height). So, the units will be cubed since the dimensions are multiplied.

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- multiplying decimals
- evaluating numerical and algebraic expressions involving whole-number exponents
- finding areas of triangles, quadrilaterals, and semicircles
- identifying the faces of three-dimensional figures
- finding volumes of prisms and pyramids

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Perimeters, Areas, and Volumes** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

Promote Growth Over Speed

Learning requires time and effort – time to think, reason, make mistakes, and learn from your mistakes and the mistakes of others. Ultimately, it's about the deep connections students make in their thinking and reasoning that matter more than the speed at which a problem is solved.

How Can I Apply It?

Have students complete the **What Will You Learn?** chart in their *Interactive Student Edition* before beginning each module and note the topics they don't know very well. At the end of each module, have them follow the **Rate Yourself!** directions in the module review by returning to this chart to view how their knowledge has increased. Encourage them to celebrate the topics with which their knowledge has increased, and take steps to strategize over how they can continue to grow in the topics about which they still might have questions.

Circumference of Circles

7.G.B.4

LESSON GOAL

Students will use radius and diameter to find circumference.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Radius and Diameter

Explore: The Distance Around a Circle

Learn: Circumference of Circles
Example 1: Find the Circumference Given the Diameter
Example 2: Find the Circumference Given the Radius
Learn: Use Circumference to Find Missing Dimensions
Example 3: Find the Diameter Given the Circumference
Example 4: Find the Radius Given the Circumference
Apply: Gardening

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

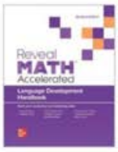
DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Perimeter of Semicircles		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 70 of the *Language Development Handbook* to help your students build mathematical language related to the circumference of circles.



ELL You can use the tips and suggestions on page T70 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry

Additional Cluster(s): In this lesson, students address additional cluster **7.G.B** by finding circumference of a circle.

Standards for Mathematical Content: **7. G.B.4,** *Also addresses 7.NS.A.3, 7.EE.B.4, 7.EE.B.4.A*

Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students found the area of two-dimensional figures and the volume of rectangular prisms.
6.G.A.1, 6.G.A.2

Now
Students find the circumference of circles.
7.G.B.4

Next
Students will find the area of circles.
7.G.B.4

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will develop an *understanding* of the radius and diameter of a circle and and how they relate to the circumference of the circle. They will gain *fluency* in finding the circumference of a circle through practice. They will *apply* their fluency in solving real-world problems.

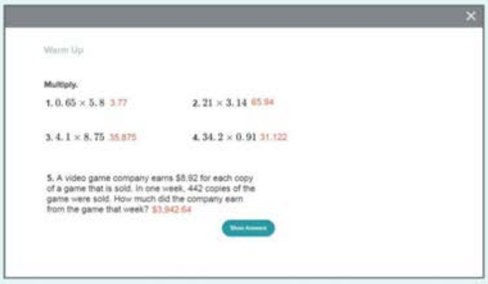
Mathematical Background

A circle has infinitely many diameters passing through its center point.

The following are true of circles.

- The circumference C is a measure of the distance around the circle.
- The ratio of the circumference to the diameter is constant and is equal to the irrational number π (pi), which has an approximate value of 3.14 or $\frac{22}{7}$. Because $\frac{C}{d} = \pi$, $C = \pi d$.
- The circumference of a circle is $C = \pi d$ or $C = 2\pi r$.

Interactive Presentation



Warm Up

Launch the Lesson

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- multiplying decimals (Exercises 1–5)

Answers

1. 3.77
2. 65.94
3. 35.875
4. 31.122
5. $3,942.64

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about pi using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What are some terms that are similar to *center* that you have used in everyday life? Sample answers: middle, median (of a highway)
- The Latin root *circ* means "ring". What are some other terms that begin with the root *circ*? Sample answers: circle, circular, circulation
- What are some other terms that begin with the prefix *dia*-? What does that prefix mean? Sample answers: diagonal, diagnostic, diagram; The prefix *dia*- means passing through.

Learn Radius and Diameter

Objective

Students will understand the relationship between the radius and diameter of a circle.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* questions on Slide 3, encourage them to reason about the relationship between the diameter and radius of a circle in order to manipulate the variables and write the equations.

Go Online to find additional teaching notes.

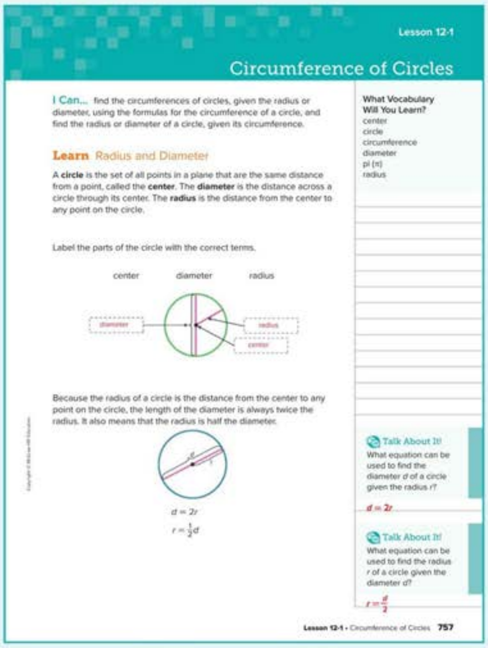
Talk About It!

SLIDE 3

Mathematical Discourse

What equation can be used to find the diameter d of a circle given the radius r? $d = 2r$

What equation can be used to find the radius r of a circle given the diameter d? $r = \frac{d}{2}$



Lesson 12-1

Circumference of Circles

I Can... find the circumferences of circles, given the radius or diameter, using the formulas for the circumference of a circle, and find the radius or diameter of a circle, given its circumference.

What Vocabulary Will You Learn?
center
circle
circumference
diameter
pi (π)
radius

Learn Radius and Diameter

A **circle** is the set of all points in a plane that are the same distance from a point, called the **center**. The **diameter** is the distance across a circle through its center. The **radius** is the distance from the center to any point on the circle.

Label the parts of the circle with the correct terms.

Because the radius of a circle is the distance from the center to any point on the circle, the length of the diameter is always twice the radius. It also means that the radius is half the diameter.

$d = 2r$

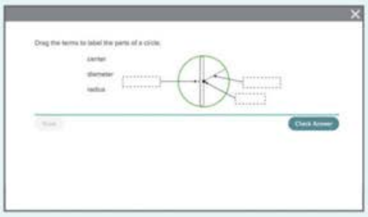
$r = \frac{1}{2}d$

Talk About It! What equation can be used to find the diameter d of a circle given the radius r? $d = 2r$

Talk About It! What equation can be used to find the radius r of a circle given the diameter d? $r = \frac{d}{2}$

Lesson 12-1 • Circumference of Circles 757

Interactive Presentation



Learn, Radius and Diameter, Slide 1 of 3

DRAG & DROP

On Slide 1, students drag terms to label the parts of a circle.

DIFFERENTIATE

Language Development Activity ELL

To help students better identify the radius or diameter of a circle in real-world settings, have them work with a partner to consider each of the following descriptions. They should determine if the radius or diameter is being described.

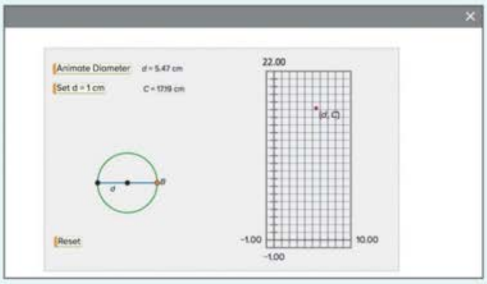
The distance across a circular bowl from rim to rim is 8 inches. diameter

The distance from the center of a circular swimming pool to the outer edge is 9.2 feet. radius

The minute hand of a clock extends to the edge of the circular face. The length of the minute hand is 4.32 centimeters. radius

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how the distance around a circle relates to its diameter.

Explore The Distance Around a Circle

Objective

Students will use Web Sketchpad to explore the relationship between the diameter of a circle and the distance around the same circle.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a sketch with an animation of a graph that shows the relationship between the diameter *d* and the distance around a circle *C*. Throughout this activity, students will use the sketch to investigate the relationship between *d* and *C*.

Inquiry Question

How does the distance around a circle relate to its diameter? Sample answer: The distance around a circle is in a proportional relationship with its diameter. Changing the length of the diameter changes the distance around the circle by a factor of about 3.14.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Is the distance around a circle proportional to the diameter? Explain your reasoning. yes; Sample answer: The graph of the relationship is a straight line through the origin.

(continued on next page)

Explore The Distance Around a Circle *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore and examine the relationship between the diameter of a circle and the distance around the circle.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

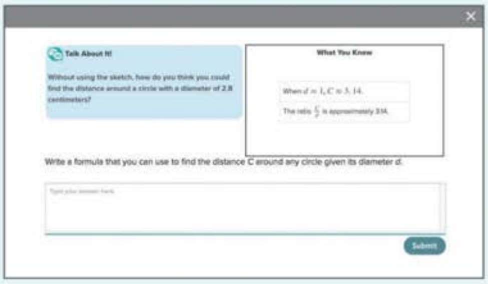
Talk About It!

SLIDE 5

Mathematical Discourse

Without using the sketch, how do you think you could find the distance around a circle with a diameter of 2.8 centimeters? Sample answer: Because the ratio $\frac{C}{d}$ is always approximately 3.14, I can multiply the diameter, 2.8 cm, by 3.14 to find the distance around the circle.

Interactive Presentation



Explore, Slide 5 of 6

TYPE

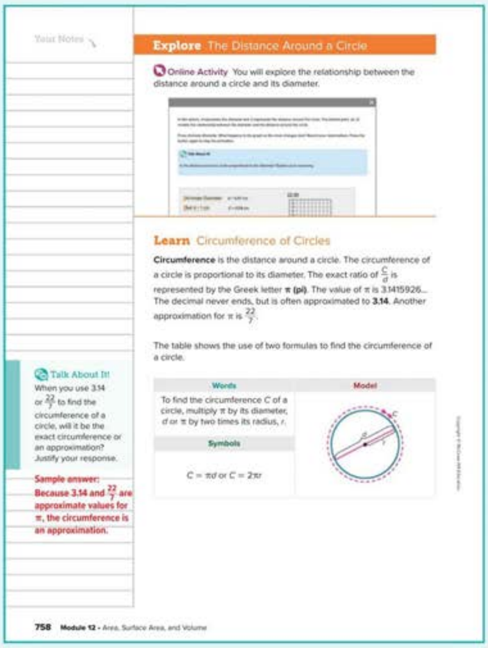
On Slide 4, students make a conjecture about the ratio of circumference to diameter as the circle changes size.

TYPE

On Slide 5, students write a formula to find the distance C around any circle given its diameter d.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.


Your Notes

Explore The Distance Around a Circle

Online Activity You will explore the relationship between the distance around a circle and its diameter.

Learn Circumference of Circles

Circumference is the distance around a circle. The circumference of a circle is proportional to its diameter. The exact ratio of $\frac{C}{d}$ is represented by the Greek letter π **(pi)**. The value of π is 3.1415926... The decimal never ends, but is often approximated to **3.14**. Another approximation for π is $\frac{22}{7}$.

The table shows the use of two formulas to find the circumference of a circle.

Words	Model
To find the circumference C of a circle, multiply π by its diameter, d or π by two times its radius, r.	
Symbols	
$C = \pi d$ or $C = 2\pi r$	

Talk About It! When you use 3.14 or $\frac{22}{7}$ to find the circumference of a circle, will it be the exact circumference or an approximation? Justify your response.

Sample answer: Because 3.14 and $\frac{22}{7}$ are approximate values for π, the circumference is an approximation.

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Interactive Presentation

Learn, Circumference of Circles, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of the formula for the circumference of a circle.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Circumference of Circles

Objective

Students will understand that the distance around a circle is called its circumference, and how the circumference is related to the circle's diameter and radius.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make a connection between proportional relationships and the formula that is used to find the circumference of a circle. Have them discuss what it means that the relationship between a circle's circumference and diameter is proportional. They should understand that the ratio for the circumference of any circle to its diameter will always be the same value, π.

As students discuss the *Talk About It!* question on Slide 2, encourage them to understand the difference between an estimate and an exact answer.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 2, remind them that the digits in the exact value of π never repeat and never terminate. Have them discuss what that means for calculations that involve π.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

When you use 3.14 or $\frac{22}{7}$ to find the circumference of a circle, will it be the exact circumference or an approximation? Justify your response.

Sample answer: Because 3.14 and $\frac{22}{7}$ are approximate values for π, the circumference is an approximation.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 1 Find the Circumference Given the Diameter

Objective

Students will find the circumference of a circle given the diameter.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 3, encourage them to understand the symbolic notation used for each symbol, = and ≈.

6 Attend to Precision Encourage students to determine and defend which formula they will use to find the circumference and express the circumference with a degree of precision appropriate for the context of the problem.

As students discuss the *Talk About It!* question on Slide 3, be sure they can precisely explain why the *equals sign* was changed to the *approximately equals* sign during the solution process.

Questions for Mathematical Discourse

SLIDE 2

AL What formula for the circumference will you use? Why? $C = \pi d$; I am given the diameter.

AL A classmate claims they can use the formula $C = 2\pi r$. Is this correct? Explain. yes; Sample answer: It will just involve an extra step to find the radius, but the answer will be the same.

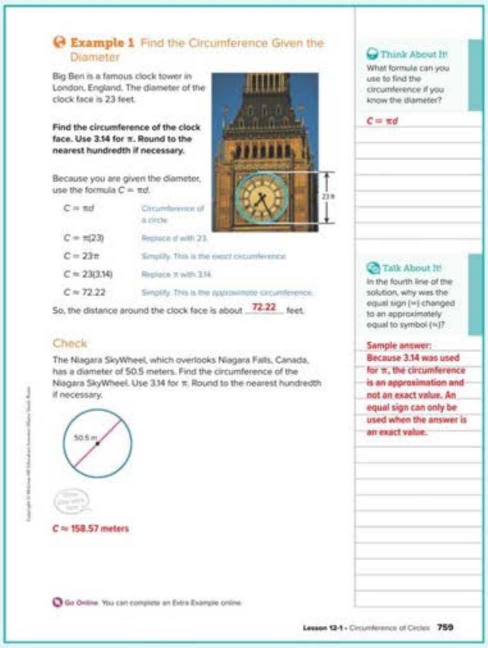
OL Why is it important to make an estimate prior to solving the problem? Sample answer: By making an estimate, I can compare my answer to the estimate to verify that my answer is reasonable.

OL A classmate claims the exact distance around the clock face is 23π feet. Is this correct? Explain. yes; Sample answer: The exact value does not use a decimal approximation.

BL How far does the tip of the minute hand of Big Ben travel in one day? Explain. about 1,733.28 feet; Sample answer: The tip travels about 72.22 feet in one hour. Multiply this by 24 hours to find how far it travels in one day.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Find the Circumference Given the Diameter

Big Ben is a famous clock tower in London, England. The diameter of the clock face is 23 feet.

Find the circumference of the clock face. Use 3.14 for π. Round to the nearest hundredth if necessary.

Because you are given the diameter, use the formula $C = \pi d$.

$C = \pi d$	Circumference of a circle
$C = \pi(23)$	Replace d with 23.
$C = 23\pi$	Simplify. This is the exact circumference.
$C \approx 23(3.14)$	Replace π with 3.14.
$C \approx 72.22$	Simplify. This is the approximate circumference.

So, the distance around the clock face is about 72.22 feet.

Think About It! What formula can you use to find the circumference if you know the diameter? $C = \pi d$

Talk About It! In the fourth line of the solution, why was the equal sign (=) changed to an approximately equal to symbol (≈)? Sample answer: Because 3.14 was used for π, the circumference is an approximation and not an exact value. An equal sign can only be used when the answer is an exact value.

Check

The Niagara SkyWheel, which overlooks Niagara Falls, Canada, has a diameter of 50.5 meters. Find the circumference of the Niagara SkyWheel. Use 3.14 for π. Round to the nearest hundredth if necessary.

50.5 m

$C \approx 158.57$ meters

Go Online You can complete an Extra Example online.

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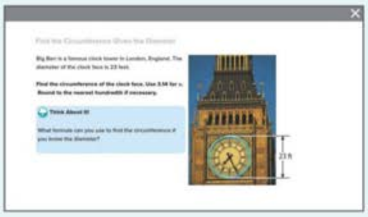
Interactive Presentation



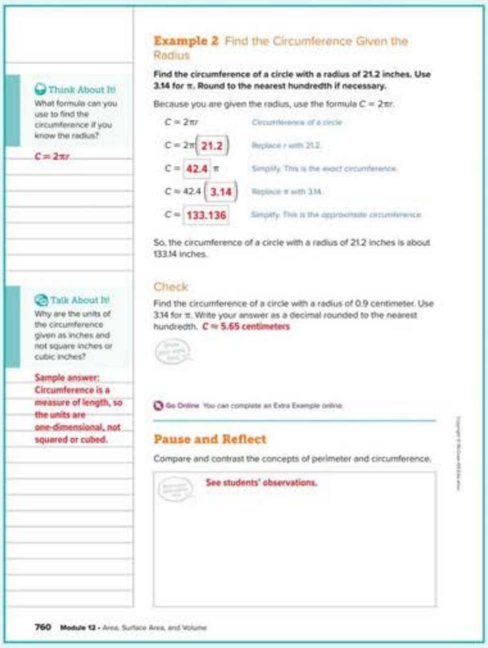
Example 1, Find the Circumference Given the Diameter, Slide 1 of 4

TYPE

On Slide 2, students enter the approximate circumference.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Interactive Presentation

Example 2, Find the Circumference Given the Radius, Slide 2 of 4

TYPE

On Slide 2, students enter the approximate circumference.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Find the Circumference Given the Radius

Objective

Students will find the circumference of a circle given the radius.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to determine and defend which formula they will use to find the circumference and express the circumference with a degree of precision appropriate for the context of the problem.

As students discuss the *Talk About It!* question on Slide 3, encourage them to use appropriate measurements and labels, keeping in mind the difference between area and perimeter or length.

Questions for Mathematical Discourse

SLIDE 2

AL What formula for the circumference will you use? Why? $C = 2\pi r$; I am given the radius.

AL A classmate claims they can use the formula $C = \pi d$. Is this correct? Explain. yes; Sample answer: It will just involve an extra step to find the diameter, but the answer will be the same.

OL Estimate the circumference. Sample answer: The radius is close to 20 inches. Pi is close to 3. So, the circumference is close to 120 inches.

OL Write a real-world problem that can be represented by these quantities. Sample answer: John is making a small circular table. The radius of the table is 21.2 inches. What will be the distance around the table?

BL Find the circumference using a value of pi rounded to 3.14159. Round the circumference to the nearest thousandth. Compare this circumference to the one using pi rounded to 3.14. about 133.203 inches; Sample answer: 133.203 is very close to 133.136, the difference is 0.067 inch, which is less than seven hundredths of an inch.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Use Circumference to Find Missing Dimensions

Objective

Students will understand how the circumference formula can be applied to find the diameter or radius of a circle.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 1

Mathematical Discourse

Why is there a 2 in the denominator for the equation to find the radius, but not in the equation to find the diameter? Sample answer: Using the formula $C = 2\pi r$ to find the radius, you divide each side by 2π. Therefore, there is a 2 in the denominator. Using the formula $C = \pi d$ to find the diameter, you divide each side by π, not 2π.

Example 3 Find the Diameter Given the Circumference

Objective

Students will find the diameter of a circle given the circumference.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the diameter of the ring

AL What are you given? the circumference of the ring, 66 meters

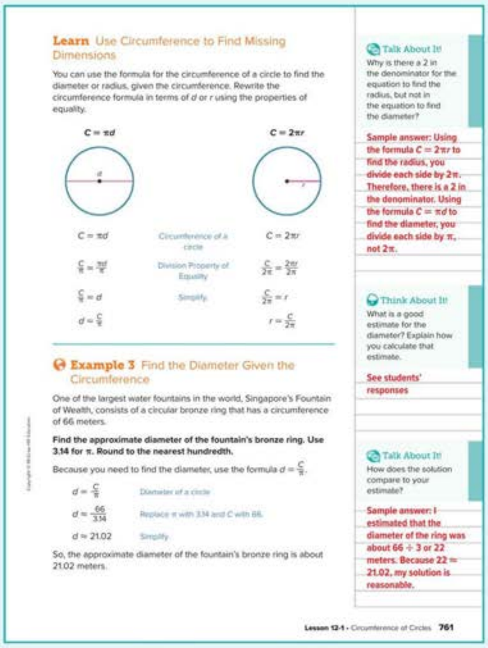
OL What formula will you use? Why? $d = \frac{C}{\pi}$; Sample answer: I need to find the diameter and I know the circumference.

OL Estimate the diameter. Sample answer: π is close to 3. Because $66 \div 3$ is 22, the diameter is close to 22 meters.

BL Suppose you wanted to make a model of the bronze ring with a scale of 1 inch = 3 meters. What will be the approximate diameter of the model? Explain. about 7 inches; Sample answer: If 1 inch = 3 meters, then the circumference of the model will be $66 \div 3$, or 22 inches. To find the approximate diameter of the model, divide 22 by 3.14. The diameter will be about 7 inches.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Learn Use Circumference to Find Missing Dimensions

You can use the formula for the circumference of a circle to find the diameter or radius, given the circumference. Rewrite the circumference formula in terms of *d* or *r* using the properties of equality.

$C = \pi d$		$C = 2\pi r$
$C = \pi d$	Circumference of a circle	$C = 2\pi r$
$\frac{C}{\pi} = \frac{\pi d}{\pi}$	Division Property of Equality	$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$
$\frac{C}{\pi} = d$	Simplify.	$\frac{C}{2\pi} = r$
$d = \frac{C}{\pi}$		$r = \frac{C}{2\pi}$

Example 3 Find the Diameter Given the Circumference

One of the largest water fountains in the world, Singapore's Fountain of Wealth, consists of a circular bronze ring that has a circumference of 66 meters.

Find the approximate diameter of the fountain's bronze ring. Use 3.14 for π. Round to the nearest hundredth.

Because you need to find the diameter, use the formula $d = \frac{C}{\pi}$.

$d = \frac{C}{\pi}$ Diameter of a circle

$d \approx \frac{66}{3.14}$ Replace π with 3.14 and C with 66.

$d \approx 21.02$ Simplify.

So, the approximate diameter of the fountain's bronze ring is about 21.02 meters.

Talk About It! Why is there a 2 in the denominator for the equation to find the radius, but not in the equation to find the diameter?

Sample answer: Using the formula $C = 2\pi r$ to find the radius, you divide each side by 2π. Therefore, there is a 2 in the denominator. Using the formula $C = \pi d$ to find the diameter, you divide each side by π, not 2π.

Think About It! What is a good estimate for the diameter? Explain how you calculate that estimate.

See students' responses

Talk About It! How does the solution compare to your estimate?

Sample answer: I estimated that the diameter of the ring was about $66 \div 3$ or 22 meters. Because $22 \approx 21.02$, my solution is reasonable.

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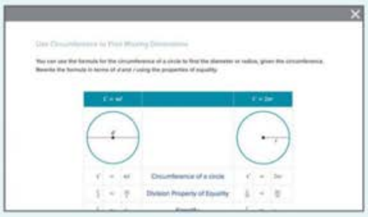
Interactive Presentation



Learn, Use Circumference to Find Missing Dimensions

TYPE

On Slide 2 of Example 3, students enter the approximate diameter.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Find the approximate diameter of a basketball hoop that has a circumference of 56.52 inches. Use 3.14 for π. Round to the nearest hundredth if necessary. $d \approx 18$ inches

Example 4 Find the Radius Given the Circumference

Find the approximate radius of a circle with a circumference of 70.82 inches. Use 3.14 for π. Round to the nearest hundredth.

Because you need to find the radius, use the formula $r = \frac{C}{2\pi}$.

$r = \frac{C}{2\pi}$ Radius of a circle

$r \approx \frac{70.82}{2(3.14)}$ Replace π with 3.14 and C with 70.82.

$r \approx 11.28$ Simplify.

So, the approximate length of the radius of a circle that has a 70.82-inch circumference is about 11.28 inches.

Think About It! What is a good estimate for the radius? Explain how you calculate that estimate.

See students' responses.

Talk About It! How does your solution compare to the estimate?

Sample answer: I estimated that the radius of the circle was about 70 ÷ 2(3) or about 11.7 inches. Because 11.28 ≈ 11.7, my solution is reasonable.

Check

Find the approximate radius of a circle with a circumference of 7.92 centimeters. Use 3.14 for π. Round to the nearest hundredth.

$r \approx 1.26$ centimeters

Go Online You can complete an Extra Example online.

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Interactive Presentation

Example 4, Find the Radius Given the Circumference, Slide 2 of 4

TYPE

On Slide 2, students enter the approximate radius.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Find the Radius Given the Circumference

Objective

Students will find the radius of a circle given the circumference.

MP Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to determine and defend which formula they will use to find the radius and express the radius with a degree of precision appropriate for the context of the problem.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the radius of the circle

AL What are you given? the circumference of the circle, 70.82 inches

OL What formula will you use? Why? $\frac{C}{2\pi}$; Sample answer: I need to find the radius and I know the circumference.

OL Estimate the diameter. Sample answer: π is close to 3, and 70.82 is close to 72. Since 72 ÷ 6 is 12, the radius is close to 12 inches.

BL Can you use the formula $C = 2\pi r$ to find the radius? Explain. yes; Sample answer: I can replace C with the value of the circumference and then divide each side of the equation by 2 and by π.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Gardening

Objective

Students will come up with their own strategy to solve an application problem involving a circular garden.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage them to draw a representation of the problem to explain their reasoning.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

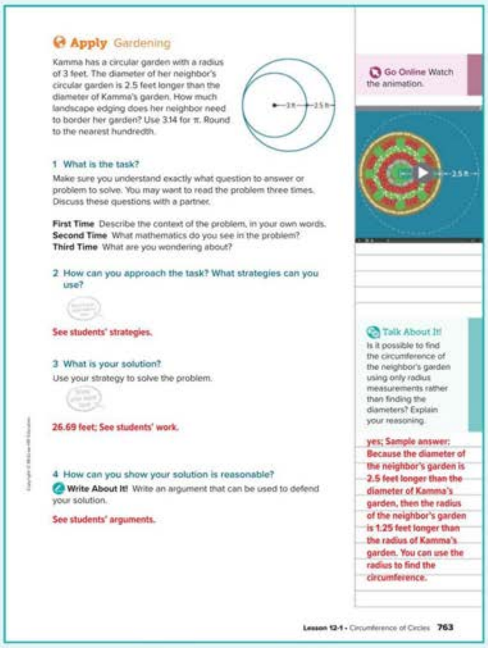
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does the circumference of a circle represent?
- What formula for circumference should you use?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Gardening

Kamma has a circular garden with a radius of 3 feet. The diameter of her neighbor's circular garden is 2.5 feet longer than the diameter of Kamma's garden. How much landscape edging does her neighbor need to border her garden? Use 3.14 for π. Round to the nearest hundredth.

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

26.69 feet; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It!
Is it possible to find the circumference of the neighbor's garden using only radius measurements rather than finding the diameters? Explain your reasoning.

yes; Sample answer: Because the diameter of the neighbor's garden is 2.5 feet longer than the diameter of Kamma's garden, then the radius of the neighbor's garden is 1.25 feet longer than the radius of Kamma's garden. You can use the radius to find the circumference.

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Interactive Presentation

Apply, Gardening

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

A nickel has a diameter that is 2.16 millimeters longer than the diameter of a penny. If the radius of a penny is 9.525 millimeters, what is the circumference of a nickel? Use 3.14 for π. Round to the nearest tenth. $C \approx 66.6$ **millimeters**

Go Online You can complete an Extra Example online.

Pause and Reflect

Create a graphic organizer that will help you choose when to use the diameter or radius to find the circumference.

See students' observations.

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Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can we measure objects to solve problems?

In this lesson, students learned how to find the circumference of circles. Encourage them to brainstorm with a partner at least two real-world situations in which they might need to find the circumference of a circle. Some examples could include the distance around a circular swimming pool or the length of a circular walking path in a park.

Exit Ticket

Refer to the Exit Ticket slide. Suppose a model of the Olympic rings is constructed and each circle has a radius of 6 feet. What is the amount of rope used in the construction of the model of the five rings? Explain how you solved the problem. Use 3.14 for π. Round to the nearest hundredth, if necessary. 188.4 feet; Sample answer: The circumference of one ring in the model is $2\pi(6)$, or about 37.68 feet. Multiply 37.68 by 5 to obtain that about 188.4 feet of rope is needed to create the model of the five rings.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–9 odd, 11–14
- Extension: Perimeter of Semicircles
- ALEKS Circumference and Area of Circles

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–9, 12
- Extension: Perimeter of Semicircles
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Areas of Parallelograms, Triangles, and Trapezoids

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Areas of Parallelograms, Triangles, and Trapezoids

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

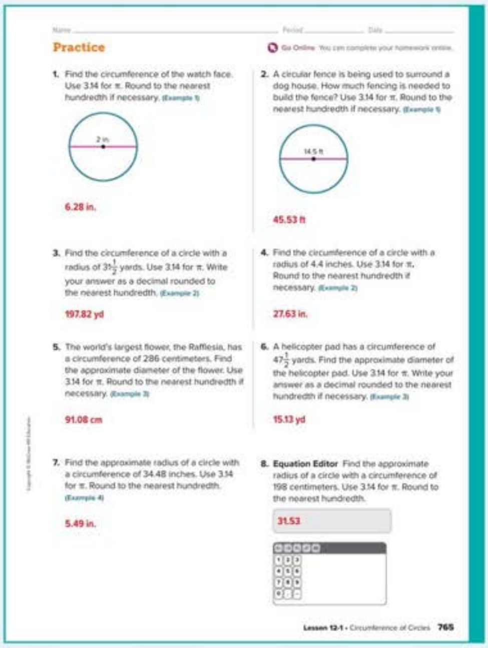
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find the circumference of a circle given the diameter	1, 2
2	find the circumference of a circle given the radius	3, 4
2	find the diameter of a circle given the circumference	5, 6
2	find the radius of a circle given the circumference	7, 8
3	solve application problems involving circumference of circles	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may reverse the relationship between the radius and diameter, thinking that the radius is twice the diameter. In this case, they may calculate the circumference as π times the radius, or π times twice the diameter. In Exercise 3, students may demonstrate this common misconception by calculating the circumference to be about 98.91 centimeters. Remind them to use reasoning about the relationship between the radius and diameter in order to check their work and determine if their answer is reasonable.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Find the circumference of the watch face. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)

6.28 in.

2. A circular fence is being used to surround a dog house. How much fencing is needed to build the fence? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)

45.53 ft

3. Find the circumference of a circle with a radius of $31\frac{1}{2}$ yards. Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth. (Example 2)

197.82 yd

4. Find the circumference of a circle with a radius of 4.4 inches. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 2)

27.63 in.

5. The world's largest flower, the Rafflesia, has a circumference of 286 centimeters. Find the approximate diameter of the flower. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 3)

91.08 cm

6. A helicopter pad has a circumference of $47\frac{1}{2}$ yards. Find the approximate diameter of the helicopter pad. Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth if necessary. (Example 3)

15.13 yd

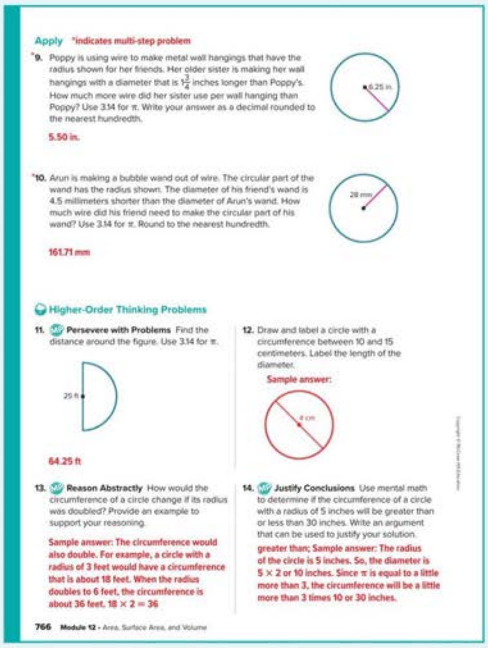
7. Find the approximate radius of a circle with a circumference of 34.48 inches. Use 3.14 for π. Round to the nearest hundredth. (Example 4)

5.49 in.

8. **Equation Editor** Find the approximate radius of a circle with a circumference of 198 centimeters. Use 3.14 for π. Round to the nearest hundredth.

31.53

Lesson 12-1 • Circumference of Circles 765



Apply *indicates multi-step problem

*9. Poppy is using wire to make metal wall hangings that have the radius shown for her friends. Her older sister is making her wall hangings with a diameter that is $1\frac{3}{4}$ inches longer than Poppy's. How much more wire did her sister use per wall hanging than Poppy? Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth.

6.25 in.

5.50 in.

*10. Arun is making a bubble wand out of wire. The circular part of the wand has the radius shown. The diameter of his friend's wand is 4.5 millimeters shorter than the diameter of Arun's wand. How much wire did his friend need to make the circular part of his wand? Use 3.14 for π. Round to the nearest hundredth.

28 mm

161.71 mm

Higher-Order Thinking Problems

11. MP **Persevere with Problems** Find the distance around the figure. Use 3.14 for π.

25 ft

64.25 ft

12. Draw and label a circle with a circumference between 10 and 15 centimeters. Label the length of the diameter.

Sample answer:

4 cm

13. MP **Reason Abstractly** How would the circumference of a circle change if its radius was doubled? Provide an example to support your reasoning.

Sample answer: The circumference would also double. For example, a circle with a radius of 3 feet would have a circumference that is about 18 feet. When the radius doubles to 6 feet, the circumference is about 36 feet. $18 \times 2 = 36$

14. MP **Justify Conclusions** Use mental math to determine if the circumference of a circle with a radius of 5 inches will be greater than or less than 30 inches. Write an argument that can be used to justify your solution.

greater than; Sample answer: The radius of the circle is 5 inches. So, the diameter is 5×2 or 10 inches. Since π is equal to a little more than 3, the circumference will be a little more than 3 times 10 or 30 inches.

766 Module 12 • Area, Surface Area, and Volume

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them In Exercise 11, students use multiple steps to find the distance around a semicircle.

2 Reason Abstractly and Quantitatively In Exercise 13, students describe how the circumference of a circle would change if the radius was doubled.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students use mental math to determine if the circumference of a circle with a radius of 5 inches will be greater than or less than 30 inches.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercise 9 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Listen and ask clarifying questions.
Use with Exercises 13–14 Have students work in pairs. Have students individually read Exercise 13 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 14.

Lesson 12-2

Area of Circles

7.G.B.4

LESSON GOAL

Students will find the area of circles.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Area of Circles

Learn: Derive the Formula for the Area of a Circle

Learn: Area of Circles
Example 1: Find the Area Given the Radius
Example 2: Find the Area Given the Diameter
Learn: Area of Semicircles
Example 3: Find Area of Semicircles
Learn: Use Circumference to Find Area
Example 4: Use Circumference to Find Area
Apply: Crafting

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

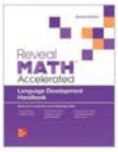
DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 71 of the *Language Development Handbook* to help your students build mathematical language related to the area of circles.



ELL You can use the tips and suggestions on page T71 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.B** by finding areas of circles.
Standards for Mathematical Content: **7. G.B.4,** *Also addresses 7.NS.A.3, 7.EE.B.4, 7.EE.B.4.A*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the circumference of circles.
7.G.B.4

Now
Students find the area of circles.
7.G.B.4

Next
Students will find the area of composite figures.
7.G.B.6

Rigor

The Three Pillars of Rigor

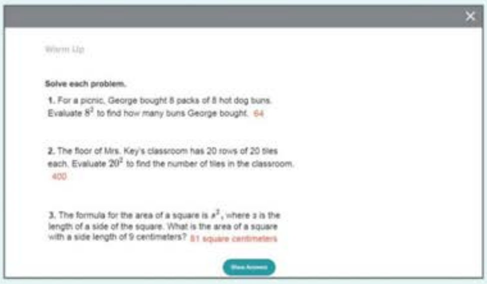
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of circles to build an *understanding* of the area of circles. Students will use their understanding to gain *fluency* while *applying* their knowledge in real-world problems. Students will also develop an understanding of finding the area of a circle given the circumference.

Mathematical Background

If a circle's radius or diameter is known, then its area can be calculated. To find the *area* of a circle, use the formula $A = \pi r^2$, where r is the radius. You can also estimate the area of a circle by rounding π to 3 and calculating mentally.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- evaluating numerical expressions involving whole-number exponents (Exercises 1–2)
- evaluating algebraic expressions involving whole-number exponents (Exercise 3)

Answers

1. 64

2. 400

3. 81 square centimeters

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the use of longe lines when training a horse in a circular enclosure.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Define the term *area* in your own words. Sample answer: Area is the amount of space within a closed figure.
- How does the prefix *semi-* help you define *semicircle*? Sample answer: The prefix *semi-* means half, so a semicircle is half of a circle.

Explore Area of Circles

Objective

Students will use Web Sketchpad to explore the formula for the area of a circle.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with two figures divided into the same number of sectors: a circle and a wavy parallelogram. Throughout this activity, students will use the figures to investigate the relationships between the attributes of the two figures.

Inquiry Question

How can you use the formula for the area of a parallelogram to help you find the area of a circle? Sample answer: I can start with the formula for the area of a parallelogram and then substitute half of the circle's circumference for the base and the radius for the height to find the area of the corresponding circle.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

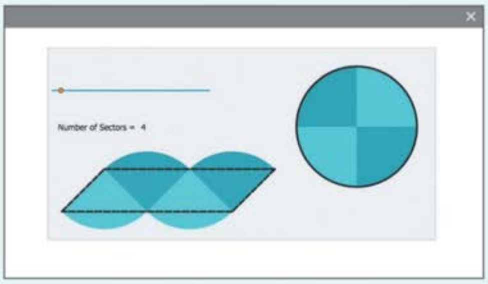
Mathematical Discourse

How are the circle and the wavy parallelogram related? What happens to the wavy parallelogram as more sectors are added? Sample answer: Both are made up of the same number of sectors of the same size; As more sectors are added, the waves become smaller and the parallelogram becomes less wavy.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 6

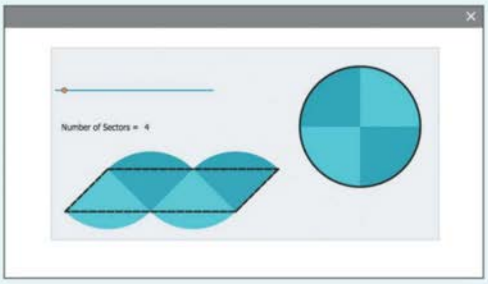


Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how the formula for the area of a parallelogram can be used to help find the area of a circle.

Interactive Presentation



Explore, Slide 4 of 6

CLICK

On Slide 5, students select from drop-down menus to complete statements.

TYPE

On Slide 5, students make a conjecture about how to find the area of a circle.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Area of Circles *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to help them gain insight into determining the formula for the area of a circle.

7 Look For and Make Use of Structure Students should analyze the structures of the figures in order to describe the relationship between the circle and the wavy parallelogram.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

SLIDE 4

Mathematical Discourse

What is the relationship between the number of sectors along the base of the parallelogram and the total number of sectors in the circle? Sample answer: The number of sectors along the base of the parallelogram is half the total number of sectors in the circle.

As the number of sector increases, how does the area of the parallelogram compare to the area of the circle? Sample answer: The area of the parallelogram gets closer and closer to the area of the circle.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

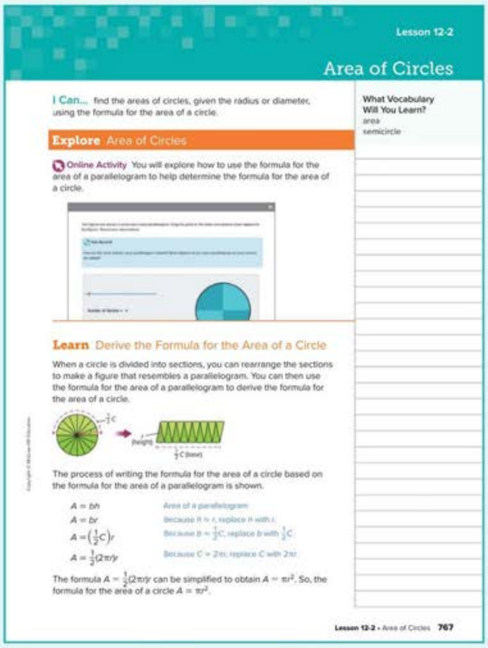
Learn Derive the Formula for the Area of a Circle

Objective

Students will understand how to derive the formula for the area of a circle from the area of a parallelogram.

Teaching Notes

Have students imagine the number of sections into which the circle can be divided increasing to an infinite number of sections. As the number of sections increases, the rearranged figure more closely resembles a parallelogram. Be sure that students understand how the labels on the parallelogram are related to the radius and circumference of the circle. Prior to having students move through the steps to derive the area formula, have them discuss with a partner what strategies they can use to come up with the formula. Then have them move through the steps to see if their strategy was similar or different, and whether or not their strategy was mathematically correct.



Lesson 12-2

Area of Circles

I Can... find the areas of circles, given the radius or diameter, using the formula for the area of a circle.

What Vocabulary Will You Learn?
area
semicircle

Explore Area of Circles

Online Activity You will explore how to use the formula for the area of a parallelogram to help determine the formula for the area of a circle.

Learn Derive the Formula for the Area of a Circle

When a circle is divided into sections, you can rearrange the sections to make a figure that resembles a parallelogram. You can then use the formula for the area of a parallelogram to derive the formula for the area of a circle.

The process of writing the formula for the area of a circle based on the formula for the area of a parallelogram is shown.

$A = bh$ Area of a parallelogram

$A = br$ Because $h = r$, replace h with r.

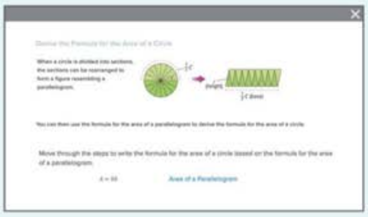
$A = \left(\frac{1}{2}C\right)r$ Because $b = \frac{1}{2}C$, replace b with $\frac{1}{2}C$

$A = \frac{1}{2}(2\pi r)r$ Because $C = 2\pi r$, replace C with $2\pi r$.

The formula $A = \frac{1}{2}(2\pi r)r$ can be simplified to obtain $A = \pi r^2$. So, the formula for the area of a circle $A = \pi r^2$.

Lesson 12-2 • Area of Circles 767

Interactive Presentation



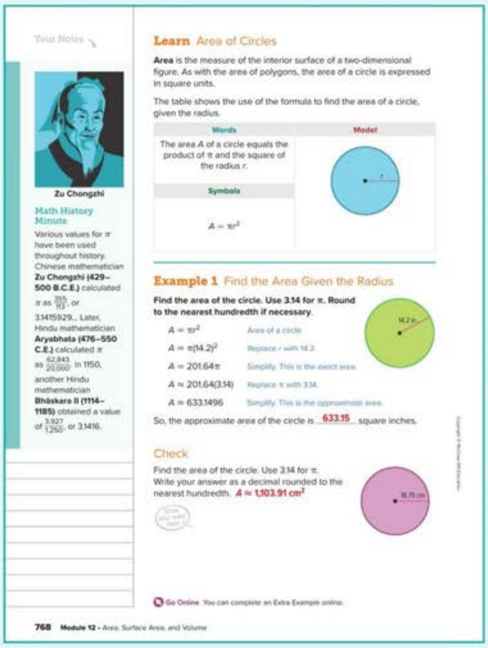
Learn, Derive the Formula for the Area of a Circle

CLICK

Students move through the steps to write the formula for the area of a circle based on the formula for the area of a parallelogram.

7.G.B.4

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Your Notes

Zu Chongzhi

Math History Minute

Various values for π have been used throughout history. Chinese mathematician **Zu Chongzhi (429–500 B.C.E.)** calculated π as $\frac{355}{113}$, or 3.1415929... Later, Hindu mathematician **Aryabhata (476–550 C.E.)** calculated π as $\frac{62,843}{20,000}$. In 1150, another Hindu mathematician **Bhāskara II (1114–1185)** obtained a value of $\frac{3,927}{1,250}$, or 3.1416.

Learn Area of Circles

Area is the measure of the interior surface of a two-dimensional figure. As with the area of polygons, the area of a circle is expressed in square units.

The table shows the use of the formula to find the area of a circle, given the radius.

Words	Model
The area A of a circle equals the product of π and the square of the radius r.	
Symbols	
$A = \pi r^2$	

Example 1 Find the Area Given the Radius

Find the area of the circle. Use 3.14 for π. Round to the nearest hundredth if necessary.

$A = \pi r^2$ Area of a circle

$A = \pi(14.2)^2$ Replace r with 14.2.

$A = 201.64\pi$ Simplify. This is the exact area.

$A \approx 201.64(3.14)$ Replace π with 3.14.

$A \approx 633.1496$ Simplify. This is the approximate area.

So, the approximate area of the circle is 633.15 square inches.

Check

Find the area of the circle. Use 3.14 for π. Write your answer as a decimal rounded to the nearest hundredth. $A \approx 1{,}103.91 \text{ cm}^2$

Go Online You can complete an Extra Example online.

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Interactive Presentation

Learn, Area of Circles

FLASHCARDS

On Slide 1 of the Learn, students use Flashcards to view multiple representations of the formula for the area of a circle.

TYPE

On Slide 1 of Example 1, students determine the approximate area of the circle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Area of Circles

Objective

Students will understand how to find the area of a circle.

Go Online to find additional teaching notes.

Example 1 Find the Area Given the Radius

Objective

Students will find the area of a circle given the radius.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to begin by making an estimate, and to use their estimate to determine if their solution is reasonable.

6 Attend to Precision Students should be precise in using the *approximately equals* sign during their solution process after they replace π with its approximation of 3.14.

Questions for Mathematical Discourse

SLIDE 1

AL What dimension is known? What are you asked to find? I know the radius, 14.2 in. I need to find the area.

AL Why is it important to make an estimate? Sample answer: By making an estimate, I can use the estimate to determine if my answer is reasonable.

OL Explain how to estimate the area of the circle. Sample answer: π is close to 3, and 14.2 is close to 14. Find the product of 3 and the square of 14, or 196, which is 588. The area is about 588 square inches.

OL Is the actual area less than or greater than 588 square inches? Explain. greater than; Sample answer: I rounded π down to 3, and I rounded 14.2 down to 14. So, the actual area is greater than my estimate.

BL If the radius doubles, what happens to the area? Explain. the area quadruples; Sample answer: If the radius doubles, the area will quadruple because the radius is squared in the area formula.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Find the Area Given the Diameter

Objective

Students will find the area of a circle given the diameter.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the relationship between a circle's diameter and radius, in order to reason that they must first find the radius.

6 Attend to Precision Students should be precise in using the *approximately equals* sign during their solution process after they replace π with its approximation of 3.14.

Questions for Mathematical Discourse

SLIDE 2

AL What dimension is known? I know the diameter.

AL How can you find the radius? Divide the diameter by 2.

OL Why do you need to find the radius? The formula for the area of a circle is given in terms of the radius.

OL How are the radius and the diameter related? The radius is half of the diameter. The diameter is twice the radius.

BL How can you change the area formula so that you can use the diameter? Sample answer: Since $r = \frac{d}{2}$ substitute $\frac{d}{2}$ for r in the formula $A = \pi\left(\frac{d}{2}\right)^2$.

SLIDE 3

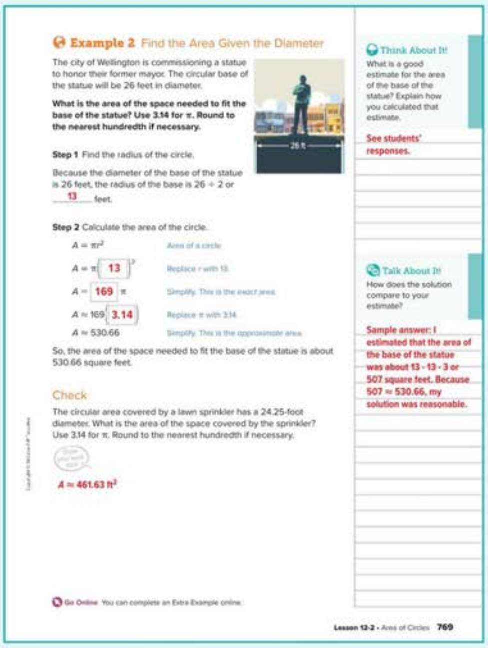
AL Why do you replace r with 13 in the formula, as opposed to 26? The radius is 13 feet, and r represents the radius. The diameter is 26 feet.

OL Explain how to estimate the solution. Sample answer: π is close to 3. Find the product of 3 and the square of 13, or 169, which is 507. The area of the circle is close to 507 square feet.

BL A classmate claims the solution, 530.66 square feet, is not close enough to the estimate of 507 square feet in order for the solution to be reasonable. How can you convince your classmate the solution is close to the estimate? Sample answer: Since π was rounded down, the estimate will be less than the area. The difference between π (169) and 3(169) is 0.14(169), which is 23.66, which is the difference between 530.66 and 507.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Find the Area Given the Diameter

The city of Wellington is commissioning a statue to honor their former mayor. The circular base of the statue will be 26 feet in diameter.

What is the area of the space needed to fit the base of the statue? Use 3.14 for π. Round to the nearest hundredth if necessary.

26 ft

Step 1 Find the radius of the circle.

Because the diameter of the base of the statue is 26 feet, the radius of the base is 26 ÷ 2 or 13 feet.

Step 2 Calculate the area of the circle.

$A = \pi r^2$ — Area of a circle

$A = \pi(13)^2$ — Replace r with 13.

$A = 169\pi$ — Simplify. This is the exact area.

$A \approx 169(3.14)$ — Replace π with 3.14.

$A \approx 530.66$ — Simplify. This is the approximate area.

So, the area of the space needed to fit the base of the statue is about 530.66 square feet.

Check

The circular area covered by a lawn sprinkler has a 24.25-foot diameter. What is the area of the space covered by the sprinkler? Use 3.14 for π. Round to the nearest hundredth if necessary.

$A \approx 461.63$ ft^2

Think About It! What is a good estimate for the area of the base of the statue? Explain how you calculated that estimate.

See students' responses.

Talk About It! How does the solution compare to your estimate?

Sample answer: I estimated that the area of the base of the statue was about 13 · 13 · 3 or 507 square feet. Because 507 ≈ 530.66, my solution was reasonable.

Go Online You can complete an Extra Example online.

Lesson 12-2 • Area of Circles 769

Interactive Presentation

Example 2, Find the Area Given the Diameter, Slide 1 of 5

TYPE

On Slide 2, students determine the radius of the base of the statue.

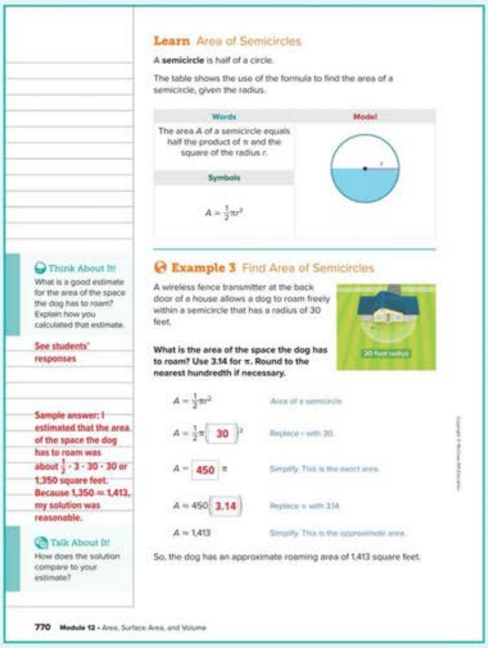
TYPE

On Slide 3, students determine the approximate area of the space needed to fit the base of the statue.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION



Learn Area of Semicircles

A **semicircle** is half of a circle.

The table shows the use of the formula to find the area of a semicircle, given the radius.

Words	Model
The area A of a semicircle equals half the product of π and the square of the radius r.	
Symbols	
$A = \frac{1}{2}\pi r^2$	

Think About It! What is a good estimate for the area of the space the dog has to roam? Explain how you calculated that estimate.

See students' responses

Example 3 Find Area of Semicircles

A wireless fence transmitter at the back door of a house allows a dog to roam freely within a semicircle that has a radius of 30 feet.

What is the area of the space the dog has to roam? Use 3.14 for π. Round to the nearest hundredth if necessary.

$A = \frac{1}{2}\pi r^2$ Area of a semicircle

$A = \frac{1}{2}\pi(30)^2$ Replace r with 30.

$A = 450\pi$ Simplify. This is the exact area.

$A \approx 450(3.14)$ Replace π with 3.14

$A \approx 1{,}413$ Simplify. This is the approximate area.

So, the dog has an approximate roaming area of 1,413 square feet.

Sample answer: I estimated that the area of the space the dog has to roam was about $\frac{1}{2} \cdot 3 \cdot 30 \cdot 30$ or 1,350 square feet. Because $1{,}350 \approx 1{,}413$, my solution was reasonable.

Talk About It! How does the solution compare to your estimate?

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Interactive Presentation

Example 3, Find Area of Semicircles, Slide 2 of 4

FLASHCARDS

In the Learn, students use Flashcards to view multiple representations for the formula for the area of a semicircle.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Area of Semicircles

Objective

Students will understand how the area of a semicircle is related to the area of a circle, with the same radius, and how that relationship can be expressed in a formula.

Teaching Notes

Have students select the *Words*, *Symbols*, and *Model* flashcards to see the area formula for a semicircle expressed in these multiple representations. You may wish to ask students what the prefix *semi-* means, and how it can help them understand the meaning of the term *semicircle*. Ask students for other examples of words that have the same prefix. Possible examples can include semifinals, semifinalists, semiprofessional, semester, or semisolids.

Example 3 Find Area of Semicircles

Objective

Students will find the area of a semicircle.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the relationship between a circle and semicircle, in order to find the area of the semicircle.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the area in which the dog can roam? Sample answer: It is a semicircle.

AL Are you given the radius or the diameter? I am given the radius.

OL If you forgot the formula for the area of a semicircle, how can you solve the problem without it? Sample answer: Find the area of the whole circle. Then divide that area by 2.

OL Suppose the dog had the entire area of the circle to roam. What would be the area of the entire circle? Use 3.14 for π. Explain how you solved. 2,826 square feet; Sample answer: I multiplied the area of the semicircle by 2.

BL Suppose the family restricted the radius to be 15 feet, as opposed to 30 feet. Explain what effect this restriction has on the area in which the dog will have to roam. Sample answer: The radius is divided by 2. Since the radius is squared in the area formula, this means the area will be divided by the square of 2, or 4.

Go Online

- Find additional teaching notes, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Use Circumference to Find Area

Objective

Students will understand how they can apply the formula for the circumference of a circle to find the circle's area, given the circumference.

Teaching Notes

Prior to having students watch the animation, you may wish to have students discuss how they might be able to find the area of a circle given its circumference. Draw a circle on the board, or have students draw a circle using notebook paper. Label the circle's circumference as approximately 18.84 meters. Encourage students to discuss with a partner some possible strategies they can use to find the area of the circle. Have them work together to determine the approximate area of the circle. Then have them watch the animation to see if their strategy was used. If not, ask them how the strategies compare and whether or not their strategy was a mathematically correct one.

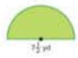
DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of how area is related to circumference, have students work with a partner to discover a formula for the area of a circle based on the circumference. They should be able to explain how they arrived at their formula. $A = \pi\left(\frac{C}{2\pi}\right)^2$ or $A = \frac{C^2}{4\pi^2}$

Check

What is the area of the semicircle? Use 3.14 for π. Write your estimate as a decimal rounded to the nearest hundredth.

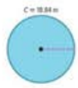


$A \approx 22.08$ or 22.07 yd^2

Go Online You can complete an Extra Example online.

Learn Use Circumference to Find Area

When you know the circumference of a circle, you can work backward to find the area of the circle.



Go Online Watch the animation to learn how to find the area of the circle, given its circumference.

Step 1 Find the radius of the circle.

$C = 2\pi r$	Circumference of a circle
$18.84 \approx 2 \cdot 3.14 \cdot r$	Replace C with 18.84 and π with 3.14.
$18.84 \approx 6.28r$	Simplify.
$\frac{18.84}{6.28} \approx \frac{6.28r}{6.28}$	Division Property of Equality
$3 \approx r$	Simplify.

Step 2 Find the area of the circle.

$A = \pi r^2$	Area of a circle
$A \approx 3.14 \cdot 3^2$	Replace π with 3.14 and r with 3.
$A \approx 28.26$	Simplify.

The area of the circle is about 28.26 m^2.

Lesson 12-2 • Area of Circles 771

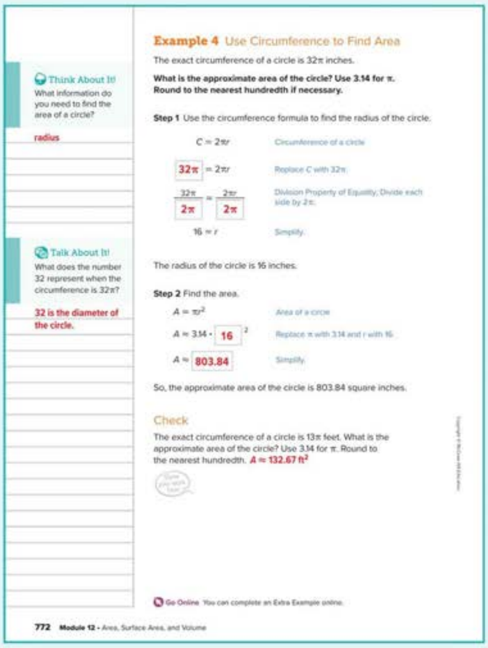
Interactive Presentation

Learn, Use Circumference to Find Area

WATCH

Students watch an animation that explains how to find the area of a circle given its circumference.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION



Example 4 Use Circumference to Find Area

The exact circumference of a circle is 32π inches.

What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

Think About It! What information do you need to find the area of a circle? radius

Step 1 Use the circumference formula to find the radius of the circle.

$C = 2\pi r$ Circumference of a circle

$32\pi = 2\pi r$ Replace C with 32π.

$\frac{32\pi}{2\pi} = \frac{2\pi r}{2\pi}$ Division Property of Equality; Divide each side by 2π.

$16 = r$ Simplify.

The radius of the circle is 16 inches.

Talk About It! What does the number 32 represent when the circumference is 32π? 32 is the diameter of the circle.

Step 2 Find the area.

$A = \pi r^2$ Area of a circle

$A \approx 3.14 \cdot 16^2$ Replace π with 3.14 and r with 16.

$A \approx 803.84$ Simplify.

So, the approximate area of the circle is 803.84 square inches.

Check

The exact circumference of a circle is 13π feet. What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth. $A \approx 132.67\ ft^2$

Go Online You can complete an Extra Example online.

772 Module 12 • Area, Surface Area, and Volume

Example 4 Use Circumference to Find Area

Objective

Students will find the area of a circle given the circumference of the circle.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to analyze the structure of the formulas for the area and circumference of a circle, noting that if they first find the radius (given the circumference), they can then find the area.

Questions for Mathematical Discourse

SLIDE 2

AL What information are you given? What do you need to find? I am given the exact circumference. I need to find the approximate area.

OL In order to find the area, what do you need to do first? Why? Sample answer: I need to find the radius. The input for the area formula is the radius, not the circumference.

OL How can you use the exact circumference to find the radius? Sample answer: Write the circumference formula, substitute 32π for C. Then solve the equation for r.

BL How can you find the radius another way? Sample answer: Because the circumference is 32π , this means the diameter of the circle is 32 inches. Divide the diameter by 2 to obtain a radius of 16 inches.

SLIDE 3

AL Now that you know the radius, what do you need to do now? Use the radius to find the area of the circle.

OL Estimate the area of the circle. Sample answer: π is close to 3. Find the product of 3 and the square of 16, or 256, which is 768. The area is close to 768 square inches.

OL What is the exact area? 256π

BL A classmate claims that since 256 divided by 32 is 8, then the area of any circle with a given circumference of $n\pi$ units, will always be $8n\pi$ square units. Is this reasoning correct? Explain. no; Sample answer: If the circumference is $n\pi$, then the diameter is n units. This means the radius is $\frac{n}{2}$ units, and the area is actually $\pi\left(\frac{n^2}{4}\right)$ square units. It just happens that when $n = 32$, $\frac{n^2}{4}$ is 256. If n was 20, then $\frac{n^2}{4}$ is 100, which is not 8 times as great as 20.

Interactive Presentation

Example 4, Use Circumference to Find Area, Slide 2 of 5

TYPE On Slide 2, students determine the radius of the circle.

TYPE On Slide 3, students determine the approximate area of the circle.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Crafting

Objective

Students will come up with their own strategy to solve an application problem involving a square scrapbook page.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question, encourage them make sense of the quantities and to explain their reasoning within the context of the problem.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them to make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What is the relationship between the side length of the square and the diameter of the circle?
- What is the formula for finding the area of a circle?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Crafting

A square scrapbook page has an area of 144 square inches. Jillian wants to cut the largest circle possible from the page to create a layered background for a new page. What is the approximate area of the paper circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

113.04 square inches; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How did you know that the radius of the circle is one-half the side length of the square?

Sample answer: The circle touches the square at the midpoint of each side of the square. Because the length of the radius of a circle is the distance from the center to the edge, the radius is also one-half of the side length of the square.

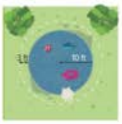
Lesson 12-2 • Area of Circles 773

Interactive Presentation

Apply, Crafting

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The Blackwells have a circular pool with a radius of 10 feet. They want to install a 3-foot wide sidewalk around the pool.

What will be the area of the sidewalk? Use 3.14 for π. Round to the nearest hundredth if necessary. $A \approx 216.66$ ft²

Go Online You can complete an Extra Example online.

Pause and Reflect

Describe when it is beneficial to use 3.14 instead of π, and when it is beneficial to use π instead of 3.14 when calculating the circumference or area of a circle.

See students' observations.

774 Module 12 • Area, Surface Area, and Volume

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can we measure objects to solve problems?

In this lesson, students learned how to find the area of circles. Encourage them to brainstorm with a partner a real-world situation in which they might need to find the area of a circle. Have them compare and contrast when they would need to find the area versus the circumference of a circle. Some examples could include the number of square feet covered by a circular floor rug (area) and the length of binding needed to bind the outside edge of a circular floor rug (circumference).

Exit Ticket

Refer to the Exit Ticket slide. Why is the amount of area a horse can roam, while attached to the 30-foot-long longe line, about 2,826 square feet? Sample answer: The radius of the circle is 30 feet. The area of a circle with radius 30 feet is about 3.14(30)(30), or about 2,286 square feet.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 3, 5, 9, 11–14
- ALEKS Circumference and Area of Circles

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–8, 10, 12
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

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1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

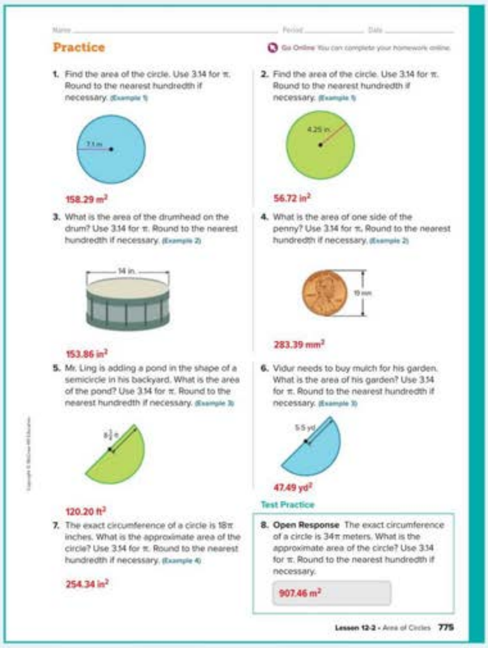
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the area of a circle given the radius	1, 2
2	find the area of a circle given the diameter	3, 4
2	find the area of a semicircle	5, 6
1	find the area of a circle given the circumference of the circle	7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving areas of circles	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may mistake the formula for the area of a circle with the formula for the circumference of a circle using the radius. Explain to students that πr^2 is not equivalent to $2\pi r$.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Find the area of the circle. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)

158.29 m²

2. Find the area of the circle. Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 1)

56.72 in²

3. What is the area of the drumhead on the drum? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 2)

153.86 in²

4. What is the area of one side of the penny? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 2)

283.39 mm²

5. Mr. Ling is adding a pond in the shape of a semicircle in his backyard. What is the area of the pond? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 3)

120.20 ft²

6. Vidur needs to buy mulch for his garden. What is the area of his garden? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 3)

47.49 yd²

7. The exact circumference of a circle is 18π inches. What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth if necessary. (Example 4)

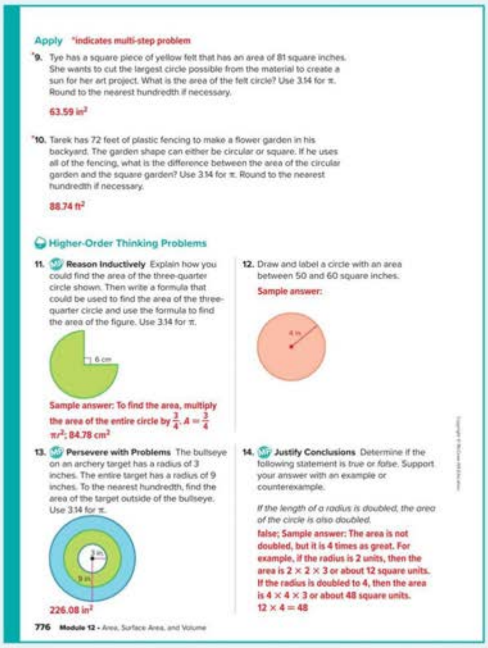
254.34 in²

Test Practice

8. **Open Response** The exact circumference of a circle is 34π meters. What is the approximate area of the circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

907.46 m²

Lesson 12-2 • Area of Circles 775



Apply *indicates multi-step problem

*9. Tye has a square piece of yellow felt that has an area of 81 square inches. She wants to cut the largest circle possible from the material to create a sun for her art project. What is the area of the felt circle? Use 3.14 for π. Round to the nearest hundredth if necessary.

63.59 in²

*10. Tarek has 72 feet of plastic fencing to make a flower garden in his backyard. The garden shape can either be circular or square. If he uses all of the fencing, what is the difference between the area of the circular garden and the square garden? Use 3.14 for π. Round to the nearest hundredth if necessary.

88.74 ft²

Higher-Order Thinking Problems

11. **Reason Inductively** Explain how you could find the area of the three-quarter circle shown. Then write a formula that could be used to find the area of the three-quarter circle and use the formula to find the area of the figure. Use 3.14 for π.

Sample answer: To find the area, multiply the area of the entire circle by $\frac{3}{4}$. $A = \frac{3}{4}\pi r^2$; 84.78 cm²

12. Draw and label a circle with an area between 50 and 60 square inches.

Sample answer:

13. **Persevere with Problems** The bullseye on an archery target has a radius of 3 inches. The entire target has a radius of 9 inches. To the nearest hundredth, find the area of the target outside of the bullseye. Use 3.14 for π.

226.08 in²

14. **Justify Conclusions** Determine if the following statement is true or false. Support your answer with an example or counterexample.

If the length of a radius is doubled, the area of the circle is also doubled.

false; Sample answer: The area is not doubled, but it is 4 times as great. For example, if the radius is 2 units, then the area is 2 × 2 × 3 or about 12 square units. If the radius is doubled to 4, then the area is 4 × 4 × 3 or about 48 square units. 12 × 4 = 48

776 Module 12 • Area, Surface Area, and Volume

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 11, students explain how to find the area of a three-quarter circle and write a formula.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 13, find the area of part of a circle.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students determine if a statement is true or false and support their answer with an example.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Make sense of the problem.

Use with Exercise 10 Have students work together to prepare a brief demonstration that illustrates how they solved this problem. For example, before they can find the difference in areas, they must first find the area of a circular garden and the area of a square garden. Have each pair or group of students present their response to the class.

Create your own higher-order thinking problem.

Use with Exercises 11–14 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Lesson 12-3

Area of Composite Figures

7.G.B.6

LESSON GOAL

Students will find the area of composite figures.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Area of Composite Figures
Example 1: Area of Composite Figures
Learn: Area of Shaded Regions
Example 2: Area of Shaded Regions
Apply: Art

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 72 of the *Language Development Handbook* to help your students build mathematical language related to the area of composite figures.

ELL You can use the tips and suggestions on page T72 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min 0.5 day
45 min 1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.B** by finding areas of composite figures.
Standards for Mathematical Content: **7 .G.B.6,** *Also addresses 7.NS.A.3, 7.EE.B.4, 7.EE.B.4.A, 7.G.B.4*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the area of circles.
7.G.B.4

Now
Students find the area of composite figures.
7.G.B.6

Next
Students will find the volume of prisms and pyramids.
7.G.B.6

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of finding the area of triangles and quadrilaterals to gain *fluency* in finding area of composite figures. They will also *apply* their fluency to real-world problems involving composite figures.

Mathematical Background

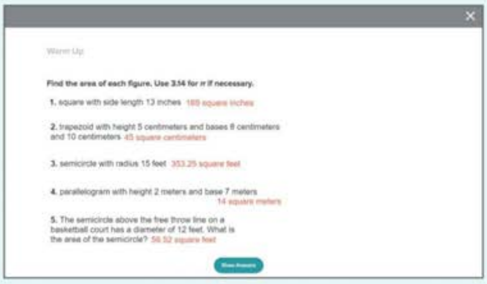
Composite figures are figures that are composed of two or more figures such as parallelograms, triangles, trapezoids, and circles. The area of a composite figure is the sum of the areas of the figures that make up the composite figure. To find the area of a composite figure, use the following steps.

Step 1 Decompose the figure into shapes with areas you know how to find.

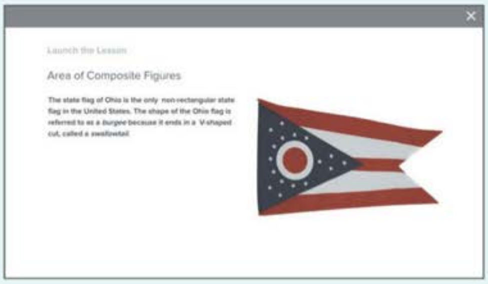
Step 2 Use a formula to find the area of each shape.

Step 3 Find the sum of the areas.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding areas of quadrilaterals and semicircles (Exercises 1–5)

Answers

1. 169 square inches

2. 45 square centimeters

3. 353.25 square feet

4. 14 square meters

5. 56.52 square feet

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the state flag of Ohio as an example of a composite figure.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *composite* comes from the French term *componere* which means *to put together*. Make a prediction as to what you think a *composite figure* might be. Sample answer: A composite figure might be a figure that is put together, or composed of other figures.

Learn Area of Composite Figures

Objective

Students will understand how to decompose a composite figure into known shapes in order to find the area.

Teaching the Mathematical Practices

6 Attend to Precision As students complete the drag and drop activity, encourage them to pay attention to characteristics of each figure when identifying its name and area formula.

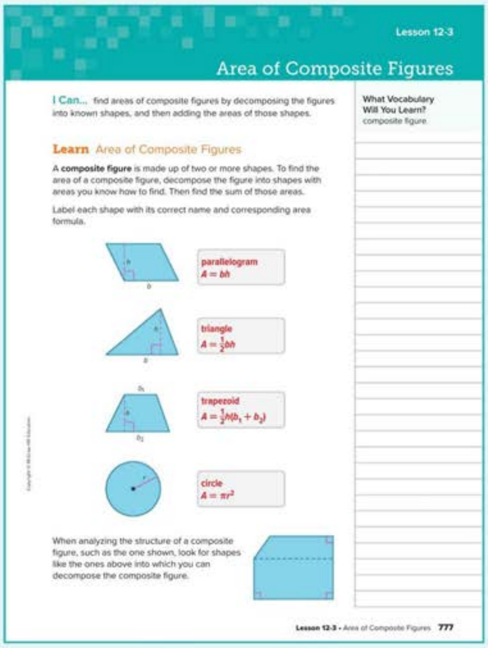
Teaching Notes

The drag and drop activity will allow students to review and practice the names of shapes they have previously studied, in this grade or prior grades, and their associated area formulas. You may wish to ask students to draw their own composite figures on a separate piece of paper. Then have students explain to a classmate why their figure is a composite figure. Have volunteers share their drawings with the class.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling to determine the shapes that compose a composite figure, have them work with a partner to draw several composite figures, each using two or more of the shapes presented in the Learn. Then have them trade drawings with another pair of students. Each pair should identify the figures into which each composite figure can be decomposed. Have them compare answers with the original pair of students, and discuss and resolve any differences.


Lesson 12-3

Area of Composite Figures

I Can... find areas of composite figures by decomposing the figures into known shapes, and then adding the areas of those shapes.

What Vocabulary Will You Learn?
composite figure

Learn Area of Composite Figures

A **composite figure** is made up of two or more shapes. To find the area of a composite figure, decompose the figure into shapes with areas you know how to find. Then find the sum of those areas.

Label each shape with its correct name and corresponding area formula.

parallelogram $A = bh$

triangle $A = \frac{1}{2}bh$

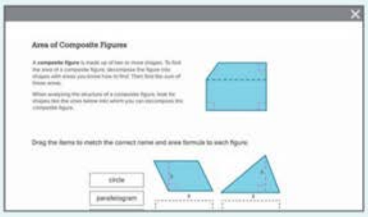
trapezoid $A = \frac{1}{2}h(b_1 + b_2)$

circle $A = \pi r^2$

When analyzing the structure of a composite figure, such as the one shown, look for shapes like the ones above into which you can decompose the composite figure.

Lesson 12-3 • Area of Composite Figures 777

Interactive Presentation

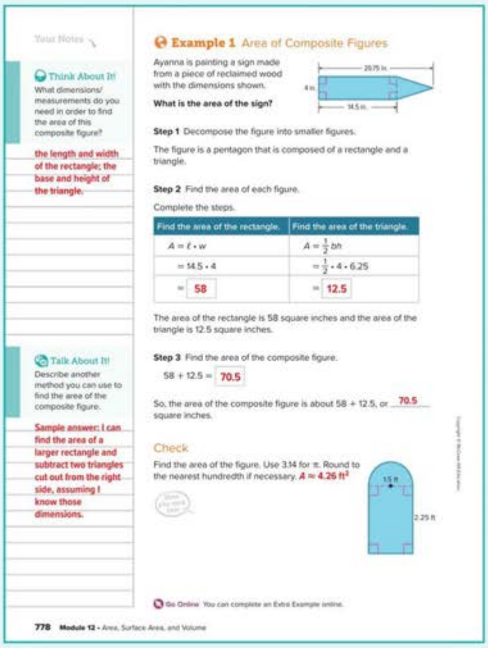


Learn, Area of Composite Figures

DRAG & DROP

Students drag items to match the correct name and area formula to each figure.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**


Your Notes

Think About It! What dimensions/measurements do you need in order to find the area of this composite figure?

the length and width of the rectangle; the base and height of the triangle.

Talk About It! Describe another method you can use to find the area of the composite figure.

Sample answer: I can find the area of a larger rectangle and subtract two triangles cut out from the right side, assuming I know those dimensions.

Example 1 Area of Composite Figures

Ayanna is painting a sign made from a piece of reclaimed wood with the dimensions shown.

What is the area of the sign?

Step 1 Decompose the figure into smaller figures.

The figure is a pentagon that is composed of a rectangle and a triangle.

Step 2 Find the area of each figure.

Complete the steps.

Find the area of the rectangle.	Find the area of the triangle.
$A = \ell \cdot w$	$A = \frac{1}{2}bh$
$= 14.5 \cdot 4$	$= \frac{1}{2} \cdot 4 \cdot 6.25$
$= 58$	$= 12.5$

The area of the rectangle is 58 square inches and the area of the triangle is 12.5 square inches.

Step 3 Find the area of the composite figure.

$58 + 12.5 = 70.5$

So, the area of the composite figure is about 58 + 12.5, or 70.5 square inches.

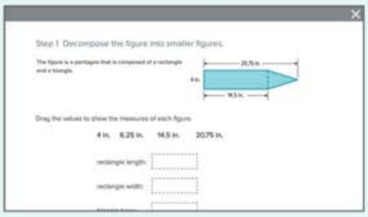
Check

Find the area of the figure. Use 3.14 for π. Round to the nearest hundredth if necessary. $A \approx 4.26$ ft²

Go Online You can complete an Extra Example online.

778 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 1, Area of Composite Figures, Slide 2 of 6

DRAG & DROP

On Slide 2, students drag values to identify dimensions of known figures.

TYPE

On Slide 4, students determine the area of the composite figure.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Area of Composite Figures

Objective

Students will find the area of composite figures.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to study the structure of the composite figure in order to decompose it into smaller figures.

Questions for Mathematical Discourse

SLIDE 2

AL Study the figure. Explain how it can be composed of a rectangle and a triangle. Sample answer: If a vertical line segment is drawn where the dotted line segment appears, the figure to the left of this line segment is the rectangle. The figure to the right is the triangle.

OL Explain how to find the length of the base of the triangle. Sample answer: The base is parallel to the width of the rectangle, labeled 4 in., and is the same length. So, the base of the triangle is 4 in.

OL Explain how to calculate the height of the triangle. Sample answer: The total length of the pentagon is 20.75 in. Subtract 14.5 in. from this length. So, the height of the triangle is 6.25 in.

BL Is there another way to decompose the figure? Explain. yes; Sample answer: I can draw a horizontal line segment cutting the pentagon directly in half. This will result in two congruent trapezoids.

SLIDE 3

AL Estimate the area of the rectangle. Sample answer: 14.5 is close to 15. Find the product of 15 and 4, which is 60. So, the area of the rectangle is close to 60 square inches.

OL Estimate the area of the triangle. Sample answer: 6.25 is close to 6. Find the product of 6 and 4, which is 24. Then divide that by 2. So, the area of the triangle is close to 12 square inches.

BL If the width of the rectangle was 5 inches, instead of 4 inches, would it affect only the area of the rectangle, or also the triangle? Explain. It would affect both figures; Sample answer: The width of the rectangle is also the base of the triangle.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Area of Shaded Regions

Objective

Students will understand how to find the area of shaded regions.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find the area of a shaded region.

Teaching Notes

SLIDE 1

You may wish to pause the animation right after the figure is shown and have students discuss with a partner what strategies they could use to find the area of the shaded region. They may use any strategy they wish; however, they should be prepared to explain their strategy and defend why it works. Then have them watch the animation to see if their strategy was used. If not, ask them how the strategies compare and whether or not their strategy was a mathematically correct one.

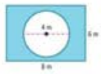
DIFFERENTIATE

Enrichment Activity 1

To further students' understanding of the area of shaded regions, have them work with a partner to draw their own figure that involves a shaded region. They should determine which measurements are necessary to include as labels, in order for another student to be able to find the area of the shaded region. Then have them trade figures with another pair of students. Each pair should discuss possible strategies that can be used to find the area of the shaded region, and then use an agreed-upon strategy to find the area. Have them compare solutions with the original pair of students, and discuss and resolve any differences.

Learn Area of Shaded Regions

Use area formulas to find the area of a shaded region. First find the area of the entire figure. Then subtract to find the area of the shaded region.



Go Online Watch the animation to see how to find the area of shaded regions.

The animation shows the following steps.

Step 1 Find the area of the entire figure.

$A = \ell w$ — Area of entire figure, a rectangle

$A = 8 \cdot 6$ — Replace ℓ with 8 and w with 6.

$A = 48$ — Simplify.

Step 2 Find the area of the unshaded region.

$A = \pi r^2$ — Area of unshaded region, a circle

$A \approx 3.14 \cdot 2^2$ — Replace π with 3.14 and r with 2.

$A \approx 12.6$ — Simplify.

Step 3 Subtract to find the area of the shaded region.

entire figure − unshaded region = shaded region

48 − 12.6 ≈ 35.4 m^2

The area of the shaded region is about 35.4 square meters.

Pause and Reflect

If the area of the unshaded region was a triangle, what dimensions of the triangle would keep the area of the shaded region about the same?

See students' observations.

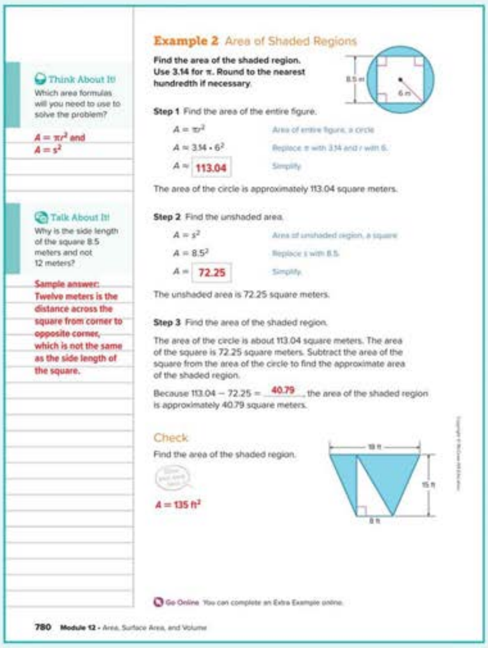
Lesson 12-3 • Area of Composite Figures 779

Interactive Presentation

Learn, Area of Shaded Regions

WATCH

Students watch an animation that demonstrates how to find the area of shaded regions.



Think About It! Which area formulas will you need to use to solve the problem?

$A = \pi r^2$ and $A = s^2$

Talk About It! Why is the side length of the square 8.5 meters and not 12 meters?

Sample answer: Twelve meters is the distance across the square from corner to opposite corner, which is not the same as the side length of the square.

Example 2 Area of Shaded Regions

Find the area of the shaded region. Use 3.14 for π. Round to the nearest hundredth if necessary.

Step 1 Find the area of the entire figure.

$A = \pi r^2$ — Area of entire figure, a circle

$A \approx 3.14 \cdot 6^2$ — Replace π with 3.14 and r with 6.

$A \approx$ 113.04 — Simplify.

The area of the circle is approximately 113.04 square meters.

Step 2 Find the unshaded area.

$A = s^2$ — Area of unshaded region, a square

$A = 8.5^2$ — Replace s with 8.5.

$A =$ 72.25 — Simplify.

The unshaded area is 72.25 square meters.

Step 3 Find the area of the shaded region.

The area of the circle is about 113.04 square meters. The area of the square is 72.25 square meters. Subtract the area of the square from the area of the circle to find the approximate area of the shaded region.

Because $113.04 - 72.25 =$ 40.79, the area of the shaded region is approximately 40.79 square meters.

Check

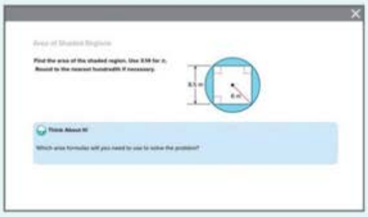
Find the area of the shaded region.

$A = 135\ ft^2$

Go Online You can complete an Extra Example online.

780 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 2, Area of Shaded Regions, Slide 1 of 6

TYPE On Slide 2, students determine the approximate area of the circle.

TYPE On Slide 4, students determine the approximate area of the shaded region.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Area of Shaded Regions

Objective

Students will find the area of shaded regions.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Encourage students to analyze the structure of the figure to determine which figure represents the entire figure, and which figure should have its area subtracted from the area of the entire figure, in order to find the area of the shaded region.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the two figures that make up this figure, and how they relate to one another. Sample answer: There is a larger circle. Inside the circle is a square. The area inside the circle that is not also inside the square is shaded.

OL Estimate the area of the entire circle. Sample answer: π is close to 3. The radius is 6. Find the product of 3 and the square of 6, or 36, which is 108. So, the area is close to 108 square meters.

BL What must be true about the four shaded regions? Explain. Sample answer: They are the same shape and size, so they have the same area.

SLIDE 3

AL Describe the unshaded area. Sample answer: The unshaded area is in the shape of a square with side length 8.5 meters.

OL What are some ways you can describe the relationship between the area of the circle and the unshaded area? Sample answers: The area of the circle is greater than the area of the unshaded area. The unshaded area plus the shaded area is equal to the area of the circle.

BL Describe how the radius of the circle and the diagonal of the square are related. Sample answer: The length of the diameter of the circle is equal to the length of the diagonal of the square. Because the diameter is twice the radius, the radius of the circle is equal to half the length of the diagonal of the square.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FUNDING | **3 APPLICATION**

Apply Art

Objective

Students will come up with their own strategy to solve an application problem involving the creation of a mosaic.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

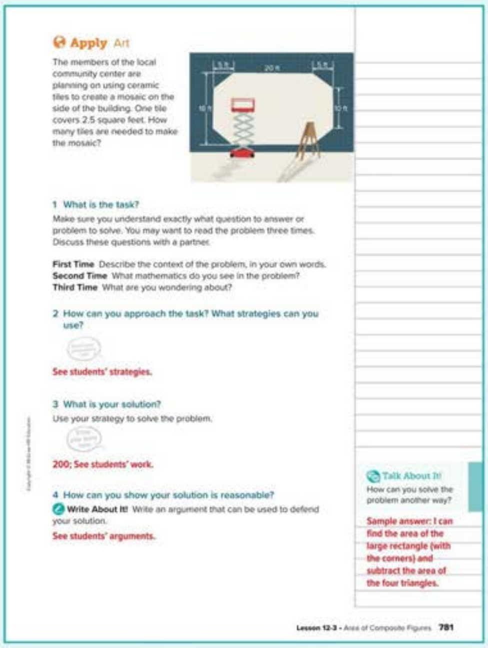
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How many shapes make up the area of the mosaic?
- What formulas will you need to find the area of the mosaic?
- How would you determine the number of tiles needed?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Art

The members of the local community center are planning on using ceramic tiles to create a mosaic on the side of the building. One tile covers 2.5 square feet. How many tiles are needed to make the mosaic?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

200; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

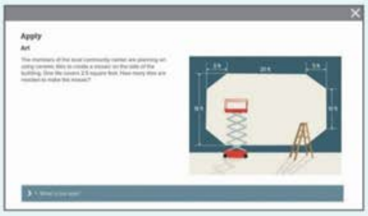
See students' arguments.

Talk About It! How can you solve the problem another way?

Sample answer: I can find the area of the large rectangle (with the corners) and subtract the area of the four triangles.

Lesson 12-3 • Area of Composite Figures 781

Interactive Presentation

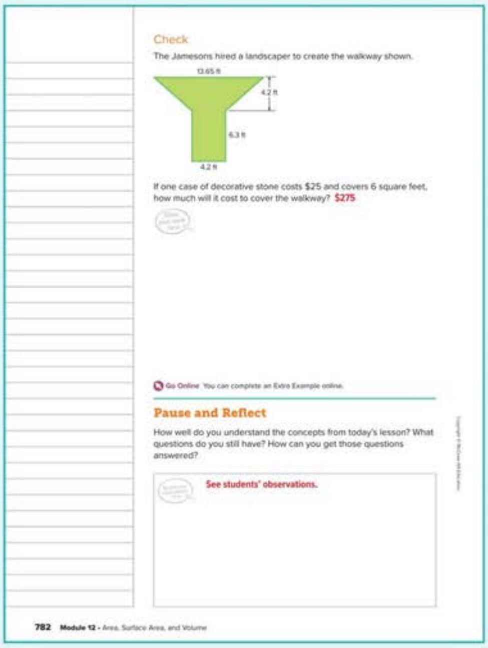


Apply, Art

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

3 REFLECT AND PRACTICE



Check

The Jamesons hired a landscaper to create the walkway shown.

If one case of decorative stone costs $25 and covers 6 square feet, how much will it cost to cover the walkway? $275

Go Online You can complete an Extra Example online.

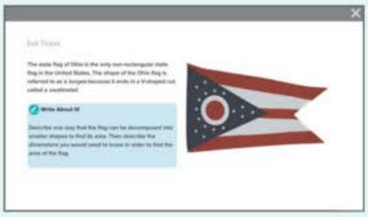
Pause and Reflect

How well do you understand the concepts from today's lesson? What questions do you still have? How can you get those questions answered?

See students' observations.

782 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Exit Ticket

7.G.B.6

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | **3 APPLICATION**

Essential Question Follow-Up

How can we measure objects to solve problems?

In this lesson, students learned how to find the area of composite figures. Encourage them to brainstorm with a partner a real-world situation in which they might need to find the area of a composite figure. For example, they might need to find the area of an L-shaped room to find how much carpet is needed to cover the floor.

Exit Ticket

Refer to the Exit Ticket slide. Describe one way that the flag can be decomposed into smaller shapes to find its area. Then describe the dimensions you would need to know in order to find the area of the flag. Sample answer: A vertical line can be drawn through the vertex where the V-shaped cut exists. This results in a trapezoid and two small triangles. I would need to know the lengths of the base and height of each triangle. I would also need to know the lengths of the two bases of the trapezoid and its height. Then I can find the area of the flag.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 9, 11–14
- ALEKS Area of Composite Figures

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–7, 10, 12
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ALEKS Area of Parallelograms, Triangles, and Trapezoids

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

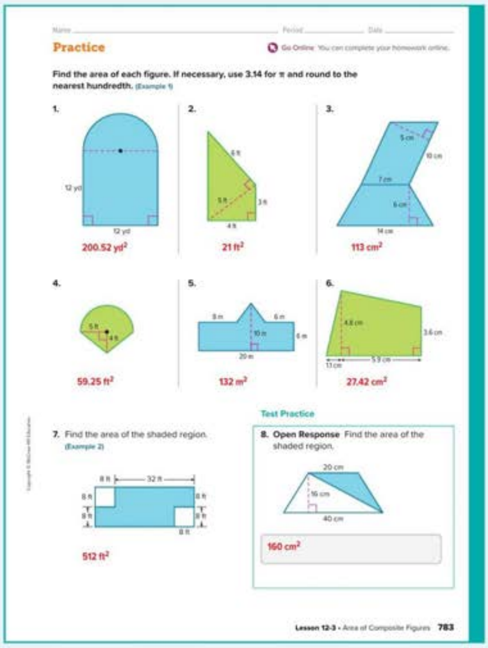
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the area of composite figures	1–6
1	find the area of shaded regions	7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving area of composite figures	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may assume that there is only one way to divide a composite figure into recognizable shapes. In Exercise 5, discuss with students the multiple ways the area of the composite figure could be found.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find the area of each figure. If necessary, use 3.14 for π and round to the nearest hundredth. (Example 1)

1. 200.52 yd^2

2. 21 ft^2

3. 113 cm^2

4. 59.25 ft^2

5. 132 m^2

6. 27.42 cm^2

Test Practice

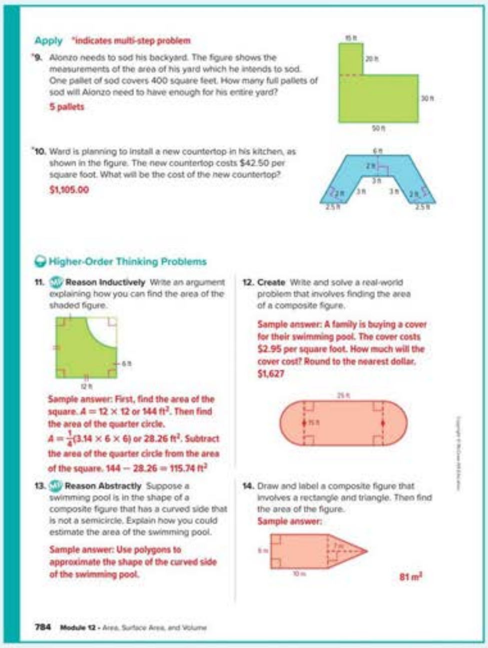
7. Find the area of the shaded region. (Example 2)

512 ft^2

8. **Open Response** Find the area of the shaded region.

160 cm^2

Lesson 12-3 • Area of Composite Figures 783



Apply *indicates multi-step problem

*9. Alonzo needs to sod his backyard. The figure shows the measurements of the area of his yard which he intends to sod. One pallet of sod covers 400 square feet. How many full pallets of sod will Alonzo need to have enough for his entire yard?

5 pallets

*10. Ward is planning to install a new countertop in his kitchen, as shown in the figure. The new countertop costs $42.50 per square foot. What will be the cost of the new countertop?

$1,105.00

Higher-Order Thinking Problems

11. **Reason Inductively** Write an argument explaining how you can find the area of the shaded figure.

Sample answer: First, find the area of the square. $A = 12 \times 12$ or $144\ ft^2$. Then find the area of the quarter circle. $A = \frac{1}{4}(3.14 \times 6 \times 6)$ or $28.26\ ft^2$. Subtract the area of the quarter circle from the area of the square. $144 - 28.26 = 115.74\ ft^2$

12. **Create** Write and solve a real-world problem that involves finding the area of a composite figure.

Sample answer: A family is buying a cover for their swimming pool. The cover costs $2.95 per square foot. How much will the cover cost? Round to the nearest dollar.
$1,627

13. **Reason Abstractly** Suppose a swimming pool is in the shape of a composite figure that has a curved side that is not a semicircle. Explain how you could estimate the area of the swimming pool.

Sample answer: Use polygons to approximate the shape of the curved side of the swimming pool.

14. Draw and label a composite figure that involves a rectangle and triangle. Then find the area of the figure.
Sample answer:

$81\ m^2$

784 Module 12 • Area, Surface Area, and Volume

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 11, students explain how to find the area of a figure involving a square and a quarter circle.

2 Reason Abstractly and Quantitatively In Exercise 13, students explain how they could estimate the area of a composite figure with a curved side that is not a semicircle.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Solve the problem another way.
Use with Exercise 10 Have students work in groups of 3–4. After completing Exercise 10, have one student from each group rotate to form a different group of students. Each student should share the solution method they previously used to solve the problem. Have students compare and contrast the different methods for solving the problem, and determine if each method is a viable solution. If the solutions were the same, have them brainstorm another way to solve the problem. Have one group present two viable solution methods to the class, and explain why each method is a correct method.

Be sure everyone understands.
Use with Exercises 11 and 13 Have students work in groups of 3–4 to solve the problem in Exercise 11. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 13.

Lesson 12-4

Volume of Prisms and Pyramids

7.G.B.6

LESSON GOAL

Students will find the volume of prisms and pyramids.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Volume of Prisms

Learn: Volume of Prisms
Example 1: Volume of Rectangular Prisms
Example 2: Volume of Triangular Prisms

Explore: Volume of Pyramids

Learn: Volume of Pyramids
Example 3: Volume of Pyramids
Learn: Use Volume to Find Missing Dimensions
Examples 4–5: Use Volume to Find Missing Dimensions
Apply: Packaging

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of the **Checks** to differentiate instruction.

Resources	AL	BL	OL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Volume of Cylinders		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 73 of the *Language Development Handbook* to help your students build mathematical language related to the volume of prisms and pyramids.

ELL You can use the tips and suggestions on page T73 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address additional cluster **7.G.B** by finding the volume of prisms and pyramids.
Standards for Mathematical Content: **7 .G.B.6,** *Also addresses 7.NS.A.3, 7.EE.B.4, 7.EE.B.4.A*
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the area of composite figures.
7.G.B.6

Now
Students find the volume of prisms and pyramids.
7.G.B.6

Next
Students will find the surface area of prisms and pyramids.
7.G.B.6

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of finding area to gain *fluency* in finding the volume of rectangular prisms, triangular prisms and pyramids. They will use their knowledge to gain an understanding of using the volume of a three-dimensional object to find a missing dimension. They will *apply* their fluency to solve real-world problems.

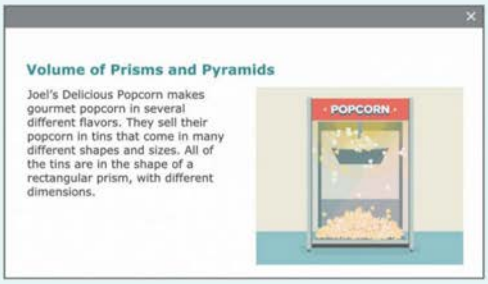
Mathematical Background

Volume is the measure of space occupied by a three-dimensional figure. It is measured in cubic units.

- Rectangular Prism: $V = Bh$ or $V = lwh$
- Triangular Prism: $V = Bh$, where $B = \frac{1}{2}bh$
- Pyramid: $V = \frac{1}{3}Bh$

Interactive Presentation

Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding areas of triangles and rectangles (Exercises 1–5)

Answers

1. 6,903 square millimeters
2. 3.15 square feet
3. 112 square inches
4. 6,283 square centimeters
5. 10.6 square feet

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the volume of various popcorn tins in the shape of rectangular prisms.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Perimeter is measured in units, sometimes called linear units. Area is measured in square units. Make a prediction as to what type of measurement is measured in *cubic units.* Volume is measured in cubic units.
- A *rectangular prism* has 6 faces that are all rectangles. What are some real-world examples of rectangular prisms? Sample answers: tissue boxes, shoe boxes, moving boxes, some buildings and skyscrapers
- Define *volume* in your own words. Sample answer: Volume is the amount of space in a solid figure.

Explore Volume of Prisms

Objective

Students will explore the relationship between the area of the base and the volume of a prism.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will construct three prisms. Throughout this activity, students will use the prisms to compare volumes and to investigate how the area of the base affects a prism's volume.

Inquiry Question

How does the base area of a prism affect the volume of a prism? Sample answer: If the heights of prisms are the same, the greater the base area, the greater the volume.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

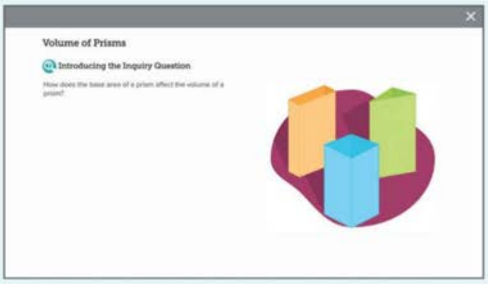
SLIDE 3

Mathematical Discourse

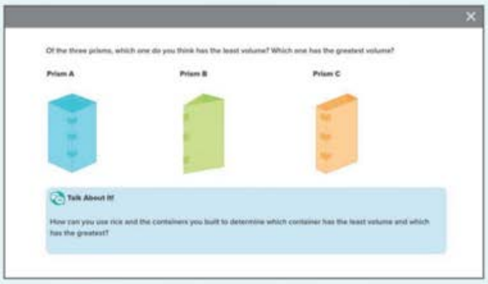
How can you use rice and the containers you built to determine which container has the least volume and which has the greatest? Sample answer: I can fill one prism with rice and then pour that rice into another container. If the rice overflows the container, the first container has a greater volume, if it doesn't fill the container, the first container has a lesser volume.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 7

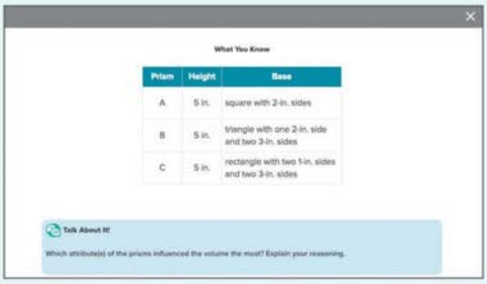


Explore, Slide 3 of 7

CLICK

On Slide 2, students select prisms to view instructions for creating them.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

a|

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Volume of Prisms *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to understand and explain the benefit of using the prisms to visualize the connection between the variation in bases and the volumes.

5 Use Appropriate Tools Strategically Students will use index cards, uncooked rice, and grid paper to explore how the area of the base of a prism affects its area.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Which attribute(s) of the prisms influenced the volume the most? Explain your reasoning. Sample answer: The shape and size of the base; Because the height of the prisms is the same, it is not a factor in determining the prism with the greatest volume. The size of the base is a factor because it is the only thing that changes.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Volume of Prisms

Objective

Students will understand how to find the volume of a prism.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure A s students discuss the *Talk About It!* question on Slide 2, encourage students to analyze the structure of the prism and the meanings of the variables in the formula $V = Bh$ in order to write a formula specifically for the volume of a rectangular prism.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

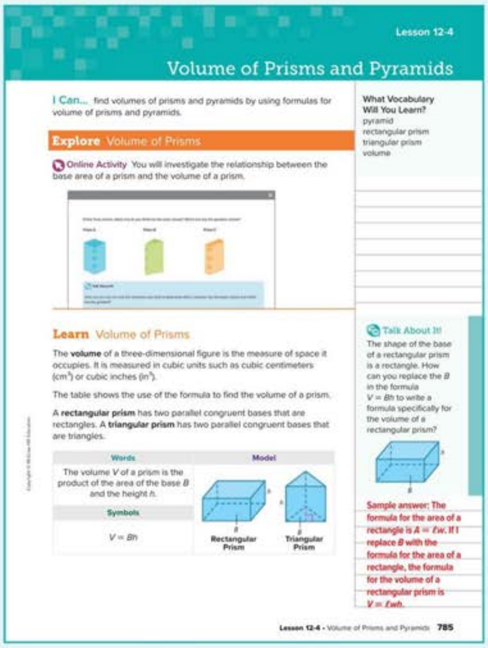
Mathematical Discourse

The shape of the base of a rectangular prism is a rectangle. How can you replace the B in the formula $V = Bh$ to write a formula specifically for the volume of a rectangular prism? Sample answer: The formula for the area of a rectangle is $A = \ell w$. If I replace B with the formula for the area of a rectangle, the formula for the volume of a rectangular prism is $V = \ell wh$.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are having difficulty understanding how to find the volume of prisms, have them work with a partner to draw at least three examples of prisms – a rectangular prism, a triangular prism, and a trapezoidal prism. They may draw more examples than these three if they choose. Have them use crayons, colored pencils, or markers to shade the two parallel and congruent bases (with the same color) of each figure. Then have them use a different color to indicate the height of each prism. Finally, have them label the dimensions that are needed to determine first the area of the base, and then the volume of the prism.



Lesson 12-4

Volume of Prisms and Pyramids

I Can... find volumes of prisms and pyramids by using formulas for volume of prisms and pyramids.

What Vocabulary Will You Learn?
pyramid
rectangular prism
triangular prism
volume

Explore Volume of Prisms

Online Activity You will investigate the relationship between the base area of a prism and the volume of a prism.

Learn Volume of Prisms

The **volume** of a three-dimensional figure is the measure of space it occupies. It is measured in cubic units such as cubic centimeters (cm^3) or cubic inches (in^3).

The table shows the use of the formula to find the volume of a prism.

A **rectangular prism** has two parallel congruent bases that are rectangles. A **triangular prism** has two parallel congruent bases that are triangles.

Words	Model
The volume V of a prism is the product of the area of the base B and the height h.	Rectangular Prism; Triangular Prism
Symbols	
$V = Bh$	

Talk About It! The shape of the base of a rectangular prism is a rectangle. How can you replace the B in the formula $V = Bh$ to write a formula specifically for the volume of a rectangular prism?

Sample answer: The formula for the area of a rectangle is $A = \ell w$. If I replace B with the formula for the area of a rectangle, the formula for the volume of a rectangular prism is $V = \ell wh$.

Lesson 12-4 • Volume of Prisms and Pyramids 785

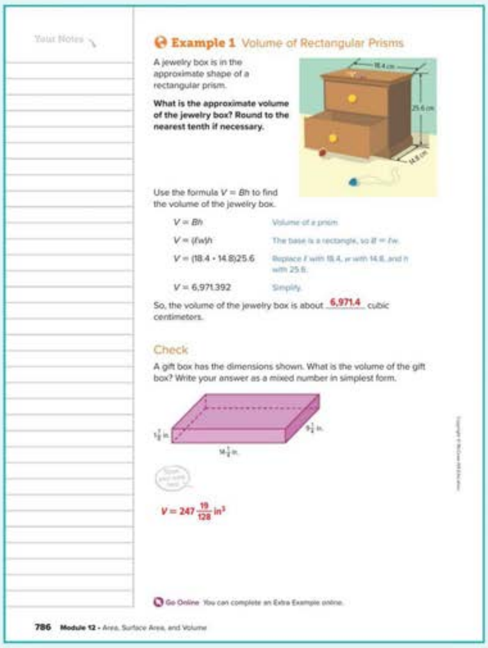
Interactive Presentation

Learn, Volume of Prisms, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view multiple representations of the formula for the volume of a prism.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION


Your Notes

Example 1 Volume of Rectangular Prisms

A jewelry box is in the approximate shape of a rectangular prism.

What is the approximate volume of the jewelry box? Round to the nearest tenth if necessary.

Use the formula $V = Bh$ to find the volume of the jewelry box.

$V = Bh$ — Volume of a prism

$V = (\ell w)h$ — The base is a rectangle, so $B = \ell w$.

$V = (18.4 \cdot 14.8)25.6$ — Replace ℓ with 18.4, w with 14.8, and h with 25.6.

$V = 6{,}971.392$ — Simplify.

So, the volume of the jewelry box is about 6,971.4 cubic centimeters.

Check

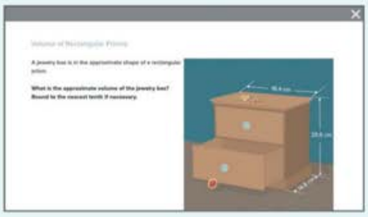
A gift box has the dimensions shown. What is the volume of the gift box? Write your answer as a mixed number in simplest form.

$V = 247\frac{19}{128}$ in^3

Go Online You can complete an Extra Example online.

786 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 1, Volume of Rectangular Prisms, Slide 1 of 2

TYPE

On Slide 1, students determine the approximate volume of the jewelry box.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Volume of Rectangular Prisms

Objective

Students will find the volume of a rectangular prism.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to begin by making an estimate, and to use their estimate to determine if their solution is reasonable. Students should attend to the meaning of each quantity in the volume formula, not just how to perform the calculations.

6 Attend to Precision Have students explain why the volume is an approximation, even though the measurements were not given as approximations. Students should be able to explain that since the jewelry box is not in the exact shape of a rectangular prism, the volume is an estimate.

Questions for Mathematical Discourse

SLIDE 1

AL What is the shape of the base of the jewelry box? rectangle

AL What are the dimensions of the base of the jewelry box? 14.8 cm and 18.4 cm

OL Explain how to estimate the volume. Sample answer: 18.4 is close to 20, 14.8 is close to 15, and 25.6 is close to 25. The product of 20(15)(25) = 7,500. So, the volume is close to 7,500 cubic centimeters.

OL Explain why the volume of the jewelry box is not exactly 6,971.4 cubic centimeters. Sample answer: The shape of the jewelry box is approximately a rectangular prism, but not exactly.

BL What is another way to think of the height of a rectangular prism? Sample answer: the number of layers of the base

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Volume of Triangular Prisms

Objective

Students will find the volume of a triangular prism.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to begin by making an estimate, and to use their estimate to determine if their solution is reasonable. As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of *B* and *b* representing two different quantities in the two volume formulas.

6 Attend to Precision Students should be able to explain why the volume is an approximation, because the problem asked them to round to the nearest hundredth.

7 Look For and Make Use of Structure Encourage students to analyze the structure of the prism to determine that it has two triangular bases.

Questions for Mathematical Discourse

SLIDE 2

AL Why is this not a rectangular prism? Not all of the faces are rectangles. The two parallel bases are triangles.

AL Why is it important to make an estimate? Sample answer: I can use the estimate to determine if my solution is reasonable.

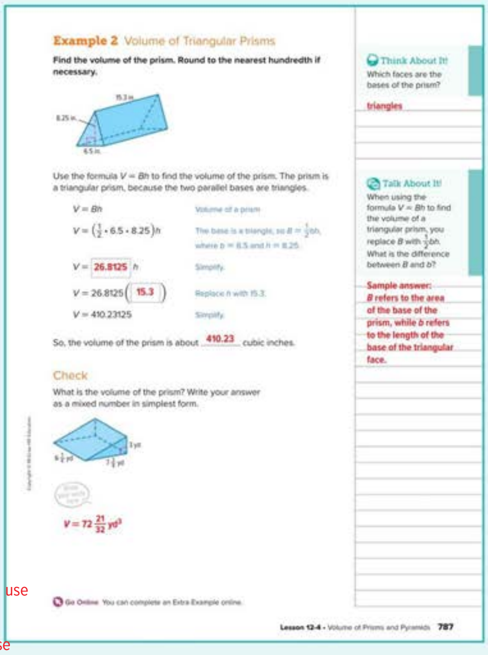
OL Explain how to estimate the volume. Sample answer: 8.25 is close to 8, 6.5 is close to 7, and 15.3 is close to 15. The volume is close to 0.5(8)(7)(15), or 420 cubic inches.

OL Why is the height of the prism not 8.25 inches? Sample answer: 8.25 inches represents the height of the triangular base. The height of the prism is the length that is perpendicular to the two parallel bases and connects them. That height is 15.3 inches.

BL Explain why the faces of a rectangular prism are all rectangles, but the faces of a triangular prism are not all triangles. Sample answer: A prism consists of two parallel bases that can be any polygon, such as rectangles and triangles. The other faces of a prism are always rectangles because they connect the two parallel bases.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Volume of Triangular Prisms

Find the volume of the prism. Round to the nearest hundredth if necessary.

Use the formula $V = Bh$ to find the volume of the prism. The prism is a triangular prism, because the two parallel bases are triangles.

$V = Bh$ — Volume of a prism

$V = \left(\frac{1}{2} \cdot 6.5 \cdot 8.25\right)h$ — The base is a triangle, so $B = \frac{1}{2}bh$, where $b = 6.5$ and $h = 8.25$.

$V = 26.8125\,h$ — Simplify.

$V = 26.8125(15.3)$ — Replace h with 15.3.

$V = 410.23125$ — Simplify.

So, the volume of the prism is about 410.23 cubic inches.

Check

What is the volume of the prism? Write your answer as a mixed number in simplest form.

$V = 72\frac{21}{32}\ yd^3$

Go Online You can complete an Extra Example online.

Think About It! Which faces are the bases of the prism?

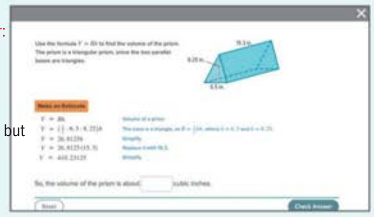
triangles

Talk About It! When using the formula $V = Bh$ to find the volume of a triangular prism, you replace B with $\frac{1}{2}bh$. What is the difference between B and b?

Sample answer: B refers to the area of the base of the prism, while b refers to the length of the base of the triangular face.

Lesson 12-4 • Volume of Prisms and Pyramids 787

Interactive Presentation



Example 2, Volume of Triangular Prisms, Slide 2 of 4

TYPE

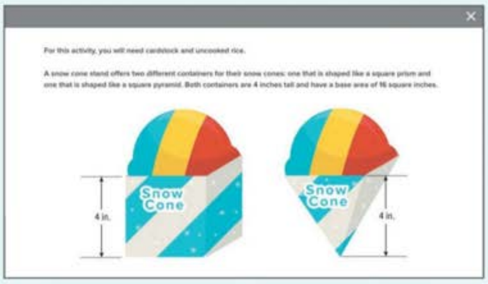
On Slide 2, students determine the volume of the prism.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Interactive Presentation

Explore, Slide 1 of 5



Explore, Slide 2 of 5

TYPE

On Slide 3, students type to indicate the number of times they poured the rice from the pyramid to completely fill the prism.

Explore Volume of Pyramids

Objective

Students will explore the relationship between the volume of a prism and the volume of a pyramid that have the same base area and height.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use nets to build prisms and pyramids that have the same heights and bases. Throughout this activity, students will use the solids to investigate how the volumes of prisms and pyramids are related.

Inquiry Question

What is the relationship between the volume of a prism and the volume of a pyramid with the same base area and height? Sample answer: The volume of the pyramid is $\frac{1}{3}$ the volume of the prism.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Suppose you were to use the pyramid-shaped container to fill the prism-shaped container with rice. How many times would you need to fill the pyramid with rice to completely fill the prism? Sample answer: about 3 times

(continued on next page)

Explore Volume of Pyramids *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to compare prisms and pyramids that have the same base area and height.

5 Use Appropriate Tools Strategically Explain to students the benefit of using cardstock figures to visualize how the volumes of these figures are related.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

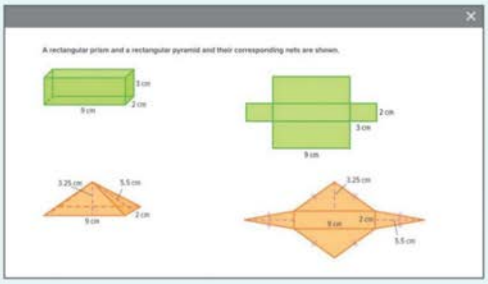
Talk About It!

SLIDE 4

Mathematical Discourse

Are the results what you expected? Explain. yes; Sample answer: Since each pyramid has the same base as its corresponding prism and the same height, I thought that it would take the same number of times to fill the prism.

Interactive Presentation



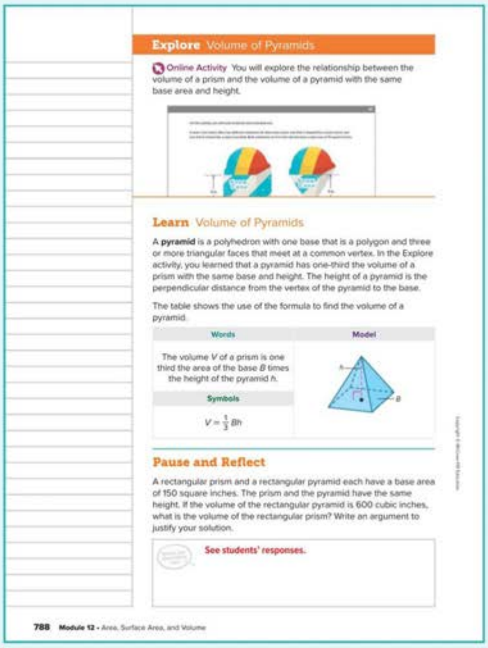
Explore, Slide 4 of 5

TYPE

On Slide 4, students type to indicate the number of times they had to pour rice from the pyramid to completely fill the prism.

TYPE

On Slide 5, students respond to the Inquiry Question and view a sample answer.



Explore Volume of Pyramids

Online Activity You will explore the relationship between the volume of a prism and the volume of a pyramid with the same base area and height.

Learn Volume of Pyramids

A **pyramid** is a polyhedron with one base that is a polygon and three or more triangular faces that meet at a common vertex. In the Explore activity, you learned that a pyramid has one-third the volume of a prism with the same base and height. The height of a pyramid is the perpendicular distance from the vertex of the pyramid to the base.

The table shows the use of the formula to find the volume of a pyramid.

Words	Model
The volume V of a prism is one third the area of the base B times the height of the pyramid h.	
Symbols	
$V = \frac{1}{3}Bh$	

Pause and Reflect

A rectangular prism and a rectangular pyramid each have a base area of 150 square inches. The prism and the pyramid have the same height. If the volume of the rectangular pyramid is 600 cubic inches, what is the volume of the rectangular prism? Write an argument to justify your solution.

See students' responses.

788 Module 12 • Area, Surface Area, and Volume

Interactive Presentation

Learn, Volume of Pyramids

FLASHCARDS

Students use Flashcards to view multiple representations of the formula for the volume of a pyramid.

Learn Volume of Pyramids

Objective

Students will understand how to find the volume of a pyramid.

Teaching Notes

Have students select the *Words*, *Symbols*, and *Model* flashcards to view how the formula to find the volume of a pyramid can be expressed in these multiple representations. You may wish to draw a prism and a pyramid (with the same base and height) and have students compare and contrast the figures. Students should note that pyramids have one base, while prisms have two congruent and parallel bases. The remaining faces of pyramids are triangular, while those of prisms are rectangular. Some students may confuse the height of a pyramid with one of the slant heights. Remind them that the height of a figure is always perpendicular to the figure's base.

You may wish to have students draw several examples of pyramids with different bases, such as rectangular pyramids, square pyramids, triangular pyramids, trapezoidal pyramids, and so on. Point out that pyramids are named by the shape of their base. Have students name real-world objects that are pyramids. Possible responses can include the Great Pyramid in Egypt, paperweights shaped as pyramids, and some buildings that are shaped as pyramids, such as the Louvre art museum in Paris, France.

Example 3 Volume of Pyramids

Objective

Students will find the volume of a pyramid.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to begin by making an estimate, and to use their estimate to determine if their solution is reasonable.

6 Attend to Precision Students should be able to perform the volume calculations with the given mixed number measurements.

7 Look For and Make Use of Structure Encourage students to analyze the structure of the pyramid to determine the dimensions of the rectangular base and the height of the pyramid.

Questions for Mathematical Discourse

SLIDE 1

AL What is the shape of the base, and what are its dimensions? The base is a rectangle with length $3\frac{1}{5}$ in. and width $1\frac{1}{2}$ in.

AL What are the shapes of the lateral faces? What does the term *lateral* mean? triangles; Lateral means side.

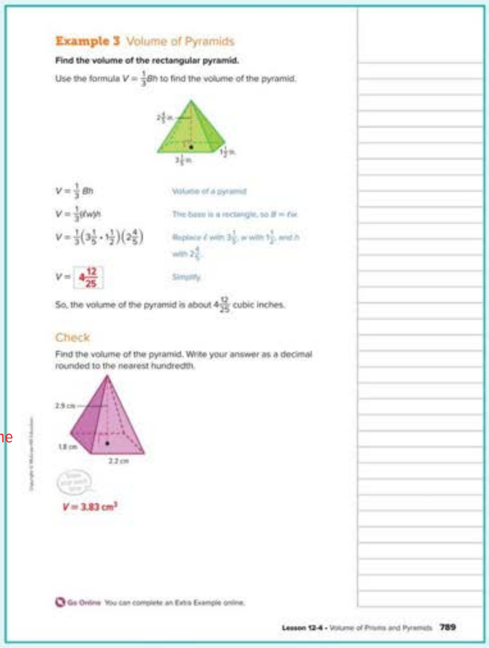
OL Explain how to estimate the volume. Sample answer: Round each fraction dimension to the nearest whole number. The area of the base is close to 3(2), or 6 square inches. The height of the pyramid is close to 3. Find 6(3), or 18, and multiply by $\frac{1}{3}$ since it is a pyramid, not a prism. The volume is close to 6 cubic inches.

OL Why do you multiply $\frac{1}{3}$? The volume of a pyramid is one-third the volume of a prism with the same base area and height.

BL If you were not told that this pyramid is a rectangular pyramid, can you assume that from the figure shown? Explain. no; Sample answer: There is no right angle symbol in the corner of the base.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Volume of Pyramids

Find the volume of the rectangular pyramid.

Use the formula $V = \frac{1}{3}Bh$ to find the volume of the pyramid.

$V = \frac{1}{3}Bh$ — Volume of a pyramid

$V = \frac{1}{3}(\ell w)h$ — The base is a rectangle, so $B = \ell w$.

$V = \frac{1}{3}\left(3\frac{1}{5} \cdot 1\frac{1}{2}\right)\left(2\frac{4}{5}\right)$ — Replace ℓ with $3\frac{1}{5}$, w with $1\frac{1}{2}$, and h with $2\frac{4}{5}$.

$V = 4\frac{12}{25}$ — Simplify.

So, the volume of the pyramid is about $4\frac{12}{25}$ cubic inches.

Check

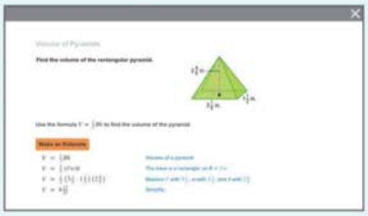
Find the volume of the pyramid. Write your answer as a decimal rounded to the nearest hundredth.

$V = 3.83\text{ cm}^3$

Go Online You can complete an Extra Example online.

Lesson 12-4 • Volume of Prisms and Pyramids 789

Interactive Presentation



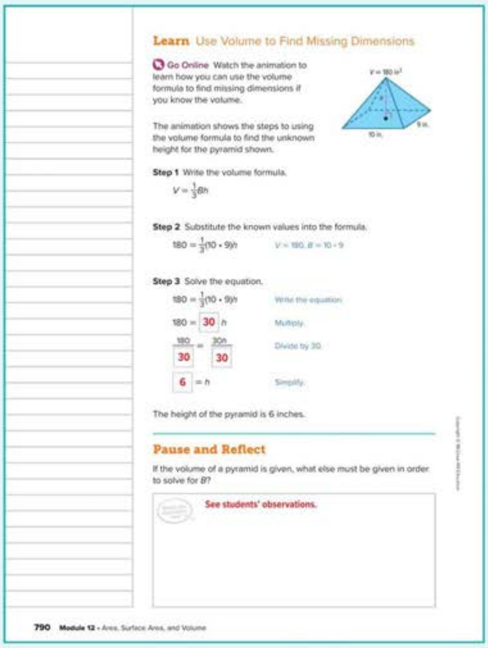
Example 3, Volume of Pyramids, Slide 1 of 2

TYPE

On Slide 1, students determine the volume of the pyramid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Use Volume to Find Missing Dimensions

Go Online Watch the animation to learn how you can use the volume formula to find missing dimensions if you know the volume.

The animation shows the steps to using the volume formula to find the unknown height for the pyramid shown.

Step 1 Write the volume formula.

$V = \frac{1}{3}Bh$

Step 2 Substitute the known values into the formula.

$180 = \frac{1}{3}(10 \cdot 9)h$ — $V = 180$, $B = 10 \cdot 9$

Step 3 Solve the equation.

$180 = \frac{1}{3}(10 \cdot 9)h$ — Write the equation.

$180 = 30h$ — Multiply.

$\frac{180}{30} = \frac{30h}{30}$ — Divide by 30.

$6 = h$ — Simplify.

The height of the pyramid is 6 inches.

Pause and Reflect

If the volume of a pyramid is given, what else must be given in order to solve for B?

See students' observations.

790 Module 12 • Area, Surface Area, and Volume

Interactive Presentation

Learn, Use Volume to Find Missing Dimensions

WATCH

Students watch an animation that demonstrates how to use volume formulas to find missing dimensions.

Learn Use Volume to Find Missing Dimensions

Objective

Students will understand how they can apply the volume formulas to find a missing dimension, given the volume and the other dimensions.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find a missing dimension of a three-dimensional figure.

Teaching Notes

SLIDE 1

You may wish to pause the animation after the rectangular pyramid is shown, and have students discuss with a partner what strategies they can use to find the unknown height h of the pyramid, given they know the volume and the dimensions of the rectangular base. Then have them watch the animation to see if their strategy was used. If not, ask them how the strategies compare and whether or not their strategy was a mathematically correct one.

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of how to apply volume formulas to find a missing dimension, have them work with a partner to draw their own prism or pyramid. Have them label all of the dimensions needed to find the volume except for one. That dimension should be the unknown the dimension. Then have them calculate the volume, given an appropriate numerical value for the unknown dimension, and label the volume on the figure. Have them trade drawings with another pair of students. Each pair should determine the unknown dimension, and compare it to the solution. Have pairs discuss and resolve any differences.

Example 4 Use Volume to Find Missing Dimensions

Objective

Students will find the area of the base of a prism given the volume.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the known and unknown quantities that represent the area of the trapezoidal base in order to formulate a plan for finding the height of the trapezoid.

6 Attend to Precision Have students explain why the quantity *B* does not represent the length of the trapezoidal base, 11.45 centimeters.

7 Look For and Make Use of Structure Encourage students to analyze the structure of the prism to determine the known and unknown dimensions, as this will help them determine what steps need to be taken to find the area of the base.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the area of the base of the prism

AL What is the shape of the base? Can you find its area by using an area formula? Explain. The base is a trapezoid. I cannot use an area formula because I only know the lengths of the two bases of the trapezoid, but not the trapezoid's height.

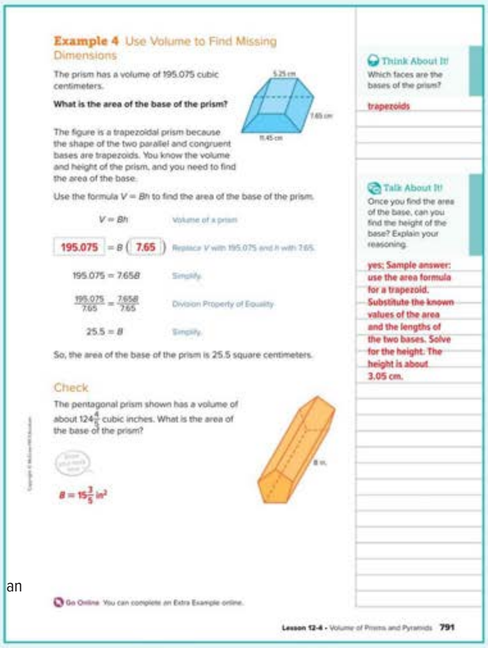
OL Other than the shown dimensions, what else do you know? The volume is 195.075 cubic centimeters.

OL Explain why you can use the volume to find the area of the base, since you do not know the trapezoid's height. Sample answer: I can use the volume formula $V = Bh$, because I do know the height of the prism and the volume of the prism. Solving for B will give me the area of the base.

BL Explain how you know the lateral faces are rectangles even though right angle symbols are not drawn on the figure. Sample answer: The figure is a prism. The lateral faces of a prism are defined to be rectangles since the lateral faces connect the two parallel bases at a distance equal to the height of the prism. The height is perpendicular to the base, so right angles can be assumed.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Use Volume to Find Missing Dimensions

The prism has a volume of 195.075 cubic centimeters.

What is the area of the base of the prism?

The figure is a trapezoidal prism because the shape of the two parallel and congruent bases are trapezoids. You know the volume and height of the prism, and you need to find the area of the base.

Use the formula $V = Bh$ to find the area of the base of the prism.

$V = Bh$ Volume of a prism

$195.075 = B(7.65)$ Replace V with 195.075 and h with 7.65.

$195.075 = 7.65B$ Simplify.

$\frac{195.075}{7.65} = \frac{7.65B}{7.65}$ Division Property of Equality

$25.5 = B$ Simplify.

So, the area of the base of the prism is 25.5 square centimeters.

Think About It! Which faces are the bases of the prism? trapezoids

Talk About It! Once you find the area of the base, can you find the height of the base? Explain your reasoning. yes; Sample answer: use the area formula for a trapezoid. Substitute the known values of the area and the lengths of the two bases. Solve for the height. The height is about 3.05 cm.

Check

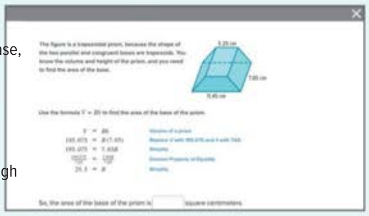
The pentagonal prism shown has a volume of about $124\frac{4}{5}$ cubic inches. What is the area of the base of the prism?

$B = 15\frac{3}{5}$ in²

Go Online You can complete an Extra Example online.

Lesson 12-4 • Volume of Prisms and Pyramids 791

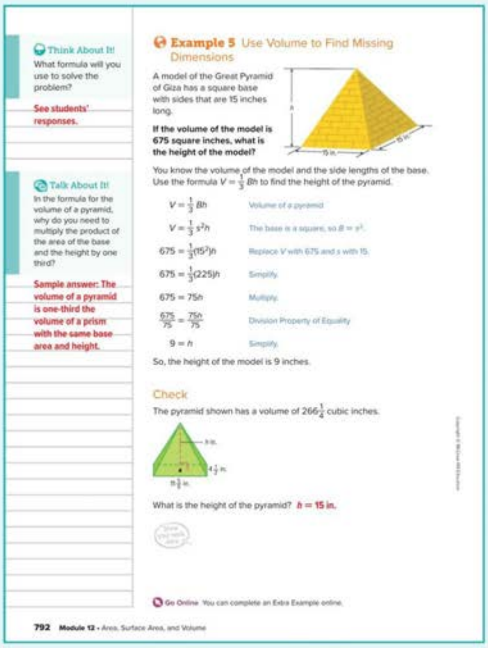
Interactive Presentation



Example 4, Use Volume to Find Missing Dimensions, Slide 2 of 4

TYPE On Slide 2, students determine the area of the base of the prism.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It!
What formula will you use to solve the problem?
See students' responses.

Talk About It!
In the formula for the volume of a pyramid, why do you need to multiply the product of the area of the base and the height by one third?
Sample answer: The volume of a pyramid is one-third the volume of a prism with the same base area and height.

Example 5 Use Volume to Find Missing Dimensions

A model of the Great Pyramid of Giza has a square base with sides that are 15 inches long.

If the volume of the model is 675 square inches, what is the height of the model?

You know the volume of the model and the side lengths of the base. Use the formula $V = \frac{1}{3}Bh$ to find the height of the pyramid.

$V = \frac{1}{3}Bh$	Volume of a pyramid
$V = \frac{1}{3}s^2h$	The base is a square, so $B = s^2$.
$675 = \frac{1}{3}(15^2)h$	Replace V with 675 and s with 15.
$675 = \frac{1}{3}(225)h$	Simplify.
$675 = 75h$	Multiply.
$\frac{675}{75} = \frac{75h}{75}$	Division Property of Equality
$9 = h$	Simplify.

So, the height of the model is 9 inches.

Check

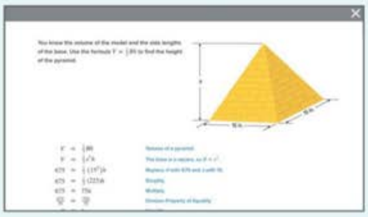
The pyramid shown has a volume of $266\frac{1}{4}$ cubic inches.

What is the height of the pyramid? $h = 15$ in.

Go Online You can complete an Extra Example online.

792 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 5, Use Volume to Find Missing Dimensions, Slide 2 of 4

TYPE

On Slide 2, students determine the height of the pyramid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Use Volume to Find Missing Dimensions

Objective

Students will find the height of a pyramid given the volume.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* question on Slide 3, encourage them to make sense of the relationship between a prism and a pyramid and how that relationship is manifested in their respective volume formulas.

6 Attend to Precision Have students precisely explain why they can replace the quantity B in the volume formula with s^2, where s is the side of the square base.

7 Look For and Make Use of Structure Encourage students to analyze the structure of the pyramid to determine the known and unknown dimensions, as this will help them determine what steps need to be taken to find the height.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know the shape of the base is a square? The problem tells me that the pyramid is a square pyramid.

OL How can you determine if your answer is correct? Sample answer: Replace h with 9 in the volume formula and determine if the volume is 675 cubic inches.

OL Instead of multiplying $\frac{1}{3}$ by 225 in the fifth step, describe another way to continue solving the equation. Sample answer: Multiply each side by 3 to eliminate the fraction. Then divide each side by 225.

BL Can the formula $V \frac{1}{3}s^2h$ be used if the base is not square? Explain. no; Sample answer: s^2 represents the area of the base. If the base is not square, then the area of the base would need to be represented by a different area formula.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Packaging

Objective

Students will come up with their own strategy to solve an application problem involving packaging a candle properly.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

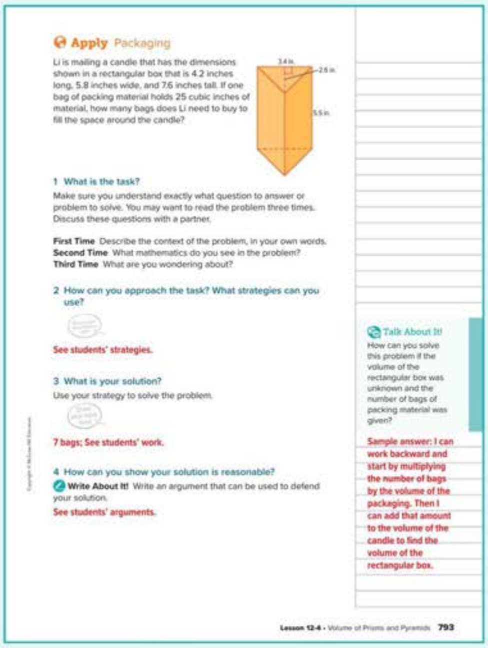
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you find the volume of the candle?
- How can you find the volume of the box?
- How do the two volumes you found relate?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Packaging

Li is mailing a candle that has the dimensions shown in a rectangular box that is 4.2 inches long, 5.8 inches wide, and 7.6 inches tall. If one bag of packing material holds 25 cubic inches of material, how many bags does Li need to buy to fill the space around the candle?

3.4 in. 2.6 in. 5.5 in.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

7 bags; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

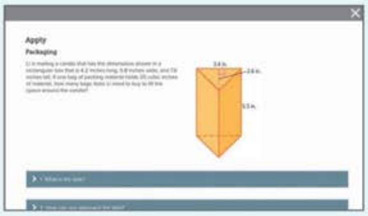
Talk About It!

How can you solve this problem if the volume of the rectangular box was unknown and the number of bags of packing material was given?

Sample answer: I can work backward and start by multiplying the number of bags by the volume of the packaging. Then I can add that amount to the volume of the candle to find the volume of the rectangular box.

Lesson 12-4 • Volume of Prisms and Pyramids 793

Interactive Presentation



Apply, Packaging

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Thema has a raised garden bed in her backyard that is shaped like a rectangular prism. It is 6 feet long, 3 feet wide, and $\frac{2}{3}$ foot deep. If a bag of garden soil holds 960 cubic inches of soil, how many bags will Thema need to fill the bed?

22 bags

Go Online You can complete an Extra Example online.

794 **Module 12** • Area, Surface Area, and Volume

Interactive Presentation

Exit Ticket

Essential Question Follow-Up

How can we measure objects to solve problems?

In this lesson, students learned how to find the volume of prisms and pyramids. Encourage them to brainstorm with a partner at least two real-world situations in which they might need to find the volume of a prism or pyramid. Some examples could include the amount of space occupied by a shipping box and the amount of sand that can fit inside a glass pyramid.

Exit Ticket

Refer to the Exit Ticket slide. The dimensions and cost for two tins of popcorn are shown. Determine which popcorn tin offers the better deal. Write a mathematical argument that can be used to defend your solution. the rectangular prism; Sample answer: The volume of the rectangular prism is 153.125 cubic inches, and the unit price is about $0.04 per cubic inch. The volume of the square pyramid is 77.08 cubic inches, and the unit price is about $0.06 per cubic inch.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 1, 7, 9, 11, 13–16
- Extension: Volume of Cylinders
- ALEKS® Volume of Prisms and Cylinders

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–9, 12, 14
- Extension: Volume of Cylinders
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Three-Dimensional Figures

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Three-Dimensional Figures

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

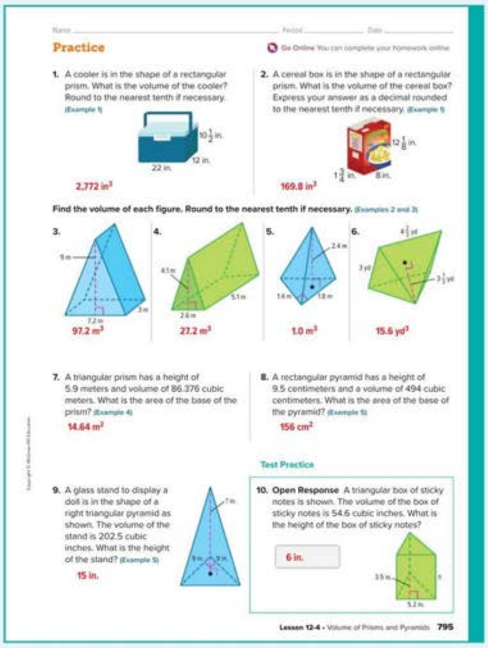
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
2	find the volume of a rectangular prism	1, 2
1	find the volume of pyramids and triangular prisms	3–6
2	find the area of the base or the height of a prism given the volume	7, 10
2	find the area of the base or the height of a pyramid given the volume	8, 9
3	solve application problems involving volume	11, 12
3	higher-order and critical thinking skills	13–16

Common Misconception

Some students may mistake a triangular prism for a triangular pyramid. In Exercise 3, students may identify the figure as a pyramid and calculate the volume to be 64.8 cubic meters rather than 97.2 cubic meters by using the formula for the volume of a pyramid. Encourage students to analyze the structure of each figure first in order to precisely identify it. A pyramid will have only one base, while a prism will have two parallel and congruent bases.



Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

1. A cooler is in the shape of a rectangular prism. What is the volume of the cooler? Round to the nearest tenth if necessary. (Example 1)
$10\frac{1}{2}$ in., 12 in., 22 in.
2,772 in^3

2. A cereal box is in the shape of a rectangular prism. What is the volume of the cereal box? Express your answer as a decimal rounded to the nearest tenth if necessary. (Example 1)
$12\frac{1}{8}$ in., $1\frac{3}{4}$ in., 8 in.
169.8 in^3

Find the volume of each figure. Round to the nearest tenth if necessary. (Examples 2 and 3)

3. 9 m, 7.2 m, 3 m — 97.2 m^3

4. 4.1 m, 2.6 m, 5.1 m — 27.2 m^3

5. 2.4 m, 1.4 m, 1.8 m — 1.0 m^3

6. $4\frac{1}{2}$ yd, 3 yd, $3\frac{1}{2}$ yd — 15.6 yd^3

7. A triangular prism has a height of 5.9 meters and volume of 86.376 cubic meters. What is the area of the base of the prism? (Example 4)
14.64 m^2

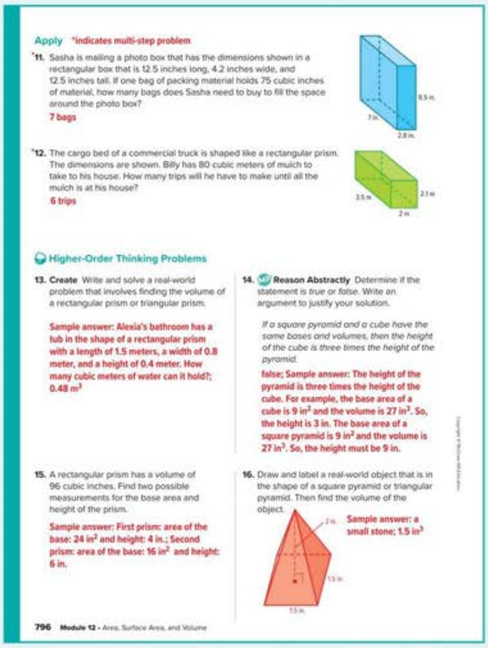
8. A rectangular pyramid has a height of 9.5 centimeters and a volume of 494 cubic centimeters. What is the area of the base of the pyramid? (Example 5)
156 cm^2

9. A glass stand to display a doll is in the shape of a right triangular pyramid as shown. The volume of the stand is 202.5 cubic inches. What is the height of the stand? (Example 5)
9 in., 9 in.
15 in.

Test Practice

10. **Open Response** A triangular box of sticky notes is shown. The volume of the box of sticky notes is 54.6 cubic inches. What is the height of the box of sticky notes?
3.5 in., 5.2 in.
6 in.

Lesson 12-4 • Volume of Prisms and Pyramids 795



Apply *indicates multi-step problem

*11. Sasha is mailing a photo box that has the dimensions shown in a rectangular box that is 12.5 inches long, 4.2 inches wide, and 12.5 inches tall. If one bag of packing material holds 75 cubic inches of material, how many bags does Sasha need to buy to fill the space around the photo box?

7 bags

*12. The cargo bed of a commercial truck is shaped like a rectangular prism. The dimensions are shown. Billy has 80 cubic meters of mulch to take to his house. How many trips will he have to make until all the mulch is at his house?

6 trips

Higher-Order Thinking Problems

13. **Create** Write and solve a real-world problem that involves finding the volume of a rectangular prism or triangular prism.

Sample answer: Alexia's bathroom has a tub in the shape of a rectangular prism with a length of 1.5 meters, a width of 0.8 meter, and a height of 0.4 meter. How many cubic meters of water can it hold?; 0.48 m^3

14. **Reason Abstractly** Determine if the statement is *true* or *false*. Write an argument to justify your solution.

If a square pyramid and a cube have the same bases and volumes, then the height of the cube is three times the height of the pyramid.

false; Sample answer: The height of the pyramid is three times the height of the cube. For example, the base area of a cube is 9 in^2 and the volume is 27 in^3. So, the height is 3 in. The base area of a square pyramid is 9 in^2 and the volume is 27 in^3. So, the height must be 9 in.

15. A rectangular prism has a volume of 96 cubic inches. Find two possible measurements for the base area and height of the prism.

Sample answer: First prism: area of the base: 24 in^2 and height: 4 in.; Second prism: area of the base: 16 in^2 and height: 6 in.

16. Draw and label a real-world object that is in the shape of a square pyramid or triangular pyramid. Then find the volume of the object.

Sample answer: a small stone; 1.5 in^3

796 Module 12 • Area, Surface Area, and Volume

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In E xercise 14, students determine if a statement is true or false and support their reasoning.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 11–12 Have students work in pairs. Have students individually read Exercise 11 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 12.

Clearly and precisely explain.

Use with Exercise 14 Have pairs of students prepare their explanations, making sure that their reasoning is clear and precise. Then call on one pair of students to explain their reasoning to the class. Encourage students to come up with a variety of responses, such as drawing a counterexample.

Lesson 12-5

Surface Area of Prisms and Pyramids

7.G.B.6

LESSON GOAL

Students will find the surface area of prisms and pyramids.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Surface Area of Prisms and Pyramids

Learn: Surface Area of Prisms
Example 1: Surface Area of Rectangular Prisms
Example 2: Surface Area of Triangular Prisms
Learn: Surface Area of Pyramids
Example 3: Surface Area of Pyramids
Apply: Painting

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Arrive**MATH** Take Another Look	●		
Extension: Surface Area of Cylinders		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 74 of the *Language Development Handbook* to help your students build mathematical language related to the surface area of prisms and pyramids.

ELL You can use the tips and suggestions on page T74 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry

Additional Cluster(s): In this lesson, students address additional cluster **7.G.B** by finding the surface area of prisms and pyramids.

Standards for Mathematical Content: **7 .G.B.6,** *Also addresses 7.NS.A.3*

Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the volume of prisms and pyramids.
7.G.B.6

Now
Students find the surface area of prisms and pyramids.
7.G.B.6

Next
Students will find the volume of cylinders.
8.G.C.9

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students will draw on their knowledge of nets and their knowledge of finding the area of two-dimensional objects to gain *fluency* in finding the surface area of prisms and pyramids. They will *apply* this fluency in solving real-world problems involving surface area.

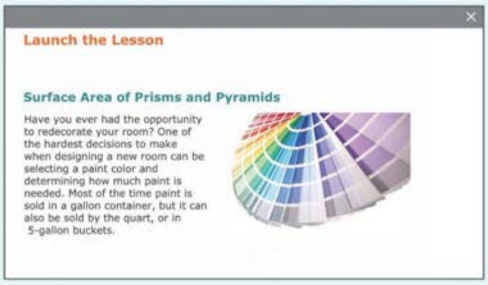
Mathematical Background

Surface area is the sum of the areas of all the surfaces, or faces, of a three-dimensional figure. It is measured in square units.

- Rectangular Prism: $S.A. = 2lh + 2lw + 2hw$
- Pyramid: $S.A. = B + 4\left(\frac{1}{2}bh\right)$ where h is the slant height

Interactive Presentation

Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- identifying the faces of three-dimensional figures (Exercises 1–5)

Answers

1. 1 rectangle, 4 triangles
2. 6 squares
3. 1 square, 4 triangles
4. 6 rectangles
5. 4 triangles

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about buying paint to redecorate a room.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Describe the *face* of a three-dimensional figure in your own words. Sample answer: The face of a three-dimensional figure is a flat surface that is formed by the figure's connecting edges.
- How might the *slant height* of a pyramid differ from the height of a pyramid? Sample answer: The height of a pyramid is perpendicular to the base. The slant height is likely not perpendicular since the adjective *slant* is describing it.
- What are some real-world examples when you might describe the *surface* of an object? Sample answers: the surface of a table, the surface of a desk, the surface of a countertop

Explore Surface Area of Prisms and Pyramids

Objective

Students will explore the relationship between nets and surface area.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a cereal box to determine different methods for finding the surface area of rectangular prisms. Throughout this activity, students will use the net of a rectangular prism to make conjectures about how to find surface area without using a net.

Inquiry Question

How can you find the surface area of prisms and pyramids without using nets? Sample answer: I can find the sum of the areas of each face and base to find the total surface area.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

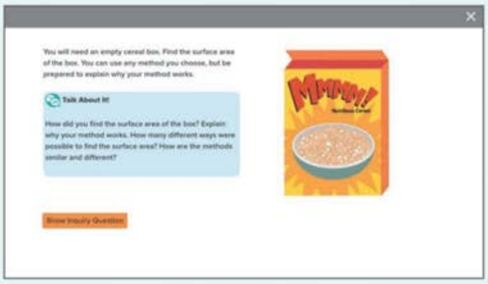
Mathematical Discourse

How did you find the surface area of the box? Explain why your method works. How many different ways were possible to find the surface area? How are the methods similar and different? Sample answer: My method works because I cut up the box to make a net. I then found the area of each side of the box and found the total area by finding the sum of the areas of all the sides.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7



Explore, Slide 2 of 7

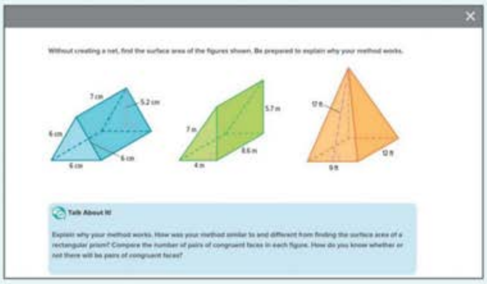
CLICK

On Slide 3, students move through the steps to create a net.

TYPE

On Slide 4, students make a conjecture for how they can find the surface area of any rectangular prism.

Interactive Presentation



Explore, Slide 6 of 7

TYPE

On Slide 5, students type to indicate the surface area of a prism.

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Surface Area of Prisms and Pyramids *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to understand the relationship between a figure and its net.

5 Use Appropriate Tools Strategically Explain to students the benefit of using a net to visualize all of the quantities that comprise the total surface area of a solid.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 6 are shown.

Talk About It!

SLIDE 6

Mathematical Discourse

Explain why your method works. How was your method similar to and different from finding the surface area of a rectangular prism? Compare the number of pairs of congruent faces in each figure. How do you know whether or not there will be pairs of congruent faces? Sample answer: I found the area of each face, and then found the sum of the areas. If I find the area of each face/base , and then find the sum of those areas, I will have the total surface area. There will be congruent faces if there side lengths of the base are congruent.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Surface Area of Prisms

Objective

Students will understand the relationship between using a net and a formula for finding the surface area of a rectangular prism.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to abstract the surface area of the prism in order to represent it symbolically.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 2, have them compare and contrast the formulas of their classmates to determine if they are equivalent.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Suppose the dimensions of the prism were unknown. Can you write a formula that would help you find the surface area of any rectangular prism with length ℓ, height h, and width w? Share your formulas with other classmates. Compare and contrast your formulas. yes; Sample answer: $S.A. = 2\ell h + 2\ell w + 2hw$; See students' responses.

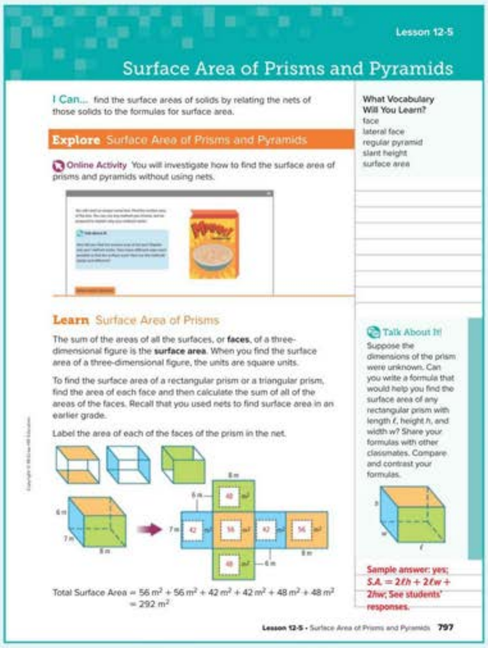
DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of surface area, have them identify whether or not the surface area of the rectangular prisms described below can be found using the information given. Have students explain their reasoning.

Three of the faces of a rectangular prism have areas 10 square meters, 20 square meters, and 12 square meters yes; Sample answer: Opposite pairs of faces have the same area. Since the three areas given are different, the areas of all faces can be determined and the surface area can be found.

Three of the faces of a rectangular prism have areas 16 square centimeters, 30 square meters, and 16 square centimeters. no; Sample answer: The two faces with areas of 16 square centimeters could be opposite faces, so the third pair of faces might have unknown areas. The surface area cannot be calculated without more information.


Lesson 12-5

Surface Area of Prisms and Pyramids

I Can... find the surface areas of solids by relating the nets of those solids to the formulas for surface area.

What Vocabulary Will You Learn?
face
lateral face
regular pyramid
slant height
surface area

Explore Surface Area of Prisms and Pyramids

Online Activity You will investigate how to find the surface area of prisms and pyramids without using nets.

Learn Surface Area of Prisms

The sum of the areas of all the surfaces, or **faces**, of a three-dimensional figure is the **surface area**. When you find the surface area of a three-dimensional figure, the units are square units.

To find the surface area of a rectangular prism or a triangular prism, find the area of each face and then calculate the sum of all of the areas of the faces. Recall that you used nets to find surface area in an earlier grade.

Label the area of each of the faces of the prism in the net.

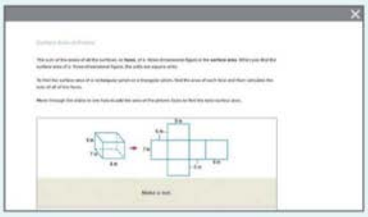
Total Surface Area = $56\text{ m}^2 + 56\text{ m}^2 + 42\text{ m}^2 + 42\text{ m}^2 + 48\text{ m}^2 + 48\text{ m}^2$
$= 292\text{ m}^2$

Talk About It! Suppose the dimensions of the prism were unknown. Can you write a formula that would help you find the surface area of any rectangular prism with length ℓ, height h, and width w? Share your formulas with other classmates. Compare and contrast your formulas.

Sample answer: yes; $S.A. = 2\ell h + 2\ell w + 2hw$; See students' responses.

Lesson 12-5 • Surface Area of Prisms and Pyramids 797

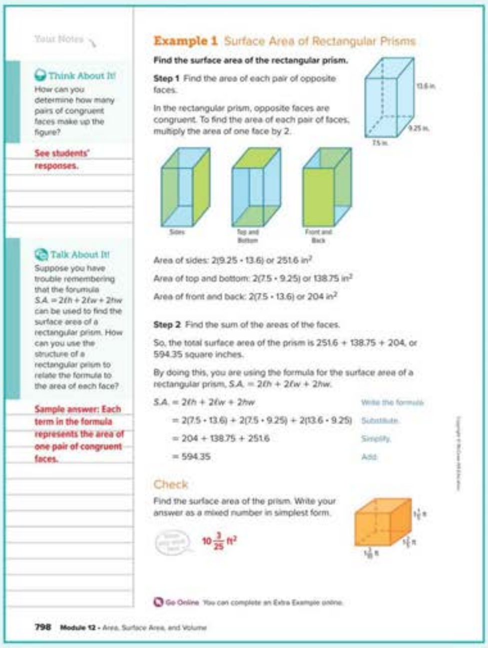
Interactive Presentation



Learn, Surface Area of Prisms, Slide 1 of 2

CLICK

On Slide 1, students move through the slides to see how to use the net to find the surface area.



Your Notes

Think About It! How can you determine how many pairs of congruent faces make up the figure?

See students' responses.

Talk About It! Suppose you have trouble remembering that the forumula $S.A. = 2\ell h + 2\ell w + 2hw$ can be used to find the surface area of a rectangular prism. How can you use the structure of a rectangular prism to relate the formula to the area of each face?

Sample answer: Each term in the formula represents the area of one pair of congruent faces.

Example 1 Surface Area of Rectangular Prisms

Find the surface area of the rectangular prism.

Step 1 Find the area of each pair of opposite faces.

In the rectangular prism, opposite faces are congruent. To find the area of each pair of faces, multiply the area of one face by 2.

Sides | Top and Bottom | Front and Back

Area of sides: $2(9.25 \cdot 13.6)$ or 251.6 in²

Area of top and bottom: $2(7.5 \cdot 9.25)$ or 138.75 in²

Area of front and back: $2(7.5 \cdot 13.6)$ or 204 in²

Step 2 Find the sum of the areas of the faces.

So, the total surface area of the prism is 251.6 + 138.75 + 204, or 594.35 square inches.

By doing this, you are using the formula for the surface area of a rectangular prism, $S.A. = 2\ell h + 2\ell w + 2hw$.

$S.A. = 2\ell h + 2\ell w + 2hw$ Write the formula.

$= 2(7.5 \cdot 13.6) + 2(7.5 \cdot 9.25) + 2(13.6 \cdot 9.25)$ Substitute.

$= 204 + 138.75 + 251.6$ Simplify.

$= 594.35$ Add.

Check

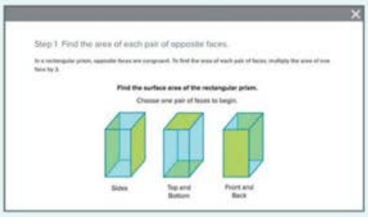
Find the surface area of the prism. Write your answer as a mixed number in simplest form.

$10\frac{3}{25}$ ft²

Go Online You can complete an Extra Example online.

798 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 1, Surface Area of Rectangular Prisms, Slide 2 of 5

CLICK

On Slide 2, students choose a pair of faces to begin finding the surface area.

TYPE

On Slide 3, students determine the total surface area of the prism.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Surface Area of Rectangular Prisms

Objective

Students will find the surface area of rectangular prisms.

MP Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the pairs of congruent faces that make up the surface area of the prism.

7 Look For and Make Use of Structure Students should analyze the structure of the prism to determine which faces can be paired together that have the same area.

Questions for Mathematical Discourse

SLIDE 2

AL Y ou can begin by choosing a pair of faces. Which pair of faces would you like to choose? See students' responses.

OL Explain how to find the area of any pair of faces. Find the area of one face, then multiply by 2.

OL Can the top face be paired with the front face? Explain. no; Sample answer: The pair of faces must be congruent in order to be able to multiply the area of one face by 2.

BL In any rectangular prism, how many pairs of congruent faces are there? What kind of rectangular prism has 6 congruent faces? Any rectangular prism has 3 pairs of congruent faces. A cube has 6 congruent faces.

SLIDE 3

AL Explain why you add the areas of the pairs of faces. The total surface area is the sum of all of the areas of the faces.

OL How can you check your answer? Sample answer: I can draw a net to find the area of each face and then add the areas together. I can also use estimation to check my calculations.

BL Suppose this rectangular prism represented a shipping box. Do you think the amount of cardboard needed to produce the box is exactly equal to the surface area? Explain. Sample answer: No, in order to seal the box closed, the top and bottom faces usually consist of extra cardboard. So, the amount of cardboard needed will be close to the surface area, but greater.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Surface Area of T riangular Prisms

Objective

Students will find the surface area of triangular prisms.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to understand that in a triangular prism, the triangular bases are congruent, but the rectangular lateral faces may not be.

7 Look For and Make Use of Structure Students should study the structure of the prism in order to determine that two of the lateral faces are congruent (3.6 inches × 14 inches), but the third lateral face has dimensions 4 inches × 14 inches.

Questions for Mathematical Discourse

SLIDE 2

AL Why are the triangles considered the bases? They are parallel and congruent.

OL Explain why not all three rectangular faces are congruent. Sample answer: Two of the rectangular faces are congruent, each with dimensions 3.6 inches by 14 inches. The third rectangular face has dimensions 4 inches by 14 inches.

BL Describe a triangular prism in which the three rectangular faces would be congruent. Sample answer: If the shape of the base is an equilateral triangle, then the three rectangular faces of the prism would be congruent.

SLIDE 3

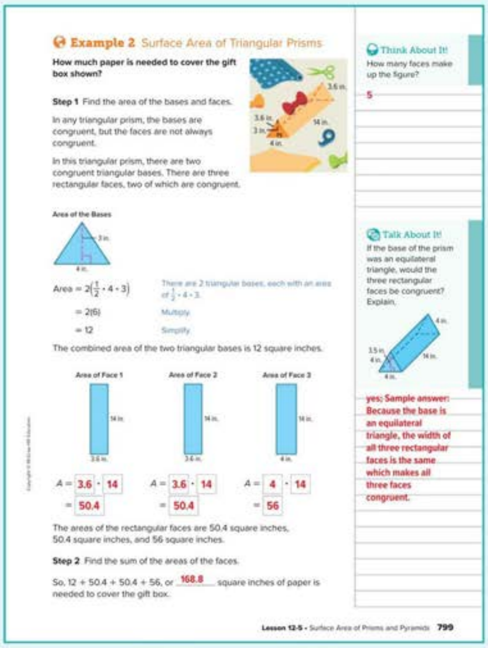
AL How do you find the total surface area? Sample answer: Add the areas of the bases and each rectangular face together.

OL Does this mean that exactly 168.8 square inches of wrapping paper is needed to wrap the gift box? Explain. no; Sample answer: When wrapping a present, there is overlap of paper needed to tape down edges. So, the amount of paper needed will actually be greater than 168.8 square inches.

BL How many square feet of wrapping paper is needed? Explain. about 1.2 square feet; Sample answer: Divide 168.8 by 144, because there are 144 (12 by 12) square inches in one square foot.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Surface Area of Triangular Prisms

How much paper is needed to cover the gift box shown?

Step 1 Find the area of the bases and faces.

In any triangular prism, the bases are congruent, but the faces are not always congruent.

In this triangular prism, there are two congruent triangular bases. There are three rectangular faces, two of which are congruent.

Area of the Bases

$\text{Area} = 2\left(\frac{1}{2} \cdot 4 \cdot 3\right)$ — There are 2 triangular bases, each with an area of $\frac{1}{2} \cdot 4 \cdot 3$.

$= 2(6)$ — Multiply

$= 12$ — Simplify

The combined area of the two triangular bases is 12 square inches.

Area of Face 1	Area of Face 2	Area of Face 3
$A = 3.6 \cdot 14$	$A = 3.6 \cdot 14$	$A = 4 \cdot 14$
$= 50.4$	$= 50.4$	$= 56$

The areas of the rectangular faces are 50.4 square inches, 50.4 square inches, and 56 square inches.

Step 2 Find the sum of the areas of the faces.

So, 12 + 50.4 + 50.4 + 56, or 168.8 square inches of paper is needed to cover the gift box.

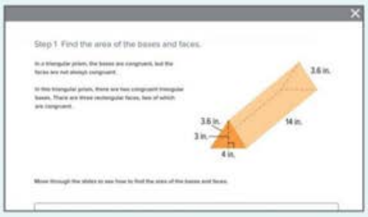
Think About It! How many faces make up the figure? 5

Talk About It! If the base of the prism was an equilateral triangle, would the three rectangular faces be congruent? Explain.

yes; Sample answer: Because the base is an equilateral triangle, the width of all three rectangular faces is the same which makes all three faces congruent.

Lesson 12-5 • Surface Area of Prisms and Pyramids 799

Interactive Presentation

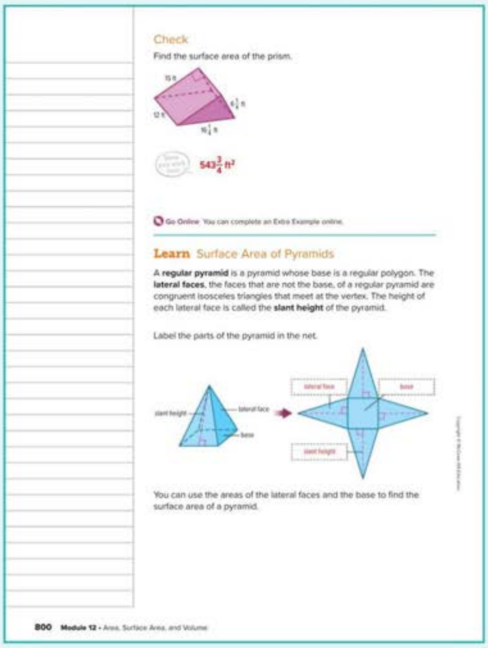


Example 2, Surface Area of Triangular Prisms, Slide 2 of 5

CLICK On Slide 2, students move through the slides to see how to find the area of the bases and faces.

TYPE On Slide 3, students determine the amount of paper needed to cover the gift box.

CHECK Students complete the Check exercise online to determine if they are ready to move on.


Check

Find the surface area of the prism.

Go Online You can complete an Extra Example online.

Learn Surface Area of Pyramids

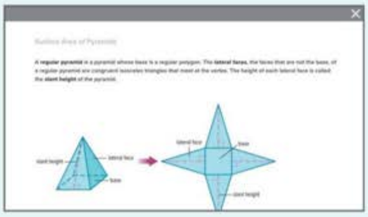
A **regular pyramid** is a pyramid whose base is a regular polygon. The **lateral faces**, the faces that are not the base, of a regular pyramid are congruent isosceles triangles that meet at the vertex. The height of each lateral face is called the **slant height** of the pyramid.

Label the parts of the pyramid in the net.

You can use the areas of the lateral faces and the base to find the surface area of a pyramid.

800 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Learn, Surface Area of Pyramids

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Surface Area of Pyramids

Objective

Students will understand the structure of a pyramid and how to find its surface area.

Teaching Notes

SLIDE 1

Have students study the structure of the pyramid shown and its corresponding net. Students have previously learned how to use nets of three-dimensional figures to find surface area. Remind students that surface area is the total area of all of the surfaces of a three-dimensional figure.

Point out the difference between the height and the slant height of a pyramid. Students should understand that the height of the pyramid is always perpendicular to the pyramid's base. The slant height is the height of a lateral face, and the slant height is perpendicular to the base of the triangular lateral face. Have students explain why the height of a pyramid is needed to determine its volume, but the slant height is needed to determine its surface area.

You may wish to ask students how the lateral faces would compare if the pyramid was not a regular pyramid. They should be able to reason that the lateral faces would not be congruent, since the lengths of the triangular bases that form each lateral face would not be equivalent.

DIFFERENTIATE

Language Development Activity ELL

If any of your students would benefit from additional support in understanding the vocabulary presented in the Learn, have them work with a partner to draw different pyramids that satisfy the following conditions. Each pyramid should be drawn on a different piece of paper. Label each pyramid A, B, C, D, or E as described.

Pyramid A should be a square pyramid.

Pyramid B should be a rectangular pyramid, but not regular.

Pyramid C should be a triangular, regular pyramid.

Pyramid D should be a triangular pyramid, but not regular.

Pyramid E should be a trapezoidal pyramid.

Then have pairs sort the pyramids into the following two categories – *Lateral Faces are Congruent*, *Lateral Faces are not Congruent*. They should be prepared to defend their choices. Ask for volunteers to explain how they sorted their pyramids into each category.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Surface Area of Pyramids

Objective

Students will find the surface area of pyramids.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to understand that in a pyramid with a square base, the lateral faces are congruent.

As students discuss the *Talk About It!* question on Slide 5, encourage them to make sense of how different bases might affect the triangular faces.

6 Attend to Precision Students should calculate accurately and efficiently, paying careful attention to use the correct dimensions to calculate the areas of the faces.

Questions for Mathematical Discourse

SLIDE 2

AL Is this a regular pyramid? Explain. yes; Sample answer: A regular pyramid has a base that has congruent side lengths.

OL Estimate the area of the base. Sample answer: Round the side length to 5 inches. The area of the base is about 25 square inches.

OL Describe what you know about the lateral faces. Sample answer: Each triangle has the same base and height, so the four lateral faces are congruent.

BL Do you think the height of the pyramid is less than, equal to, or greater than the slant height? Explain. Sample answer: The height of the pyramid should be less than the slant height because the distance traveled from the base along the lateral face to the opposite vertex is greater than the perpendicular distance traveled from the base to the opposite vertex.

SLIDE 3

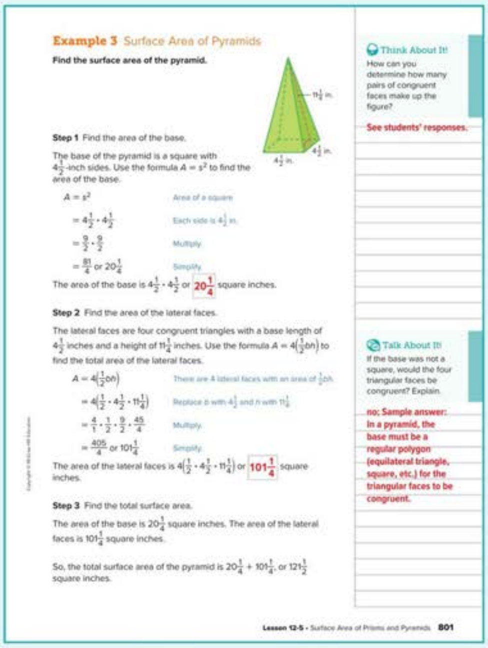
AL How many lateral faces are there? There are four lateral faces.

OL In the formula $A = 4\left(\frac{1}{2}bh\right)$, what does the 4 represent? There are 4 congruent triangles.

BL Would a trapezoidal pyramid have congruent lateral faces? Explain. no; Sample answer: A trapezoid is not a regular polygon, so the lateral faces will not be congruent.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Surface Area of Pyramids

Find the surface area of the pyramid.

Step 1 Find the area of the base.

The base of the pyramid is a square with $4\frac{1}{2}$-inch sides. Use the formula $A = s^2$ to find the area of the base.

$A = s^2$ — Area of a square

$= 4\frac{1}{2} \cdot 4\frac{1}{2}$ — Each side is $4\frac{1}{2}$ in.

$= \frac{9}{2} \cdot \frac{9}{2}$ — Multiply.

$= \frac{81}{4}$ or $20\frac{1}{4}$ — Simplify.

The area of the base is $4\frac{1}{2} \cdot 4\frac{1}{2}$ or $20\frac{1}{4}$ square inches.

Step 2 Find the area of the lateral faces.

The lateral faces are four congruent triangles with a base length of $4\frac{1}{2}$ inches and a height of $11\frac{1}{4}$ inches. Use the formula $A = 4\left(\frac{1}{2}bh\right)$ to find the total area of the lateral faces.

$A = 4\left(\frac{1}{2}bh\right)$ — There are 4 lateral faces with an area of $\frac{1}{2}bh$.

$= 4\left(\frac{1}{2} \cdot 4\frac{1}{2} \cdot 11\frac{1}{4}\right)$ — Replace b with $4\frac{1}{2}$ and h with $11\frac{1}{4}$.

$= \frac{4}{1} \cdot \frac{1}{2} \cdot \frac{9}{2} \cdot \frac{45}{4}$ — Multiply.

$= \frac{405}{4}$ or $101\frac{1}{4}$ — Simplify.

The area of the lateral faces is $4\left(\frac{1}{2} \cdot 4\frac{1}{2} \cdot 11\frac{1}{4}\right)$ or $101\frac{1}{4}$ square inches.

Step 3 Find the total surface area.

The area of the base is $20\frac{1}{4}$ square inches. The area of the lateral faces is $101\frac{1}{4}$ square inches.

So, the total surface area of the pyramid is $20\frac{1}{4} + 101\frac{1}{4}$, or $121\frac{1}{2}$ square inches.

Think About It! How can you determine how many pairs of congruent faces make up the figure?

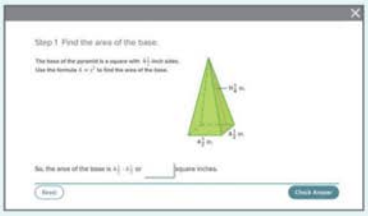
See students' responses.

Talk About It! If the base was not a square, would the four triangular faces be congruent? Explain.

no; Sample answer: In a pyramid, the base must be a regular polygon (equilateral triangle, square, etc.) for the triangular faces to be congruent.

Lesson 12-5 • Surface Area of Prisms and Pyramids 801

Interactive Presentation



Example 3, Surface Area of Pyramids, Slide 2 of 6

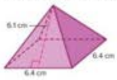
TYPE On Slide 2, students determine the area of the base.

TYPE On Slide 4, students determine the total surface area.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

Check

Find the surface area of the pyramid. **119.04 cm²**



Go Online You can complete an Extra Example online.

Pause and Reflect

How will you study the concepts in today's lesson? Describe some steps you can take.

See students' observations.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | **3 APPLICATION**

Apply Painting

Objective

Students will come up with their own strategy to solve an application problem involving painting a toy box.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How many inches are in a foot?
- What symbol should be used to show that there is enough paint?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Painting

Domingo built a toy box for his little brother that is 42 inches long, 21 inches wide, and 24 inches tall. He has 1 quart of paint that covers about 87 square feet. Does he have enough paint to cover the outside of the toy box with two coats of paint?

24 in.
42 in.
21 in.
1 qt
covers 87 square feet

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?
See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.
yes; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.
See students' arguments.

Go Online Watch the animation.

Talk About It! How can you solve this problem another way?

Sample answer: I can convert 87 square feet into square inches using the conversion 1 square foot = 144 square inches.

Lesson 12-5 • Surface Area of Prisms and Pyramids 803

Interactive Presentation

Apply, Painting

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

Lucia is covering boxes with fabric to sell at a craft fair. The boxes are shaped like rectangular prisms and measure $10\frac{1}{2}$ inches wide, $14\frac{1}{2}$ inches long, and 3 inches tall. If she has 100 square feet of fabric, how many boxes can she cover? **31 boxes**

Go Online You can complete an Extra Example online.

804 Module 12 • Area, Surface Area, and Volume

Essential Question Follow-Up

How can we measure objects to solve problems?

In this lesson, students learned how to find the surface area of prisms and pyramids. Encourage them to brainstorm with a partner at least two real-world situations in which they might need to find the surface area of a three-dimsenional figure. Some examples could include the amount of paint needed to paint a garage and the amount of glass on the surface of the Louvre pyramid in Paris, France.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 3, 7, 9–12
- Extension: Surface Area of Cylinders
- ALEKS Surface Area

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 7, 10
- Extension: Surface Area of Cylinders
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Three-Dimensional Figures

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Three-Dimensional Figures

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

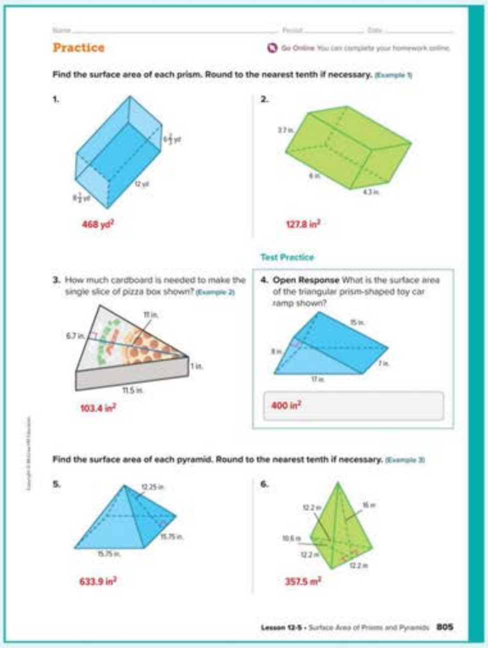
Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the surface area of rectangular prisms	1, 2
2	find the surface area of triangular prisms	3, 4
1	find the surface area of pyramids	5, 6
3	solve application problems involving surface area	7, 8
3	higher-order and critical thinking skills	9–12

Exit Ticket

Refer to the Exit Ticket slide. Suppose you decide to paint your bedroom. It is in the shape of a rectangular prism. It is 13 feet long, 9 feet wide, and 8 feet high. Suppose the door, windows, and closet cover an area of 80 square feet. A gallon of paint covers 400 square feet. How many gallons of paint do you need to cover the walls with two coats of paint? Write a mathematical argument that can be used to defend your solution.

2 gallons; Sample answer: The total surface area of the four walls is 2(13)(8) + 2(9)(8), or 352 square feet. Subtract 80 square feet to obtain 272 square feet. Multiply by two since I need two coats. The total area to be painted is 544 square feet. Since one gallon covers 400 square feet and will not be enough, I need to purchase two gallons.



Practice

Go Online You can complete your homework online.

Find the surface area of each prism. Round to the nearest tenth if necessary. (Example 1)

1. 468 yd^2

2. 127.8 in^2

Test Practice

3. How much cardboard is needed to make the single slice of pizza box shown? (Example 2)

103.4 in^2

4. **Open Response** What is the surface area of the triangular prism-shaped toy car ramp shown?

400 in^2

Find the surface area of each pyramid. Round to the nearest tenth if necessary. (Example 3)

5. 633.9 in^2

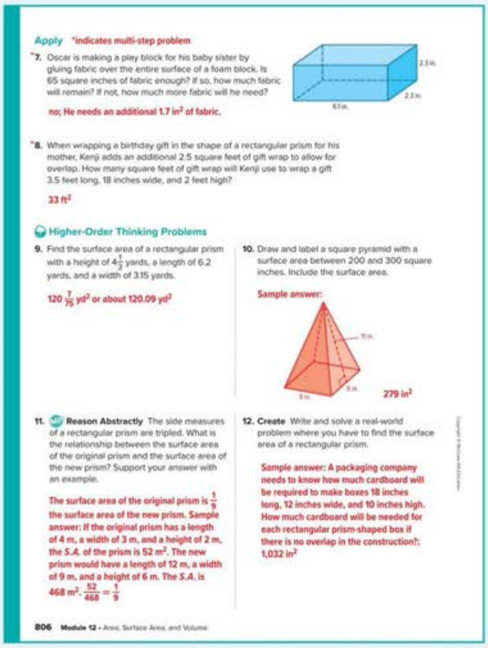
6. 357.5 m^2

Lesson 12-5 • Surface Area of Prisms and Pyramids 805

Interactive Presentation

Exit Ticket

3 REFLECT AND PRACTICE



Apply *indicates multi-step problem

*7. Oscar is making a play block for his baby sister by gluing fabric over the entire surface of a foam block. Is 65 square inches of fabric enough? If so, how much fabric will remain? If not, how much more fabric will he need?

no; He needs an additional 1.7 in^2 of fabric.

*8. When wrapping a birthday gift in the shape of a rectangular prism for his mother, Kenji adds an additional 2.5 square feet of gift wrap to allow for overlap. How many square feet of gift wrap will Kenji use to wrap a gift 3.5 feet long, 18 inches wide, and 2 feet high?

33 ft^2

Higher-Order Thinking Problems

9. Find the surface area of a rectangular prism with a height of $4\frac{1}{3}$ yards, a length of 6.2 yards, and a width of 3.15 yards.

$120\frac{7}{75}$ yd^2 or about 120.09 yd^2

10. Draw and label a square pyramid with a surface area between 200 and 300 square inches. Include the surface area.

Sample answer:

279 in^2

11. **Reason Abstractly** The side measures of a rectangular prism are tripled. What is the relationship between the surface area of the original prism and the surface area of the new prism? Support your answer with an example.

The surface area of the original prism is $\frac{1}{9}$ the surface area of the new prism. Sample answer: If the original prism has a length of 4 m, a width of 3 m, and a height of 2 m, the S.A. of the prism is 52 m^2. The new prism would have a length of 12 m, a width of 9 m, and a height of 6 m. The S.A. is 468 m^2. $\frac{52}{468} = \frac{1}{9}$

12. **Create** Write and solve a real-world problem where you have to find the surface area of a rectangular prism.

Sample answer: A packaging company needs to know how much cardboard will be required to make boxes 18 inches long, 12 inches wide, and 10 inches high. How much cardboard will be needed for each rectangular prism-shaped box if there is no overlap in the construction?; 1,032 in^2

806 Module 12 • Area, Surface Area, and Volume

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively In Exercise 11, students describe how the surface area of a rectangular prism would change if the side lengths are tripled.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.

Use with Exercise 7 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.

Use with Exercise 9 Have students work in groups of 3–4 to solve the problem in Exercise 9. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class.

Lesson 12-6

Volume of Cylinders

8.G.C.9

LESSON GOAL

Students will find the volume of cylinders.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Volume of Cylinders

Learn: Volume of Cylinders
Example 1: Find Volume of Cylinders Given the Radius
Example 2: Find Volume of Cylinders Given the Diameter
Example 3: Solve Problems Involving the Volume of Cylinders
Apply: Swimming

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 75 of the *Language Development Handbook* to help your students build mathematical language related to the volume of cylinders.

ELL You can use the tips and suggestions on page T75 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In t his lesson, students address the additional cluster **8.G.C** by finding the volume of cylinders.
Standards for Mathematical Content: **8.G.C.9**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the surface area of prisms and pyramids.
7.G.B.6

Now
Students find the volume of cylinders.
8.G.C.9

Next
Students will find the volume of cones.
8.G.C.9

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of finding volume of prisms to develop understanding of how to find the volume of cylinders. They use this understanding to build *fluency* with calculating volume. They *apply* their fluency to solve multi-step real-world problems.

Mathematical Background

A *cylinder* is a three-dimensional figure with two parallel congruent circular bases. *Volume* is a measure of three-dimensional space, and the volume of a cylinder can be found by multiplying the area of the base by the height.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying with decimals (Exercise 1)
- finding squares of numbers (Exercises 2–3)
- using the formula for the area of a circle (Exercises 4–5)

Answers

1. 25.19972
2. 25
3. 144
4. 28.27 $units^2$
5. 153.94 ft^2

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about estimating the number of objects in a cylindrical jar.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- A soup can is in the shape of a *cylinder*. What other objects in everyday life have shapes similar to cylinders? Sample answers: batteries, candles, drinking glasses
- When you pour water into a drinking glass, the amount of water is measured by volume. What other quantities are measured by volume? Sample answers: the amount of water in a fish tank, the amount of cement in a cement truck, the amount of wax needed to make a candle

Explore Volume of Cylinders

Objective

Students will explore how the volumes of cylinders and prisms are related.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will find the volume of a rectangular and triangular prism, and then apply that reasoning to determine how to find the volume of a cylinder.

Inquiry Question

How can you determine the volume of a cylinder? Sample answer: I can find the area of the base and multiply by the height.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

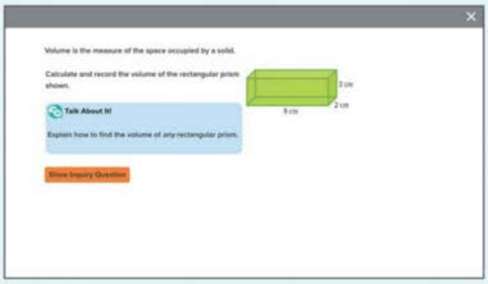
Mathematical Discourse

Explain how to find the volume of *any* rectangular prism. Sample answer: To find the volume of any rectangular prism, I can find the area of the base and multiply by the height.

(continued on next page)

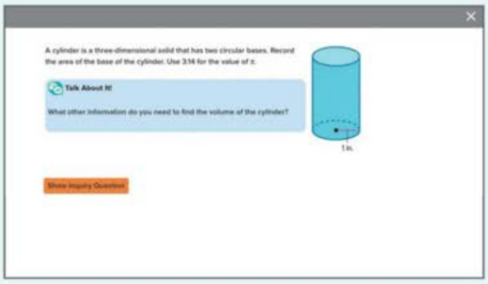
Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 2 of 8

Interactive Presentation



Explore, Slide 5 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.

Explore Volume of Cylinders *(continued)*

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Talk About It!* questions, encourage them to make conjectures and be able to justify them using mathematical reasoning.

2 Reason Abstractly and Quantitatively Encourage students to examine the similarities between finding the volume of a prism and finding the volume of a cylinder.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

What other information do you need to find the volume of the cylinder?
Sample answer: The height of the cylinder must be known to find its volume.

Learn Volume of Cylinders

Objective

Students will learn how to find the volume of cylinders.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others While discussing the *Talk About It!* questions on Slide 2, encourage students to analyze and compare the various representations of π before explaining their reasoning.

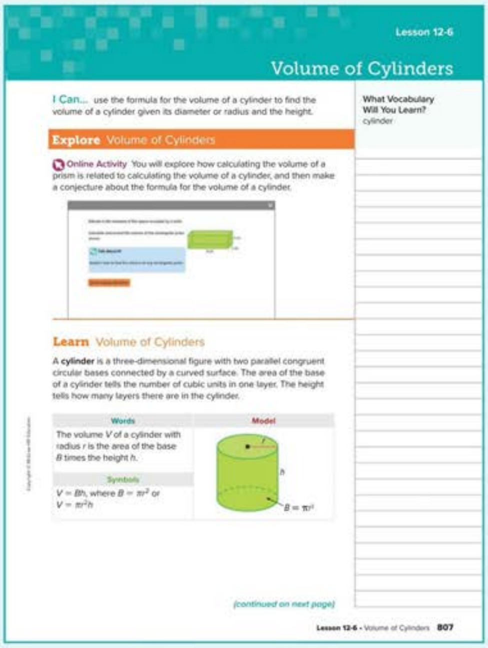
6 Attend to Precision While discussing the *Talk About It!* questions on Slide 2, students should pay close attention to the dimensions and units they use in their explanation.

Teaching Notes

SLIDE 1

Students will learn the definition of a cylinder and how to find the volume of a cylinder. Have students select the *Words, Symbols,* and *Model* flashcards to view how the volume formula of a cylinder can be expressed in these multiple representations.

(continued on next page)


Lesson 12-6

Volume of Cylinders

I Can... use the formula for the volume of a cylinder to find the volume of a cylinder given its diameter or radius and the height.

What Vocabulary Will You Learn?
cylinder

Explore Volume of Cylinders

Online Activity You will explore how calculating the volume of a prism is related to calculating the volume of a cylinder, and then make a conjecture about the formula for the volume of a cylinder.

Learn Volume of Cylinders

A **cylinder** is a three-dimensional figure with two parallel congruent circular bases connected by a curved surface. The area of the base of a cylinder tells the number of cubic units in one layer. The height tells how many layers there are in the cylinder.

Words	Model
The volume V of a cylinder with radius r is the area of the base B times the height h.	
Symbols	
$V = Bh$, where $B = \pi r^2$ or $V = \pi r^2 h$	

(continued on next page)

Lesson 12-6 • Volume of Cylinders 807

Interactive Presentation

Learn, Volume of Cylinders, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view the formula for the volume of a cylinder expressed in multiple representations.

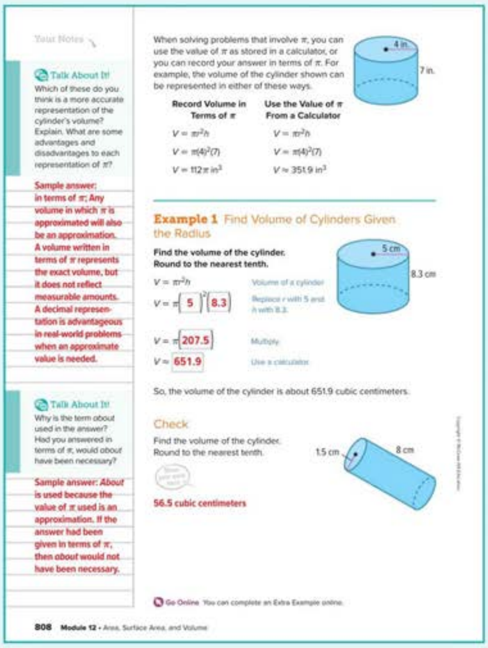
DIFFERENTIATE

Reteaching Activity AL

If students have difficulty using the formula for the volume of a cylinder, have students write clear, step-by-step instructions for finding the volume of a cylinder, that contains the following criteria.

- Instructions for when the radius is given
- Instructions for when the diameter is given
- A drawing of a cylinder with the radius, diameter, and height labeled

Students should refer to and possibly alter their instructions as needed while working through the problems in this lesson.



Your Notes

Talk About It!
Which of these do you think is a more accurate representation of the cylinder's volume? Explain. What are some advantages and disadvantages to each representation of π?

Sample answer: in terms of π; Any volume in which π is approximated will also be an approximation. A volume written in terms of π represents the exact volume, but it does not reflect measurable amounts. A decimal representation is advantageous in real-world problems when an approximate value is needed.

When solving problems that involve π, you can use the value of π as stored in a calculator, or you can record your answer in terms of π. For example, the volume of the cylinder shown can be represented in either of these ways.

Record Volume in Terms of π	Use the Value of π From a Calculator
$V = \pi r^2h$	$V = \pi r^2h$
$V = \pi(4)^2(7)$	$V = \pi(4)^2(7)$
$V = 112\pi$ in³	$V \approx 351.9$ in³

Example 1 Find Volume of Cylinders Given the Radius

Find the volume of the cylinder. Round to the nearest tenth.

$V = \pi r^2h$ Volume of a cylinder

$V = \pi(5)^2(8.3)$ Replace r with 5 and h with 8.3.

$V = \pi(207.5)$ Multiply.

$V \approx 651.9$ Use a calculator.

So, the volume of the cylinder is about 651.9 cubic centimeters.

Talk About It!
Why is the term *about* used in the answer? Had you answered in terms of π, would *about* have been necessary?

Sample answer: *About* is used because the value of π used is an approximation. If the answer had been given in terms of π, then *about* would not have been necessary.

Check

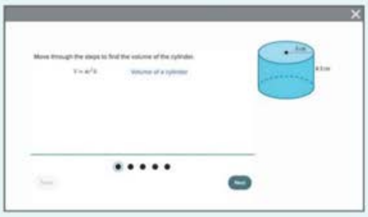
Find the volume of the cylinder. Round to the nearest tenth.

56.5 cubic centimeters

Go Online You can complete an Extra Example online.

808 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 1, Find Volume of Cylinders Given the Radius, Slide 2 of 4

CLICK

On Slide 2 of Example 1, students move through the steps to find the volume of the cylinder.

TYPE

On Slide 2 of Example 1, students determine the approximate volume of the cylinder.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Volume of Cylinders *(continued)*

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Which of these do you think is a more accurate representation of the cylinder's volume? Explain. Sample answer: The most accurate representation of the cylinder's volume is the volume written in terms of π. Since π is an irrational number, any volume in which it is approximated will also be an approximation.

What are some advantages and disadvantages to each representation of π? Sample answer: A volume written in terms of π represents the exact volume, but it does not reflect measurable amounts since π is irrational. A decimal representation is advantageous in real-world problems when an approximate value is needed, though the volume written in this form is not exact.

Example 1 Find Volume of Cylinders Given the Radius

Objective

Students will find the volume of a cylinder, given the radius.

Questions for Mathematical Discourse

SLIDE 2

AL Why is it important to know how to find the area of the base? Sample answer: The volume of the cylinder can be found by multiplying the area of the base by the height of the cylinder.

OL What are some different ways you can represent π in order to find the volume? Sample answer: I can use the π key on the calculator and round my final answer. I can write the volume in terms of π. I can use an approximation of π, such as 3.14.

OL Which representation of π do you prefer? Explain. See students' preferences.

BL What is the difference in volume between using the π key on the calculator and rounding to the nearest tenth, and using 3.14? 651.9 – 651.55, or 0.35 cubic centimeters

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Find Volume of Cylinders Given the Diameter

Objective

Students will find the volume of a cylinder in terms of π, given the diameter.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students make sense of the relationship between the radius and diameter of a circle, in order to first find the radius.

6 Attend to Precision Students should be able to efficiently and accurately calculate the volume, expressing their answer in terms of π.

While discussing the *Talk About It!* question on Slide 3, encourage students to use clear and precise mathematical terms when explaining why $1{,}280\pi$ is considered the exact volume of the cylinder.

Questions for Mathematical Discourse

SLIDE 2

AL Explain why you need to first find the radius. The volume formula uses the radius and height as the input values.

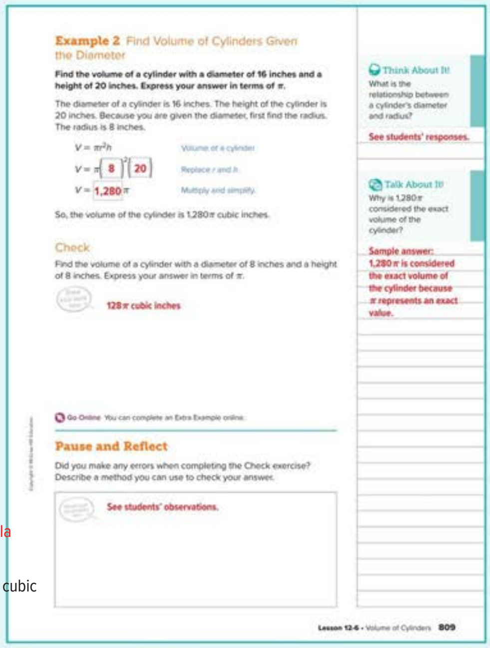
OL Why do you not need to further simplify the volume of $1{,}280\pi$ cubic inches? I am asked to find the volume in terms of π.

OL Describe one advantage and one disadvantage to writing volume in terms of π. Sample answer: One advantage is that the volume in terms of π is the exact value, it has not been approximated or rounded. One disadvantage is that it can be difficult to make sense of the volume; $1{,}280\pi$ is about 4,021 cubic inches, which is easier to grasp mentally.

BL How can you find the volume in cubic feet? Is this volume an exact value? Explain. Sample answer: Divide $1{,}280\pi$ by 12^3, which is about 0.74π cubic foot. It is not an exact value, because I had to round when dividing 1,280 by 12^3.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Find Volume of Cylinders Given the Diameter

Find the volume of a cylinder with a diameter of 16 inches and a height of 20 inches. Express your answer in terms of π.

The diameter of a cylinder is 16 inches. The height of the cylinder is 20 inches. Because you are given the diameter, first find the radius. The radius is 8 inches.

$V = \pi r^2 h$ Volume of a cylinder

$V = \pi(8)^2(20)$ Replace r and h.

$V = 1{,}280\pi$ Multiply and simplify.

So, the volume of the cylinder is $1{,}280\pi$ cubic inches.

Think About It! What is the relationship between a cylinder's diameter and radius? See students' responses.

Talk About It! Why is $1{,}280\pi$ considered the exact volume of the cylinder? Sample answer: $1{,}280\pi$ is considered the exact volume of the cylinder because π represents an exact value.

Check

Find the volume of a cylinder with a diameter of 8 inches and a height of 8 inches. Express your answer in terms of π.

128π cubic inches

Go Online You can complete an Extra Example online.

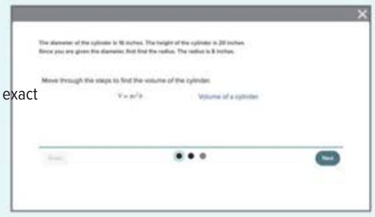
Pause and Reflect

Did you make any errors when completing the Check exercise? Describe a method you can use to check your answer.

See students' observations.

Lesson 12-6 • Volume of Cylinders 809

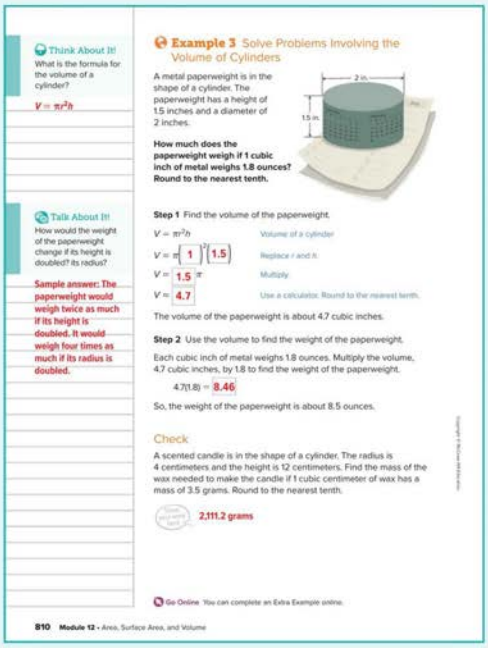
Interactive Presentation



Example 2, Find Volume of Cylinders Given the Diameter, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the volume of the cylinder.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Think About It!
What is the formula for the volume of a cylinder?

$V = \pi r^2 h$

Talk About It!
How would the weight of the paperweight change if its height is doubled? its radius?

Sample answer: The paperweight would weigh twice as much if its height is doubled. It would weigh four times as much if its radius is doubled.

Example 3 Solve Problems Involving the Volume of Cylinders

A metal paperweight is in the shape of a cylinder. The paperweight has a height of 1.5 inches and a diameter of 2 inches.

How much does the paperweight weigh if 1 cubic inch of metal weighs 1.8 ounces? Round to the nearest tenth.

Step 1 Find the volume of the paperweight.

$V = \pi r^2 h$ — Volume of a cylinder

$V = \pi(1)^2(1.5)$ — Replace r and h.

$V = 1.5\pi$ — Multiply

$V \approx 4.7$ — Use a calculator. Round to the nearest tenth.

The volume of the paperweight is about 4.7 cubic inches.

Step 2 Use the volume to find the weight of the paperweight.

Each cubic inch of metal weighs 1.8 ounces. Multiply the volume, 4.7 cubic inches, by 1.8 to find the weight of the paperweight.

$4.7(1.8) = 8.46$

So, the weight of the paperweight is about 8.5 ounces.

Check

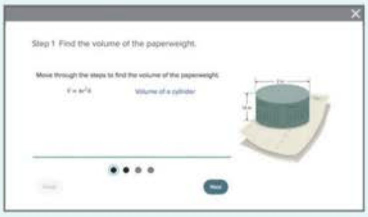
A scented candle is in the shape of a cylinder. The radius is 4 centimeters and the height is 12 centimeters. Find the mass of the wax needed to make the candle if 1 cubic centimeter of wax has a mass of 3.5 grams. Round to the nearest tenth.

2,111.2 grams

Go Online You can complete an Extra Example online.

810 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 3, Solve Problems Involving the Volume of Cylinders, Slide 2 of 5

CLICK

On Slide 2, students move through the steps to find the volume.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Solve Problems Involving the Volume of Cylinders

Objective

Students will solve a real-world problem involving the volume of a cylinder.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to use the formula for the volume of a cylinder to explain and analyze how the weight of the paperweight changes if the height or radius is doubled.

Questions for Mathematical Discourse

SLIDE 2

AL What are the known dimensions of the paperweight? The diameter is 2 inches, and the height is 1.5 inches.

OL Why do you need to find the volume of the paperweight first? Sample answer: Since I know the weight of one cubic inch of metal, I can multiply that by the volume of the paperweight to find the weight of the paperweight.

OL Are you done solving the problem? Explain. no; Sample answer: I need to find the weight of the paperweight. Right now, all I know is the volume of the paperweight.

BL Explain, without calculating, whether the weight of the paperweight will be less than or greater than 1 pound. less than; Sample answer: The volume is 4.7 cubic inches, and each cubic inch weighs 1.8 ounces. A pound is 16 ounces. The product of 4.7 and 1.8 is much less than 16 ounces.

SLIDE 3

AL Explain why multiplication is the operation needed to find the weight. Sample answer: I know the weight of each cubic inch of metal, and the number of cubic inches. The product will give me the weight of all of the cubic inches.

OL What is the weight of the paperweight in pounds? about 0.53 pound

BL If each 0.5 ounce of a paperweight costs $1.75, what is the approximate price of the paperweight? $29.75

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Swimming

Objective

Students will come up with their own strategy to solve an application problem that involves the amount of time it takes to fill a pool.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

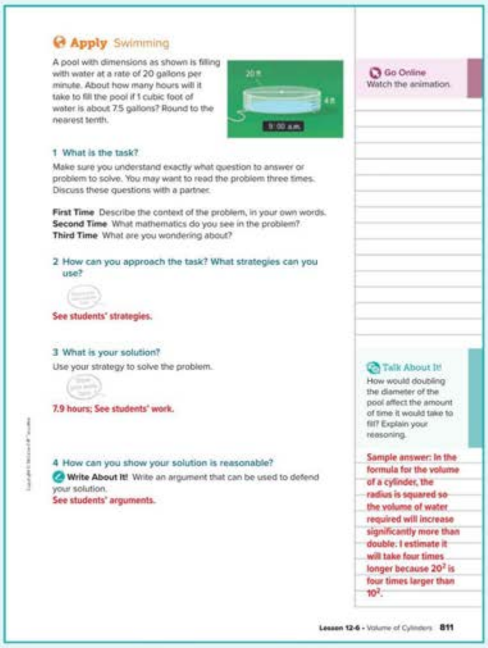
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What dimensions are given?
- What dimensions are needed to find the volume of the pool?
- What information is needed in order to convert cubic feet to gallons?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Swimming

A pool with dimensions as shown is filling with water at a rate of 20 gallons per minute. About how many hours will it take to fill the pool if 1 cubic foot of water is about 7.5 gallons? Round to the nearest tenth.

20 ft
4 ft
9:00 a.m.

Go Online Watch the animation.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

7.9 hours; See students' work.

Talk About It! How would doubling the diameter of the pool affect the amount of time it would take to fill? Explain your reasoning.

Sample answer: In the formula for the volume of a cylinder, the radius is squared so the volume of water required will increase significantly more than double. I estimate it will take four times longer because 20^2 is four times larger than 10^2.

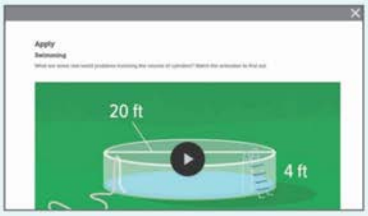
4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Lesson 12-6 • Volume of Cylinders 811

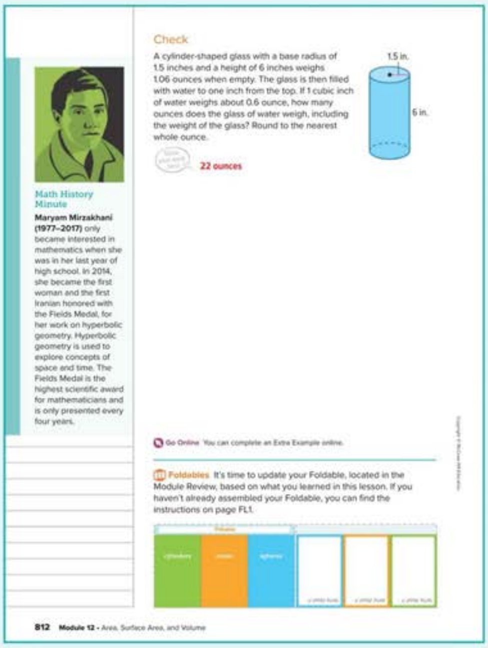
Interactive Presentation



Apply, Swimming

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

A cylinder-shaped glass with a base radius of 1.5 inches and a height of 6 inches weighs 1.06 ounces when empty. The glass is then filled with water to one inch from the top. If 1 cubic inch of water weighs about 0.6 ounce, how many ounces does the glass of water weigh, including the weight of the glass? Round to the nearest whole ounce.

1.5 in.

6 in.

22 ounces

Math History Minute

Maryam Mirzakhani (1977–2017) only became interested in mathematics when she was in her last year of high school. In 2014, she became the first woman and the first Iranian honored with the Fields Medal, for her work on hyperbolic geometry. Hyperbolic geometry is used to explore concepts of space and time. The Fields Medal is the highest scientific award for mathematicians and is only presented every four years.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

812 Module 12 • Area, Surface Area, and Volume

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of volume of cylinders. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Suppose the cylindrical jar has a radius of 6 inches and a height of 5.2 inches. Suppose each marble has a volume of 0.5 cubic inch. Estimate the number of marbles that will fill the cylinder. Write a mathematical argument that can be used to defend your solution.

about 1,176 marbles; Sample answer: The volume of the jar is about 588 in^3, and 588 $in^3 \div 0.5\ in^3 \approx 1{,}176$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 9–13
- ALEKS® Volume of Prisms and Cylinders

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 9, 12, 13
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Volume of Prisms and Cylinders

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Volume of Prisms and Cylinders

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

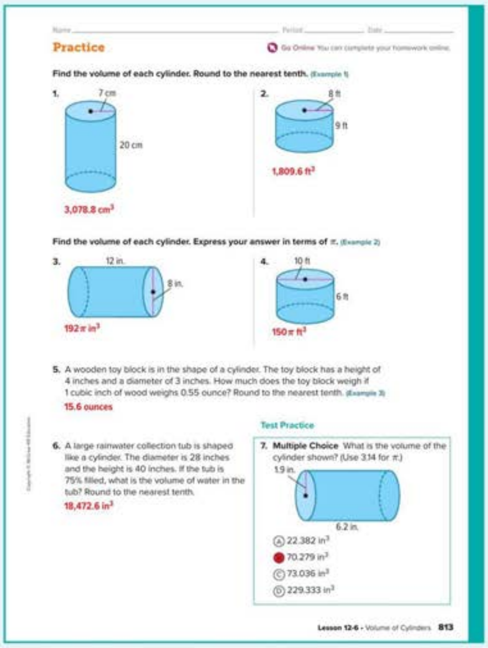
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find volume of cylinders, given the radius	1, 2
1	find the volume of cylinders in terms of π, given the diameter	3, 4
2	solve real-world problems involving the volume of cylinders	5
2	extend concepts learned in class to apply them in new contexts	6, 7
3	solve application problems involving the volume of cylinders	8, 9
3	higher-order and critical thinking skills	10–13

Common Misconception

Some students may forget to square the radius when using the formula for the volume of a cylinder. Encourage students to use estimation to check their answers for reasonableness.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find the volume of each cylinder. Round to the nearest tenth. (Example 1)

1. 7 cm, 20 cm
3,078.8 cm^3

2. 8 ft, 9 ft
1,809.6 ft^3

Find the volume of each cylinder. Express your answer in terms of π. (Example 2)

3. 12 in., 8 in.
192π in^3

4. 10 ft, 6 ft
150π ft^3

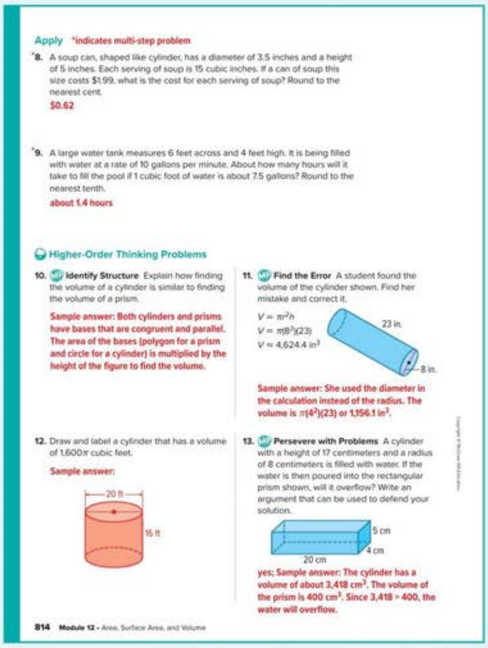
5. A wooden toy block is in the shape of a cylinder. The toy block has a height of 4 inches and a diameter of 3 inches. How much does the toy block weigh if 1 cubic inch of wood weighs 0.55 ounce? Round to the nearest tenth. (Example 3)
15.6 ounces

6. A large rainwater collection tub is shaped like a cylinder. The diameter is 28 inches and the height is 40 inches. If the tub is 75% filled, what is the volume of water in the tub? Round to the nearest tenth.
18,472.6 in^3

Test Practice

7. **Multiple Choice** What is the volume of the cylinder shown? (Use 3.14 for π.)
1.9 in., 6.2 in.
(A) 22.382 in^3
(B) 70.279 in^3
(C) 73.036 in^3
(D) 229.333 in^3

Lesson 12-6 • Volume of Cylinders 813



Apply *indicates multi-step problem

*8. A soup can, shaped like cylinder, has a diameter of 3.5 inches and a height of 5 inches. Each serving of soup is 15 cubic inches. If a can of soup this size costs $1.99, what is the cost for each serving of soup? Round to the nearest cent.
$0.62

*9. A large water tank measures 6 feet across and 4 feet high. It is being filled with water at a rate of 10 gallons per minute. About how many hours will it take to fill the pool if 1 cubic foot of water is about 7.5 gallons? Round to the nearest tenth.
about 1.4 hours

Higher-Order Thinking Problems

10. **Identify Structure** Explain how finding the volume of a cylinder is similar to finding the volume of a prism.
Sample answer: Both cylinders and prisms have bases that are congruent and parallel. The area of the bases (polygon for a prism and circle for a cylinder) is multiplied by the height of the figure to find the volume.

11. **Find the Error** A student found the volume of the cylinder shown. Find her mistake and correct it.
$V = \pi r^2 h$
$V = \pi(8^2)(23)$
$V \approx 4{,}624.4$ in^3

Sample answer: She used the diameter in the calculation instead of the radius. The volume is $\pi(4^2)(23)$ or 1,156.1 in^3.

12. Draw and label a cylinder that has a volume of $1{,}600\pi$ cubic feet.
Sample answer:

13. **Persevere with Problems** A cylinder with a height of 17 centimeters and a radius of 8 centimeters is filled with water. If the water is then poured into the rectangular prism shown, will it overflow? Write an argument that can be used to defend your solution.

yes; Sample answer: The cylinder has a volume of about 3,418 cm^3. The volume of the prism is 400 cm^3. Since 3,418 > 400, the water will overflow.

814 Module 12 • Area, Surface Area, and Volume

Teaching the Mathematical Practices

7 Look For and Make Use of Structure In E xercise 10, students explain how finding the volume of a cylinder is similar to finding the volume of a prism. Encourage students to use the structure of the figures to support their explanation.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students find the mistake in the problem and correct it. Encourage students to locate the error and then explain how to find the correct answer.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 13, students determine if the water will overflow. Encourage students to make a list of what they know and what they need to find in order to solve the problem.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 8–9 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Interview a student.
Use with Exercises 11–12 Have pairs of students interview each other as they complete these problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 11 might be, "Are we given the radius or diameter of the cylinder?"

Lesson 12-7

Volume of Cones

8.G.C.9

LESSON GOAL

Students will find the volume of cones.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Volume of Cones

Learn: Volume of Cones
Example 1: Find Volume of Cones
Example 2: Find Volume of Cones
Apply: Popcorn

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 76 of the *Language Development Handbook* to help your students build mathematical language related to the volume of cones.

ELL You can use the tips and suggestions on page T76 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address the additional cluster **8.G.C** by finding the volume of cones.
Standards for Mathematical Content: **8. G.C.9**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the volume of cylinders.
8.G.C.9

Now
Students find the volume of cones.
8.G.C.9

Next
Students will find the volume of spheres and hemispheres.
8.G.C.9

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of finding volume of cylinders to develop understanding of how to find the volume of cones. They use this understanding to build *fluency* with calculating volume. They *apply* their fluency to solve multi-step real-world problems.

Mathematical Background

A *cone* is a three-dimensional figure with one circular base connected by a curved surface to a single point. The volume of a cone is related to the volume of a cylinder with the same dimensions. The volume of a cone is one-third the volume of a cylinder with the same radius and height.

Interactive Presentation

Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying and dividing with decimals (Exercises 1–4)
- using the area of a circle formula (Exercise 5)

Answers

1. 386.952 **4.** 481.425

2. 534.4 **5.** $A \approx 66.48$ cm²

3. 623

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about filling conical bags with treats.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What are some real-world examples of *cones* in everyday life? Sample answer: an ice cream cone, a cone you put around a dog or cat's head, a traffic cone

Explore Volume of Cones

Objective

Students will explore the relationship between the volume of cones and the volume of cylinders.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will compare a cylinder and a cone with the same height and base area. Using rice, students will explore how the cylinder and cones are related and how this relationship can help form the formula used to find the volume of a cone.

Inquiry Question

How can you determine the volume of a cone? Sample answer: Multiply the area of the base by the height. Then multiply the product by $\frac{1}{3}$ or divide it by three.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 3 are shown.

Talk About It!

SLIDE 3

Mathematical Discourse

How can you use the uncooked rice and the containers you built to determine which container has the lesser volume and which has the greater? Sample answer: I can fill the container that I believe to have a lesser volume to the top with rice. Then, I can transfer that amount of rice to the container that I believe has the greater volume. If this amount of rice does not fill the cylinder, then I can conclude its volume is greater. If this amount overfills the container, then I can conclude that its volume is less than the container from which the rice is poured.

Which object do you think will hold the most rice? the least rice? Explain your reasoning. See students' responses.

(continued on next page)

Interactive Presentation

Explore, Slide 1of 6

Explore, Slide 2 of 6

Interactive Presentation

Explore, Slide 4 of 6

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.

Explore Volume of Cones *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to examine the similarity between the cylinder and cone using rice and the volume formulas.

5 Use Appropriate Tools Strategically Students will use paper cups and uncooked rice to compare the volume of cylinders and cones.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

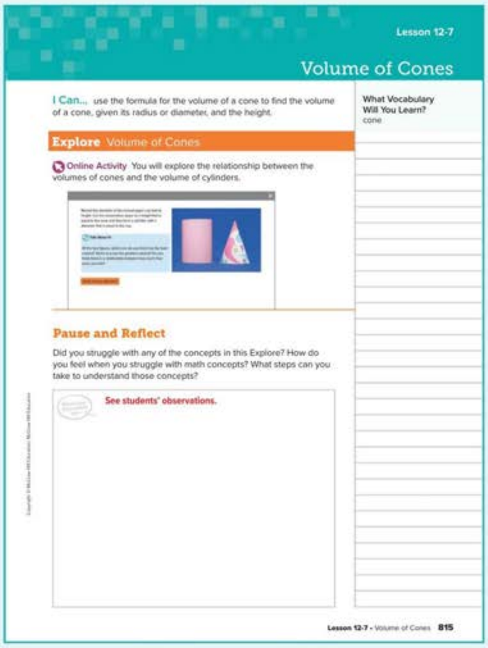
SLIDE 4

Mathematical Discourse

Based on your results, what is the relationship between the volume of a cone and the volume of a cylinder with the same dimensions? Sample answer: The volume of a cylinder is three times greater than the volume of a cone with the same dimensions.

Teaching Notes

Before moving from the *Explore, Volume of Cones*, to the *Learn, Volume of Cones*, have students discuss the *Pause and Reflect* question with a partner. Encourage each student to openly talk about whether or not they struggled with any part of the Explore activity, and if so, what they did to help overcome any struggle. Have them write down any remaining questions they have, and share those with their partner. Pairs should work to resolve any outstanding questions. If they are unable to come to a resolution, have them meet with other pairs of students in the class. Walk around the room, listening to the conversations and encourage students to assist each other before stepping in.



Lesson 12-7

Volume of Cones

I Can... use the formula for the volume of a cone to find the volume of a cone, given its radius or diameter, and the height.

What Vocabulary Will You Learn?
cone

Explore Volume of Cones

Online Activity You will explore the relationship between the volumes of cones and the volume of cylinders.

Pause and Reflect

Did you struggle with any of the concepts in this Explore? How do you feel when you struggle with math concepts? What steps can you take to understand those concepts?

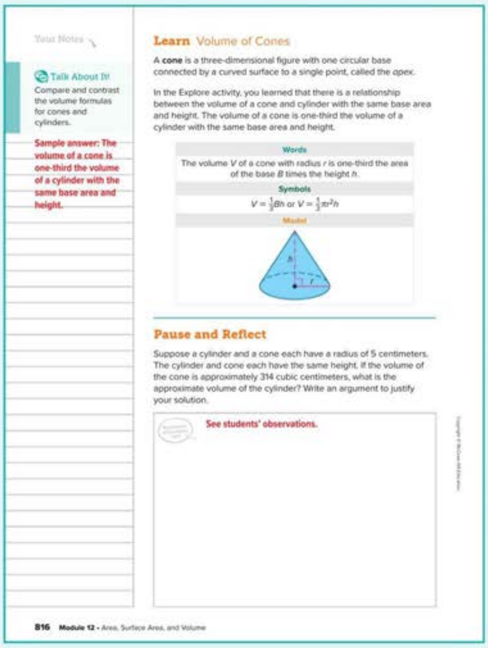
See students' observations.

Lesson 12-7 • Volume of Cones 815

DIFFERENTIATE

Language Development Activity ELL

Some students may struggle with identifying the diameter, radius, and height of a cone. In pairs have students write the definitions for diameter, radius, and height of a cone. If they have difficulty writing the definitions, remind them that the radius is half of the diameter and the length of the altitude (the segment from the apex to the center of the circular base) is called the height. Have students draw and label the parts of several cones, including some labeled with the diameter and some with the radius. It is important for students to understand that the radius is used in the formula $V = \frac{1}{3}\pi r^2 h$, for the volume of a cone.


Your Notes

Talk About It!
Compare and contrast the volume formulas for cones and cylinders.

Sample answer: The volume of a cone is one-third the volume of a cylinder with the same base area and height.

Learn Volume of Cones

A **cone** is a three-dimensional figure with one circular base connected by a curved surface to a single point, called the *apex*.

In the Explore activity, you learned that there is a relationship between the volume of a cone and cylinder with the same base area and height. The volume of a cone is one-third the volume of a cylinder with the same base area and height.

Words
The volume V of a cone with radius r is one-third the area of the base B times the height h.

Symbols
$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2h$

Model

Pause and Reflect

Suppose a cylinder and a cone each have a radius of 5 centimeters. The cylinder and cone each have the same height. If the volume of the cone is approximately 314 cubic centimeters, what is the approximate volume of the cylinder? Write an argument to justify your solution.

See students' observations.

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Interactive Presentation

Learn, Volume of Cones, Slide 1 of 2

FLASHCARDS

On Slide 1, students use Flashcards to view the formula for the volume of a cone expressed in multiple representations.

8.G.C.9

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Volume of Cones

Objective

Students will learn how to find the volume of cones.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure W hile discussing the *Talk About It!* question on Slide 2, encourage students to explain the similarities and differences between the formula for the volume of a cone and the formula for the volume of a cylinder. Encourage students to make connections between the formula for the volume of a cylinder and the formula for the volume of a cone with the same radius and height.

Teaching Notes

SLIDE 1

Students will learn the definition of a *cone*. Have them select the *Words, Symbols*, and *Model* flashcards to view how the volume of a cone can be expressed in these representations.

Talk About It!

SLIDE 2

Mathematical Discourse

Compare and contrast the volume formulas for cones and cylinders.

Sample answer: The volume of a cone is one-third the volume of a cylinder with the same base area and height.

Example 1 Find Volume of Cones

Objective

Students will find the volume of a cone in terms of π.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively W hile discussing the *Talk About It!* question on Slide 3, encourage students to make sense of the quantities in the volume formula, as opposed to just calculating with them. This will help them remember what volume means conceptually.

6 Attend to Precision Students should use precision in order to calculate the volume in terms of π.

7 Look For and Make Use of Structure Encourage students to study the figure to correctly determine which volume formula to use, based on the information given.

Questions for Mathematical Discourse

SLIDE 2

AL In the formula $V = \frac{1}{3}\pi r^2 h$, what does πr^2 represent? the area of the circular base

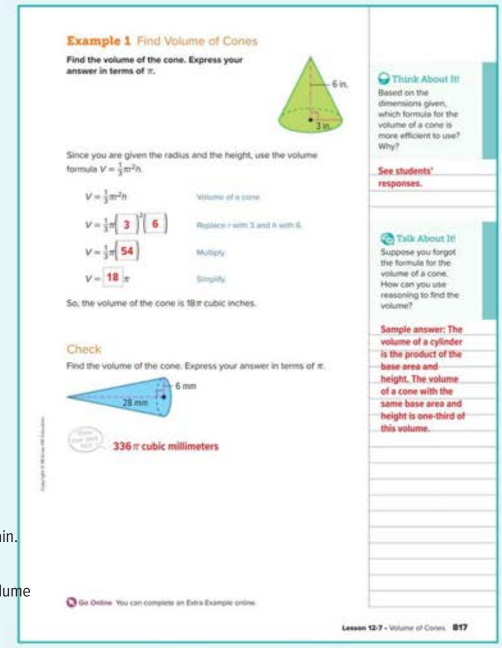
OL Is the volume 18π an exact volume or an approximation? Explain. It is the exact volume because is not rounded.

OL Without calculating, compare the volume of the cone to the volume of a cylinder with the same radius and height. The volume of the cone is one-third the volume of the cylinder. The volume of the cylinder is 54π cubic inches.

BL How many cones of this size will fit into a cylinder with a base area of 18π square inches and a height of 12 inches? Explain. 12 cones; Sample answer: The volume of the cylinder is 216π cubic inches. Divide 216π by 18π.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 1 Find Volume of Cones

Find the volume of the cone. Express your answer in terms of π.

6 in.
3 in.

Since you are given the radius and the height, use the volume formula $V = \frac{1}{3}\pi r^2 h$.

$V = \frac{1}{3}\pi r^2 h$ — Volume of a cone

$V = \frac{1}{3}\pi(3)^2(6)$ — Replace r with 3 and h with 6.

$V = \frac{1}{3}\pi(54)$ — Multiply

$V = 18\pi$ — Simplify

So, the volume of the cone is 18π cubic inches.

Think About It! Based on the dimensions given, which formula for the volume of a cone is more efficient to use? Why?

See students' responses.

Talk About It! Suppose you forgot the formula for the volume of a cone. How can you use reasoning to find the volume?

Sample answer: The volume of a cylinder is the product of the base area and height. The volume of a cone with the same base area and height is one-third of this volume.

Check

Find the volume of the cone. Express your answer in terms of π.

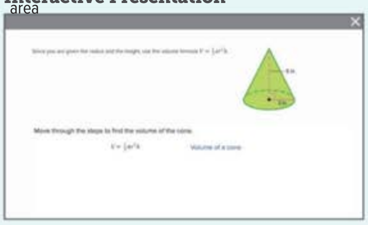
6 mm
28 mm

336π cubic millimeters

Go Online You can complete an Extra Example online.

Lesson 12-7 • Volume of Cones 817

Interactive Presentation



Example 1, Find Volume of Cones, Slide 2 of 4

CLICK

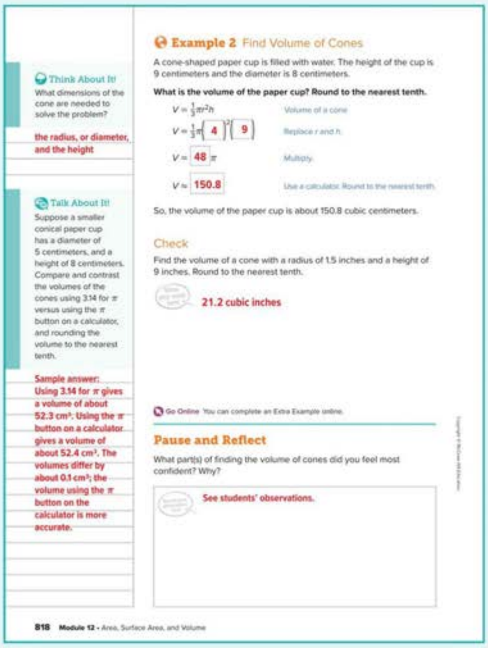
On Slide 2, students move through the steps to find the volume.

TYPE

On Slide 2, students determine the volume of the cone.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Example 2 Find Volume of Cones

A cone-shaped paper cup is filled with water. The height of the cup is 9 centimeters and the diameter is 8 centimeters.

What is the volume of the paper cup? Round to the nearest tenth.

$V = \frac{1}{3}\pi r^2 h$ — Volume of a cone

$V = \frac{1}{3}\pi(4)^2(9)$ — Replace r and h.

$V = 48\pi$ — Multiply.

$V \approx 150.8$ — Use a calculator. Round to the nearest tenth.

So, the volume of the paper cup is about 150.8 cubic centimeters.

Think About It! What dimensions of the cone are needed to solve the problem? **the radius, or diameter, and the height**

Talk About It! Suppose a smaller conical paper cup has a diameter of 5 centimeters, and a height of 8 centimeters. Compare and contrast the volumes of the cones using 3.14 for π versus using the π button on a calculator, and rounding the volume to the nearest tenth.

Sample answer: Using 3.14 for π gives a volume of about 52.3 cm³. Using the π button on a calculator gives a volume of about 52.4 cm³. The volumes differ by about 0.1 cm³; the volume using the π button on the calculator is more accurate.

Check

Find the volume of a cone with a radius of 1.5 inches and a height of 9 inches. Round to the nearest tenth.

21.2 cubic inches

Go Online You can complete an Extra Example online.

Pause and Reflect

What part(s) of finding the volume of cones did you feel most confident? Why?

See students' observations.

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Interactive Presentation

Example 2, Find Volume of Cones, Slide 2 of 4

CLICK On Slide 2, students move through the steps to find the volume.

TYPE On Slide 2, students determine the approximate volume of the paper cup.

CHECK Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Example 2 Find Volume of Cones

Objective

Students will find the volume of a cone in a real-world context.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of the given information to determine they must first find the radius.

6 Attend to Precision Students should be able to efficiently and accurately calculate the volume, using the π button on a calculator, and rounding to the desired degree of precision.

While discussing the *Talk About It!* question on Slide 3, encourage students to understand and be able to compare and contrast the different approaches for generating two approximations.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you replace 4 for *r* in the volume formula? I am given the diameter is 8 centimeters, but the formula uses the radius. The radius is 4 centimeters.

OL Is the final volume an approximation or an exact answer? Explain. approximation; Sample answer: I used the π button on the calculator and rounded to the nearest tenth.

OL Suppose it takes 14 paper cups to fill a pitcher. What is the volume of the pitcher? The volume of the pitcher is about 2,111.2 cubic centimeters.

BL One milliliter of fluid is approximately equal to one cubic centimeter. How many milliliters does one paper cup hold? About how many liters is this? about 150.8 milliliters; about 0.15 liter

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Popcorn

Objective

Students will come up with their own strategy to solve an application problem that involves the costs of popcorn.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What dimensions are given?
- What dimensions are needed to find the volume of the cylindrical container? the conical container?
- How can you find the cost of the popcorn in each container?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Popcorn

A family-owned movie theater offers popcorn in the sizes shown. Their cost for the popcorn is $0.09 per cubic inch. If each container is filled to the top, what is the difference between the costs of the popcorn in the two containers?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

$6.79 − $2.26 = $4.53; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

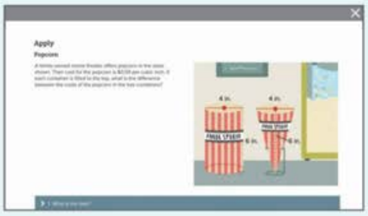
See students' arguments.

Talk About It!
What do you notice about the relationship between the cost of a cylindrical container and the cost of a conical container?

Sample answer: The cost of a cylindrical container is three times the cost of a conical container with the same radius and height.

Lesson 12-7 • Volume of Cones 819

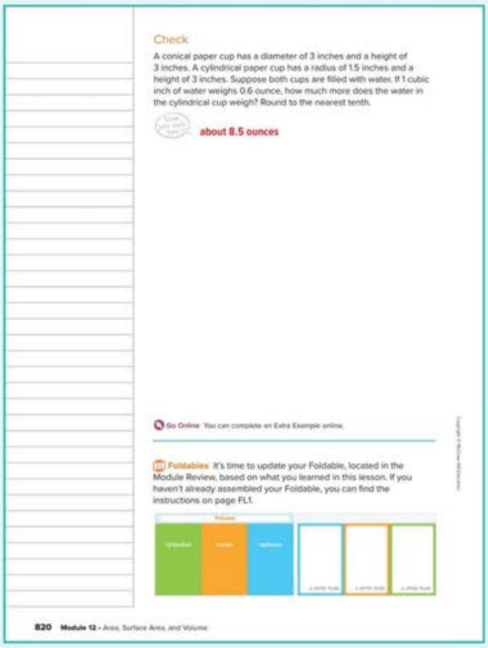
Interactive Presentation



Apply, Popcorn

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

A conical paper cup has a diameter of 3 inches and a height of 3 inches. A cylindrical paper cup has a radius of 1.5 inches and a height of 3 inches. Suppose both cups are filled with water. If 1 cubic inch of water weighs 0.6 ounce, how much more does the water in the cylindrical cup weigh? Round to the nearest tenth.

about 8.5 ounces

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of volume of cones. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5–9 odd, 11–14
- ALEKS Volume of Pyramids, Cones, and Spheres

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–7, 9, 11, 13
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS Volume of Pyramids, Cones, and Spheres

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Volume of Pyramids, Cones, and Spheres

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Exit Ticket

Refer to the Exit Ticket slide. Suppose the conical treat bags have a diameter of 2 inches and a height of 5 inches. Each jelly bean has a volume of 0.12 cubic inch and each piece of popcorn has a volume of 0.2 cubic inch. About how many more jelly beans can fit in a treat bag than pieces of popcorn? Assume no gaps of space between the jelly beans or pieces of popcorn in each bag. Write a mathematical argument that can be used to defend your solution. Sample answer: Find the volume of the conical treat bag, which is about 5.2 cubic inches. Divide the volume by 0.12 to find the approximate number of jelly beans that will fit in a treat bag, which is about 43 jelly beans. Divide the volume by 0.2 to find the approximate number of pieces of popcorn that will fit in a treat bag, which is about 26 pieces of popcorn. Then subtract 26 from 43, which is 17. So, assuming no gaps, about 17 more jelly beans will fit in a treat bag than number of pieces of popcorn.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

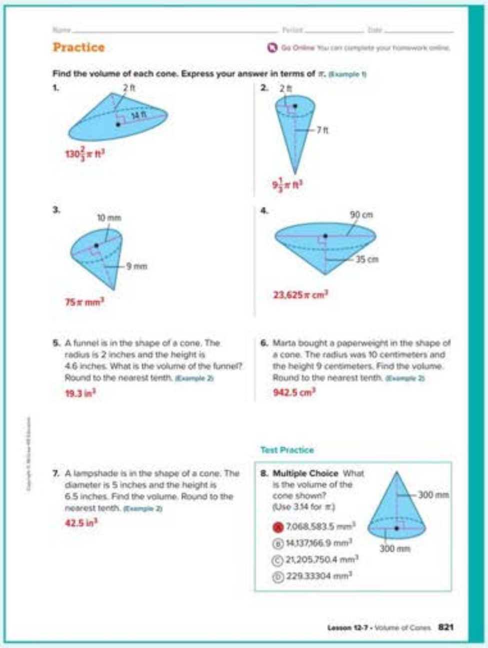
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the volume of cones in terms of π	1–4
2	find the volume of cones in real-world contexts	5–7
2	extend concepts learned in class to apply them in new contexts	8
3	solve application problems involving the volume of cones	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

In Exercises 3, 4, 7, and 8, some students may incorrectly substitute the value of the diameter, instead of the radius, in the formula $V = \frac{1}{3}\pi r^2 h$. Encourage students to use caution when substituting values into the volume formula. Have them first identfiy the information they are given, and determine whether they first need to find the radius.



Name ____ Period ____ Date ____

Practice

Go Online You can complete your homework online.

Find the volume of each cone. Express your answer in terms of π. (Example 1)

1. (2 ft, 14 ft) $130\frac{2}{3}\pi$ ft³

2. (2 ft, 7 ft) $9\frac{1}{3}\pi$ ft³

3. (10 mm, 9 mm) 75π mm³

4. (90 cm, 35 cm) $23{,}625\pi$ cm³

5. A funnel is in the shape of a cone. The radius is 2 inches and the height is 4.6 inches. What is the volume of the funnel? Round to the nearest tenth. (Example 2)
19.3 in³

6. Marta bought a paperweight in the shape of a cone. The radius was 10 centimeters and the height 9 centimeters. Find the volume. Round to the nearest tenth. (Example 2)
942.5 cm³

7. A lampshade is in the shape of a cone. The diameter is 5 inches and the height is 6.5 inches. Find the volume. Round to the nearest tenth. (Example 2)
42.5 in³

Test Practice

8. **Multiple Choice** What is the volume of the cone shown? (Use 3.14 for π.) (300 mm, 300 mm)
   - Ⓐ 7,068,583.5 mm³
   - Ⓑ 14,137,166.9 mm³
   - Ⓒ 21,205,750.4 mm³
   - Ⓓ 229.33304 mm³

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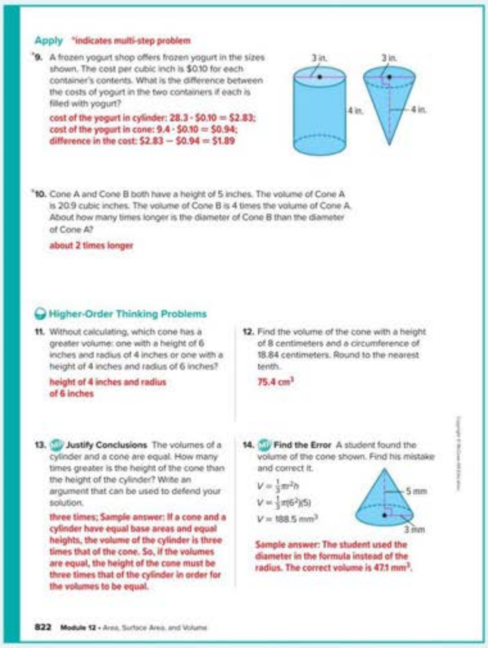
Interactive Presentation

Exit Ticket



Apply *indicates multi-step problem

*9. A frozen yogurt shop offers frozen yogurt in the sizes shown. The cost per cubic inch is $0.10 for each container's contents. What is the difference between the costs of yogurt in the two containers if each is filled with yogurt?

cost of the yogurt in cylinder: 28.3 · $0.10 = $2.83; cost of the yogurt in cone: 9.4 · $0.10 = $0.94; difference in the cost: $2.83 − $0.94 = $1.89

*10. Cone A and Cone B both have a height of 5 inches. The volume of Cone A is 20.9 cubic inches. The volume of Cone B is 4 times the volume of Cone A. About how many times longer is the diameter of Cone B than the diameter of Cone A?

about 2 times longer

Higher-Order Thinking Problems

11. Without calculating, which cone has a greater volume: one with a height of 6 inches and radius of 4 inches or one with a height of 4 inches and radius of 6 inches?

height of 4 inches and radius of 6 inches

12. Find the volume of the cone with a height of 8 centimeters and a circumference of 18.84 centimeters. Round to the nearest tenth.

75.4 cm^3

13. **MP Justify Conclusions** The volumes of a cylinder and a cone are equal. How many times greater is the height of the cone than the height of the cylinder? Write an argument that can be used to defend your solution.

three times; Sample answer: If a cone and a cylinder have equal base areas and equal heights, the volume of the cylinder is three times that of the cone. So, if the volumes are equal, the height of the cone must be three times that of the cylinder in order for the volumes to be equal.

14. **MP Find the Error** A student found the volume of the cone shown. Find his mistake and correct it.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(6^2)(5)$

$V = 188.5\ mm^3$

Sample answer: The student used the diameter in the formula instead of the radius. The correct volume is 47.1 mm^3.

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MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 13, students determine how many times greater the height of the cone is compared to the height of the cylinder. Encourage students to construct a response that supports their answer.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students find the student's mistake and correct it. Encourage students to locate the error and then explain how to find the correct answer.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.

Use with Exercises 9–10 Have students work in pairs. Have students individually read Exercise 9 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 10.

Make sense of the problem.

Use with Exercise 14 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that the diameter can be used in the formula for the volume of a cone. Have each pair or group of students present their explanations to the class.

Lesson 12-8

Volume of Spheres

8.G.C.9

LESSON GOAL

Students will find the volume of spheres and hemispheres.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Volume of Spheres
Example 1: Find Volume of Spheres
Example 2: Find Volume of Spheres
Example 3: Find Volume of Spheres
Learn: Volume of Hemispheres
Example 4: Find Volume of Hemispheres
Apply: Packaging

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Surface Area of Cylinders, Cones, and Spheres		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 77 of the *Language Development Handbook* to help your students build mathematical language related to the volume of spheres.

ELL You can use the tips and suggestions on page T77 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Additional Cluster(s): In this lesson, students address the additional cluster **8.G.C** by finding the volume of spheres and hemispheres.
Standards for Mathematical Content: **8. G.C.9**
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the volume of cones.
8.G.C.9

Now
Students find the volume of spheres and hemispheres.
8.G.C.9

Next
Students will find the volume and surface area of composite solids.
7.G.B.6, 8.G.C.9

Rigor

The Three Pillars of Rigor

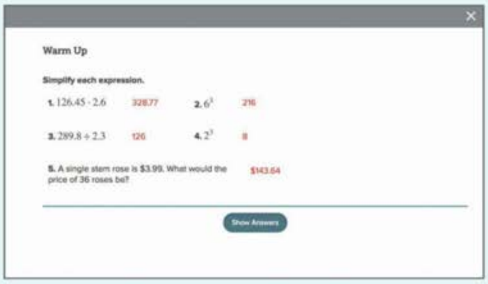
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to expand on their *understanding* of volume, by finding the volume of spheres and hemispheres. They use this understanding to build *fluency* with calculating volume. They *apply* their fluency to solve multi-step real-world problems.

Mathematical Background

A *sphere* is a three-dimensional figure in which every point on the surface is the same distance from a center point. The volume of a sphere is four-thirds the product of π and the cube of the radius. Cross sections of spheres are circles. A *hemisphere* is one of two congruent halves of a sphere. The volume of a hemisphere is half the volume of the corresponding sphere.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- multiplying and dividing with decimals (Exercises 1, 3, 5)
- finding cubes of numbers (Exercises 2, 4)

Answers

1. 328.77
2. 216
3. 126
4. 8
5. $143.64

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about different circumferences of basketballs.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The prefix *hemi-* means *half*. What does this tell you about the term *hemisphere*? A hemisphere is half of a sphere.
- An example of a *sphere* is a basketball. A non-example of a sphere is a picture of a basketball. Describe how these are different. Sample answer: A basketball is three-dimensional and a picture of a basketball is two-dimensional.

Learn Volume of Spheres

Objective

Students will learn how to find the volume of spheres.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- Find sample answers for the *Talk About It!* questions.

Example 1 Find Volume of Spheres

Objective

Students will find the volume of a sphere in terms of π.

Teaching the Mathematical Practices

6 Attend to Precision Encourage students to calculate the volume of the sphere efficiently and accurately, paying careful attention to the units for volume, and expressing the exact volume in terms of π.

Questions for Mathematical Discourse

SLIDE 1

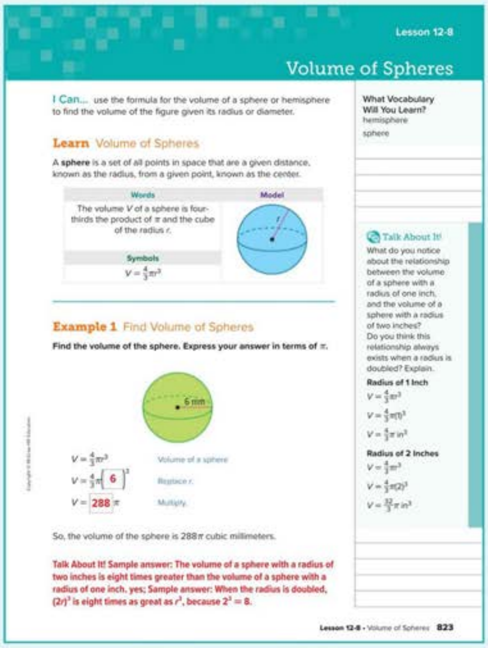
AL Why is the radius cubed when determining the volume? Sample answer: Volume is measured in cubic units, so three dimensions are needed.

OL Describe a disadvantage to writing the volume in terms of π. Sample answer: It might be difficult to conceptualize how large the sphere is, when the volume is written in terms of π.

BL A classmate found the volume to be 216π cubic millimeters. What was the likely mistake? Sample answer: The classmate forgot to multiply by $\frac{4}{3}$ in the formula.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 12-8

Volume of Spheres

I Can... use the formula for the volume of a sphere or hemisphere to find the volume of the figure given its radius or diameter.

What Vocabulary Will You Learn?
hemisphere
sphere

Learn Volume of Spheres

A **sphere** is a set of all points in space that are a given distance, known as the radius, from a given point, known as the center.

Words	Model
The volume V of a sphere is four-thirds the product of π and the cube of the radius r.	
Symbols	
$V = \frac{4}{3}\pi r^3$	

Example 1 Find Volume of Spheres

Find the volume of the sphere. Express your answer in terms of π.

6 mm

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3}\pi(6)^3$ Replace r.

$V = 288\pi$ Multiply.

So, the volume of the sphere is 288π cubic millimeters.

Talk About It! What do you notice about the relationship between the volume of a sphere with a radius of one inch, and the volume of a sphere with a radius of two inches? Do you think this relationship always exists when a radius is doubled? Explain.

Radius of 1 Inch

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1)^3$

$V = \frac{4}{3}\pi \text{ in}^3$

Radius of 2 Inches

$V = \frac{4}{3}\pi r^3$

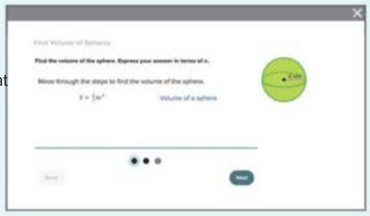
$V = \frac{4}{3}\pi(2)^3$

$V = \frac{32}{3}\pi \text{ in}^3$

Talk About It! Sample answer: The volume of a sphere with a radius of two inches is eight times greater than the volume of a sphere with a radius of one inch. yes; Sample answer: When the radius is doubled, $(2r)^3$ is eight times as great as r^3, because $2^3 = 8$.

Lesson 12-8 • Volume of Spheres 823

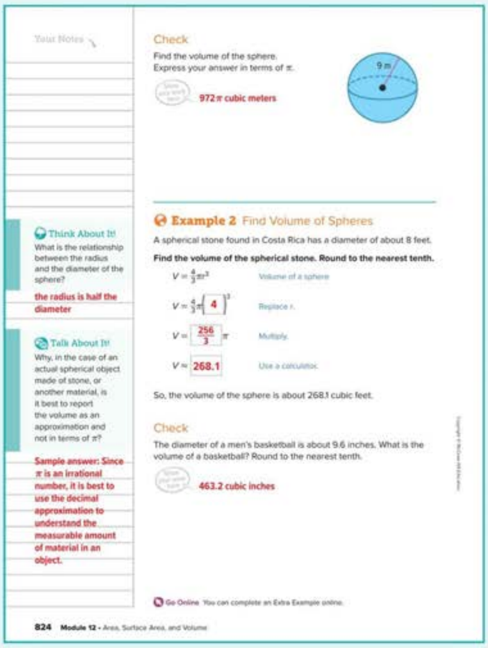
Interactive Presentation



Example 1, Find Volume of Spheres, Slide 1 of 2

CLICK On Slide 1, students move through the steps to find the volume.

CHECK Students complete the Check exercise online to determine if they are ready to move on.



Your Notes

Check

Find the volume of the sphere. Express your answer in terms of π.

972π cubic meters

9 m

Think About It!

What is the relationship between the radius and the diameter of the sphere?

the radius is half the diameter

Example 2 Find Volume of Spheres

A spherical stone found in Costa Rica has a diameter of about 8 feet.

Find the volume of the spherical stone. Round to the nearest tenth.

$V = \frac{4}{3}\pi r^3$ Volume of a sphere

$V = \frac{4}{3}\pi(4)^3$ Replace r.

$V = \frac{256}{3}\pi$ Multiply.

$V \approx 268.1$ Use a calculator.

So, the volume of the sphere is about 268.1 cubic feet.

Talk About It!

Why, in the case of an actual spherical object made of stone, or another material, is it best to report the volume as an approximation and not in terms of π?

Sample answer: Since π is an irrational number, it is best to use the decimal approximation to understand the measurable amount of material in an object.

Check

The diameter of a men's basketball is about 9.6 inches. What is the volume of a basketball? Round to the nearest tenth.

463.2 cubic inches

Go Online You can complete an Extra Example online.

824 Module 12 • Area, Surface Area, and Volume

Interactive Presentation

Example 2, Find Volume of Spheres, Slide 2 of 4

CLICK

On Slide 2, students move through the steps to find the volume.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Find Volume of Spheres

Objective

Students will find the volume of a sphere in a real-world context.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discus sing the *Talk About It!* question on Slide 3, encourage students to make sense of real-world situations in which it is more meaningful to report an approximation of volume, rather than the exact volume.

6 Attend to Precision Encourage students to calculate the volume efficiently and accurately, by first finding the radius, and then expressing their answer with the appropriate degree of precision.

Questions for Mathematical Discourse

SLIDE 2

AL What measurement is given in the figure? What does this tell you? the diameter; I need to first find the radius.

OL What is the exact volume of the sphere? $\frac{256}{3}\pi$ cubic feet

OL Explain why cubic feet are used as the unit of measure. Sample answer: When finding volume, three dimensions are multiplied. In this case, the radius is cubed. So, the units are cubic feet.

BL If you wanted to paint the sphere, can you use the volume to determine how much paint you need? Explain. no; Sample answer: The volume tells how much space is contained in an object. I would need to find the surface area of the sphere in order to find how much paint is needed to cover the surface of the sphere.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 3 Find Volume of Spheres

Objective

Students will solve a real-world problem involving the volume of a sphere.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discussing the *Talk About It!* questions on Slide 4, encourage students to make sense of the quantities given in the proportion and to reason why it will take longer than one minute to inflate the volleyball, based on the quantities given in the proportion.

Questions for Mathematical Discourse

SLIDE 2

AL Before determining the time, what do you need to find? the volume of the volleyball

OL Explain why $\frac{4}{3}\pi(5)^3$ represents the volume of the volleyball. The diameter is 10 inches, so the radius is 5 inches. This expression represents the volume formula with 5 replacing r.

BL Why do you think the volume is not written in terms of π? Sample answer: Since the volume will be used for further calculations, it makes sense to round the volume instead of writing it in terms of π.

SLIDE 3

AL What information do you know? What do you need to find? I know the two volumes, and I know the time to inflate to one of the volumes. I need to find the time to inflate to the other volume.

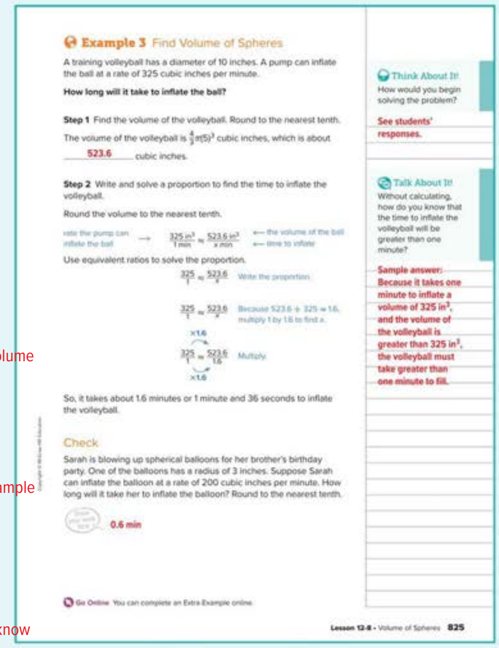
OL What does x represent in the proportion? the amount of time it will take to inflate the ball

OL Without calculating, estimate the time to inflate the volleyball. Sample answer: 523.6 $\text{in}^3 > 325\ \text{in}^3$, but less than 2(325), or 650 in^3. So, it will take somewhere between 1–2 minutes to inflate the ball.

BL About how many minutes will it take to inflate a basket of 30 volleyballs? Assume no resting time is needed between volleyballs. 48 minutes

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Find Volume of Spheres

A training volleyball has a diameter of 10 inches. A pump can inflate the ball at a rate of 325 cubic inches per minute.

How long will it take to inflate the ball?

Step 1 Find the volume of the volleyball. Round to the nearest tenth.

The volume of the volleyball is $\frac{4}{3}\pi(5)^3$ cubic inches, which is about 523.6 cubic inches.

Step 2 Write and solve a proportion to find the time to inflate the volleyball.

Round the volume to the nearest tenth.

rate the pump can inflate the ball → $\frac{325\ \text{in}^3}{1\ \text{min}} \approx \frac{523.6\ \text{in}^3}{x\ \text{min}}$ ← the volume of the ball ← time to inflate

Use equivalent ratios to solve the proportion.

$\frac{325}{1} \approx \frac{523.6}{x}$ Write the proportion.

$\frac{325}{1} \approx \frac{523.6}{x}$ Because 523.6 ÷ 325 ≈ 1.6, multiply 1 by 1.6 to find x.

$\frac{325}{1} \approx \frac{523.6}{1.6}$ (×1.6) Multiply.

So, it takes about 1.6 minutes or 1 minute and 36 seconds to inflate the volleyball.

Think About It! How would you begin solving the problem?

See students' responses.

Talk About It! Without calculating, how do you know that the time to inflate the volleyball will be greater than one minute?

Sample answer: Because it takes one minute to inflate a volume of 325 in^3, and the volume of the volleyball is greater than 325 in^3, the volleyball must take greater than one minute to fill.

Check

Sarah is blowing up spherical balloons for her brother's birthday party. One of the balloons has a radius of 3 inches. Suppose Sarah can inflate the balloon at a rate of 200 cubic inches per minute. How long will it take her to inflate the balloon? Round to the nearest tenth.

0.6 min

Go Online You can complete an Extra Example online.

Lesson 12-8 • Volume of Spheres 825

Interactive Presentation

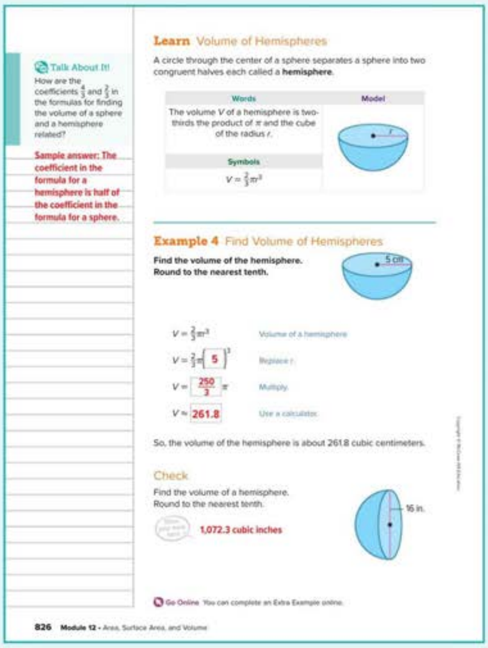
Example 3, Find Volume of Spheres, Slide 3 of 5

TYPE

On Slide 3, students determine the time to inflate the volleyball.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Learn Volume of Hemispheres

A circle through the center of a sphere separates a sphere into two congruent halves each called a **hemisphere**.

Words	Model
The volume V of a hemisphere is two-thirds the product of π and the cube of the radius r.	
Symbols	
$V = \frac{2}{3}\pi r^3$	

Talk About It! How are the coefficients $\frac{4}{3}$ and $\frac{2}{3}$ in the formulas for finding the volume of a sphere and a hemisphere related?

Sample answer: The coefficient in the formula for a hemisphere is half of the coefficient in the formula for a sphere.

Example 4 Find Volume of Hemispheres

Find the volume of the hemisphere. Round to the nearest tenth.

$V = \frac{2}{3}\pi r^3$ Volume of a hemisphere

$V = \frac{2}{3}\pi(5)^3$ Replace r.

$V = \frac{250}{3}\pi$ Multiply.

$V \approx 261.8$ Use a calculator.

So, the volume of the hemisphere is about 261.8 cubic centimeters.

Check

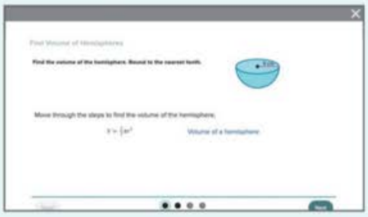
Find the volume of a hemisphere. Round to the nearest tenth.

1,072.3 cubic inches

Go Online You can complete an Extra Example online.

826 Module 12 • Area, Surface Area, and Volume

Interactive Presentation



Example 4, Find Volume of Hemispheres, Slide 1 of 2

CLICK

On Slide 1 of Example 4, students move through the steps to find the volume.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Learn Volume of Hemispheres

Objective

Students will learn how to find the volume of hemispheres.

Go Online to find additional teaching notes and Teaching the Mathematical Practices.

Talk About It!

SLIDE 2

Mathematical Discourse

How are the coefficients $\frac{4}{3}$ and $\frac{2}{3}$ in the formula for a sphere and a hemisphere related? Sample answer: The coefficient in the formula for a hemisphere is half of the coefficient in the formula for a sphere.

Example 4 Find Volume of Hemispheres

Objective

Students will find the volume of a hemisphere.

Questions for Mathematical Discourse

SLIDE 1

AL What is a hemisphere? half of a sphere

AL What measure is the input value in the formula for the volume of a hemisphere? the radius

OL If you forgot the formula for the volume of a hemisphere, what could you do? Sample answer: Find the volume of the entire sphere, then divide by 2.

BL When the radius of a sphere is doubled, its volume increases by a factor of 8. Is the same true for a hemisphere? Explain. yes; Sample answer: The volume formula for a hemisphere cubes the radius. This means that doubling the radius will increase the volume by a factor of 2^3 or 8.

Go Online

- Find additional teaching notes and Teaching the Mathematical Practices.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Language Development Activity ELL

Hemi- is a prefix, from the Greek language, that means *half*. Students may be more familiar with the Latin prefix *semi-* that also means *half*, as this prefix is more common in the English language. Ask students to generate terms that begin with the prefix *semi-*, such as *semicircle*, *semiannual*, *semifinal*, *semicolon*, etc. While the prefixes *hemi-* and *semi-* are different, they both mean *half*. In mathematics, half of a sphere is referred to as a *hemisphere*.

Apply Packaging

Objective

Students will come up with their own strategy to solve an application problem that involves the volume of a cylindrical container filled with bouncy balls.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What dimensions are given?
- What is the volume of the container?
- What is the volume of one bouncy ball?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Packaging

Brad is packing 3 bouncy balls in a cylindrical container. The radius of each bouncy ball is 10 centimeters. The cylinder has a base area of 315 square centimeters and a height of 65 centimeters. What is the volume of empty space in the container? Round to the nearest tenth.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

7,908.6 cm^3; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

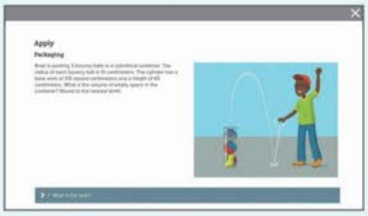
See students' arguments.

Talk About It! Will another whole bouncy ball fit in the container? Explain why or why not.

Sample answer: Even though there is enough empty space in the cylinder, another bouncy ball will not fit. Without any space between the bouncy balls and the container, the only possible way to add another ball is to stack it, which will not work because the container is 65 cm tall and the height of three stacked bouncy balls is 60 cm.

Lesson 12-8 • Volume of Spheres 827

Interactive Presentation



Apply, Packaging

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The radius of a table tennis ball is 2 centimeters. Olivia is packing 30 table tennis balls in a box with length of 24 centimeters, a width of 20 centimeters, and a height of 4 centimeters. What is the volume of the empty space? Round to the nearest tenth.

Ⓐ 914.7 cubic centimeters
Ⓑ 1,003.1 cubic centimeters
Ⓒ 1,005.3 cubic centimeters
Ⓓ 1,920 cubic centimeters

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

828 Module 12 • Area, Surface Area, and Volume

Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson students, could record examples of volume of spheres. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Exit Ticket

Refer to the Exit Ticket slide. Find the amount of air that each size basketball will hold. Use the π button on a calculator, and round each volume to the nearest hundredth. Write a mathematical argument that can be used to defend your solution. Size 5: about 351.97 cubic inches; Size 6: about 391.97 cubic inches; Size 7: about 434.89 cubic inches; Sample answer: Find each volume using the formula $V = \frac{4}{3}\pi r^3$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **BL**
THEN assign:

- Practice, Exercises 3, 7, 9, 11–14
- Extension: Surface Area of Cylinders, Cones, and Spheres
- ALEKS® Volume of Pyramids, Cones, and Spheres

IF students score 66–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–6, 9, 12
- Extension: Surface Area of Cylinders, Cones, and Spheres
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Volume of Pyramids, Cones, and Spheres

IF students score 65% or below on the Checks, **AL**
THEN assign:

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Volume of Pyramids, Cones, and Spheres

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

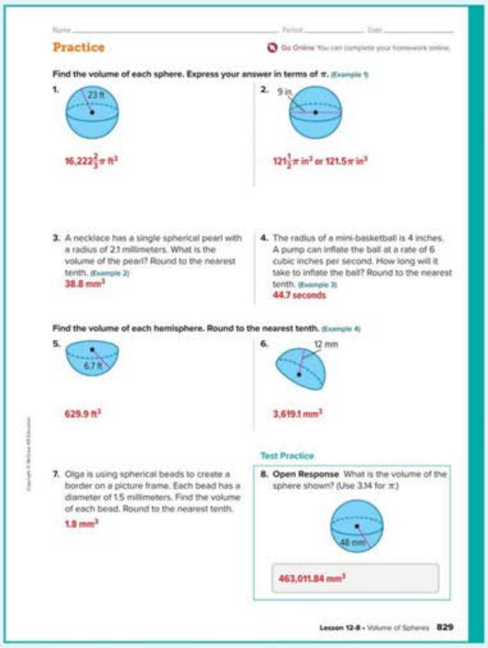
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the volume of spheres in terms of π	1, 2
2	find the volume of spheres in real-world contexts	3
2	solve real-world problems involving the volume of spheres	4
1	find the volume of hemispheres	5, 6
2	extend concepts learned in class to apply them in new contexts	7, 8
3	solve application problems involving the volume of spheres	9, 10
3	higher-order and critical thinking skills	11–14

Common Misconception

Some students may confuse the formula for the volume of a sphere and the formula for the volume of a hemisphere with the volume formulas they have previously learned. Encourage students to solve each Exercise by first writing the appropriate formula needed to solve the problem, making sure the formula is written correctly.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Find the volume of each sphere. Express your answer in terms of π. (Example 1)

1. [sphere, 23 ft] $16,222\frac{2}{3}\pi$ ft^3

2. [sphere, 9 in.] $121\frac{1}{2}\pi$ in^3 or 121.5π in^3

3. A necklace has a single spherical pearl with a radius of 2.1 millimeters. What is the volume of the pearl? Round to the nearest tenth. (Example 2)
38.8 mm^3

4. The radius of a mini-basketball is 4 inches. A pump can inflate the ball at a rate of 6 cubic inches per second. How long will it take to inflate the ball? Round to the nearest tenth. (Example 3)
44.7 seconds

Find the volume of each hemisphere. Round to the nearest tenth. (Example 4)

5. [hemisphere, 6.7 ft] 629.9 ft^3

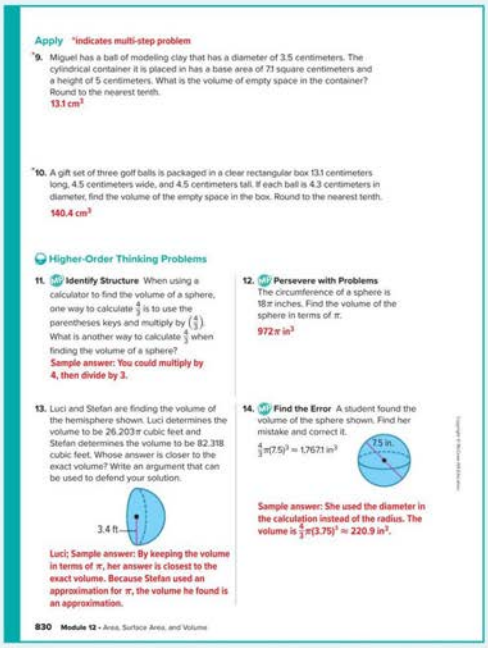
6. [hemisphere, 12 mm] 3,619.1 mm^3

7. Olga is using spherical beads to create a border on a picture frame. Each bead has a diameter of 1.5 millimeters. Find the volume of each bead. Round to the nearest tenth.
1.8 mm^3

Test Practice

8. **Open Response** What is the volume of the sphere shown? (Use 3.14 for π.)
[sphere, 48 mm]
463,011.84 mm^3

Lesson 12-8 • Volume of Spheres 829



Apply *indicates multi-step problem

*9. Miguel has a ball of modeling clay that has a diameter of 3.5 centimeters. The cylindrical container it is placed in has a base area of 7.1 square centimeters and a height of 5 centimeters. What is the volume of empty space in the container? Round to the nearest tenth.
13.1 cm^3

*10. A gift set of three golf balls is packaged in a clear rectangular box 13.1 centimeters long, 4.5 centimeters wide, and 4.5 centimeters tall. If each ball is 4.3 centimeters in diameter, find the volume of the empty space in the box. Round to the nearest tenth.
140.4 cm^3

Higher-Order Thinking Problems

11. **Identify Structure** When using a calculator to find the volume of a sphere, one way to calculate $\frac{4}{3}$ is to use the parentheses keys and multiply by $\left(\frac{4}{3}\right)$. What is another way to calculate $\frac{4}{3}$ when finding the volume of a sphere?
Sample answer: You could multiply by 4, then divide by 3.

12. **Persevere with Problems** The circumference of a sphere is 18π inches. Find the volume of the sphere in terms of π.
$972\pi \text{ in}^3$

13. Luci and Stefan are finding the volume of the hemisphere shown. Luci determines the volume to be 26.203π cubic feet and Stefan determines the volume to be 82.318 cubic feet. Whose answer is closer to the exact volume? Write an argument that can be used to defend your solution.

Luci; Sample answer: By keeping the volume in terms of π, her answer is closest to the exact volume. Because Stefan used an approximation for π, the volume he found is an approximation.

14. **Find the Error** A student found the volume of the sphere shown. Find her mistake and correct it.
$\frac{4}{3}\pi(7.5)^3 \approx 1{,}767.1 \text{ in}^3$

Sample answer: She used the diameter in the calculation instead of the radius. The volume is $\frac{4}{3}\pi(3.75)^3 \approx 220.9 \text{ in}^3$.

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Teaching the Mathematical Practices

7 Look for and Make Use of Structure In Exercise 11, students describe another way to calculate $\frac{4}{3}$ when finding the volume of a sphere. Encourage students to use the structure of fractions in their explanation.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 12, students find the volume of the described sphere in terms of π. Encourage students to use the formulas for the circumference and volume of a sphere to answer the question.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 14, students find the student's error and correct it. Encourage students to identify the error and then construct a response that explains how to fix the error.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 9 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.
Use with Exercises 12–13 Have students work in groups of 3–4 to solve the problem in Exercise 12. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 13.

Lesson 12-9

7.G.B.6, 8.G.C.9

Volume and Surface Area of Composite Solids

LESSON GOAL

Students will find the volume and surface area of composite solids.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Composite Solids
Learn: Volume of Composite Solids
Example 1: Find Volume of Composite Solids
Example 2: Find Volume of Composite Solids
Example 3: Volume of Composite Solids
Learn: Surface Area of Composite Solids
Example 4: Surface Area of Composite Solids
Apply: Art

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the Checks after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Extension: Platonic Solids		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 78 of the *Language Development Handbook* to help your students build mathematical language related to the volume and surface area of composite Solids.

ELL You can use the tips and suggestions on page T78 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry

Additional Cluster(s): In this lesson, students address additional clusters **7.G.B** and **8.G.C** by finding the volume and surface area of composite solids.

Standards for Mathematical Content: **7 .G.B.6, 8.G.C.9,** *Also addresses 7.NS.A.3, 7.G.A.1*

Standards for Mathematical Practice: **MP 1, MP3, MP4, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the volume of spheres and hemispheres.
8.G.C.9

Now
Students find the volume and surface area of composite solids.
7.G.B.6, 8.G.C.9

Next
Students will translate figures and describe translations on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

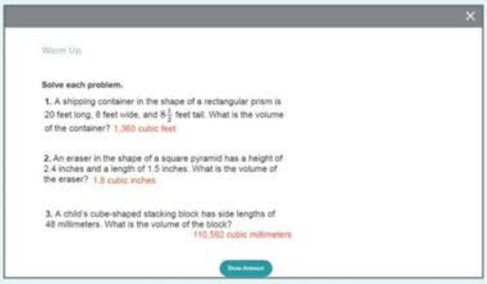
Conceptual Bridge In this lesson, students will draw on their knowledge of volume and surface area to gain *fluency* in finding the volume and surface area of composite solids. They will *apply* this fluency to solve real-world problems involving the volume and surface area of composite solids.

Mathematical Background

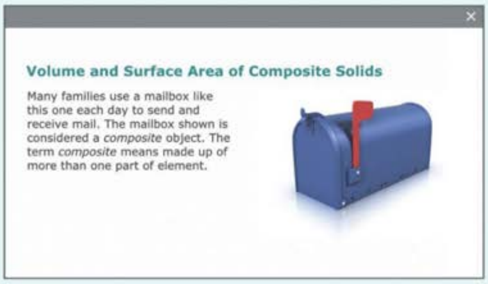
A three-dimensional composite solid is made up of two or more three-dimensional solids.

The volume of a composite solid is found by separating the solid into solids with volumes that can be calculated, finding those volumes, and adding. The surface area of a composite solid is found by adding the areas of all the faces.

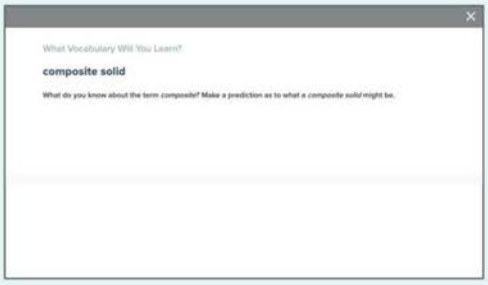
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- finding volumes of prisms and pyramids (Exercises 1–3)

Answers

1. 1,360 cubic feet

2. 43.2 cubic centimeters

3. 110,592 cubic millimeters

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a mailbox as an example of a composite object.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What do you know about the term *composite*? Make a prediction as to what a *composite solid* might be. Sample answer: *Composite* means to be made up of more than one part or element. A composite solid might be a three-dimensional figure that is composed of multiple three-dimensional figures.

Learn Composite Solids

Objective

Students will learn about composite solids.

Teaching Notes

SLIDE 1

Students will learn the definition for a *composite solid*. Have them explore the interactive activity to see the solids that make up each composite solid. You may wish to have them make a prediction as to which solids make up the composite solid prior to completing each activity.

Learn Volume of Composite Solids

Objective

Students will learn how to find the volume of composite solids.

Go Online to have your students watch the animation on Slide 1. The animation illustrates how to find the volume of composite solids.

Teaching Notes

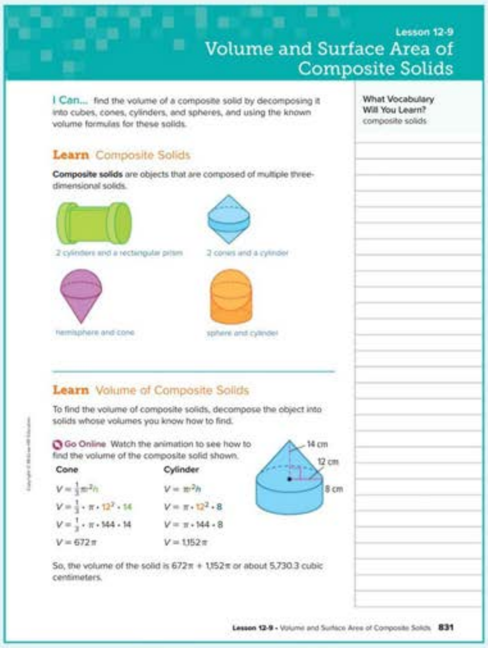
SLIDE 1

You may wish to pause the animation after the composite solid is shown. Have students work with a partner to determine the volume of the composite solid. They may use any strategy they wish, but must be prepared to explain their strategy and defend why it works. Have students share their strategies with the class. Some students may find the exact volume, while others may find approximate volumes. Have them continue to watch the animation to compare their strategy and volume with the one shown. Be sure they can explain why the individual volumes of the cylinder and cone were added.

DIFFERENTIATVE

Reteaching Activity AL

Some students may struggle to understand the concept of composite solids. Lead a short classroom discussion so students can brainstorm different composite solids they have seen. Encourage students to mention objects found around the classroom so that a visual is accessible. For each composite solid, have students describe the solids that make up the composite solid.



Lesson 12-9

Volume and Surface Area of Composite Solids

I Can... find the volume of a composite solid by decomposing it into cubes, cones, cylinders, and spheres, and using the known volume formulas for these solids.

What Vocabulary Will You Learn?
composite solids

Learn Composite Solids

Composite solids are objects that are composed of multiple three-dimensional solids.

2 cylinders and a rectangular prism

2 cones and a cylinder

hemisphere and cone

sphere and cylinder

Learn Volume of Composite Solids

To find the volume of composite solids, decompose the object into solids whose volumes you know how to find.

Go Online Watch the animation to see how to find the volume of the composite solid shown.

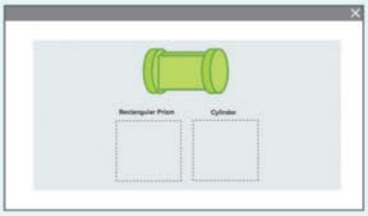
14 cm
12 cm
8 cm

Cone	Cylinder
$V = \frac{1}{3}\pi r^2 h$	$V = \pi r^2 h$
$V = \frac{1}{3} \cdot \pi \cdot 12^2 \cdot 14$	$V = \pi \cdot 12^2 \cdot 8$
$V = \frac{1}{3} \cdot \pi \cdot 144 \cdot 14$	$V = \pi \cdot 144 \cdot 8$
$V = 672\pi$	$V = 1{,}152\pi$

So, the volume of the solid is $672\pi + 1{,}152\pi$ or about 5,730.3 cubic centimeters.

Lesson 12-9 • Volume and Surface Area of Composite Solids 831

Interactive Presentation



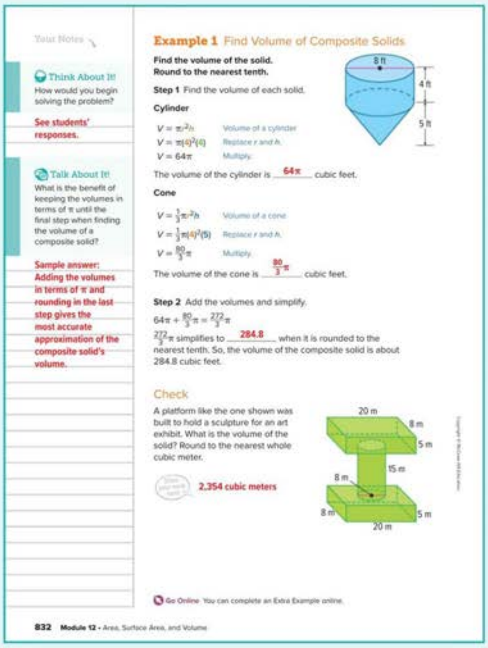
Learn, Composite Solids

DRAG & DROP

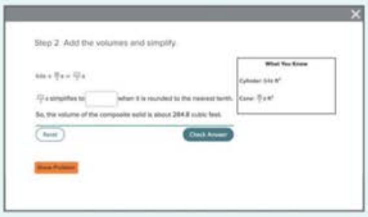
On Slide 1 of Learn 1, students drag the objects that make up the composite solids to the appropriate bins that describe them.

WATCH

On Slide 1 of Learn 2, students watch the animation to see how to find the volume of composite solids.



Interactive Presentation



Example 1, Find Volume of Composite Solids, Slide 3 of 5

CLICK

On Slide 2, students determine the volume of each solid.

TYPE

On Slide 3, students determine the total volume.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Find Volume of Composite Solids

Objective

Students will find the volume of composite solids.

Questions for Mathematical Discourse

SLIDE 2

AL What figures make up the composite figure? a cylinder and a cone

AL What is the volume formula for a cylinder? a cone? $V = \pi r^2 h$; $V = \frac{1}{3}\pi r^2 h$

OL Explain why the radius is 4 feet. The diameter is 8 feet, so the radius is half the diameter, or 4 feet.

OL Did you find the exact volumes or approximations of the cylinder and cone? Explain. the exact volumes; Sample answer: The volumes are in terms of π.

BL A classmate wrote the volume of the cone as 26.7π. Is this still an exact volume? Explain. no; Sample answer: $\frac{80}{3}$ does not simplify to a whole number or terminating decimal. So, to write $\frac{80}{3}\pi$ as 26.7π, you need to round. Therefore, it does not represent the exact volume.

SLIDE 3

AL What quantities will be added together? What are those values? the volume of the cylinder and the volume of the cone; the volume of the cylinder is 64π cubic feet, and the volume of the cone is $\frac{80}{3}\pi$ cubic feet.

OL How can you add the volumes without needing to use an approximation for π first? Sample answer: Use the Distributive Property to factor π from the expression for the sum of the areas. $64\pi + \frac{80}{3}\pi = \left(64 + \frac{80}{3}\right)\pi$ or $\frac{272}{3}\pi$.

BL How many cubic yards is equivalent to $\frac{272}{3}\pi$ cubic feet? Explain. $\frac{272}{81}\pi$ cubic yards, or about 10.5 cubic yards; Sample answer: Divide $\frac{272}{3}\pi$ by 3^3, or 27, since there are 27 cubic feet in one cubic yard.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Find Volume of Composite Solids

Objective

Students will find the volume of composite solids.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 4, encourage them to explain both numerically and verbally why the volume of the spherical bead will be less than the volume of the cube-shaped bead. Students may use a visual of both types of figures to help justify their answer.

Questions for Mathematical Discourse

SLIDE 2

AL Study the structure of the diagram. What do you notice? Sample answer: There is a hole in the center of the cube.

OL Explain why the radius is 1 millimeter. The diameter is 2 millimeters, so the radius is half the diameter, or 1 millimeter.

OL Did you find the exact volumes or approximations of the cylinder and cube? Explain. the exact volumes; Sample answer: The volume of the cube is an exact volume. The volume of the cylinder is also exact, because it is written in terms of π.

BL If the cylindrical hole had a larger radius, such as 3 millimeters, how would that impact the total volume of the composite solid? Sample answer: The total volume of the composite solid will be less, since a greater volume of the cylindrical hole will be subtracted from the volume of the cube.

SLIDE 3

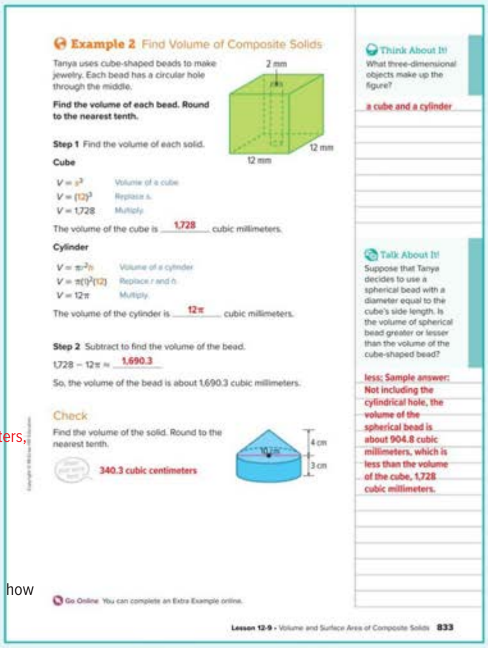
AL Explain why subtraction is used, instead of addition, to find the total volume. Sample answer: I need to subtract the volume of the hole from the volume of the cube.

OL Can you find the exact volume by subtracting, or must you approximate the difference? Explain. Sample answer: I need to approximate the difference because I cannot subtract 12π from 1,728 without using an approximation of π.

BL What is the volume of each bead in cubic centimeters? about 1.7 cm^3

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Find Volume of Composite Solids

Tanya uses cube-shaped beads to make jewelry. Each bead has a circular hole through the middle.

Find the volume of each bead. Round to the nearest tenth.

Step 1 Find the volume of each solid.

Cube

$V = s^3$ Volume of a cube
$V = (12)^3$ Replace s.
$V = 1{,}728$ Multiply.

The volume of the cube is 1,728 cubic millimeters.

Cylinder

$V = \pi r^2 h$ Volume of a cylinder
$V = \pi(1)^2(12)$ Replace r and h.
$V = 12\pi$ Multiply.

The volume of the cylinder is 12π cubic millimeters.

Step 2 Subtract to find the volume of the bead.

$1{,}728 - 12\pi \approx$ 1,690.3

So, the volume of the bead is about 1,690.3 cubic millimeters.

Check

Find the volume of the solid. Round to the nearest tenth.

340.3 cubic centimeters

Go Online You can complete an Extra Example online.

Think About It! What three-dimensional objects make up the figure?

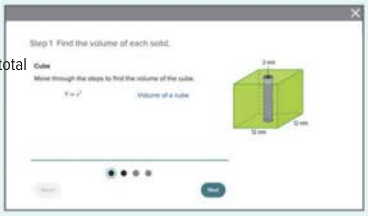
a cube and a cylinder

Talk About It! Suppose that Tanya decides to use a spherical bead with a diameter equal to the cube's side length. Is the volume of spherical bead greater or lesser than the volume of the cube-shaped bead?

less; Sample answer: Not including the cylindrical hole, the volume of the spherical bead is about 904.8 cubic millimeters, which is less than the volume of the cube, 1,728 cubic millimeters.

Lesson 12-9 • Volume and Surface Area of Composite Solids 833

Interactive Presentation



Example 2, Find Volume of Composite Solids, Slide 2 of 5

CLICK

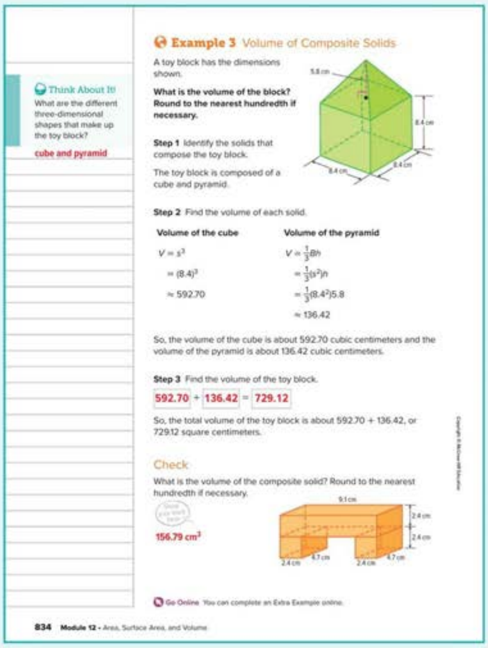
On Slide 2, students determine the volume of the cube and cylinder.

TYPE

On Slide 3, students subtract to find the volume of the solid.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 3 Volume of Composite Solids

A toy block has the dimensions shown.

What is the volume of the block? Round to the nearest hundredth if necessary.

Step 1 Identify the solids that compose the toy block.

The toy block is composed of a cube and pyramid.

Step 2 Find the volume of each solid.

Volume of the cube	Volume of the pyramid
$V = s^3$	$V = \frac{1}{3}Bh$
$= (8.4)^3$	$= \frac{1}{3}(s^2)h$
≈ 592.70	$= \frac{1}{3}(8.4^2)5.8$
	≈ 136.42

So, the volume of the cube is about 592.70 cubic centimeters and the volume of the pyramid is about 136.42 cubic centimeters.

Step 3 Find the volume of the toy block.

592.70 + 136.42 = 729.12

So, the total volume of the toy block is about 592.70 + 136.42, or 729.12 square centimeters.

Check

What is the volume of the composite solid? Round to the nearest hundredth if necessary.

156.79 cm³

Think About It! What are the different three-dimensional shapes that make up the toy block? cube and pyramid

Go Online You can complete an Extra Example online.

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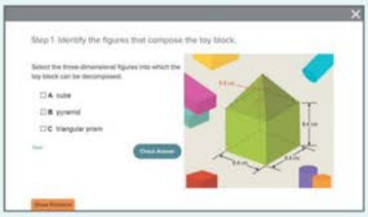
Interactive Presentation



Example 1, Volume of Composite Solids, Slide 2 of 5

CLICK

On Slide 2, students select the three-dimensional solids into which the toy block can be decomposed.

TYPE

On Slide 4, students determine the volume of the toy block.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

7.G.B.6, 8.G.C.9

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 3 Volume of Composite Solids

Objective

Students will find the volume of three-dimensional composite solids.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to study the structure of the composite solid in order to determine how to decompose it into smaller solids.

Questions for Mathematical Discourse

SLIDE 2

AL Do you need to find the volume or surface area of the toy block? volume

OL Explain why a triangular prism is not one of the figures into which the toy block can be decomposed. Sample answer: A triangular prism has two triangular bases. Neither figure has triangular bases.

BL Do you need to know the areas of the lateral faces of the pyramid? Explain. no; Sample answer: I am not finding the surface area. I am finding the volume. I only need to know the area of the base and the height of the figure.

SLIDE 3

AL Explain why you can use the volume formula $V = s^3$ to find the volume of the cube. Sample answer: The volume of a prism is found by multiplying the area of the base by the height of the prism. When the prism is a cube, all of these dimensions are equivalent.

OL Explain why it makes sense that the volume of the pyramid is less than one third the volume of the cube. Sample answer: The volume of a pyramid is one third the volume of a cube with the same base area and height. In this case, the pyramid has the same base area, but a lesser height.

BL Design your own toy block that is a composite solid and find its volume. See students' drawings and volumes.

Go Online

- Find additional teaching notes and discussion questions.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Surface Area of Composite Solids

Objective

Students will learn how to find the surface area of three-dimensional composite solids.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure As students discuss the *Talk About It!* question on Slide 2, encourage them to study the structure of the composite solid in order to understand that the shared face is not part of the exterior surface area of the composite solid.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to find the surface area of a composite solid.

Talk About It!

SLIDE 2

Mathematical Discourse

One of the faces of each solid is not included when finding the surface area of the composite solid. Explain why. Sample answer: The shared face of each solid is not included because it is not part of the surface of the composite solid.

DIFFERENTIATE

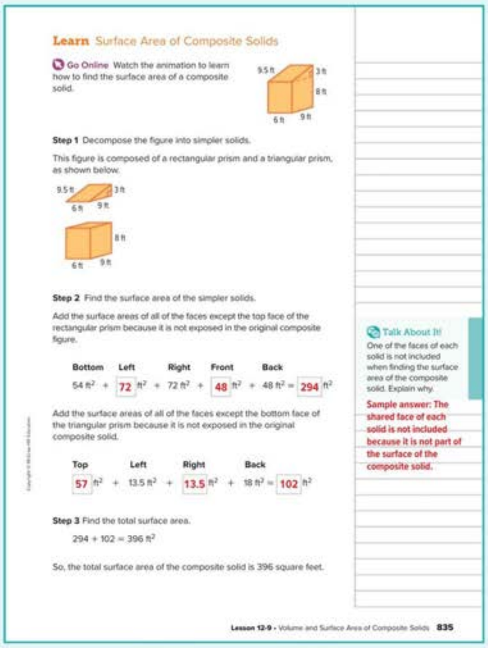
Enrichment Activity BL

To strengthen students' understanding of volume and surface area of composite solids, have students work with a partner to compare and contrast finding the volume of a composite solid and finding the surface area of the composite solid. Have them prepare a brief presentation, using drawings or other illustrations, that summarizes the similarities and differences. Have each pair of students present their findings to another pair, or to the whole class. Some students may be uncomfortable speaking in front of others. Encourage them to make appropriate eye contact, and articulate their thoughts clearly and loudly enough for others to hear. You may wish to provide a rubric for students' presentations. A sample scoring rubric can include, but is not limited to, the following:

Volume is measured in cubic units, while surface area is measured in square units.

When finding either volume or surface area, decompose the solid into known solids.

The composite solid's volume is the sum of the individual solids' volumes. However, when finding the composite solid's surface area, any shared surfaces should not be included in the total surface area.



Learn Surface Area of Composite Solids

Go Online Watch the animation to learn how to find the surface area of a composite solid.

Step 1 Decompose the figure into simpler solids.

This figure is composed of a rectangular prism and a triangular prism, as shown below.

Step 2 Find the surface area of the simpler solids.

Add the surface areas of all of the faces except the top face of the rectangular prism because it is not exposed in the original composite figure.

Bottom	Left	Right	Front	Back	
$54\ ft^2$ +	$72\ ft^2$ +	$72\ ft^2$ +	$48\ ft^2$ +	$48\ ft^2$ =	$294\ ft^2$

Add the surface areas of all of the faces except the bottom face of the triangular prism because it is not exposed in the original composite solid.

Top	Left	Right	Back	
$57\ ft^2$ +	$13.5\ ft^2$ +	$13.5\ ft^2$ +	$18\ ft^2$ =	$102\ ft^2$

Step 3 Find the total surface area.

$294 + 102 = 396\ ft^2$

So, the total surface area of the composite solid is 396 square feet.

Talk About It! One of the faces of each solid is not included when finding the surface area of the composite solid. Explain why.

Sample answer: The shared face of each solid is not included because it is not part of the surface of the composite solid.

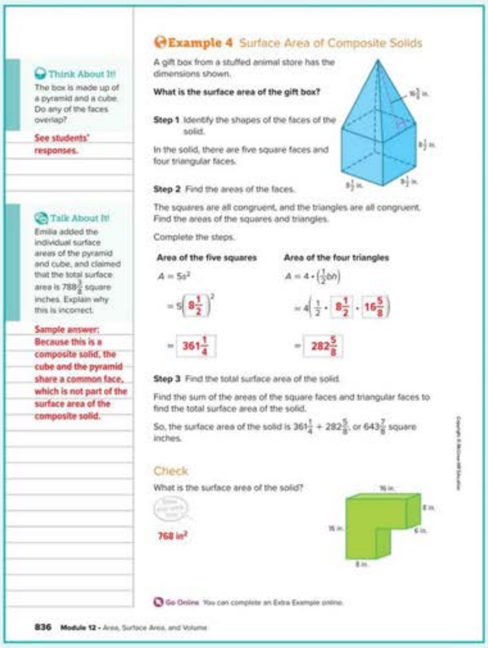
Lesson 12-9 • Volume and Surface Area of Composite Solids 835

Interactive Presentation

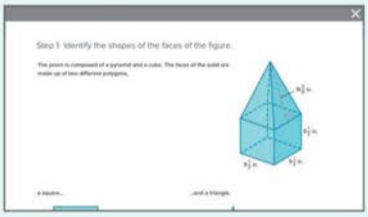
Learn, Surface Area of Composite Solids, Slide 2 of 2

WATCH

On Slide 1, students watch an animation that demonstrates how to find the surface area of a composite solid.



Interactive Presentation



Example 2, Surface Area of Composite Solids, Slide 2 of 6

CLICK

On Slide 2, students select the number of squares and triangles that compose the exterior of the solid.

TYPE

On Slide 4, students determine the total surface area of the gift box.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Surface Area of Composite Solids

Objective

Students will find the surface area of three-dimensional composite solids.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 5, encourage them to use the structure of the solid to explain why Emilia is incorrect.

7 Look For and Make Use of Structure Encourage students to study the structure of the composite solid in order to determine how to decompose it into smaller solids.

Questions for Mathematical Discourse

SLIDE 2

AL Explain why the faces are squares and triangles. Sample answer: A cube has square faces. A square pyramid has a square base and triangular faces.

OL If the solids were separated, how many faces would each solid have? Explain. A cube has six faces. A square pyramid has five faces.

OL Describe the shared face. The shared face is a square.

OL What faces remain, since the shared face is not part of the exterior of the composite solid? Four triangular faces remain and five square faces remain.

BL What is true about the lateral faces of the pyramid? They are all congruent because the pyramid is a regular pyramid.

SLIDE 3

AL How do you know that the base of the triangle is $8\frac{1}{2}$ inches? It has the same length as the side length of the cube.

OL Explain why the squares are congruent, and why the triangles are congruent. Sample answer: The squares are congruent because the solid is a cube. The triangles are congruent because the base of the pyramid is a regular polygon.

BL Explain why you don't include the shared face when finding surface area. The shared face is not part of the exterior surface of the composite solid.

Go Online

- Find additional teaching notes, discussion questions, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Art

Objective

Students will come up with their own strategy to solve an application problem that involves the amount of concrete needed to make a concrete sculpture.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

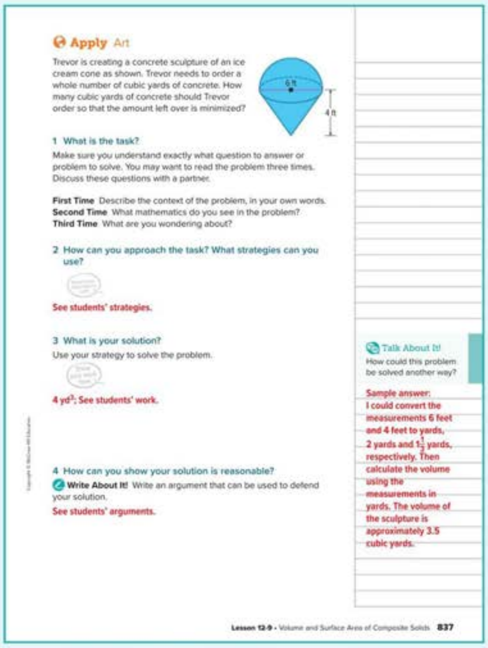
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What solids make up the ice cream cone sculpture?
- What dimensions are given?
- How can you calculate the number of cubic yards of concrete needed for the sculpture?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.


Apply Art

Trevor is creating a concrete sculpture of an ice cream cone as shown. Trevor needs to order a whole number of cubic yards of concrete. How many cubic yards of concrete should Trevor order so that the amount left over is minimized?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

4 yd^3; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! How could this problem be solved another way?

Sample answer: I could convert the measurements 6 feet and 4 feet to yards, 2 yards and $1\frac{1}{3}$ yards, respectively. Then calculate the volume using the measurements in yards. The volume of the sculpture is approximately 3.5 cubic yards.

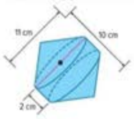
Lesson 12-9 • Volume and Surface Area of Composite Solids 837

Interactive Presentation

Apply, Art

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

A jeweler is thinking about using this solid silver bead in a necklace he is designing. He only has 5,000 grams of silver. If pure silver's mass is 10.5 grams per cubic centimeter, can he make this bead? If so, how many grams of silver will the jeweler have left?

Ⓐ no
● yes; 343.4 g
Ⓒ yes; 1,267 g
Ⓓ yes; 1,673.9 g

Go Online You can complete an Extra Example online.

Pause and Reflect

Did you have difficulty finding the volume of composite solids? If so, what can you do to get help? If not, how could you explain the process to another student?

See students' observations.

838 Module 12 • Area, Surface Area, and Volume

Interactive Presentation

Exit Ticket

Exit Ticket

Refer to the Exit Ticket slide. Find the volume of the mailbox. Use the π button on a calculator. Round the volume to the nearest hundredth. Write a mathematical argument that can be used to defend your solution.

1,170.12 cubic inches; Sample answer: Find the volume of the rectangular prism by finding the product of 8 inches, 16 inches, and 6 inches. The volume of the rectangular prism is 768 cubic inches. Find the volume of the half cylinder by finding the area of the circular base (16π), multiplying by the height of the cylinder (16 inches), and then dividing by 2 since it is only a half cylinder. The volume of the half cylinder is 128π cubic inches. Add the two volumes, $768 + 128\pi$ is about 1,170.12 cubic inches.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 9–12
- Extension: Platonic Solids
- ALEKS Volume of Cylinders, Volume of Pyramids, Cones, and Spheres

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 9–11
- Extension: Platonic Solids
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Volume of Cylinders, Volume of Pyramids, Cones, and Spheres

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS Volume of Cylinders, Volume of Pyramids, Cones, and Spheres

ayzek/123RF

Practice and Homework

The Independent Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

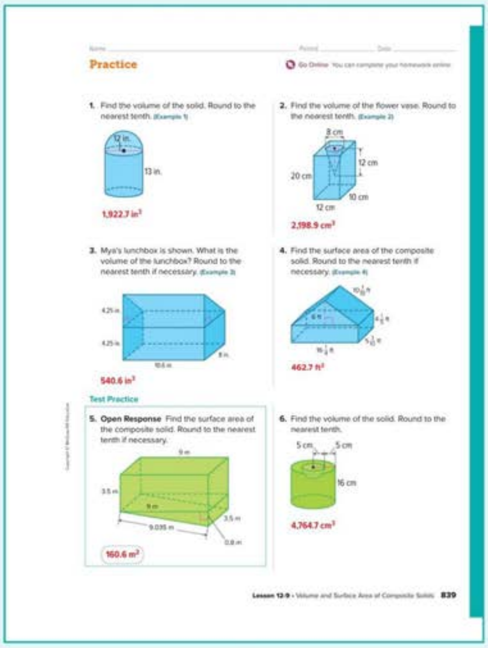
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	find the volume of three-dimensional composite solids	1
2	find the volume of three-dimensional composite solids	2, 3
2	find the surface area of three-dimensional composite solids	4
2	extend concepts learned in class to apply them in new contexts	5, 6
3	solve application problems involving volume and surface area of composite solids	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

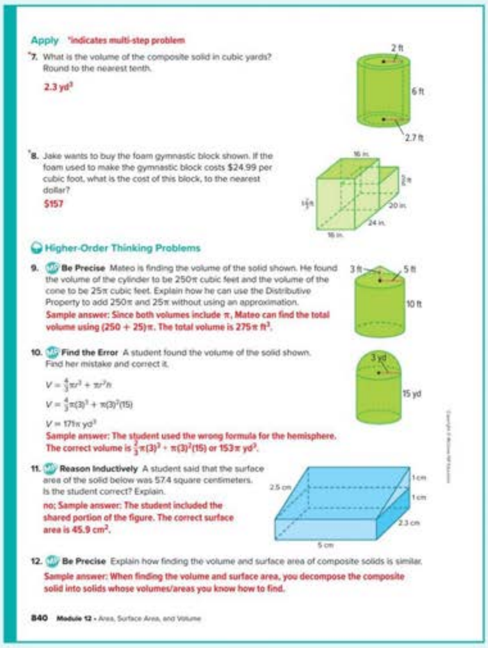
Some students may find surface area of composite solids by adding the surface areas of the decomposed solids. In Exercise 4, students may find the surface area by finding the surface area of the rectangular prism and the surface area of the triangular prism and adding the results. This will include two faces that are not on the exterior of the composite solid. Remind students to subtract any surfaces that are not on the exterior of the composite solid.



3 REFLECT AND PRACTICE

7.G.B.6, 8.G.C.9

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*7. What is the volume of the composite solid in cubic yards? Round to the nearest tenth.

2.3 yd^3

*8. Jake wants to buy the foam gymnastic block shown. If the foam used to make the gymnastic block costs $24.99 per cubic foot, what is the cost of this block, to the nearest dollar?

$157

Higher-Order Thinking Problems

9. **Be Precise** Mateo is finding the volume of the solid shown. He found the volume of the cylinder to be 250π cubic feet and the volume of the cone to be 25π cubic feet. Explain how he can use the Distributive Property to add 250π and 25π without using an approximation.

Sample answer: Since both volumes include π, Mateo can find the total volume using $(250 + 25)\pi$. The total volume is 275π ft^3.

10. **Find the Error** A student found the volume of the solid shown. Find her mistake and correct it.

$V = \frac{4}{3}\pi r^3 + \pi r^2 h$

$V = \frac{4}{3}\pi(3)^3 + \pi(3)^2(15)$

$V = 171\pi$ yd^3

Sample answer: The student used the wrong formula for the hemisphere. The correct volume is $\frac{2}{3}\pi(3)^3 + \pi(3)^2(15)$ or 153π yd^3.

11. **Reason Inductively** A student said that the surface area of the solid below was 57.4 square centimeters. Is the student correct? Explain.

no; Sample answer: The student included the shared portion of the figure. The correct surface area is 45.9 cm^2.

12. **Be Precise** Explain how finding the volume and surface area of composite solids is similar.

Sample answer: When finding the volume and surface area, you decompose the composite solid into solids whose volumes/areas you know how to find.

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Teaching the Mathematical Practices

6 Attend to Precision In E xercise 9, students explain how the Distributive Property can be used in the problem. Encourage students to use precision to explain how the Distributive Property can be used.

3 Construct Viable Arguments and Critique the Reasoning of others In Exercise 10, students find the student's mistake and correct it. Encourage students to supply a well-constructed response in order to explain how to fix the error.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students critique the reasoning of another student about the surface area of a solid.

6 Attend to Precision In Exercise 12, students explain how finding the volume and surface area of composite solids is similar.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.

Use with Exercises 7–8 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 7 might be "Are the measurements given in yards?"

Make sense of the problem.

Use with Exercise 11 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise included the shared side of both solids when calculating the surface area. Have each pair or group of students present their explanations to the class.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of calculating volume of cylinders, cones, and spheres. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

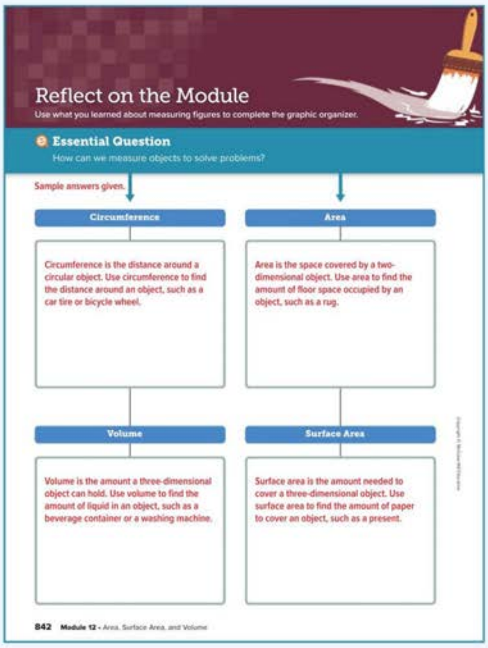
Assessment Resources

Put It All Together 1: Lessons 12-1, 12-2, and 12-3
Put It All Together 2: Lessons 12-4 through 12-9
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with this topic for **Geometry**.

- Area
- Volume
- Surface Area
- Volume of Solids


Reflect on the Module

Use what you learned about measuring figures to complete the graphic organizer.

Essential Question

How can we measure objects to solve problems?

Sample answers given.

Circumference

Circumference is the distance around a circular object. Use circumference to find the distance around an object, such as a car tire or bicycle wheel.

Area

Area is the space covered by a two-dimensional object. Use area to find the amount of floor space occupied by an object, such as a rug.

Volume

Volume is the amount a three-dimensional object can hold. Use volume to find the amount of liquid in an object, such as a beverage container or a washing machine.

Surface Area

Surface area is the amount needed to cover a three-dimensional object. Use surface area to find the amount of paper to cover an object, such as a present.

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

How can we measure objects to solve problems? See students' graphic organizers.

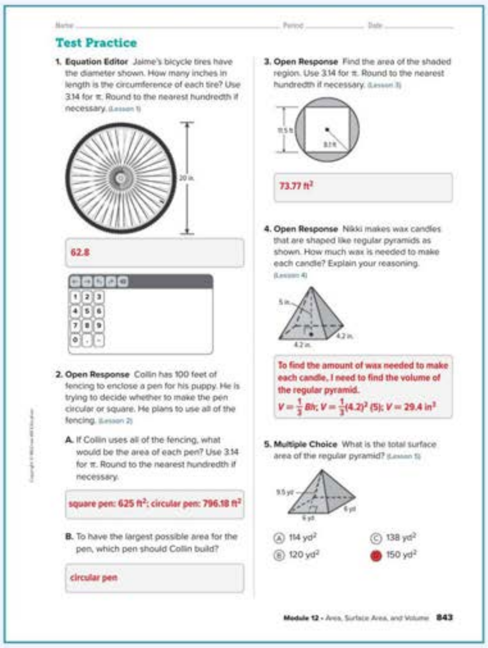
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–9 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	5, 6
Multiselect	Multiple answers may be correct. Students must select all correct answers.	7
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	1
Open Response	Students construct their own response in the area provided.	2, 3, 4, 8, 9

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
7.G.B.4	12-1, 12-2	1, 2
7.G.B.6	12–3, 12–4, 12–5, 12–6	3–5, 9
8.G.C.9	12–6, 12–7, 12–8, 12–9	6–9



Name ______ Period ______ Date ______

Test Practice

1. **Equation Editor** Jaime's bicycle tires have the diameter shown. How many inches in length is the circumference of each tire? Use 3.14 for π. Round to the nearest hundredth if necessary. (Lesson 1)

62.8

2. **Open Response** Collin has 100 feet of fencing to enclose a pen for his puppy. He is trying to decide whether to make the pen circular or square. He plans to use all of the fencing. (Lesson 2)

   A. If Collin uses all of the fencing, what would be the area of each pen? Use 3.14 for π. Round to the nearest hundredth if necessary.

   square pen: 625 ft^2; circular pen: 796.18 ft^2

   B. To have the largest possible area for the pen, which pen should Collin build?

   circular pen

3. **Open Response** Find the area of the shaded region. Use 3.14 for π. Round to the nearest hundredth if necessary. (Lesson 3)

73.77 ft^2

4. **Open Response** Nikki makes wax candles that are shaped like regular pyramids as shown. How much wax is needed to make each candle? Explain your reasoning. (Lesson 4)

To find the amount of wax needed to make each candle, I need to find the volume of the regular pyramid.
$V = \frac{1}{3}Bh$; $V = \frac{1}{3}(4.2)^2(5)$; $V = 29.4 \text{ in}^3$

5. **Multiple Choice** What is the total surface area of the regular pyramid? (Lesson 5)

Ⓐ 114 yd^2
Ⓑ 120 yd^2
Ⓒ 138 yd^2
● 150 yd^2

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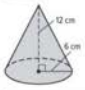
6. **Multiple Choice** A galvanized stock tank with the dimensions shown is filling with water at a rate of 25 gallons per minute. About how many minutes will it take to fill the stock tank if 1 cubic foot is about 7.5 gallons? Round to the nearest minute. (Lesson 6)

Ⓐ 10 minutes
● 17 minutes
Ⓒ 34 minutes
Ⓓ 68 minutes

7. **Multiselect** Which of the following statements regarding the hemisphere are accurate? Select all that apply. (Lesson 8)

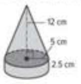
- [x] The diameter of the hemisphere is 14 centimeters.
- [ ] The volume of a sphere is half the volume of a hemisphere that has the same radius.
- [x] The volume of a hemisphere is half the volume of a sphere that has the same radius.
- [x] The volume of the hemisphere, rounded to the nearest tenth, is 718.4 cubic centimeters.
- [ ] The volume of the hemisphere, rounded to the nearest tenth, is 205.3 cubic centimeters.

8. **Open Response** Find the volume of the cone. Express your answer in terms of π. (Lesson 7)



144π cubic centimeters

9. **Open Response** The awards received by the students at the journalism banquet were shaped as shown. (Lesson 9)



A. Find the total volume of the award. Round to the nearest tenth.

510.5 cubic centimeters

B. Suppose the award is made from a high density polyethylene that has a density of 0.95 gram per cubic centimeter. What is the mass of the award? Round to the nearest whole number.

Ⓐ 265 grams
Ⓑ 342 grams
Ⓒ 430 grams
● 485 grams

Module 13

Transformations, Congruence, and Similarity

Module Goal

Analyze translations, rotations, reflections, and dilations. Analyze and use similar and congruent figures using transformations.

Focus

Domain: Geometry
Major Cluster(s):
8.G.A Under stand congruence and similarity using physical models, transparencies, or geometry software.
Standards for Mathematical Content:
8.G.A.3 Describ e the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
Also addresses 8.G.A.1, 8.G.A.2, 8.G.A.4, 8.G.A.5
Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Be Sure to Cover

Students need to have a thorough understanding of the prerequisite skills required for this module.

- graph points with rational number coordinates in the coordinate plane
- use a protractor to find the measure of an angle

Use the Module Pretest to diagnose students' readiness for this module. You may wish to spend more time on the Warm Up for each lesson to fully review these concepts.

Coherence

Vertical Alignment

Previous
Students solved real-world and mathematical problems involving area, volume, and surface area.
7.G.B.4, 7.G.B.6, 8.G.C.9

Now
Students analyze translations, rotations, reflections, dilations, and use similar and congruent figures using transformation.
8.G.A.1, 8.G.A.3

Next
Students will represent transformations and describe transformations as functions.
HSG.COA.2

Rigor

The Three Pillars of Rigor

In this module, students draw on their knowledge of graphing in the coordinate plane to develop *understanding* of transformations. They use their understanding to build *fluency* with graphing and describing translations, reflections, rotations, and dilations using coordinates. They develop *understanding* that two figures are congruent or similar if the second figure can be obtained from the first by a series of transformations. They *apply* their understanding to solve real-world indirect measurement problems.

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE

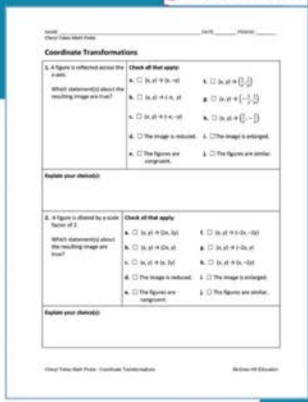
Suggested Pacing

Lesson		Standard(s)	45-min classes	90-min classes
Module Pretest and Launch the Module Video			1	0.5
13-1	Translations	8.G.A.1, 8.G.A.1.A, 8.G.A.3	1	0.5
13-2	Reflections	8.G.A.1, 8.G.A.1.A, 8.G.A.3	1	0.5
Put It All Together 1: Lessons 13-1 and 13-2			0.5	0.25
13-3	Rotations	8.G.A.1, 8.G.A.1.A, 8.G.A.3	1	0.5
13-4	Dilations	8.G.A.3	1	0.5
Put It All Together 2: Lessons 13-1 through 13-4			0.5	0.25
13-5	Congruence and Transformations	8.G.A.1, 8.G.A.1.A, 8.G.A.1.B, 8.G.A.1.C, 8.G.A.2	2	1
13-6	Similarity and Transformations	8.G.A.4, 8.G.A.5	2	1
13-7	Indirect Measurement	8.G.A.4, 8.G.A.5	1	0.5
Module Review			1	0.5
Module Assessment			1	0.5
		Total Days	13	6.5

CHERYL TOBEY MATH PROBES

Formative Assessment Math Probe
Coordinate Transformations

Module Resource



Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will select all of the true statements about each transformation, and explain their choices.

Targeted Concepts Understand how geometric transformations appear in algebraic notation.

Targeted Misconceptions

- Students may misinterpret the meaning of "reflection across the *x*-axis," and think that this means that the sign of the *x*-coordinates change.
- Students may incorrectly believe that scale factors only affect one coordinate (either *x* or *y*).

Assign the probe after Lesson 4.

Correct Answers:
1. a, e, j
2. a, i, j

Collect and Assess Student Work

If the student selects...	Then the student likely...
1. b, c, g, h Does not choose a or f; other choices may or may not be included.	misinterprets a reflection across the *x*-axis as meaning that signs of the *x*-coordinates change.
2. b, c, g, h Does not choose a or f; other choices may or may not be included.	believes that the scale factor only affects one coordinate (either *x* or *y*).

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Transformations
- Lesson 2, Examples 1–4
- Lesson 4, Examples 1–3

Revisit the probe at the end of the module to be sure your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students will complete a graphic organizer to help them answer the Essential Question.

What does it mean to perform a transformation on a figure? See students' graphic organizers.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. At the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Foldables are three-dimensional graphic organizers that help students create study guides for each module.

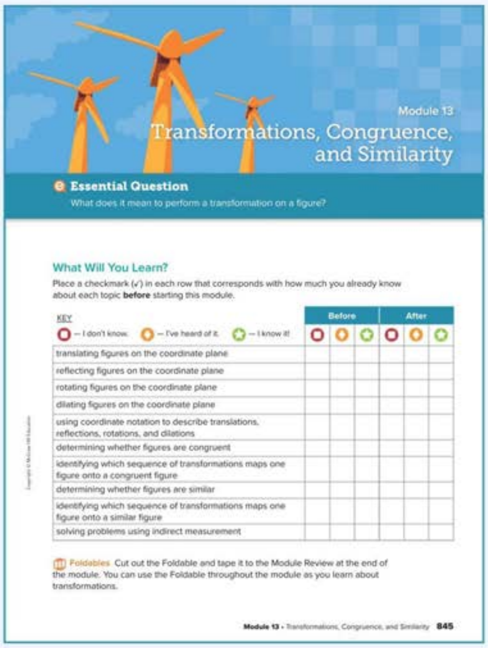
Step 1 Have students locate the module Foldable at the back of the *Interactive Student Edition*. They should follow the cutting and assembly instructions at the top of the page.

Step 2 Have students attach their Foldable to the first page of the Module Review, by matching up the tabs. Dotted tabs indicate where to place the Foldable. Striped tabs indicate where to tape the Foldable.

When to Use It Students add information to their Foldables as they complete selected lessons. Once they've completed their Foldable, they can use it to help them study for the module assessment.

Launch the Module

The Launch the Module video uses the topics of chess, mirrors, wind turbines, and photographs to introduce the idea of transformations. Use the video to engage students before starting the module.



Module 13

Transformations, Congruence, and Similarity

Essential Question

What does it mean to perform a transformation on a figure?

What Will You Learn?

Place a checkmark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

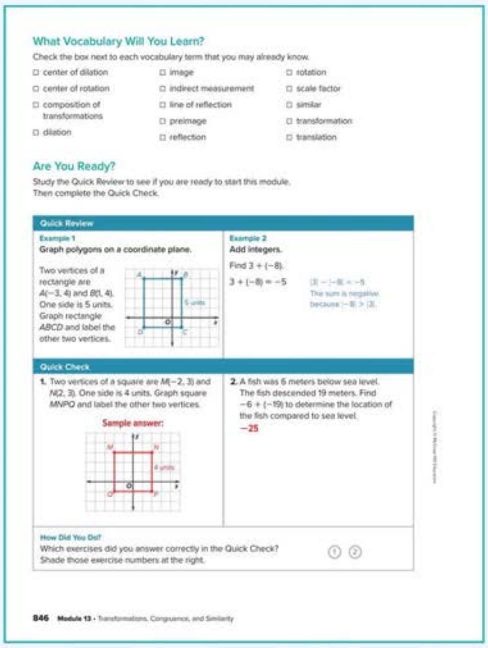
KEY: — I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know.	I've heard of it.	I know it!	I don't know.	I've heard of it.	I know it!
translating figures on the coordinate plane						
reflecting figures on the coordinate plane						
rotating figures on the coordinate plane						
dilating figures on the coordinate plane						
using coordinate notation to describe translations, reflections, rotations, and dilations						
determining whether figures are congruent						
identifying which sequence of transformations maps one figure onto a congruent figure						
determining whether figures are similar						
identifying which sequence of transformations maps one figure onto a similar figure						
solving problems using indirect measurement						

Foldables Cut out the Foldable and tape it to the Module Review at the end of the module. You can use the Foldable throughout the module as you learn about transformations.

Module 13 • Transformations, Congruence, and Similarity 845

Interactive Student Presentation



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ center of dilation
- ☐ center of rotation
- ☐ composition of transformations
- ☐ dilation
- ☐ image
- ☐ indirect measurement
- ☐ line of reflection
- ☐ preimage
- ☐ reflection
- ☐ rotation
- ☐ scale factor
- ☐ similar
- ☐ transformation
- ☐ translation

Are You Ready?

Study the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Graph polygons on a coordinate plane.

Two vertices of a rectangle are $A(-3, 4)$ and $B(1, 4)$. One side is 5 units. Graph rectangle $ABCD$ and label the other two vertices.

Example 2

Add integers.

Find $3 + (-8)$.

$3 + (-8) = -5$

$|3| - |-8| = -5$ The sum is negative because $|-8| > |3|$.

Quick Check

1. Two vertices of a square are $M(-2, 3)$ and $N(2, 3)$. One side is 4 units. Graph square $MNPQ$ and label the other two vertices.

Sample answer:

2. A fish was 6 meters below sea level. The fish descended 19 meters. Find $-6 + (-19)$ to determine the location of the fish compared to sea level.

-25

How Did You Do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers at the right. ① ②

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What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce each vocabulary term using the following routine. Ask the students to say each term aloud after you say it.

Define A **translation** is a transformation that slides a figure from one position to another without turning it.

Example Triangle *ABC* has coordinates $A(-1, 1)$, $B(0, 2)$, and $C(1, -2)$. After a translation down 3 units and 2 units to the left, the coordinates of the image are $A'(-3, -2)$, $B'(-2, -1)$, and $C'(-1, -5)$.

Ask Suppose Triangle *RST* with coordinates $R(-4, 1)$, $S(-3, 4)$, and $T(1, 3)$ is translated 4 units up and 5 units to the right. What are the coordinates of the image? $R'(1, 5)$, $S'(2, 8)$, and $T'(6, 7)$

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- graphing in the coordinate plane
- using a protractor to measure angles

ALEKS

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the concepts in the **Transformations** topic – who is ready to learn these concepts and who isn't quite ready to learn them yet – in order to adjust your instruction as appropriate.

Mindset Matters

View Challenges as Opportunities

Part of cultivating a growth mindset in math involves viewing challenging problems or tasks as opportunities to learn and make new connections in your brain.

How Can I Apply It?

Encourage students to embrace challenges by trying problems that are thought provoking, such as the **Apply Problems** and **Higher-Order Thinking Problems** in the **Practice** section of each lesson. Remember to regularly remind students that each new challenge is an opportunity to grow!

Lesson 13-1

Translations

8.G.A.3

LESSON GOAL

Students will translate figures and describe translations on the coordinate plane.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Transformations
Learn: Translations on a Coordinate Plane
Example 1: Translate Figures on the Coordinate Plane

Explore: Translate Using Coordinates

Learn: Translations Using Coordinates
Example 2: Translate Using Coordinates
Example 3: Use Coordinate Notation to Describe Translations
Apply: Map Reading

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 79 of the *Language Development Handbook* to help your students build mathematical language related to translations.

ELL You can use the tips and suggestions on page T79 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by translating figures and describing translations on the coordinate plane.
Standards for Mathematical Content: **8. G.A.1, 8.G.A.1.A, 8.G.A.3**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students found the volume and surface area of composite solids.
7.G.B.6, 8.G.C.9

Now
Students translate figures and describe translations on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Next
Students will reflect figures and describe reflections on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Rigor

The Three Pillars of Rigor

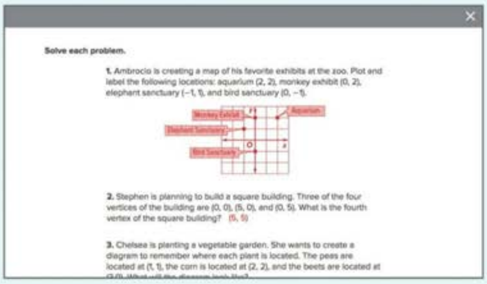
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of translations on the coordinate plane. Students use their understanding to build *fluency* with translating figures using a graph and using coordinates. Students come to understand how to describe a translation using coordinate notation.

Mathematical Background

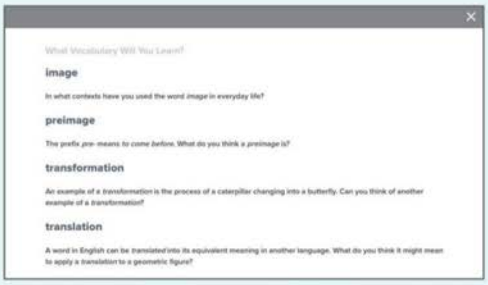
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- graphing in the coordinate plane (Exercises 1–3)

1–3. See W arm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about a GPS using coordinates to identify certain locations.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- In what contexts have you used the word *image* in everyday life? Sample answer: I use the word *image* when talking about pictures.
- The prefix *pre-* means *to come before*. What do you think a *preimage* is? Sample answer: A preimage could be an image before it is manipulated in some way.
- An example of a *transformation* is the process of a caterpillar changing into a butterfly. Can you think of another example of a *transformation*? Sample answer: a tadpole transforming into a frog; a vacant lot being transformed into a community garden
- A word in English can be *translated* into its equivalent meaning in another language. What do you think it might mean to apply a *translation* to a geometric figure? Sample answer: A translation might be the process to move a geometric figure without changing its properties (side lengths, angle measures).

Learn Transformations

Objective

Students will understand that transformations map one geometric figure onto another.

Go Online to find additional teaching notes.

Learn Translations on a Coordinate Plane

Objective

Students will understand that translating a figure on the coordinate plane slides the figure in one or two directions.

Teaching the Mathematical Practices

6 Attend to Precision While discussing the *Talk About It!* question on Slide 2, encourage students to think about the direction of the translation and what the image of the line will look like.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 2

Mathematical Discourse

Describe the image if the line shown is translated 2 units up. Sample answer: The line would still be a line, with the same slope, just two units above the preimage.

DIFFERENTIATE

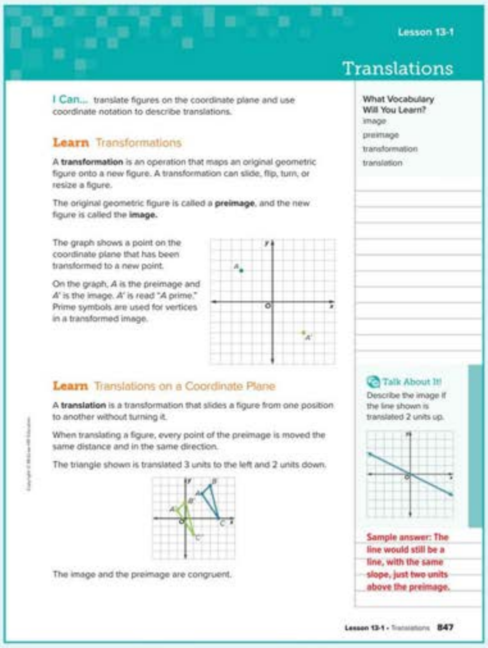
Enrichment Activity BL

For students who need more of a challenge, use the following activity.

Provide students with the function $y = x + 1$.

- Have students investigate what happens to the slope and y-intercept of the line when the line is translated 5 units right. Then ask them to write an equation of the translated line. The slope remains the same and the y-intercept changes; $y = x - 4$
- Have students investigate what happens to the slope and y-intercept of the line when the line is translated 3 units left. Then ask them to write an equation of the translated line. The slope remains the same and the y-intercept changes; $y = x + 2$.

Ask students if they can predict what will happen to the equation of the line $y = x + b$ if the line is translated a units to the right. Sample answer: The slope remains the same and you would subtract a from the y-intercept; $y = x + (b - a)$.



Lesson 13-1

Translations

I Can... translate figures on the coordinate plane and use coordinate notation to describe translations.

What Vocabulary Will You Learn?
image
preimage
transformation
translation

Learn Transformations

A **transformation** is an operation that maps an original geometric figure onto a new figure. A transformation can slide, flip, turn, or resize a figure.

The original geometric figure is called a **preimage**, and the new figure is called the **image.**

The graph shows a point on the coordinate plane that has been transformed to a new point.

On the graph, A is the preimage and A' is the image. A' is read "A prime." Prime symbols are used for vertices in a transformed image.

Learn Translations on a Coordinate Plane

A **translation** is a transformation that slides a figure from one position to another without turning it.

When translating a figure, every point of the preimage is moved the same distance and in the same direction.

The triangle shown is translated 3 units to the left and 2 units down.

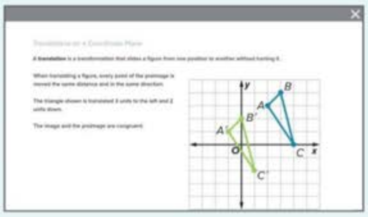
The image and the preimage are congruent.

Talk About It! Describe the image if the line shown is translated 2 units up.

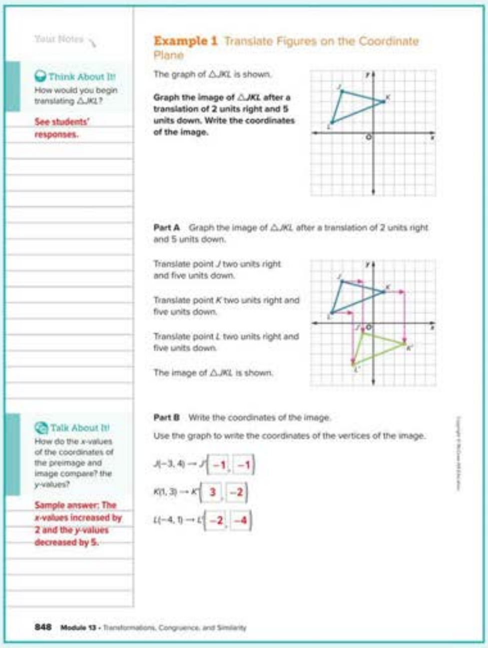
Sample answer: The line would still be a line, with the same slope, just two units above the preimage.

Lesson 13-1 • Translations 847

Interactive Presentation



Learn, Translations on a Coordinate Plane, Slide 1 of 2


Your Notes

Think About It! How would you begin translating △JKL?

See students' responses.

Example 1 Translate Figures on the Coordinate Plane

The graph of △JKL is shown.

Graph the image of △JKL after a translation of 2 units right and 5 units down. Write the coordinates of the image.

Part A Graph the image of △JKL after a translation of 2 units right and 5 units down.

Translate point J two units right and five units down.

Translate point K two units right and five units down.

Translate point L two units right and five units down.

The image of △JKL is shown.

Part B Write the coordinates of the image.

Use the graph to write the coordinates of the vertices of the image.

J(−3, 4) → J′(−1, −1)

K(1, 3) → K′(3, −2)

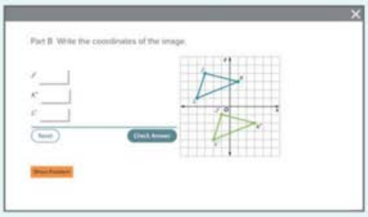
L(−4, 1) → L′(−2, −4)

Talk About It! How do the x-values of the coordinates of the preimage and image compare? the y-values?

Sample answer: The x-values increased by 2 and the y-values decreased by 5.

848 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 1, Translate Figures on the Coordinate Plane, Slide 3 of 5

CLICK

On Slide 2, students move through the slides to translate the triangle.

TYPE

On Slide 3, students determine the coordinates of the image.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

8.G.A.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 1 Translate Figures on the Coordinate Plane

Objective

Students will translate figures on the coordinate plane, and determine the coordinates of the image.

Teaching the Mathematical Practices

6 Attend to Precision Enc ourage students to use precision in translating *each* vertex of the triangle according to the description of the translation.

While discussing the *Talk About It!* question on Slide4, encourage students to use precise mathematical language as they explain how the x- and y-values of the preimage and image compare.

Questions for Mathematical Discourse

SLIDE 2

AL What is a translation? Sample answer: A transformation that slides a figure from one position to another.

OL Why is it important that each vertex of the triangle is translated the same number of units and in the same direction? Sample answer: In order to keep the preimage and the image congruent, each vertex has to be moved the same number of units and in the same direction.

BL In which quadrant(s) is the image? Quadrants III and IV

SLIDE 3

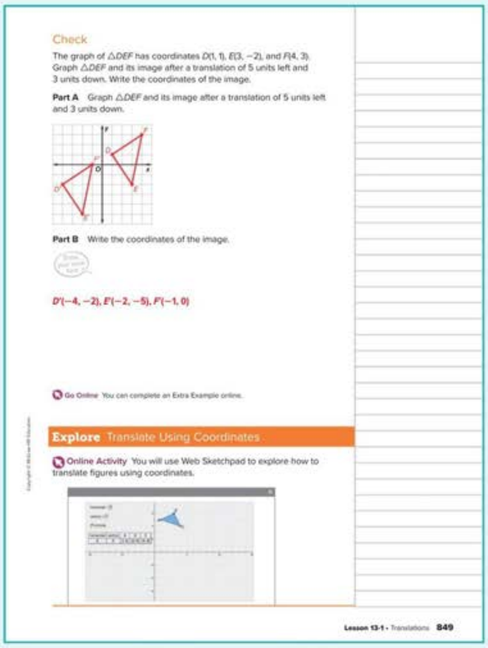
AL What is the general form of an ordered pair? (x, y)

OL If you translated each vertex down 5 units first, and then 2 units to the right, how would the ordered pairs of the image compare? They would be the same.

BL Describe a translation of △*JKL* with an image that lies entirely in the first quadrant. Sample answer: 5 units to the right

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Check

The graph of $\triangle DEF$ has coordinates $D(1, 1)$, $E(3, -2)$, and $F(4, 3)$. Graph $\triangle DEF$ and its image after a translation of 5 units left and 3 units down. Write the coordinates of the image.

Part A Graph $\triangle DEF$ and its image after a translation of 5 units left and 3 units down.

Part B Write the coordinates of the image.

$D'(-4, -2)$, $E'(-2, -5)$, $F'(-1, 0)$

Go Online You can complete an Extra Example online.

Explore Translate Using Coordinates

Online Activity You will use Web Sketchpad to explore how to translate figures using coordinates.

Lesson 13-1 • Translations 849

DIFFERENTIATE

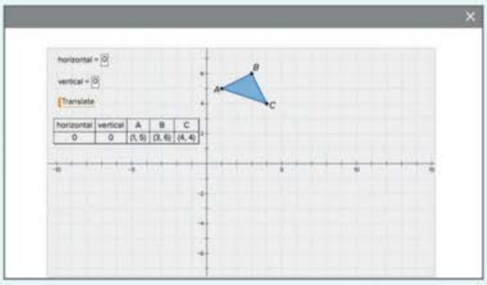
Language Development Activity ELL

Students may confuse the terms *preimage* and *image*. They may refer to the original figure as the *image*. Have them discuss with a partner what the preposition *pre-* means. Because it means *previous to* or *before*, the *preimage* is the figure that was *before* the image. The only way that can be true is for the preimage to be the original figure. Have them generate other terms they may have heard in their everyday lives that begin with *pre-*. Have them create and complete a table like the one shown to reinforce their understanding that a *preimage* is the figure that comes *before* the image. Thus, a *preimage* is the original figure. Some sample terms are shown.

This word "comes before".....	.....this word
preimage	image
preschool	school
preadolescence	adolescence
premature	mature
preheat	heat
presoak	soak

Interactive Presentation

Explore, Slide 1 of 6



Explore, Slide 2 of 6

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to translate figures using coordinates.

Explore Translate Using Coordinates

Objective

Students will use Web Sketchpad to explore how to translate figures using coordinates.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to investigate how translations affect the coordinates of a figure. Students should then apply what they found to complete the sentences that state how different translations affect the coordinates.

Inquiry Question

How do the coordinates of a figure change after a translation? Sample answer: When a figure is translated right, a positive value is added to the *x*-coordinates, and when it is translated left, a positive value is subtracted from the *x*-coordinates. When a figure is translated up, a positive value is added to the *y*-coordinates, and when it is translated down, a positive value is subtracted from the *y*-coordinates.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

What do you notice about the new coordinates of points *A, B,* and *C* and the translation values you entered? Sample answer: The *x*-coordinate for each point increased by 5 and this is the same as the horizontal translation. The *y*-coordinate for each point decreased by 4 and this is the same as the vertical translation.

In what direction(s) did the figure move? The figure moved to the right and then down.

(continued on next page)

Explore Translate Using Coordinates *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore how the coordinates of a figure change after a translation.

7 Look For and Make Use of Structure Encourage students to examine the structure of the x- and y-coordinates in order to determine how they are affected after a translation.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 4 are shown.

Talk About It!

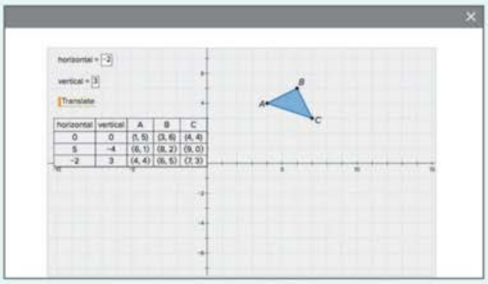
SLIDE 4

Mathematical Discourse

What happens to the coordinates when a figure translates to the right? to the left? Sample answer: The x-coordinates increase when a figure translates to the right and they decrease when a figure translates to the left.

What happens to the coordinates when a figure translates up? down? Sample answer: The y-coordinates increase when a figure translates up and they decrease when a figure translates down.

Interactive Presentation



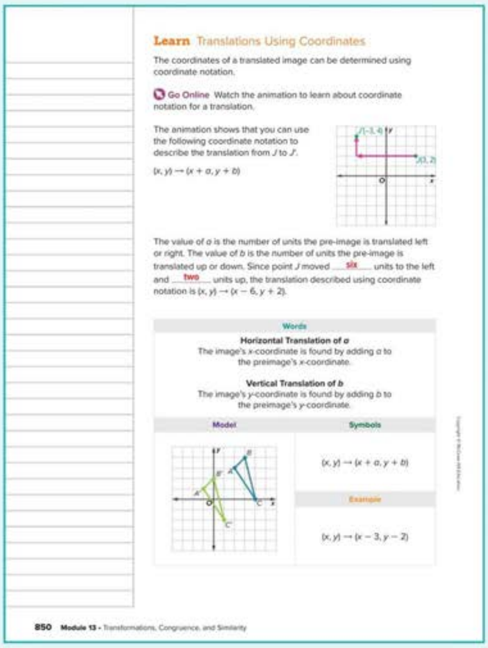
Explore, Slide 4 of 6

CLICK

On Slide 5, students select the correct words to complete statements about translations.

TYPE

On Slide 6, students respond to the Inquiry Question and view a sample answer.



Learn Translations Using Coordinates

The coordinates of a translated image can be determined using coordinate notation.

Go Online Watch the animation to learn about coordinate notation for a translation.

The animation shows that you can use the following coordinate notation to describe the translation from J to J'.

$(x, y) \rightarrow (x + a, y + b)$

The value of a is the number of units the pre-image is translated left or right. The value of b is the number of units the pre-image is translated up or down. Since point J moved __six__ units to the left and __two__ units up, the translation described using coordinate notation is $(x, y) \rightarrow (x - 6, y + 2)$.

Words

Horizontal Translation of a
The image's x-coordinate is found by adding a to the preimage's x-coordinate.

Vertical Translation of b
The image's y-coordinate is found by adding b to the preimage's y-coordinate.

Model	Symbols
	$(x, y) \rightarrow (x + a, y + b)$
	Example
	$(x, y) \rightarrow (x - 3, y - 2)$

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Interactive Presentation

Learn, Translations Using Coordinates, Slide 2 of 2

WATCH

On Slide 1, students watch an animation that illustrates how to use coordinate notation for a translation.

FLASHCARDS

On Slide 2, students use Flashcards to view multiple representations of the coordinate notation used when translating a figure on the coordinate plane.

Learn Translations Using Coordinates

Objective

Students will learn how to use coordinate notation to find the coordinates of an image after a translation.

Go Online to have students watch the animation on Slide 1. The animation illustrates coordinate notation for translations.

Teaching Notes

SLIDE 1

Play the animation for the class. You may wish to pause the animation when the notation $(x, y) \longrightarrow (x + a, y + b)$ first appears. Ask students to conjecture what the variables a and b might represent. Some students may say a represents the number of units for the horizontal translation and b represents the number of units for the vertical translation, while others reverse the descriptions. Point out that the value for a will affect the x-coordinate, causing a horizontal move, and the value for b will affect the y-coordinate, causing a vertical move.

Students may have difficulty when determining the coordinate notation for translations that involve translating a figure to the left or down. Point out to students that the general form of the coordinate notation for a translation is $(x, y) \longrightarrow (x + a, y + b)$. If a figure is translated to the left, then the value of a is negative, because the translation *decreases* the x-coordinate by a units. If a figure is translated down, then the value of b is negative, because the translation *decreases* the y-coordinate by b units.

SLIDE 2

Have students select the *Words, Symbols, Example*, and *Model* flashcards to view the multiple ways a translation can be represented. You may wish to have students discuss how they can determine the signs of a and b just by looking at the graphic representation. Students should notice that if the figure moves to the right, a is positive, and to the left, a is negative. If the figure moves up, b is positive, and down, b is negative.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Translate Using Coordinates

Objective

Students will write the coordinate notation for a translation and use it to find the coordinates of a figure's image.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to use the structure of coordinate notation to understand that a translation to the left means the part of the coordinate notation that describes what happens to the *x*-coordinate must be written as either $x + (-2)$ or $x - 2$.

Questions for Mathematical Discourse

SLIDE 2

AL What does the general form of coordinate notation look like? $(x, y) \rightarrow (x + a, y + b)$

AL What do the variables *a* and *b* represent in the coordinate notation? Sample answer: *a* represents the movement in the direction to the left or right, and *b* represents the movement in the direction upwards or down

OL Why is the value of *a* negative? Sample answer: *a* represents the value of the movement in the direction to the right. Since the movement is to the left, *a* must be negative.

BL Describe a translation whose coordinate notation is $(x, y) \rightarrow (x - 1, y - 5)$. 1 unit to the left, 5 units down

SLIDE 3

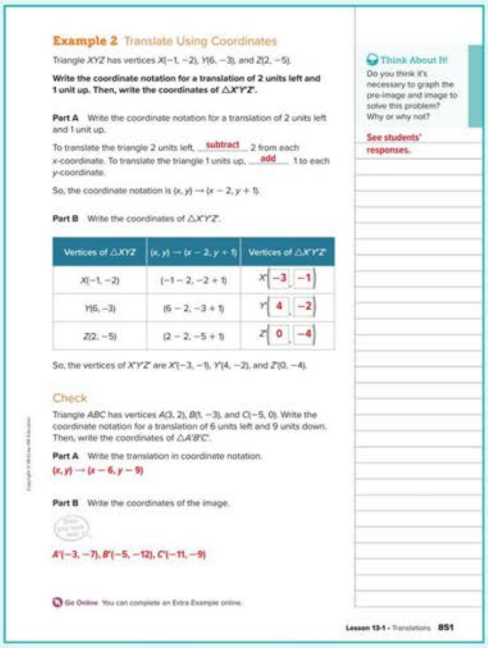
AL How can you find the *x*-coordinate of each vertex of the image? the *y*-coordinate? To find each *x*-coordinate of the image, subtract 2 from each *x*-coordinate of the preimage. To find each *y*-coordinate of the image, add 1 to each *y*-coordinate of the preimage.

OL How can you check your coordinate notation? Sample answer: I can graph the preimage and the image and then verify that the translation is 2 units to the left and 1 unit up.

BL Describe a real-world situation that can be represented by this example. Sample answer: Joe is painting a mural of different triangles of the same size. He wants the second triangle to be located 2 units to the left and 1 unit above the first triangle. Where will the second triangle be located?

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Translate Using Coordinates

Triangle *XYZ* has vertices *X*(−1, −2), *Y*(6, −3), and *Z*(2, −5).

Write the coordinate notation for a translation of 2 units left and 1 unit up. Then, write the coordinates of △*X*′*Y*′*Z*′.

Part A Write the coordinate notation for a translation of 2 units left and 1 unit up.

To translate the triangle 2 units left, subtract 2 from each *x*-coordinate. To translate the triangle 1 units up, add 1 to each *y*-coordinate.

So, the coordinate notation is $(x, y) \rightarrow (x - 2, y + 1)$.

Part B Write the coordinates of △*X*′*Y*′*Z*′.

Vertices of △XYZ	$(x, y) \rightarrow (x - 2, y + 1)$	Vertices of △X′Y′Z′
X(−1, −2)	(−1 − 2, −2 + 1)	X′(−3, −1)
Y(6, −3)	(6 − 2, −3 + 1)	Y′(4, −2)
Z(2, −5)	(2 − 2, −5 + 1)	Z′(0, −4)

So, the vertices of *X*′*Y*′*Z*′ are *X*′(−3, −1), *Y*′(4, −2), and *Z*′(0, −4).

Think About It! Do you think it's necessary to graph the pre-image and image to solve this problem? Why or why not?

See students' responses.

Check

Triangle *ABC* has vertices *A*(3, 2), *B*(1, −3), and *C*(−5, 0). Write the coordinate notation for a translation of 6 units left and 9 units down. Then, write the coordinates of △*A*′*B*′*C*′.

Part A Write the translation in coordinate notation.

$(x, y) \rightarrow (x - 6, y - 9)$

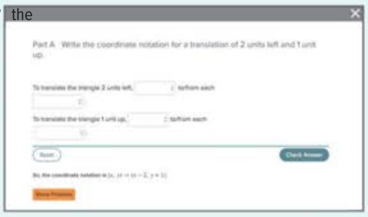
Part B Write the coordinates of the image.

A′(−3, −7), *B*′(−5, −12), *C*′(−11, −9)

Go Online You can complete an Extra Example online.

Lesson 13-1 • Translations 851

Interactive Presentation



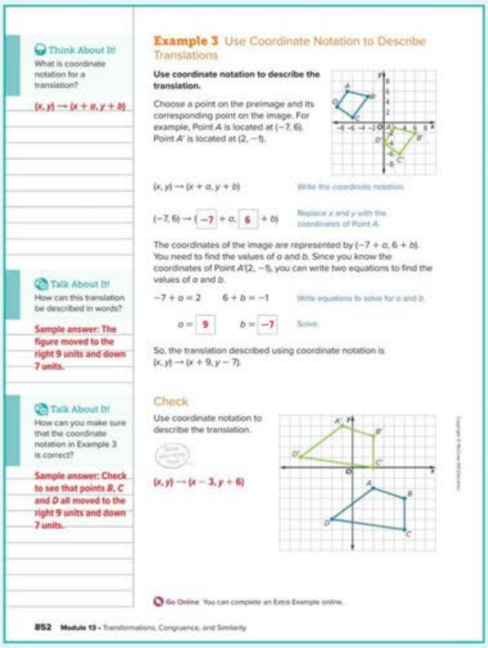
Example 2, Translate Using Coordinates, Slide 2 of 4

CLICK

On Slide 2, students select the correct words to complete the sentences.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Think About It! What is coordinate notation for a translation? $(x, y) \rightarrow (x + a, y + b)$

Talk About It! How can this translation be described in words? Sample answer: The figure moved to the right 9 units and down 7 units.

Talk About It! How can you make sure that the coordinate notation in Example 3 is correct? Sample answer: Check to see that points B, C and D all moved to the right 9 units and down 7 units.

Example 3 Use Coordinate Notation to Describe Translations

Use coordinate notation to describe the translation.

Choose a point on the preimage and its corresponding point on the image. For example, Point A is located at (−7, 6). Point A′ is located at (2, −1).

$(x, y) \rightarrow (x + a, y + b)$ Write the coordinate notation.

$(-7, 6) \rightarrow (-7 + a, 6 + b)$ Replace x and y with the coordinates of Point A.

The coordinates of the image are represented by $(-7 + a, 6 + b)$. You need to find the values of a and b. Since you know the coordinates of Point A′(2, −1), you can write two equations to find the values of a and b.

$-7 + a = 2$ $6 + b = -1$ Write equations to solve for a and b.

$a = 9$ $b = -7$ Solve.

So, the translation described using coordinate notation is $(x, y) \rightarrow (x + 9, y - 7)$.

Check

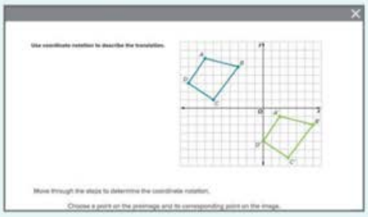
Use coordinate notation to describe the translation.

$(x, y) \rightarrow (x - 3, y + 6)$

Go Online You can complete an Extra Example online.

852 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 3, Use Coordinate Notation to Describe Translations, Slide 2 of 4

TYPE

On Slide 2, students determine the coordinate notation used to describe the translation.

CLICK

On Slide 2, students move through the steps to determine the coordinate notation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Use Coordinate Notation to Describe Translations

Objective

Students will describe translations shown on the coordinate plane using coordinate notation.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively While discussing the *Talk About It!* questions on Slide 3, encourage students to explain the translation in words and describe how they could check to make sure their coordinate notation is correct.

6 Attend to Precision Encourage students to precisely describe the translation using coordinate notation.

7 Look For and Make Use of Structure Encourage students to use the structure of coordinate notation to describe the translation.

Questions for Mathematical Discourse

SLIDE 2

AL How can you describe the translation in words? Sample answer: 9 units to the right and 7 units down

OL The x-coordinate of point A is -7. The x-coordinate of point A' is 2. What equation relates these two coordinates and the value of a in the coordinate notation? $-7 + a = 2$

OL The y-coordinate of point A is 6. The y-coordinate of point A' is -1. What equation relates these two coordinates and the value of b in the coordinate notation? $6 + b = -1$

OL How can you solve each equation for a and b? To solve $-7 + a = 2$, add 7 to each side, so $a = 9$. To solve $6 + b = -1$, subtract 6 from each side, so $b = -7$.

BL Generate a coordinate notation that can be used to describe a translation that moves the original figure to be located entirely within Quadrant I. Sample answer: $(x, y) \rightarrow (x + 10, y)$

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Map Reading

Objective

Students will come up with their own strategy to solve an application problem involving distance on the coordinate plane.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

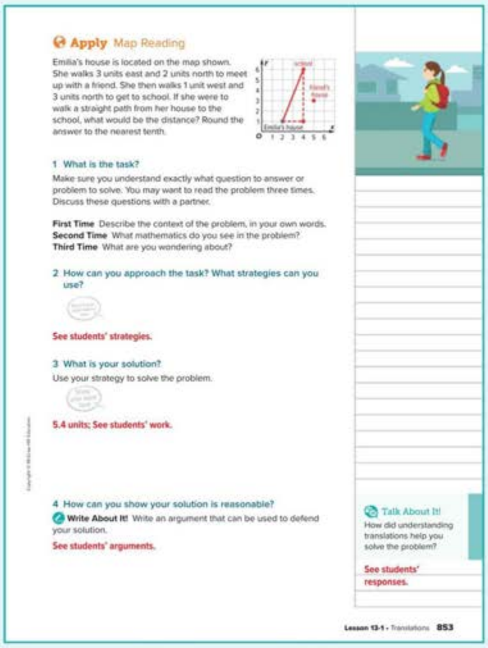
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How are the cardinal directions represented on the coordinate plane?
- How can you find the location of the friend's house? the school?
- How can you use what you know about right triangles to find the distance?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Map Reading

Emilia's house is located on the map shown. She walks 3 units east and 2 units north to meet up with a friend. She then walks 1 unit west and 3 units north to get to school. If she were to walk a straight path from her house to the school, what would be the distance? Round the answer to the nearest tenth.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

5.4 units; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

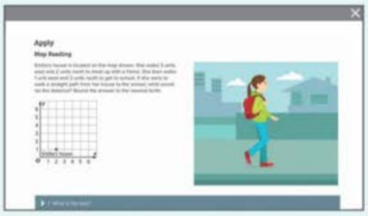
See students' arguments.

Talk About It!
How did understanding translations help you solve the problem?

See students' responses.

Lesson 13-1 • Translations 853

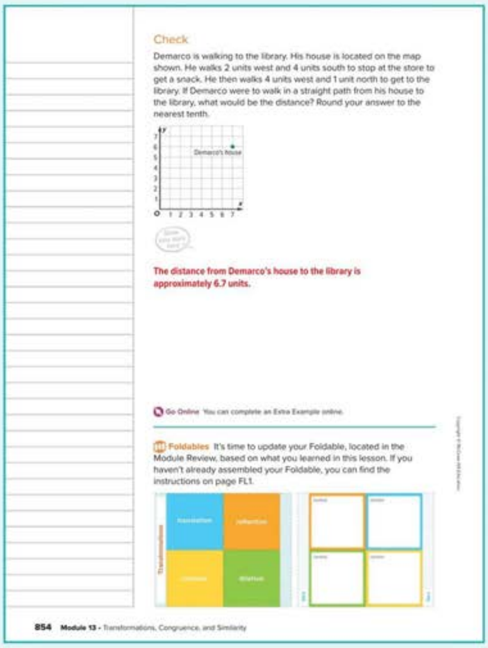
Interactive Presentation



Apply, Map Reading

CHECK

Students complete the Check exercise online to determine if they are ready to move on.


Check

Demarco is walking to the library. His house is located on the map shown. He walks 2 units west and 4 units south to stop at the store to get a snack. He then walks 4 units west and 1 unit north to get to the library. If Demarco were to walk in a straight path from his house to the library, what would be the distance? Round your answer to the nearest tenth.

The distance from Demarco's house to the library is approximately 6.7 units.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, could record an example of a translation. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What does it mean to perform a transformation on a figure?

In this lesson, students learned how to translate figures on the coordinate plane. Encourage them to research the word "isometry", and discuss with a partner if a translation is an isometry. Some students may find an isometry is a transformation that preserves the size of a figure, so a translation is an isometry.

Exit Ticket

Refer to the Exit Ticket slide. A geographic coordinate system such as latitude and longitude can be used to help a person navigate to different locations on Earth. A coordinate plane can also be used to move points and figures to a certain location. Use coordinate notation to describe the transformation from (3, 4) to (−1, −3). $(x, y) \rightarrow (x - 4, y - 7)$

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: **BL**

- Practice, Exercises 6, 7, 9–12
- ALEKS Translations

IF students score 66–89% on the Checks, **THEN** assign: **OL**

- Practice, Exercises 1–5, 7, 9, 10
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Translations

IF students score 65% or below on the Checks, **THEN** assign: **AL**

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Translations

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

- **AL** Practice Form B
- **OL** Practice Form A
- **BL** Practice Form C

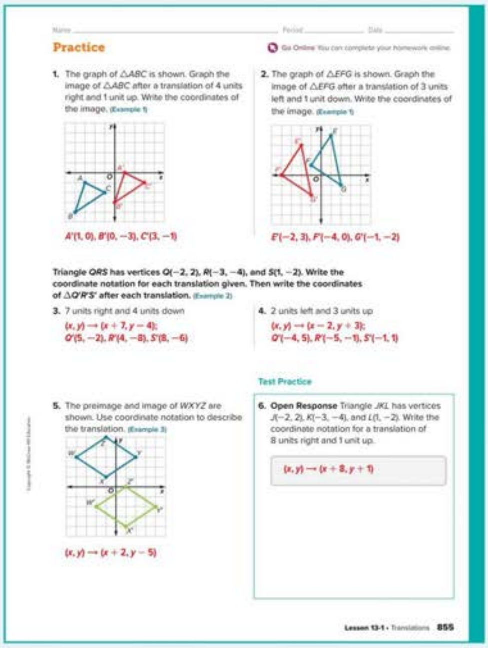
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	translate figures on the coordinate plane and determine the coordinates of the image	1, 2
1	write the coordinate notation for a translation and use it to find the coordinates of a figure's image	3, 4
1	describe translations using coordinate notation	5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving translations	7, 8
3	higher-order and critical thinking skills	9–12

Common Misconception

Some students may confuse how to represent the preimage moving left and right and up and down. In Exercise 4, students may incorrectly use "+2" to represent "2 units left" and "−3" to represent "3 units up". Help students to correlate moving to the right or up as positive and moving left or down as negative.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The graph of $\triangle ABC$ is shown. Graph the image of $\triangle ABC$ after a translation of 4 units right and 1 unit up. Write the coordinates of the image. (Example 1)

$A'(1, 0), B'(0, -3), C'(3, -1)$

2. The graph of $\triangle EFG$ is shown. Graph the image of $\triangle EFG$ after a translation of 3 units left and 1 unit down. Write the coordinates of the image. (Example 1)

$E'(-2, 3), F'(-4, 0), G'(-1, -2)$

Triangle QRS has vertices $Q(-2, 2)$, $R(-3, -4)$, and $S(1, -2)$. Write the coordinate notation for each translation given. Then write the coordinates of $\triangle Q'R'S'$ after each translation. (Example 2)

3. 7 units right and 4 units down

$(x, y) \rightarrow (x + 7, y - 4)$;
$Q'(5, -2), R'(4, -8), S'(8, -6)$

4. 2 units left and 3 units up

$(x, y) \rightarrow (x - 2, y + 3)$;
$Q'(-4, 5), R'(-5, -1), S'(-1, 1)$

5. The preimage and image of $WXYZ$ are shown. Use coordinate notation to describe the translation. (Example 3)

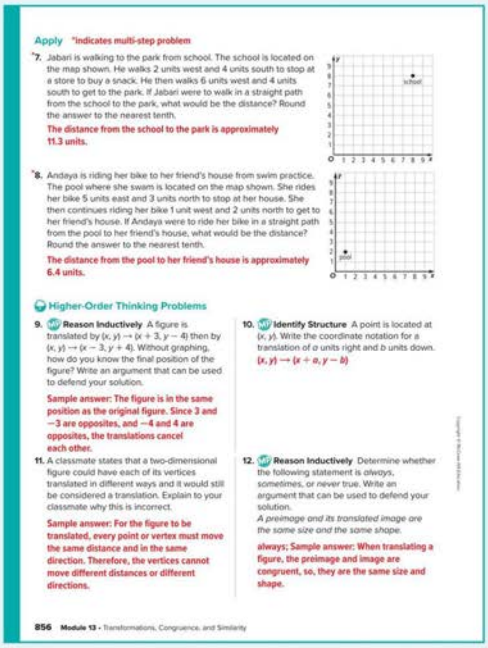
$(x, y) \rightarrow (x + 2, y - 5)$

Test Practice

6. **Open Response** Triangle JKL has vertices $J(-2, 2)$, $K(-3, -4)$, and $L(1, -2)$. Write the coordinate notation for a translation of 8 units right and 1 unit up.

$(x, y) \rightarrow (x + 8, y + 1)$

Lesson 13-1 • Translations 855



Apply *Indicates multi-step problem

*7. Jabari is walking to the park from school. The school is located on the map shown. He walks 2 units west and 4 units south to stop at a store to buy a snack. He then walks 6 units west and 4 units south to get to the park. If Jabari were to walk in a straight path from the school to the park, what would be the distance? Round the answer to the nearest tenth.

The distance from the school to the park is approximately 11.3 units.

*8. Andaya is riding her bike to her friend's house from swim practice. The pool where she swam is located on the map shown. She rides her bike 5 units east and 3 units north to stop at her house. She then continues riding her bike 1 unit west and 2 units north to get to her friend's house. If Andaya were to ride her bike in a straight path from the pool to her friend's house, what would be the distance? Round the answer to the nearest tenth.

The distance from the pool to her friend's house is approximately 6.4 units.

Higher-Order Thinking Problems

9. **Reason Inductively** A figure is translated by $(x, y) \rightarrow (x + 3, y - 4)$ then by $(x, y) \rightarrow (x - 3, y + 4)$. Without graphing, how do you know the final position of the figure? Write an argument that can be used to defend your solution.

Sample answer: The figure is in the same position as the original figure. Since 3 and −3 are opposites, and −4 and 4 are opposites, the translations cancel each other.

10. **Identify Structure** A point is located at (x, y). Write the coordinate notation for a translation of *a* units right and *b* units down.

$(x, y) \rightarrow (x + a, y - b)$

11. A classmate states that a two-dimensional figure could have each of its vertices translated in different ways and it would still be considered a translation. Explain to your classmate why this is incorrect.

Sample answer: For the figure to be translated, every point or vertex must move the same distance and in the same direction. Therefore, the vertices cannot move different distances or different directions.

12. **Reason Inductively** Determine whether the following statement is *always*, *sometimes*, or *never* true. Write an argument that can be used to defend your solution.

A preimage and its translated image are the same size and the same shape.

always; Sample answer: When translating a figure, the preimage and image are congruent, so, they are the same size and shape.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students will explain how they know the final position of the figure without graphing it. Encourage students to use reasoning to explain how they can determine the position of the figure.

7 Look For and Make Use of Structure In Exercise 10, students will write the coordinate notation for a translation of *a* units right and *b* units down. Encourage students to use the generic structure of coordinate notation to write the described translation.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 12, students will determine whether the given statement is always, sometimes, or never true. Encourage students to identify the important pieces of the statement and use that information to help determine the validity of the statement.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 7–8 After completing the application problems, have students write their own real-world application problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 11 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that the direction and distance that each of the vertices is translated can be different. Have each pair or group of students present their explanations to the class.

Lesson 13-2

Reflections

8.G.A.3

LESSON GOAL

Students will reflect figures and describe reflections on the coordinate plane.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Reflections on a Coordinate Plane
Example 1: Reflect Figures on the Coordinate Plane
Example 2: Reflect Figures on the Coordinate Plane

Explore: Reflect Using Coordinates

Learn: Reflect Using Coordinates
Example 3: Reflect Using Coordinates
Example 4: Describe Reflections

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Arrive**MATH** Take Another Look	●		
Extension: Extension Resources		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 80 of the *Language Development Handbook* to help your students build mathematical language related to reflections.

ELL You can use the tips and suggestions on page T80 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by reflecting figures and describing reflections on the coordinate plane.
Standards for Mathematical Content: **8. G.A.1, 8.G.A.1.A, 8.G.A.3**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students translated figures and described translations on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Now
Students reflect figures and describe reflections on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Next
Students will rotate figures and describe rotations on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Rigor

The Three Pillars of Rigor

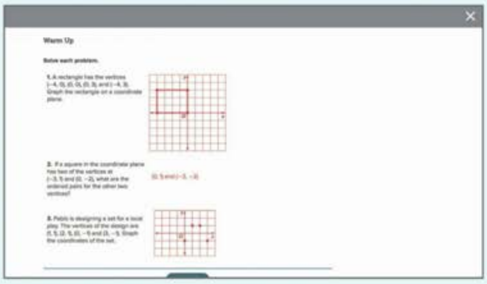
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of reflections on the coordinate plane. Students use their understanding to build *fluency* with reflecting figures using a graph and using coordinates. Students come to understand how to describe a reflection using coordinate notation.

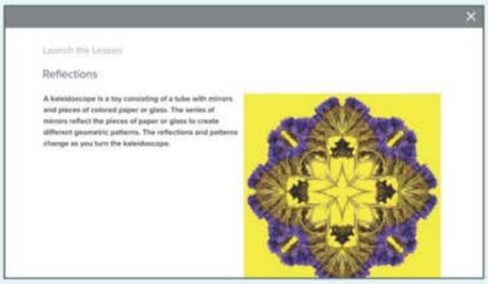
Mathematical Background

A *reflection* is a mirror image of a figure resulting from a transformation across a *line of reflection*. Each point of the preimage and its image are the same distance from the line of reflection, and the image and preimage are congruent. Coordinate notation for reflections across the x- and y-axes are $(x, y) \longrightarrow (x, -y)$ and $(x, y) \longrightarrow (-x, y)$ respectively.

Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- graphing in the coordinate plane (Exercises 1–3)

1–3. See W arm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about reflections and pattern changes when using a kaleidoscope.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- What do you think a *line of reflection* might be, based on the term *line* and the term *reflection*? Sample answer: A line of reflection might be a straight line that separates an image and its reflection.
- What do you know about reflections in your everyday life? Sample answer: When I look in a mirror, I see a reflection of myself.

Learn Reflections on a Coordinate Plane

Objective

Students will understand that reflecting a figure on the coordinate plane results in a mirror image of that figure across a line of reflection.

Teaching the Mathematical Practices

6 Attend to Precision While discussing the *Talk About It!* question on Slide 2, encourage students to use clear and precise mathematical language to describe the image after the reflection.

Teaching Notes

SLIDE 1

Students should note in a reflection, that each point of the preimage and its image are the same distance from the line of reflection, and that the image and the preimage are congruent.

Have students select the buttons to view the given triangle reflected across each line of reflection. You may wish to have them predict what the reflection will look like prior to selecting the buttons and then compare the reflection to the prediction.

Talk About It!

SLIDE 2

Mathematical Discourse

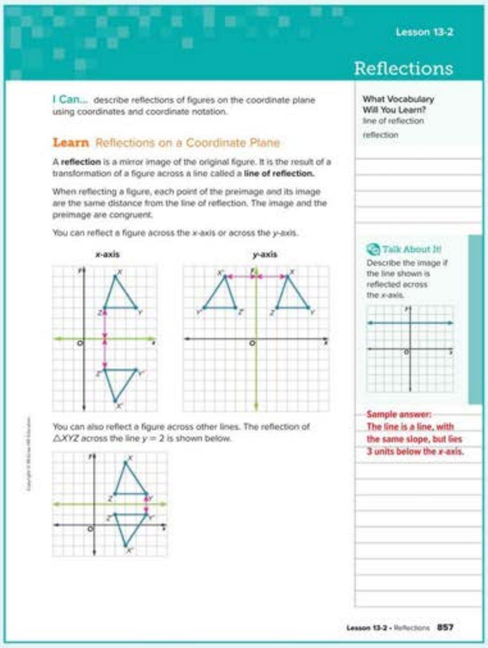
Describe the image if the line shown is reflected across the x-axis. Sample answer: The line is a line, with the same slope, but lies 3 units below the x-axis.

DIFFERENTIATE

Reteaching Activity AL

When the line of reflection is not the x- or y-axis, some students may struggle relating the equations of horizontal and vertical lines with their graphs. Use the following activity to support student learning.

- Instruct students to draw a horizontal line on the coordinate plane. Remind students that the equation of a horizontal line can be written as $y = b$, where b is the value of the y-coordinates. Then have them write the equation of their line.
- Instruct students to draw a vertical line on the coordinate plane. Remind students that the equation of a vertical line can be written as $x = a$, where a is the value of the x-coordinates. Then have them write the equation of their line.


Lesson 13-2

Reflections

I Can... describe reflections of figures on the coordinate plane using coordinates and coordinate notation.

What Vocabulary Will You Learn?
line of reflection
reflection

Learn Reflections on a Coordinate Plane

A **reflection** is a mirror image of the original figure. It is the result of a transformation of a figure across a line called a **line of reflection.**

When reflecting a figure, each point of the preimage and its image are the same distance from the line of reflection. The image and the preimage are congruent.

You can reflect a figure across the x-axis or across the y-axis.

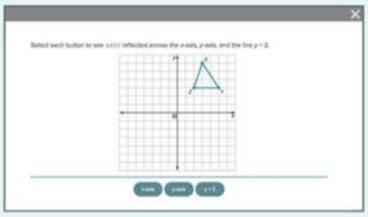
You can also reflect a figure across other lines. The reflection of $\triangle XYZ$ across the line $y = 2$ is shown below.

Talk About It!
Describe the image if the line shown is reflected across the x-axis.

Sample answer:
The line is a line, with the same slope, but lies 3 units below the x-axis.

Lesson 13-2 • Reflections 857

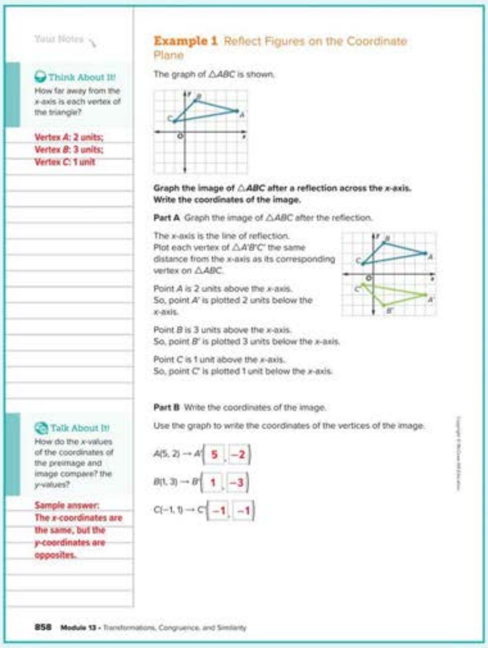
Interactive Presentation



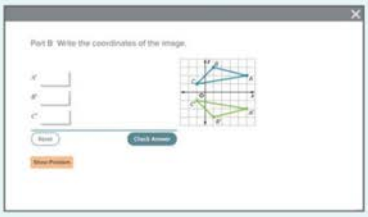
Learn, Reflections on a Coordinate Plane, Slide 1 of 2

CLICK

On Slide 1, students select each button to see the triangle reflected across the x-axis, y-axis, and the line $y = 2$.



Interactive Presentation



Example 1, Reflect Figures on the Coordinate Plane, Slide 3 of 5

TYPE

On Slide 3, students determine the coordinates of the image.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Reflect Figures on the Coordinate Plane

Objective

Students will determine the coordinates of an image after a reflection across an axis on the coordinate plane.

MP Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* question on Slide 4, encourage students to clearly and precisely explain how the *x*- and *y*-values of the preimage and image compare.

7 Look For and Make Use of Structure Encourage students to use the structure of the coordinate plane to know how the graph of a reflection across the *x*-axis compares to the preimage.

Questions for Mathematical Discourse

SLIDE 2

AL How far away from the *x*-axis is point *A*? point *B*? point *C*? 2 units; 3 units; 1 unit

OL Compare the distance from each vertex of the preimage to the *x*-axis to the distance from each vertex of the image to the *x*-axis. What do you notice? The distances are the same, respectively for each vertex.

BL In which quadrant(s) does the image lie? Does this make sense, given the line of reflection is the *x*-axis? Explain. Quadrants III and IV; Sample answer: Yes, since the preimage lies above the *x*-axis in QI and QII, the image will lie below the *x*-axis in QIII and QIV.

SLIDE 3

AL Imagine the graph as a flat piece of paper. If you were to fold the paper in half where the fold line is the *x*-axis, what would happen to the two triangles? Sample answer: They would be on top of each other and match exactly, since they are mirror images.

OL Why are the *x*-coordinates for the image and preimage the same? Sample answer: When a point is reflected across the *x*-axis, the *x*-coordinates of the image and preimage lie on the same vertical line. So, the *x*-coordinates are the same.

BL Explain why it makes sense that the *y*-coordinates of the preimage and image will be opposites. Sample answer: The *x*-axis is the line of reflection. So, when a point is reflected across this line, it will have the same vertical distance to the *x*-axis, but be on the other side.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Reflect Figures on the Coordinate Plane

Objective

Students will determine the coordinates of an image after a reflection across horizontal or vertical lines on the coordinate plane.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* questions, they should be able to explain whether this relationship will persist when the line of reflection is a vertical line.

6 Attend to Precision Students should be able to explain why the x-coordinates of the preimage and image are not opposites of one another, as they are when the line of reflection is the y-axis.

7 Look For and Make Use of Structure Encourage students to use the structure of the coordinate plane to know how the graph of a reflection across a vertical line of reflection, other than the y-axis, compares to the preimage.

As students discuss the *Talk About It!* questions, remind students to make sense of the comparison of the x- and y-coordinates between the preimage and image.

Questions for Mathematical Discourse

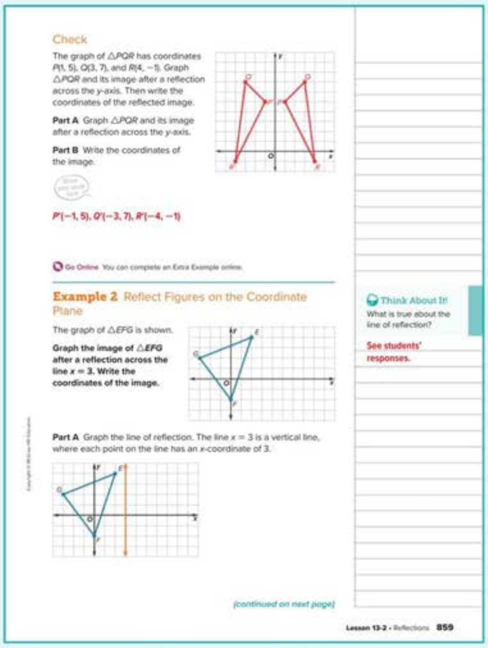
SLIDE 2

AL Describe two things you know about the line of reflection in this example. Sample answer: The line of reflection, $x = 3$, is a vertical line. Every point on the line has an x-coordinate of 3.

OL Will reflecting a figure across this line be more similar to reflecting a figure across the y-axis or the x-axis? Explain. Sample answer: It will be more similar to reflecting a figure across the y-axis, because both the y-axis and this line $x = 3$ are vertical lines.

BL What vertex of the image will be the farthest away from the line of reflection? Explain. G'; Sample answer: G is the vertex farthest away from the line prior to the reflection. So, its image will be the farthest away after the reflection.

(continued on next page)



Check

The graph of $\triangle PQR$ has coordinates $P(1, 5)$, $Q(3, 7)$, and $R(4, -1)$. Graph $\triangle PQR$ and its image after a reflection across the y-axis. Then write the coordinates of the reflected image.

Part A Graph $\triangle PQR$ and its image after a reflection across the y-axis.

Part B Write the coordinates of the image.

$P'(-1, 5)$, $Q'(-3, 7)$, $R'(-4, -1)$

Go Online You can complete an Extra Example online.

Example 2 Reflect Figures on the Coordinate Plane

The graph of $\triangle EFG$ is shown.

Graph the image of $\triangle EFG$ after a reflection across the line $x = 3$. Write the coordinates of the image.

Part A Graph the line of reflection. The line $x = 3$ is a vertical line, where each point on the line has an x-coordinate of 3.

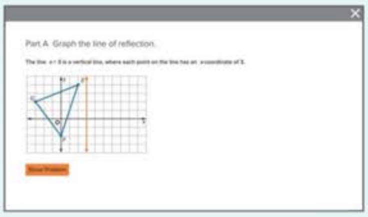
Think About It! What is true about the line of reflection?

See students' responses.

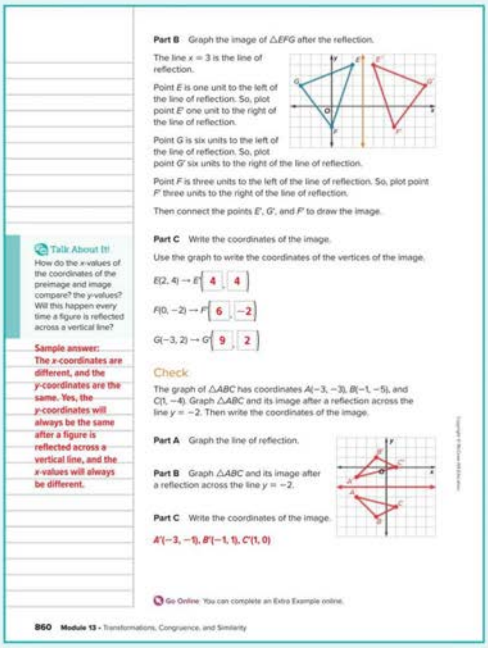
(continued on next page)

Lesson 13-2 • Reflections 859

Interactive Presentation



Example 2, Reflect Figures on the Coordinate Plane, Slide 2 of 6



Part B Graph the image of $\triangle EFG$ after the reflection.

The line $x = 3$ is the line of reflection.

Point E is one unit to the left of the line of reflection. So, plot point E' one unit to the right of the line of reflection.

Point G is six units to the left of the line of reflection. So, plot point G' six units to the right of the line of reflection.

Point F is three units to the left of the line of reflection. So, plot point F' three units to the right of the line of reflection.

Then connect the points E', G', and F' to draw the image.

Part C Write the coordinates of the image.

Use the graph to write the coordinates of the vertices of the image.

$E(2, 4) \rightarrow E'(4, 4)$

$F(0, -2) \rightarrow F'(6, -2)$

$G(-3, 2) \rightarrow G'(9, 2)$

Talk About It!

How do the x-values of the coordinates of the preimage and image compare? the y-values? Will this happen every time a figure is reflected across a vertical line?

Sample answer: The x-coordinates are different, and the y-coordinates are the same. Yes, the y-coordinates will always be the same after a figure is reflected across a vertical line, and the x-values will always be different.

Check

The graph of $\triangle ABC$ has coordinates $A(-3, -3)$, $B(-1, -5)$, and $C(1, -4)$. Graph $\triangle ABC$ and its image after a reflection across the line $y = -2$. Then write the coordinates of the image.

Part A Graph the line of reflection.

Part B Graph $\triangle ABC$ and its image after a reflection across the line $y = -2$.

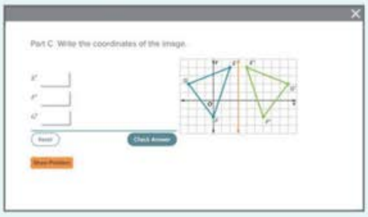
Part C Write the coordinates of the image.

$A'(-3, -1)$, $B'(-1, 1)$, $C'(1, 0)$

Go Online You can complete an Extra Example online.

860 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 2, Reflect Figures on the Coordinate Plane, Slide 4 of 6

CLICK

On Slide 3, students select points in order to see the triangle reflected over the line $x = 3$.

TYPE

On Slide 4, students determine the coordinates of the image.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Reflect Figures on the Coordinate Plane *(continued)*

Questions for Mathematical Discourse

SLIDE 3

AL How far away from the line of reflection is point E? point F? point G? 1 unit; 3 units; 6 units

OL Why do we not need to determine how many units each vertex is from the x-axis or the y-axis, in this example? Sample answer: The line of reflection is neither the x-axis nor the y-axis. The line of reflection is the line $x = 3$.

BL How many units will be between G and G'? Explain without counting. 12 units; Sample answer: There are 6 units from G to the line of reflection, plus another 6 units from the line of reflection to G'.

SLIDE 4

AL Compare the y-coordinates of each vertex of the preimage to the y-coordinates of each vertex of the image. What do you notice? Sample answer: They are the same, respectively for each vertex.

BL Compare the x-coordinates of each vertex of the preimage to the x-coordinates of each vertex of the image. Why are they not opposites of one another? Sample answer: The x-values are not opposites of one another because they are not being reflected across the y-axis, where $x = 0$. They are reflected across a different vertical line of reflection.

BL What would be the coordinates of E' if the line of reflection was $x = 4$? $E'(6, 4)$

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Explore Reflect Using Coordinates

Objective

Students will use Web Sketchpad to explore how to reflect two-dimensional figures using coordinates.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore the idea of reflecting a polygon across the *x*- and *y*-axes. They will make conjectures about the coordinates of the preimage and image and test to see if their conjectures are true. Students should be looking to find connections between reflections over the axes and the coordinates used in each situation.

Inquiry Question

How can you determine the coordinates of a figure after a reflection across either axis? Sample answer: If reflecting across the *x*-axis, keep the *x*-coordinate and take the opposite of the *y*-coordinate. If reflecting across the *y*-axis, take the opposite of the *x*-coordinate and keep the *y*-coordinate.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 3 is shown.

Talk About It!

SLIDE 3

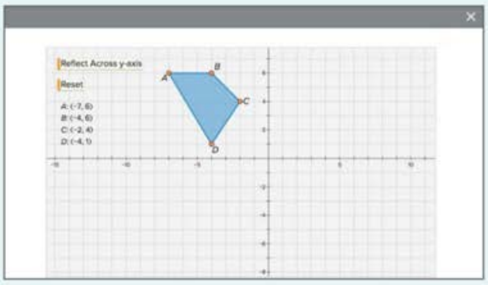
Mathematical Discourse

Press *Reflect Across y-axis*. The coordinates of the reflected image appear. Compare the coordinates of the preimage and the image. How do the coordinates change? Sample answer: The *x*-coordinates are opposites, the *y*-coordinates are the same.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 9

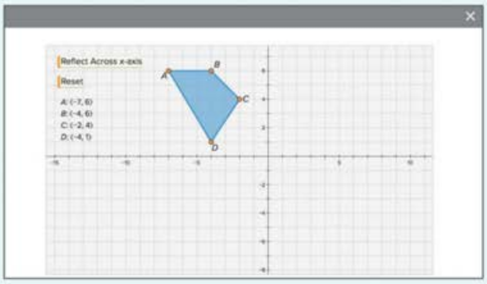


Explore, Slide 3 of 9

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to reflect two-dimensional figures using coordinates.

Interactive Presentation



Explore, Slide 8 of 9

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Reflect Using Coordinates *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore the relationship between the coordinates of a preimage and the coordinates of an image after reflections across the x- and y-axes.

6 Attend to Precision While discussing the *Inquiry Question*, encourage students to make sense of the similarities and differences that occur when reflecting an image over either axis. Remind students to use precise mathematical language in their explanations.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 8 is shown.

Talk About It!

SLIDE 8

Mathematical Discourse

Test your conjecture with quadrilaterals of different sizes and shapes. Did your experiments support your conjecture? Explain. Sample answer: Yes. Anytime I reflect the quadrilateral across the x-axis, the x-coordinates are the same, the y-coordinates are opposites.

Learn Reflect Using Coordinates

Objective

Students will learn how to use coordinate notation to find the coordinates of an image after a reflection across the *x*- or *y*-axis.

Go Online to have students watch the animation on Slide 1. The animation illustrates coordinate notation for reflections.

Teaching Notes

SLIDE 1

Play the animation for the class. You may wish to pause the animation when the notation $C'(4, -1)$ first appears. Ask students to make a conjecture about how the reflection over the *x*-axis affects the coordinates of the points. Some students may notice that the *x*-coordinates remain the same, but the *y*-coordinates are opposites. Continue playing the animation. You may wish to pause the animation after the screen showing the coordinate notation for a reflection over the *x*-axis is complete. Ask students to make a conjecture about how a reflection over the *y*-axis affects the coordinates of the points. Some students may choose to write coordinate notation for this reflection, $(x, y) \rightarrow (-x, y)$ based on what they saw for a reflection across the *x*-axis.

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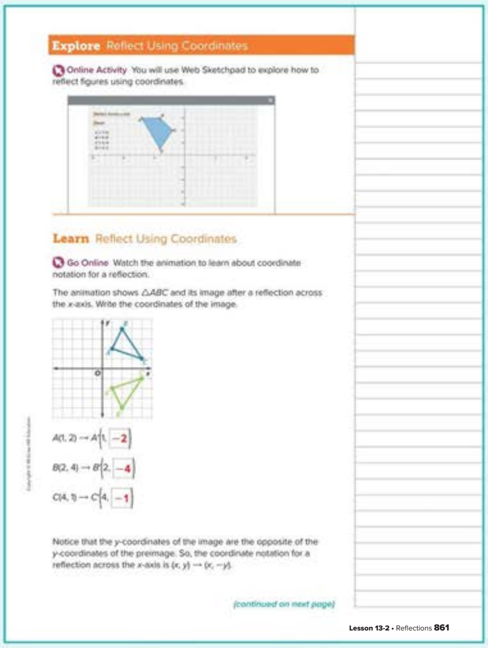


Explore Reflect Using Coordinates

Online Activity You will use Web Sketchpad to explore how to reflect figures using coordinates.

Learn Reflect Using Coordinates

Go Online Watch the animation to learn about coordinate notation for a reflection.

The animation shows $\triangle ABC$ and its image after a reflection across the *x*-axis. Write the coordinates of the image.

$A(1, 2) \rightarrow A'(1, -2)$

$B(2, 4) \rightarrow B'(2, -4)$

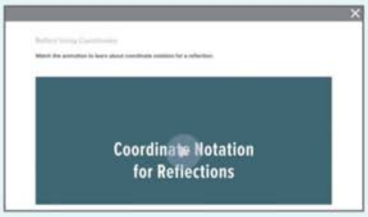
$C(4, 1) \rightarrow C'(4, -1)$

Notice that the *y*-coordinates of the image are the opposite of the *y*-coordinates of the preimage. So, the coordinate notation for a reflection across the *x*-axis is $(x, y) \rightarrow (x, -y)$.

(continued on next page)

Lesson 13-2 • Reflections **861**

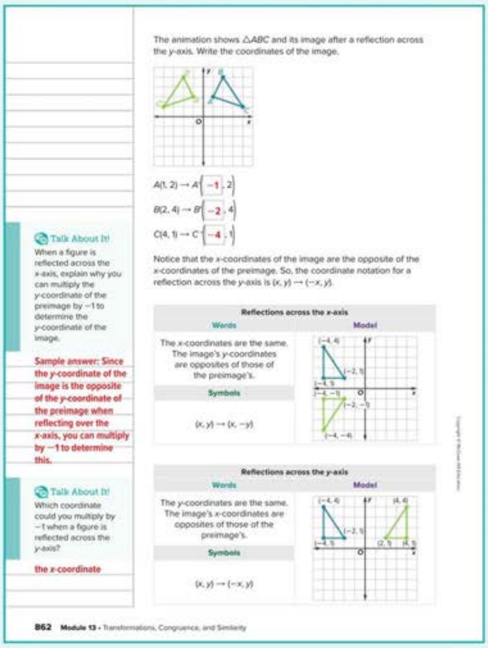
Interactive Presentation



Learn, Reflect Using Coordinates, Slide 1 of 3

WATCH

On Slide 1, students watch the animation to learn about coordinate notation for a reflection.



The animation shows $\triangle ABC$ and its image after a reflection across the *y*-axis. Write the coordinates of the image.

$A(1, 2) \rightarrow A'(-1, 2)$

$B(2, 4) \rightarrow B'(-2, 4)$

$C(4, 1) \rightarrow C'(-4, 1)$

Notice that the *x*-coordinates of the image are the opposite of the *x*-coordinates of the preimage. So, the coordinate notation for a reflection across the *y*-axis is $(x, y) \rightarrow (-x, y)$.

Reflections across the *x*-axis

Words	Model
The *x*-coordinates are the same. The image's *y*-coordinates are opposites of those of the preimage's.	(−4, 4), (−2, 1), (−4, 1), (−4, −1), (−2, −1), (−4, −4)
Symbols	
$(x, y) \rightarrow (x, -y)$	

Reflections across the *y*-axis

Words	Model
The *y*-coordinates are the same. The image's *x*-coordinates are opposites of those of the preimage's.	(−4, 4), (−2, 1), (−4, 1), (4, 4), (2, 1), (4, 1)
Symbols	
$(x, y) \rightarrow (-x, y)$	

Talk About It! When a figure is reflected across the *x*-axis, explain why you can multiply the *y*-coordinate of the preimage by −1 to determine the *y*-coordinate of the image.

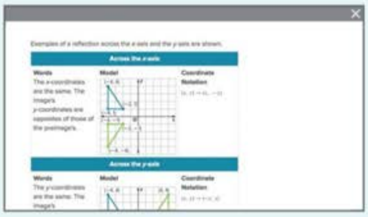
Sample answer: Since the *y*-coordinate of the image is the opposite of the *y*-coordinate of the preimage when reflecting over the *x*-axis, you can multiply by −1 to determine this.

Talk About It! Which coordinate could you multiply by −1 when a figure is reflected across the *y*-axis?

the *x*-coordinate

862 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Learn, Reflect Using Coordinates, Slide 2 of 3

Learn Reflect Using Coordinates *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively W hile discussing the *Talk About It!* questions on Slide 3, encourage students to identify how the coordinates change after the reflection across the *x*-axis.

7 Look For and Make Use of Structure While discussing the *Talk About It!* questions on Slide 3, encourage students to hypothesize how they could use a similar strategy to find the coordinates of a figure reflected across the *y*-axis.

Teaching Notes

SLIDE 2

Have students study the table to learn more about the coordinate notation used for reflections when the line of reflection is either the *x*-axis or the *y*-axis.

Ask students to formulate a plan for how they can remember which coordinate changes signs. Some students may say that a reflection across the *x*-axis results in a vertical change so the *y*-coordinate changes signs, while a reflection across the *y*-axis results in a horizontal change so the *x*-coordinate changes signs.

Talk About It!

SLIDE 3

Mathematical Discourse

When a figure is reflected across the *x*-axis, explain why you can multiply the *y*-coordinate of the preimage by −1 to determine the *y*-coordinate of the image. Sample answer: Since the *y*-coordinate of the image is the opposite of the *y*-coordinate of the preimage when reflecting over the *x*-axis, you can multiply by −1 to determine this.

Which coordinate could you multiply by −1 when a figure is reflected across the *y*-axis? the *x*-coordinate

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Reflect Using Coordinates

Objective

Students will write the coordinate notation for a reflection and use it to find the coordinates of a two-dimensional figure's image.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to write? the coordinate notation for a reflection across the x-axis

OL What happens to the x-coordinate when a point is reflected across the x-axis? Explain why this makes sense. Sample answer: The x-coordinates remain the same. Reflecting a point across the horizontal x-axis does not change how far that point is away from the y-axis (x-coordinate). It only changes how far the point is away from the x-axis (y-coordinate).

BL Is the transformation $(x, y) \longrightarrow (x, -y)$ true for horizontal lines of reflection other than the x-axis? Explain. no; Sample answer: A point and its image must be the same distance from the line of reflection. (x, y) and $(x, -y)$ are only the same distance from the horizontal line $y = 0$ (the x-axis).

SLIDE 3

AL Since the x-coordinates remain the same, what will be the x-coordinate of each point? The x-coordinate of R' is 2. The x-coordinate of S' is -1. The x-coordinate of T' is -2.

AL How will you find the y-coordinates of the image? Find the opposite of the y-coordinates of each vertex of the preimage.

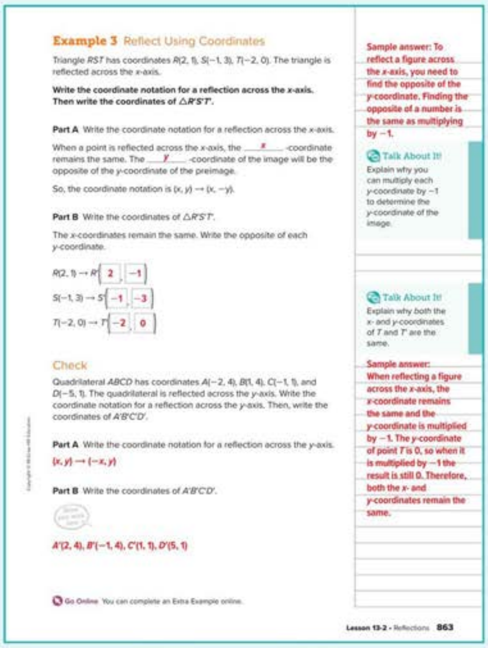
OL How can you verify that you found the correct coordinates? Sample answer: Graph the preimage and image on the same coordinate plane to verify that the image is a reflection of the preimage across the x-axis.

OL Explain how you can remember that the y-coordinates are opposites when the line of reflection is the x-axis. Sample answer: I can mentally picture a point, such as (2, 3) and reflect it across the x-axis to verify that the reflection is (2, −3), not (−2, 3).

BL If the preimage of a point has the ordered pair $(-r, -t)$, what is the ordered pair that represents the image after the preimage is reflected across the x-axis? $(-r, t)$

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 3 Reflect Using Coordinates

Triangle RST has coordinates $R(2, 1)$, $S(-1, 3)$, $T(-2, 0)$. The triangle is reflected across the x-axis.

Write the coordinate notation for a reflection across the x-axis. Then write the coordinates of $\triangle R'S'T'$.

Part A Write the coordinate notation for a reflection across the x-axis.

When a point is reflected across the x-axis, the x-coordinate remains the same. The y-coordinate of the image will be the opposite of the y-coordinate of the preimage.

So, the coordinate notation is $(x, y) \rightarrow (x, -y)$.

Part B Write the coordinates of $\triangle R'S'T'$.

The x-coordinates remain the same. Write the opposite of each y-coordinate.

$R(2, 1) \rightarrow R'(2, -1)$

$S(-1, 3) \rightarrow S'(-1, -3)$

$T(-2, 0) \rightarrow T'(-2, 0)$

Check

Quadrilateral $ABCD$ has coordinates $A(-2, 4)$, $B(1, 4)$, $C(-1, 1)$, and $D(-5, 1)$. The quadrilateral is reflected across the y-axis. Write the coordinate notation for a reflection across the y-axis. Then, write the coordinates of $A'B'C'D'$.

Part A Write the coordinate notation for a reflection across the y-axis.

$(x, y) \rightarrow (-x, y)$

Part B Write the coordinates of $A'B'C'D'$.

$A'(2, 4)$, $B'(-1, 4)$, $C'(1, 1)$, $D'(5, 1)$

Go Online You can complete an Extra Example online.

Sample answer: To reflect a figure across the x-axis, you need to find the opposite of the y-coordinate. Finding the opposite of a number is the same as multiplying by −1.

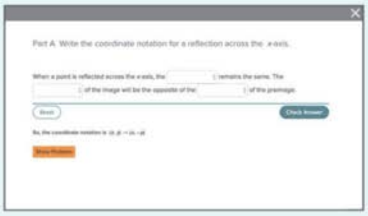
Talk About It! Explain why you can multiply each y-coordinate by −1 to determine the y-coordinate of the image.

Talk About It! Explain why *both* the x- and y-coordinates of T and T' are the same.

Sample answer: When reflecting a figure across the x-axis, the x-coordinate remains the same and the y-coordinate is multiplied by −1. The y-coordinate of point T is 0, so when it is multiplied by −1 the result is still 0. Therefore, both the x- and y-coordinates remain the same.

Lesson 13-2 • Reflections 863

Interactive Presentation



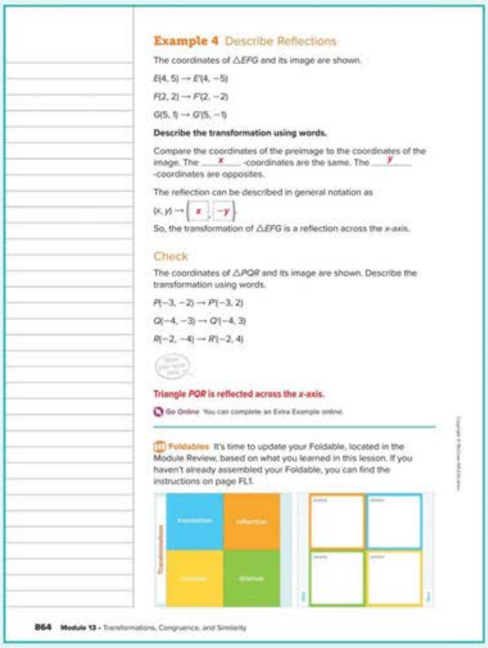
Example 3, Reflect Using Coordinates, Slide 2 of 5

CLICK

On Slide 2, students select the correct words to complete the sentences.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 4 Describe Reflections

The coordinates of $\triangle EFG$ and its image are shown.

$E(4, 5) \rightarrow E'(4, -5)$

$F(2, 2) \rightarrow F'(2, -2)$

$G(5, 1) \rightarrow G'(5, -1)$

Describe the transformation using words.

Compare the coordinates of the preimage to the coordinates of the image. The x -coordinates are the same. The y -coordinates are opposites.

The reflection can be described in general notation as

$(x, y) \rightarrow (x, -y)$

So, the transformation of $\triangle EFG$ is a reflection across the x-axis.

Check

The coordinates of $\triangle PQR$ and its image are shown. Describe the transformation using words.

$P(-3, -2) \rightarrow P'(-3, 2)$

$Q(-4, -3) \rightarrow Q'(-4, 3)$

$R(-2, -4) \rightarrow R'(-2, 4)$

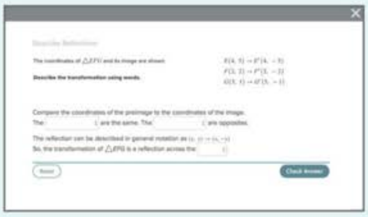
Triangle PQR is reflected across the x-axis.

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

864 **Module 13** • Transformations, Congruence, and Similarity

Interactive Presentation



Example 4, Describe Reflections, Slide 1 of 2

CLICK

On Slide 1, students select the correct words to complete the sentences.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 4 Describe Reflections

Objective

Students will use coordinate notation and words to describe reflections by analyzing the coordinates of a reflected image.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to analyze the coordinates before and after the transformation to describe the transformation using coordinate notation.

Questions for Mathematical Discourse

SLIDE 1

AL What do you notice about the coordinates of the preimage and the image? The x-coordinates are the same, and the y-coordinates are opposites.

OL What must be true when the x-coordinates are the same and the y-coordinates are opposites? The transformation is a reflection and the line of reflection is the horizontal x-axis.

OL How can you check your answer? Sample answer: I can graph the preimage and the image to verify the transformation is a reflection across the x-axis.

BL In a reflection across the x-axis, what would be the coordinates of the image if the preimage coordinates were (5, 0)? Explain. (5, 0); Zero is its own opposite.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record an example of a reflectionoY may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What does it mean to perform a transformation on a figure?

In this lesson, students learned how to reflect figures on the coordinate plane. Encourage them to work with a partner to compare and contrast the algebraic representations of reflections over the x-axis and over the y-axis. For example, the absolute values of the coordinates of the preimage and the image are the same for both reflections. In a reflection across the x-axis, the x-coordinates are the same. The image's y-coordinates are opposites of those of the preimage's. In a reflection across the y-axis, the y-coordinates are the same. The image's x-coordinates are opposites of those of the preimage's.

Exit Ticket

Refer to the Exit Ticket slide. What are the coordinates of the vertices of the triangle if it is reflected across the *y*-axis? What about the *x*-axis? Write a mathematical argument that can be used to defend your solution. *y*-axis: (−1, 7), (−1, 1), (−6, 1); *x*-axis: (1, −1), (6, −1), (1, −7); Sample answer: In a reflection across the *y*-axis, the *x*-coordinates of the image and preimage are opposites. In a reflection across the *x*-axis, the *y*-coordinates of the image and preimage are opposites.

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

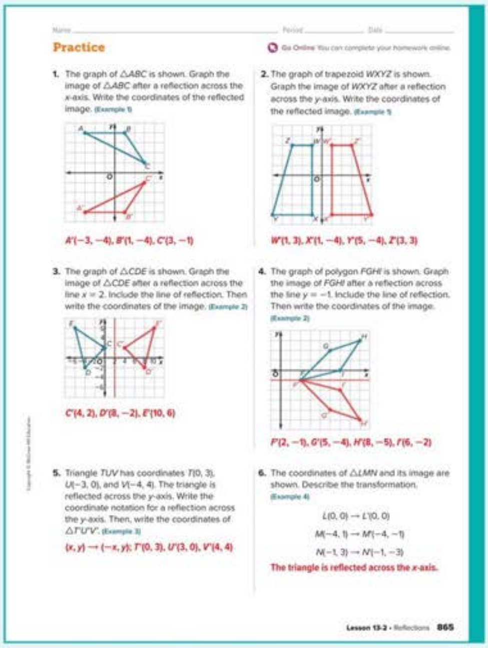
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	determine the coordinates of an image after a reflection across an axis on the coordinate plane	1, 2
1	determine the coordinates of an image after a reflection across horizontal or vertical lines on the coordinate plane	3, 4
1	write the coordinate notation for a reflection and use it to find the coordinates of a two-dimensional figure's image	5
1	describe reflections by analyzing the coordinates of a reflected image	6
2	extend concepts learned in class to apply them in new contexts	7
3	higher-order and critical thinking skills	8–11

Common Misconception

Students may associate prime notation with the coordinates of the *preimage* and fail to apply prime notation to the *image*. Help students avoid this mistake by showing them a series of transformations, with the coordinates of each new *image* notated by an additional prime symbol.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. The graph of △ABC is shown. Graph the image of △ABC after a reflection across the *x*-axis. Write the coordinates of the reflected image. (Example 1)
A′(−3, −4), B′(1, −4), C′(3, −1)

2. The graph of trapezoid WXYZ is shown. Graph the image of WXYZ after a reflection across the *y*-axis. Write the coordinates of the reflected image. (Example 1)
W′(1, 3), X′(1, −4), Y′(5, −4), Z′(3, 3)

3. The graph of △CDE is shown. Graph the image of △CDE after a reflection across the line $x = 2$. Include the line of reflection. Then write the coordinates of the image. (Example 2)
C′(4, 2), D′(8, −2), E′(10, 6)

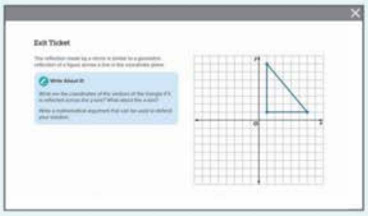
4. The graph of polygon FGHI is shown. Graph the image of FGHI after a reflection across the line $y = -1$. Include the line of reflection. Then write the coordinates of the image. (Example 2)
F′(2, −1), G′(5, −4), H′(8, −5), I′(6, −2)

5. Triangle TUV has coordinates T(0, 3), U(−3, 0), and V(−4, 4). The triangle is reflected across the *y*-axis. Write the coordinate notation for a reflection across the *y*-axis. Then, write the coordinates of △T′U′V′. (Example 3)
(x, y) → (−x, y); T′(0, 3), U′(3, 0), V′(4, 4)

6. The coordinates of △LMN and its image are shown. Describe the transformation. (Example 4)
L(0, 0) → L′(0, 0)
M(−4, 1) → M′(−4, −1)
N(−1, 3) → N′(−1, −3)
The triangle is reflected across the *x*-axis.

Lesson 13-2 • Reflections 865

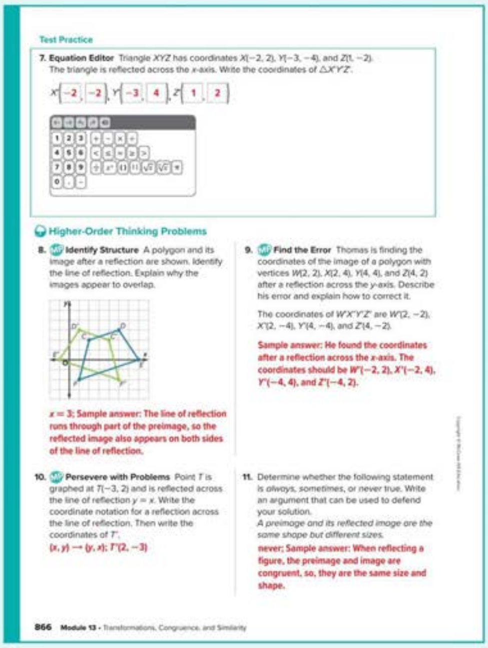
Interactive Presentation



Exit Ticket

8.G.A.3

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**


Test Practice

7. Equation Editor Triangle XYZ has coordinates $X(-2, 2)$, $Y(-3, -4)$, and $Z(1, -2)$. The triangle is reflected across the x-axis. Write the coordinates of $\triangle X'Y'Z'$.

$X'(-2, -2)$, $Y'(-3, 4)$, $Z'(1, 2)$

Higher-Order Thinking Problems

8. Identify Structure A polygon and its image after a reflection are shown. Identify the line of reflection. Explain why the images appear to overlap.

$x = 3$; Sample answer: The line of reflection runs through part of the preimage, so the reflected image also appears on both sides of the line of reflection.

9. Find the Error Thomas is finding the coordinates of the image of a polygon with vertices $W(2, 2)$, $X(2, 4)$, $Y(4, 4)$, and $Z(4, 2)$ after a reflection across the y-axis. Describe his error and explain how to correct it.

The coordinates of $W'X'Y'Z'$ are $W'(2, -2)$, $X'(2, -4)$, $Y'(4, -4)$, and $Z'(4, -2)$.

Sample answer: He found the coordinates after a reflection across the x-axis. The coordinates should be $W'(-2, 2)$, $X'(-2, 4)$, $Y'(-4, 4)$, and $Z'(-4, 2)$.

10. Persevere with Problems Point T is graphed at $T(-3, 2)$ and is reflected across the line of reflection $y = x$. Write the coordinate notation for a reflection across the line of reflection. Then write the coordinates of T'.

$(x, y) \rightarrow (y, x)$; $T'(2, -3)$

11. Determine whether the following statement is *always*, *sometimes*, or *never* true. Write an argument that can be used to defend your solution.
A preimage and its reflected image are the same shape but different sizes.

never; Sample answer: When reflecting a figure, the preimage and image are congruent, so, they are the same size and shape.

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Teaching the Mathematical Practices

7 Look For and Make Use of Structure In Exercise 8, students identify the line of reflection and explain why the images appear to overlap. Encourage students to examine the preimage and image in order to determine the line of reflection.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students will describe the error and how to correct it. Encourage students to find the error and then use a well constructed response that explains how to fix it.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 10, students write the coordinate notation and the coordinates of the image. Encourage students to plan a solution pathway they can implement to answer the question.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercise.

Create your own higher-order thinking problem.
Use with Exercises 8–11 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner, solve them, and check each other's work.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 7–11
- Extension: Reflections Over the Line $y = x$
- ALEKS® Reflections

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–6, 8, 11
- Extension: Reflections Over the Line $y = x$
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS® Reflections

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Reflections

Lesson 13-3

Rotations

8.G.A.3

LESSON GOAL

Students will rotate figures and describe rotations on the coordinate plane.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Rotations About a Vertex
Example 1: Rotate Figures About a Vertex

Explore: Rotate Using Coordinates

Learn: Rotations About the Origin
Example 2: Rotate Using Coordinates
Example 3: Describe Rotations
Apply: Arranging Furniture

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

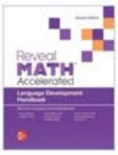
View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL BL	
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Rotations About Other Points		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 81 of the *Language Development Handbook* to help your students build mathematical language related to rotations.

ELL You can use the tips and suggestions on page T81 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min: 0.5 day
45 min: 1 day

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by rotating figures and describing rotations on the coordinate plane.
Standards for Mathematical Content: **8. G.A.1, 8.G.A.1.A, 8.G.A.3**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students reflected figures and described reflections on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Now
Students rotate figures and describe rotations on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Next
Students will dilate figures and describe dilations on the coordinate plane.
8.G.A.3

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

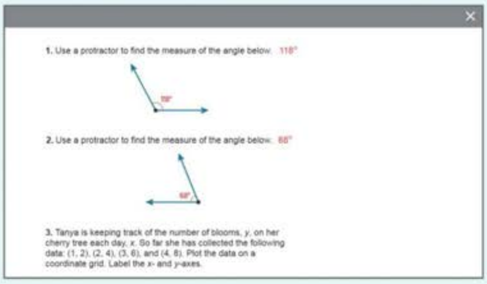
Conceptual Bridge In this lesson, students develop *understanding* of rotations on the coordinate plane. Students use their understanding to build *fluency* with rotating figures about a vertex and about the origin. Students come to understand how to describe a rotation using coordinate notation.

Mathematical Background

A *rotation* is a transformation in which a figure is rotated about a fixed point called the *center of rotation*. Because the size and shape of the figure are unchanged, the preimage and image of a rotation are congruent. Coordinate notation can be used to specify clockwise rotations about the origin.

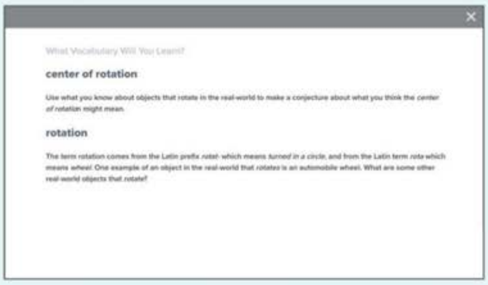
- 90° clockwise: $(x, y) \longrightarrow (y, -x)$
- 180° clockwise: $(x, y) \longrightarrow (-x, -y)$
- 270° clockwise: $(x, y) \longrightarrow (-y, x)$

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- using a protractor to measure angles (Exercises 1–2)
- graphing in the coordinate plane (Exercise 3)

1–3. See W arm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the rotation of Ferris Wheels.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Use what you know about objects that rotate in the real-world to make a conjecture about what you think the *center of rotation* might mean. Sample answer: The center of rotation might be a fixed point about which an object rotates.
- The term rotation comes from the Latin prefix *rotat-* which means *turned in a circle*, and from the Latin term *rota* which means *wheel*. One example of an object in the real-world that *rotates* is an automobile wheel. What are some other real-world objects that *rotate?* Sample answers: Earth, a windmill, rotating drill, Ferris wheel, merry-go round, fans

Learn Rotations About a Vertex

Objective

Students understand how to rotate two-dimensional figures about a vertex on the coordinate plane.

Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* question on Slide 2, encourage students to visualize the line being rotated 90° clockwise about the origin and to draw the image of the line after the rotation to validate their results.

Go Online to have students watch the animation on Slide 1. The animation illustrates rotations of figures about a vertex.

Teaching Notes

SLIDE 1

Students will learn that a *rotation* is a transformation in which a figure is rotated, or turned, about a fixed point. Students should note that the *center of rotation* is the fixed point around which the figure rotates. It is important that students realize that rotations can be described in degrees and direction, such as clockwise or counterclockwise. Point out that a full circle is 360°. Play the animation for the class. You may wish to pause the animation when the notation "90° Clockwise Rotation" first appears. Ask students to determine what point remains fixed and what points will rotate. Then ask them in what direction will point *B* move. Have students make a conjecture about the coordinates of *B′*. You may wish to ask these questions for a 180° and a 270° rotation.

Talk About It!

SLIDE 2

Mathematical Discourse

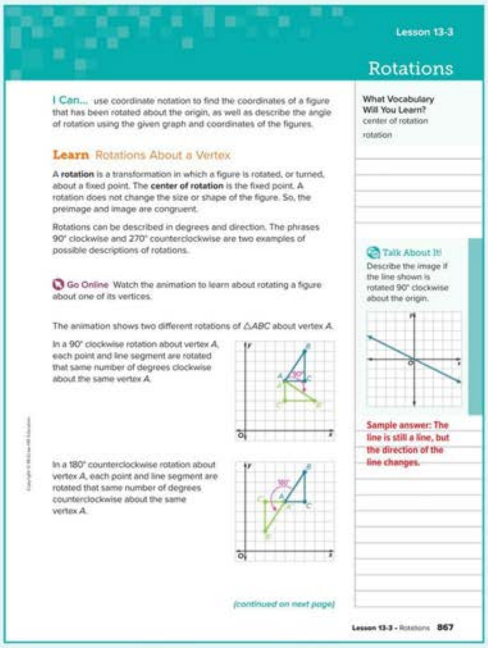
Describe the image if the line shown is rotated 90° clockwise about the origin.

Sample answer: The line is still a line, but the direction of the line changes.

DIFFERENTIATE

Language Development Activity ELL

Review the difference between a clockwise rotation and a counterclockwise rotation using an image of an analog clock or a Ferris Wheel. Students may also struggle with visualizing the image after a rotation. Suggest they trace the preimage on a piece of thin paper. Then have them place the traced image over the original image, and place the end of a pencil on the point of rotation. This allows them to rotate the traced image. Remind them that a 90° rotation is a quarter turn, a 180° rotation is a half turn, and a 270° rotation is a three-quarter turn. Some students may discover that a 270° clockwise rotation is the same as a 90° counterclockwise rotation and a 270° counterclockwise rotation is the same as a 90° clockwise rotation.



Lesson 13-3

Rotations

I Can... use coordinate notation to find the coordinates of a figure that has been rotated about the origin, as well as describe the angle of rotation using the given graph and coordinates of the figures.

What Vocabulary Will You Learn?
center of rotation
rotation

Learn Rotations About a Vertex

A **rotation** is a transformation in which a figure is rotated, or turned, about a fixed point. The **center of rotation** is the fixed point. A rotation does not change the size or shape of the figure. So, the preimage and image are congruent.

Rotations can be described in degrees and direction. The phrases 90° clockwise and 270° counterclockwise are two examples of possible descriptions of rotations.

Go Online Watch the animation to learn about rotating a figure about one of its vertices.

The animation shows two different rotations of $\triangle ABC$ about vertex *A*.

In a 90° clockwise rotation about vertex *A*, each point and line segment are rotated that same number of degrees clockwise about the same vertex *A*.

In a 180° counterclockwise rotation about vertex *A*, each point and line segment are rotated that same number of degrees counterclockwise about the same vertex *A*.

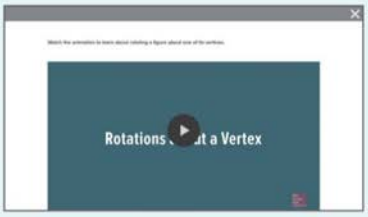
Talk About It! Describe the image if the line shown is rotated 90° clockwise about the origin.

Sample answer: The line is still a line, but the direction of the line changes.

(continued on next page)

Lesson 13-3 • Rotations 867

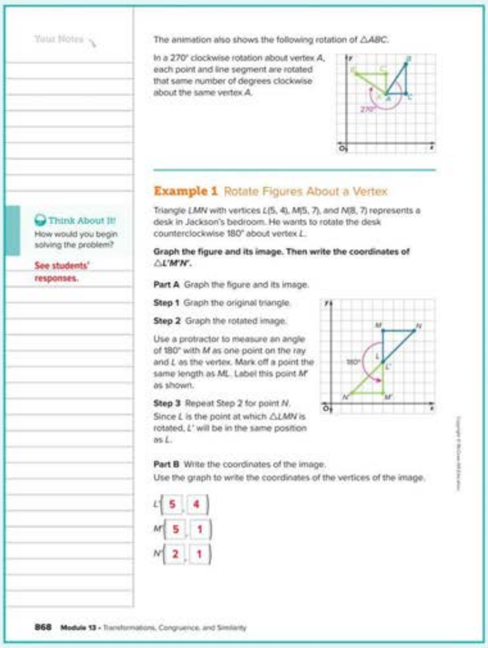
Interactive Presentation



Learn, Rotations About a Vertex, Slide 1 of 2

WATCH

On Slide 1, students watch the animation to learn about rotating a figure about one of its vertices.



Your Notes

The animation also shows the following rotation of $\triangle ABC$.

In a 270° clockwise rotation about vertex A, each point and line segment are rotated that same number of degrees clockwise about the same vertex A.

Example 1 Rotate Figures About a Vertex

Triangle LMN with vertices $L(5, 4)$, $M(5, 7)$, and $N(8, 7)$ represents a desk in Jackson's bedroom. He wants to rotate the desk counterclockwise 180° about vertex L.

Think About It! How would you begin solving the problem?

See students' responses.

Graph the figure and its image. Then write the coordinates of $\triangle L'M'N'$.

Part A Graph the figure and its image.

Step 1 Graph the original triangle.

Step 2 Graph the rotated image.

Use a protractor to measure an angle of 180° with M as one point on the ray and L as the vertex. Mark off a point the same length as ML. Label this point M' as shown.

Step 3 Repeat Step 2 for point N. Since L is the point at which $\triangle LMN$ is rotated, L' will be in the same position as L.

Part B Write the coordinates of the image.

Use the graph to write the coordinates of the vertices of the image.

$L'(5, 4)$

$M'(5, 1)$

$N'(2, 1)$

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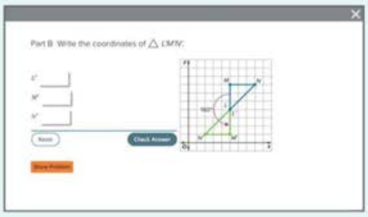
Interactive Presentation



Example 1, Rotate Figures About a Vertex, Slide 3 of 4

CLICK

On Slide 2, students move through the slides to rotate the triangle.

TYPE

On Slide 3, students determine the coordinates of the image.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Rotate Figures About a Vertex

Objective

Students will rotate two-dimensional figures about a vertex on the coordinate plane, and determine the coordinates of the image.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to make sense of what a 180° counterclockwise rotation means prior to rotating the figure.

6 Attend to Precision Students should be precise when determining the coordinates of the image after the rotation.

Questions for Mathematical Discourse

SLIDE 2

AL Describe in your own words what a 180° counterclockwise rotation means. Sample answer: A 180° rotation is a rotation halfway around a full circular rotation of 360°. Since the direction is counterclockwise, the rotation will occur in the opposite direction that a clock's hands rotate.

OL What kind of angle is 180°? How can this help you locate point M'? a straight angle; Sample answer: Point M' is located along the same straight line as the line segment between points M and L, the same distance as this line segment, but on the other side of point L.

BL If the triangle was rotated 180° clockwise instead of counterclockwise, how would this affect the image? Sample answer: The image will have the same coordinates as a 180° counterclockwise rotation because a 180° rotation represents half a full circle.

SLIDE 3

AL Why are the coordinates of point L' the same as the coordinates of point L? Sample answer: Point L is the center of rotation. If a point rotates about itself, it remains in the same location.

OL How do the coordinates of point M and M' compare? Why do you think this is? Sample answer: They have the same x-coordinate, but different y-coordinates. This is because points M and M' lie on a vertical line. So, the x-coordinates are the same.

BL Find the area of Triangle $L'M'N'$. 4.5 square units

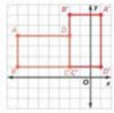
Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Check

Rectangle $ABCD$ with vertices $A(-7, 4)$, $B(-7, 1)$, $C(-2, 1)$, and $D(-2, 4)$ represents the bed in Jackson's room. Graph the figure and its image after a clockwise rotation of 90° about vertex C. Then write the coordinates of the rectangle $A'B'C'D'$.

Part A Graph the figure and its image after a clockwise rotation of 90° about vertex C.



Part B Write the coordinates of rectangle $A'B'C'D'$.

$A'(1, 6)$, $B'(-2, 6)$, $C'(-2, 1)$, $D'(1, 1)$

Go Online You can complete an Extra Example online.

Math History Minute

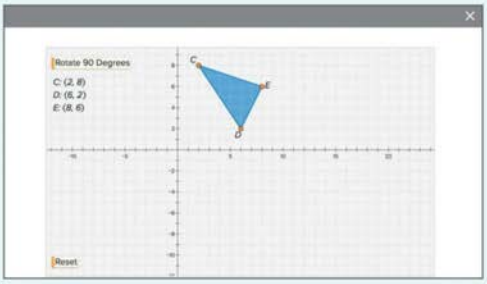
Sofia Kovalevskaya (1850–1891) was a Russian mathematician and the first European woman to receive a doctorate in mathematics since the Renaissance. She received an award from the French Academy of Science for her paper "On the Problem of the Rotation of a Solid Body About a Fixed Point". The Academy was so impressed with her paper, that they increased the monetary amount of the award.

Explore Rotate Using Coordinates

Online Activity You will use Web Sketchpad to explore how to rotate figures using coordinates.

Interactive Presentation

Explore, Slide 1 of 11



Explore, Slide 4 of 11

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to rotate two-dimensional figures using coordinates.

Explore Rotate Using Coordinates

Objective

Students will use Web Sketchpad to explore how to rotate two-dimensional figures using coordinates.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore 90°, 180°, and 270° clockwise rotations about the origin. Students can change the coordinates ofthe preimage before it is rotated. Students will use their observations to make conjectures about how to determine the coordinates of an image after these types of rotations.

Inquiry Question

How can you determine the coordinates of an image after a 90°, 180°, or 270° clockwise rotation about the origin? Sample answer: The coordinates of a preimage and an image after a 90°, 180°, and 270° clockwise rotation about the origin each have a special relationship. I can use these relationships to determine the coordinates of the image.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

SLIDE 4

Mathematical Discourse

Test your conjecture by rotating different triangles created by dragging points *C, D,* and *E*. Does your conjecture hold true? Sample answer: Yes. In the image, the *x*-coordinate is the same as the *y*-coordinate of the preimage. The *y*-coordinate of the image is the opposite of the *x*-coordinate of the preimage.

(continued on next page)

Explore Rotate Using Coordinates *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore the correspondences between the coordinates of the image and the coordinates of the preimage after 90°, 180°, and 270° clockwise rotations about the origin.

7 Look For and Make Use of Structure Encourage students to examine the structure of the coordinates before and after each rotation to discover patterns for 90°, 180°, and 270° clockwise rotations about the origin.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 10 is shown.

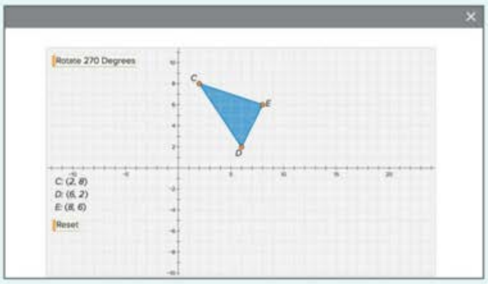
Talk About It!

SLIDE 10

Mathematical Discourse

Test your conjecture by rotating different triangles created by dragging points *C*, *D*, and *E*. Does your conjecture hold true? Sample answer: Yes. In the image, the *x*-coordinate is the opposite of the *y*-coordinate of the preimage. The *y*-coordinate of the image is the same as the *x*-coordinate of the preimage.

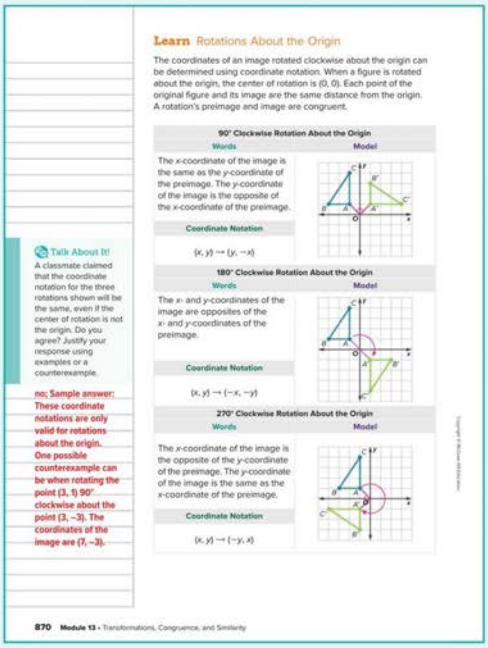
Interactive Presentation



Explore, Slide 10 of 11

TYPE

On Slide 11, students respond to the Inquiry Question and view a sample answer.



Learn Rotations About the Origin

The coordinates of an image rotated clockwise about the origin can be determined using coordinate notation. When a figure is rotated about the origin, the center of rotation is (0, 0). Each point of the original figure and its image are the same distance from the origin. A rotation's preimage and image are congruent.

90° Clockwise Rotation About the Origin	
Words	**Model**
The x-coordinate of the image is the same as the y-coordinate of the preimage. The y-coordinate of the image is the opposite of the x-coordinate of the preimage.	
Coordinate Notation	
$(x, y) \rightarrow (y, -x)$	

180° Clockwise Rotation About the Origin	
Words	**Model**
The x- and y-coordinates of the image are opposites of the x- and y-coordinates of the preimage.	
Coordinate Notation	
$(x, y) \rightarrow (-x, -y)$	

270° Clockwise Rotation About the Origin	
Words	**Model**
The x-coordinate of the image is the opposite of the y-coordinate of the preimage. The y-coordinate of the image is the same as the x-coordinate of the preimage.	
Coordinate Notation	
$(x, y) \rightarrow (-y, x)$	

Talk About It!
A classmate claimed that the coordinate notation for the three rotations shown will be the same, even if the center of rotation is not the origin. Do you agree? Justify your response using examples or a counterexample.

no; Sample answer: These coordinate notations are only valid for rotations about the origin. One possible counterexample can be when rotating the point (3, 1) 90° clockwise about the point (3, −3). The coordinates of the image are (7, −3).

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Interactive Presentation

Learn, Rotations About the Origin, Slide 1 of 2

8.G.A.3

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Rotations About the Origin

Objective

Students will understand how to use coordinate notation to rotate two-dimensional figures on the coordinate plane about the origin.

MP Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others While discussing the *Talk About It!* question on Slide 2, encourage students to create a plausible argument and draw a counterexample to illustrate why these coordinate notations are only valid for rotations about the origin.

Teaching Notes

SLIDE 1

Students will learn how they can determine the coordinates of an image after 90°, 180°, and 270° clockwise rotations about the origin using coordinate notation. Have them study the chart to further investigate each rotation and the coordinate notation for each.

Remind students that the table shows the clockwise rotation coordinate notation. Ask them to make a conjecture about coordinate notation for counterclockwise rotations and how the table can help them. Some students may recognize how clockwise and counterclockwise rotations are related:
- 90° clockwise is the same as 270° counterclockwise,
- 180° rotation is the same for both,
- 270° clockwise is the same as 90° counterclockwise.

Talk About It!

SLIDE 2

Mathematical Discourse

A classmate claimed that the coordinate notation for the rotations shown will be the same, even if the center of rotation is not the origin. Do you agree? Justify your response using examples or a counterexample. no; Sample answer: These coordinate notations are only valid for rotations about the origin. One possible counterexample can be when reflecting the point (3, 1) 90° clockwise about the point (3, −3). The coordinates of the image are (7, −3).

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 2 Rotate Using Coordinates

Objective

Students will write the coordinate notation for a rotation and use it to find the coordinates of a two-dimensional figure's image.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically A s students discuss the *Talk About It!* question, encourage them to understand that they can use a protractor to measure the angle of rotation in order to verify it.

Questions for Mathematical Discourse

SLIDE 2

AL Describe the rotation in your own words. Sample answer: The rotation is 90°, which is a right angle, in the same direction that a clock's hands rotate, and about the origin.

OL If you do not remember the coordinate notation for a 90° rotation clockwise about the origin, how can you determine the coordinates or check your answer? Sample answer: Plot a point on the coordinate plane, rotate it 90° clockwise about the origin, and verify the coordinates of the image and how they relate to the coordinates of the preimage.

BL Suppose the image, with coordinates $(y, -x)$ is rotated an additional 90° clockwise about the origin. Describe how to find the coordinates of the final image. Sample answer: The x-coordinate of the final image will be the same as the y-coordinate of $(y, -x)$. The y-coordinate of the final image will be the opposite of the x-coordinate of $(y, -x)$. So, the coordinates will be $(-x, -y)$. Note that $(-x, -y)$ is the same as the notation for a rotation of 180°.

SLIDE 3

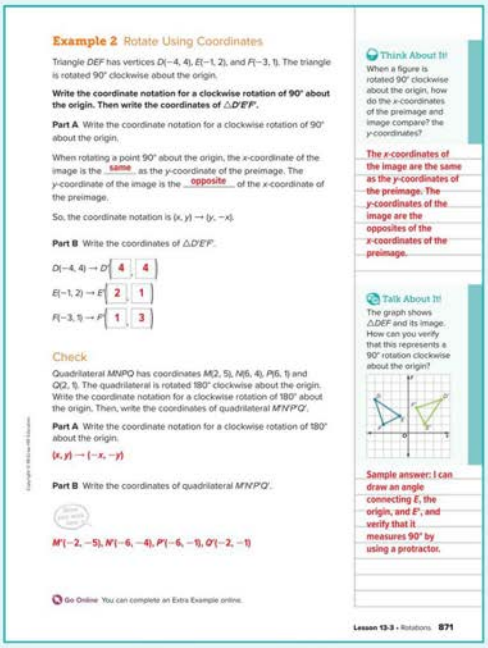
AL Describe how to find the x-coordinate of each vertex of the image. Sample answer: The x-coordinate of each image will be the same as the y-coordinate of the preimage.

OL Describe how to find the y-coordinate of each vertex of the image. Sample answer: The y-coordinate of each image will be the opposite of the x-coordinate of the preimage.

BL What does it mean that all of the coordinates of the image are positive? Sample answer: The image lies entirely within Quadrant I.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Rotate Using Coordinates

Triangle DEF has vertices $D(-4, 4)$, $E(-1, 2)$, and $F(-3, 1)$. The triangle is rotated 90° clockwise about the origin.

Write the coordinate notation for a clockwise rotation of 90° about the origin. Then write the coordinates of $\triangle D'E'F'$.

Part A Write the coordinate notation for a clockwise rotation of 90° about the origin.

When rotating a point 90° about the origin, the x-coordinate of the image is the __same__ as the y-coordinate of the preimage. The y-coordinate of the image is the __opposite__ of the x-coordinate of the preimage.

So, the coordinate notation is $(x, y) \rightarrow (y, -x)$.

Part B Write the coordinates of $\triangle D'E'F'$.

$D(-4, 4) \rightarrow D'(4, 4)$

$E(-1, 2) \rightarrow E'(2, 1)$

$F(-3, 1) \rightarrow F'(1, 3)$

Check

Quadrilateral $MNPQ$ has coordinates $M(2, 5)$, $N(6, 4)$, $P(6, 1)$ and $Q(2, 1)$. The quadrilateral is rotated 180° clockwise about the origin. Write the coordinate notation for a clockwise rotation of 180° about the origin. Then, write the coordinates of quadrilateral $M'N'P'Q'$.

Part A Write the coordinate notation for a clockwise rotation of 180° about the origin.

$(x, y) \rightarrow (-x, -y)$

Part B Write the coordinates of quadrilateral $M'N'P'Q'$.

$M'(-2, -5)$, $N'(-6, -4)$, $P'(-6, -1)$, $Q'(-2, -1)$

Go Online You can complete an Extra Example online.

Think About It! When a figure is rotated 90° clockwise about the origin, how do the x-coordinates of the preimage and image compare? the y-coordinates?

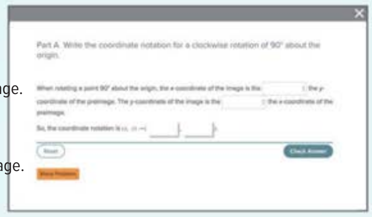
The x-coordinates of the image are the same as the y-coordinates of the preimage. The y-coordinates of the image are the opposites of the x-coordinates of the preimage.

Talk About It! The graph shows $\triangle DEF$ and its image. How can you verify that this represents a 90° rotation clockwise about the origin?

Sample answer: I can draw an angle connecting E, the origin, and E', and verify that it measures 90° by using a protractor.

Lesson 13-3 • Rotations 871

Interactive Presentation



Example 2, Rotate Using Coordinates, Slide 2 of 5

TYPE

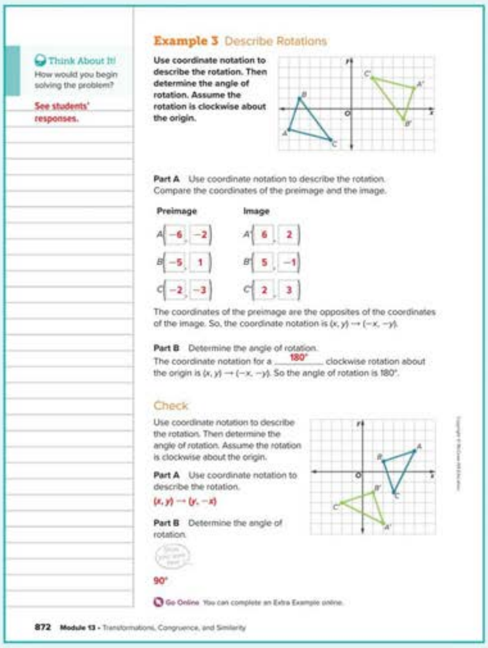
On Slide 2, students determine the correct coordinate notation.

CLICK

On Slide 2, students select the correct words that complete the sentences.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! How would you begin solving the problem?

See students' responses.

Example 3 Describe Rotations

Use coordinate notation to describe the rotation. Then determine the angle of rotation. Assume the rotation is clockwise about the origin.

Part A Use coordinate notation to describe the rotation. Compare the coordinates of the preimage and the image.

Preimage	Image
$A(-6, -2)$	$A'(6, 2)$
$B(-5, 1)$	$B'(5, -1)$
$C(-2, -3)$	$C'(2, 3)$

The coordinates of the preimage are the opposites of the coordinates of the image. So, the coordinate notation is $(x, y) \rightarrow (-x, -y)$.

Part B Determine the angle of rotation.

The coordinate notation for a 180° clockwise rotation about the origin is $(x, y) \rightarrow (-x, -y)$. So the angle of rotation is 180°.

Check

Use coordinate notation to describe the rotation. Then determine the angle of rotation. Assume the rotation is clockwise about the origin.

Part A Use coordinate notation to describe the rotation.

$(x, y) \rightarrow (y, -x)$

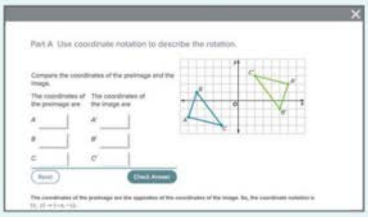
Part B Determine the angle of rotation.

90°

Go Online You can complete an Extra Example online.

872 **Module 13** • Transformations, Congruence, and Similarity

Interactive Presentation



Example 3, Describe Rotations, Slide 2 of 4

TYPE

On Slide 2, students determine the coordinates of the preimage and image. On Slide 3, students enter the degree of rotation.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Describe Rotations

Objective

Students will describe rotations using coordinate notation, and determine the angle of rotation.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure Enc ourage students to study the structure of the coordinates of the image and how they compare to the preimage to determine the angle of rotation.

Questions for Mathematical Discourse

SLIDE 2

AL After listing the coordinates, what do you notice? Sample answer: The image coordinates are the opposite of the preimage coordinates.

OL If you do not remember the coordinate notation for a rotation, how can you determine the angle of rotation? Sample answer: Draw the angle that connects point B, the origin, and point B'. Since that angle is a straight angle, the angle of rotation is 180°.

BL A classmate claimed that the transformation is a reflection across the line $y = x$, and not a rotation. How would you respond? Sample answer: The transformation is not a reflection across the line $y = x$. If it was, then the coordinates of the image would be $A'(-2, -6)$, $B'(1, -5)$, and $C'(-3, 2)$.

SLIDE 3

AL Just by looking at the coordinate notation, how do you know this is not a 90° rotation? Sample answer: The x- and y-coordinates are not reversed.

OL How can you check your answer? Sample answer: I can plot another point and use a protractor to draw the image after a 180° rotation clockwise about the origin, and then compare the coordinates of the image to the coordinate notation.

BL Is the coordinate notation for a 180° *counterclockwise* rotation about the origin the same as a 180° *clockwise* rotation about the origin? Explain. yes; Sample answer: The image will end up in the same place regardless of whether the direction is clockwise or counterclockwise.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Arranging Furniture

Objective

Students will come up with their own strategy to solve an application problem involving a sequence of transformations.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- Which transformation should you perform first?
- What does a clockwise rotation about the origin mean?
- How can using coordinate notation help you solve the problem?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Arranging Furniture

Before moving furniture in her bedroom, Jasmine made a diagram of the current arrangement. She drew rectangle *ABCD* to represent her desk with vertices *A*(2, 4), *B*(6, 4), *C*(6, 1), and *D*(2, 1). She moved the desk twice, first translating it 3 units left and 2 units down, and then rotating it 90° clockwise about the origin. What are the coordinates of the vertices of the final image after these transformations were applied?

1 What is the task?
Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?
Use your strategy to solve the problem.

$A''(2, 1)$, $B''(2, -3)$, $C''(-1, -3)$, and $D''(-1, 1)$; See students' work.

4 How can you show your solution is reasonable?
Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It! Will the results be the same if the figure was rotated first, then translated? Explain.

no; Sample answer: By using the coordinate rules to rotate first and then translate, the resulting coordinates are $A''(1, -4)$, $B''(1, -8)$, $C''(-2, -8)$, and $D''(-2, -4)$, which are not the same.

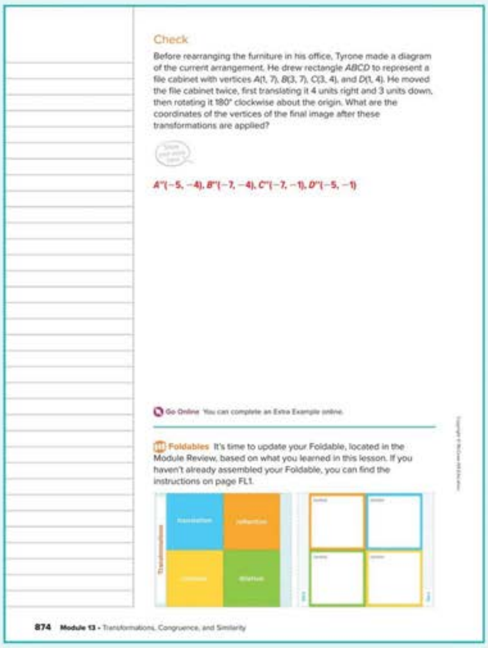
Lesson 13-3 • Rotations 873

Interactive Presentation

Apply, Arranging Furniture

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

Before rearranging the furniture in his office, Tyrone made a diagram of the current arrangement. He drew rectangle $ABCD$ to represent a file cabinet with vertices $A(1, 7)$, $B(3, 7)$, $C(3, 4)$, and $D(1, 4)$. He moved the file cabinet twice, first translating it 4 units right and 3 units down, then rotating it 180° clockwise about the origin. What are the coordinates of the vertices of the final image after these transformations are applied?

$A''(-5, -4)$, $B''(-7, -4)$, $C''(-7, -1)$, $D''(-5, -1)$

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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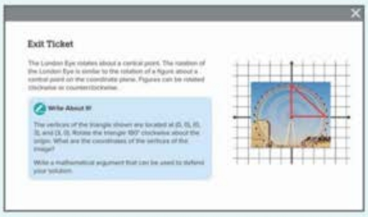
Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record an example of a rotation. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What does it mean to perform a transformation on a figure?

In this lesson, students learned how to rotate a figure about a vertex and about the origin. Encourage them to discuss with a partner how they find the coordinates when a figure is rotated about the origin. Some students may state that they prefer to use the coordinate notation when the rotation is a multiple of 90°.

Exit Ticket

Refer to the Exit Ticket slide. The vertices of the triangle are located at (0, 0), (0, 3), and (3, 0). Rotate the triangle 180° clockwise about the origin. What are the coordinates of the vertices of the image? Write a mathematical argument that can be used to defend your solution.

(0, 0), (0, −3), (−3, 0); Sample answer: The coordinates of the image and preimage are opposites in a 180° clockwise rotation about the origin.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 6, 7, 9–11
- Extension: Rotations About Other Points
- ALEKS® Rotations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 7, 10, 11
- Extension: Rotations About Other Points
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS® Rotations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Rotations

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

Suggested Assignments

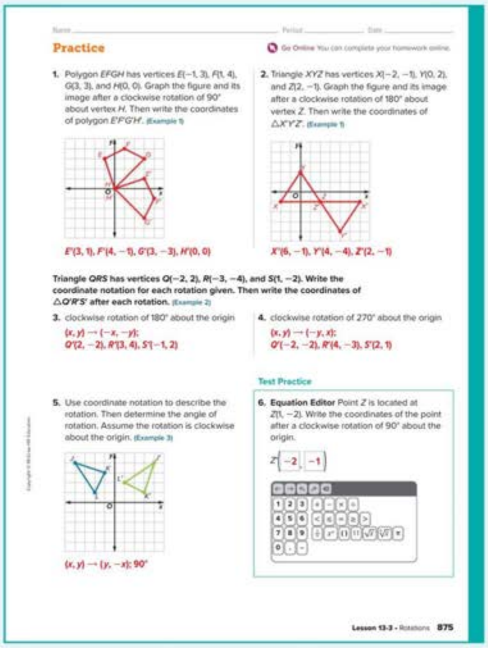
Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	rotate two-dimensional figures about a vertex on the coordinate plane, and determine the coordinates of the image	1, 2
1	write the coordinate notation for a clockwise rotation about the origin and use it to find the coordinates of a two-dimensional figure's image	3, 4
1	describe clockwise rotations using coordinate notation, and determine the angle of rotation about the origin	5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving rotations	7, 8
3	higher-order and critical thinking skills	9–11

Common Misconception

Some students may incorrectly use coordinate notation when rotating a figure about a vertex that is not the origin. In Exercises 1 and 2, students may incorrectly list the coordinates of the rotations. Remind students that coordinate notation only applies to clockwise rotations about the origin, and that they need to use 90°, 180°, or 270° angles when finding the coordinates after the rotations.

Some students may also incorrectly write the coordinate notation for a given rotation. In Exercise 3, students may forget to write the negative sign in front of the coordinates in the coordinate notation. Remind students that a 180° clockwise rotation about the origin results in both coordinates of the image being the opposite of the coordinates of the preimage.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Polygon *EFGH* has vertices *E*(−1, 3), *F*(1, 4), *G*(3, 3), and *H*(0, 0). Graph the figure and its image after a clockwise rotation of 90° about vertex *H*. Then write the coordinates of polygon *E′F′G′H′*. (Example 1)

   E′(3, 1), *F′*(4, −1), *G′*(3, −3), *H′*(0, 0)

2. Triangle *XYZ* has vertices *X*(−2, −1), *Y*(0, 2), and *Z*(2, −1). Graph the figure and its image after a clockwise rotation of 180° about vertex *Z*. Then write the coordinates of △*X′Y′Z′*. (Example 1)

   X′(6, −1), *Y′*(4, −4), *Z′*(2, −1)

Triangle *QRS* has vertices *Q*(−2, 2), *R*(−3, −4), and *S*(1, −2). Write the coordinate notation for each rotation given. Then write the coordinates of △*Q′R′S′* after each rotation. (Example 2)

3. clockwise rotation of 180° about the origin

   (*x*, *y*) → (−*x*, −*y*);
   Q′(2, −2), *R′*(3, 4), *S′*(−1, 2)

4. clockwise rotation of 270° about the origin

   (*x*, *y*) → (−*y*, *x*);
   Q′(−2, −2), *R′*(4, −3), *S′*(2, 1)

5. Use coordinate notation to describe the rotation. Then determine the angle of rotation. Assume the rotation is clockwise about the origin. (Example 3)

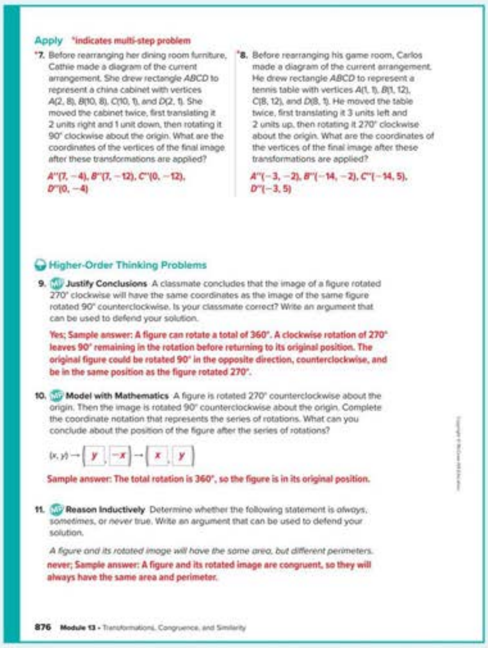
   (*x*, *y*) → (*y*, −*x*); 90°

Test Practice

6. **Equation Editor** Point *Z* is located at *Z*(1, −2). Write the coordinates of the point after a clockwise rotation of 90° about the origin.

   Z′(−2, −1)

Lesson 13-3 • Rotations 875



Apply *indicates multi-step problem

*7. Before rearranging her dining room furniture, Cathie made a diagram of the current arrangement. She drew rectangle $ABCD$ to represent a china cabinet with vertices $A(2, 8)$, $B(10, 8)$, $C(10, 1)$, and $D(2, 1)$. She moved the cabinet twice, first translating it 2 units right and 1 unit down, then rotating it 90° clockwise about the origin. What are the coordinates of the vertices of the final image after these transformations are applied?

$A''(7, -4)$, $B''(7, -12)$, $C''(0, -12)$, $D''(0, -4)$

*8. Before rearranging his game room, Carlos made a diagram of the current arrangement. He drew rectangle $ABCD$ to represent a tennis table with vertices $A(1, 1)$, $B(1, 12)$, $C(8, 12)$, and $D(8, 1)$. He moved the table twice, first translating it 3 units left and 2 units up, then rotating it 270° clockwise about the origin. What are the coordinates of the vertices of the final image after these transformations are applied?

$A''(-3, -2)$, $B''(-14, -2)$, $C''(-14, 5)$, $D''(-3, 5)$

Higher-Order Thinking Problems

9. **Justify Conclusions** A classmate concludes that the image of a figure rotated 270° clockwise will have the same coordinates as the image of the same figure rotated 90° counterclockwise. Is your classmate correct? Write an argument that can be used to defend your solution.

Yes; Sample answer: A figure can rotate a total of 360°. A clockwise rotation of 270° leaves 90° remaining in the rotation before returning to its original position. The original figure could be rotated 90° in the opposite direction, counterclockwise, and be in the same position as the figure rotated 270°.

10. **Model with Mathematics** A figure is rotated 270° counterclockwise about the origin. Then the image is rotated 90° counterclockwise about the origin. Complete the coordinate notation that represents the series of rotations. What can you conclude about the position of the figure after the series of rotations?

$(x, y) \rightarrow (y, -x) \rightarrow (x, y)$

Sample answer: The total rotation is 360°, so the figure is in its original position.

11. **Reason Inductively** Determine whether the following statement is *always*, *sometimes*, or *never* true. Write an argument that can be used to defend your solution.

A figure and its rotated image will have the same area, but different perimeters.

never; Sample answer: A figure and its rotated image are congruent, so they will always have the same area and perimeter.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students will determine whether the classmate is correct. Encourage students to explain how they know they knew the classmate was correct using details from the problem.

4 Model with Mathematics In Exercise 10, students will state what they can conclude about the position of the figure after the series of rotations. Encourage students to model the problem, using a real-world situation to help them work through the series of rotations.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 11, students will determine the validity of the statement. Encourage students to identify the pieces of information that make the statement never true.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.
Use with Exercises 7–8 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 7 might be, "What is the coordinate notation for a translation?"

Listen and ask clarifying questions.
Use with Exercises 9–10 Have students work in pairs. Have students individually read Exercise 9 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 10.

Lesson 13-4

Dilations

8.G.A.3

LESSON GOAL

Students will dilate figures and describe dilations on the coordinate plane.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Dilations and Scale Factor

Explore: Dilate Figures on the Coordinate Plane

Learn: Dilations on a Coordinate Plane
Example 1: Graph Dilations
Example 2: Graph Dilations
Example 3: Describe Dilations
Apply: Consumer Science

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Extension Resoures		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 82 of the *Language Development Handbook* to help your students build mathematical language related to dilations.

ELL You can use the tips and suggestions on page T82 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by dilating figures and describing dilations on the coordinate plane.
Standards for Mathematical Content: **8. G.A.3**
Standards for Mathematical Practice: **MP 1, MP3, MP4, MP5, MP6, MP7, MP8**

Coherence

Vertical Alignment

Previous
Students rotated figures and described rotations on the coordinate plane.
8.G.A.1, 8.G.A.1.A, 8.G.A.3

Now
Students dilate figures and describe dilations on the coordinate plane.
8.G.A.3

Next
Students will use a sequence of transformations to describe congruency between figures.
8.G.A.1, 8.G.A.1.A, 8.G.A.1.B, 8.G.A.1.C, 8.G.A.2

Rigor

The Three Pillars of Rigor

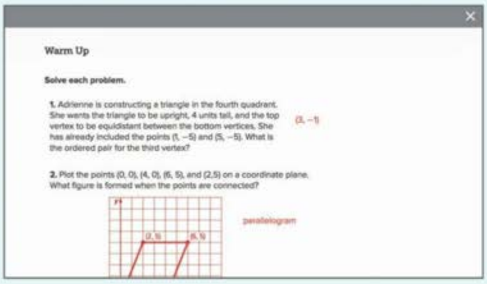
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop *understanding* of dilations on the coordinate plane. Students use their understanding to build *fluency* with graphing dilations. Students come to understand how to describe a dilation using coordinate notation.

Mathematical Background

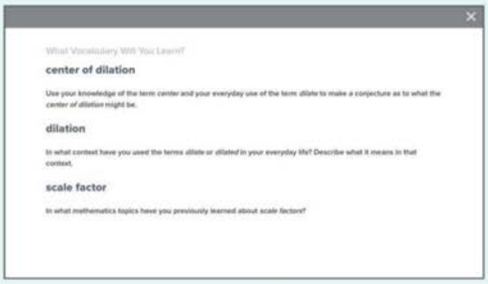
Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- graphing on the coordinate plane (Exercises 1–3)

1–3. See W arm Up slide online for correct answers.

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the use of microscopes to enlarge objects that are too small for the human eye to see.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- Use your knowledge of the term *center* and your everyday use of the term *dilate* to make a conjecture as to what the *center of dilation* might be. Sample answer: The center of dilation might be a fixed point about which a figure is enlarged.
- In what context have you used the terms *dilate* or *dilated* in your everyday life? Describe what it means in that context. Sample answer: My eye doctor dilates the pupils of my eyes so she can better examine the inside of my eye. In this context, dilate means to enlarge or widen.
- In what mathematics topics have you previously learned about *scale factors*? Sample answer: Scale factors are used in scale drawings, such as blueprints or maps.

Learn Dilations and Scale Factor

Objective

Students will understand that a dilation is a transformation that can enlarge or reduce a figure proportionally.

Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* question on Slide 3, encourage students to use clear and precise mathematical language to explain why the ratios are equivalent.

Teaching Notes

SLIDE 1

Remind students that "proportionally" means the ratios of the corresponding sides of the preimage and the image are the same for all of the sides of the figure.

Point out that a scale factor greater than one results in an *enlargement*. You may wish to ask students for real-world examples of enlargements. Examples may include an enlargement of a photo or how a building is an enlargement of the blueprints/models.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 3

Mathematical Discourse

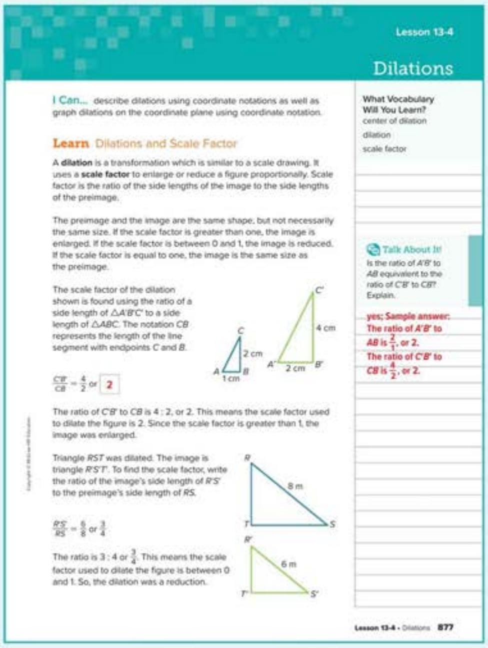
Is the ratio of $A'B'$ to AB equivalent to the ratio of $C'B'$ to CB? Explain. yes; Sample answer: The ratio of $A'B'$ to AB is $\frac{2}{1}$, or 2. The ratio of $C'B'$ to CB is $\frac{4}{2}$, or 2.

DIFFERENTIATE

Enrichment Activity BL

If students need more of a challenge, use the following activity.

Have students find the area and perimeter of triangle ABC and triangle $A'B'C'$ in the Learn. Remind them to use the Pythagorean Theorem to find the lengths of the hypotenuses of the triangles. Ask students to use that information to make a conjecture about how a dilation affects the area and the perimeter of the preimage. Some students may notice that the perimeter of the image is equal to the scale factor times the perimeter of the preimage, and the area of the image is equal to the square of the scale factor times the area of the preimage. Ask students to sketch an additional example with a scale factor not equal to 1 to support their conjecture.



Lesson 13-4

Dilations

I Can... describe dilations using coordinate notations as well as graph dilations on the coordinate plane using coordinate notation.

What Vocabulary Will You Learn?
center of dilation
dilation
scale factor

Learn Dilations and Scale Factor

A **dilation** is a transformation which is similar to a scale drawing. It uses a **scale factor** to enlarge or reduce a figure proportionally. Scale factor is the ratio of the side lengths of the image to the side lengths of the preimage.

The preimage and the image are the same shape, but not necessarily the same size. If the scale factor is greater than one, the image is enlarged. If the scale factor is between 0 and 1, the image is reduced. If the scale factor is equal to one, the image is the same size as the preimage.

The scale factor of the dilation shown is found using the ratio of a side length of $\triangle A'B'C'$ to a side length of $\triangle ABC$. The notation CB represents the length of the line segment with endpoints C and B.

$\frac{C'B'}{CB} = \frac{4}{2}$ or 2

The ratio of $C'B'$ to CB is 4 : 2, or 2. This means the scale factor used to dilate the figure is 2. Since the scale factor is greater than 1, the image was enlarged.

Triangle RST was dilated. The image is triangle $R'S'T'$. To find the scale factor, write the ratio of the image's side length of $R'S'$ to the preimage's side length of RS.

$\frac{R'S'}{RS} = \frac{6}{8}$ or $\frac{3}{4}$

The ratio is 3 : 4 or $\frac{3}{4}$. This means the scale factor used to dilate the figure is between 0 and 1. So, the dilation was a reduction.

Talk About It!
Is the ratio of $A'B'$ to AB equivalent to the ratio of $C'B'$ to CB? Explain.

yes; Sample answer: The ratio of $A'B'$ to AB is $\frac{2}{1}$, or 2. The ratio of $C'B'$ to CB is $\frac{4}{2}$, or 2.

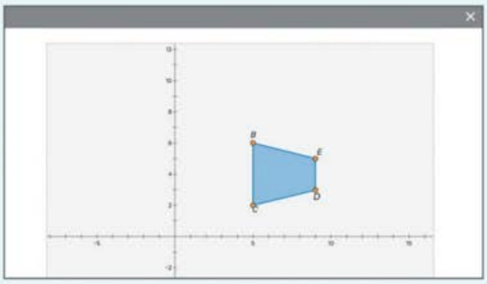
Lesson 13-4 • Dilations 877

Interactive Presentation

Learn, Dilations and Scale Factor, Slide 1 of 3

Interactive Presentation

Explore, Slide 1 of 8



Explore, Slide 2 of 8

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to dilate two-dimensional figures on the coordinate plane when the origin is the center of dilation.

Explore Dilate Figures on the Coordinate Plane

Objective

Students will use Web Sketchpad to explore how to dilate two-dimensional figures on the coordinate plane when the origin is the center of dilation.

Ideas for Use

Recommended Use Pr esent the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to investigate how scale factors impact the coordinates and overall size of figures.

Inquiry Question

How does the scale factor change the size and coordinates of a figure after a dilation relative to the origin? Sample answer: The scale factor decreases or increases the size of the figure. The coordinates of the image are the product of the scale factor and the coordinates of the preimage.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Suppose you dilate the figure by a scale factor of 0.5. Do you think the image after the dilation will be larger or smaller than the preimage? Be prepared to explain your answer. Sample answer: I think the image will be smaller after the dilation because the coordinates will be 0.5 times as great.

(continued on next page)

Explore Dilate Figures on the Coordinate Plane *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore how scale factors less than 1 and scale factors greater than 1 affect the size of the image after the dilation.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

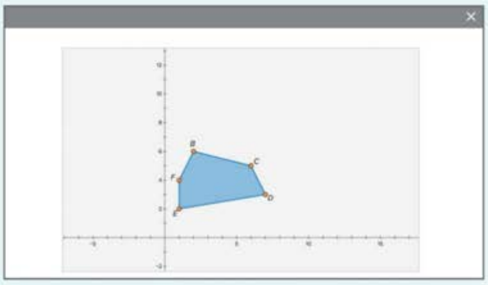
Talk About It!

SLIDE 5

Mathematical Discourse

Suppose you dilate the figure by a scale factor of 2.0. Do you think the image after the dilation will be larger or smaller than the preimage? Be prepared to explain your answer. Sample answer: I think the image will be larger than the preimage because the coordinates will be twice as great.

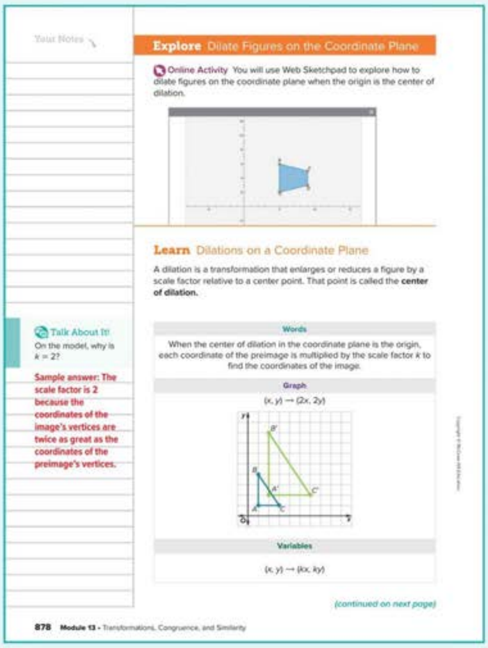
Interactive Presentation



Explore, Slide 5 of 8

TYPE

On Slide 8, students respond to the Inquiry Question and view a sample answer.


Your Notes

Explore Dilate Figures on the Coordinate Plane

Online Activity You will use Web Sketchpad to explore how to dilate figures on the coordinate plane when the origin is the center of dilation.

Learn Dilations on a Coordinate Plane

A dilation is a transformation that enlarges or reduces a figure by a scale factor relative to a center point. That point is called the **center of dilation.**

Words

When the center of dilation in the coordinate plane is the origin, each coordinate of the preimage is multiplied by the scale factor k to find the coordinates of the image.

Graph

$(x, y) \rightarrow (2x, 2y)$

Variables

$(x, y) \rightarrow (kx, ky)$

Talk About It! On the model, why is $k = 2$?

Sample answer: The scale factor is 2 because the coordinates of the image's vertices are twice as great as the coordinates of the preimage's vertices.

(continued on next page)

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Interactive Presentation

Learn, Dilations on a Coordinate Plane, Slide 1 of 4

FLASHCARDS

On Slide 1, students use Flashcards to learn about coordinate notation for a dilation on the coordinate plane when the center of dilation is the origin.

Learn Dilations on a Coordinate Plane

Objective

Students will understand how to dilate two-dimensional figures on the coordinate plane using coordinate notation.

Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* questions on Slide 2 and Slide 4, encourage students to use clear and precise mathematical language in their explanations.

Teaching Notes

SLIDE 1

Point out that dilations on the coordinate plane are relative to a center point, called the *center of dilation*. A common center of dilation is the origin. Have students select the *Words, Variables*, and *Graph* flashcards to view how a dilation, centered at the origin, can be expressed in multiple ways.

You may wish to ask students how the graph shows the connection between the origin, the coordinates of the points, and the scale factor. Some students may say that the distance from each point on the image to the origin is twice the distance from the corresponding point on the preimage to the origin.

Talk About It!

SLIDE 2

Mathematical Discourse

On the model, why is $k = 2$? Sample answer: The scale factor is 2 because the coordinates of the image's vertices are twice as great as the coordinates of the preimage's vertices.

(continued on next page)

Learn Dilations on a Coordinate Plane *(continued)*

Teaching Notes

SLIDE 3

Some students may describe the dilations in the table as "the image is double the size of the preimage", or "the image is half the size of the preimage". Remind students that in two-dimensional figures, size can indicate the area of a figure. The areas of the preimages and the images in the table are related, but not equal to the scale factors.

Talk About It!

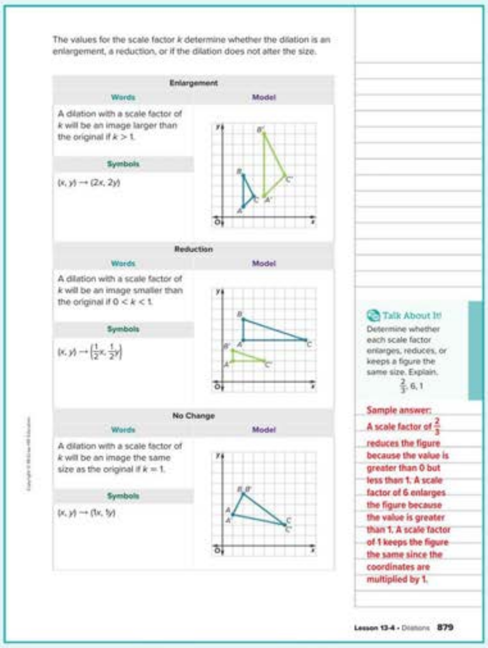
SLIDE 4

Mathematical Discourse

Determine whether each scale factor would enlarge, reduce, or keep the figure the same. Explain.

- $\frac{2}{3}$
- 6
- 1

Sample answer: A scale factor of $\frac{2}{3}$ reduces the figure because the value is greater than 0 but less than 1. A scale factor of 6 enlarges the figure because the value is greater than 1. A scale factor of 1 keeps the figure the same since the coordinates are multiplied by 1.


The values for the scale factor k determine whether the dilation is an enlargement, a reduction, or if the dilation does not alter the size.

Enlargement	
Words	Model
A dilation with a scale factor of k will be an image larger than the original if $k > 1$.	
Symbols	
$(x, y) \rightarrow (2x, 2y)$	

Reduction	
Words	Model
A dilation with a scale factor of k will be an image smaller than the original if $0 < k < 1$.	
Symbols	
$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$	

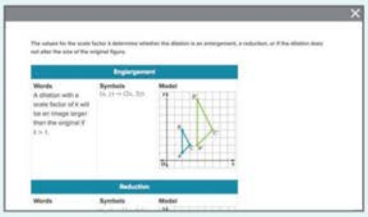
No Change	
Words	Model
A dilation with a scale factor of k will be an image the same size as the original if $k = 1$.	
Symbols	
$(x, y) \rightarrow (1x, 1y)$	

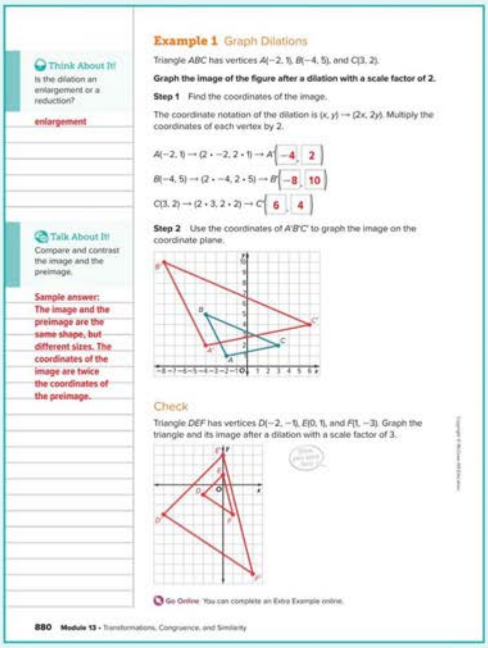
Talk About It! Determine whether each scale factor enlarges, reduces, or keeps a figure the same size. Explain. $\frac{2}{3}$, 6, 1

Sample answer: A scale factor of $\frac{2}{3}$ reduces the figure because the value is greater than 0 but less than 1. A scale factor of 6 enlarges the figure because the value is greater than 1. A scale factor of 1 keeps the figure the same since the coordinates are multiplied by 1.

Lesson 13-4 • Dilations 879

Interactive Presentation


Learn, Dilations on a Coordinate Plane, Slide 3 of 4



Example 1 Graph Dilations

Think About It! Is the dilation an enlargement or a reduction?

enlargement

Triangle *ABC* has vertices *A*(−2, 1), *B*(−4, 5), and *C*(3, 2).

Graph the image of the figure after a dilation with a scale factor of 2.

Step 1 Find the coordinates of the image.

The coordinate notation of the dilation is $(x, y) \rightarrow (2x, 2y)$. Multiply the coordinates of each vertex by 2.

$A(-2, 1) \rightarrow (2 \cdot -2, 2 \cdot 1) \rightarrow A'(-4, 2)$

$B(-4, 5) \rightarrow (2 \cdot -4, 2 \cdot 5) \rightarrow B'(-8, 10)$

$C(3, 2) \rightarrow (2 \cdot 3, 2 \cdot 2) \rightarrow C'(6, 4)$

Step 2 Use the coordinates of *A′B′C′* to graph the image on the coordinate plane.

Talk About It! Compare and contrast the image and the preimage.

Sample answer: The image and the preimage are the same shape, but different sizes. The coordinates of the image are twice the coordinates of the preimage.

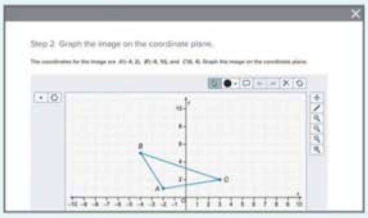
Check

Triangle *DEF* has vertices *D*(−2, −1), *E*(0, 1), and *F*(1, −3). Graph the triangle and its image after a dilation with a scale factor of 3.

Go Online You can complete an Extra Example online.

880 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 1, Graph Dilations, Slide 3 of 5

eTOOLS

On Slide 3, students use the Coordinate Graphing eTool to graph the image on the coordinate plane.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Graph Dilations

Objective

Students will dilate two-dimensional figures with a scale factor greater than 1 on the coordinate plane.

MP Teaching the Mathematical Practices

6 Attend to Precision Encourage students to accurately and efficiently use the scale factor to write the coordinates of the image and graph the dilation on the coordinate plane.

While discussing the *Talk About It!* question on Slide 4, encourage students to use proper mathematical language as they discuss the differences and similarities of the preimage and image.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you multiply the coordinates of each vertex by 2? The scale factor is 2.

OL Classify the dilation as an enlargement or reduction. enlargement

OL Suppose a classmate only multiplied the *x*-coordinate of each vertex by 2. How can you explain to them why it is important that both the *x*- and *y*-coordinates are multiplied by 2? Sample answer: If only the *x*-coordinates are multiplied by 2, the image will not have the same shape as the preimage. A dilation applies to both the *x*- and *y*-coordinates.

BL If the scale factor was $\frac{3}{2}$ instead of 2, what would the coordinates of point *A′* be? $\left(-3, \frac{3}{2}\right)$

SLIDE 3

AL What do you notice about the locations of the vertices of the image compared to the preimage? Why does this make sense? Sample answer: The vertices of the image are farther away from the origin than the preimage. This makes sense because the scale factor is greater than 1.

OL How can you check that your graph is reasonable? Sample answer: Since the scale factor is greater than 1, the image should be larger than the preimage.

BL How do you think the perimeter of the image and preimage compare? Explain. Sample answer: The perimeter of the image is twice that of the preimage. Because each coordinate of each vertex is twice as great, the distance between each vertex is twice as great.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Graph Dilations

Objective

Students will dilate two-dimensional figures with a scale factor between 0 and 1 on the coordinate plane.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you multiply the coordinates of each vertex by $\frac{1}{2}$? The scale factor is $\frac{1}{2}$.

OL What coordinate notation can you use? $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$

OL Instead of multiplying each coordinate by the fraction $\frac{1}{2}$, what can you do to mentally find the coordinates of each vertex of the image? Sample answer: Find half of each coordinate. For example, half of 3 is 1.5.

BL In what quadrant will the image lie? How do you know this? Quadrant I; Sample answer: The preimage lies within Quadrant I. Since the scale factor is between 0 and 1, the image will lie inside Quadrant I also.

SLIDE 3

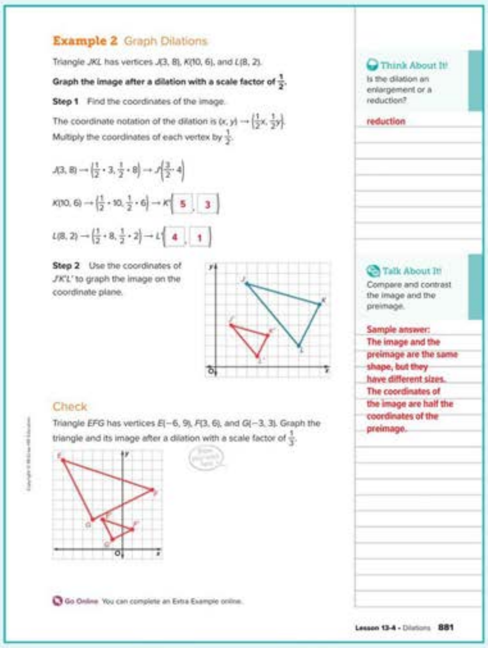
AL What do you notice about the locations of the vertices of the image compared to the preimage? Why does this make sense? Sample answer: The vertices of the image are closer to the origin than that of the preimage. This makes sense because the scale factor is between 0 and 1.

OL How can you check that your graph is reasonable? Sample answer: Since the scale factor is between 0 and 1, the image should be smaller than the preimage.

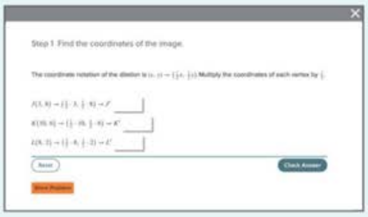
BL How do you think the perimeter of the image and preimage compare? Explain. Sample answer: The perimeter of the image is half that of the preimage. Because each coordinate of each vertex is half as great, the distance between each vertex is half as great.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Interactive Presentation



Example 2, Graph Dilations, Slide 2 of 5

TYPE

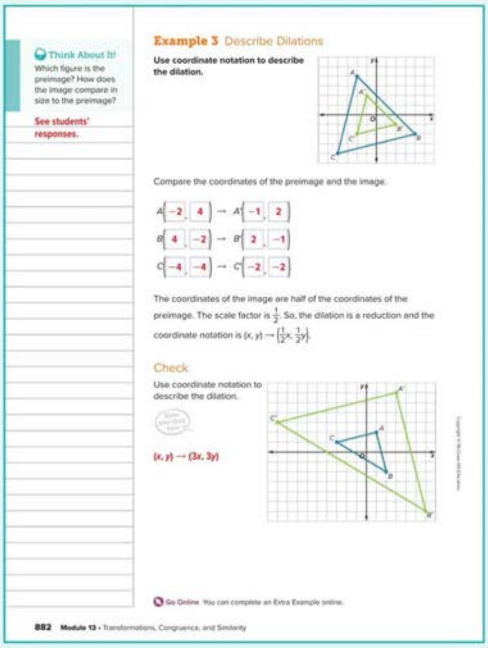
On Slide 2, students determine the coordinates of the image.

eTOOLS

On Slide 3, students use the Coordinate Graphing eTool to graph the image on the coordinate plane.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Think About It! Which figure is the preimage? How does the image compare in size to the preimage?

See students' responses.

Example 3 Describe Dilations

Use coordinate notation to describe the dilation.

Compare the coordinates of the preimage and the image.

$A(-2, 4) \rightarrow A'(-1, 2)$

$B(4, -2) \rightarrow B'(2, -1)$

$C(-4, -4) \rightarrow C'(-2, -2)$

The coordinates of the image are half of the coordinates of the preimage. The scale factor is $\frac{1}{2}$. So, the dilation is a reduction and the coordinate notation is $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.

Check

Use coordinate notation to describe the dilation.

$(x, y) \rightarrow (3x, 3y)$

Go Online You can complete an Extra Example online.

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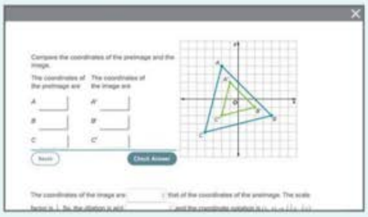
Interactive Presentation



Example 3, Describe Dilations, Slide 2 of 3

TYPE

On Slide 2, students determine the coordinates of the preimage and image.

CLICK

On Slide 2, students select the correct words to complete the sentences.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Describe Dilations

Objective

Students will describe dilations using coordinate notation.

Teaching the Mathematical Practices

6 Attend to Precision S tudents should be able to precisely classify the dilation as a reduction.

7 Look For and Make Use of Structure Encourage students to use the structure of the coordinate plane, the coordinates of the preimage and image, and coordinate notation to describe the dilation.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know this is a dilation rather than a translation, rotation, or reflection? Sample answer: The figure and its image are different sizes.

AL What do you notice about the preimage and the image? Sample answer: The image is smaller than the preimage.

OL What is true about the coordinates of the image compared to the preimage? Sample answer: The coordinates of the image are half of the coordinates of the preimage.

OL Classify the dilation as an enlargement or a reduction. reduction

BL In this example, the image is located in the interior of the preimage. Will this always be true of dilations that are reductions? Explain or provide a counterexample. no; Sample answer: If the preimage is located within one quadrant, the image's coordinates will be closer to the origin than the preimage's coordinates, which does not mean that the image will be located in the interior of the preimage. If the preimage is located within all four quadrants, then the image will be located within the interior of the preimage. This is because the image's coordinates will be closer to the origin.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply Consumer Science

Objective

Students will come up with their own strategy to solve an application problem that involves finding the cost of fencing.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

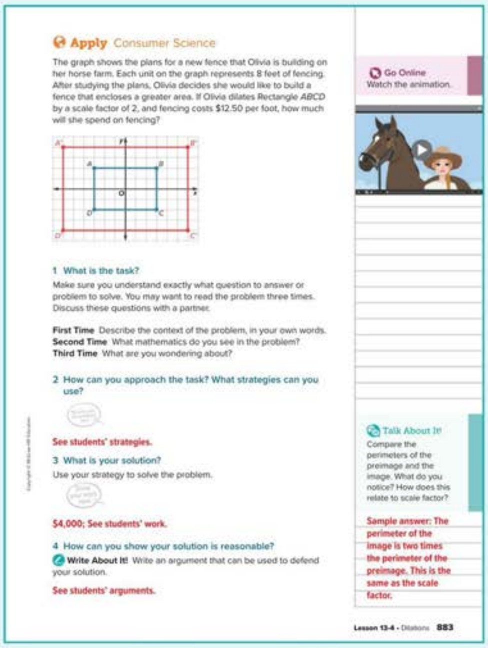
Encourage Productive Struggle
As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle
If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How does a dilation with a scale factor of 2 change the rectangle?
- How can you find the perimeter of the dilated rectangle?
- How can you use the scale of the graph to find the length of the fence?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply Consumer Science

The graph shows the plans for a new fence that Olivia is building on her horse farm. Each unit on the graph represents 8 feet of fencing. After studying the plans, Olivia decides she would like to build a fence that encloses a greater area. If Olivia dilates Rectangle *ABCD* by a scale factor of 2, and fencing costs $12.50 per foot, how much will she spend on fencing?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

$4,000; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

Talk About It! Compare the perimeters of the preimage and the image. What do you notice? How does this relate to scale factor?

Sample answer: The perimeter of the image is two times the perimeter of the preimage. This is the same as the scale factor.

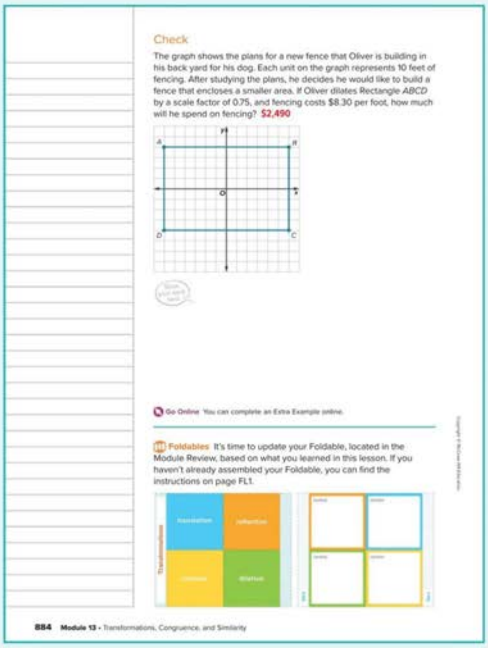
Lesson 13-4 • Dilations 883

Interactive Presentation

Apply, Consumer Science

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

The graph shows the plans for a new fence that Oliver is building in his back yard for his dog. Each unit on the graph represents 10 feet of fencing. After studying the plans, he decides he would like to build a fence that encloses a smaller area. If Oliver dilates Rectangle *ABCD* by a scale factor of 0.75, and fencing costs $8.30 per foot, how much will he spend on fencing? $2,490

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.

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Interactive Presentation

Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record an example of a dilation. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

What does it mean to perform a transformation on a figure?

In this lesson, students learned how to dilate figures on the coordinate plane with a given scale factor. Encourage them to discuss with a partner if a dilation is an isometry. Some students may state that dilations are only isometries if the scale factor is one.

Exit Ticket

Refer to the Exit Ticket slide. Classify the magnification of a bacteria cell under a microscope as an enlargement or reduction. enlargement

Suppose Triangle *ABC* has vertices $A(-2, 1)$, $B(-4, 5)$, and $C(3, 2)$. It is dilated with a scale factor of $\frac{1}{4}$. What are the vertices of the image after the dilation? Write a mathematical argument that can be used to defend your solution. $A'\left(-\frac{1}{2}, \frac{1}{4}\right)$, $B'\left(-1, 1\frac{1}{4}\right)$, $C'\left(\frac{3}{4}, \frac{1}{2}\right)$; Sample answer: Multiply the coordinates of each vertex of $\triangle ABC$ by $\frac{1}{4}$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 6, 7, 9–11
- Extension: Dilations About Other Points
- ALEKS Dilations

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 7, 9, 11
- Extension: Dilations About Other Points
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–3
- ALEKS Dilations

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Prerequisite Skill topics

1 CONCEPTUAL UNDERSTANDING **2 FLUENCY** **3 APPLICATION**

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition.*

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

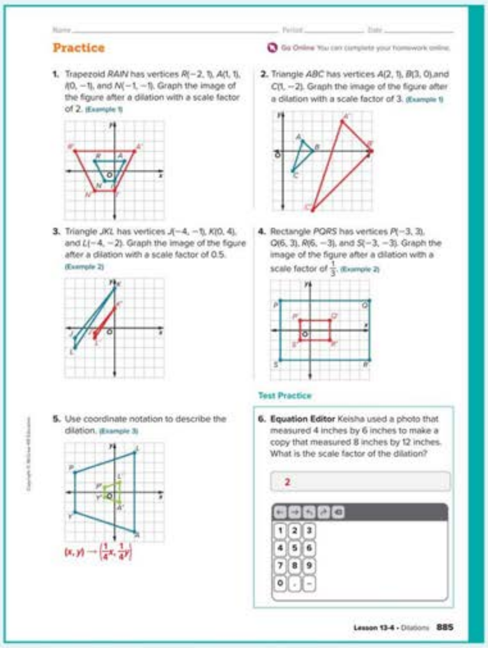
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	dilate two-dimensional figures with a scale factor greater than 1 on the coordinate plane	1, 2
1	dilate two-dimensional figures with a scale factor between 0 and 1 on the coordinate plane	3, 4
1	describe dilations using coordinate notation	5
2	extend concepts learned in class to apply them in new contexts	6
3	solve application problems involving dilations	7, 8
3	higher-order and critical thinking skills	9–11

Common Misconception

When the dilation is a *reduction*, students may incorrectly think that *dividing* the coordinates of the preimage by the scale factor will result in the coordinates of the image. Remind students that when dilating a figure on the coordinate plane, they should always multiply the coordinates of the preimage by the scale factor to obtain the coordinates of the image (when the center of dilation is the origin).


Name _______ Period _______ Date _______

Practice

Go Online You can complete your homework online.

1. Trapezoid $RAIN$ has vertices $R(-2, 1)$, $A(1, 1)$, $I(0, -1)$, and $N(-1, -1)$. Graph the image of the figure after a dilation with a scale factor of 2. (Example 1)

2. Triangle ABC has vertices $A(2, 1)$, $B(3, 0)$, and $C(1, -2)$. Graph the image of the figure after a dilation with a scale factor of 3. (Example 1)

3. Triangle JKL has vertices $J(-4, -1)$, $K(0, 4)$, and $L(-4, -2)$. Graph the image of the figure after a dilation with a scale factor of 0.5. (Example 2)

4. Rectangle $PQRS$ has vertices $P(-3, 3)$, $Q(6, 3)$, $R(6, -3)$, and $S(-3, -3)$. Graph the image of the figure after a dilation with a scale factor of $\frac{1}{3}$. (Example 2)

5. Use coordinate notation to describe the dilation. (Example 3)

$(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$

Test Practice

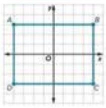
6. **Equation Editor** Keisha used a photo that measured 4 inches by 6 inches to make a copy that measured 8 inches by 12 inches. What is the scale factor of the dilation?

2

Lesson 13-4 • Dilations 885

Apply *indicates multi-step problem

For Exercises 7 and 8, use the graph of Rectangle *ABCD*.



*7. Suppose the graph represents the plans for a fence that Tara is building for a new city dog park. Each unit on the graph represents 12 yards. After studying the plans, Tara decides to build a fence that encloses a smaller area. If Tara dilates Rectangle *ABCD* by a scale factor of 0.75, and fencing costs $6.39 per yard, how much will she spend on fencing?

$1,610.28

*8. Suppose the graph of Rectangle *ABCD* shows the plans for a safety fence that Kenny is setting up around a construction area. Each unit on the graph represents 25 feet. After studying the plans, Kenny decides to build a fence that encloses a larger area. If Kenny dilates Rectangle *ABCD* by a scale factor of 2.5, and fencing costs $5.25 per foot, how much will he spend on fencing?

$9,187.50

Higher-Order Thinking Problems

9. **Persevere with Problems** The coordinates of two triangles are shown in the table. Is *XYZ* a dilation of *JKL*? Write an argument that can be used to defend your solution.

△JKL		△XYZ	
J	(*a*, *b*)	*X*	(3*a*, 6*b*)
K	(*c*, *d*)	*Y*	(3*c*, 6*d*)
L	(*a*, *d*)	*Z*	(3*a*, 6*d*)

no; Sample answer: Both coordinates of all points must be multiplied by the same scale factor.

10. **Find the Error** Kelly is finding the coordinates of the image of a polygon with vertices *W*(2, 2), *X*(2, 4), *Y*(4, 4), and *Z*(4, 2) after a dilation with a scale factor of 3. Describe her error and explain how to correct it.

The coordinates of *W'X'Y'Z'* are *W'*(2, 6), *X'*(2, 12), *Y'*(4, 12), and *Z'*(4, 6).

Sample answer: Kelly multiplied the *y*-coordinates by 3, but forgot to multiply the *x*-coordinates by 3 as well. The coordinates should be *W'*(6, 6), *X'*(6, 12), *Y'*(12, 12), and *Z'*(12, 6).

11. Determine whether the following statement is *always*, *sometimes*, or *never* true. Write an argument that can be used to defend your solution.

A preimage and its dilated image are the same shape but different sizes.

sometimes; Sample answer: A dilation results in an image that is similar to the preimage. It will always be the same shape. A dilation by a scale factor other than 1 results in an image with a different size. The preimage and image will be the same size and shape if the scale factor is 1.

886 Module 13 • Transformations, Congruence, and Similarity

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them
In Exercise 9, students determine if a dilation took place. Encourage students to use what they know about dilations to explain why a dilation did not take place.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students will find the error and correct it. Encourage students to find the error and then construct a response that precisely describes it and how to fix it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Clearly explain your strategy.
Use with Exercise 7 Have students work in pairs. Give students 1–2 minutes to individually consider the problem and formulate their strategy. Then ask them to clearly explain their strategy to their partner how they would solve the problem, without actually solving it. Have each student use their partner's strategy to solve the problem. Have them compare and contrast strategies to determine if one or both strategies were viable, and discuss and resolve any differences.

Be sure everyone understands.
Use with Exercises 9–10 Have students work in groups of 3–4 to solve the problem in Exercise 9. Assign each student in the group a number. The entire group is responsible to ensure that every group member understands how to solve the problem. Group members should ask each other clarifying questions and check each other's understanding. Call on a randomly numbered student from one group to share their group's solution to the class. Repeat the process for Exercise 10.

Lesson 13-5

Congruence and Transformations

8.G.A.1, 8.G.A.2

LESSON GOAL

Students will use a sequence of transformations to describe congruency between figures.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Congruence and Transformations

Learn: Congruence and Transformations
Example 1: Determine Congruence
Example 2: Determine Congruence
Learn: Identify Transformations
Example 3: Identify Transformations
Example 4: Identify Transformations

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Congruent Triangles by SSS and SAS		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 83 of the *Language Development Handbook* to help your students build mathematical language related to congruence and transformations.

ELL You can use the tips and suggestions on page T83 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by using a sequence of transformations to describe congruency between figures.
Standards for Mathematical Content: **8. G.A.1, 8.G.A.1.A, 8.G.A.1.B, 8.G.A.1.C, 8.G.A.2**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students described the effect of translations, rotations, and reflections on two-dimensional figures.
8.G.A.3

Now
Students use a sequence of transformations to describe congruency between figures.
8.G.A.1, 8.G.A.1.A, 8.G.A.1.B, 8.G.A.1.C, 8.G.A.2

Next
Students will use a sequence of transformations to describe similarity between figures.
8.G.A.4

Rigor

The Three Pillars of Rigor

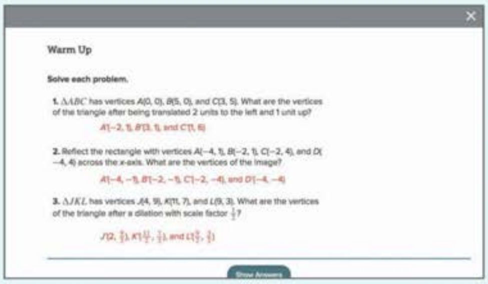
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of transformations to develop *understanding* that two figures are congruent if the second can be obtained from the first by a series of translations, reflections, and rotations. They build *fluency* with identifying what series of transformations determines congruence.

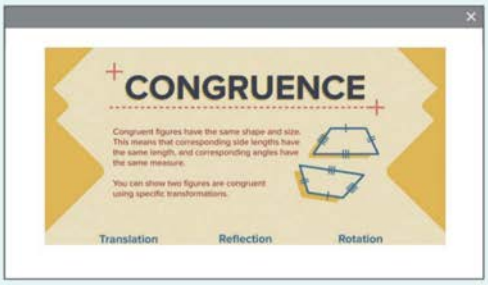
Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

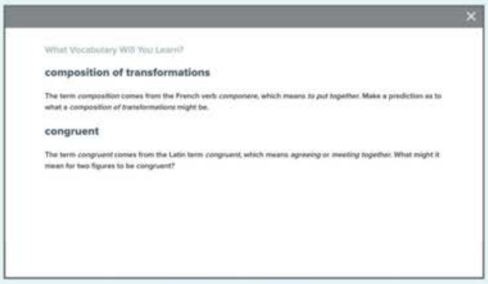
Interactive Presentation



Warm Up



Launch the Lesson



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding translations (Exercise 1)
- understanding reflections (Exercise 2)
- understanding dilations (Exercise 3)

Answers

1. $A'(-2, 1)$, $B'(3, 1)$, and $C'(1, 6)$

2. $A'(-4, -1)$, $B'(-2, -1)$, $C'(-2, -4)$, and $D'(-4, -4)$

3. $J'\left(2, \frac{9}{2}\right)$, $K'\left(\frac{11}{2}, \frac{7}{2}\right)$, and $L'\left(\frac{9}{2}, \frac{3}{2}\right)$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about congruence, using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Learn?

Use the following questions to engage students and facilitate a class discussion.

Ask:

- The term *composition* comes from the French verb *componere,* which means *to put together*. Make a prediction as to what a *composition of transformations* might be. Sample answer: A composition of transformations might be a combination of two or more translations, rotations, reflections or dilations.
- The term *congruent* comes from the Latin term *congruent*, which means *agreeing* or *meeting together*. What might it mean for two figures to be congruent? Sample answer: The figures are the same shape and size, which might mean they have the same size and shape.

Explore Congruence and T ransformations

Objective

Students will use Web Sketchpad to explore the properties of translations, reflections, and rotations.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? Y ou may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to explore the effects of transformations on figures. Students will explore whether the angle measures and side lengths change and then infer if the same conjectures could be made about all translations, reflections, or rotations.

Inquiry Question

What happens to a figure when you translate, reflect, or rotate it? Sample answer: When I translate a figure, the position of the figure changes, but the side lengths, angle measures, and parallel lines all remain the same. When I reflect a figure across an axis, it creates a mirror image of it. The side lengths, angle measures, and parallel lines all remain the same. When I rotate a figure about the origin, the figure does not change. The position of the figure is different, but the side lengths, angle measures, and parallel lines all remain the same.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 5 is shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Do the opposite sides of a square remain parallel after a translation? Explain. yes; Sample answer: Translating the square does not change its shape or size, and every point of the preimage is moved the same distance and in the same direction; therefore, the opposite sides remain parallel.

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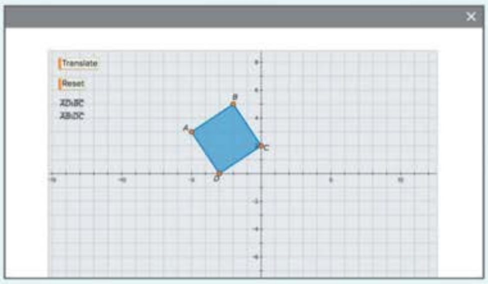
Interactive Presentation

Explore, Slide 1 of 14



Explore, Slide 5 of 14

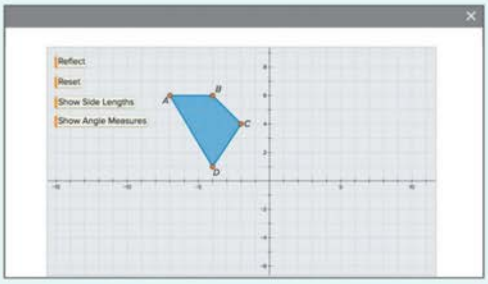
WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore the properties of translations, reflections, and rotations.

TYPE

On Slide 4, students make a conclusion about the effect of a translation on a figure's size and shape.

Interactive Presentation



Explore, Slide 7 of 14

TYPE

On Slide 8 and 12, students make conclusions about the effect of a reflection or rotation on a figure's size and shape.

TYPE

On Slide 14, students respond to the Inquiry Question and view a sample answer.

Explore Congruence and T ransformations *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to create a compare/contrast chart that will help them examine the effects translations, rotations, and reflections have on figures.

3 Construct Viable Arguments and Critique the Reasoning of Others Students should be able to communicate their conclusions about the changes to the figures after transformations.

5 Use Appropriate Tools Strategically Students will use Web Sketchpad to explore the effects of transformations on figures.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 7 is shown.

Talk About It!

SLIDE 7

Mathematical Discourse

Do you think a reflection will always produce a figure that has the same side lengths and angle measures? Explain. yes; Sample answer: A reflection produces a mirror image of the original figure, and does not alter its size or shape.

Learn Congruence and T ransformations

Objective

Students will understand the properties of translations, reflections, and rotations and how these transformations are used to show that a pair of two-dimensional figures is congruent.

Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* questions on Slide 4, encourage students to use clear and precise language in their explanations.

7 Look For and Make Use of Structure While discussing the *Talk About It!* questions on Slide 4, encourage students to think about the structure of figures before and after these transformations to explain why some transformations preserve congruence and others do not.

Go Online

- Have students watch the videos on Slide 1. The videos illustrate properties of transformations.
- Have students watch the animation on Slide 3. The animation illustrates congruence and transformations.

Teaching Notes

SLIDE 1

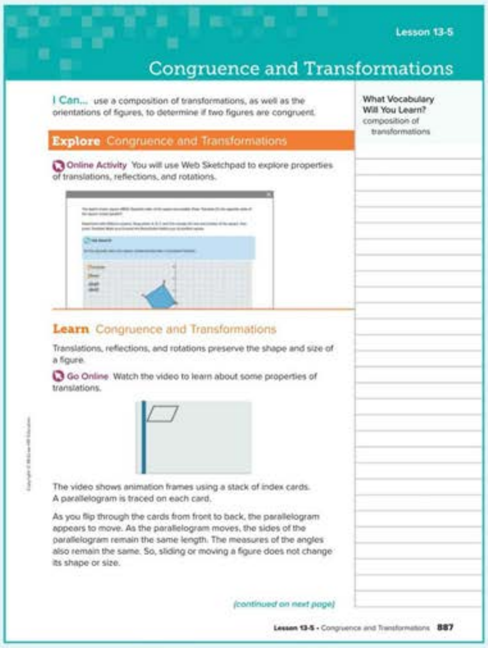
Play the *Model Translations* video for the class which demonstrates that translations preserve the shape and size of a figure. You may wish to ask students how they know that the shape and size haven't changed. Some students may say that the exact figure was traced on different notecards and the size did not change. Some students may say that a dilation is the only transformation that changes the size of a figure by a scale factor.

(continued on next page)

DIFFERENTIATE

Enrichment Activity BL

To further students' understanding of congruence and transformations, have them work with a partner to prepare a brief presentation that illustrates the properties shown in the videos. Encourage them to use the observations they discovered in the Explore activity. Have each pair of students prepare their presentation to another pair, or to the whole class. Some students may be uncomfortable speaking in front of others. Encourage them to make appropriate eye contact, and articulate their thoughts clearly and loudly enough for others to hear.



Lesson 13-5

Congruence and Transformations

I Can... use a composition of transformations, as well as the orientations of figures, to determine if two figures are congruent.

What Vocabulary Will You Learn?
composition of transformations

Explore Congruence and Transformations

Online Activity You will use Web Sketchpad to explore properties of translations, reflections, and rotations.

Learn Congruence and Transformations

Translations, reflections, and rotations preserve the shape and size of a figure.

Go Online Watch the video to learn about some properties of translations.

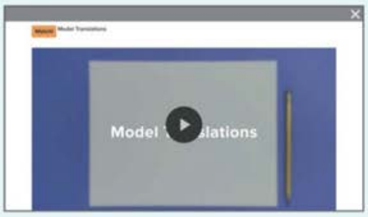
The video shows animation frames using a stack of index cards. A parallelogram is traced on each card.

As you flip through the cards from front to back, the parallelogram appears to move. As the parallelogram moves, the sides of the parallelogram remain the same length. The measures of the angles also remain the same. So, sliding or moving a figure does not change its shape or size.

(continued on next page)

Lesson 13-5 • Congruence and Transformations **887**

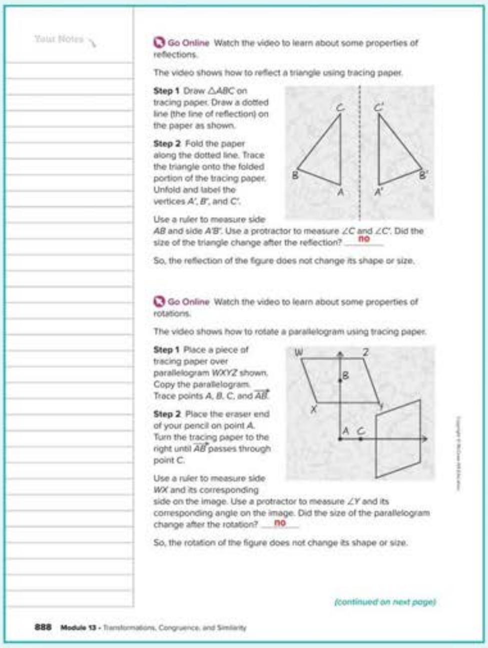
Interactive Presentation



Learn, Congruence and Transformations, Slide 1 of 4

WATCH

On Slide 1, students watch the video that models properties of translations.



Your Notes

Go Online Watch the video to learn about some properties of reflections.

The video shows how to reflect a triangle using tracing paper.

Step 1 Draw $\triangle ABC$ on tracing paper. Draw a dotted line (the line of reflection) on the paper as shown.

Step 2 Fold the paper along the dotted line. Trace the triangle onto the folded portion of the tracing paper. Unfold and label the vertices A', B', and C'.

Use a ruler to measure side AB and side $A'B'$. Use a protractor to measure $\angle C$ and $\angle C'$. Did the size of the triangle change after the reflection? no

So, the reflection of the figure does not change its shape or size.

Go Online Watch the video to learn about some properties of rotations.

The video shows how to rotate a parallelogram using tracing paper.

Step 1 Place a piece of tracing paper over parallelogram $WXYZ$ shown. Copy the parallelogram. Trace points A, B, C, and $\overrightarrow{AB}$.

Step 2 Place the eraser end of your pencil on point A. Turn the tracing paper to the right until $\overrightarrow{AB}$ passes through point C.

Use a ruler to measure side WX and its corresponding side on the image. Use a protractor to measure $\angle Y$ and its corresponding angle on the image. Did the size of the parallelogram change after the rotation? no

So, the rotation of the figure does not change its shape or size.

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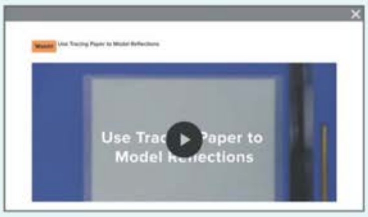
888 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Learn, Congruence and Transformations, Slide 1 of 4

WATCH

On Slide 1, students watch the videos that model properties of reflections and rotations.

Learn Congruence and T ransformations *(continued)*

Teaching Notes

SLIDE 1

Play the *Use Tracing Paper to Model Reflections* and the *Use Tracing Paper to Model Rotations* videos for the class which demonstrate that reflections and rotations both preserve the shape and size of a figure. You may wish to have a class discussion around the following questions:

- Do these transformations map line segments to corresponding segments of the same length? How do you know?
- Do these transformations map angles to corresponding angles of the same measure? How do you know?
- Do these transformations map parallel sides to corresponding parallel sides? How do you know?

(continued on next page)

Learn Congruence and T ransformations *(continued)*

Teaching Notes

SLIDE 2

Explain to students that composition of transformations is a sequence of transformations. Prime notation is used to indicate a single transformation, and that double prime notation is used to indicate a second in a sequence of transformations.

You may wish to ask students how they know that triangle *A″B″C″* isn't the result of a single transformation. Some students may say that since double prime notation was used, there must be another transformation that maps triangle *ABC* to triangle *A′B′C′*.

Point out that the size and shape of triangle *ABC* has been preserved in the composition of transformations since each individual transformation preserves size and shape.

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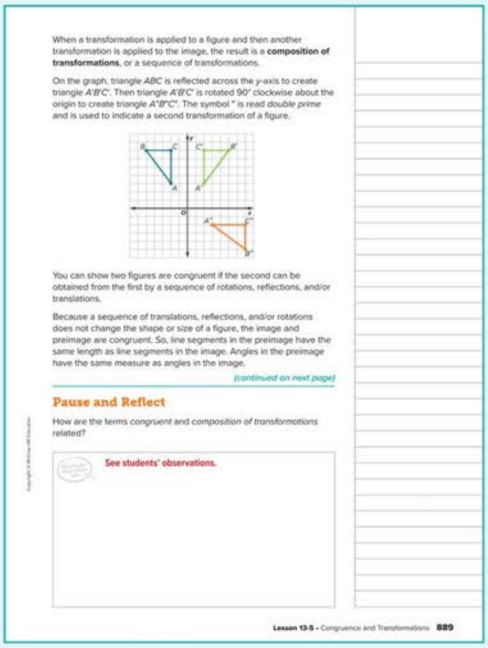

When a transformation is applied to a figure and then another transformation is applied to the image, the result is a **composition of transformations**, or a sequence of transformations.

On the graph, triangle *ABC* is reflected across the *y*-axis to create triangle *A′B′C′*. Then triangle *A′B′C′* is rotated 90° clockwise about the origin to create triangle *A″B″C″*. The symbol ″ is read *double prime* and is used to indicate a second transformation of a figure.

You can show two figures are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and/or translations.

Because a sequence of translations, reflections, and/or rotations does not change the shape or size of a figure, the image and preimage are congruent. So, line segments in the preimage have the same length as line segments in the image. Angles in the preimage have the same measure as angles in the image.

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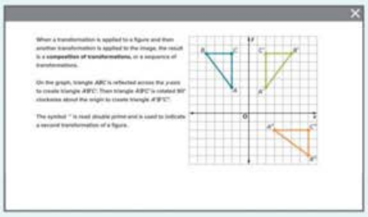
Pause and Reflect

How are the terms *congruent* and *composition of transformations* related?

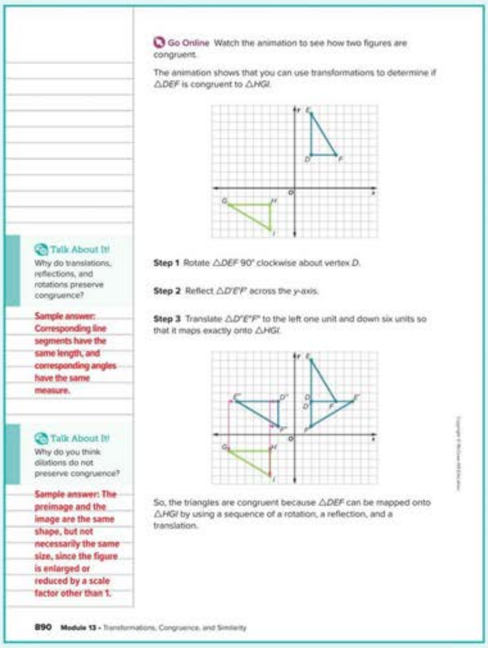
See students' observations.

Lesson 13-5 • Congruence and Transformations 889

Interactive Presentation



Learn, Congruence and Transformations, Slide 2 of 4



Go Online Watch the animation to see how two figures are congruent.

The animation shows that you can use transformations to determine if $\triangle DEF$ is congruent to $\triangle HGI$.

Talk About It!
Why do translations, reflections, and rotations preserve congruence?

Sample answer: Corresponding line segments have the same length, and corresponding angles have the same measure.

Step 1 Rotate $\triangle DEF$ 90° clockwise about vertex D.

Step 2 Reflect $\triangle D'E'F'$ across the y-axis.

Step 3 Translate $\triangle D''E''F''$ to the left one unit and down six units so that it maps exactly onto $\triangle HGI$.

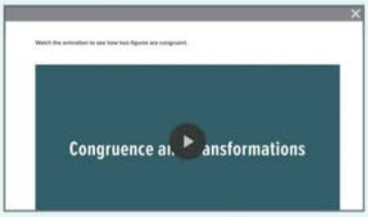
Talk About It!
Why do you think dilations do not preserve congruence?

Sample answer: The preimage and the image are the same shape, but not necessarily the same size, since the figure is enlarged or reduced by a scale factor other than 1.

So, the triangles are congruent because $\triangle DEF$ can be mapped onto $\triangle HGI$ by using a sequence of a rotation, a reflection, and a translation.

890 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Learn, Congruence and Transformations, Slide 3 of 4

WATCH

On Slide 3, students watch the animation to see how two figures are congruent.

Learn Congruence and Transformations *(continued)*

Teaching Notes

SLIDE 3

Play the *Congruence and Transformations* animation for the class. After viewing, you may wish to remind students of the order of the transformations used in the animation:

- a 90° clockwise rotation about point D,
- a reflection across the y-axis,
- a translation of one unit left and six units down.

Have students discuss with a partner if they can determine a different composition of transformations that will map triangle DEF onto triangle HGI. Some possible combinations are:

- a reflection across the y-axis, 90° counterclockwise rotation about point D, translation of one unit left and six units down
- a translation of one unit right and six units down, a 90° clockwise rotation about point D, a reflection across the y-axis.

Encourage students to use coordinate notation or to draw a diagram to explain their reasoning.

Talk About It!

SLIDE 4

Mathematical Discourse

Why do translations, reflections, and rotations preserve congruence? Sample answer: Corresponding line segments have the same length, and corresponding angles have the same measure.

Why do you think dilations do not preserve congruence? Sample answer: The preimage and the image are the same shape, but not necessarily the same size, since the figure is enlarged or reduced by a scale factor other than 1.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 1 Determine Congruence

Objective

Students will determine that a pair of two-dimensional figures is congruent by applying a sequence of rotations, reflections, and translations.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively W hile discussing the *Talk About It!* questions, encourage students to use reasoning to understand that these transformations can be performed in the reverse order.

5 Use Appropriate Tools Strategically Students will use the Transformations eTool to help determine the series of transformations that maps one triangle onto another.

6 Attend to Precision While discussing the *Talk About It!* questions, students should use clear and precise mathematical language to explain why the transformations that are used demonstrate that the triangles are congruent.

Questions for Mathematical Discourse

SLIDE 2

AL How do you know that a series of translations cannot be the transformations that map △*ABC* onto △*XYZ*? Sample answer: △*ABC* appears to be either reflected or rotated, not just moved to the right and up.

OL What does it mean for △*ABC* to be reflected over the line $x = 0$? The line $x = 0$ is the *y*-axis, so it means to reflect △*ABC* across the *y*-axis.

BL If you start with △*XYZ*, how can △*XYZ* be mapped onto △*ABC*? Sample answer: Reflect △*XYZ* across the line $x = 0$. Then translate △*XYZ* down 4 units.

SLIDE 3

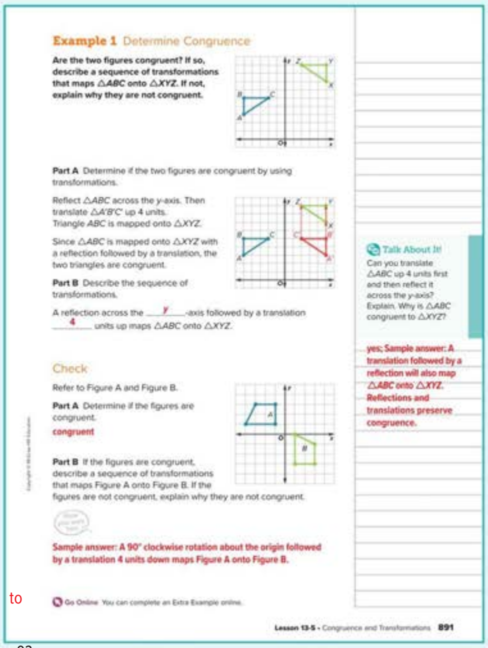
AL What is the length $\overline{AB}$? the length of $\overline{XY}$? What do you notice? The lengths are the same, 2 units.

OL Which vertex in △*XYZ* corresponds to vertex *A* in △*ABC*? Explain. vertex *X*; Reflecting the point (−5, 3) across the *y*-axis and translating it up 4 units becomes the point (5, 7), which corresponds to vertex *X*.

BL Write the coordinate notation that can be used to map △*ABC* onto △*XYZ*. Sample answer: $(x, y) \rightarrow (-x, y + 4)$

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.


Example 1 Determine Congruence

Are the two figures congruent? If so, describe a sequence of transformations that maps △ABC onto △XYZ. If not, explain why they are not congruent.

Part A Determine if the two figures are congruent by using transformations.

Reflect △*ABC* across the *y*-axis. Then translate △*A′B′C′* up 4 units. Triangle *ABC* is mapped onto △*XYZ*.

Since △*ABC* is mapped onto △*XYZ* with a reflection followed by a translation, the two triangles are congruent.

Part B Describe the sequence of transformations.

A reflection across the __y__-axis followed by a translation __4__ units up maps △*ABC* onto △*XYZ*.

Talk About It! Can you translate △*ABC* up 4 units first and then reflect it across the *y*-axis? Explain. Why is △*ABC* congruent to △*XYZ*?

yes; Sample answer: A translation followed by a reflection will also map △*ABC* onto △*XYZ*. Reflections and translations preserve congruence.

Check

Refer to Figure A and Figure B.

Part A Determine if the figures are congruent.

congruent

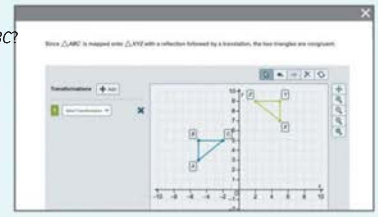
Part B If the figures are congruent, describe a sequence of transformations that maps Figure A onto Figure B. If the figures are not congruent, explain why they are not congruent.

Sample answer: A 90° clockwise rotation about the origin followed by a translation 4 units down maps Figure A onto Figure B.

Go Online You can complete an Extra Example online.

Lesson 13-5 • Congruence and Transformations 891

Interactive Presentation



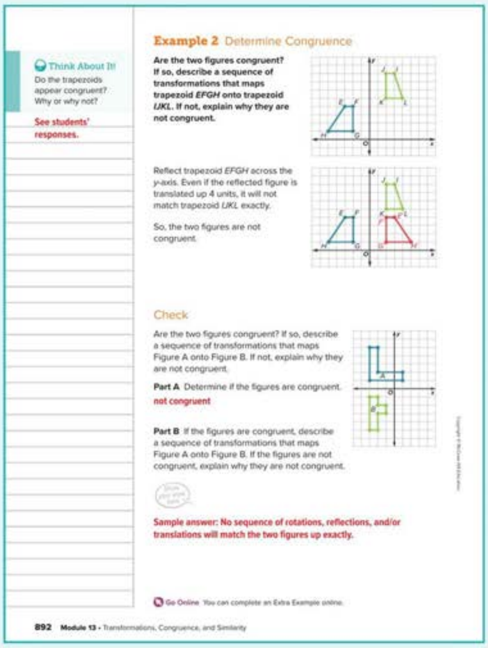
Example 1, Determine Congruence, Slide 2 of 5

eTOOLS

On Slide 2, students use the Transformations eTool to determine if the two figures are congruent.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 2 Determine Congruence

Think About It! Do the trapezoids appear congruent? Why or why not?

See students' responses.

Are the two figures congruent? If so, describe a sequence of transformations that maps trapezoid *EFGH* onto trapezoid *IJKL*. If not, explain why they are not congruent.

Reflect trapezoid *EFGH* across the *y*-axis. Even if the reflected figure is translated up 4 units, it will not match trapezoid *IJKL* exactly.

So, the two figures are not congruent.

Check

Are the two figures congruent? If so, describe a sequence of transformations that maps Figure A onto Figure B. If not, explain why they are not congruent.

Part A Determine if the figures are congruent.

not congruent

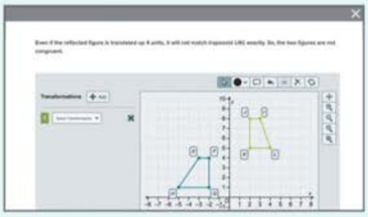
Part B If the figures are congruent, describe a sequence of transformations that maps Figure A onto Figure B. If the figures are not congruent, explain why they are not congruent.

Sample answer: No sequence of rotations, reflections, and/or translations will match the two figures up exactly.

Go Online You can complete an Extra Example online.

892 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 2, Determine Congruence, Slide 2 of 3

eTOOLS

On Slide 2, students use the Transformations eTool to determine if the figures are congruent.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Determine Congruence

Objective

Students will determine that a pair of two-dimensional figures is not congruent by applying a sequence of rotations, reflections, and translations.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents' use of the transformation tool should increase their understanding and confidence of determining congruence by using transformations.

6 Attend to Precision Encourage students to use precise mathematical language to explain why the figures are not congruent.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the figures? Sample answer: The figures do not appear congruent.

OL How can you use transformations to determine if the trapezoids are congruent? Sample answer: I can try to use a series of translations, reflections, and rotations to map the trapezoids onto each other.

BL If a dilation was part of the series of transformations, could trapezoid *EFGH* be mapped onto trapezoid *IJKL*? Explain. no; Sample answer: Sides *EF* and *IJ* are the same length, $\overline{GH}$ and $\overline{KL}$ are not the same length. If a scale factor was applied, it would be applied to every side length.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Learn Identify T ransformations

Objective

Students will understand how the orientation of two congruent figures can be used to identify the sequence of transformations between them.

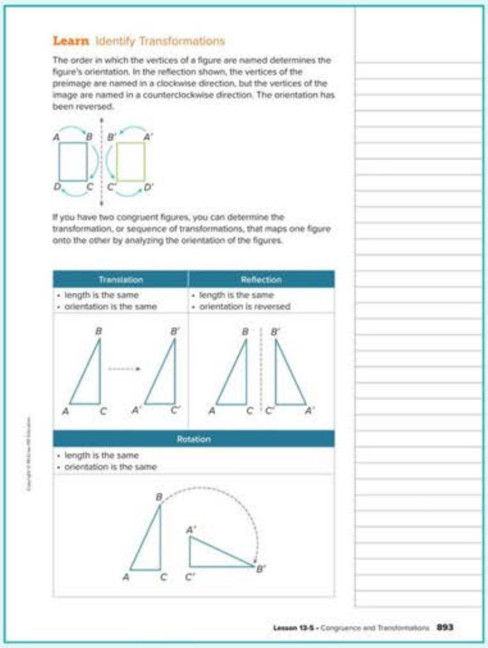
Teaching Notes

SLIDE 1

Remind students that when naming a figure, you follow the vertices in order, either clockwise or counterclockwise. If you name the rectangle shown as *ABCD*, you list the vertices *clockwise* in order. Translations and rotations preserve this orientation. However, to name the reflected rectangle in order, *A′B′C′D′*, you list the vertices *counterclockwise*, so the orientation is reversed.

SLIDE 2

Before students play the short animations, you may wish to ask them how analyzing the orientation can help them determine the transformation used to map the preimage on to the image. Some students may say if the orientation is reversed, then they know a reflection has occurred.



Learn Identify Transformations

The order in which the vertices of a figure are named determines the figure's orientation. In the reflection shown, the vertices of the preimage are named in a clockwise direction, but the vertices of the image are named in a counterclockwise direction. The orientation has been reversed.

If you have two congruent figures, you can determine the transformation, or sequence of transformations, that maps one figure onto the other by analyzing the orientation of the figures.

Translation	Reflection
• length is the same • orientation is the same	• length is the same • orientation is reversed

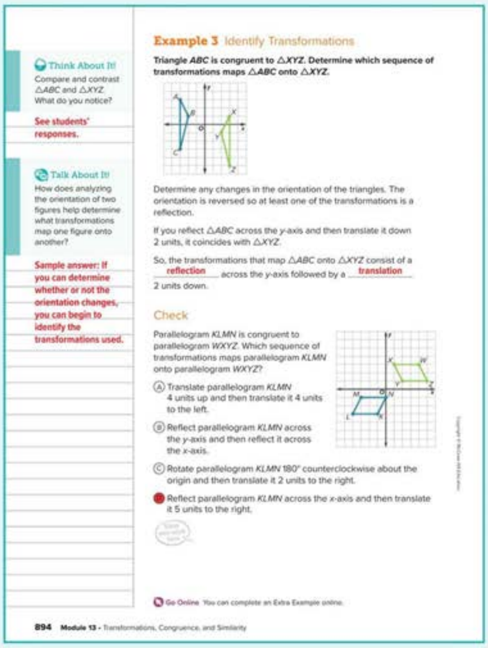
Rotation
• length is the same • orientation is the same

Interactive Presentation

Learn, Identify Transformations, Slide 2 of 2

WATCH

On Slide 2, students watch the brief animations that illustrate which transformations preserve orientation.



Think About It! Compare and contrast $\triangle ABC$ and $\triangle XYZ$. What do you notice?

See students' responses.

Talk About It! How does analyzing the orientation of two figures help determine what transformations map one figure onto another?

Sample answer: If you can determine whether or not the orientation changes, you can begin to identify the transformations used.

Example 3 Identify Transformations

Triangle *ABC* is congruent to $\triangle XYZ$. Determine which sequence of transformations maps $\triangle ABC$ onto $\triangle XYZ$.

Determine any changes in the orientation of the triangles. The orientation is reversed so at least one of the transformations is a reflection.

If you reflect $\triangle ABC$ across the *y*-axis and then translate it down 2 units, it coincides with $\triangle XYZ$.

So, the transformations that map $\triangle ABC$ onto $\triangle XYZ$ consist of a ___reflection___ across the *y*-axis followed by a ___translation___ 2 units down.

Check

Parallelogram *KLMN* is congruent to parallelogram *WXYZ*. Which sequence of transformations maps parallelogram *KLMN* onto parallelogram *WXYZ*?

(A) Translate parallelogram *KLMN* 4 units up and then translate it 4 units to the left.

(B) Reflect parallelogram *KLMN* across the *y*-axis and then reflect it across the *x*-axis.

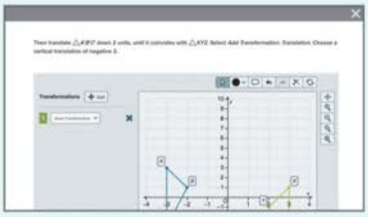
(C) Rotate parallelogram *KLMN* 180° counterclockwise about the origin and then translate it 2 units to the right.

(D) Reflect parallelogram *KLMN* across the *x*-axis and then translate it 5 units to the right.

Go Online You can complete an Extra Example online.

894 **Module 13** • Transformations, Congruence, and Similarity

Interactive Presentation



Example 3, Identify Transformations, Slide 2 of 4

eTOOLS

On Slide 2, students use the Transformations eTool to determine the transformations that map one triangle onto the other.

CLICK

On Slide 2, students select the correct words to complete the sentence.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Identify T ransformations

Objective

Students will describe a sequence of transformations between a pair of two-dimensional congruent figures.

MP Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use the Transformations eTool to determine the transformations that map one triangle onto the other.

6 Attend to Precision While discussing the *Talk About It!* question on Slide 3, encourage students to precisely explain why analyzing the orientation can help determine the transformations used to map one figure onto another.

Questions for Mathematical Discourse

SLIDE 2

AL What are the corresponding vertices of the two triangles? *A* and *X*, *B* and *Y*, *C* and *Z*

OL Are the orientations of the figures the same? Explain. no; Sample answer: When naming $\triangle ABC$, the vertices are named going clockwise. When naming $\triangle XYZ$, the vertices are named going counterclockwise.

BL Describe another sequence of transformations you can use to map $\triangle ABC$ onto $\triangle XYZ$. Sample answer: I can translate $\triangle ABC$ down 2 units, and then reflect it across the *y*-axis.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are struggling with identifying the transformations between the two figures, you may wish to have them trace one of the figures on a small piece of transparent or translucent paper. They can use that image to test different transformations to determine which ones will map one image on to the other. Suggest that they keep track of the transformations in a table like the one shown, noting the effect of the transformation and if it will work to show congruence.

Transformation	Effect	Does it work?
reflection over the *y*-axis	reversed orientation	yes

Example 4 Identify T ransformations

Objective

Students will describe a sequence of transformations between two congruent real-world figures.

Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* question on Slide 4, encourage students to clearly and precisely explain another set of transformations that can map the letter "d" onto the letter "p".

7 Look For and Make Use of Structure Encourage students to use the structure and placement of the letters to identify the transformations.

Questions for Mathematical Discourse

SLIDE 2

AL What do you notice about the "d" and "p" in the logo? Sample answer: "d" and "p" look like they have the same shape and size.

OL What should be done after rotating the letter "d" about point *A*? It needs to be translated up.

BL Would the transformations be the same if point *A* was placed at the top right point of the "d" instead? Explain. no; Sample answer: The "d" could still be rotated about point *A*, but then it would need to be translated down instead of up.

SLIDE 3

AL How can you prove that two figures are congruent? Sample answer: If a sequence of translations, rotations, and/or reflections can be used to map one figure onto the other, then the figures are congruent.

OL If the letter "d" was dilated at some point, would the letters be congruent? Explain. no; Sample answer: A dilation changes the size of a figure.

BL Create your own logo that uses a combination of transformations. Share your logo with a classmate, and describe the transformations in each logo. See students' logos.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 4 Identify Transformations

Ms. Martinez created the logo shown for Diamond Plumbing.

What transformations did she use if the letter "d" is the preimage and the letter "p" is the image? Are the two figures congruent?

Part A Identify the transformations.

Step 1 Start with the preimage.
Determine which transformation can be used to create the letter "p".

Step 2 Rotate the letter "d" 180° about point *A*.

Step 3 Translate the new image up.

So, Ms. Martinez could have used a rotation and a translation to create the logo.

Part B Determine congruence.

The letters are congruent because images produced by a rotation and a translation have the same shape and size.

Check the solution.

Copy the logo onto a piece of paper. Then trace the letter "d" with tracing paper. Rotate the letter 180° about Point *A*. Slide it up to line up with the letter "p". The letters are the same shape and size. They are congruent.

Think About It! Compare and contrast the letters "d" and "p" in the logo. What do you notice?

See students' responses.

Talk About It! Describe another set of transformations that can map the letter "d" onto the letter "p".

Sample answer: Reflect "d" over a vertical line, then reflect the image over a horizontal line. Finally, translate the letter up.

Lesson 13-5 • Congruence and Transformations **895**

Interactive Presentation

Example 4, Identify Transformations, Slide 2 of 5

CLICK

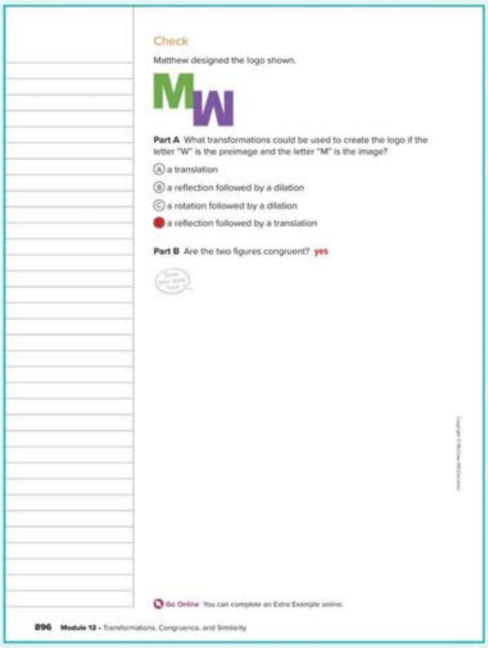
On Slide 2, students move through the slides to identify the transformations.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

8.G.A.1, 8.G.A.2

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION



Check

Matthew designed the logo shown.

Part A What transformations could be used to create the logo if the letter "W" is the preimage and the letter "M" is the image?

Ⓐ a translation

Ⓑ a reflection followed by a dilation

Ⓒ a rotation followed by a dilation

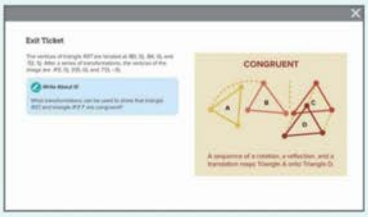
● a reflection followed by a translation

Part B Are the two figures congruent? yes

Go Online You can complete an Extra Example online.

896 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Exit Ticket

Exit Ticket

What transformations can be used to show that triangle *RST* and triangle *R′S′T′* on the Exit Ticket slide are congruent? Sample answer: Translate △*RST* one unit to the right and then reflect it across the *x*-axis.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7–10
- Extension: Congruent Triangles by SSS and SAS
- ALEKS Congruence and Similarity

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–5, 7
- Extension: Congruent Triangles by SSS and SAS
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Congruence and Similarity

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Congruence and Similarity

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

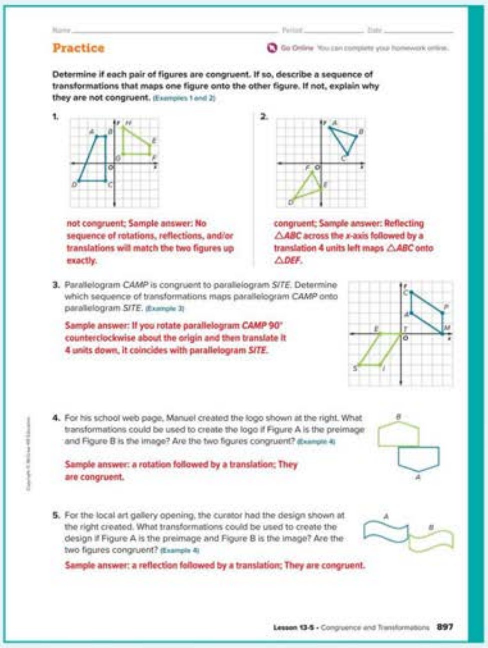
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	determine that a pair of two-dimensional figures is not congruent by applying a sequence of rotations, reflections, and translations	1
1	determine that a pair of two-dimensional figures is congruent by applying a sequence of rotations, reflections, and translations	2
1	describe a sequence of transformations between a pair of two-dimensional congruent figures	3
2	describe a sequence of transformations between two congruent real-world figures	4, 5
2	extend concepts learned in class to apply them in new contexts	6
3	higher-order and critical thinking skills	7–10

Common Misconception

Some students may have difficulty identifying congruent figures when the orientation of the figures is reversed. If this is the case, remind students that they can first perform a reflection on the preimage. Then they can apply other transformations, as needed, to map one figure onto the other figure.


Name ______ Period ______ Date ______

Practice — Go Online You can complete your homework online.

Determine if each pair of figures are congruent. If so, describe a sequence of transformations that maps one figure onto the other figure. If not, explain why they are not congruent. (Examples 1 and 2)

1. not congruent; Sample answer: No sequence of rotations, reflections, and/or translations will match the two figures up exactly.

2. congruent; Sample answer: Reflecting $\triangle ABC$ across the x-axis followed by a translation 4 units left maps $\triangle ABC$ onto $\triangle DEF$.

3. Parallelogram *CAMP* is congruent to parallelogram *SITE*. Determine which sequence of transformations maps parallelogram *CAMP* onto parallelogram *SITE*. (Example 3)

Sample answer: If you rotate parallelogram *CAMP* 90° counterclockwise about the origin and then translate it 4 units down, it coincides with parallelogram *SITE*.

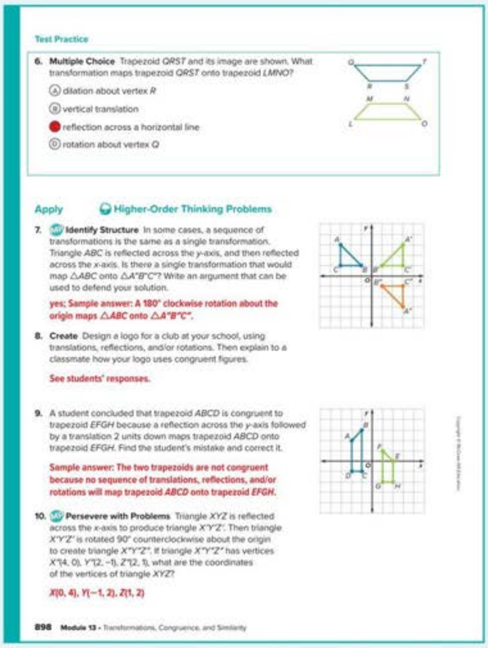
4. For his school web page, Manuel created the logo shown at the right. What transformations could be used to create the logo if Figure A is the preimage and Figure B is the image? Are the two figures congruent? (Example 4)

Sample answer: a rotation followed by a translation; They are congruent.

5. For the local art gallery opening, the curator had the design shown at the right created. What transformations could be used to create the design if Figure A is the preimage and Figure B is the image? Are the two figures congruent? (Example 4)

Sample answer: a reflection followed by a translation; They are congruent.

Lesson 13-5 • Congruence and Transformations 897



Test Practice

6. **Multiple Choice** Trapezoid *QRST* and its image are shown. What transformation maps trapezoid *QRST* onto trapezoid *LMNO*?

Ⓐ dilation about vertex *R*

Ⓑ vertical translation

● reflection across a horizontal line

Ⓓ rotation about vertex *Q*

Apply **Higher-Order Thinking Problems**

7. **Identify Structure** In some cases, a sequence of transformations is the same as a single transformation. Triangle *ABC* is reflected across the *y*-axis, and then reflected across the *x*-axis. Is there a single transformation that would map $\triangle ABC$ onto $\triangle A''B''C''$? Write an argument that can be used to defend your solution.

yes; Sample answer: A 180° clockwise rotation about the origin maps $\triangle ABC$ onto $\triangle A''B''C''$.

8. **Create** Design a logo for a club at your school, using translations, reflections, and/or rotations. Then explain to a classmate how your logo uses congruent figures.

See students' responses.

9. A student concluded that trapezoid *ABCD* is congruent to trapezoid *EFGH* because a reflection across the *y*-axis followed by a translation 2 units down maps trapezoid *ABCD* onto trapezoid *EFGH*. Find the student's mistake and correct it.

Sample answer: The two trapezoids are not congruent because no sequence of translations, reflections, and/or rotations will map trapezoid *ABCD* onto trapezoid *EFGH*.

10. **Persevere with Problems** Triangle *XYZ* is reflected across the *x*-axis to produce triangle *X'Y'Z'*. Then triangle *X'Y'Z'* is rotated 90° counterclockwise about the origin to create triangle *X''Y''Z''*. If triangle *X''Y''Z''* has vertices $X''(4, 0)$, $Y''(2, -1)$, $Z''(2, 1)$, what are the coordinates of the vertices of triangle *XYZ*?

$X(0, 4)$, $Y(-1, 2)$, $Z(1, 2)$

898 Module 13 • Transformations, Congruence, and Similarity

Teaching the Mathematical Practices

7 Look For and Make Use of Structure In E xercise 7, students determine if a single transformation would map one triangle to the other. Encourage students to use the structure of the triangles to determine which transformation was used.

1 Make Sense of Problems and Persevere in Solving Them In Exercise 10, students will determine the coordinates of the vertices of triangle *XYZ*. Encourage students to identify the important information in the problem and then decide a starting point to solve the problem.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Explore the truth of statements created by others.
Use with Exercise 6 Have students work in pairs. After completing the problem, have students write two true statements and one false statement about the situation. An example of a true statement might be "A rotation followed by a translation would map the trapezoid *QRST* onto trapezoid *LMNO*." An example of a false statement might be, "The two trapezoids have different perimeters." Have them trade statements with another pair or group. Each pair identifies which statements are true and which are false. Have them discuss and resolve any differences.

Listen and ask clarifying questions.
Use with Exercises 9–10 Have students work in pairs. Have students individually read Exercise 9 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 10.

Lesson 13-6

Similarity and Transformations

8.G.A.4

LESSON GOAL

Students will use a sequence of transformations to describe similarity between figures.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Learn: Similarity
Example 1: Determine Similarity
Example 2: Determine Similarity
Learn: Identify Transformations
Example 3: Identify Transformations
Example 4: Use the Scale Factor
Apply: Careers

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Extension: Similar Triangles by SAS		●	●
Collaboration Strategies	●	●	●

Language Development Support

Assign page 84 of the *Language Development Handbook* to help your students build mathematical language related to similarity and transformations.

ELL You can use the tips and suggestions on page T84 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min: 1 day
45 min: 2 days

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by using a sequence of transformations to describe similarities between figures.
Standards for Mathematical Content: **8. G.A.4**
Standards for Mathematical Practice: **MP 1, MP2, MP3, MP4, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students used a sequence of transformations to describe congruency between figures.
8.G.A.1, 8.G.A.1.A, 8.G.A.1.B, 8.G.A.1.C, 8.G.A.2

Now
Students use a sequence of transformations to describe similarity between figures.
8.G.A.4

Next
Students will solve problems involving similar triangles.
8.G.A.4, 8.G.A.5

Rigor

The Three Pillars of Rigor

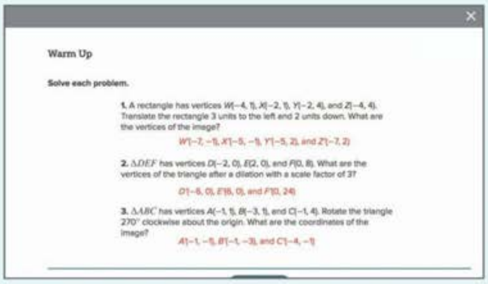
1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Conceptual Bridge In this lesson, students use their knowledge of transformations to develop *understanding* that two figures are similar if the second can be obtained from the first by a series of translations, reflections, rotations, and dilations. They build *fluency* with identifying what series of transformations determines similarity.

Mathematical Background

Go Online to find the mathematical background for the topics that are covered in this lesson.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2

What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- performing translations (Exercise 1)
- performing dilations (Exercise 2)
- performing rotations (Exercise 3)

Answers

1. $W'(-7, -1)$, $X'(-5, -1)$, $Y'(-5, 2)$, and $Z'(-7, 2)$

2. $D'(-6, 0)$, $E'(6, 0)$, and $F'(0, 24)$

3. $A'(-1, -1)$, $B'(-1, -3)$, and $C'(-4, -1)$

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the similarity of Russian nesting dolls, known as *Matryoshka* dolls.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet this standard?* and *How can I use these practices?*, and connect these to the standard.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The adjective *similar*, when used in everyday life, means *resembling without being identical*. Describe two everyday objects that might be similar. Sample answer: Two cars might be similar by both being four-door sedans that are the same color, but made by two different manufacturers, and thus, not identical.

Learn Similarity

Objective

Students will understand how dilations, translations, reflections, and rotations are used to show that a pair of two-dimensional figures is similar.

Teaching the Mathematical Practices

6 Attend to Precision W hile discussing the *Talk About It!* question on Slide 4, encourage students to use clear and precise mathematical language to compare and contrast using transformations to prove congruency versus similarity.

Teaching Notes

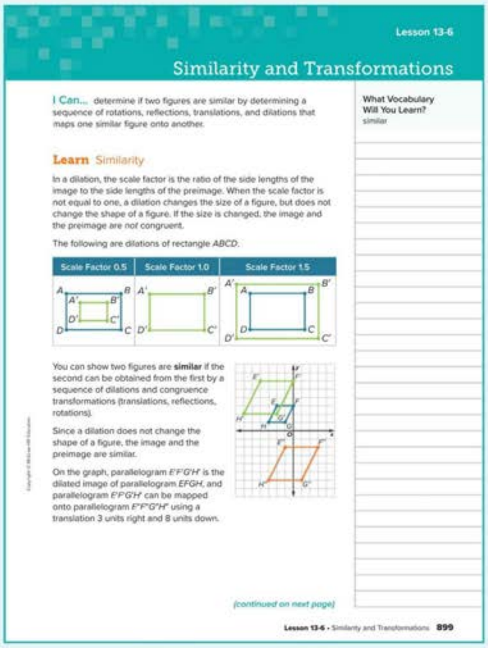
SLIDE 1

Before students choose a scale factor on the number line, ask them to predict how the image will look compared to the preimage. Some students may say that a scale factor of 0.5 will reduce the preimage, a scale factor of 1 does not change the preimage, and a scale factor of 1.5 and 2.0 will enlarge the preimage. Students should also note that when the scale factor is not equal to one, dilations change the size of a figure, but do not change the shape of the figure. It is also important that students understand that dilations do not preserve congruence, unless the scale factor is 1.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 3. The animation illustrates how to show two figures are similar using transformations.

(continued on next page)



Lesson 13-6

Similarity and Transformations

I Can... determine if two figures are similar by determining a sequence of rotations, reflections, translations, and dilations that maps one similar figure onto another.

What Vocabulary Will You Learn?
similar

Learn Similarity

In a dilation, the scale factor is the ratio of the side lengths of the image to the side lengths of the preimage. When the scale factor is not equal to one, a dilation changes the size of a figure, but does not change the shape of a figure. If the size is changed, the image and the preimage are not congruent.

The following are dilations of rectangle *ABCD*.

Scale Factor 0.5	Scale Factor 1.0	Scale Factor 1.5

You can show two figures are **similar** if the second can be obtained from the first by a sequence of dilations and congruence transformations (translations, reflections, rotations).

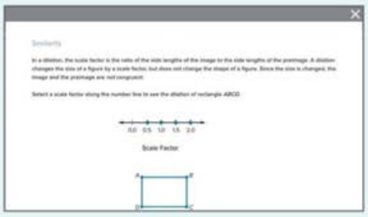
Since a dilation does not change the shape of a figure, the image and the preimage are similar.

On the graph, parallelogram *E'F'G'H'* is the dilated image of parallelogram *EFGH*, and parallelogram *E'F'G'H'* can be mapped onto parallelogram *E''F''G''H''* using a translation 3 units right and 8 units down.

(continued on next page)

Lesson 13-6 • Similarity and Transformations 899

Interactive Presentation



Learn, Similarity, Slide 1 of 4

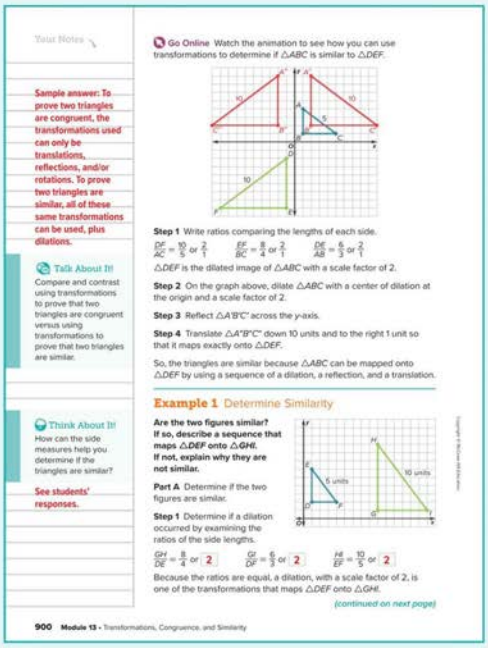
CLICK

On Slide 1, students select a scale factor along the number line to see the dilation of rectangle *ABCD*.

DIFFERENTIATE

Reteaching Activity AL

If any of your students are having difficulty understanding similarity transformations, have them work with a partner to create a flowchart that guides them through the process of determining if two figures are similar. Suggest the first step in the flowchart might be “Did a dilation between the two figures occur?” When students have finished their flowcharts, pair them with another student to discuss any similarities and differences between their flowcharts. You may choose to hang the flowcharts around the classroom.



Your Notes

Sample answer: To prove two triangles are congruent, the transformations used can only be translations, reflections, and/or rotations. To prove two triangles are similar, all of these same transformations can be used, plus dilations.

Talk About It! Compare and contrast using transformations to prove that two triangles are congruent versus using transformations to prove that two triangles are similar.

Think About It! How can the side measures help you determine if the triangles are similar?

See students' responses.

Go Online Watch the animation to see how you can use transformations to determine if $\triangle ABC$ is similar to $\triangle DEF$.

Step 1 Write ratios comparing the lengths of each side.

$\frac{DF}{AC} = \frac{10}{5}$ or $\frac{2}{1}$ $\frac{EF}{BC} = \frac{8}{4}$ or $\frac{2}{1}$ $\frac{DE}{AB} = \frac{6}{3}$ or $\frac{2}{1}$

$\triangle DEF$ is the dilated image of $\triangle ABC$ with a scale factor of 2.

Step 2 On the graph above, dilate $\triangle ABC$ with a center of dilation at the origin and a scale factor of 2.

Step 3 Reflect $\triangle A'B'C'$ across the y-axis.

Step 4 Translate $\triangle A''B''C''$ down 10 units and to the right 1 unit so that it maps exactly onto $\triangle DEF$.

So, the triangles are similar because $\triangle ABC$ can be mapped onto $\triangle DEF$ by using a sequence of a dilation, a reflection, and a translation.

Example 1 Determine Similarity

Are the two figures similar? If so, describe a sequence that maps $\triangle DEF$ onto $\triangle GHI$. If not, explain why they are not similar.

Part A Determine if the two figures are similar.

Step 1 Determine if a dilation occurred by examining the ratios of the side lengths.

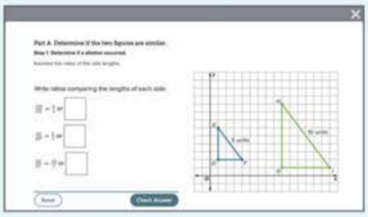
$\frac{GH}{DE} = \frac{8}{4}$ or 2 $\frac{GI}{DF} = \frac{6}{3}$ or 2 $\frac{HI}{EF} = \frac{10}{5}$ or 2

Because the ratios are equal, a dilation, with a scale factor of 2, is one of the transformations that maps $\triangle DEF$ onto $\triangle GHI$.

(continued on next page)

900 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 1, Determine Similarity, Slide 2 of 6

TYPE

On Slide 2 of Example 1, students determine if the ratios are equal.

Learn Similarity *(continued)*

Talk About It!

SLIDE 4

Mathematical Discourse

Compare and contrast using transformations to prove that two triangles are congruent versus using transformations to prove that two triangles are similar. Sample answer: To prove two triangles are congruent, the transformations used can only be translations, reflections, and/or rotations. To prove two triangles are similar, all of these same transformations can be used, plus dilations.

Example 1 Determine Similarity

Objective

Students will determine that a pair of two-dimensional figures is similar by applying a sequence of dilations, rotations, reflections, and translations.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to use reasoning about the ratios of side lengths to determine that a dilation has occurred.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question, encourage them to construct an argument to justify their reasoning using clear and precise mathematical language.

5 Use Appropriate Tools Strategically Students will use the Transformations eTool to confirm the transformations needed to map one triangle onto the other.

Questions for Mathematical Discourse

SLIDE 2

AL Why is it important to know whether or not a dilation has occurred? If a dilation occurred with a scale factor that is not one, then the figures are not congruent, but are similar.

OL How will examining the side lengths help you know if a dilation has occurred? Sample answer: If the corresponding side lengths are proportional (have the same ratio), then a scale factor has been applied.

BL If a dilation with a scale factor of 2, centered at the origin, was the only transformation that was performed on Triangle *DEF*, what would the coordinates of the image be? (2, 4), (2, 12), (8, 4)

(continued on next page)

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 1 Determine Similarity *(continued)*

SLIDE 3

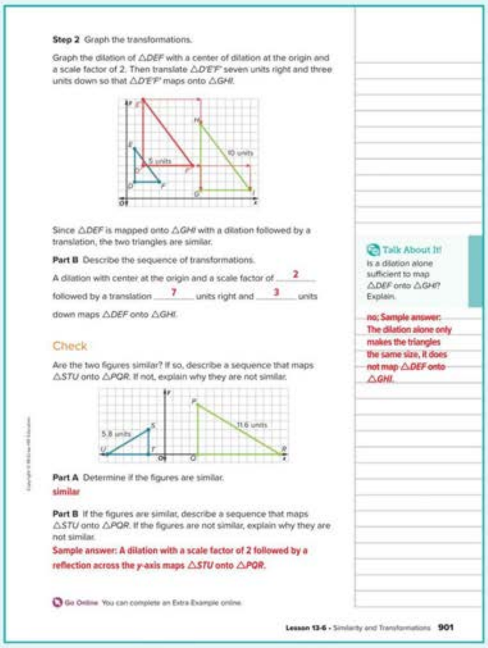
AL Does a scale factor of 2 represent a reduction or an enlargement? enlargement

OL Why is the dilation not the only transformation that maps Triangle *DEF* onto Triangle *GHI*? After the dilation, the triangles are not located at the same coordinates.

BL If the translation occurred first, followed by the dilation, would the final image be the same? Explain. No; translating the figure first means the dilation is performed on different coordinates, so the final image is not the same.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Step 2 Graph the transformations.

Graph the dilation of $\triangle DEF$ with a center of dilation at the origin and a scale factor of 2. Then translate $\triangle D'E'F'$ seven units right and three units down so that $\triangle D'E'F'$ maps onto $\triangle GHI$.

Since $\triangle DEF$ is mapped onto $\triangle GHI$ with a dilation followed by a translation, the two triangles are similar.

Part B Describe the sequence of transformations.

A dilation with center at the origin and a scale factor of 2 followed by a translation 7 units right and 3 units down maps $\triangle DEF$ onto $\triangle GHI$.

Talk About It! Is a dilation alone sufficient to map $\triangle DEF$ onto $\triangle GHI$? Explain.

no; Sample answer: The dilation alone only makes the triangles the same size, it does not map $\triangle DEF$ onto $\triangle GHI$.

Check

Are the two figures similar? If so, describe a sequence that maps $\triangle STU$ onto $\triangle PQR$. If not, explain why they are not similar.

Part A Determine if the figures are similar.

similar

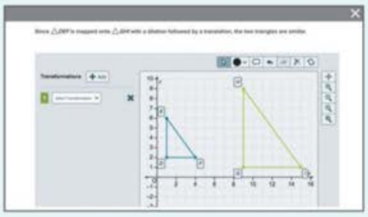
Part B If the figures are similar, describe a sequence that maps $\triangle STU$ onto $\triangle PQR$. If the figures are not similar, explain why they are not similar.

Sample answer: A dilation with a scale factor of 2 followed by a reflection across the *y*-axis maps $\triangle STU$ onto $\triangle PQR$.

Go Online You can complete an Extra Example online.

Lesson 13-6 • Similarity and Transformations 901

Interactive Presentation



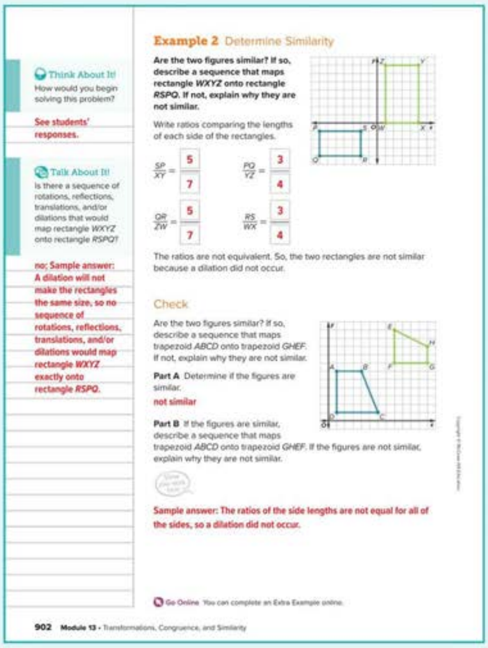
Example 1, Determine Similarity, Slide 3 of 6

eTOOLS

On Slide 3, students use the Transformations eTool to graph the transformations.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 2 Determine Similarity

Are the two figures similar? If so, describe a sequence that maps rectangle *WXYZ* onto rectangle *RSPQ*. If not, explain why they are not similar.

Write ratios comparing the lengths of each side of the rectangles.

$\frac{SP}{XY} = \frac{5}{7}$ $\frac{PQ}{YZ} = \frac{3}{4}$

$\frac{QR}{ZW} = \frac{5}{7}$ $\frac{RS}{WX} = \frac{3}{4}$

The ratios are not equivalent. So, the two rectangles are not similar because a dilation did not occur.

Think About It! How would you begin solving this problem?

See students' responses.

Talk About It! Is there a sequence of rotations, reflections, translations, and/or dilations that would map rectangle *WXYZ* onto rectangle *RSPQ*?

no; Sample answer: A dilation will not make the rectangles the same size, so no sequence of rotations, reflections, translations, and/or dilations would map rectangle *WXYZ* exactly onto rectangle *RSPQ*.

Check

Are the two figures similar? If so, describe a sequence that maps trapezoid *ABCD* onto trapezoid *GHEF*. If not, explain why they are not similar.

Part A Determine if the figures are similar.

not similar

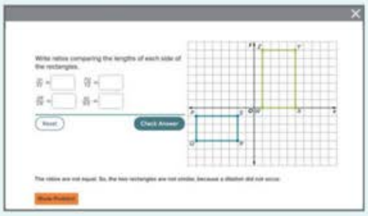
Part B If the figures are similar, describe a sequence that maps trapezoid *ABCD* onto trapezoid *GHEF*. If the figures are not similar, explain why they are not similar.

Sample answer: The ratios of the side lengths are not equal for all of the sides, so a dilation did not occur.

Go Online You can complete an Extra Example online.

902 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 2, Determine Similarity, Slide 2 of 4

TYPE

On Slide 2, students determine if the ratios are equal.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 2 Determine Similarity

Objective

Students will determine that a pair of two-dimensional figures is not similar by determining if a dilation occurred.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Enc ourage students to use reasoning about the ratios of the corresponding side lengths of the quadrilaterals to determine the figures are not similar, because a dilation has not occurred.

6 Attend to Precision While discussing the *Talk About It!* question on Slide 3, encourage students to provide a clear and precise justification for why there is no sequence of these transformations that would map rectangle *WXYZ* onto rectangle *RSPQ*.

Questions for Mathematical Discourse

SLIDE 2

AL Why do you find the ratios of the corresponding side lengths? to determine if a dilation has occurred

OL What does it mean that the ratios are not all equivalent? A dilation has not occurred, so the quadrilaterals are not similar.

BL If you knew the area of rectangle *WXYZ*, but did not know the coordinates for rectangle *RSPQ*, could you find the area of rectangle *RSPQ*? Explain. no; Sample answer: Since the figures are not similar, I cannot use the scale factor to help me determine the dimensions or area of rectangle *RSPQ*.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING 2 FLUENCY 3 APPLICATION

Learn Identify T ransformations

Objective

Students will understand how the sizes of two similar figures are related by the scale factor of the dilation.

Teaching Notes

SLIDE 1

The process for determining the sequence of transformations used for similar figures is the same as the process used for determining the transformations used for congruent figures, except for the addition of a dilation. It may be helpful for students to identify the dilation first before identifying the congruence transformations used. Point out to students that a scale factor can be 1 and that if the scale factor is 1, the image is congruent to the preimage.

Example 3 Identify T ransformations

Objective

Students will describe a sequence of transformations between a pair of two-dimensional similar figures.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively A s students discuss the *Talk About It!* questions, encourage them to use reasoning about similarity and transformations to explain why all squares are similar.

5 Use Appropriate Tools Strategically Students will use the Transformations eTool to determine which sequence of transformations maps the first figure onto the second figure.

7 Look For and Make Use of Structure Students should study the structure of the two figures, paying close attention to the locations of the corresponding vertices.

Questions for Mathematical Discourse

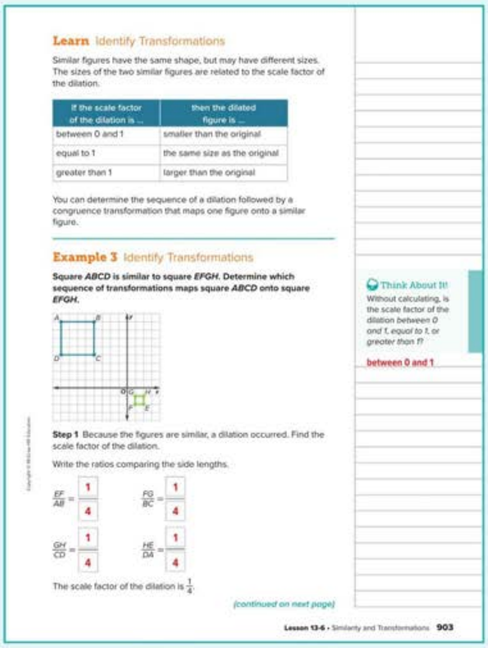
SLIDE 2

AL Is square *EFGH* smaller or larger than square *ABCD*? What should this tell you about the scale factor? smaller; The scale factor is between 0 and 1.

OL How do you know that more than just a dilation occurred? Sample answer: If just the dilation occurred, the image would still be located in Quadrant II, just closer to the origin.

BL How can you determine the dilation has a scale factor of $\frac{1}{4}$, without finding *each* ratio? Sample answer: Both figures are squares, so I can just compare one of the side lengths of the original figure to one of the side lengths in the reduced figure.

(continued on next page)



Learn Identify Transformations

Similar figures have the same shape, but may have different sizes. The sizes of the two similar figures are related to the scale factor of the dilation.

If the scale factor of the dilation is ...	then the dilated figure is ...
between 0 and 1	smaller than the original
equal to 1	the same size as the original
greater than 1	larger than the original

You can determine the sequence of a dilation followed by a congruence transformation that maps one figure onto a similar figure.

Example 3 Identify Transformations

Square *ABCD* is similar to square *EFGH*. Determine which sequence of transformations maps square *ABCD* onto square *EFGH*.

Step 1 Because the figures are similar, a dilation occurred. Find the scale factor of the dilation.

Write the ratios comparing the side lengths.

$\frac{EF}{AB} = \frac{1}{4}$ $\frac{FG}{BC} = \frac{1}{4}$

$\frac{GH}{CD} = \frac{1}{4}$ $\frac{HE}{DA} = \frac{1}{4}$

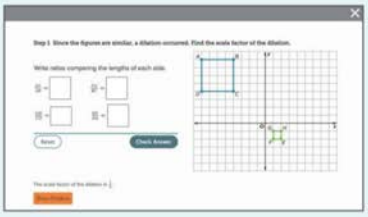
The scale factor of the dilation is $\frac{1}{4}$.

(continued on next page)

Think About It! Without calculating, is the scale factor of the dilation between 0 and 1, equal to 1, or greater than 1? between 0 and 1

Lesson 13-6 • Similarity and Transformations 903

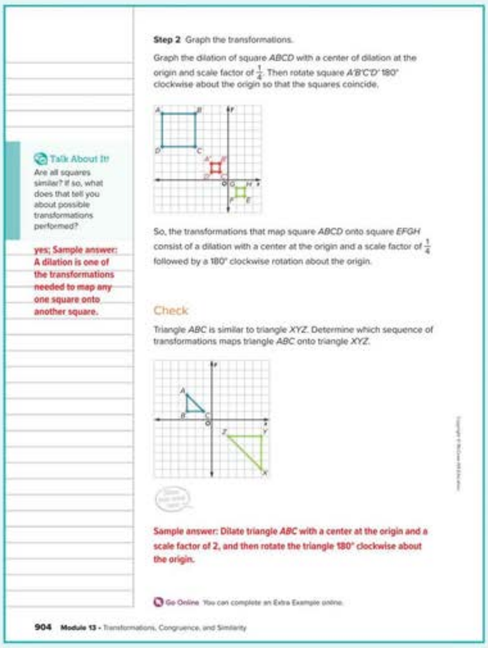
Interactive Presentation



Example 3, Identify Transformation, Slide 2 of 5

TYPE

On Slide 2 of Example 3, students determine the scale factor of the dilation.



Step 2 Graph the transformations.

Graph the dilation of square *ABCD* with a center of dilation at the origin and scale factor of $\frac{1}{4}$. Then rotate square *A'B'C'D'* 180° clockwise about the origin so that the squares coincide.

So, the transformations that map square *ABCD* onto square *EFGH* consist of a dilation with a center at the origin and a scale factor of $\frac{1}{4}$ followed by a 180° clockwise rotation about the origin.

Talk About It!
Are all squares similar? If so, what does that tell you about possible transformations performed?

yes; Sample answer: A dilation is one of the transformations needed to map any one square onto another square.

Check

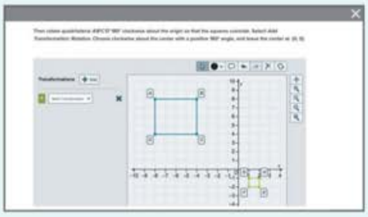
Triangle *ABC* is similar to triangle *XYZ*. Determine which sequence of transformations maps triangle *ABC* onto triangle *XYZ*.

Sample answer: Dilate triangle *ABC* with a center at the origin and a scale factor of 2, and then rotate the triangle 180° clockwise about the origin.

Go Online You can complete an Extra Example online.

904 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 3, Identify Transformation, Slide 3 of 5

eTOOLS

On Slide 3, students use the Transformations eTool to graph the transformations.

CLICK

On Slide 3, students select the correct words to complete the sentence.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 3 Identify Transformations *(continued)*

SLIDE 3

AL After dilating the figure, what transformation do you need to apply next? Sample answer: rotation

OL A classmate claims that the second transformation, after the dilation, is a translation down 2 units and to the right 3 units. How can you explain that this is incorrect? Sample answer: While the translation seems to move the figure into the correct location, the corresponding vertices are not lined up.

BL Can the transformations be applied in the reverse order and obtain the same result? Yes, if the rotation occurred first, then followed by the dilation, the final image is in the same location.

Go Online

- Find additional teaching notes and the *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 4 Use the Scale Factor

Objective

Students will determine missing dimensions of similar figures by using the scale factor.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others While discussing the *Talk About It!* question on Slide 4, encourage students to construct a mathematical argument to defend their reasoning.

Questions for Mathematical Discourse

SLIDE 2

AL What is the size of the original photo? 2 inches by 3 inches

OL How will you find the dimensions of the print? multiply the web page photo dimensions by 1.5.

BL How are the dimensions of the print related to the original dimensions? The print dimensions are 3 times the dimensions of the original.

SLIDE 3

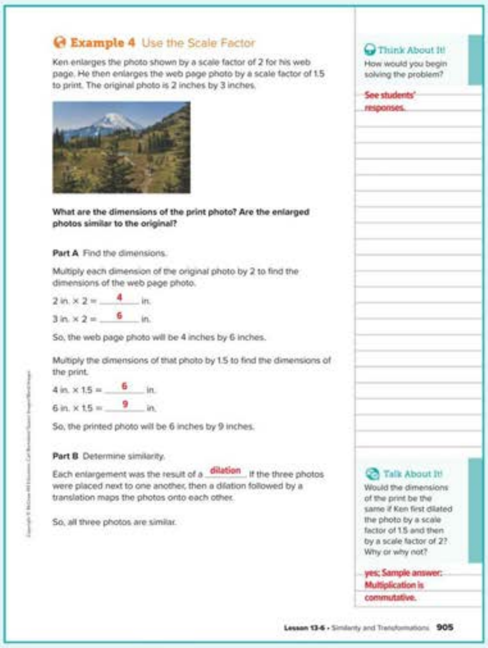
AL What type of transformation changes the size of a figure? dilation

OL How can you prove the photos are similar? Sample answer: I can describe a sequence of transformations that maps the photos onto each other.

BL Would a representation of the photo with dimensions 10 inches by 16 inches be similar to the original? Explain. no; Sample answer: The ratio of 10 to 2 is 5 but the ratio of 16 to 3 is not 5.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Use the Scale Factor

Ken enlarges the photo shown by a scale factor of 2 for his web page. He then enlarges the web page photo by a scale factor of 1.5 to print. The original photo is 2 inches by 3 inches.

What are the dimensions of the print photo? Are the enlarged photos similar to the original?

Part A Find the dimensions.

Multiply each dimension of the original photo by 2 to find the dimensions of the web page photo.

2 in. × 2 = 4 in.

3 in. × 2 = 6 in.

So, the web page photo will be 4 inches by 6 inches.

Multiply the dimensions of that photo by 1.5 to find the dimensions of the print.

4 in. × 1.5 = 6 in.

6 in. × 1.5 = 9 in.

So, the printed photo will be 6 inches by 9 inches.

Part B Determine similarity.

Each enlargement was the result of a dilation. If the three photos were placed next to one another, then a dilation followed by a translation maps the photos onto each other.

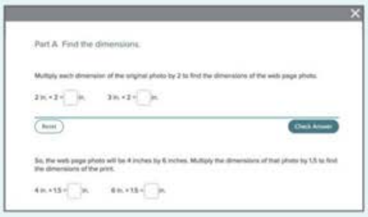
So, all three photos are similar.

Think About It! How would you begin solving the problem? See students' responses.

Talk About It! Would the dimensions of the print be the same if Ken first dilated the photo by a scale factor of 1.5 and then by a scale factor of 2? Why or why not? yes; Sample answer: Multiplication is commutative.

Lesson 13-6 • Similarity and Transformations 905

Interactive Presentation



Example 4, Use the Scale Factor, Slide 2 of 5

TYPE

On Slide 2, students determine the missing dimensions of the photos.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

An art show offers different-sized prints of the same painting. The original print measures 24 centimeters by 30 centimeters. A printer enlarges the original by a scale factor of 1.5, and then enlarges the second image by a scale factor of 3.

Part A

What are the dimensions of the largest print?

108 centimeters by 135 centimeters

Part B

Are both of the enlarged prints similar to the original? **yes**

Go Online You can complete an Extra Example online.

Pause and Reflect

Compare and contrast *congruent figures* and *similar figures*. How are they the alike? How are they different?

See students' observations.

Apply Careers

Objective

Students will come up with their own strategy to solve an application problem involving similar figures.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the *Write About It!* prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does scale factor mean?
- What transformations were applied to the image?
- What do you know about the lengths of the sides of each image?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Apply Careers

A designer enlarges an image with a length of 6 centimeters and width of 9 centimeters by a scale factor of 3. The designer decides that the enlarged image is too large and reduces it by a scale factor of 0.5. Will the final image fit into a rectangular space that has an area of 121 square centimeters? Explain your answer.

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Because the area of the final image is 121.5 square centimeters, it will not fit into a space that has an area of 121 square centimeters; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Talk About It!

How did the properties of similar figures help you solve the problem?

See students' responses.

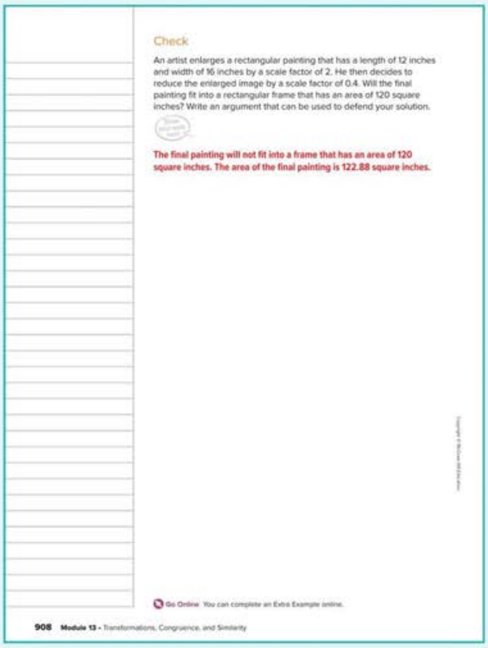
Lesson 13-6 • Similarity and Transformations 907

Interactive Presentation

Apply, Careers

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

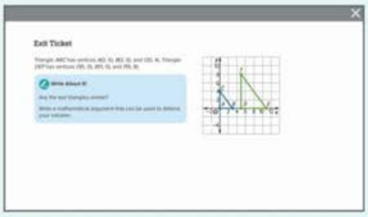
An artist enlarges a rectangular painting that has a length of 12 inches and width of 16 inches by a scale factor of 2. He then decides to reduce the enlarged image by a scale factor of 0.4. Will the final painting fit into a rectangular frame that has an area of 120 square inches? Write an argument that can be used to defend your solution.

The final painting will not fit into a frame that has an area of 120 square inches. The area of the final painting is 122.88 square inches.

Go Online You can complete an Extra Example online.

908 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Exit Ticket

Exit Ticket

Are the two triangles on the Exit Ticket slide similar? Write a mathematical argument that can be used to defend your solution. The triangles are similar, because the ratios of the sides are equivalent.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 4, 5, 7–10
- Extension: Similar Triangles by SAS
- ALEKS Congruence and Similarity

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4, 7, 8
- Extension: Similar Triangles by SAS
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–4
- ALEKS Similar Figures

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS Similar Figures

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

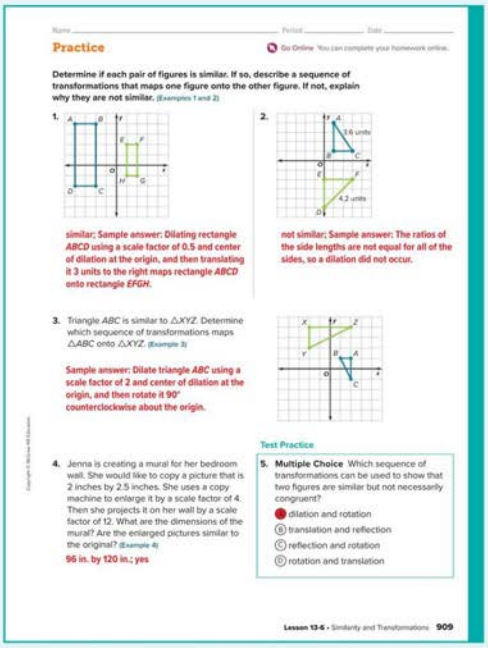
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	determine that a pair of two-dimensional figures is similar by applying a sequence of dilations, rotations, reflections, and translations	1
1	determine that a pair of two-dimensional figures is not similar by determining if a dilation occurred	2
1	describe a sequence of transformations between a pair of two-dimensional similar figures	3
2	determine missing dimensions of similar figures by using the scale factor	4
2	extend concepts learned in class to apply them in new contexts	5
3	solve application problems involving similar figures	6, 7
3	higher-order and critical thinking skills	8–10

Common Misconception

In Exercises 1 and 3, some students may have difficulty identifying a sequence of transformations for the pair of similar figures. Remind students that any combination of rotations, reflections, and/or translations and a dilation can be used to map one figure onto another similar figure. So, more than one sequence of transformations is possible when describing the transformations.



Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

Determine if each pair of figures is similar. If so, describe a sequence of transformations that maps one figure onto the other figure. If not, explain why they are not similar. (Examples 1 and 2)

1. similar; Sample answer: Dilating rectangle *ABCD* using a scale factor of 0.5 and center of dilation at the origin, and then translating it 3 units to the right maps rectangle *ABCD* onto rectangle *EFGH*.

2. not similar; Sample answer: The ratios of the side lengths are not equal for all of the sides, so a dilation did not occur.

3. Triangle *ABC* is similar to △*XYZ*. Determine which sequence of transformations maps △*ABC* onto △*XYZ*. (Example 3)

Sample answer: Dilate triangle *ABC* using a scale factor of 2 and center of dilation at the origin, and then rotate it 90° counterclockwise about the origin.

4. Jenna is creating a mural for her bedroom wall. She would like to copy a picture that is 2 inches by 2.5 inches. She uses a copy machine to enlarge it by a scale factor of 4. Then she projects it on her wall by a scale factor of 12. What are the dimensions of the mural? Are the enlarged pictures similar to the original? (Example 4)

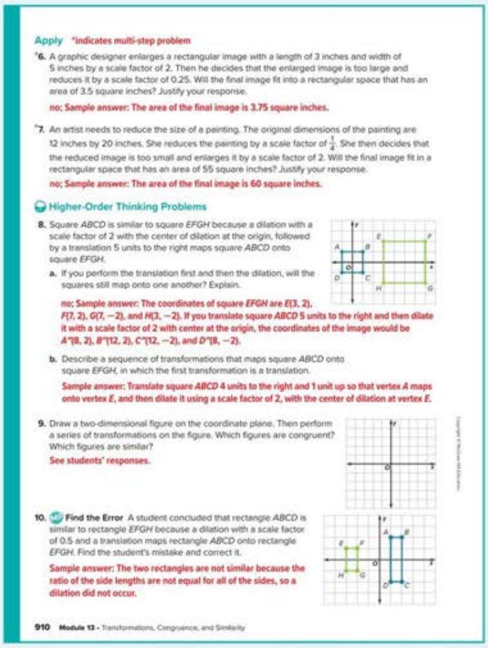
96 in. by 120 in.; yes

Test Practice

5. **Multiple Choice** Which sequence of transformations can be used to show that two figures are similar but not necessarily congruent?

(A) dilation and rotation
(B) translation and reflection
(C) reflection and rotation
(D) rotation and translation

Lesson 13-6 • Similarity and Transformations 909



Apply *indicates multi-step problem

*6. A graphic designer enlarges a rectangular image with a length of 3 inches and width of 5 inches by a scale factor of 2. Then he decides that the enlarged image is too large and reduces it by a scale factor of 0.25. Will the final image fit into a rectangular space that has an area of 3.5 square inches? Justify your response.

no; Sample answer: The area of the final image is 3.75 square inches.

*7. An artist needs to reduce the size of a painting. The original dimensions of the painting are 12 inches by 20 inches. She reduces the painting by a scale factor of $\frac{1}{4}$. She then decides that the reduced image is too small and enlarges it by a scale factor of 2. Will the final image fit in a rectangular space that has an area of 55 square inches? Justify your response.

no; Sample answer: The area of the final image is 60 square inches.

Higher-Order Thinking Problems

8. Square *ABCD* is similar to square *EFGH* because a dilation with a scale factor of 2 with the center of dilation at the origin, followed by a translation 5 units to the right maps square *ABCD* onto square *EFGH*.

a. If you perform the translation first and then the dilation, will the squares still map onto one another? Explain.

no; Sample answer: The coordinates of square *EFGH* are *E*(3, 2), *F*(7, 2), *G*(7, −2), and *H*(3, −2). If you translate square *ABCD* 5 units to the right and then dilate it with a scale factor of 2 with center at the origin, the coordinates of the image would be *A*″(8, 2), *B*″(12, 2), *C*″(12, −2), and *D*″(8, −2).

b. Describe a sequence of transformations that maps square *ABCD* onto square *EFGH*, in which the first transformation is a translation.

Sample answer: Translate square *ABCD* 4 units to the right and 1 unit up so that vertex *A* maps onto vertex *E*, and then dilate it using a scale factor of 2, with the center of dilation at vertex *E*.

9. Draw a two-dimensional figure on the coordinate plane. Then perform a series of transformations on the figure. Which figures are congruent? Which figures are similar?

See students' responses.

10. **Find the Error** A student concluded that rectangle *ABCD* is similar to rectangle *EFGH* because a dilation with a scale factor of 0.5 and a translation maps rectangle *ABCD* onto rectangle *EFGH*. Find the student's mistake and correct it.

Sample answer: The two rectangles are not similar because the ratio of the side lengths are not equal for all of the sides, so a dilation did not occur.

910 Module 13 • Transformations, Congruence, and Similarity

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 10, students find the mistake and correct it. Encourage students to explain why the rectangles are not similar.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Create your own application problem.
Use with Exercises 6–7 After completing the application problems, have students write their own real-world problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Make sense of the problem.
Use with Exercise 10 Have students work together to prepare a brief explanation that illustrates the flawed reasoning. For example, the student in the exercise thinks that a scale factor of 0.5 will make rectangle *ABCD* the same size as rectangle *EFGH*. Have each pair or group of students present their explanations to the class.

Lesson 13-7

Indirect Measurement

8.G.A.4, 8.G.A.5

LESSON GOAL

Students will solve problems involving similar triangles.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Similar Triangles and Indirect Measurement

Learn: Indirect Measurement
Example 1: Use Indirect Measurement
Example 2: Use Indirect Measurement

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
Collaboration Strategies	●	●	●

Language Development Support

Assign page 85 of the *Language Development Handbook* to help your students build mathematical language related to indirect measurement.

ELL You can use the tips and suggestions on page T85 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min 0.5 day
45 min 1 day

Focus

Domain: Geometry
Major Cluster(s): In this lesson, students address the major cluster **8.G.A** by solving problems involving similar triangles.
Standards for Mathematical Content: **8. G.A.4, 8.G.A.5**
Standards for Mathematical Practice: **MP 3, MP5, MP6, MP7**

Coherence

Vertical Alignment

Previous
Students used a sequence of transformations to describe similarity between figures.
8.G.A.4

Now
Students solve problems involving similar triangles.
8.G.A.4, 8.G.A.5

Next
Students will prove theorems involving similarity.
HSG.SRT.B.4, HSG.SRT.B.5

Rigor

The Three Pillars of Rigor

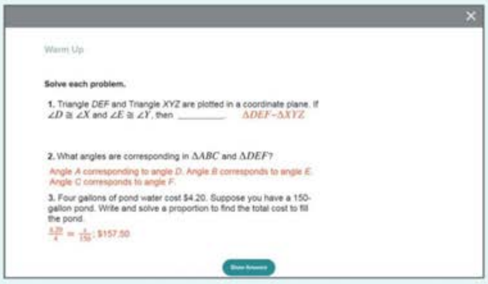
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students continue to expand on their *understanding* of similar figures by solving problems using indirect measurement. They build *fluency* with finding missing measures using indirect measurement, and *apply* their fluency to real-world problems.

Mathematical Background

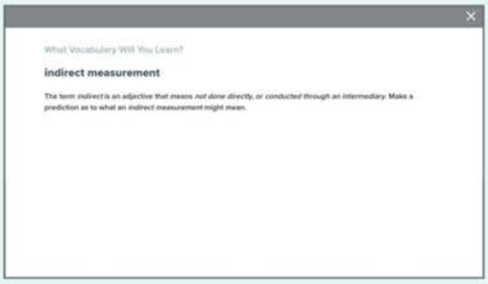
Indirect measurement is a method of finding unknown dimensions of a figure by using the properties of similar polygons. To use indirect measurement with two similar triangles, first identify the corresponding parts. Set up a proportion involving the unknown value using the corresponding parts. To find the unknown value, solve the proportion.

Interactive Presentation



Warm Up

Launch the Lesson, Slide 1 of 2



What Vocabulary Will You Learn?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skills for this lesson:

- understanding angle-angle similarity (Exercise 1)
- understanding corresponding parts (Exercise 2)
- writing and solving proportions (Exercise 3)

Answers

1. $\triangle DEF \sim \triangle XYZ$
2. Angle *A* corresponding to angle *D*. Angle *B* corresponds to angle *E*. Angle *C* corresponds to angle *F*.
3. $\frac{4.20}{4} = \frac{x}{150}$; \$157.50

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about the length of a shadow being proportional to a person's height.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

What Vocabulary Will You Learn?

Use the following question to engage students and facilitate a class discussion.

Ask:

- The term *indirect* is an adjective that means *not done directly*, or *conducted through an intermediary*. Make a prediction as to what *indirect measurement* might mean. Sample answer: An indirect measurement might be a measurement that was not measured directly, or it was measured via an intermediary.

Explore Similar Triangles and Indirect Measurement

Objective

Students will use Web Sketchpad to explore how to use similar triangles to solve problems involving indirect measurement.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use Web Sketchpad to investigate the how the Sun and the shadows cast from figures can be related to similar triangles to find measurements that are hard to measure.

Inquiry Question

How can you find lengths that are difficult to measure directly? Sample answer: I can use the properties of similar triangles to write and solve a proportion to find the missing lengths.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 2 are shown.

Talk About It!

SLIDE 2

Mathematical Discourse

What objects in the sketch can you easily measure in real life? Sample answer: my height, my shadow, and the tree's shadow

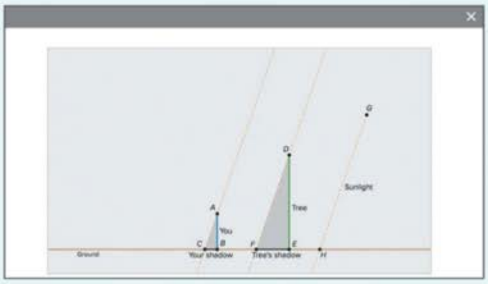
In the sketch, how can you show that $\triangle ABC$ is similar to $\triangle DEF$? Sample answer: It is given that the two triangles are right triangles, and the rays of the sun are parallel. Using the ground as a transversal, I can say that $m\angle ACB = m\angle DFE$ (corresponding angles have equal measures). Since the measures of the angles are equal, the two angles are congruent. I can then use Angle-Angle Similarity to show that $\triangle ABC \sim \triangle DEF$.

If the triangles are similar, what do you know about the lengths of the sides? Sample answer: If two triangles are similar, the lengths of their corresponding sides are proportional.

(continued on next page)

Interactive Presentation

Explore, Slide 1 of 7

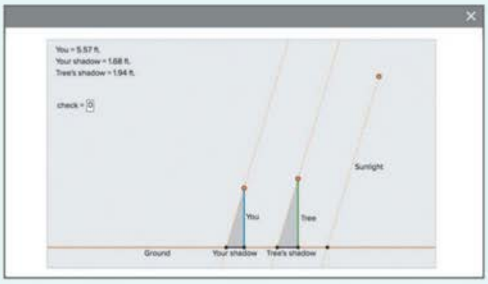


Explore, Slide 2 of 7

WEB SKETCHPAD

Throughout the Explore, students use Web Sketchpad to explore how to use similar triangles to solve problems involving indirect measurement.

Interactive Presentation



Explore, Slide 5 of 7

TYPE

On Slide 7, students respond to the Inquiry Question and view a sample answer.

Explore Similar T riangles and Indirect Measurement *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically S tudents will use Web Sketchpad to explore this activity.

7 Look For and Make Use of Structure Encourage students to examine the correspondences between the height of figures, the position of the Sun, and the length of the shadows.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* question on Slide 5 are shown.

Talk About It!

SLIDE 5

Mathematical Discourse

Share your tree height with a partner and explain how you found the answer. See students' repsonses.

How well do your calculations match the measurement for the tree's height? If they don't match, explain any errors you may have made and try the calculations again. See students' repsonses.

8.G.A.4, 8.G.A.5

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Indirect Measurement

Objective

Students will understand how properties of similar triangles can be used to solve problems involving indirect measurement.

Go Online to find additional teaching notes.

DIFFERENTIATE

Reteaching Activity L

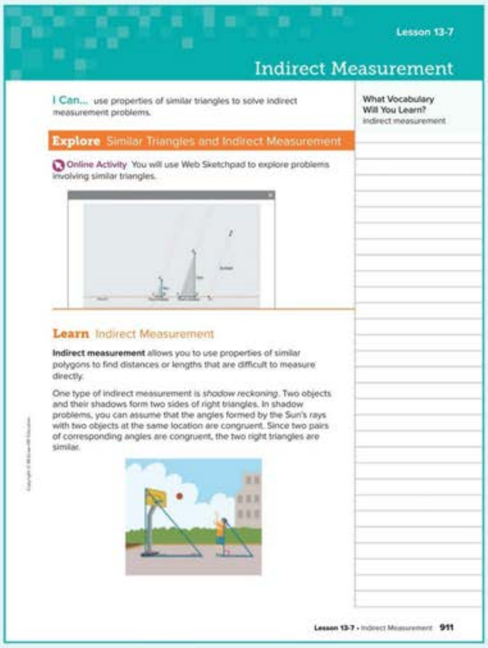
If any of your students are struggling with indirect measurement, have them work with a partner to complete the following hands-on shadow reckoning activity outdoors on a sunny morning or afternoon.

One student stands upright casting a shadow while the other student measures their height and the length of the shadow cast. Students should draw a scalene right triangle using these measurements with the horizontal leg representing the shadow and the vertical leg representing the person's height. The pair then measures the shadow created by a tree, building, or other object. Students should draw a similar triangle to the one already drawn, and label the triangle using these measurements.

You may wish to ask students to answer the following questions:

- How do you know the two triangles are similar? Sample answer: I know one set of corresponding angles are right angles, and since the measurements were done at the same time, the angles formed by the light of the sun have the same measure. Using Angle-Angle Similarity, the two triangles are similar.
- Do you need to find the length of the hypotenuse? Explain. No; Sample answer: I only need to find the length of the missing leg since that is the height of the object I want to find.
- How will you find the height of the object? Sample answer: I can write and solve a proportion comparing the corresponding legs.

You may wish to have students explain why, when using shadow reckoning, shadows must be measured at the same time, and why shadow reckoning does not work when the sun is directly overhead. Ask them to include an example of what might happen if the shadows were measured several hours apart.


Lesson 13-7

Indirect Measurement

I Can... use properties of similar triangles to solve indirect measurement problems.

What Vocabulary Will You Learn?
indirect measurement

Explore Similar Triangles and Indirect Measurement

Online Activity You will use Web Sketchpad to explore problems involving similar triangles.

Learn Indirect Measurement

Indirect measurement allows you to use properties of similar polygons to find distances or lengths that are difficult to measure directly.

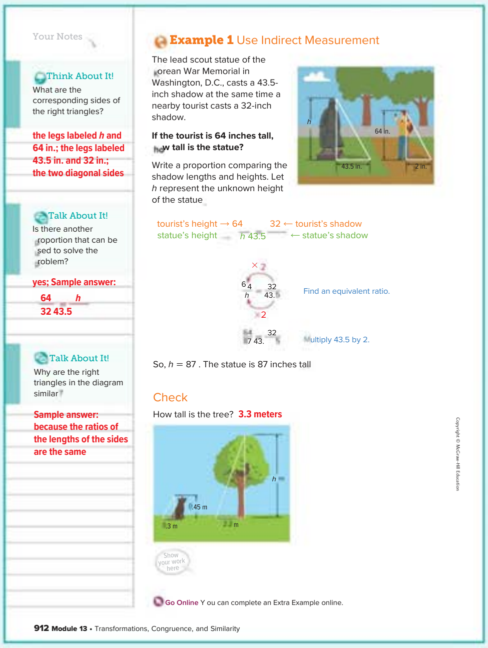
One type of indirect measurement is *shadow reckoning*. Two objects and their shadows form two sides of right triangles. In shadow problems, you can assume that the angles formed by the Sun's rays with two objects at the same location are congruent. Since two pairs of corresponding angles are congruent, the two right triangles are similar.

Lesson 13-7 • Indirect Measurement 911

Interactive Presentation


Learn, Indirect Measurement

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Your Notes

Think About It!
What are the corresponding sides of the right triangles?

the legs labeled h and 64 in.; the legs labeled 43.5 in. and 32 in.; the two diagonal sides

Talk About It!
Is there another proportion that can be used to solve the problem?

yes; Sample answer: $\frac{64}{32} = \frac{h}{43.5}$

Talk About It!
Why are the right triangles in the diagram similar?

Sample answer: because the ratios of the lengths of the sides are the same

Example 1 Use Indirect Measurement

The lead scout statue of the Korean War Memorial in Washington, D.C., casts a 43.5-inch shadow at the same time a nearby tourist casts a 32-inch shadow.

If the tourist is 64 inches tall, how tall is the statue?

Write a proportion comparing the shadow lengths and heights. Let h represent the unknown height of the statue.

tourist's height → 64 / statue's height → h = 32 ← tourist's shadow / 43.5 ← statue's shadow

$\frac{64}{h} = \frac{32}{43.5}$ (×2) Find an equivalent ratio.

$\frac{64}{87} = \frac{32}{43.5}$ Multiply 43.5 by 2.

So, $h = 87$. The statue is 87 inches tall

Check

How tall is the tree? **3.3 meters**

Go Online You can complete an Extra Example online.

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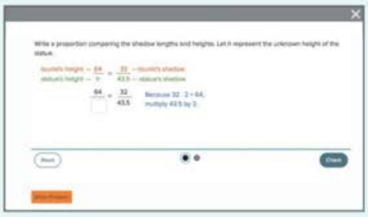
912 Module 13 • Transformations, Congruence, and Similarity

Interactive Presentation



Example 1, Use Indirect Measurement, Slide 2 of 4

TYPE

On Slide 2, students determine the height of a statue.

CHECK

Student complete the Check exercise online to determine if they are ready to move on.

Example 1 Use Indirect Measurement

Objective

Students will use similar triangles to solve problems involving indirect measurement with shadows.

Teaching the Mathematical Practices

6 Attend to Precision While discussing the *Talk About It!* questions on Slide 3, students should be able to use clear and precise mathematical language in their explanations.

Questions for Mathematical Discourse

SLIDE 2

AL What are you trying to find? How will you represent it in the proportion? the height of the statue; with a variable

OL Write the statue's height in feet and inches. The statue is 7 feet 3 inches tall.

OL What is the ratio of the length of the tourist's shadow to the height of the tourist? $\frac{1}{2}$

BL Use this ratio, or set up a proportion, to find the length of a tourist's shadow if the tourist's height is 6 feet 2 inches. Assume the same scale factor as this problem. The length of the tourist's shadow is 37 inches, or 3 feet 1 inch.

Teaching Notes

You may want to point out that another way to solve proportions is to use cross products. In the proportion $\frac{64}{h} = \frac{32}{43.5}$, the product of h and 32 and the product of 64 and 43.5 are called the cross products. The cross products of a proportion are equivalent. To solve a proportion by using cross products, write the corresponding equation stating the cross products are equivalent. Then solve the equation.

$\frac{64}{h} = \frac{32}{43.5}$	Write the proportion.
$32h = 64(43.5)$	Find the cross products.
$\frac{32h}{32} = \frac{2{,}784}{32}$	Simplify.
$h = 87$	Divide each side by 32.

Have students compare and contrast using ratio reasoning and cross products to solve proportions. Ask them to decide which method is more meaningful and which method may be more efficient to use with more difficult values.

Go Online

- Find additional teaching notes and *Talk About It!* questions to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 2 Use Indirect Measurement

Objective

Students will use similar triangles to solve problems involving indirect measurement.

Teaching the Mathematical Practices

7 Look For and Make Use of Structure S tudents should analyze the structure of the two triangles in the diagram in order to identify the corresponding side lengths accurately.

Questions for Mathematical Discourse

SLIDE 2

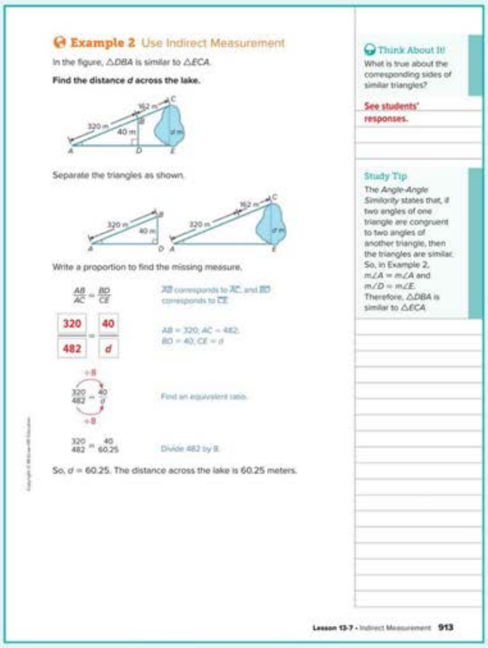
AL What are you trying to find? How will you represent it in the proportion? the distance across the lake; with a variable

OL A classmate set up the proportion $\frac{320}{162} = \frac{40}{d}$, solved for d, and stated the distance across the lake is 20.25 meters. What mistake did they make? Sample answer: They assumed that $\overline{AB}$ corresponds with $\overline{BC}$,but $\overline{AB}$ corresponds with $\overline{AC}$.

BL AD is approximately 317.5 m. What is AE? about 478.2 meters

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 2 Use Indirect Measurement

In the figure, △DBA is similar to △ECA.

Find the distance d across the lake.

Separate the triangles as shown.

Write a proportion to find the missing measure.

$\frac{AB}{AC} = \frac{BD}{CE}$ — $\overline{AB}$ corresponds to $\overline{AC}$, and $\overline{BD}$ corresponds to $\overline{CE}$.

$\frac{320}{482} = \frac{40}{d}$ — AB = 320; AC = 482; BD = 40; CE = d

$\frac{320}{482} = \frac{40}{d}$ (÷8) — Find an equivalent ratio.

$\frac{320}{482} = \frac{40}{60.25}$ — Divide 482 by 8.

So, d = 60.25. The distance across the lake is 60.25 meters.

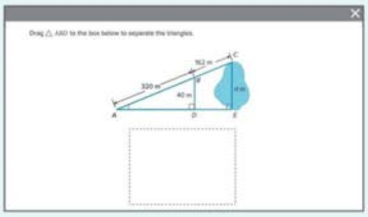
Think About It! What is true about the corresponding sides of similar triangles?

See students' responses.

Study Tip The *Angle-Angle Similarity* states that, if two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar. So, in Example 2, m∠A = m∠A and m∠D = m∠E. Therefore, △DBA is similar to △ECA.

Lesson 13-7 • Indirect Measurement 913

Interactive Presentation



Example 2, Use Indirect Measurement, Slide 2 of 3

DRAG & DROP

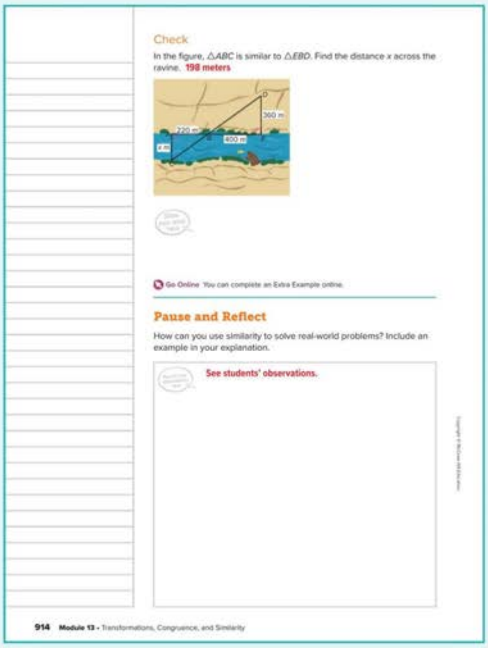
On Slide 2, students drag to separate the triangles.

TYPE

On Slide 2, students determine the missing distance.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Check

In the figure, △ABC is similar to △EBD. Find the distance x across the ravine. 198 meters

Go Online You can complete an Extra Example online.

Pause and Reflect

How can you use similarity to solve real-world problems? Include an example in your explanation.

See students' observations.

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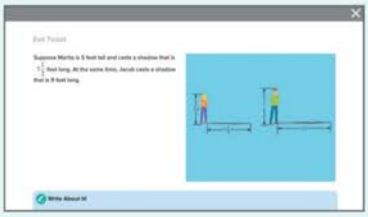
Interactive Presentation



Exit Ticket

Exit Ticket

Use the properties of similar right triangles to find Jacob's height, as shown on the Exit Ticket slide. Write a mathematical argument that can be used to defend your solution. Jacob is 6 feet tall. Sample answer: Write and solve a proportion. $\frac{5}{h} = \frac{7\frac{1}{2}}{9}$, where h represents Jacob's height. So, $h = 6$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 5, 7, 9–11
- ALEKS® Congruence and Similarity

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–4
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1 and 2
- ALEKS® Similar Figures

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ALEKS® Similar Figures

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

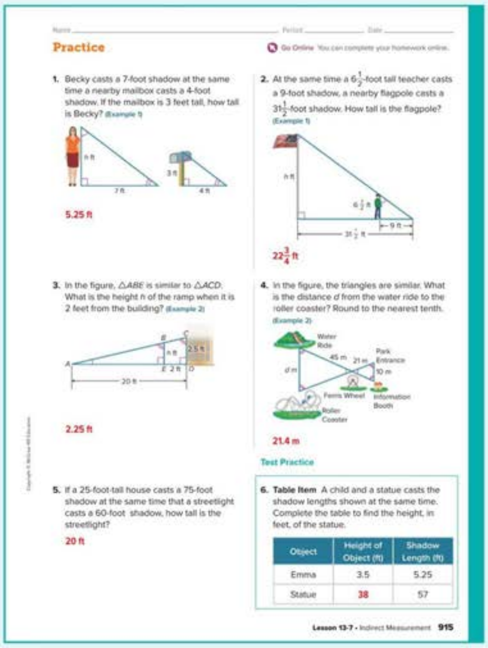
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	solve problems involving indirect measurement with shadows	1, 2
1	solve problems involving indirect measurement	3, 4
2	extend concepts learned in class to apply them in new contexts	5-8
3	higher-order and critical thinking skills	9–11

Common Misconception

Some students may incorrectly set up the proportion to solve the problems. Remind students to identify the corresponding parts in the similar triangles to help them write a correct proportion.


Name ______ Period ______ Date ______

Practice

Go Online You can complete your homework online.

1. Becky casts a 7-foot shadow at the same time a nearby mailbox casts a 4-foot shadow. If the mailbox is 3 feet tall, how tall is Becky? (Example 1)

5.25 ft

2. At the same time a $6\frac{1}{2}$-foot tall teacher casts a 9-foot shadow, a nearby flagpole casts a $31\frac{1}{2}$-foot shadow. How tall is the flagpole? (Example 1)

$22\frac{3}{4}$ ft

3. In the figure, $\triangle ABE$ is similar to $\triangle ACD$. What is the height h of the ramp when it is 2 feet from the building? (Example 2)

2.25 ft

4. In the figure, the triangles are similar. What is the distance d from the water ride to the roller coaster? Round to the nearest tenth. (Example 2)

21.4 m

5. If a 25-foot-tall house casts a 75-foot shadow at the same time that a streetlight casts a 60-foot shadow, how tall is the streetlight?

20 ft

Test Practice

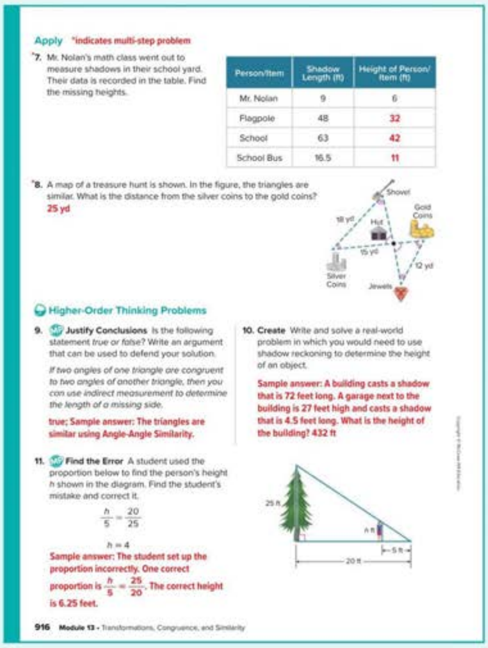
6. **Table Item** A child and a statue casts the shadow lengths shown at the same time. Complete the table to find the height, in feet, of the statue.

Object	Height of Object (ft)	Shadow Length (ft)
Emma	3.5	5.25
Statue	38	57

Lesson 13-7 • Indirect Measurement 915

8.G.A.4, 8.G.A.5

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**



Apply *indicates multi-step problem

*7. Mr. Nolan's math class went out to measure shadows in their school yard. Their data is recorded in the table. Find the missing heights.

Person/Item	Shadow Length (ft)	Height of Person/ Item (ft)
Mr. Nolan	9	6
Flagpole	48	32
School	63	42
School Bus	16.5	11

*8. A map of a treasure hunt is shown. In the figure, the triangles are similar. What is the distance from the silver coins to the gold coins?
25 yd

Higher-Order Thinking Problems

9. Justify Conclusions Is the following statement *true* or *false*? Write an argument that can be used to defend your solution.

If two angles of one triangle are congruent to two angles of another triangle, then you can use indirect measurement to determine the length of a missing side.

true; Sample answer: The triangles are similar using Angle-Angle Similarity.

10. Create Write and solve a real-world problem in which you would need to use shadow reckoning to determine the height of an object.

Sample answer: A building casts a shadow that is 72 feet long. A garage next to the building is 27 feet high and casts a shadow that is 4.5 feet long. What is the height of the building? 432 ft

11. Find the Error A student used the proportion below to find the person's height h shown in the diagram. Find the student's mistake and correct it.

$$\frac{h}{5} = \frac{20}{25}$$

$$h = 4$$

Sample answer: The student set up the proportion incorrectly. One correct proportion is $\frac{h}{5} = \frac{25}{20}$. The correct height is 6.25 feet.

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Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 9, students determine the validity of the statement. Encourage students to determine the statement is true using the Angle-Angle Similarity.

In Exercise 11 , students will find the mistake in the problem and correct it. Encourage students to pinpoint the mistake and then determine how it should be fixed.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Listen and ask clarifying questions.
Use with Exercises 7–8 Have students work in pairs. Have students individually read Exercise 7 and formulate their strategy for solving the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 8.

Create your own higher-order thinking problem.
Use with Exercises 9–11 After completing the higher-order thinking problems, have students write their own higher-order thinking problem that involves the concepts from this lesson. Have them trade their problems with a partner and solve them. Then have them check each other's work, and discuss and resolve any differences.

Review

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include examples of various transformations. Have students share their completed Foldables with a partner, comparing the similarities and differences in the examples recorded. Students can use their completed Foldables to study for the module assessment.

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Interactive Student Edition* and share their responses with a partner.

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Vocabulary Activity
Module Review

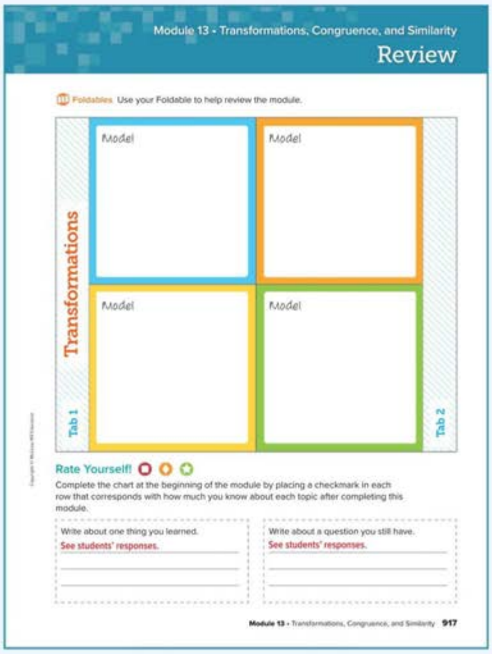
Assessment Resources

Put It All Together 1: Lessons 13-1 and 13-2
Put It All Together 2: Lessons 13-1, 13-2, 13-3, and 13-4
Vocabulary Test
AL Module Test Form B
OL Module Test Form A
BL Module Test Form C
Performance Task*

*The module-level performance task is available online as a printable and editable document. A scoring rubric is included.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice with these topics for **Geometry**.

- Understand Congruence and Similarity
- Lines, Angles, and Triangles


Module 13 • Transformations, Congruence, and Similarity

Review

Foldables Use your Foldable to help review the module.

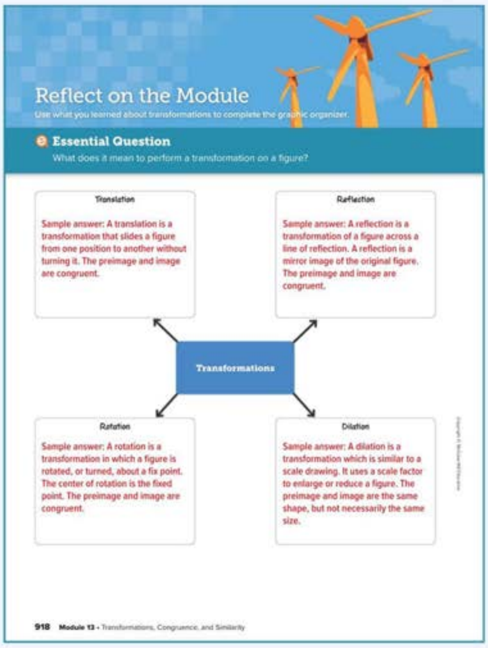
Rate Yourself!

Complete the chart at the beginning of the module by placing a checkmark in each row that corresponds with how much you know about each topic after completing this module.

Write about one thing you learned.
See students' responses.

Write about a question you still have.
See students' responses.

Module 13 • Transformations, Congruence, and Similarity 917


Reflect on the Module

Use what you learned about transformations to complete the graphic organizer.

Essential Question

What does it mean to perform a transformation on a figure?

Transformations

Translation

Sample answer: A translation is a transformation that slides a figure from one position to another without turning it. The preimage and image are congruent.

Reflection

Sample answer: A reflection is a transformation of a figure across a line of reflection. A reflection is a mirror image of the original figure. The preimage and image are congruent.

Rotation

Sample answer: A rotation is a transformation in which a figure is rotated, or turned, about a fix point. The center of rotation is the fixed point. The preimage and image are congruent.

Dilation

Sample answer: A dilation is a transformation which is similar to a scale drawing. It uses a scale factor to enlarge or reduce a figure. The preimage and image are the same shape, but not necessarily the same size.

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Essential Question

ELL Have students complete the graphic organizer to organize their thoughts related to the Essential Question. You may wish to have students work in pairs or groups to answer the Essential Question, or facilitate a whole class discussion. You may wish to have students watch the Launch the Module video again in which the module Essential Question was first presented.

What does it mean to perform a transformation on a figure? See students' graphic organizers.

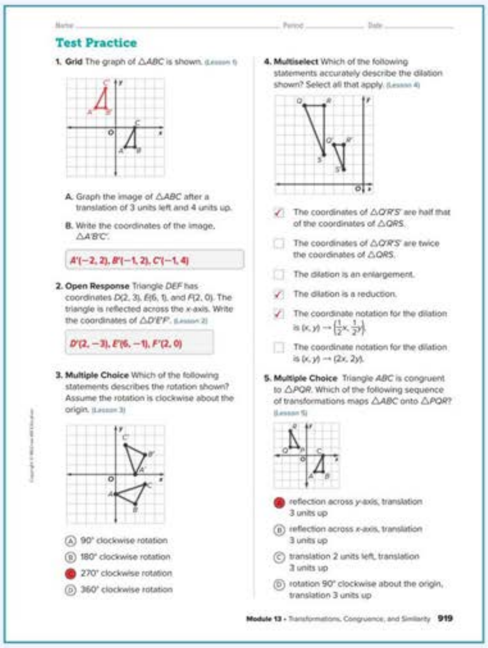
Test Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–8 mirror the types of questions your students will see on the online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	3, 5
Multiselect	Multiple answers may be correct. Students must select all correct answers.	4
Grid	Students will graph a figure on an online coordinate plane.	1
Open Response	Students construct their own response in the area provided.	2, 6, 7
Equation Editor	Students use an online equation editor to construct their response, often using math notation and symbols.	8

To ensure that students understand the standards, check students' success on individual exercises.

Standard(s)	Lesson(s)	Exercise(s)
8.G.A.1	13-1, 13-2, 13-3, 13-5	1, 2, 3, 5, 6
8.G.A.1.A	13-1, 13-2, 13-3, 13-5	1, 2, 3, 5, 6
8.G.A.1.B	13-5	5, 6
8.G.A.1.C	13-5	5, 6
8.G.A.2	13-5	5, 6
8.G.A.3	13-1, 13-2, 13-3, 13-4	1–4
8.G.A.4	13-6, 13-7	7, 8
8.G.A.5	13-6, 13-7	7, 8


Name Period Date

Test Practice

1. Grid The graph of $\triangle ABC$ is shown. (Lesson 1)

A. Graph the image of $\triangle ABC$ after a translation of 3 units left and 4 units up.

B. Write the coordinates of the image, $\triangle A'B'C'$.

$A'(-2, 2), B'(-1, 2), C'(-1, 4)$

2. Open Response Triangle DEF has coordinates $D(2, 3)$, $E(6, 1)$, and $F(2, 0)$. The triangle is reflected across the x-axis. Write the coordinates of $\triangle D'E'F'$. (Lesson 2)

$D'(2, -3), E'(6, -1), F'(2, 0)$

3. Multiple Choice Which of the following statements describes the rotation shown? Assume the rotation is clockwise about the origin. (Lesson 3)

(A) 90° clockwise rotation
(B) 180° clockwise rotation
(C) 270° clockwise rotation
(D) 360° clockwise rotation

4. Multiselect Which of the following statements accurately describe the dilation shown? Select all that apply. (Lesson 4)

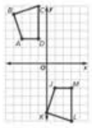
- [x] The coordinates of $\triangle Q'R'S'$ are half that of the coordinates of $\triangle QRS$.
- [ ] The coordinates of $\triangle Q'R'S'$ are twice the coordinates of $\triangle QRS$.
- [ ] The dilation is an enlargement.
- [x] The dilation is a reduction.
- [x] The coordinate notation for the dilation is $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.
- [ ] The coordinate notation for the dilation is $(x, y) \rightarrow (2x, 2y)$.

5. Multiple Choice Triangle ABC is congruent to $\triangle PQR$. Which of the following sequence of transformations maps $\triangle ABC$ onto $\triangle PQR$? (Lesson 5)

(A) reflection across y-axis, translation 3 units up
(B) reflection across x-axis, translation 3 units up
(C) translation 2 units left, translation 3 units up
(D) rotation 90° clockwise about the origin, translation 3 units up

Module 13 • Transformations, Congruence, and Similarity 919

6. Open Response Consider quadrilaterals *ABCD* and *JKLM* as shown. (Lesson 5)



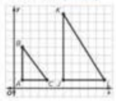
A. Are the quadrilaterals congruent?

yes

B. If the quadrilaterals are congruent, describe the sequence of transformations that maps quadrilateral *ABCD* onto quadrilateral *JKLM*. If the figures are not congruent, explain why they are not congruent.

Sample answer: A translation 4 units right, followed by a reflection across the *x*-axis maps quadrilateral *ABCD* onto quadrilateral *JKLM*.

7. Open Response Consider triangles *ABC* and *JKL* as shown. (Lesson 6)



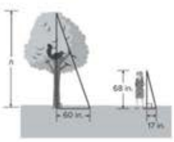
A. Are the triangles similar?

no

B. If the triangles are similar, describe the sequence of transformations that maps triangle *ABC* onto triangle *JKL*. If the figures are not similar, explain why they are not similar.

Sample answer: $\frac{AB}{JK} = \frac{4}{8}$ **or** $\frac{1}{2}$; $\frac{AC}{JL} = \frac{3}{5}$; **and** $\frac{BC}{KL} = \frac{5}{9.4}$. **Since the ratios between the side lengths are not equal, the two triangles are not similar.**

8. Open Response A tree in the school yard casts a 60-inch shadow at the same time a nearby student casts a 17-inch shadow. If the student is 68 inches tall, how many <u>feet</u> tall is the tree? (Lesson 7)



20

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Foldables Study Organizers

What Are Foldables and How Do I Create Them?

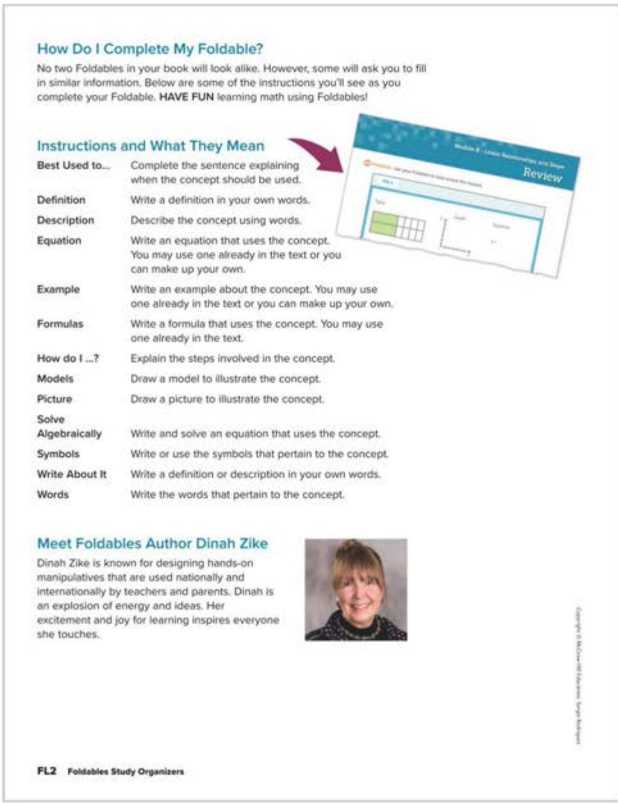
Foldables are three-dimensional graphic organizers that help you create study guides for each module in your book.

Step 1 Go to the back of your book to find the Foldable for the module you are currently studying. Follow the cutting and assembly instructions at the top of the page.

Step 2 Go to the Module Review at the end of the module you are currently studying. Match up the tabs and attach your Foldable to this page. Dotted tabs show where to place your Foldable. Striped tabs indicate where to tape the Foldable.

How Will I Know When to Use My Foldable?

You will be directed to work on your Foldable at the end of selected lessons. This lets you know that it is time to update it with concepts from that lesson. Once you've completed your Foldable, use it to study for the module test.



How Do I Complete My Foldable?

No two Foldables in your book will look alike. However, some will ask you to fill in similar information. Below are some of the instructions you'll see as you complete your Foldable. **HAVE FUN** learning math using Foldables!

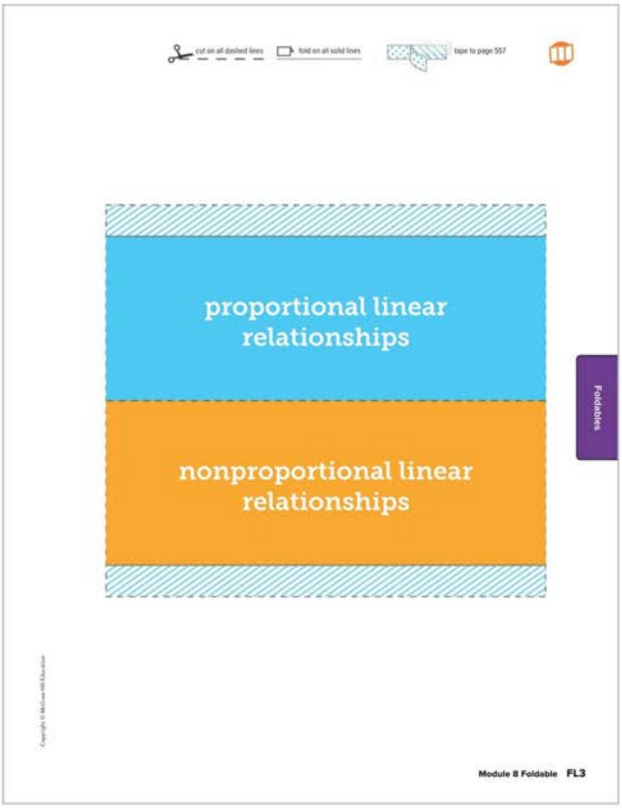
Instructions and What They Mean

Best Used to...	Complete the sentence explaining when the concept should be used.
Definition	Write a definition in your own words.
Description	Describe the concept using words.
Equation	Write an equation that uses the concept. You may use one already in the text or you can make up your own.
Example	Write an example about the concept. You may use one already in the text or you can make up your own.
Formulas	Write a formula that uses the concept. You may use one already in the text.
How do I ...?	Explain the steps involved in the concept.
Models	Draw a model to illustrate the concept.
Picture	Draw a picture to illustrate the concept.
Solve Algebraically	Write and solve an equation that uses the concept.
Symbols	Write or use the symbols that pertain to the concept.
Write About It	Write a definition or description in your own words.
Words	Write the words that pertain to the concept.

Meet Foldables Author Dinah Zike

Dinah Zike is known for designing hands-on manipulatives that are used nationally and internationally by teachers and parents. Dinah is an explosion of energy and ideas. Her excitement and joy for learning inspires everyone she touches.

FL2 **Foldables Study Organizers**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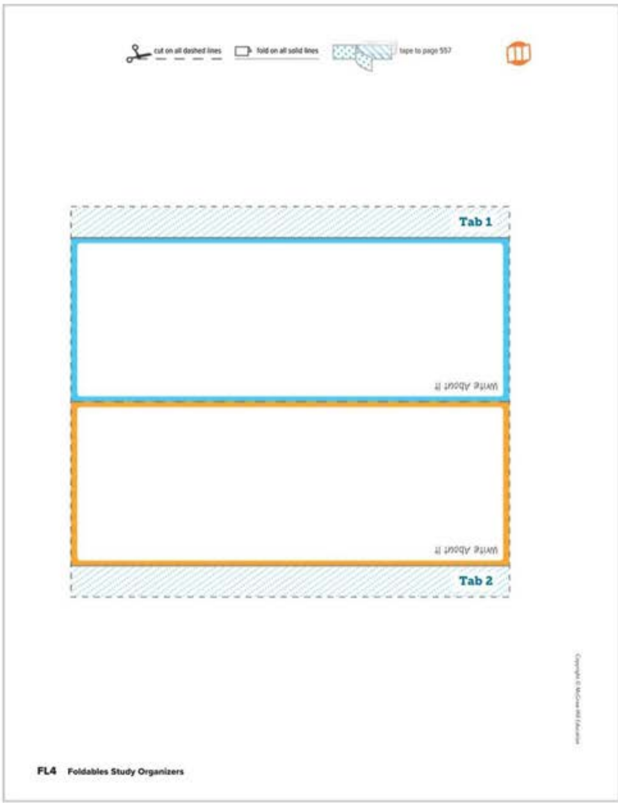
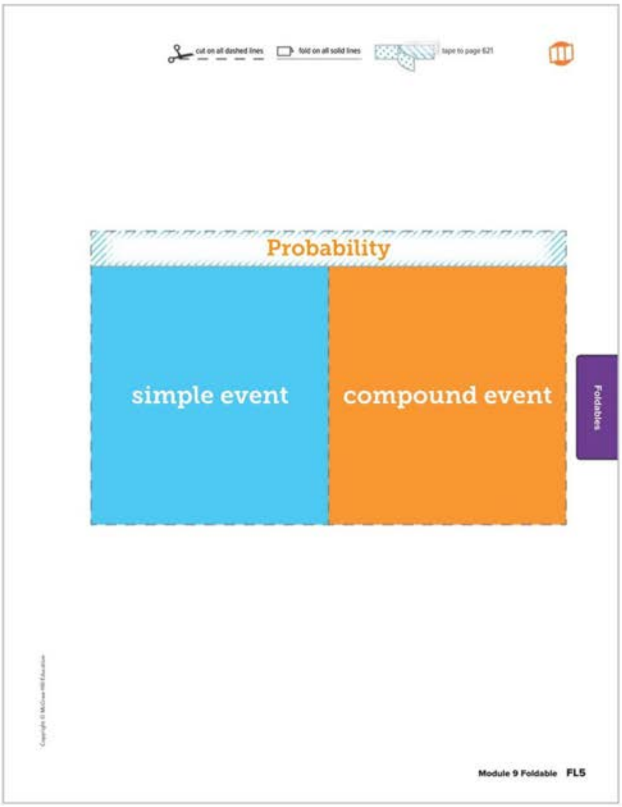
fold on all solid lines
tape to page 557
proportional linear relationships
nonproportional linear relationships
Foldables
Module 8 Foldable FL3

Foldables


cut on all dashed lines
fold on all solid lines
Tab 1
Tab 2
FL4 Foldables Study Organizers



cut on all dashed lines
fold on all solid lines
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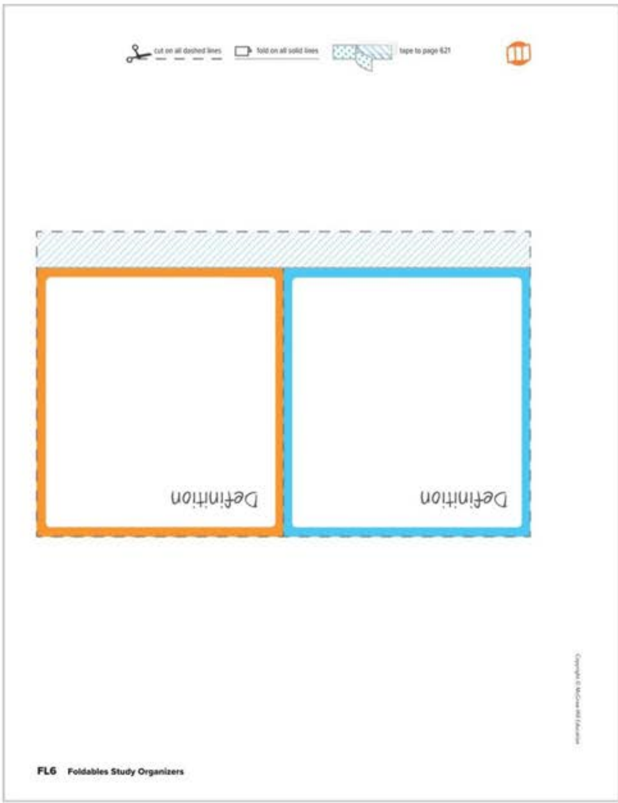
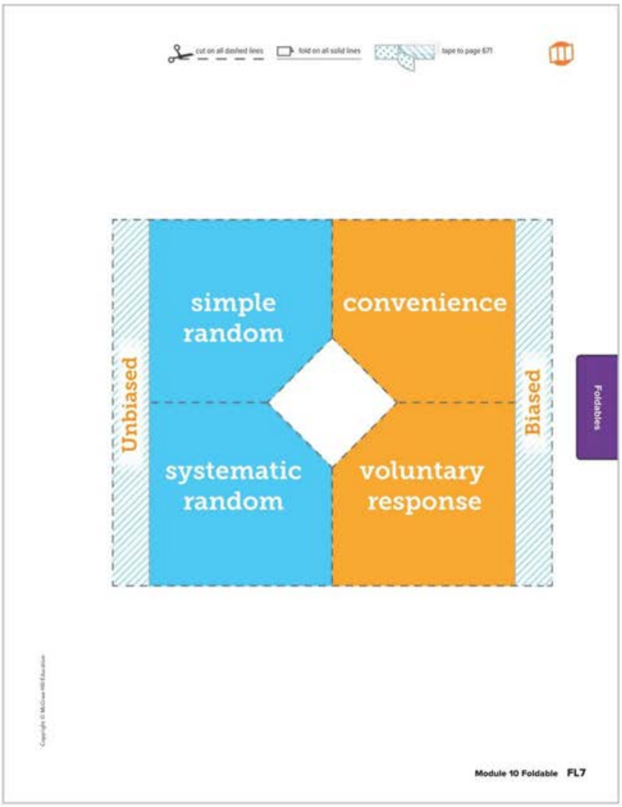
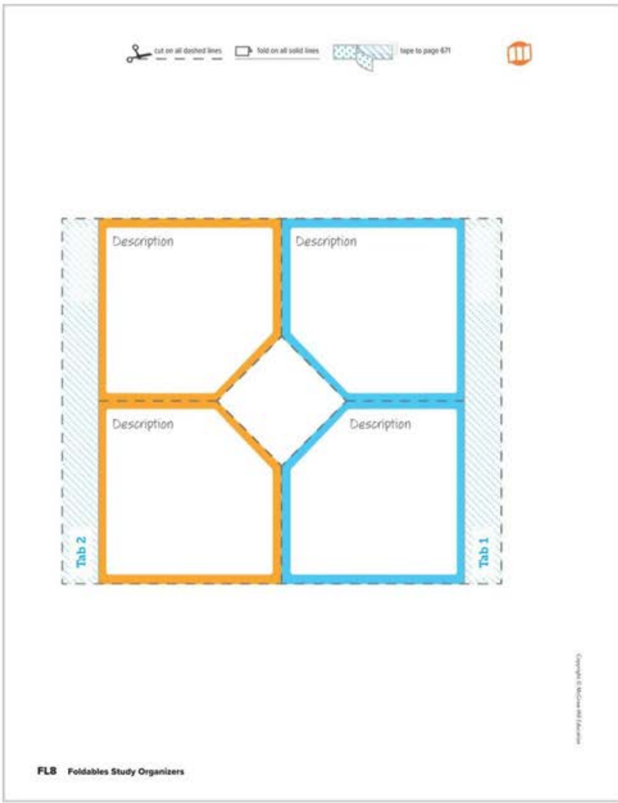
Probability
simple event
compound event
Foldables
Module 9 Foldable FL5

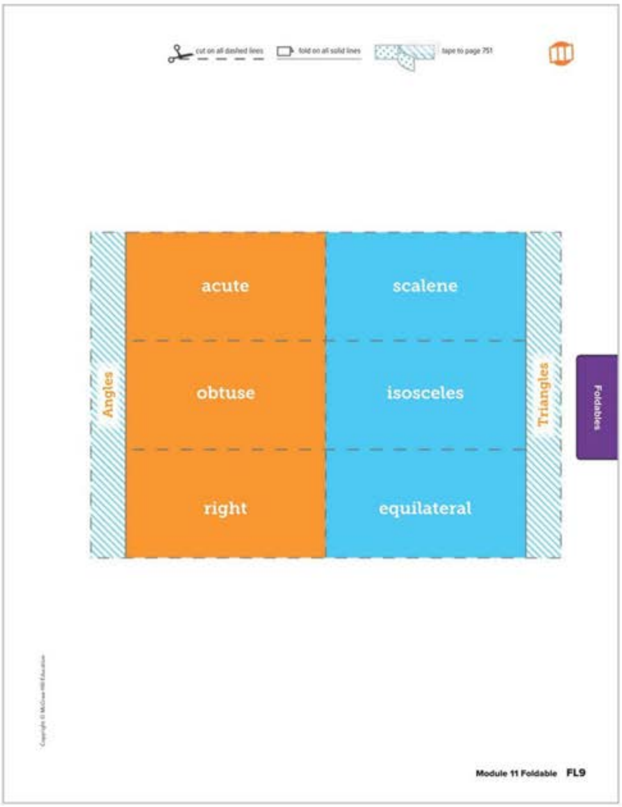
Foldables


cut on all dashed lines
fold on all solid lines
tape to page 621
Definition
Definition
FL6 Foldables Study Organizers


simple random
convenience
Unbiased
Biased
systematic random
voluntary response
Foldables
Module 10 Foldable FL7

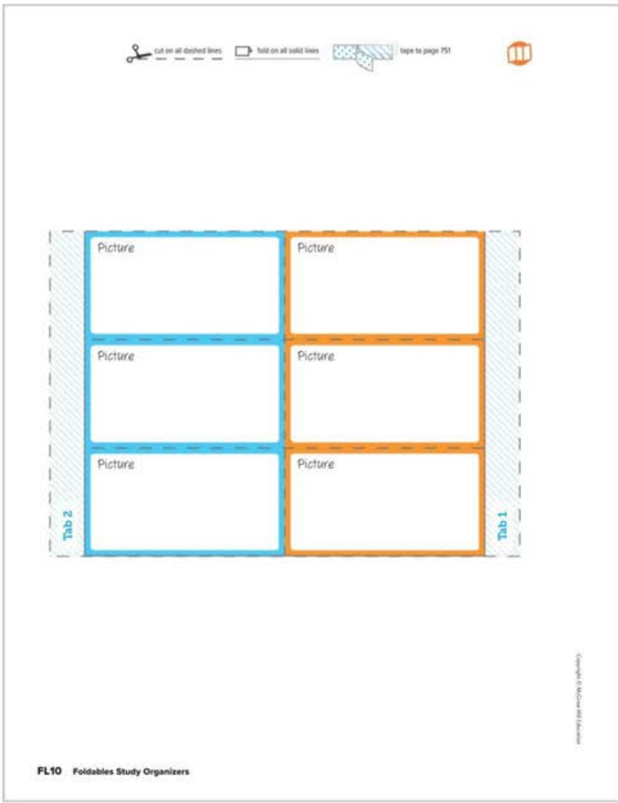
Foldables

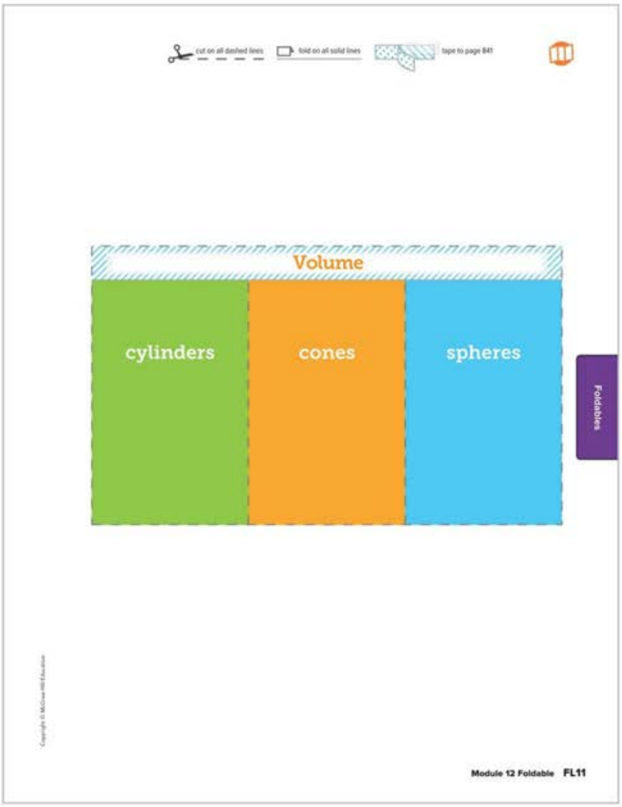

Description
Description
Description
Description
Tab 2
Tab 1
FL8 Foldables Study Organizers



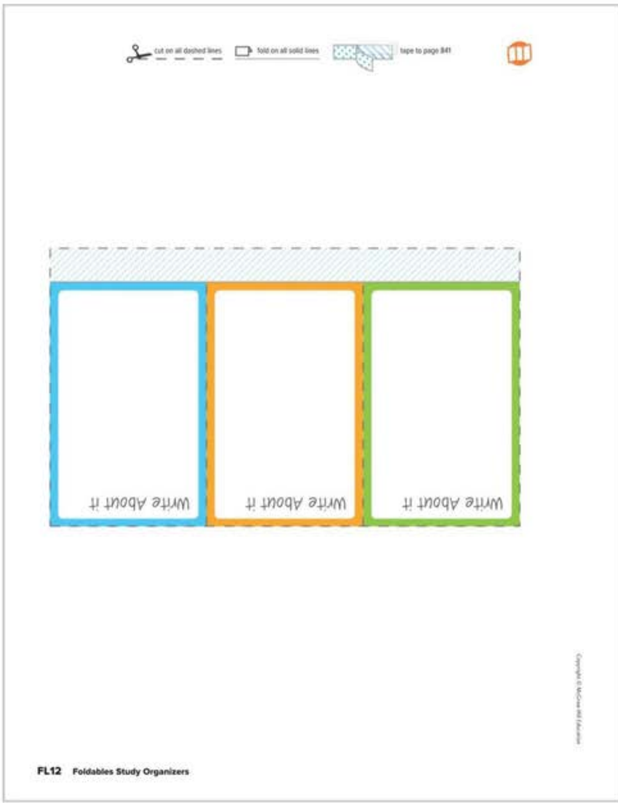
Angles
acute
obtuse
right
scalene
isosceles
equilateral
Triangles
Foldables
Module 11 Foldable FL9

Foldables




Volume
cylinders
cones
spheres
Foldables
Module 12 Foldable FL11

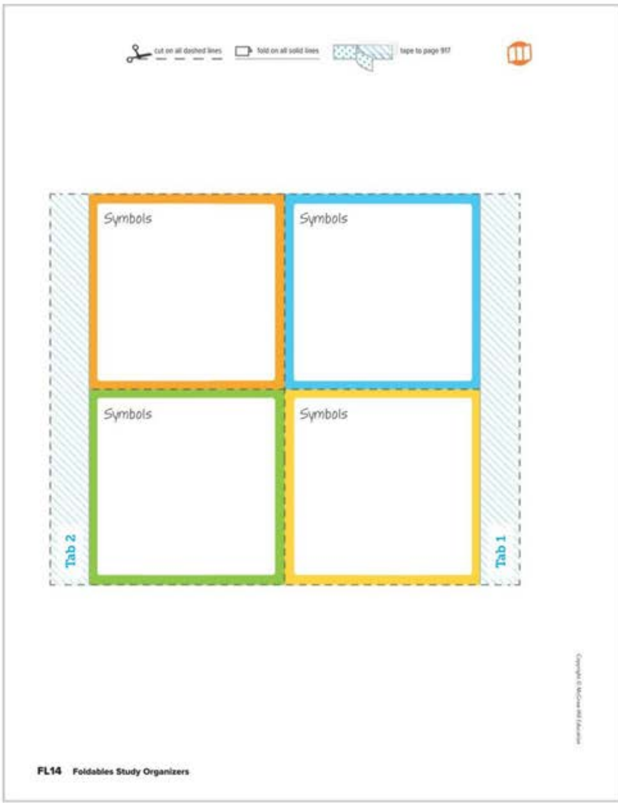
Foldables



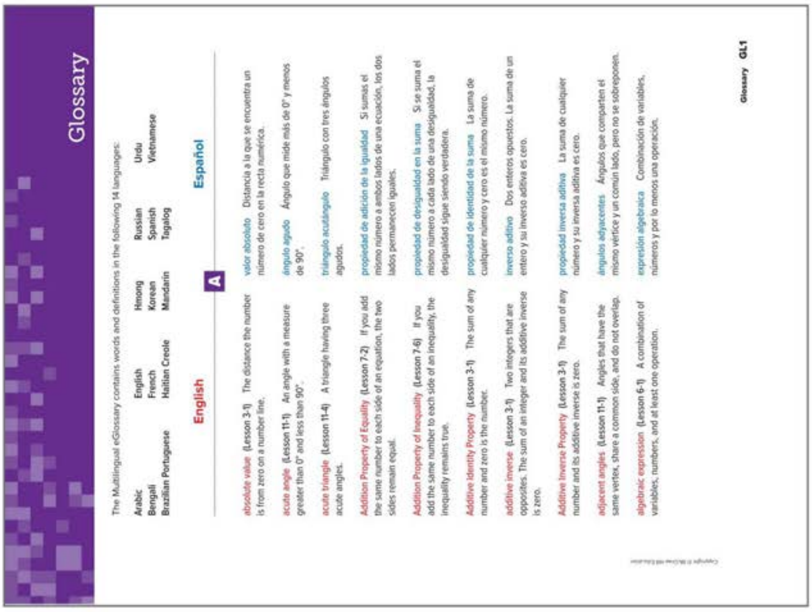
cut on all dashed lines
fold on all solid lines
tape to page 841
Write About it
Write About it
Write About it
FL12 Foldables Study Organizers


cut on all dashed lines
fold on all solid lines
Transformations
translation
reflection
rotation
dilation
Foldables
Module 13 Foldable FL13

Foldables


cut on all dashed lines
fold on all solid lines
tape to page 917
Symbols
Symbols
Symbols
Symbols
Tab 2
Tab 1
FL14 Foldables Study Organizers

Glossary



Glossary

The Multilingual eGlossary contains words and definitions in the following 14 languages:

Arabic	English	Hmong	Russian	Urdu
Bengali	French	Korean	Spanish	Vietnamese
Brazilian Portuguese	Haitian Creole	Mandarin	Tagalog	

English	Español
A	
absolute value (Lesson 3-1) The distance the number is from zero on a number line.	**valor absoluto** Distancia a la que se encuentra un número de cero en la recta numérica.
acute angle (Lesson 11-1) An angle with a measure greater than 0° and less than 90°.	**ángulo agudo** Ángulo que mide más de 0° y menos de 90°.
acute triangle (Lesson 11-4) A triangle having three acute angles.	**triángulo acutángulo** Triángulo con tres ángulos agudos.
Addition Property of Equality (Lesson 7-2) If you add the same number to each side of an equation, the two sides remain equal.	**propiedad de adición de la igualdad** Si sumas el mismo número a ambos lados de una ecuación, los dos lados permanecen iguales.
Addition Property of Inequality (Lesson 7-6) If you add the same number to each side of an inequality, the inequality remains true.	**propiedad de desigualdad en la suma** Si se suma el mismo número a cada lado de una desigualdad, la desigualdad sigue siendo verdadera.
Additive Identity Property (Lesson 3-1) The sum of any number and zero is the number.	**propiedad de identidad de la suma** La suma de cualquier número y cero es el mismo número.
additive inverse (Lesson 3-1) Two integers that are opposites. The sum of an integer and its additive inverse is zero.	**inverso aditivo** Dos enteros opuestos. La suma de un entero y su inverso aditivo es cero.
Additive Inverse Property (Lesson 3-1) The sum of any number and its additive inverse is zero.	**propiedad inversa aditiva** La suma de cualquier número y su inversa aditiva es cero.
adjacent angles (Lesson 11-1) Angles that have the same vertex, share a common side, and do not overlap.	**ángulos adyacentes** Ángulos que comparten el mismo vértice y un común lado, pero no se sobreponen.
algebraic expression (Lesson 6-1) A combination of variables, numbers, and at least one operation.	**expresión algebraica** Combinación de variables, números y por lo menos una operación.

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Glossary GL1

alternate exterior angles (Lesson 11-3) Exterior angles that lie on opposite sides of the transversal.

ángulos alternos externos Ángulos externos que se encuentran en lados opuestos de la transversal.

alternate interior angles (Lesson 11-3) Interior angles that lie on opposite sides of the transversal.

ángulos alternos internos Ángulos internos que se encuentran en lados opuestos de la transversal.

amount of error (Lesson 2-4) The positive difference between the estimate and the actual amount.

cantidad de error La diferencia positiva entre la estimación y la cantidad real.

angle (Lesson 11-1) Two rays with a common endpoint form an angle. The rays and vertex are used to name the angle.

ángulo Dos rayos con un extremo común forman un ángulo. Los rayos y el vértice se usan para nombrar el ángulo.

angle of rotation (Lesson 13-3) The degree measure of the angle through which a figure is rotated.

ángulo de rotación Medida en grados del ángulo sobre el cual se rota una figura.

area (Lesson 12-2) The measure of the interior surface of a two-dimensional figure.

área La medida de la superficie interior de una figura bidimensional.

asymmetric distribution (Lesson 10-4) A distribution in which the shape of the graph on one side of the center is very different than the other side, or it has outliers that might affect the average.

distribución asimétrica Una distribución en la que la forma del gráfico en un lado del centro es muy diferente del otro lado, o tiene valores atípicos que pueden afectar al promedio.

B

bar notation (Lesson 3-6) In repeating decimals, the line or bar placed over the digits that repeat. For example, $2.\overline{63}$ indicates that the digits 63 repeat.

notación de barra Línea o barra que se coloca sobre los dígitos que se repiten en decimales periódicos. Por ejemplo, $2.\overline{63}$ indica que los dígitos 63 se repiten.

base (Lesson 4-1) In a power, the number that is the common factor. In 10^3, the base is 10. That is, $10^3 = 10 \cdot 10 \cdot 10$.

base En una potencia, el número que es el factor común. En 10^3, la base es 10. Es decir, $10^3 = 10 \cdot 10 \cdot 10$.

base (Lesson 11-7) One of the two parallel congruent faces of a prism.

base Una de las dos caras paralelas congruentes de un prisma.

biased sample (Lesson 10-1) A sample drawn in such a way that one or more parts of the population are favored over others.

muestra sesgada Muestra en que se favorece una o más partes de una población.

box plot (Lesson 10-4) A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

diagrama de caja Un método de mostrar visualmente una distribución de valores usando la mediana, cuartiles y extremos del conjunto de datos. Una caja muestra el 50% del medio de los datos.

C

center (Lesson 12-1) The point from which all points on a circle are the same distance.

centro El punto desde el cual todos los puntos en una circunferencia están a la misma distancia.

center of dilation (Lesson 13-4) The center point from which dilations are performed.

centro de la homotecia Punto fijo en torno al cual se realizan las homotecias.

center of rotation (Lesson 13-3) A fixed point around which shapes move in a circular motion to a new position.

centro de rotación Punto fijo alrededor del cual se giran las figuras en movimiento circular alrededor de un punto fijo.

circle (Lesson 12-1) The set of all points in a plane that are the same distance from a given point called the center.

círculo Conjunto de todos los puntos de un plano que están a la misma distancia de un punto dado denominado centro.

circumference (Lesson 12-1) The distance around a circle.

circunferencia Distancia en torno a un círculo.

coefficient (Lesson 6-1) The numerical factor of a term that contains a variable.

coeficiente El factor numérico de un término que contiene una variable.

commission (Lesson 2-5) A payment equal to a percent of the amount of goods or services that an employee sells for the company.

comisión Un pago igual a un porcentaje de la cantidad de bienes o servicios que un empleado vende para la empresa.

common denominator (Lesson 3-6) A common multiple of the denominators of two or more fractions. 24 is a common denominator for $\frac{1}{3}$, $\frac{5}{8}$, and $\frac{3}{4}$ because 24 is the LCM of 3, 8, and 4.

común denominador El múltiplo común de los denominadores de dos o más fracciones. 24 es un denominador común para $\frac{1}{3}$, $\frac{5}{8}$, y $\frac{3}{4}$ porque 24 es el mcm de 3, 8 y 4.

complementary angles (Lesson 11-2) Two angles are complementary if the sum of their measures is 90°.

ángulos complementarios Dos ángulos son complementarios si la suma de sus medidas es 90°.

complementary events (Lesson 9-3) Two events in which either one or the other must happen, but they cannot happen at the same time. The sum of the probability of an event and its complement is 1 or 100%.

eventos complementarios Dos eventos en los cuales uno o el otro debe suceder, pero no pueden ocurrir al mismo tiempo. La suma de la probabilidad de un evento y su complemento es 1 o 100%.

complex fraction (Lesson 1-1) A fraction $\frac{A}{B}$ where A and/or B are fractions and B does not equal zero.

fracción compleja Una fracción $\frac{A}{B}$ en la cual A y/o B son fracciones y B no es igual a cero.

composite figure (Lesson 12-3) A figure that is made up of two or more figures.

figura compuesta Figura formada por dos o más figuras.

composite solid (Lesson 12-9) An object made up of more than one solid.

sólido complejo Cuerpo compuesto de más de un sólido.

composition of transformations (Lesson 13-5) The resulting transformation when a transformation is applied to a figure and then another transformation is applied to its image.

composición de transformaciones Transformación que resulta cuando se aplica una transformación a una figura y luego se le aplica otra transformación a su imagen.

compound event (Lesson 9-5) An event consisting of two or more simple events.

evento compuesto Un evento que consiste en dos o más eventos simples.

cone (Lesson 11-7) A three-dimensional figure with one circular base connected by a curved surface to a single point.

cono Una figura tridimensional con una base circular conectada por una superficie curva para un solo punto.

congruent (Lesson 13-5) Having the same measure; if one image can be obtained from another by a sequence of rotations, reflections, or translations.

congruente Que tienen la misma medida; si una imagen puede obtenerse de otra por una secuencia de rotaciones, reflexiones o traslaciones.

congruent angles (Lesson 11-1) Angles that have the same measure.

ángulos congruentes Ángulos que tienen la misma medida.

congruent figures (Lesson 11-4) Figures that have the same size and same shape and corresponding sides and angles with equal measure.

figuras congruentes Figuras que tienen el mismo tamaño y la misma forma y los lados y los ángulos correspondientes tienen igual medida.

congruent segments (Lesson 11-4) Sides with the same length.

segmentos congruentes Lados con la misma longitud.

constant (Lesson 6-1) A term that does not contain a variable.

constante Término que no contiene ninguna variable.

constant of proportionality (Lesson 1-3) A constant ratio or unit rate of two variable quantities. It is also called the constant of variation.

constante de proporcionalidad Una razón constante o tasa por unidad de dos cantidades variables. También se llama constante de variación.

constant of variation (Lesson 1-3) The constant ratio in a direct variation. It is also called the constant of proportionality.

constante de variación Una razón constante o tasa por unidad de dos cantidades variables. También se llama constante de proporcionalidad.

constant rate of change (Lesson 1-3) The rate of change between any two points in a linear relationship is the same or *constant*.

tasa constante de cambio La tasa de cambio entre dos puntos cualesquiera en una relación lineal permanece igual o *constante*.

convenience sample (Lesson 10-1) A sample which consists of members of a population that are easily accessed.

muestra de conveniencia Muestra que incluye miembros de una población fácilmente accesibles.

corresponding angles (Lesson 11-3) Angles that are in the same position on two parallel lines in relation to a transversal.

ángulos correspondientes Ángulos que están en la misma posición sobre dos rectas paralelas en relación con la transversal.

corresponding parts (Lesson 8-3) Parts of congruent or similar figures that are in the same relative position.

partes correspondientes Partes de figuras congruentes o similares que están en la misma posición relativa.

counterexample (Lesson 5-3) A statement or example that shows a conjecture is false.

contraejemplo Ejemplo o enunciado que demuestra que una conjetura es falsa.

cross section (Lesson 11-7) The intersection of a solid and a plane.

sección transversal Intersección de un sólido con un plano.

cube root (Lesson 4-1) One of three equal factors of a number. If $a^3 = b$, then a is the cube root of b. The cube root of 64 is 4 since $4^3 = 64$.

raíz cúbica Uno de tres factores iguales de un número. Si $a^3 = b$, entonces a es la raíz cúbica de b. La raíz cúbica de 64 es 4, dado que $4^3 = 64$.

cylinder (Lesson 11-7) A three-dimensional figure with two parallel congruent circular bases connected by a curved surface.

cilindro Una figura tridimensional con dos paralelas congruentes circulares bases conectados por una superficie curva.

D

degrees (Lesson 11-1) The most common unit of measure for angles. If a circle were divided into 360 equal-sized parts, each part would have an angle measure of 1 degree.

grados La unidad más común para medir ángulos. Si un círculo se divide en 360 partes iguales, cada parte tiene una medida angular de 1 grado.

diameter (Lesson 12-1) The distance across a circle through its center.

diámetro Segmento que pasa por el centro de un círculo y lo divide en dos partes iguales.

dilation (Lesson 13-4) A transformation that enlarges or reduces a figure by a scale factor.

homotecia Transformación que produce la ampliación o reducción de una imagen por un factor de escala.

dimensional analysis (Lesson 1-2) The process of including units of measurement when you compute.

análisis dimensional Proceso que incluye las unidades de medida al hacer cálculos.

direct variation (Lesson 8-4) A relationship between two variable quantities with a constant ratio. A proportional linear relationship.

variación directa Relación entre dos cantidades variables con una razón constante. Una relación lineal proporcional.

discount (Lesson 2-8) The amount by which the regular price of an item is reduced.

descuento Cantidad que se le rebaja al precio regular de un artículo.

distribution (Lesson 10-4) The shape of a graph of data.

distribución La forma de un gráfico de datos.

Distributive Property (Lesson 3-3) To multiply a sum by a number, multiply each addend of the sum by the number outside the parentheses. For any numbers a, b, and c, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

Example: $2(5 + 3) = (2 \cdot 5) + (2 \cdot 3)$ and $2(5 - 3) = (2 \cdot 5) - (2 \cdot 3)$

propiedad distributiva Para multiplicar una suma por un número, multiplíquese cada sumando de la suma por el número que está fuera del paréntesis. Sean cuales fuere los números a, b, y c, $a(b + c) = ab + ac$ y $a(b - c) = ab - ac$.

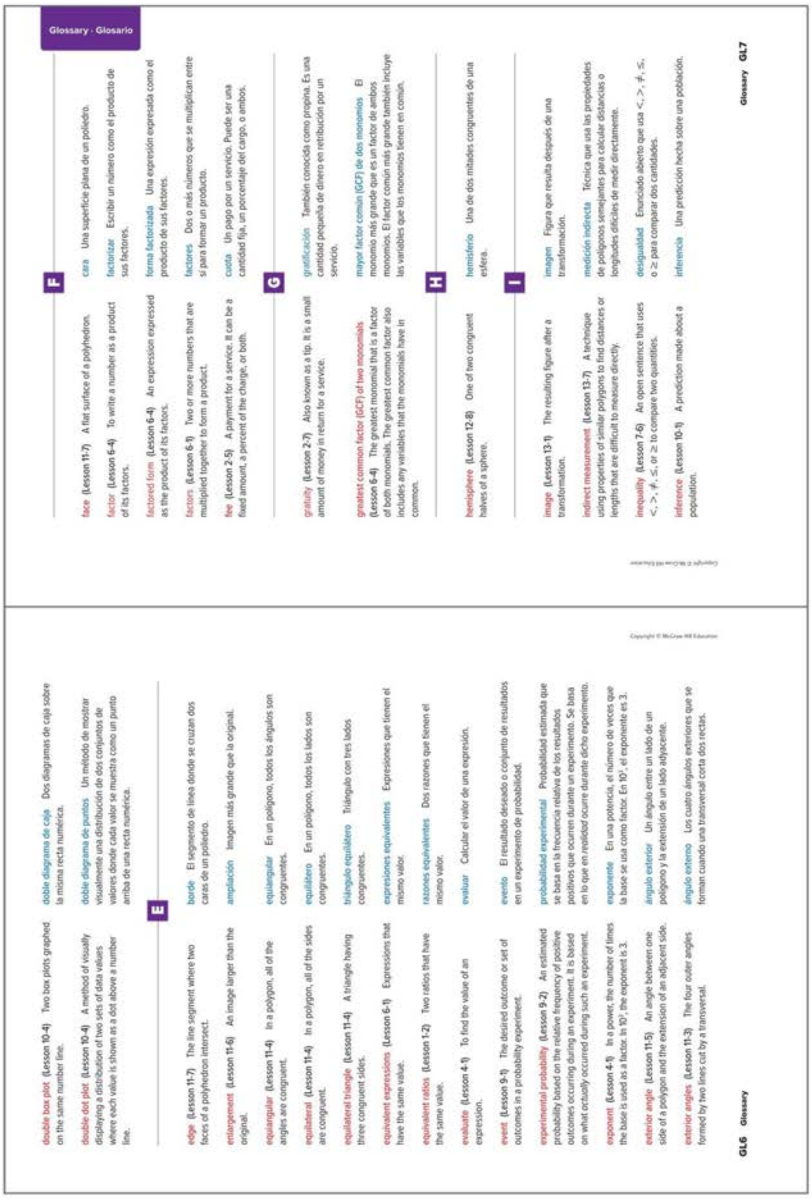
Ejemplo: $2(5 + 3) = (2 \cdot 5) + (2 \cdot 3)$ y $2(5 - 3) = (2 \cdot 5) - (2 \cdot 3)$

Division Property of Equality (Lesson 7-1) If you divide each side of an equation by the same nonzero number, the two sides remain equal.

propiedad de igualdad de la división Si divides ambos lados de una ecuación entre el mismo número no nulo, los lados permanecen iguales.

Division Property of Inequality (Lesson 7-7) When you divide each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

propiedad de desigualdad en la división Cuando se divide cada lado de una desigualdad entre un número negativo, el símbolo de desigualdad debe invertirse para que la desigualdad siga siendo verdadera.



Glossary · Glosario

double box plot (Lesson 10-4) Two box plots graphed on the same number line.

doble diagrama de caja Dos diagramas de caja sobre la misma recta numérica.

double dot plot (Lesson 10-4) A method of visually displaying a distribution of two sets of data values where each value is shown as a dot above a number line.

doble diagrama de puntos Un método de mostrar visualmente una distribución de dos conjuntos de valores donde cada valor se muestra como un punto arriba de una recta numérica.

E

edge (Lesson 11-7) The line segment where two faces of a polyhedron intersect.

borde El segmento de línea donde se cruzan dos caras de un poliedro.

enlargement (Lesson 11-6) An image larger than the original.

ampliación Imagen más grande que la original.

equiangular (Lesson 11-4) In a polygon, all of the angles are congruent.

equiangular En un polígono, todos los ángulos son congruentes.

equilateral (Lesson 11-4) In a polygon, all of the sides are congruent.

equilátero En un polígono, todos los lados son congruentes.

equilateral triangle (Lesson 11-4) A triangle having three congruent sides.

triángulo equilátero Triángulo con tres lados congruentes.

equivalent expressions (Lesson 6-1) Expressions that have the same value.

expresiones equivalentes Expresiones que tienen el mismo valor.

equivalent ratios (Lesson 1-2) Two ratios that have the same value.

razones equivalentes Dos razones que tienen el mismo valor.

evaluate (Lesson 4-1) To find the value of an expression.

evaluar Calcular el valor de una expresión.

event (Lesson 9-1) The desired outcome or set of outcomes in a probability experiment.

evento El resultado deseado o conjunto de resultados en un experimento de probabilidad.

experimental probability (Lesson 9-2) An estimated probability based on the relative frequency of positive outcomes occurring during an experiment. It is based on what *actually* occurred during such an experiment.

probabilidad experimental Probabilidad estimada que se basa en la frecuencia relativa de los resultados positivos que ocurren durante un experimento. Se basa en lo que en *realidad* ocurre durante dicho experimento.

exponent (Lesson 4-1) In a power, the number of times the base is used as a factor. In 10^3, the exponent is 3.

exponente En una potencia, el número de veces que la base se usa como factor. En 10^3, el exponente es 3.

exterior angle (Lesson 11-5) An angle between one side of a polygon and the extension of an adjacent side.

ángulo exterior Un ángulo entre un lado de un polígono y la extensión de un lado adyacente.

exterior angles (Lesson 11-3) The four outer angles formed by two lines cut by a transversal.

ángulo externo Los cuatro ángulos exteriores que se forman cuando una transversal corta dos rectas.

GL6 Glossary

F

face (Lesson 11-7) A flat surface of a polyhedron.

cara Una superficie plana de un poliedro.

factor (Lesson 6-4) To write a number as a product of its factors.

factorizar Escribir un número como el producto de sus factores.

factored form (Lesson 6-4) An expression expressed as the product of its factors.

forma factorizada Una expresión expresada como el producto de sus factores.

factors (Lesson 6-1) Two or more numbers that are multiplied together to form a product.

factores Dos o más números que se multiplican entre sí para formar un producto.

fee (Lesson 2-5) A payment for a service. It can be a fixed amount, a percent of the charge, or both.

cuota Un pago por un servicio. Puede ser una cantidad fija, un porcentaje del cargo, o ambos.

G

gratuity (Lesson 2-7) Also known as a tip. It is a small amount of money in return for a service.

gratificación También conocida como propina. Es una cantidad pequeña de dinero en retribución por un servicio.

greatest common factor (GCF) of two monomials (Lesson 6-4) The greatest monomial that is a factor of both monomials. The greatest common factor also includes any variables that the monomials have in common.

mayor factor común (GCF) de dos monomios El monomio más grande que es un factor de ambos monomios. El factor común más grande también incluye las variables que los monomios tienen en común.

H

hemisphere (Lesson 12-8) One of two congruent halves of a sphere.

hemisferio Una de dos mitades congruentes de una esfera.

I

image (Lesson 13-1) The resulting figure after a transformation.

imagen Figura que resulta después de una transformación.

indirect measurement (Lesson 13-7) A technique using properties of similar polygons to find distances or lengths that are difficult to measure directly.

medición indirecta Técnica que usa las propiedades de polígonos semejantes para calcular distancias o longitudes difíciles de medir directamente.

inequality (Lesson 7-6) An open sentence that uses $<$, $>$, $\neq$, $\leq$, or $\geq$ to compare two quantities.

desigualdad Enunciado abierto que usa $<$, $>$, $\neq$, $\leq$, o $\geq$ para comparar dos cantidades.

inference (Lesson 10-1) A prediction made about a population.

inferencia Una predicción hecha sobre una población.

Glossary GL7

initial value (Lesson 8-5) The starting value in a real-world situation in which an equation can be written. The y-intercept of a linear function.

valor inicial El valor inicial en una situación real en la que se puede escribir una ecuación. La intersección y de una función lineal.

integer (Lesson 3-1) Any number from the set {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}, where ... means continues without end.

entero Cualquier número del conjunto {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}, donde ... significa que continúa sin fin.

interest (Lesson 2-9) The amount paid or earned for the use of the principal.

interés La cantidad pagada o ganada por el uso del principal.

interior angle (Lesson 11-5) An angle inside a polygon.

ángulo interno Ángulo dentro de un polígono.

interior angles (Lesson 11-3) The four inside angles formed by two lines cut by a transversal.

ángulo interno Los cuatro ángulos internos formados por dos rectas intersecadas por una transversal.

interquartile range (IQR) (Lesson 10-4) A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set.

rango intercuartil (RIQ) El rango intercuartil, una medida de la variacion en un conjunto de datos numéricos, es la distancia entre el primer y el tercer cuartil del conjunto de datos

invalid inference (Lesson 10-1) An inference that is based on a biased sample or makes a conclusion not supported by the results of the sample.

inferencia inválida Una inferencia que se basa en una muestra sesgada o hace una conclusión no apoyada por los resultados de la muestra.

inverse operations (Lesson 5-1) Pairs of operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

peraciones inversas Pares de operaciones que se anulan mutuamente. La adición y la sustracción son operaciones inversas. La multiplicación y la división son operaciones inversas.

irrational number (Lesson 5-2) A number that cannot be expressed as the ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

números irracionales Número que no se puede expresar como la proporción $\frac{a}{b}$, donde a y b son enteros y $b \neq 0$.

isosceles triangle (Lesson 11-4) A triangle having at least two congruent sides.

triángulo isósceles Triángulo que tiene por lo menos dos lados congruentes.

L

lateral face (Lesson 12-5) In a polyhedron, a face that is not a base.

cara lateral En un poliedro, las caras que no forman las bases.

lateral surface area (Lesson 12-5) The sum of the areas of all of the lateral faces of a solid.

área de superficie lateral Suma de las áreas de todas las caras de un sólido.

least common denominator (LCD) (Lesson 3-6) The least common multiple of the denominators of two or more fractions. You can use the LCD to compare fractions.

mínimo común denominador (mcd) El menor de los múltiplos de los denominadores de dos o más fracciones. Puedes usar el mínimo común denominador para comparar fracciones.

like terms (Lesson 6-1) Terms that contain the same variable(s) raised to the same power. Example: 5x and 6x are like terms.

términos semejante Términos que contienen las mismas variable(s) elevadas a la misma potencia. Ejemplo: 5x y 6x son términos semejante.

likelihood (Lesson 9-1) The chance of an event occurring.

probabilidad La probabilidad de que ocurra un evento.

linear (Lesson 8-1) To fall in a straight line.

lineal Que cae en una línea recta.

linear equation (Lesson 8-1) An equation with a graph that is a straight line.

ecuación lineal Ecuación cuya gráfica es una recta.

linear expression (Lesson 6-2) An algebraic expression in which the variable is raised to the first power, and variables are neither multiplied nor divided.

expresión lineal Expresión algebraica en la cual la variable se eleva a la primera potencia.

linear relationship (Lesson 1-4) A relationship for which the graph is a straight line.

relación lineal Una relación para la cual la gráfica es una línea recta.

line of reflection (Lesson 13-2) The line over which a figure is reflected in a transformation.

línea de reflexión Línea a través de la cual se refleja una figura en una transformación.

line segment (Lesson 11-5) Part of a line containing two endpoints and all of the points between them.

segmento de línea Parte de una línea que contiene dos extremos y todos los puntos entre ellos.

M

markdown (Lesson 2-8) An amount by which the regular price of an item is reduced.

rebaja Una cantidad por la cual el precio regular de un artículo se reduce.

markup (Lesson 2-7) The amount the price of an item is increased above the price the store paid for the item.

margen de utilidad Cantidad de aumento en el precio de un artículo por encima del precio que paga la tienda por dicho artículo.

mean (Lesson 10-4) The sum of the data divided by the number of items in the data set.

media La suma de los datos dividida entre el número total de artículos en el conjunto de datos.

mean absolute deviation (MAD) (Lesson 10-4) A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.

desviación media absoluta Una medida de variación en un conjunto de datos numéricos que se calcula sumando las distancias entre el valor de cada dato y la media, y luego dividiendo entre el número de valores.

measures of center (Lesson 10-4) Numbers that are used to describe the center of a set of data. These measures include the mean, median, and mode.

medidas del centro Números que se usan para describir el centro de un conjunto de datos. Estas medidas incluyen la media, la mediana y la moda.

measures of variation (Lesson 10-4) A measure used to describe the distribution of data.

medidas de variación Medida usada para describir la distribución de los datos.

median (Lesson 10-4) A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values.

mediana Una medida del centro en un conjunto de dados numéricos. La mediana de una lista de valores es el valor que aparace en el centro de una versión ordenada de la lista, o la media de dos valores centrales si la lista contiene un número par de valores.

monomial (Lesson 4-2) A number, variable, or product of a number and one or more variables.

monomio Número, variable o producto de un número y una o más variables.

Multiplication Property of Equality (Lesson 7-1) If you multiply each side of an equation by the same nonzero number, the two sides remain equal.

propiedad de multiplicación de la igualdad Si multiplicas ambos lados de una ecuación por el mismo número no nulo, lo lados permanecen iguales.

Multiplication Property of Inequality (Lesson 7-7) When you multiply each side of an inequality by a negative number, the inequality symbol must be reversed for the inequality to remain true.

propiedad de desigualdad en la multiplicación Cuando se multiplica cada lado de una desigualdad por un número negativo, el símbolo de desigualdad debe invertirse para que la desigualdad siga siendo verdadera.

Multiplicative Identity Property (Lesson 3-3) The product of any number and one is the number.

propiedad de identidad de la multiplicación El producto de cualquier número y uno es el mismo número.

multiplicative inverse (Lesson 3-8) Two numbers with a product of 1. For example, the multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$.

inverso multiplicativo Dos números cuyo producto es 1. Por ejemplo, el inverso multiplicativo de $\frac{2}{3}$ es $\frac{3}{2}$.

Multiplicative Property of Zero (Lesson 3-3) The product of any number and zero is zero.

propiedad del cero en la multiplicación El producto de cualquier número y cero es cero.

N

natural numbers (Lesson 5-1) The set of numbers used for counting.

números naturales El conjunto de números utilizado para el recuento.

negative exponent (Lesson 4-4) The result of repeated division used to represent very small numbers.

exponente negative El resultado de la división repetida se utiliza para representar números muy pequeños.

negative integer (Lesson 3-1) An integer that is less than zero. Negative integers are written with a − sign.

entero negativo Número menor que cero. Se escriben con el signo −.

net (Lesson 12-5) A two-dimensional figure that can be used to build a three-dimensional figure.

red Figura bidimensional que sirve para hacer una figura tridimensional.

nonproportional (Lesson 1-3) The relationship between two ratios with a rate or ratio that is not constant.

no proporcional Relación entre dos razones cuya tasa o razón no es constante.

numerical expression (Lesson 6-1) A combination of numbers and operations.

expresión numérica Combinación de números y operaciones.

O

obtuse angle (Lesson 11-1) Any angle that measures greater than 90° but less than 180°.

ángulo obtuso Cualquier ángulo que mide más de 90° pero menos de 180°.

obtuse triangle (Lesson 11-4) A triangle having one obtuse angle.

triángulo obtusángulo Triángulo que tiene un ángulo obtuso.

opposites (Lesson 3-1) Two integers are opposites if they are represented on the number line by points that are the same distance from zero, but on opposite sides of zero. The sum of two opposites is zero.

opuestos Dos enteros son opuestos si, en la recta numérica, están representados por puntos que equidistan de cero, pero en direcciones opuestas. La suma de dos opuestos es cero.

order of operations (Lesson 3-9) The rules to follow when more than one operation is used in a numerical expression.

1. Evaluate the expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

orden de las operaciones Reglas a seguir cuando se usa más de una operación en una expresión numérica.

1. Primero, evalúa las expresiones dentro de los símbolos de agrupación.
2. Evalúa todas las potencias.
3. Multiplica y divide en orden de izquierda a derecha.
4. Suma y resta en orden de izquierda a derecha.

ordered pair (Lesson 8-1) A pair of numbers used to locate a point in the coordinate plane. The ordered pair is written in this form: (*x*-coordinate, *y*-coordinate).

par ordenado Par de números que se utiliza para ubicar un punto en un plano de coordenadas. Se escribe de la siguiente forma: (coordenada *x*, coordenada *y*).

origin (Lesson 8-1) The point of intersection of the *x*-axis and *y*-axis in a coordinate plane.

origen Punto en que el eje *x* y el eje *y* se intersecan en un plano de coordenadas.

outcome (Lesson 9-1) Any one of the possible results of an action. For example, 4 is an outcome when a number cube is rolled.

resultado Cualquiera de los resultados posibles de una acción. Por ejemplo, 4 puede ser un resultado al lanzar un cubo numerado.

P

parallel lines (Lesson 11-3) Lines in the same plane that never intersect or cross. The symbol || means parallel.

rectas paralelas Rectas que yacen en un mismo plano y que no se intersecan. El símbolo || significa paralela a.

parallelogram (Lesson 12-3) A quadrilateral with opposite sides parallel and opposite sides congruent.

paralelogramo Cuadrilátero cuyos lados opuestos son paralelos y congruentes.

percent error (Lesson 2-4) A ratio that compares the inaccuracy of an estimate (amount of error) to the actual amount.

porcentaje de error Una razón que compara la inexactitud de una estimación (cantidad del error) con la cantidad real.

percent of change (Lesson 2-3) A ratio that compares the change in a quantity to the original amount.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}} \cdot 100$$

porcentaje de cambio Razón que compara el cambio en una cantidad a la cantidad original.

$$\text{porcentaje de cambio} = \frac{\text{cantidad del cambio}}{\text{cantidad original}} \cdot 100$$

percent of decrease (Lesson 2-3) A negative percent of change.

porcentaje de disminución Porcentaje de cambio negativo.

percent of increase (Lesson 2-3) A positive percent of change.

porcentaje de aumento Porcentaje de cambio positivo.

perfect cube (Lesson 5-1) A number whose cube root is an integer. 27 is a perfect cube because its cube root is 3.

cubo perfecto Número cuya raíz cúbica es un número entero. 27 es un cubo perfecto porque su raíz cúbica es 3.

perfect square (Lesson 5-1) A number whose square root is a whole number. 25 is a perfect square because its square root is 5.

cuadrados perfectos Número cuya raíz cuadrada es un número entero. 25 es un cuadrado perfecto porque su raíz cuadrada es 5.

perpendicular lines (Lesson 11-3) Two lines that intersect to form right angles.

rectas perpendiculares Dos rectas que se intersecan formando ángulos rectos.

pi (Lesson 12-1) The ratio of the circumference of a circle to its diameter. The Greek letter π represents this number. The value of pi is 3.1415926.... Approximations for pi are 3.14 and $\frac{22}{7}$.

pi Relación entre la circunferencia de un círculo y su diámetro. La letra griega π representa este número. El valor de pi es 3.1415926.... Las aproximaciones de pi son 3.14 y $\frac{22}{7}$.

plane (Lesson 11-7) A two-dimensional flat surface that extends in all directions.

plano Superficie bidimensional que se extiende en todas direcciones.

polygon (Lesson 12-3) A simple closed figure formed by three or more straight line segments.

polígono Figura cerrada simple formada por tres o más segmentos de recta.

polyhedron (Lesson 11-7) A three-dimensional figure with faces that are polygons.

poliedro Una figura tridimensional con caras que son polígonos.

population (Lesson 10-1) The entire group of items or individuals from which the samples under consideration are taken.

población El grupo total de individuos o de artículos del cual se toman las muestras bajo estudio.

positive integer (Lesson 3-1) An integer that is greater than zero. They are written with or without a + sign.

entero positivo Entero que es mayor que cero; se escribe con o sin el signo +.

power (Lesson 4-1) A product of repeated factors using an exponent and a base. The power 7^3 is read *seven to the third power, or seven cubed.*

potencia Producto de factores repetidos con un exponente y una base. La potencia 7^3 se lee *siete a la tercera potencia o siete al cubo.*

Power of a Power Property (Lesson 4-3) A property that states to find the power of a power, multiply the exponents.

potencia de una propiedad de potencia Una propiedad que declara encontrar el poder de un poder, multiplicar los exponentes.

Power of a Product Property (Lesson 4-3) A property that states to find the power of a product, find the power of each factor and multiply.

potencia de una propiedad de producto Una propiedad que declara encontrar el poder de un producto, encuentra el poder de cada factor y se multiplica.

preimage (Lesson 13-1) The original figure before a transformation.

preimagen Figura original antes de una transformación.

principal (Lesson 2-9) The amount of money deposited or borrowed.

capital Cantidad de dinero que se deposita o se toma prestada.

principal square root (Lesson 5-1) The positive square root of a number.

raíz cuadrada principal La raíz cuadrada positiva de un número.

prism (Lesson 11-7) A polyhedron with two parallel congruent faces called bases.

prisma Un poliedro con dos caras congruentes paralelas llamadas bases.

probability (Lesson 9-2) The chance that some event will happen. It is the ratio of the number of favorable outcomes to the number of possible outcomes.

probabilidad La posibilidad de que suceda un evento. Es la razón del número de resultados favorables al número de resultados posibles.

probability experiment (Lesson 9-2) When you perform an event to find the likelihood of an event.

experimento de probabilidad Cuando realiza un evento para encontrar la probabilidad de un evento.

probability model (Lesson 9-3) A model used to assign probabilities to outcomes of a chance process by examining the nature of the process.

modelo de probabilidad Un modelo usado para asignar probabilidades a resultados de un proceso aleatorio examinando la naturaleza del proceso.

Product of Powers Property (Lesson 4-2) A property that states to multiply powers with the same base, add their exponents.

producto de la propiedad de los poderes Una propiedad que declara multiplicar poderes con la misma base, añade sus exponentes.

properties (Lesson 3-1) Statements that are true for any number or variable.

propiedades Enunciados que son verdaderos para cualquier número o variable.

proportion (Lesson 1-5) An equation stating that two ratios or rates are equivalent.

proporción Ecuación que indica que dos razones o tasas son equivalentes.

proportional (Lesson 1-3) The relationship between two ratios with a constant rate or ratio.

proporcional Relación entre dos razones con una tasa o razón constante.

pyramid (Lesson 11-7) A polyhedron with one base that is a polygon and three or more triangular faces that meet at a common vertex.

pirámide Un poliedro con una base que es un polígono y tres o más caras triangulares que se encuentran en un vértice común.

Q

quadrants (Lesson 8-1) The four sections of the coordinate plane.

cuadrantes Las cuatro secciones del plano de coordenadas.

quadrilateral (Lesson 12-3) A closed figure having four sides and four angles.

cuadrilátero Figura cerrada que tiene cuatro lados y cuatro ángulos.

Quotient of Powers Property (Lesson 4-2) A property that states to divide powers with the same base, subtract their exponents.

propiedad del cociente de poderes Una propiedad que declara dividir poderes con la misma base, resta sus exponentes.

R

radical sign (Lesson 5-1) The symbol used to indicate a positive square root, $\sqrt{}$.

signo radical Símbolo que se usa para indicar una raíz cuadrada no positiva, $\sqrt{}$.

radius (Lesson 12-1) The distance from the center of a circle to any point on the circle.

radio Distancia desde el centro de un círculo hasta cualquiera de sus puntos.

random (Lesson 9-2) Outcomes occur at random if each outcome occurs by chance. For example, rolling a number on a number cube occurs at random.

azar Los resultados ocurren aleatoriamente si cada resultado ocurre por casualidad. Por ejemplo, sacar un número en un cubo numerado ocurre al azar.

rate (Lesson 1-1) A special kind of ratio in which the units are different.

tasa Un tipo especial de relación en el que las unidades son diferentes.

rate of change (Lesson 8-1) A rate that describes how one quantity changes in relation to another quantity.

tasa de cambio Una tasa que describe cómo cambia una cantidad en relación con otra cantidad.

ratio (Lesson 1-1) A comparison between two quantities, in which for every *a* units of one quantity, there are *b* units of another quantity.

razón Una comparación entre dos cantidades, en la que por cada *a* unidades de una cantidad, hay unidades *b* de otra cantidad.

rational numbers (Lesson 3-6) The set of numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Examples: $1 = \frac{1}{1}, \frac{2}{9}, -2.3 = \frac{-23}{10}$

números racionales Conjunto de números que puede escribirse en la forma $\frac{a}{b}$ donde a y b son números enteros y $b \neq 0$.

Ejemplos: $1 = \frac{1}{1}, \frac{2}{9}, -2.3 = \frac{-23}{10}$

real numbers (Lesson 5-2) The set of rational numbers together with the set of irrational numbers.

números reales El conjunto de números racionales junto con el conjunto de números irracionales.

reciprocal (Lesson 3-8) The multiplicative inverse of a number.

recíproco El inverso multiplicativo de un número.

rectangular prism (Lesson 12-4) A prism that has two parallel congruent bases that are rectangles.

prisma rectangular Un prisma con dos bases paralelas congruentes que son rectángulos.

reduction (Lesson 11-6) An image smaller than the original.

reducción Imagen más pequeña que la original.

reflection (Lesson 13-2) A transformation where a figure is flipped over a line. Also called a flip.

reflexión Transformación en la cual una figura se voltea sobre una recta. También se conoce como simetría de espejo.

regular polygon (Lesson 12-3) A polygon that has all sides congruent and all angles congruent.

polígono regular Polígono con todos los lados y todos los ángulos congruentes.

regular pyramid (Lesson 12-5) A pyramid whose base is a regular polygon and in which the segment from the vertex to the center of the base is the altitude.

pirámide regular Pirámide cuya base es un polígono regular y en la cual el segmento desde el vértice hasta el centro de la base es la altura.

relative frequency (Lesson 9-2) A ratio that compares the frequency of each category to the total.

frecuencia relativa Razón que compara la frecuencia de cada categoría al total.

relative frequency graph (Lesson 9-2) A graph used to organize occurrences compared to a total.

gráfico de frecuencia relativa Gráfico utilizado para organizar las ocurrencias en comparación con un total.

relative frequency table (Lesson 9-2) A table used to organize occurrences compared to a total.

tabla de frecuencia relativa Una tabla utilizada para organizar las ocurrencias en comparación con un total.

remote interior angles (Lesson 11-5) The angles of a triangle that are not adjacent to a given exterior angle.

ángulos internos no adyacentes Ángulos de un triángulo que no son adya centes a un ángulo exterior dado.

repeating decimal (Lesson 3-6) A decimal in which 1 or more digits repeat.

decimal periódico Un decimal en el que se repiten 1 o más dígitos.

rhombus (Lesson 12-3) A parallelogram having four congruent sides.

rombo Paralelogramo que tiene cuatro lados congruentes.

right angle (Lesson 11-1) An angle that measures exactly 90°.

ángulo recto Ángulo que mide exactamente 90°.

right triangle (Lesson 11-4) A triangle having one right angle.

triángulo rectángulo Triángulo que tiene un ángulo recto.

rise (Lesson 8-2) The vertical change between any two points on a line.

elevación El cambio vertical entre cualquier par de puntos en una recta.

rotation (Lesson 13-3) A transformation in which a figure is turned about a fixed point.

rotación Transformación en la cual una figura se gira alrededor de un punto fijo.

run (Lesson 8-2) The horizontal change between any two points on a line.

carrera El cambio horizontal entre cualquier par de puntos en una recta.

S

sales tax (Lesson 2-6) An additional amount of money charged on items that people buy.

impuesto sobre las ventas Cantidad de dinero adicional que se cobra por los artículos que se compran.

sample (Lesson 10-1) A randomly selected group chosen for the purpose of collecting data.

muestra Grupo escogido al azar o aleatoriamente que se usa con el propósito de recoger datos.

sample space (Lesson 9-3) The set of all possible outcomes of a probability experiment.

espacio muestral Conjunto de todos los resultados posibles de un experimento probabilístico.

scale (Lesson 11-6) The scale that gives the ratio that compares the measurements of a drawing or model to the measurements of the real object.

escala Razón que compara las medidas de un dibujo o modelo a las medidas del objeto real.

scale drawing (Lesson 11-6) A drawing that is used to represent objects that are too large or too small to be drawn at actual size.

dibujo a escala Dibujo que se usa para representar objetos que son demasiado grandes o demasiado pequeños como para dibujarlos de tamaño natural.

scale factor (Lesson 11-6) A scale written as a ratio without units in simplest form.

factor de escala Escala escrita como una razón sin unidades en forma simplificada.

scale factor (Lesson 13-4) The ratio of the lengths of two corresponding sides of two similar polygons.

factor de escala La razón de las longitudes de dos lados correspondientes de dos polígonos semejantes.

scale model (Lesson 11-6) A model used to represent objects that are too large or too small to be built at actual size.

modelo a escala Réplica de un objeto real, el cual es demasiado grande o demasiado pequeño como para construirlo de tamaño natural.

scalene triangle (Lesson 11-4) A triangle having no congruent sides.

triángulo escaleno Triángulo sin lados congruentes.

scientific notation (Lesson 4-5) A compact way of writing numbers with absolute values that are very large or very small. In scientific notation, 5,500 is 5.5×10^3.

notación científica Manera abreviada de escribir números con valores absolutos que son muy grandes o muy pequeños. En notación científica, 5,500 es 5.5×10^3.

selling price (Lesson 2-7) The amount the customer pays for an item.

precio de venta Cantidad de dinero que paga un consumidor por un artículo.

semicircle (Lesson 12-2) Half of a circle. The formula for the area of a semicircle is $A = \frac{1}{2}\pi r^2$.

semicírculo Medio círculo. La fórmula para el área de un semicírculo es $A = \frac{1}{2}\pi r^2$.

similar (Lesson 13-6) If one image can be obtained from another by a sequence of transformations and dilations.

similar Si una imagen puede obtenerse de otra mediante una secuencia de transformaciones y dilataciones.

similar figures (Lesson 8-3) Figures that have the same shape but not necessarily the same size.

figuras semejantes Figuras que tienen la misma forma, pero no necesariamente el mismo tamaño.

simple event (Lesson 9-2) One outcome or a collection of outcomes.

eventos simples Un resultado o una colección de resultados.

simple interest (Lesson 2-9) The amount paid or earned for the use of money. The formula for simple interest is $I = prt$.

interés simple Cantidad que se paga o que se gana por el uso del dinero. La fórmula para calcular el interés simple es $I = prt$.

simple random sample (Lesson 10-1) An unbiased sample where each item or person in the population is as likely to be chosen as any other.

muestra aleatoria simple Muestra de una población que tiene la misma probabilidad de escogerse que cualquier otra.

simplest form (Lesson 6-1) An expression is in simplest form when it is replaced by an equivalent expression having no like terms or parentheses.

expresión mínima Expresión en su forma más simple cuando es reemplazada por una expresión equivalente que no tiene términos similares ni paréntesis.

simplify (Lesson 6-5) Write an expression in simplest form.

simplificar Escribir una expresión en su forma más simple.

simulation (Lesson 9-6) An experiment that is designed to model the action in a given situation.

simulación Un experimento diseñado para modelar la acción en una situación dada.

slant height (Lesson 12-5) The height of each lateral face.

altura oblicua Altura de cada cara lateral.

slope (Lesson 8-1) The rate of change between any two points on a line. The ratio of the rise, or vertical change, to the run, or horizontal change.

pendiente Razón de cambio entre cualquier par de puntos en una recta. La razón de la altura, o cambio vertical, a la carrera, o cambio horizontal.

slope-intercept form (Lesson 8-5) An equation written in the form $y = mx + b$, where m is the slope and b is the y-intercept.

forma pendiente intersección Ecuación de la forma $y = mx + b$, donde m es la pendiente y b es la intersección y.

slope triangles (Lesson 8-3) Right triangles that fall on the same line on the coordinate plane.

triángulos de pendiente Triángulos rectos que caen en la misma línea en el plano de coordenadas.

solid (Lesson 12-6) A three-dimensional figure formed by intersecting planes.

sólido Figura tridimensional formada por planos que se intersecan.

sphere (Lesson 12-8) The set of all points in space that are a given distance from a given point called the center.

esfera Conjunto de todos los puntos en el espacio que están a una distancia dada de un punto dado llamado centro.

square root (Lesson 5-1) One of the two equal factors of a number. If $a^2 = b$, then a is the square root of b. A square root of 144 is 12 since $12^2 = 144$.

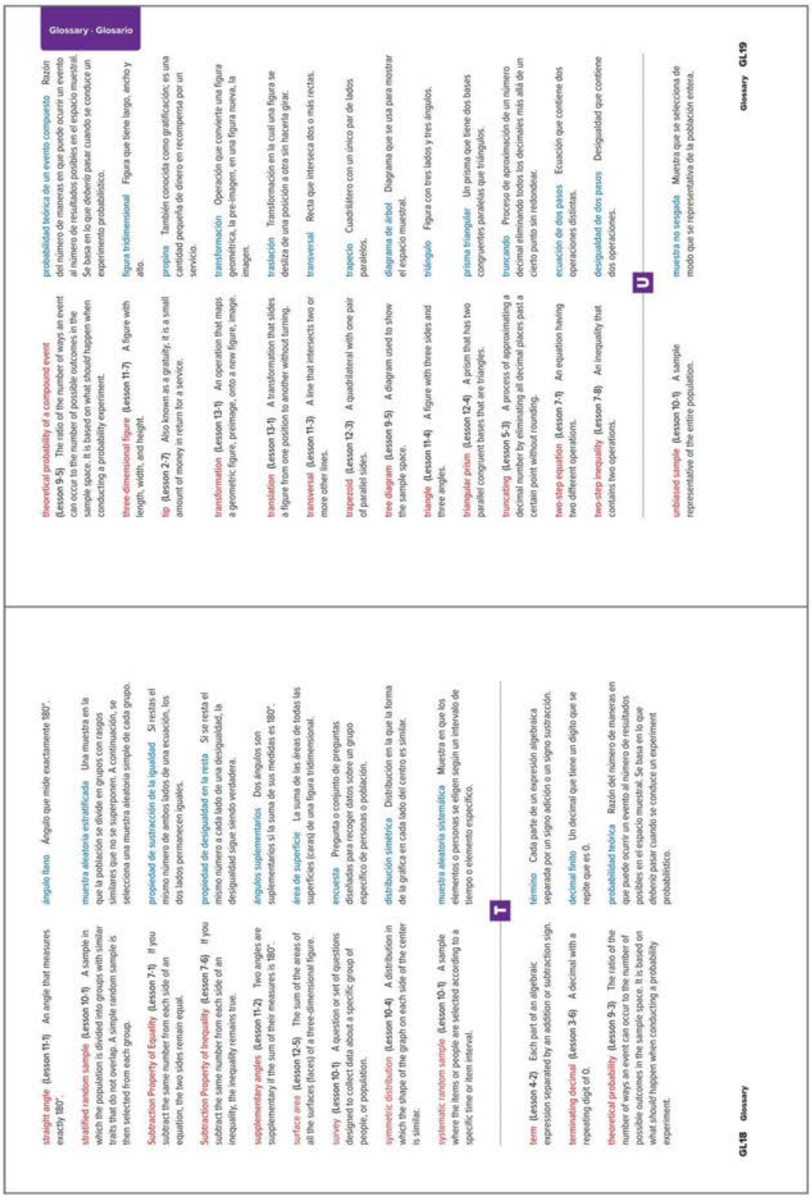
raíz cuadrada Uno de dos factores iguales de un número. Si $a^2 = b$, la a es la raíz cuadrada de b. Una raíz cuadrada de 144 es 12 porque $12^2 = 144$.

standard form (Lesson 4-5) Numbers written without exponents.

forma estándar Números escritos sin exponentes.

statistics (Lesson 10-1) The study of collecting, organizing, and interpreting data.

estadística Estudio que consiste en recopilar, organizar e interpretar datos.



Glossary · Glosario

straight angle (Lesson 11-1) An angle that measures exactly 180°.

ángulo llano Ángulo que mide exactamente 180°.

stratified random sample (Lesson 10-1) A sample in which the population is divided into groups with similar traits that do not overlap. A simple random sample is then selected from each group.

muestra aleatoria estratificada Una muestra en la que la población se divide en grupos con rasgos similares que no se superponen. A continuación, se selecciona una muestra aleatoria simple de cada grupo.

Subtraction Property of Equality (Lesson 7-1) If you subtract the same number from each side of an equation, the two sides remain equal.

propiedad de sustracción de la igualdad Si restas el mismo número de ambos lados de una ecuación, los dos lados permanecen iguales.

Subtraction Property of Inequality (Lesson 7-6) If you subtract the same number from each side of an inequality, the inequality remains true.

propiedad de desigualdad en la resta Si se resta el mismo número a cada lado de una desigualdad, la desigualdad sigue siendo verdadera.

supplementary angles (Lesson 11-2) Two angles are supplementary if the sum of their measures is 180°.

ángulos suplementarios Dos ángulos son suplementarios si la suma de sus medidas es 180°.

surface area (Lesson 12-5) The sum of the areas of all the surfaces (faces) of a three-dimensional figure.

área de superficie La suma de las áreas de todas las superficies (caras) de una figura tridimensional.

survey (Lesson 10-1) A question or set of questions designed to collect data about a specific group of people, or population.

encuesta Pregunta o conjunto de preguntas diseñadas para recoger datos sobre un grupo específico de personas o población.

symmetric distribution (Lesson 10-4) A distribution in which the shape of the graph on each side of the center is similar.

distribución simétrica Distribución en la que la forma de la gráfica en cada lado del centro es similar.

systematic random sample (Lesson 10-1) A sample where the items or people are selected according to a specific time or item interval.

muestra aleatoria sistemática Muestra en que los elementos o personas se eligen según un intervalo de tiempo o elemento específico.

T

term (Lesson 4-2) Each part of an algebraic expression separated by an addition or subtraction sign.

término Cada parte de un expresión algebraica separada por un signo adición o un signo sustracción.

terminating decimal (Lesson 3-6) A decimal with a repeating digit of 0.

decimal finito Un decimal que tiene un dígito que se repite que es 0.

theoretical probability (Lesson 9-3) The ratio of the number of ways an event can occur to the number of possible outcomes in the sample space. It is based on what *should* happen when conducting a probability experiment.

probabilidad teórica Razón del número de maneras en que puede ocurrir un evento al número de resultados posibles en el espacio muestral. Se basa en lo que *debería* pasar cuando se conduce un experiment probabilístico.

theoretical probability of a compound event (Lesson 9-5) The ratio of the number of ways an event can occur to the number of possible outcomes in the sample space. It is based on what *should* happen when conducting a probability experiment.

probabilidad teórica de un evento compuesto Razón del número de maneras en que puede ocurrir un evento al número de resultados posibles en el espacio muestral. Se basa en lo que *debería* pasar cuando se conduce un experimento probabilístico.

three-dimensional figure (Lesson 11-7) A figure with length, width, and height.

figura tridimensional Figura que tiene largo, ancho y alto.

tip (Lesson 2-7) Also known as a gratuity, it is a small amount of money in return for a service.

propina También conocida como gratificación; es una cantidad pequeña de dinero en recompensa por un servicio.

transformation (Lesson 13-1) An operation that maps a geometric figure, preimage, onto a new figure, image.

transformación Operación que convierte una figura geométrica, la pre-imagen, en una figura nueva, la imagen.

translation (Lesson 13-1) A transformation that slides a figure from one position to another without turning.

traslación Transformación en la cual una figura se desliza de una posición a otra sin hacerla girar.

transversal (Lesson 11-3) A line that intersects two or more other lines.

transversal Recta que interseca dos o más rectas.

trapezoid (Lesson 12-3) A quadrilateral with one pair of parallel sides.

trapecio Cuadrilátero con un único par de lados paralelos.

tree diagram (Lesson 9-5) A diagram used to show the sample space.

diagrama de árbol Diagrama que se usa para mostrar el espacio muestral.

triangle (Lesson 11-4) A figure with three sides and three angles.

triángulo Figura con tres lados y tres ángulos.

triangular prism (Lesson 12-4) A prism that has two parallel congruent bases that are triangles.

prisma triangular Un prisma que tiene dos bases congruentes paralelas que triángulos.

truncating (Lesson 5-3) A process of approximating a decimal number by eliminating all decimal places past a certain point without rounding.

truncando Proceso de aproximación de un número decimal eliminando todos los decimales más allá de un cierto punto sin redondear.

two-step equation (Lesson 7-1) An equation having two different operations.

ecuación de dos pasos Ecuación que contiene dos operaciones distintas.

two-step inequality (Lesson 7-8) An inequality that contains two operations.

desigualdad de dos pasos Desigualdad que contiene dos operaciones.

U

unbiased sample (Lesson 10-1) A sample representative of the entire population.

muestra no sesgada Muestra que se selecciona de modo que se representativa de la población entera.

uniform probability model (Lesson 9-3) A probability model which assigns equal probability to all outcomes.

modelo de probabilidad uniforme Un modelo de probabilidad que asigna igual probabilidad a todos los resultados.

unit rate (Lesson 1-1) A rate in which the first quantity is compared to 1 unit of the second quantity.

tasa unitaria Una tasa en la que la primera candidad se compara con 1 unidad de la segunda candidad.

unit ratio (Lesson 1-2) A ratio in which the first quantity is compared to every 1 unit of the second quantity.

razón unitaria Una relación en la que la primera cantidad se compara con cada 1 unidad de la segunda cantidad.

V

valid inference (Lesson 10-1) A prediction, made about a population, based on an unbiased sample that is representative of the population.

inferencia válida Una predicción, hecha sobre una población, basada en una muestra imparcial que es representativa de la población.

valid sampling method (Lesson 10-1) A sampling method that is: representative of the population selected at random, where each member has an equal chance of being selected, and large enough to provide accurate data.

método de muestreo válido Un método de muestreo que es: representativo de la población seleccionada al azar, donde cada miembro tiene la misma oportunidad de ser seleccionado y suficientemente grande para proporcionar datos precisos.

variability (Lesson 10-3) A measure that describes the amount of diversity in values within a sample or samples.

variabilidad Medida que describe la cantidad de diversidad en valores dentro de una muestra o muestras.

vertex (Lesson 11-1) A vertex of an angle is the common endpoint of the rays forming the angle.

vértice El vértice de un ángulo es el extremo común de los rayos que lo forman.

vertex (Lesson 11-7) The point where three or more faces of a polyhedron intersect.

vértice El punto donde tres o más caras de un poliedro se cruzan.

vertical angles (Lesson 11-1) Opposite angles formed by the intersection of two lines. Vertical angles are congruent.

ángulos opuestos por el vértice Ángulos opuestos formados por la intersección de dos rectas. Los ángulos opuestos por el vértice son congruentes.

vertices (Lesson 11-7) Plural of vertex.

vértices Plural de vertice.

visual overlap (Lesson 10-4) A visual demonstration that compares the centers of two distributions with their variation, or spread.

superposición visual Una demostración visual que compara los centros de dos distribuciones con su variación, o magnitud.

volume (Lesson 12-4) The measure of the space occupied by a solid. Standard units of measure are cubic units such as in^3 or ft^3.

volumen Medida del espacio que ocupa un sólido. Unidades de medida estándar son unidades cúbicas tales como $pulg^3$ o $pies^3$.

voluntary response sample (Lesson 10-1) A sample which involves only those who want to participate in the sampling.

muestra de respuesta voluntaria Muestra que involucra sólo aquellos que quieren participar en el muestreo.

W

wholesale cost (Lesson 2-7) The amount the store pays for an item.

coste al por mayor La cantidad que la tienda paga por un artículo.

X

x-axis (Lesson 8-1) The horizontal number line that helps to form the coordinate plane.

eje x La recta numérica horizontal que ayuda a formar el plano de coordenadas.

x-coordinate (Lesson 8-1) The first number of an ordered pair.

coordenada x El primer número de un par ordenado.

x-intercept (Lesson 8-1) The x-coordinate of the point where the line crosses the x-axis.

intersección x La coordenada x del punto donde cruza la gráfica el eje x.

Y

y-axis (Lesson 8-1) The vertical number line that helps to form the coordinate plane.

eje y La recta numérica vertical que ayuda a formar el plano de coordenadas.

y-coordinate (Lesson 8-1) The second number of an ordered pair.

coordenada y El segundo número de un par ordenado.

y-intercept (Lesson 8-5) The y-coordinate of the point where the line crosses the y-axis.

intersección y La coordenada y del punto donde cruza la gráfica el eje y.

Z

zero angle (Lesson 11-1) An angle that measures exactly 0 degrees.

ángulo cero Un ángulo que mide exactamente 0 grados.

Zero Exponent Rule (Lesson 4-4) A rule that states that any nonzero number to the zero power is equivalent to 1.

regla de exponente cero Una regla que establece que cualquier número diferente de cero a la potencia cero es equivalente a 1.

zero pair (Lesson 3-1) The result when one positive counter is paired with one negative counter. The value of a zero pair is 0.

par nulo Resultado de hacer coordinar una ficha positiva con una negativa. El valor de un par nulo es 0.

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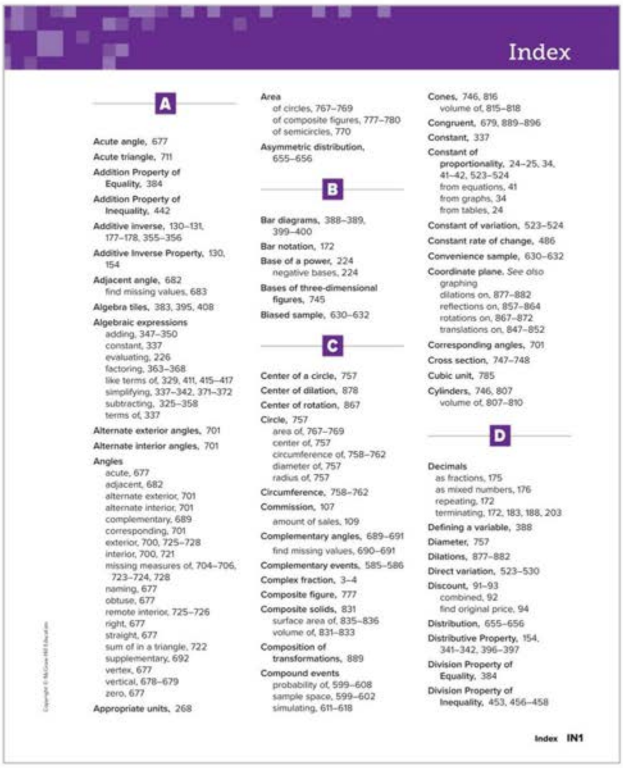

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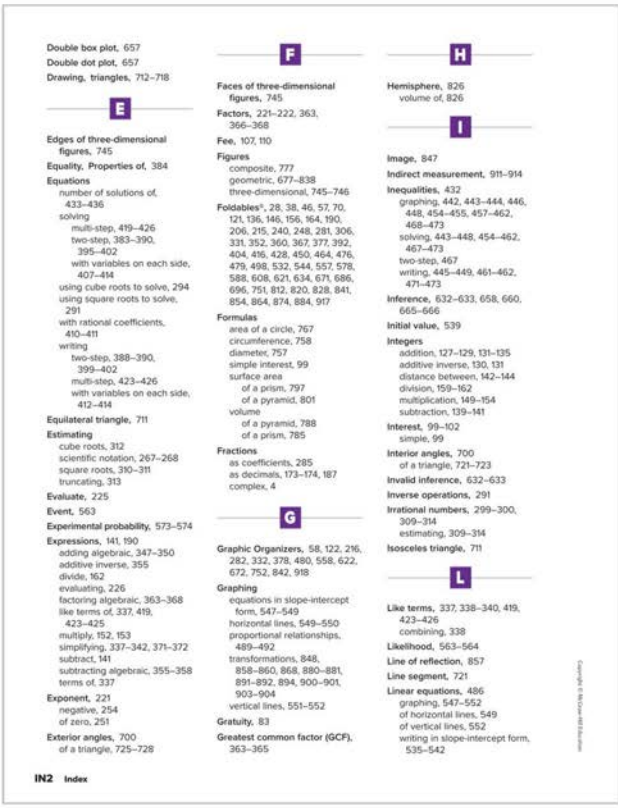

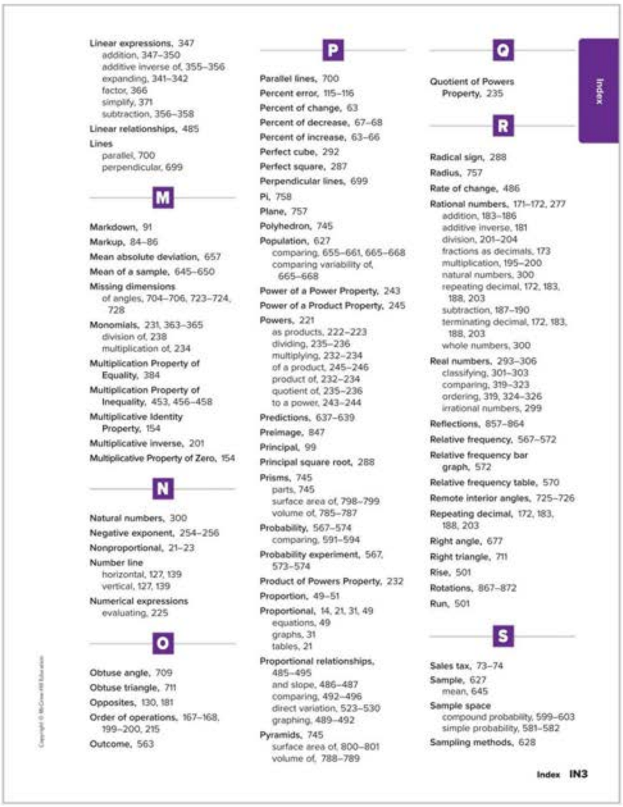

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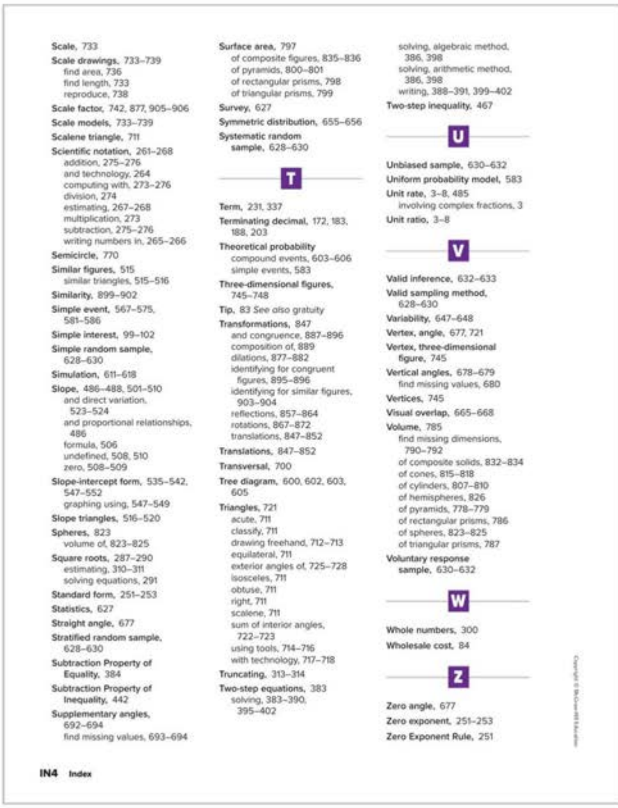

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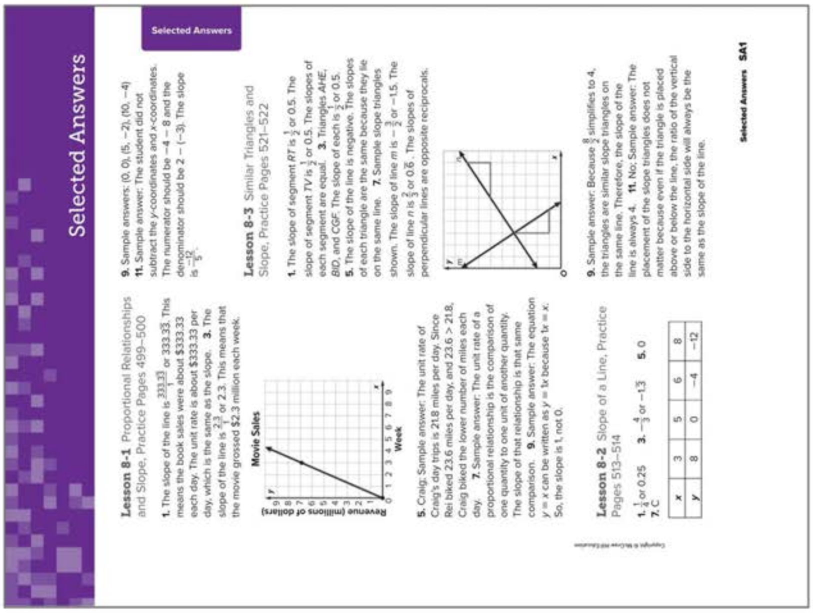

Selected Answers

Lesson 8-1 Proportional Relationships and Slope, Practice Pages 499–500

1. The slope of the line is $\frac{333.\overline{3}}{1}$ or $333.\overline{3}$. This means the book sales were about \$333.33 each day. The unit rate is about \$333.33 per day, which is the same as the slope. **3.** The slope of the line is $\frac{2.3}{1}$ or 2.3. This means that the movie grossed \$2.3 million each week.

5. Craig; Sample answer: The unit rate of Craig's day trips is 21.8 miles per day. Since Rei biked 23.6 miles per day, and 23.6 > 21.8, Craig biked the lower number of miles each day. **7.** Sample answer: The unit rate of a proportional relationship is the comparison of one quantity to one unit of another quantity. The slope of that relationship is that same comparison. **9.** Sample answer: The equation $y = x$ can be written as $y = 1x$ because $1x = x$. So, the slope is 1, not 0.

Lesson 8-2 Slope of a Line, Practice Pages 513–514

1. $\frac{1}{4}$ or 0.25 **3.** $-\frac{4}{3}$ or $-1.\overline{3}$ **5.** 0
7. C

x	3	5	6	8
y	8	0	−4	−12

9. Sample answers: (0, 0), (5, −2), (10, −4)
11. Sample answer: The student did not subtract the *y*-coordinates and *x*-coordinates. The numerator should be −4 − 8 and the denominator should be 2 − (−3). The slope is $\frac{-12}{5}$.

Lesson 8-3 Similar Triangles and Slope, Practice Pages 521–522

1. The slope of segment *RT* is $\frac{1}{2}$ or 0.5. The slope of segment *TV* is $\frac{1}{2}$ or 0.5. The slopes of each segment are equal. **3.** Triangles *AHE*, *BID*, and *CGF*. The slope of each is $\frac{1}{2}$ or 0.5. **5.** The slope of the line is negative. The slopes of each triangle are the same because they lie on the same line. **7.** Sample slope triangles shown. The slope of line *m* is $-\frac{3}{2}$ or −1.5. The slope of line *n* is $\frac{2}{3}$ or $0.\overline{6}$. The slopes of perpendicular lines are opposite reciprocals.

9. Sample answer: Because $\frac{8}{2}$ simplifies to 4, the triangles are similar slope triangles on the same line. Therefore, the slope of the line is always 4. **11.** No; Sample answer: The placement of the slope triangles does not matter because even if the triangle is placed above or below the line, the ratio of the vertical side to the horizontal side will always be the same as the slope of the line.

Selected Answers SA1

Selected Answers

Lesson 8-4 Direct Variation, Practice Pages 533–534

1. $y = 8x$; The constant of variation is 8. This means that the cost per ticket is \$8.
3. $y = 2.55x$; The constant of variation is 2.55. This means that the cost per ream is \$2.55.
5. $y = 62.5x$; The constant of variation is 62.5. This means that the bus travels 62.5 miles per hour. **7.** $y = 4x$; 26 gal **9.** It is not a direct variation relationship. Sample answer: The ratio between the y-values and x-values is not constant. The points form a line, but the line does not pass through the origin. **11.** Sample answer: The constant of variation in a direct variation equation is the constant multiplied by x. The unit rate is also the constant multiplied by x.

Lesson 8-5 Slope-Intercept Form, Practice Pages 545–546

1. slope: $\frac{1}{2}$; y-intercept: -5 **3.** $y = -\frac{1}{3}x + 4$
5. $y = 4x - 2$ **7.** $y = 2x - 3$ **9.** $y = 30x + 30$
11. Coral Snorkeling; \$5 less **13.** Sample answer: $y = -3$ **15.** Sample answer: rise = 18, run = 5

Lesson 8-6 Graph Linear Equations, Practice Pages 555–556

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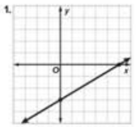

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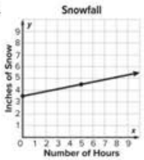

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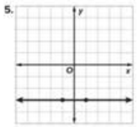
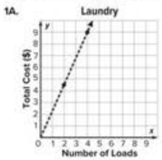


7. The slope of the line is negative. The equation of the line is $y = -\frac{7}{3}x - 7$.
9. Sample answer: Vertical lines will never have a y-intercept, unless the vertical line is the y-axis. So, there will be no b. Vertical lines have an undefined slope, so they will not have an m. **11.** Sample answer: If the slope is used to start graphing an equation, then the slope is comparing the first point plotted to the origin, and not to the y-intercept.

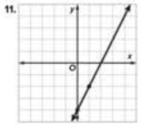
Module 8 Review Pages 559–560

1A.



1B. Sample answer: The unit rate is represented by the point (1, 2.25), which lies on the line. **3.** B **5.** The ratio of the rise to the run of each triangle is the same. The smaller triangle and the larger triangle shown are similar. The slope of the line is -2. **7.** \$44.10
9. $y = -4x + 2$; I used the slope formula to find $m = -4$. The y-intercept was given.

11.



Lesson 9-1 Find Likelihoods, Practice Pages 565–566

1. equally likely **3.** likely **5.** certain
7. selecting the letter B, selecting the letter Q or R **9.** key ring, yo-yo, cap **11.** Sample answer: likelihood of flipping a red–yellow counter and it landing on yellow **13.** likely; Sample answer: Because about 95% of Americans are not vegetarians, it is likely the person is not a vegetarian.

Lesson 9-2 Relative Frequency of Simple Events, Practice Pages 579–580

1. 68%; 10% **3.** $\frac{13}{25}$ **5.** 204 sundaes
7. \$55.20 **9.** Sample answer: Based on last year's class, a teacher determines that if a student plays a sport, the probability that they are also in a club is 75%. If there are 24 students who play a sport in this year's class, how many students would you expect to also be in a club?; about 18 students **11.** 80 tosses

Lesson 9-3 Theoretical Probability of Simple Events, Practice Pages 589–590

1. 1, 2, 3, 4, 5 **3.** $\frac{1}{2}$, 0.5, 50% **5.** $\frac{3}{8}$, 0.375, 37.5%; The complement is choosing a ride that lasts at least 200 seconds. **7.** 0.25, 25%, $\frac{1}{4}$
9. 525 times **11.** The student found the complement of spinning a multiple of 3. The correct probability is $\frac{3}{8}$. **13.** Sample answer: The weather reporter says that there is a 65% chance that it will rain tomorrow. What is the chance it will not rain tomorrow? 35%

Lesson 9-4 Compare Probabilities of Simple Events, Practice Pages 597–598

1. Jayden **3.** Graph 2; Graph 1 **5.** $\frac{1}{3}$; $\frac{1}{2}$; They are not close. Sample answer: There were not enough trials performed. **7.** Sample answer: The spinner is expected to land on each section a total of 10 times.

Lesson 9-5 Probability of Compound Events, Practice Pages 609–610

1. Sample space: lime plain, lime ice cream, lime drink, cherry plain, cherry ice cream, cherry drink, blueberry plain, blueberry ice cream, blueberry drink, watermelon plain, watermelon ice cream, watermelon drink.
3. $\frac{1}{9}$; 11.1% **5.** 6.3% **7.** 12 more **9.** no; Sample answer: The probability that Kimiko will win is $\frac{7}{12}$. Because $\frac{7}{12}$ is greater than $\frac{5}{12}$, Kimiko has a greater chance of winning. **11.** Sample answer: Choosing a hamburger or hot dog and then potato salad or macaroni salad. Sample space: hamburger, potato salad; hamburger, macaroni salad; hot dog, potato salad; hot dog, macaroni salad

Lesson 9-6 Simulate Chance Events, Practice Pages 619–620

1A. Sample answer: Use a spinner with five equal-size sections. Label two sections "R" for rain and three sections "N" for no rain. The spinner is spun twice for each trial. **1B.** $\frac{3}{10}$, 0.3, 30% **3.** $\frac{1}{20}$, 0.05, 5% **5.** Sample answer: A fair game could consist of tossing a coin, and winning the game is represented by tossing heads. A game that is not fair could consist of rolling a number cube labeled 1–6, and winning the game is represented by landing on the numbers 1 or 2. **7.** Sample answer: If you do not clearly define each of these, it can be difficult to interpret the results of the simulation in order to find the experimental probability of a favorable outcome.

Module 9 Review Pages 623–624

1. C **3.** 0.2, $\frac{1}{5}$, 20% **5.** I can expect a number greater than 3 to be spun 250 times, because $\frac{5}{8}$ of the numbers on the spinner are greater than 3, and $\frac{5}{8} \cdot 400 = 250$. **7.** 20% **9.** 3

Lesson 10-1 Biased and Unbiased Samples, Practice Pages 635–636

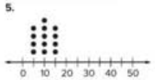
1a. stratified random sample **1b.** simple random sample **1c.** systematic random sample **3.** voluntary response sample; biased; The inference is not valid. **5.** This scenario is a systematic random sample; The inference is valid; The sampling method is unbiased. **7.** yes; 60% **9.** no; Sample answer: Marc used a voluntary response sample. The results are biased and therefore invalid because only those that wanted to respond were included. **11.** Sample answer: Every 25th student is chosen from an alphabetical listing of all students. The chosen students are then surveyed.

Lesson 10-2 Make Predictions, Practice Pages 643–644

1. 30 science fiction books **3.** about 1,850 students **5.** 6,820 customers **7.** $3,510 **9.** Sample answer: A random survey of high school students with jobs were asked whether they saved some of the money they earned. 82% of the students said they saved some money. Out of 340 students, predict how many would save some of their earnings.; about 279 students **11.** false; The survey's sample must be unbiased.

Lesson 10-3 Generate Multiple Samples, Practice Pages 653–654

1a. 2.5 sports **1b.** 0.54 sports; Sample answer: The majority of the sample means are within 0.5 sport of the mean. This means our estimate is likely not far off from the true mean. **3a.** The majority of the data are clustered between 14 and 19 pounds. **3b.** Sample answer: The data would be more tightly clustered between 15 and 18 pounds. **3c.** Sample answer: The majority of the data appear to be within 3 pounds of the center, so the company can expect the sample mean of 18 pounds to be within 3 pounds of the population mean. The mean decaf sales for stores in this company is likely to be between 15 and 21 pounds per week. **3d.** Sample answer: Due to the increased sample size, there will be less discrepancy between the sample mean and the population mean. The store might expect to sell between 16 and 18 pounds of decaf coffee.
5.



Sample answer: The mean is 10; the MAD is about 3, which means that the average distance each data value is from the mean is 3.

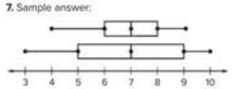
Lesson 10-4 Compare Two Populations, Practice Pages 663–664

1. Sample answer: The mean for the housecat data is 11 lb with a variation of about 0.9 lb. The mean for the small dog data is 9 lb with a variation of 1.3 lb. Overall, the housecats weigh more with less variation. You can infer that a randomly selected housecat is likely to weigh more than a randomly selected small dog.

3.

	Middle School	High School
The median is greater.		X
The IQR is 2.5.	X	
The data has greater variability.		X
A person from this sample is more likely to have more than 7 hours of homework a week.		X
The data are more symmetric.	X	

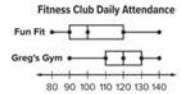
5. Summerville Jaguars; The median number of years of experience for Southwest Broncos is 5 years while the median number of years of experience for Summerville Jaguars is 6 years. It is more likely that the player belongs to the Jaguars.
7. Sample answer:



Lesson 10-5 Assess Visual Overlap, Practice Pages 669–670

1a. about 4.4 measures of variability **1b.** Sample answer: A ratio greater than two suggests the population means are likely to be different. Because 4.4 > 2, the populations in this situation are likely to be different. **3.** about 1.1 measures of variability; This means that a randomly selected registered pet could either be a dog or cat. Because 1.1 < 2, it is likely the population means are the same. **5.** Sample answer: Because the ratio is less than 2, it is likely that the populations could have the same mean.

7. Sample answer:



Module 10 Review Pages 673–674

1A. systemic random sample **1B.** The population is the 15,600 LED monitors. The sample is representative of the population because it is selected randomly, and it is large enough to provide accurate data. **3.** 96 **5A.** 2 **5B.** The data vary as low as 1 and as high as 4. To the left, the data vary 1 off of the center, and to the right, the data vary 2 off of the center. **7.** B

Lesson 11-1 Vertical and Adjacent Angles, Practice Pages 687–688

1. $\angle 3$, $\angle F$, $\angle HFG$, $\angle GFH$ **3.** $\angle 8$ and $\angle 11$, $\angle 7$ and $\angle 10$, $\angle 9$ and $\angle 12$; $\angle 9$ and $\angle 11$ **5.** $115 = 2x + 5$; $x = 55$ **7.** $5x + 95 = 180$; $x = 17$ **9.** 42°
11. Sample answer:

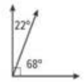
13. no; Sample answer: A pair of adjacent angles must share a common vertex, share a common side, and not overlap. They may equal 180° but do not have to.

Lesson 11-2 Complementary and Supplementary Angles, Practice Pages 697–698

1. 45° **3.** 36° **5.** 20° **7.** $22 + 4x = 90$; $x = 17$ **9.** $43 + (x - 7) = 90$; $x = 54$
11. $72 + (x + 4) = 180$; $x = 104$ **13.** 27.6

15. Sample answer:

17. no; Sample answer: A pair of complementary angles must equal 90°. Adjacent angles share a common side and vertex. A pair of complementary angles can also be adjacent.

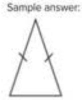


Lesson 11-3 Angle Relationships and Parallel Lines, Practice Pages 709–710

1. alternate interior **3.** alternate exterior **5.** $m\angle 2 = 60°$; Since $\angle 1$ and $\angle 2$ are alternate interior angles, they are equal. $m\angle 3 = 120°$; Because $\angle 2$ and $\angle 3$ are supplementary, the sum of their measures is 180°. **7.** alternate exterior angles; $m\angle 2 = 108°$ **9.** $x = 7$; Corresponding angles are congruent, so $4x = 3x + 7$. Solving the equation for x gives $x = 7$. **11.** Sample answer: Interior angles that are on the same side of the transversal are supplementary. One of the interior angles is supplementary to the corresponding angle of the other interior angle. **13.** 63°; Sample answer: The measure of either of the angles adjacent to the 148° angle is 32° because it is supplementary to 148°. $85° + W + 32° = 180°$, so $m\angle W$ is 63°.

Lesson 11-4 Triangles, Practice Pages 719–720

1. acute, isosceles triangle; more than one; Sample answer:



3. no; Sample answer: The sum of the angle measures is greater than 180°, so the endpoints of the sides cannot meet.
5. Sample answer:

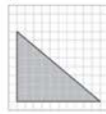
7. no; Sample answer: The sum of the two sides is not greater than the third side. **9.** 41; acute, scalene **11.** Sample answer: The sum of the two given angle measures is greater than 180° and the sum of the measures of the angles of a triangle is 180°. **13.** Sample answer: The sum of the interior angles of a triangle equal 180°. Three acute angles can have a sum of 180°. For example, 60°, 60°, and 60° are all acute angles and 60° + 60° + 60° is 180°.

Lesson 11-5 Angle Relationships and Triangles, Practice Pages 731–732

1. $x = 120$ **3.** $m\angle F = 40°$, $m\angle G = 40°$, $m\angle H = 100°$ **5.** $m\angle 1 = 31°$, $m\angle 2 = 106°$
7. $m\angle ADC = 100°$, $m\angle DCB = 39°$ **9.** Sample answer: After finding the value of x, the value should have been substituted into each expression. 4(12) = 48, 7(12) = 84. So, the three angles measure 48°, 48°, and 84°.
11. false; Sample answer: If an angle of a triangle is obtuse, then the exterior angle that is supplementary to it will be an acute angle.

Lesson 11-6 Scale Drawings, Practice Pages 743–744

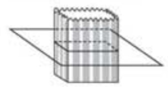
1. about 52 mi **3.** 90 ft^2
5.



7. $101.08 **9.** false; Sample answer: The scale drawing will be greater than the object. For example, if the scale factor is 2 this means that 2 units on the drawing equal 1 unit of the object. This makes the scale drawing greater than the object. **11.** 1 inch is about 19.7 mi

Lesson 11-7 Three-Dimensional Figures, Practice Pages 749–750

1. 6; 10; 6 **3.** triangle; triangle; triangle
5. square; square; rectangle **7.** rectangular prism; 6, 12, 8; Sample answer: a tissue box
9. rectangle; Sample answer:



11. true

Module 11 Review Pages 753–754

1.

	vertical	adjacent	neither
∠1 and ∠3	X		
∠5 and ∠6		X	
∠5 and ∠4	X		
∠1 and ∠2			X

3A. $(12x + 9) + 105 = 180$; The angles are supplementary, so their sum is equal to 180.
3B. 5.5; 75° **5.** D **7A.** 5, 3 **7B.** 10, 6 **9.** A

Lesson 12-1 Circumference of Circles, Practice Pages 765–766

1. 6.28 in. **3.** 197.82 yd **5.** 91.08 cm
7. 5.49 in. **9.** 5.50 in. **11.** 64.25 ft
13. Sample answer: The circumference would also double. For example, a circle with a radius of 3 feet would have a circumference that is about 18 feet. When the radius doubles to 6 feet, the circumference is about 36 feet.
$18 \times 2 = 36$

Lesson 12-2 Area of Circles, Practice Pages 775–776

1. 158.29 m^2 **3.** 153.86 in^2 **5.** 120.20 ft^2
7. 254.34 in^2 **9.** 63.59 in^2 **11.** Sample answer: To find the area, multiply the area of the entire circle by $\frac{3}{4}$. $A = \frac{3}{4}\pi r^2$; 84.78cm^2 **13.** 226.08 in^2

Lesson 12-3 Area of Composite Figures, Practice Pages 783–784

1. 200.52 yd^2 **3.** 113 cm^2 **5.** 132 m^2
7. 512 ft^2 **9.** 5 pallets **11.** Sample answer: First find the area of the square. $A = 12 \times 12$ or 144 ft^2. Then find the area of the quarter circle. $A = \frac{1}{4}(3.14 \times 6 \times 6)$ or 28.26 ft^2. Subtract the area of the quarter circle from the area of the square. $144 - 28.26 = 115.74$ ft^2 **13.** Sample answer: Use polygons to approximate the shape of the curved side of the swimming pool.

Lesson 12-4 Volume of Prisms and Pyramids, Practice Pages 795–796

1. 2,772 in^3 **3.** 97.2 m^3 **5.** 1.0 m^3
7. 14.64m^3 **9.** 15 in. **11.** 7 bags
13. Sample answer: Alexia's bathroom has a tub in the shape of a rectangular prism with a length of 1.5 meters, a width of 0.8 meter, and

a height of 0.4 meter. How many cubic meters of water can it hold?; 0.48 m^3 **15.** Sample answer: First prism: area of the base: 24 in^2 and height: 4 in.; Second prism: area of the base: 16 in^2 and height: 6 in.

Lesson 12-5 Surface Area of Prisms and Pyramids, Practice Pages 805–806

1. 468 yd^2 **3.** 103.4 in^2 **5.** 633.9 in^2 **7.** no; He needs an additional 1.7 in^2 of fabric.
9. $120\frac{7}{75}$ yd^2 or about 120.09 yd^2 **11.** The surface area of the original prism is $\frac{1}{9}$ the surface area of the new prism. Sample answer: If the original prism has a length of 4 m, a width of 3 m, and a height of 2 m, the S.A. of the prism is 52 m^2. The new prism would have a length of 12 m, a width of 9 m, and a height of 6 m. The S.A. is 468 m^2. $\frac{52}{468} = \frac{1}{9}$

Lesson 12-6 Volume of Cylinders, Practice Pages 813–814

1. 3,078.8 cm^3 **3.** 192π in^3 **5.** 15.6 ounces
7. B **9.** about 1.4 hours **11.** Sample answer: She used the diameter in the calculation instead of the radius. The volume is $\pi(4^2)(23)$ or 1,156.1 in^3. **13.** yes; Sample answer: The cylinder has a volume of about 3,418 cm^3. The volume of the prism is 400 cm^3. Since 3,418 > 400, the water will overflow.

Lesson 12-7 Volume of Cones, Practice Pages 821–822

1. $130\frac{2}{3}\pi$ ft^3 **3.** 75π mm^3 **5.** 19.3 in^3
7. 42.5 in^3 **9.** cost of the yogurt in cylinder: $28.3 \cdot \$0.10 = \$2.83$; cost of the yogurt in cone: $9.4 \cdot \$0.10 = \$0.94$; difference in the cost: $\$2.83 - \$0.94 = \$1.89$ **11.** height of 4 inches and radius of 6 inches **13.** three times; Sample answer: If a cone and a cylinder have equal base areas and equal heights, the volume of the cylinder is three times that of the cone. So, if the volumes are equal, the height of the cone must be three times that of the cylinder in order for the volumes to be equal.

Lesson 12-8 Volume of Spheres, Practice Pages 829–830

1. $16{,}222\frac{2}{3}\pi$ ft^3 **3.** 38.8 mm^3 **5.** 629.9 ft^3
7. 1.8 mm^3 **9.** 13.1 cm^3 **11.** Sample answer: You could multiply by 4, then divide by 3. **13.** Luci; Sample answer: By keeping the volume in terms of π, her answer is closest to the exact volume. Because Stefan used an approximation for π, the volume he found is an approximation.

Lesson 12-9 Volume and Surface Area of Composite Solids, Practice Pages 839–840

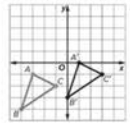
1. 1,922.7 in^3 **3.** 540.6 in^3 **5.** 160.6m^2
7. 2.3yd^3 **9.** Sample answer: Since both volumes include π, Mateo can find the total volume using $(250 + 25)\pi$. The total volume is 275π ft^3. **11.** no; Sample answer: The student included the shared portion of the figure. The correct surface area is 45.9 cm^2.

Module 12 Review Pages 843–844

1. 62.8 **3.** 73.77 ft^2 **5.** D
7. The diameter of the hemisphere is 14 centimeters. The volume of the hemisphere is half the volume of a sphere that has the same radius. The volume of the hemisphere, rounded to the nearest tenth, is 718.4 cubic centimeters. **9A.** 510.5 cubic centimeters **9B.** D

Lesson 13-1 Translations, Practice Pages 855–856

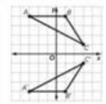
1. $A'(1, 0)$, $B'(0, -3)$, $C'(3, -1)$



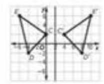
3. $(x, y) \to (x + 7, y - 4)$; $Q'(5, -2)$, $R'(4, -8)$, $S'(8, -6)$ **5.** $(x, y) \to (x + 2, y - 5)$
7. The distance from the school to the park is approximately 11.3 units. **9.** Sample answer: The figure is in the same position as the original figure. Since 3 and −3 are opposites, and −4 and 4 are opposites, the translations cancel each other. **11.** Sample answer: For the figure to be translated, every point or vertex must move the same distance and in the same direction. Therefore, the vertices cannot move different distances or different directions.

Lesson 13-2 Reflections, Practice Pages 865–866

1. $A'(-3, -4)$, $B'(1, -4)$, $C'(3, -1)$



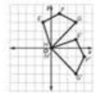
3. $C'(4, 2)$, $D'(8, -2)$, $E'(10, 6)$



5. $(x, y) \to (-x, y)$; $T'(0, 3)$, $U'(3, 0)$, $V'(4, 4)$
7. $X'(-2, -2)$, $Y'(-3, 4)$, $Z'(1, 2)$ **9.** Sample answer: He found the coordinates after a reflection across the x-axis. The coordinates should be $W'(-2, 2)$, $X'(-2, 4)$, $Y'(-4, 4)$, and $Z'(-4, 2)$. **11.** never; Sample answer: When reflecting a figure, the preimage and image are congruent, so, they are the same size and shape.

Lesson 13-3 Rotations, Practice Pages 875–876

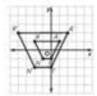
1. $E'(3, 1)$, $F'(4, -1)$, $G'(3, -3)$, $H'(0, 0)$



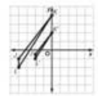
3. $(x, y) \to (-x, -y)$; $Q'(2, -2)$, $R'(3, 4)$, $S'(-1, 2)$
5. $(x, y) \to (y, -x)$; 90° **7.** $A''(7, -4)$, $B''(7, -12)$, $C''(0, -12)$, $D''(0, -4)$ **9.** Yes; Sample answer: A figure can rotate a total of 360°. A clockwise rotation of 270° leaves 90° remaining in the rotation before returning to its original position. The original figure could be rotated 90° in the opposite direction, counterclockwise, and be in the same position as the figure rotated 270°. **11.** never; Sample answer: A figure and its rotated image are congruent, so they will always have the same area and perimeter.

Lesson 13-4 Dilations, Practice Pages 885–886

1.



3.



5. $(x, y) \rightarrow \left(\frac{1}{4}x, \frac{1}{4}y\right)$ **7.** $1,610.28 **9.** no; Sample answer: Both coordinates of all points must be multiplied by the same scale factor. **11.** sometimes; Sample answer: A dilation results in an image that is similar to the preimage. It will always be the same shape. A dilation by a scale factor other than 1 results in an image with a different size. The preimage and image will be the same size and shape if the scale factor is 1.

Lesson 13-5 Congruence and Transformations, Practice Pages 897–898

1. not congruent; Sample answer: No sequence of rotations, reflections, and/or translations will match the two figures up exactly.
3. Sample answer: If you rotate parallelogram *CAMP* 90° counterclockwise about the origin and then translate it 4 units down, it coincides with parallelogram *SITE*. **5.** Sample answer: a reflection followed by a translation; They are congruent. **7.** yes; Sample answer: A 180° clockwise rotation about the origin maps $\triangle ABC$ onto $\triangle A''B''C''$. **9.** Sample answer: The two trapezoids are not congruent because no sequence of translations, reflections, and/or rotations will map trapezoid *ABCD* onto trapezoid *EFGH*.

Lesson 13-6 Similarity and Transformations, Practice Pages 909–910

1. similar; Sample answer: Dilating rectangle *ABCD* using a scale factor of 0.5 and center of dilation at the origin, and then translating it 3 units to the right maps rectangle *ABCD* onto rectangle *EFGH*. **3.** Sample answer: Dilate triangle ABC using a scale factor of 2 and center of dilation at the origin, and then rotate it 90° counterclockwise about the origin.
5. A **7.** no; Sample answer: The area of the final image is 60 square inches. **9.** See students' responses.

Lesson 13-7 Indirect Measurement, Practice Pages 915–916

1. 5.25 ft **3.** 2.25 ft **5.** 20 ft
7.

Person/Item	Shadow Length (ft)	Height of Person/Item (ft)
Mr. Nolan	9	6
Flagpole	48	32
School	63	42
School bus	16.5	11

9. true; Sample answer: The triangles are similar using Angle-Angle Similarity. **11.** Sample answer: The student set up the proportion incorrectly. One correct proportion is $\frac{x}{5} = \frac{25}{20}$. The correct height is 6.25 feet.

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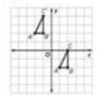
1A.



1B. $A'(-2, 2)$, $B'(-1, 2)$, $C'(-1, 4)$ **3.** C **5.** A
7A. no **7B.** Sample answer: $\frac{AB}{JK} = \frac{4}{8}$ or $\frac{1}{2}$; $\frac{AC}{JL} = \frac{3}{5}$; and $\frac{BC}{KL} = \frac{5}{9.4}$. Since the ratios between the side lengths are not equal, the two triangles are not similar.

Mathematics Reference Sheet

Formulas				
Perimeter	Square	$P = 4s$	Rectangle	$P = 2\ell + 2w$ or $P = 2(\ell + w)$
Circumference	Circle	$C = 2\pi r$ or $C = \pi d$		
Area	Square	$A = s^2$	Rectangle	$A = \ell w$
	Parallelogram	$A = bh$	Triangle	$A = \frac{b}{2}h$
	Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$	Circle	$A = \pi r^2$
Surface Area	Cube	$S = 6s^2$	Rectangular Prism	$S = 2\ell w + 2\ell h + 2wh$
	Cylinder	$S = 2\pi rh + 2\pi r^2$		
Volume	Cube	$V = s^3$	Prism	$V = \ell wh$ or Bh
	Cylinder	$V = \pi r^2 h$	Cone	$V = \frac{1}{3}\pi r^2 h$ or $\frac{1}{3}Bh$
	Pyramid	$V = \frac{1}{3}Bh$	Sphere	$V = \frac{4}{3}\pi r^3$
Temperature	Fahrenheit to Celsius $C =$	$\frac{5}{9}(F - 32)$	Celsius to Fahrenheit $F =$	$\frac{9}{5}C + 32$

Measurement Conversions		
Length	1 kilometer (km) = 1,000 meters (m) 1 meter = 100 centimeters (cm) 1 centimeter = 10 millimeters (mm)	1 foot (ft) = 12 inches (in.) 1 yard (yd) = 3 feet or 36 inches 1 mile (mi) = 1,760 yards or 5,280 feet
Volume and Capacity	1 liter (L) = 1,000 milliliters (mL) 1 kiloliter (kL) = 1,000 liters	1 cup (c) = 8 fluid ounces (fl oz) 1 pint (pt) = 2 cups 1 quart (qt) = 2 pints 1 gallon (gal) = 4 quarts
Weight and Mass	1 kilogram (kg) = 1,000 grams (g) 1 gram = 1,000 milligrams (mg) 1 metric ton = 1,000 kilograms	1 pound (lb) = 16 ounces (oz) 1 ton (T) = 2,000 pounds
Time	1 minute (min) = 60 seconds (s) 1 hour (h) = 60 minutes 1 day (d) = 24 hours	1 week (wk) = 7 days 1 year (yr) = 12 months (mo) or 52 weeks or 365 days 1 leap year = 366 days
Metric to Customary	1 meter ≈ 39.37 inches 1 kilometer ≈ 0.62 mile 1 centimeter ≈ 0.39 inch	1 kilogram ≈ 2.2 pounds 1 gram ≈ 0.035 ounce 1 liter ≈ 1.057 quarts

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